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A Model of Trickle Down Through Learning

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Abstract

This paper presents an analysis of income distribution based on an overlapping generations model of imperfect capital markets, technological non-convexities and information acquisition. Heterogeneous, altruistic agents apply for loans from …nancial intermediaries to undertake risky investment projects. Borrowing is prohibited below a critical level of wealth that depends on agents' evaluation of risk which is updated over time according to the arrival of new information. This process of learning governs the transition of lineage wealth and, with it, the dynamics of income distribution. In general, limiting outcomes depend on initial conditions that determine the extent to which class divisions persist in multiple steady state equilibria. Such divisions may vanish if the the initial distribution satis…es certain criteria.

1 Introduction

After a period of some neglect, the study of income distribution has once again began to occupy a great deal of attention among macroeconomists. At the empirical level, new evidence has been brought to bear on the national and international trends in inequality, on the sources of inequality and on the relationships between inequality and macroeconomic outcomes.[1] At the theoretical level, modern dynamic general equilibrium analysis has been used to provide a variety of perspectives on the extent of class mobility in arti…cial

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[1]See, for example, Alesina and Rodrik (1994), Atkinson (1997), Atkinson et al. (1995) and Persson and Tabellini (1994).

economies. This paper is a contribution to the theoretical literature, offering a further perspective that has not, to our knowledge, been considered before.

One of the most prominent approaches among contemporary theories of income distribution - exemplified in the work of Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira (1993) and Piketty (1997) - is based on an appeal to capital market imperfections and non-convex technologies as a means of explaining why limiting distributions may depend on initial conditions.[2] The basic analytical framework consists of overlapping generations of altruistic agents who derive utility from their own consumption and the bequests they make to their offspring. Offspring face a choice between investing in a high cost, but high yielding, project (for example, education), or a low cost, but low yielding, project (such as a subsistence activity) with consequences for their future income. The cost of investment can be financed either through bequests or through borrowing on the capital market. Imperfections in this market imply that the amount of borrowing permitted (and therefore the choice of project) depends on the size of an agent's inheritance. Individuals whose inheritance is greater than some critical level are able to take on the more profitable venture, while individuals whose inheritance is below this critical level are excluded from such an opportunity. The former are therefore in a position to bequeath relatively more to their offspring who, in turn, are relatively more able to undertake better investments, and so on and so forth. In the long-run the distribution of income across successive generations reproduces itself exactly and agents end up belonging to one of two classes - a high income class or a low income class. Thus hysteresis occurs such that initial inequalities affect long-run outcomes and the dynamics of income distribution are non-ergodic.

The model developed in this paper shares many of the characteristics of the basic framework described above. There is an infinite sequence of overlapping generations of two-period-lived, altruistic agents. Each agent, when young, would like to invest in a risky, indivisible project, the outcome of which is a random quality (or grade) of intermediate good for use in the production of final output during old-age. The alternative to this is simply to save in a safe, divisible asset and to subsequently produce at some lower, subsistence level. Not all agents, however, are necessarily able to undertake project investment. As in Banerjee and Newman (1993) and Galor and Zeira (1993), we allow for the possibility that an agent who borrows can default on a loan by severing ties with her lender. Such a possibility introduces frictions

[2] The other main approaches are those based on political considerations (e.g., Alesina and Rodrik 1994; Persson and Tabellini 1994; Perotti 1993) and neighbourhood effects (e.g., Benabou 1992; Durlauf 1993; Fernandez and Rogerson 1992).

into the capital market with the result that credit rationing occurs for those agents whose inherited level of wealth falls below some critical value. In this way, defaulting is prevented and only a subset of the population is eligible for loans.

The innovation of our analysis lies in the fact that the critical value of wealth is state-dependent, being determined each period by the level of economic activity. This is in sharp contrast to previous studies and re‡ects the linchpin of our analysis, which is the modelling of learning behaviour under conditions of uncertainty. At any point in time, the critical value of wealth depends on agents' perceived riskiness of the economic environment, as measured by the forecast error variance of future technology shocks (which determine future project outcomes). This perception is based on publicly available information about past states of technology and is updated over time as new information becomes available. Signi…cantly, the accuracy of information, and therefore the precision of forecasts, depends on the number of projects undertaken. Thus, as this number changes and forecasts are revised, so the critical level of wealth changes as well.

Allowing for endogenous variations in critical wealth, and doing so in the way described above, adds an extra dimension to the dynamics of income distribution with some interesting and novel implications. In particular, we are able to capture the informational externality that might plausibly arise from past experiences with risky enterprises. In turn, this enables us to identify a potentially important mechanism whereby the activities of agents in the upper income strata may a¤ect the fortunes of those in the lower income brackets. For example, if, at any point in time, there is a su¢cient number of agents undertaking risky investments, then the critical level of wealth may fall far enough such that all agents become project investors in the long-run. This process of trickle-down is signi…cantly di¤erent form the processes that appear elsewhere in the literature. In Aghion and Bolton (1997) it is the capital accumulation of the rich, which lowers the interest rate on loans, that allows the poor to take on high yielding ventures. In Perotti (1993) it is an aggregate externality arising from investment in education, together with a redistributive policy based on the median voter principle, that creates the opportunity for upward mobility. In our model it is the knowledge-spillover associated with learning behaviour that provides the basis for class divisions to be eliminated. This characterisation of trickle-down in terms of the production and processing of information is the novelty of our approach.

Based on our analysis, we show how the limiting distribution of income depends fundamentally on the initial distribution of income. In particular, we identify conditions on the initial sizes of the upper and middle income classes for di¤erent degrees of trickle-down to occur. At one extreme, no trickle-down occurs if the initial size of the upper class lies below a minimum a value, in which case the population remains polarised in its initial class divisions. At the other extreme, complete trickle-down occurs if both the initial size of the upper class and the initial size of the middle class exceed critical values, in which case the economy evolves into a classless society. Between these extremes, trickle-down takes partially and there are multiple steady state equilibria associated with different sized groups of rich and poor. These results are established both analytically and through numerical simulations of the model.

The paper is organised as follows. Section 2 presents a description of the model. Section 3 contains the main analysis of income distribution. Section 4 offers some concluding remarks.

2 The Model

2.1 Demographics, Preferences and Technologies

We consider a small open economy in which a continuum of mortal, reproductive agents make up a constant population of unit mass. Each agent lives for two periods and belongs to a dynastic family of overlapping generations connected by altruism. Each agent has one parent and one child, inheriting wealth from the former and bequeathing wealth to the latter. Each agent produces a single final commodity which is consumed exclusively during adulthood.

Agents are risk-neutral and have identical preferences defined over consumption and bequests.[3] The utility function of an agent born in period t is given as

$$U(c_{t+1}, b_{t+1}) = c_{t+1}^{\alpha} b_{t+1}^{1-\alpha}; \quad \alpha \in (0, 1) \tag{1}$$

where c_{t+1} denotes consumption and b_{t+1} denotes bequests. Let y_{t+1} be the total realised income available to the agent over her lifetime. Then the allocations of consumption and bequests that maximise (1) are $c_{t+1} = \alpha y_{t+1}$ and $b_{t+1} = (1-\alpha)y_{t+1}$, implying $U(\cdot) = u(y_{t+1}) = Ay_{t+1}$, where $A = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

[3] As in other models, we account for intergenerational altruism in the simplest way by assuming that parents derive utility from the size of their bequests, as opposed to the utility of their offspring (see Andreoni (1989) for further discussion). As in other models, as well, we keep the analysis tightly-focused and maintain tractability by assuming that dynasties last forever with given proclivities towards both altruism and fertility.

In the first period of life, an agent inherits w_t amount of wealth from her parent and takes advantage of any investment opportunties. Trivially, an agent can always invest in a safe, divisible asset which yields a fixed rate of return of r, equal to the exogenously given world rate of interest. Less trivially, an agent may also be able to invest in a risky, indivisible project which she operates, herself, but which delivers an uncertain outcome due to unforeseen events. We say that an agent is either active or idle according to whether or not she is a project investor.

At any point in time, there is a finite number of projects, indexed by j, each of which is able to support a certain mass of investors. The total number of projects undertaken at time t is denoted by N_t which maps onto the total population of project investors at that time.[4] To operate a project, each agent needs to make a fixed initial outlay of k units of capital. Thus any agent for whom $w_t < k$ must acquire a loan from the capital market if she is to become active. The outcome of a project is a random quality of intermediate good which is realised next period and which serves as an input to the production of final output during that period. We assume that the operation of any project, j, entails a current decision, z_{jt}, which yields a future quality (or grade) of input, x_{jt+1}, according to

$$x_{jt+1} = \lambda[1 - (\psi_{jt+1} - z_{jt})^2]; \quad \lambda > 0 \tag{2}$$

where ψ_{jt+1} is a random variable (a technology shock) with known probability distribution. This specification of technology has proved useful for its intuition and tractability in other applications concerned with uncertainty and learning (e.g., Jovanovic and Nyarko 1996; Wilson 1975).[5] The problem for an agent who has access to such a technology is to choose a value of z_{jt} that maximises the expected value of x_{jt+1}. With perfect information about ψ_{jt+1}, this problem is solved by setting $z_{jt} = \psi_{jt+1}$ which renders $x_{jt+1} = \lambda$ (the maximum quality attainable). With imperfect information about ψ_{jt+1}, which is the case that we consider, the solution is given by $z_{jt} = E_t(\psi_{jt+1})$, implying $E_t(x_{jt+1}) = \lambda\{1 - E_t[(\psi_{jt+1} - E_t(\psi_{jt+1}))^2]\}$, where $E_t(\cdot)$ denotes the conditional expectation at time t. We imagine that ψ_{jt+1} is comprised of

[4] For example, one may wish to think of each project type as being tied to a particular location which can accommodate any population of agents up to a maximum capacity. The precise mapping from the number of projects to the mass of project investors is specified later.

[5] In terms of the present set-up, one may wish to think of a machine which produces a grade of intermediate good (x_{jt+1}) with random imprecision (ψ_{jt+1}), subject to the setting of a control dial (z_{jt}). Since we do not impose bounds on the realisations of shocks, our index of quality and measure of control are understood to be continuous scales formed by the real line.

both a project-specific shock, ϵ_{jt+1}, and an economy-wide shock, ϕ_{t+1}, which are governed by known independent processes. Formally,

$$\psi_{jt+1} = \epsilon_{jt+1} + \phi_{t+1}; \tag{3}$$

where ϵ_{jt+1} is normally distributed with mean zero and variance σ_ϵ^2, while ϕ_{t+1} follows the first-order autoregressive process

$$\phi_{t+1} = \rho\phi_t + \eta_{t+1}; \tag{4}$$

where η_{t+1} is normally distributed with mean zero and variance σ_η^2.[6] Let $m_{t+1jt} = E_t(\phi_{t+1})$, the conditional expected value of ϕ_{t+1}, and $V_{t+1jt} = E_t[(\phi_{t+1} - m_{t+1jt})^2]$, the conditional variance of ϕ_{t+1}. Then, given the above, we may write the optimal decision as $z_{jt} = m_{t+1jt}$ and the corresponding expected quality of input as

$$E_t(x_{jt+1}) = \lambda(1 - V_{t+1jt} - \sigma_\epsilon^2): \tag{5}$$

The expression in (5) shows that the expected outcome of a project in period $t + 1$ depends on the forecast error variance of the aggregate technology shock, conditional on information at the beginning of period t. This variance measures the globally-oriented riskiness of the economy environment, as perceived by agents at the time when projects are undertaken. One of the main innovations of our analysis is the treatment of this risk factor as an endogenous, dynamic variable which evolves over time due to learning behaviour. Each period, agents make forecasts about future states of technology on the basis of available information. Movements in perceived risk occur naturally as a result of changes in the precision of these forecasts which are systematically updated according to the arrival of new information. The precise mechanism by which this process of learning takes place is described shortly.

In the second period of life, an agent realises the outcomes of any previous investments and supplies one unit labour to the home production of final output. Depending on whether she was idle or active during her youth, this labour is either used on its own to produce the subsistence amount $\theta > 0$, or combined with an intermediate good of realised quality x_{jt+1} to produce at the level $\max(\Theta; \Gamma + \gamma x_{jt+1})$ $(\Theta; \Gamma; \gamma > 0)$. Thus output is positive for any realisation of x_{jt+1}, striking a lower bound of Θ for all $x_{jt+1} < \frac{\Theta - \Gamma}{\gamma}$,

[6] The role of (4) is to infuse technology shocks with a degree of serial correlation that can be exploited by agents when forming their predictions about future project outcomes. Our assumption of a simple AR(1) process is made for convenience and may be extended to include other stochastic processes that display persistence.

and increasing linearly above this bound for all $x_{jt+1} > \frac{£ ¡ ¡}{°}$.[7] In addition to receiving income, an old agent who was active when young incurs the …xed cost of $(1 + r)k$, which is the opportunity cost or loan repayment associated with a risky investment.

Given the above, we can determine the total income available at time $t + 1$ to each agent of generation t, conditional on the agent's circumstances during period t:

$$y_{t+1} = \text{½} \begin{array}{l} (1 + r)w_t + µ; \ \text{if idle} \\ (1 + r)(w_t \ ¡ \ k) + \max(£; \ ¡ \ + °x_{jt+1}); \ \text{if active} \end{array} \tag{6}$$

We assume that $£ \ ¡ \ (1 + r)k > µ$ which has two implications - namely, that loans can always be repaid and that being active is always preferred to being idle. The sole concern of an agent, therefore, is the extent to which she is actually eligible for a loan should she require one, a matter to which we now turn.

2.2 The Credit Market

Our characterisation of events in the capital market follows closely the framework of Banerjee and Newman (1993). Borrowing and lending take place through competitive …nancial intermediaries which operate at zero pro…t, given the interest rate r. While there are no problems of bankruptcy (since loans can always be repaid), capital market imperfections arise because of the prospect that debt payments may be reneged upon - that is, a borrower may abscond with a loan by ‡eeing from her current location and running a project in hiding elsewhere. Suppose that an agent puts up all of her inherited wealth, w_t, as collateral against a loan for project investment. If the agent takes ‡ight, then any income accruing to her is inaccessible to lenders who either fail to track her down, or fail to apprehend her before she has the opportunity of disposing of her income. At the same time, the agent loses all of her collateral, $(1 + r)w_t$, and is able to produce only a fraction, $¯ \ 2 \ (0; 1)$, of the output that she would have otherwise been able to produce had she not absconded (e.g., because e¤ort must be spent on avoiding arrest, or because suitable hiding places are equipped with a less e¢cient technology).[8] Evidently, the

[7] This speci…cation of technology, as well as ensuring non-negative output, captures the plausible idea that there is unlikely to be much to choose between di¤erent grades of input below a certain a point. The minimum level of output, $£$, is the most that can be produced from these grades which may be regarded as being of equally poor quality in terms of their contribution to …nal production.

[8] In Banerjee and Newman (1993) there is no loss in productivity from taking ‡ight which entails, instead, a …xed disutility or transport cost. Nothing substantial in our

expected payoff from defaulting must be no greater than the expected payoff from not defaulting if defaulting is not, in fact, to occur. That is, from (6), $\beta \max[\Theta, - + \gamma E_t(x_{jt+1})] \leq (1 + r)(w_t - k) + \max[\Theta, - + \gamma E_t(x_{jt+1})]$. This incentive condition defines a critical value of wealth, $\hat{w}_t$, below which borrowing is prohibited. By virtue of (5), we may write this critical value as

$$\hat{w}_t = w(V_{t+1jt}) = k - \frac{(1-\beta)\max[\Theta, - + \gamma\lambda(1 - V_{t+1jt} - \sigma_2^2)]}{1+r}, \qquad (7)$$

where $w'(\cdot) > 0$ for $\max[\cdot] \neq \Theta$. In summary, only if an agent inherits a level of wealth at least equal to $\hat{w}_t$ is she able to acquire a loan and invest in a project. Otherwise, she is denied any credit and excluded from such investment.

The important property of $\hat{w}_t$ is that, subject to the condition given above, it is an increasing function of V_{t+1jt}, the conditional risk factor defined previously. Under such circumstances, $\hat{w}_t$ will change if ever V_{t+1jt} changes, as it does in our model. In this way, our analysis has the distinction of allowing the critical level of wealth to be time-dependent, being determined each period by the level of perceived risk associated with agents' forecasts of future shocks. Naturally, we shall impose appropriate parameter restrictions to ensure that there are instances in which the above condition (i.e., $\max[\cdot] \neq \Theta$) is satisfied.

2.3 Learning

We imagine that, at the beginning of each period, potential investors receive information about the technological conditions governing the outcomes of projects that were undertaken in the previous period. Specifically, we assume that they are able to observe the average state of technology across all of last period's projects. Based on this information, they then proceed to make forecasts about the states of technology of future projects for which loans are currently being sought. This is a dynamic signal-extraction problem, the solution to which is provided by the Kalman filter in the form of a set of updating equations for processing new information about the economic environment and revising forecasts about future project outcomes.

The number of projects undertaken in period $t - 1$ is N_{t-1}. The state of technology of each of these projects at the beginning of period t is defined by ψ_{jt} in accordance with (3). The average state of technology across all such projects is therefore

analysis would be altered by incorporating such a cost. Similarly, our results would be unchanged if we were to assume that lenders could claim some fraction of an absconder's income were she to be caught.

$$\tilde{\psi}_t = \epsilon_t + \phi_t; \quad \tilde{\psi}_t = \frac{\sum_{j=1}^{N_{t-1}} \tilde{\psi}_{jt}}{N_{t-1}}; \quad \epsilon_t = \frac{\sum_{j=1}^{N_{t-1}} \epsilon_{jt}}{N_{t-1}}; \tag{8}$$

where we note that ϵ_t has a variance of $\frac{\sigma_\epsilon^2}{N_{t-1}}$. Our informational assumption is that potential investors are able to observe $\tilde{\psi}_t$ but not its individual components, ϵ_t and ϕ_t.[9] Recall that $V_{t+1|t} = E_t[(\phi_{t+1} - m_{t+1|t})^2]$, where $m_{t+1|t} = E_t(\phi_{t+1})$. Equivalently, given (4), we have

$$V_{t+1|t} = \rho^2 V_{t|t} + \sigma_\eta^2; \tag{9}$$

where $V_{t|t} = E_t[(\phi_t - m_{t|t})^2]$ and $m_{t|t} = E_t(\phi_t)$. Thus the problem of predicting ϕ_{t+1}, the future realisation of the aggregate shock, is a problem of extracting information about ϕ_t, the current realisation of this shock. This problem is solved by the following updating rule for $m_{t|t}$ which is computed as the minimum mean squared error estimator of ϕ_t conditional on information at time t:

$$m_{t|t} = m_{t|t-1} + \mu_t(\tilde{\psi}_t - m_{t|t-1}); \quad \mu_t = \frac{N_{t-1}V_{t|t-1}}{N_{t-1}V_{t|t-1} + \sigma_\epsilon^2}. \tag{10}$$

Correspondingly, $V_{t|t} = \frac{\sigma_\epsilon^2 V_{t|t-1}}{N_{t-1}V_{t|t-1} + \sigma_\epsilon^2}$ so that (9) becomes

$$V_{t+1|t} = v(V_{t|t-1}; N_{t-1}) = \frac{\rho^2\sigma_\epsilon^2 V_{t|t-1}}{N_{t-1}V_{t|t-1} + \sigma_\epsilon^2} + \sigma_\eta^2; \tag{11}$$

where $v_1(\cdot) \in (0; 1)$ and $v_2(\cdot) < 0$.

The expression in (10) describes a dynamic process for $V_{t+1|t}$ conditional on N_{t-1}. This process is stable and converges to a stationary point at which $V_{t+1|t} = V_{t|t-1}$, as shown in Figure 1. The influence of N_{t-1} on this process plays a significant role in our subsequent analysis. The important implication is that, for a given value of $V_{t|t-1}$, an increase in N_{t-1} reduces $V_{t+1|t}$ (meaning that the $v(\cdot)$ schedule shifts down): that is, the forecast error variance of period t aggregate technology shocks is a decreasing function of the number of period $t - 1$ projects. The reason for this is that a larger sample of projects in one period increases the precision of information about the state of technology in the next period so that the perceived level of risk in the economy is reduced. It is this link between the amount of past investment activity and the evaluation of current risk that forms the linchpin of our analysis and allows us to offer a new perspective on the long-run dynamics of income distribution.

[9] The problem we study is similar to that addressed by Lang and Nakamura (1990) in a different context. The structure of the problem is such that nothing would change if one were to replace the assumption that agents are able to observe only $\tilde{\psi}_t$ with the assumption that they are able to observe each and every $\tilde{\psi}_{jt}$. In either case the information content of signals is the same so that agents make exactly the same inferences.

3 The Dynamics of Wealth Distribution

3.1 Transition of Lineage Wealth

The first step in determining the evolution of income distribution is to determine the rules governing changes in the fortunes of each dynasty. These lineage dynamics will tell us about the transition of individual wealth from one generation to the next. Then, given any initial distribution of income, we may use this information to infer the dynamic processes operating at the aggregate level and thereby deduce possible long-run distribution outcomes.

We know that an agent of generation t is able to operate a risky project only if she inherits a level of wealth at least equal to the critical level, $\hat{w}_t$ in (7). We also know that each agent of generation t leaves the fraction $1-\alpha$ of her realised income, y_{t+1} in (6), as a bequest to her own offspring. On the basis of these obsevations, we may conclude that the intergenerational evolution of wealth for an individual dynasty satisfies

$$w_{t+1} = \begin{cases} (1-\alpha)[(1+r)w_t + \theta]; & \text{if } w_t < \hat{w}_t \\ (1-\alpha)[(1+r)(w_t - k) + \max(\Theta, \Gamma + \gamma x_{jt+1})]; & \text{if } w_t \geq \hat{w}_t \end{cases} \tag{12}$$

These lineage transition equations are portrayed in Figure 2 for a particular configuration of parameters. Throughout the remainder of our analysis, we assume that $(1-\alpha)(1+r) \in (0, 1)$ so that the transition process is stable in each case. Given this, we define

$$w^* = \frac{(1-\alpha)\theta}{1-(1-\alpha)(1+r)}; \quad w^{**} = \frac{(1-\alpha)[\Theta-(1+r)k]}{1-(1-\alpha)(1+r)}; \tag{13}$$

where w^* (w^{**}) is the unique (mininum) steady state value of wealth for a representative dynasty whose members across all generations are idle (active). To make our analysis non-trivial, we suppose that $w^* < \hat{w}_0$.[10]

It is evident that the transition of dynastic wealth depends crucially on the critical level of wealth, $\hat{w}_t$, which is related to the conditional risk factor, $V_{t+1|t}$, through (7). In turn, $V_{t+1|t}$ is a function of the number of projects operated last period, N_{t-1}, by virtue of (10). In general, therefore, the critical level of wealth at any moment in time depends on the population of active agents in the past. It is this feature that provides the basis for the trickle-down effect in our model. Given that $w'(\cdot) > 0$, an increase in N_{t-1}, which reduces $V_{t+1|t}$, implies a lower value of $\hat{w}_t$. Ceteris paribus, this lower critical

[10] Evidently, $w^* < w^{**}$ by virtue of our previous assumption that $\Theta - (1+r)k > \theta$. The condition $w^* < \hat{w}_0$ is satisfied by imposing other parameter restrictions. If this condition was not satisfied, then everyone would automaticaly end up as a project investor.

value of wealth means that more agents become eligible for loans such that the number of projects undertaken increases next period to N_t. This leads to a further reduction in perceived risk and, with it, a further reduction in critical wealth. In principle, this process could continue until the economy reaches a long-run equilibrium in which all agents are investors in projects. Such an outcome is not inevitable, however, but depends acutely on the initial distribution of wealth , as we shall demonstrate shortly.

The wealth of a dynasty that always remains idle converges to the steady state value w^*, defined in (12). But as the foregoing dicussion makes clear, a dynasty that is idle at one point in time may well be active at a future point in time. This will be so if its level of wealth becomes greater than the critical level, in which case its wealth may converge to a long-run value quite different from w^*. As the model presently stands, there is also the possibility for a dynasty that is currently active to become subsequently idle if the realised outcome of its project is sufficiently low (i.e., low enough to cause its wealth to fall below the critical value). In general, therfore, the model admits the prospects of two-way class mobility and two-way movements in the critical level of wealth. Such prospects make an analysis of the transitional dynamics extremely complicated, if not intractable, unless one solves the model numerically (as we do later on). Fortunately, the dimensions of the problem are reduced considerably in the steady state and we are able to determine precisely what circumstances must prevail for alternative limiting distributions to be feasible. Central to deducing these circumstances is the determination of the steady state critical value of wealth which is found to depend solely on the steady state mass of project investors. As we shall show, for any given steady state level of critical wealth to be attained, there must be a certain mass of potential project investors to begin with, a result that holds regardless of the precise transition process towards the steady state. Given this, it is possible to sharpen the analysis, without losing generality, by focusing on the case in which any decrease in the critical level of wealth is never, in fact, reversed. This can be ensured by imposing a single restriction on initial conditions, namely $(1 - \alpha)[(1 + r)(\hat{w}_0 - k) + \theta] > \hat{w}_0$. Since $(1 - \alpha)(1 + r) \in (0, 1)$, it then follows that $(1 - \alpha)[(1 + r)(\hat{w}_t - k) + \theta] > \hat{w}_t$ for all t. In terms of Figure 2, the restriction guarantees that the wealth transition path for active agents always lies above the 45^0 line to the left of $\hat{w}_0$. The implication is that any lineage that succeeds in becoming active will never return to being idle: once a project investor, always a project investor. Moreover, since this means that $V_{t+1|t}$ can only decrease over time, the analysis can be tightened further with the aid of a second parameter restriction, namely $\theta < - + \gamma\lambda(1 - V_{0|-1} - \sigma_\epsilon^2)$ which ensures that $w'(\cdot) > 0$ for all t. Our results are not confined to this set of circumstances but the intuition underlying them is most transparent in this instance.

3.2 Distribution Outcomes

Let $G_t(w_t) = \int g_t(w_t)dw_t$ be the cumulative distribution function of wealth at time t so that $\int_{\underline{w}}^{\overline{w}} g_t(w_t)dw_t$ provides a measure of the population with $w_t \in (\underline{w}, \overline{w})$. Given an initial distribution, $G_0(w_0)$, together with an initial critical value of wealth, $\hat{w}_0$, we may divide the population into three initial income classes - a lower class, a middle class and an upper class. The sizes of these initial income classes are given, respectively, by

$$n_0^L = \int_0^{w^*} g_0(w_0)dw_0; \quad n_0^M = \int_{w^*}^{\hat{w}_t} g_0(w_0)dw_0; \quad n_0^U = 1 - \int_0^{\hat{w}_t} g_0(w_0)dw_0. \quad (14)$$

The joint population of initial middle and upper class agents is located on the real line of length $n_0^M + n_0^U$, which is divided into equal segments of length x denoting the maximum mass of investors that each project is able to support. The actual mass of agents from the initial middle and upper classes who are operating projects at time t is defined as n_t. Clearly, the total mass of project investors at time $t = 0$ is simply the mass of the initial upper class, $n_0 = n_0^U$. For any $t > 0$ and any given N_t (the number projects at time t), we have $n_t \in (x(N_t - 1), xN_t]$. The reverse mapping from n_t to N_t is given by

$$N_t = N(n_t) = \overline{N} - \text{int}\left[\overline{N} - \frac{n_t}{x}\right]; \quad \overline{N} = \frac{1}{x} - \text{int}\left[\frac{1 - (n_0^M + n_0^U)}{x}\right]; \quad (15)$$

where $\text{int}[\cdot]$ denotes the integer value and $\overline{N}$ is understood to be the maximum number of projects that can be undertaken by the joint middle and upper class population as a whole.[11]

The long-run distribution of wealth in the economy would be straightforward to characterise if the critical level of wealth did not change from its initial value (i.e., if $\hat{w}_t = \hat{w}_0$ for all t), as in other models (e.g., Banerjee and Newman 1993; Galor and Zeira 1993). Under such circumstances, the only investors in projects would be the initial upper class agents: all other agents belonging to the initial lower and middle classes would remain forever

[11] For example, let $x = 0.01$. Then $\overline{N} = 1$ for any $n_0^M + n_0^U \in (0, 0.01]$, $\overline{N} = 2$ for any $n_0^M + n_0^U \in (0.01, 0.02]$, and so and so forth up to $\overline{N} = 100$ for any $n_0^M + n_0^U \in (0.99, 1.00]$. Suppose that $\overline{N} = 70$. Then $N_t = 1$ for any $n_t \in (0, 0.01]$, $N_t = 2$ for any $n_t \in (0.01, 0.02]$, and so an so forth up to $N_t = 70$ for any $n_t \in (0.69, 0.70]$.

as non-project investors and the wealth of all such agents would converge to w^*. As it is, the problem is not so simple because the critical level of wealth does not, in fact, remain constant over time. On the contrary, $\hat{w}_t$ changes with the distribution of income, as reflected in changes in the population of active agents. In general, what matters for the limiting wealth distribution is the value of $\hat{w}_t$ in the long-run relative to w^*, and it is this aspect of the steady state equilibrium on which our analysis focuses.

A steady state is characterised by $V_{t+1|t} = V_{t|t-1} = V$, $\hat{w}_t = \hat{w}_{t-1} = \hat{w}$, $N_t = N_{t-1} = N$ and $n_t = n_{t-1} = n$. Consider the case in which $\hat{w} \leq w^*$. In this instance, every member of the population, including everyone in the initial lower class, ends up being a project investor and the initial class divisions are eliminated. By contrast, if $\hat{w} > w^*$, then some members of the population, meaning all those in the initial lower class plus a fraction of those in the initial middle class, remain permanently idle and the economy evolves into a two class society. Our aim is to determine the precise conditions under which these different outcomes will occur.

From (11), we obtain $V = V(N)$, where $V'(\cdot) < 0$.[12] Inserting this into (7) gives us $\hat{w} = w[V(N)] = W(N)$, where $W'(\cdot) < 0$. Trivially, $N = N(n)$ by virtue of (15). We may now state the following result.

Proposition 1 *There exists a critical initial mass of project investors, n_0^{Uc}, such that $n > n_0^U$ if and only if $n_0^U > n_0^{Uc}$.*

This result gives the necessary and sufficient condition for any amount of trickle-down to take place. The intuition underlying it is found in the dynamic process of adjustment pertaining to the critical level of wealth. The strength of this process depends, in part, on the size of the initial upper class (i.e., the mass of initial project investors) which determines the size of the initial fall in critical wealth through the externality effect in learning behaviour. Put simply, if the upper class is too small to begin with, then the process may never gain enough momentum to cause the critical level of wealth to fall below the wealth of any middle class agent. To prove the result formally, define the sequences $\hat{W}_\infty = \{\hat{w}_t\}_{t=1}^{\infty}$ and $W_\infty = \{w_t : w_0 = \hat{w}_0\}_{t=1}^{\infty}$, where $\hat{w}_t = w(V_{t+1|t}) = w[v(V_{t|t-1}; N_{t-1})]$ (from (7) and (11) and $w_t = (1-\alpha)[(1+r)w_{t-1} + \theta]$ (from (12). Thus $\hat{W}_\infty$ describes the time profile of critical wealth, while W_∞ describes the time profile of actual wealth for the richest member of the initial middle class. For any trickle-down to take place (i.e., for $n_t > n_0^U$ at some t), then at least some elements

[12] The value of V is given by the positive root of the quadratic equation $V^2 + \left[\frac{(1-\rho^2)\sigma_\varepsilon^2}{N} - \sigma_\theta^2\right] V - \frac{\sigma_\varepsilon^2\sigma_\theta^2}{N} = 0$.

of the sequence $\hat{\mathbf{W}}_\infty - \mathbf{W}_\infty = \{\hat{w}_t - w_t\}_{t=1}^{\infty}$ must be non-positive. In particular, for trickle-down to take place at time t, but not before, then $\hat{w}_t - w_t \leq 0$ and $\hat{w}_\tau - w_\tau > 0$ for $\tau = 1, ..., t-1$, in which case $n_\tau = n_0^U$ and $N_\tau = N_0$, and we may write $\hat{w}_t = \omega_t(N_0)$, where $\omega_t'(\cdot) < 0$.[13] Now, let N_0^1 denote the minimum initial number of projects for which $\omega_1(N_0^1) \leq w_1$, corresponding to which is an initial mass of upper class agents, $n_0^{U1} \in (x(N_0^1 - 1), N_0^1]$. Then $\omega_1(N_0) - w_1 > 0$ for any $N_0 < N_0^1$, implying the absence of trickle-down at $t = 1$. Moreover, if it is also true that $\omega_t(N_0) - w_t > 0$ for each $t \geq 2$ and for any $N_0 < N_0^1$, then $x(N_0^1 - 1)$ is precisely the critical initial mass of project investors, n_0^{Uc}, that must be exceeded for any trickle-down to occur. But suppose, less trivially, that this is not the case. In particular, suppose that $\omega_1(N_0) - w_1 > 0$ but $\omega_2(N_0) - w_2 < 0$ for some $N_0 < N_0^1$. In this case, however, we may simply find the minimum $N_0^2 < N_0^1$, together with a corresponding $n_0^{U2} \in (x(N_0^2 - 1), N_0^1] < n_0^{U1}$, such that $\omega_2(N_0^2) \leq w_2$ and that, for any $N_0 < N_0^2$, $\omega_\tau(N_0) - w_\tau > 0$ for $\tau = 1, 2$. As before, if it also true that $\omega_t(N_0) - w_t > 0$ for each $t \geq 3$, then we may claim that $x(N_0^2 - 1) = n_0^{Uc}$. Again, one could suppose the contrary and contemplate the case in which $\omega_\tau(N_0) - w_\tau > 0$ but $\omega_3(N_0) - w_3 < 0$ for $\tau = 1, 2$ and $N_0 < N_0^2$. One is then left to determine an $N_0^3 < N_0^2$, with corresponding $n_0^{U3} \in (x(N_0^3 - 1), N_0^3] < n_0^2$, for which $\omega_3(N_0^3) \leq w_3$ and $\omega_\tau(N_0) - w_\tau > 0$ for $\tau = 1, 2, 3$ and any $N_0 < N_0^3$, with the implication that $x(N_0^3 - 1) = n_0^{Uc}$ if $\omega_t(N_0) - w_t > 0$ for every $t \geq 4$. Proceeding in this way, it is possible to construct the sequence $\{x(N_0^t - 1)\}_{t=1}^{\infty}$, where $N_0^t \geq N_0^{t+1}$, such that, if $N_0 < N_0^t$, trickle-down does not occur until after time t. The critical value n_0^{Uc} is the smallest element in this sequence.

Given that trickle-down can occur, our next step is to identify conditions which determine how far this process may progress.

Proposition 2 Assume that $n_0^U > n_0^{Uc}$. Then for any arbitrary steady state level of wealth, $w_i \geq w^*$, there exists a corresponding steady state mass of project investors, n_i^c, such that $\hat{w} \leq w_i$ only if $n_0^M + n_0^U > n_i^c$.

This result can be established by simple reasoning. Since $W'(\cdot) < 0$, we may define an N_i^c such that $W(N) > w_i$ for any $N \leq N_i^c$, and $W(N) \geq w_i$ for any $N > N_i^c$. Thus N_i^c is the steady state number of projects that must be exceeded to ensure that the steady state critical value of wealth is no greater than w_i. The corresponding threshold steady state mass of project investors is $n_i^c = xN_i^c$. Clearly, $\hat{w} \leq w_i$ is feasible only if there is a sufficient population

[13] This follows from the fact that we may write $V_{t+1|t} = \psi_t(N_0)$, and hence $\hat{w}_t = w[\psi_t(N_0)]$, when $N_\tau = N_0$ for $\tau = 1, ..., t-1$. For example, $V_{2|1} = v(V_{1|0}, N_0) = \psi_1(N_0)$, $V_{3|2} = v(V_{2|1}, N_0) = v[\psi_1(N_0), N_0] = \psi_2(N_0)$, and so on and so forth, where $V_{1|0}$ is given.

of potential project investors to begin with - that is, if $n_0^M + n_0^U > n_{-}^c$. If not - if $n_0^M + n_0^U \leq n_{-}^c$ - then $\hat{w} > w_{-}$ and the steady state mass of active agents must be strictly less than n_{-}^c. Now, the fact that this argument applies to any arbitrary level of wealth means that we can apply it, in particular, to the level of wealth w^*. We then have the result that there exists a critical steady state mass of project investors, n^c, for which $\hat{w} \leq w^*$ only if $n_0^M + n_0^U > n^c$. This is the necessary condition for complete trickle-down to take place.

The results obtained above show clearly how the limiting distribution of wealth depends fundamentally on the initial distribution of wealth and, in particular, on the sizes of the initial middle and upper classes. If $n_0^U \leq n_0^{Uc}$, then no amount of trickle-down is possible because no amount of learning is sufficient to drive the critical level of wealth below the wealth of any middle class citizen. As such, the population remains polarised in its two initial groups of active and idle agents. If $n_0^U > n_0^{Uc}$, but $n_0^M + n_0^U \leq n^c$, then some trickle-down occurs but the process is only partial and stops short of capturing the entire middle class. In this case there is some persistence, and some erosion, of intitial inequalities and the economy displays multiple long-run outcomes associated with different sized groups of rich and poor. Only if both $n_0^U > n_0^{Uc}$ and $n_0^M + n_0^U > n^c$, is it possible for complete trickle-down to occur and for initial class divisions to be eliminated in the long-run. A sufficient (rather than necessary) condition for this case to arise is difficult to establish analytically, but immediately becomes transparent during the course of our final investigations that follow.

3.3 Numerical Simulations

We illustrate our results using dynamic simulations of a numerical version of the model. As well as giving an idea of the orders of magnitude involved, these simulations allow us to observe the process of transition towards the steady state and to experiment with alternative forms of initial wealth distribution. The transition process is generated by (7), (11), (12) and (15), together with an equation describing the dynamics of n_t, which we determine as follows.

Given our previous assumptions, it must be true that the mass of project investors at time t is at least equal to the mass of project investors at time $t - 1$, or $n_t \geq n_{t-1}$. The population of any new project investors appearing at time t, $n_t - n_{t-1}$, is understood to be the population of agents in the initial middle class who become eligible for loans at that time, having been denied loans at times $t - 1$ and before. These are agents whose level of wealth satisfies $w_t \in (\hat{w}_t; (1-\alpha)[(1+r)\hat{w}_{t-1}+\theta])$.[14] If $\hat{w}_t > (1-\alpha)[(1+r)\hat{w}_{t-1}+\theta]$, then $n_t = n_{t-1}$ since not even the richest middle class agent in period $t-1$ will have sufficient wealth in period t by which to secure a loan. By contrast, if $\hat{w}_t \leq (1-\alpha)[(1+r)\hat{w}_{t-1}+\theta]$, then

$$n_t = n_{t-1} + \int_{\hat{w}_t}^{(1-\alpha)[(1+r)\hat{w}_{t-1}+\theta]} g_t(w_t)dw_t. \qquad (16)$$

This expression relates the current mass of project investors to the current distribution of wealth. A straightforward transformation enables us to work with a more convenient formulation that relates the current mass of project investors to the initial distribution of wealth. That is,[15]

$$\begin{aligned} n_t &= n_{t-1} + \int_{\frac{\hat{w}_t - \{1-[(1-\alpha)(1+r)]^t\}w^*}{[(1-\alpha)(1+r)]^t}}^{\frac{\hat{w}_{t-1} - \{1-[(1-\alpha)(1+r)]^{t-1}\}w^*}{[(1-\alpha)(1+r)]^{t-1}}} g_0(w_0)dw_0 \\ &= n_0^U + \int_{\frac{\hat{w}_t - \{1-[(1-\alpha)(1+r)]^t\}w^*}{[(1-\alpha)(1+r)]^t}}^{\hat{w}_0} g_0(w_0)dw_0. \qquad (17) \end{aligned}$$

As above, the change in the population of project investors between periods is equal to the additional population of agents from the initial middle class who become eligible for loans from one period to the next. Thus the total mass of project investors at any given time is equal to the mass of initial upper class agents plus the cumulative mass of initial middle class agents whose level of wealth has become greater than (or equal to) the critical level by that time. Evidently, $n_0 = n_0^U$, while $n_t = n_0^M + n_0^U$ if $\hat{w}_t \leq w^*$.

Our benchmark set of parameter values is given by $\{\alpha = 0.50, \theta = 0.25, \Theta = 2.00, \gamma = 2.50, \Gamma = 0.00, \lambda = 5.00, r = 0.10, k = 1.25, \beta = 0.65, \rho = 1.00, \sigma_\varepsilon^2 = 0.75, \sigma_\eta^2 = 0.01, x = 0.01\}$. These values satisfy the parameter restrictions that we have assumed up to now and are sufficient for illustrative purposes.[16] With the same purposes in mind, we choose a simple,

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[14] That is, $w_t > \hat{w}_t$ and $w_{t-1} < \hat{w}_{t-1}$, where the latter condition may be expressed as $w_t < (1-\alpha)[(1+r)\hat{w}_{t-1}+\theta]$ by virtue of (11).

[15] The transformation makes use of the fact that $w_t = [(1-\alpha)(1+r)]^t w_0 + \{1-[(1-\alpha)(1+r)]^t\}w^*$ for all idle agents, where w^* is defined in (12). For example, the condition $w_t \geq \hat{w}_t$ may be written as $w_0 \leq \frac{\hat{w}_t - \{1-[(1-\alpha)(1+r)]^t\}w^*}{[(1-\alpha)(1+r)]^t}$.

[16] The value for x implies a total of 100 projects that can be taken on by the entire population.

but flexible, specification of initial wealth distribution that allows straightforward comparison of different cases. This specification is

$$g_0(w_0) = \begin{cases} \frac{1-g_1-g_2}{w^*}, & \text{for } w_0 \in (0; w^*) \\ \frac{g_1}{\hat{w}_0 - w^*}, & \text{for } w_0 \in (w^*; \hat{w}_0) \end{cases} \tag{18}$$

for some $g_1; g_2 \in (0; 1)$. Thus $n_0^L = 1 - g_1 - g_2$, $n_0^M = g_1$, and $n_0^U = g_2$ so that, by simple manipulation of g_1 and g_2, we are able to vary the initial sizes of all income classes. Our procedure now runs as follows. We begin by fixing the initial forecast error variance of aggregate shocks, $V_{1|0}$, which determines the initial critical value of wealth, $\hat{w}_0$. Next, we choose values for g_1 and g_2 so as to establish the initial size of each income class. We then iterate on the system of equations until the dynamic process converges and subsequently repeat the exercise for other values of g_1 and g_2:

Table 1 reports a selection of results which are indicative of our overall findings. We confirm that there is a value of n_0^U (i.e., 0.11) which must be exceeded if any amount of trickle-down is to take place. The is the critical value n_0^{Uc} in Proposition 1. We also determine the value of $n_0^M + n_0^U$ (i.e., 0.54) which must be exceeded for complete trickle-down to be possible. This is the critical value n^c associated with Proposition 2. In cases where trickle-down takes place partially, there are multiple long-run outcomes with the steady state mass of project investors, n, increasing in both n_0^M and n_0^U. This makes sense since a larger initial population of either actual or prospective project investors implies potentially greater externality effects of learning through which the process of trickle-down is perpetuated. As such, either a larger size of initial upper class or a larger size of inital middle class is conducive to the eradication of initial inequalities. The complete elimination of these inequalities occurs when the externality effects are strong enough such that the trickle-down process creates its own self-sustaining momentum. As indicated above, one of the main contributions of our numerical analysis is the identification of a sufficent condition for this case to arise. Our results suggest that, for each value of $n_0^M + n_0^U > n^c$, there is another critical value of initial project investors, n_0^{Ucc} say, such that $n = n_0^M + n_0^U$ if $n_0^U > n_0^{Ucc}$.[17] For example, setting $n_0^M + n_0^U = 0{:}60$ gives $n_0^{Ucc} = 0{:}31$, while setting $n_0^M + n_0^U = 0{:}70$ implies $n_0^{Ucc} = 0{:}21$. In general, the value of n_0^{Ucc} is monotonically decreasing (at a decreasing rate) in $n_0^M + n_0^U$ such that $\lim_{n_0^M + n_0^U \to n^c} n_0^{Ucc} = n^c$ and $\lim_{n_0^M + n_0^U \to 1} n_0^{Ucc} = 0{:}14$. Thus, for a smaller initial size of lower class, a

[17] The reader is reminded that n denotes the mass of agents from the initial middle and upper classes who are project investors in the steady state. In the case of complete trickle-down $\hat{w} \leq w^*$ so that $n = n_0^M + n_0^U$ and the total mass of project investors is $n_0^L + n = 1$ (i.e., the entire population, including the initial lower class agents).

smaller initial size of upper class is needed for all initial inequalities to vanish in the long-run.

Based on our results, both analytical and numerical, we present Figure 3 as a complete characterisation of the different possible outcomes in the model. Regions a and b are separated from other regions by the lower critical value for n_0^U; n_0^{Uc}. Regions a and c are separated from other regions by the critical value for $n_0^M + n_0^U$, n^c. And region e is separated from other regions by the upper critical value for n_0^U; n_0^{Ucc}. Accordingly, regions a and b are the regions of no trickle-down, regions c and d are the regions of partial trickle-down and region e is the region of complete trickle-down.

4 Conclusions

This paper has presented an analysis of income distribution based on a model of capital market imperfections, technological non-convexities and information acquisition. The novel feature of the analysis is its focus on the role of learning behaviour as a determinant of long-run distribution outcomes. This behaviour, together with its positive externality effects, can account for changes in individual investment opportunities and provide the mechanism by which wealth may trickle down from the rich to the poor. The precise extent to which such trickle-down takes place depends fundamentally on initial conditions.

Persistence of initial inequalities implies persistence of poverty for those dynasties who are never able to invest in projects. Such dynasties may be said to be in a poverty trap and the differences between steady state equilibria may be viewed in terms of the extent to which the population, as a whole, is trapped in this way. Equilibria with relatively high critical values of wealth are equilibria with relatively high levels of credit rationing and high populations of poverty-trapped agents. A larger size of middle or upper class offers a chance for some of these agents to benefit from trickle-down and escape from their predicament.

Although we have not considered policy explicitly, our analysis gives rise to some clear policy implications. Of course, there is no reason to presume that governments could do better than private institutions in resolving the types of market imperfection that we have contemplated. But there may be an important redistributive role for government in enabling a greater proportion of the population to take advantage of investment opportunities. An appropriate lump-sum manipulation of the wealth distribution could make fewer agents credit constrained which may have a knock-on effect through the process of trickle-down. Such a policy is likely to be more effective in some circumstances than in others. Suppose, for example, that the economy has reached a steady state in which $\bar{w} > w^*$. The size of the gap between $\bar{w}$ and w^* will be instrumental in determining the effects of policy. The more that $\bar{w}$ exceeds w^*, the less likely will any given redistribution succeed in eliminating the class divisions. Thus the limiting distribution, itself, influences the extent to which a government could change an equilibrium from one of relative poverty to one of relative prosperity.

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Table 1

	n_0^M	n_0^U	$n_0^M + n_0^U$	n
Zero trickle-down		· 0:11		n_0^U
Partial trickle-down	0:15	0:15	0:30	0:18
	0:15	0:25	0:40	0:33
	0:15	0:39	0:54	0:51
	0:39	0:15	0:54	0:23
	0:45	0:15	0:60	0:25
	0:55	0:15	0:70	0:29
Complete trickle-down	< 0:08	> 0:47	0:55	$n_0^M + n_0^U$
	< 0:29	> 0:31	0:60	$n_0^M + n_0^U$
	< 0:42	> 0:23	0:65	$n_0^M + n_0^U$
	< 0:49	> 0:21	0:70	$n_0^M + n_0^U$
	< 0:56	> 0:19	0:75	$n_0^M + n_0^U$