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Policy Variability in Models of Endogenous Growth with Productive Spending

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Abstract:

Existing theoretical analyses have shown that if policy variables affect investment decisions in either physical or human capital then an increase in policy variability results in higher trend output growth as individuals respond to higher uncertainty with a precautionary increase in these types of investment. In this paper I present two models in which policy variability arises from randomness in the provision of productive spending. In the first model, public spending enters as an input in the production technology of the economy. In this case I find that the sign of the policy variability-growth relationship depends critically on the technological parameters of the production function. In the second model, public spending is an input on the education sector of the economy. In this case I find that policy variability is always growth retarding as individuals respond to increased uncertainty by actually reducing rather than increasing their investment in human capital.

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1 Introduction

It is now well established that macroeconomic policies display an erratic behaviour over time. Various reasons have been proposed as an explanation for the observed variability in such policies: Political instability (e.g., due to frequent elections or social unrest) may lead to frequent changes in policy objectives; Policy makers may change their behaviour in response to changes in the economic environment; They may even deliberately try to create policy surprises as means of achieving their targets. Whatever the reason, policy variability can have profound effects on aggregate outcomes such as the long-run growth rate of an economy.

Intuitively, we can think of two possible channels through which volatile policies can influence the economic environment and, especially, the output growth trend. On the one hand, the erratic pattern in policy variables is a source of uncertainty to which individuals may respond by altering their optimal decisions, more importantly those decisions concerning activities that are of great importance for the process of technological change (i.e., saving and investment, education etc.). On the other hand, given that many of these policies target at enhancing the productive efficiency of the economy (e.g., through the provision of public infrastructure, law and order, publicly provided education etc.) then by affecting the actual growth rate directly, the statistical properties of their stochastic representation (e.g., both mean and variance) may alter the long-run trend of output growth.

Despite the above, the growth implications emerging from policy variability have not received the deserved attention and only a handful of theoretical analyses have examined this issue formally. The conclusion emerging from the majority of these studies is that as long as either fiscal or monetary policies affect the investment decisions of individuals, their variability enhances growth as the uncertainty associated with them induces individuals to undertake precautionary investment in either physical capital (e.g., Hopenhayn and Muniaguria 1996, Dotsey and Sarte 2000) or human capital (e.g., Gomme 1993).[1]

This paper presents a further investigation into this issue. The analysis is positive rather than normative, focusing exclusively on the effects of policy variability on the growth trend of an economy. It differs from the aforementioned analyses as it considers policy variability generated from randomness in the provision of productive public goods and services. The motivation for such an analysis can become transparent once we think of theoretical contributions that have shown that once it is assumed that government spending is used as to enhance the productivity of the output sector (e.g., Barro 1990) or the education sector (e.g., Glomm and Ravikumar 1997) then the overall effects that policy variables transmit in the long-run growth rate become substantially richer compared with situations in which government spending is used purely unproductively (e.g., government consumption or lump-sum transfers). Given this, introducing productive public spending may generate additional and important aspects on the policy variability-growth nexus that, to the best of my knowledge, have not been considered before in the literature.[2] My purpose is to examine the extent to which this conjecture can be verified. This is done in the context of two stochastic growth models with endogenous technologies and the stochastic element arising from randomness in the provision of productive government spending, specifically in the government spending to output ratio. The first model is in the spirit of Barro (1990) and assumes that the provision of productive goods and services is included as an input in the output production technology of the economy. The second model is in the spirit of Glomm and

[1] Blackburn and Pelloni (2004) find that variability in monetary policy has a negative effect on growth. Their model, however, does not capture the effect of policy variability on the investment decisions of individuals as the random money growth rate does not affect the equilibrium investment in physical capital. The effects of policy variability on long-run growth are transmitted solely through the adverse effect that the variance of the money growth rate has on aggregate employment.

[2] Turnovsky (1999) has studied productive spending in a stochastic growth model. However, in his analysis the parameters of government policy are deterministic and policy variability emerges from productivity shocks that cause output volatility.

Ravikumar (1992, 1997), by modifying the Lucas (1988) framework as to consider a situation where the provision of productive spending enhances the quality of education.

The results of the first model indicate that whether a more volatile policy results in either higher or lower trend growth depends on the technological parameter describing the relative importance of public inputs in the output production technology of the economy. In particular, when the value of this parameter is relatively low (high) then an increase in policy variability tends to decrease (increase) the long-run growth rate of output. The reason is because this technological parameter determines the actual (or temporary) growth rate's curvature with respect to the random policy variable. However, by Jensen's inequality, the curvature determines whether a mean-preserving spread in the distribution of the random policy variable increases or decreases the average (or trend) value of the growth rate.

In the model with human capital accumulation, the overall results are different. Rather than being ambiguous or depending on different values of technological parameters, in this scenario increased policy variability results always in lower trend output growth. Once more the effects depend, partially, on the curvature of the actual growth rate with respect to the random policy variable, through which a negative relationship emerges. Furthermore, a very interesting result of this model is that although human capital accumulation is a form of investment, individuals respond to the increased income uncertainty associated with policy variability by reducing the time they spend for activities that enhance their knowledge. This is a second channel through which volatile policies dampen growth in this model and it comes in stark contrast with the findings of the previous literature on the issue as it shows that the presumption that policy variability induces a precautionary increase in any form of investment is misleading.

The rest of the paper is organised as follows: Section 2 describes the role of the government. Section 3 presents the model with physical capital accumulation while section 4 presents the model with human capital accumulation. In section 5 I conclude.

2 The Government

The two models presented in this paper are linked by the assumption of the presence of a government whose single role in the economy is to produce and provide productive goods and services. The government utilises a production technology that transforms units of the economy's output into units of public goods and services in an one-to-one basis. The total amount of public goods and services, denoted by G_t, constitutes a fraction $\gamma_t \in (0,1)$ of total output in the economy. Therefore

$$G_t = \gamma_t Y_t, \tag{1}$$

where Y_t denotes total output. I assume that in order to finance its spending, the government resorts to proportional taxation and that it follows a balanced-budget rule each period.

With the purpose of studying the effects of policy variability on long-run output growth, I will allow randomness in the policy variable γ_t. Specifically, I assume that $\{\gamma_t\}_{t=0}^{\infty}$ is a sequence of random variables that are identically and independently distributed across time. To simplify matters, I specify a two-state Bernoulli distribution such as

$$prob\{\gamma_t = \tilde{\gamma} - \sigma\} = prob\{\gamma_t = \tilde{\gamma} + \sigma\} = 0.5. \tag{2}$$

An increase in σ corresponds to a mean-preserving spread in the distribution of the policy variables and, for the purpose of this paper, indicates an increase policy variability. The restrictions $\tilde{\gamma} > \sigma$ and $\tilde{\gamma} + \sigma < 1$ ensure that each period the random variable has support on the interval $(0,1)$.

3 Public Spending as an Input to Production

In this section, I use a discrete time variant of the Barro (1990) model of endogenous growth in which government spending is used as an input in the production of the economy's output. Apart from the government, there are two other types of agents in the economy, firms and households, which are described below.

3.1 Firms

There is a large number of perfectly competitive firms that, each period, produce units of the economy's single consumption good. Firms are identical and, without loss of generality, their total number is normalised to unity. I assume that firms, although owned by households, are separate entities with the objective of maximising their profits. During the production process, each firm employs K_t units of physical capital, rent from households at a per unit cost of r_t. Additionally, the productivity of each firm is enhanced by the provision of productive goods and services provided by the government. I assume that these goods and services are non-rival and non-excludable, affecting the productivity of each firm equally. I also postulate that firms do not internalise the benefits accrued from the provision of productive public spending.

Denoting aggregate spending by G_t, the production function can be written as

$$Y_t = \Lambda K_t^{1-\lambda} G_t^{\lambda},\ \lambda \in (0,1),\ \Lambda > 0. \tag{3}$$

As firms do not make any intertemporal decisions, their problem is reduced, effectively, to a static one. Each firm chooses the level of capital employed in production as to maximise its period t profits, $\Pi_t = Y_t - r_t K_t$, taking r_t and G_t as given. Profit maximisation requires that the marginal cost of capital equates its marginal product. That is

$$r_t = (1-\lambda)\Lambda K_t^{-\lambda} G_t^{\lambda} = (1-\lambda)\frac{Y_t}{K_t}. \tag{4}$$

Given the above, we get

$$\Pi_t = \lambda Y_t, \tag{5}$$

which gives the firm's profits. Therefore, combining (4) with (5) yields

$$Y_t = \Pi_t + r_t K_t, \tag{6}$$

i.e., total output in the economy is distributed between capital income and profits.

3.2 Households

The economy is populated by a large number of identical households. All households are of equal size. For brevity, I assume that there is no population growth and, without loss of generality, I normalise the total population size to unity.

There is a single asset in the economy, denoted by A_t, through which households claim ownership to physical capital. At any given period, the household receives rental income from its capital ownership and profits from its firm ownership. Total income from these two sources is subject to taxation, with households foregoing a fraction $\tau_t \in (0,1)$ (e.g., the tax rate) of their total resources. What is left as disposable income, is divided between consumption of goods, C_t, and asset holdings accumulated as to be carried onto the next period. Given the above, a household's budget constraint can be written as

$$C_t + A_{t+1} = (1-\tau_t)(r_t A_t + \Pi_t). \tag{7}$$

Each household derives lifetime utility from consumption, according to the following

$$V = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t)\right], \tag{8}$$

where $\beta \in (0,1)$ is the subjective discount rate.

A household's objective is to choose sequences for C_t and A_{t+1} as to maximise the expected value of its lifetime utility, given in (8), subject to the sequence of budget constraints in (7) and taking $\{\tau_t\}_{t=0}^{\infty}$ and $\{\Pi_t\}_{t=0}^{\infty}$ as given. The first order conditions associated with this problem are

$$\xi_t = \frac{1}{C_t}, \tag{9}$$

$$\xi_t = \beta E_t[\xi_{t+1}(1-\tau_{t+1})r_{t+1}], \tag{10}$$

where ξ_t is the Lagrange multiplier associated with (7) and E_t is the expectations operator. Equation (9) shows that the shadow value of wealth, ξ_t, is equal to the marginal utility of consumption. Equation (10) is the dynamic optimality condition for the real asset, A_t. It equates the marginal cost of increments in this asset's holdings, i.e., the utility loss from foregoing current consumption, with the marginal benefit, i.e., the expected discounted value of extra utility in the future resulting from the additional consumption possible through the after-tax return that holdings of this asset generate.

3.3 General Equilibrium

To obtain the general equilibrium in this economy, I will combine the results of the previous section together with a set of equilibrium conditions. A first condition describes the equilibrium in the asset market, i.e.,

$$A_t = K_t, \tag{11}$$

as I have assumed that all the capital stock, hence claims to it, belongs to the households. The second condition describes the equilibrium in the goods market by imposing a resources constraint through which total income is divided between consumption, investment and government spending.[3] That is

$$Y_t = C_t + K_{t+1} + G_t. \tag{12}$$

[3] Notice that full depreciation of capital has been assumed for each period.

Given equation (1), the model can be solved analytically by guessing that consumption and capital accumulation are (stochastically) proportional to total income. Denoting the saving rate by s_t and using (12), we can write

$$C_t = (1 - s_t - \gamma_t)Y_t, \tag{13}$$

$$K_{t+1} = s_t Y_t, \tag{14}$$

where $s_t \in (0,1)$ will be satisfied in equilibrium.

Substituting (4), (9) and (11) in (10) and multiplying both sides with K_{t+1} yields

$$\frac{K_{t+1}}{C_t} = \beta(1-\lambda)E_t\left(\frac{(1-\tau_{t+1})Y_{t+1}}{C_{t+1}}\right). \tag{15}$$

Given that the government follows a balanced budget rule each period, then (1), (6) and (11) imply that

$$\tau_t = \gamma_t, \tag{16}$$

i.e., the tax rate is equal to the share of government spending in total output or, put it differently, to the fraction of total resources that the government utilises for the production of public goods and services.

Using (1), the resources constraint in (12) can be written as

$$(1-\gamma_t)Y_t = C_t + K_{t+1}. \tag{17}$$

Substituting (16) and (17) in (15) yields

$$\frac{K_{t+1}}{C_t} = \beta(1-\lambda) + \beta(1-\lambda)E_t\left(\frac{K_{t+2}}{C_{t+1}}\right). \tag{18}$$

The expression in (18) is an expectations-difference equation which can be solved with the method of repeated substitution. Imposing the transversality condition on capital, $\lim_{\tau\to\infty}\beta^{\tau}E_t\left(\frac{K_{t+\tau+1}}{C_{t+\tau}}\right)=\lim_{\tau\to\infty}\beta^{\tau}E_t\left(\frac{A_{t+\tau+1}}{C_{t+\tau}}\right)=0$, yields the solution to (18) which is

$$\frac{K_{t+1}}{C_t}=\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}, \tag{19}$$

as $\beta(1-\lambda)<1$ by assumption.[4] Now, substitute (13) and (14) in (19) and solve for the saving rate s_t. Eventually, one gets

$$s_t=\beta(1-\lambda)(1-\gamma_t)\equiv s(\gamma_t). \tag{20}$$

It is clear that the saving rate is a function of the government's random share in total output. Predictably, $s'(\cdot)<0$, i.e., a temporary increase (decrease) in the government spending-output ratio (or, equivalently, an increase (decrease) in the provision of productive spending) leads to a decrease (increase) in capital accumulation merely by reallocating resources from (to) the private to (from) the public sector of the economy.

3.4 Growth

The equilibrium growth rate can be derived as follows. We begin by writing the production function in period $t+1$. That is

$$Y_{t+1}=\Lambda K_{t+1}^{1-\lambda}G_{t+1}^{\lambda}. \tag{21}$$

Substitute (1) in (21) and solve the resulting expression for Y_{t+1}. It yields

$$Y_{t+1}=\Lambda^{\frac{1}{1-\lambda}}K_{t+1}(\gamma_{t+1})^{\frac{\lambda}{1-\lambda}}. \tag{22}$$

[4] This solution can be verified with direct substitution of the result back in equation (18).

Now it is just a matter of using (20) in (14), substituting the result in (22) and dividing both sides with Y_t. Eventually, one gets

$$\frac{Y_{t+1}}{Y_t} = \beta(1-\lambda)\Lambda^{\frac{1}{1-\lambda}}(1-\gamma_t)(\gamma_{t+1})^{\frac{\lambda}{1-\lambda}}. \tag{23}$$

As we can see, the growth rate depends on different realisations of the random policy variable both at time t, through savings, and time $t+1$, through government spending. Particularly, as we have argued previously, an increase (decrease) in γ_t reduces (increases) output growth as it leads to a decrease (increase) in capital accumulation. In addition, an increase (decrease) in γ_{t+1} leads to higher (lower) output growth as it corresponds to an increase (decrease) in the amount of productive spending.

Effectively, equation (23) shows that the *actual* growth rate is itself a random variable. This economy, rather than growing perpetually at a constant rate, it grows through temporary growth rates each period, depending on different realisations of the policy variable. To obtain the *trend* growth rate of output we need to obtain the mean value of the growth rate in (23). Given the properties specified for the probability distribution of the policy shocks, one can derive the following

$$Mean\left(\frac{Y_{t+1}}{Y_t}\right) = \frac{\beta(1-\lambda)\Lambda^{\frac{1}{1-\lambda}}}{2}(1-\tilde{\gamma})\left[(\tilde{\gamma}-\sigma)^{\frac{\lambda}{1-\lambda}} + (\tilde{\gamma}+\sigma)^{\frac{\lambda}{1-\lambda}}\right] \equiv \tilde{\mathrm{M}}. \tag{24}$$

From the trend growth equation above, it is straightforward to establish the long-run effects of policy variability by computing the derivative $\partial\tilde{\mathrm{M}}/\partial\sigma$. It can be easily verified that,

$$sign\left(\frac{\partial\tilde{\mathrm{M}}}{\partial\sigma}\right) = sign\left[(\tilde{\gamma}+\sigma)^{\frac{2\lambda-1}{1-\lambda}} - (\tilde{\gamma}-\sigma)^{\frac{2\lambda-1}{1-\lambda}}\right]. \tag{25}$$

As $\tilde{\gamma} + \sigma < 1$ then it is easy to check from (25) that $\partial \tilde{\mathrm{M}} / \partial \sigma > 0$ (< 0) depending on whether $\lambda > 1/2$ ($<1/2$). The intuition for the result is as follows: Inspection of the actual growth rate in (23) reveals that the time t realisation of the random policy variable affects the actual growth rate linearly while the time $t+1$ realisation of the random policy variable has a non-linear effect on actual growth, as long as $\lambda \neq 1/2$. Given the i.i.d. property of the policy shock, its variability will not have any effect through the presence of the saving rate, due to the linearity in which it enters in the growth equation, but it will have an effect through the presence of the government's share in the growth equation. The direction of this effect depends on the curvature of (23) with respect to γ_{t+1}. If, on the one hand, $\lambda > 1/2$ then the actual growth rate is a convex function of the shock and a mean-preserving spread on the distribution of γ_{t+1} will increase trend growth. If, on the other hand, $\lambda < 1/2$ then the actual growth rate is a concave function of the shock and a mean-preserving spread on the distribution of γ_{t+1} will decrease trend growth.[5] Thus, by introducing public spending as an input in the production function of the economy, we are able to identify that technological factors may be critical in determining the sign of the underlying relationship between policy variability and long-run growth.

4 Public Spending as an Input to Education

In this section I introduce a model in the spirit of Glomm and Ravikumar (1992, 1997). Apart from the government, there is only one type of agents in the economy, i.e., the households. These households spend resources for accumulating human capital while the government provides public expenditures that can be thought as enhancing the quality of education.

4.1 Households

Like previously, I assume that the economy is populated by identical, infinitely-lived households of equal size and that the total population size is normalised to unity. However, in this model it is assumed that each household is both a

[5] This is merely an application of Jensen's inequality.

producer and a consumer of the economy's single commodity. Each period the members of a representative household produce Y_t units of output utilising a technology that is linear in their human capital, H_t. That is

$$Y_t = \Phi H_t, \ \Phi>0. \tag{26}$$

There are two possible ways in which the members of each household can learn and accumulate human capital. The first is by combining e_t units of their own time with their own existing stock of knowledge, H_t, to acquire further skills and expertise for themselves (e.g., through formal education, training and research). The second is by exploiting publicly provided expenditures, denoted by G_t, that improve the quality of education. Once more, I assume that expenditures on education are non-rival and non-excludable and that individuals do not internalise the benefits accruing from the provision of these expenditures. Combining the above assumptions, the process governing the evolution of human capital can be written, formally, as

$$H_{t+1} = \Lambda(e_t H_t)^{1-\lambda} G_t^{\lambda}, \ \Lambda > 0, \ \lambda \in (0,1). \tag{27}$$

The representative household derives lifetime utility from consumption and leisure according to the following

$$V = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \delta(\mathrm{T} - e_t)\right], \tag{28}$$

where T is the amount of the units of time that the representative household is endowed with each period.[6] The budget constraint facing each household is given by

[6] The assumption of linearity in the utility that households receive from leisure activities is innocuous for the results of this model and used here purely for computational simplicity. It can be shown that the same results apply with any increasing function $\Theta(l_t)$ where $l_t = \mathrm{T} - e_t$.

$$C_t = (1-\tau_t)Y_t + T_t, \tag{29}$$

where $\tau_t \in (0,1)$ is the income tax rate and T_t is a lump-sum transfer.[7]

The household's objective is to choose sequences for C_t, e_t and H_{t+1} as to maximise the expected value of (28) subject to sequences of (26), (27) and (29) and taking $\{\tau_t\}_{t=0}^{\infty}$, $\{T_t\}_{t=0}^{\infty}$ and $\{G_t\}_{t=0}^{\infty}$ as given. The first order conditions associated with this problem are

$$\xi_t = \frac{1}{C_t}, \tag{30}$$

$$\delta = \psi_t(1-\lambda)\Lambda(e_t H_t)^{-\lambda} H_t G_t^{\lambda}, \tag{31}$$

$$\psi_t = \beta(1-\lambda)\Lambda E_t\left[\psi_{t+1}(e_{t+1}H_{t+1})^{-\lambda} e_{t+1} G_{t+1}^{\lambda}\right] + \beta\Phi E_t[(1-\tau_{t+1})\xi_{t+1}], \tag{32}$$

where ξ_t, ψ_t are the Lagrange multipliers associated with (29) and (27) respectively, while E_t is the expectations operator. Equation (30) gives the familiar result that the marginal utility of consumption equals the shadow value of wealth. Equations (31) is the static optimality condition for the allocation of time towards learning, equating the marginal cost and marginal benefit of an additional unit of time spent on this activity. The marginal cost is associated with a reduction in leisure. The marginal benefit of learning is associated with an improvement in the future levels of human capital. Equation (32) is the dynamic

[7] It is well known that with logarithmic preferences for consumption and no other endowment apart from output production, the introduction of a proportional income tax would leave the equilibrium time allocations unaffected as the magnitude of the income and substitution effects from either an increase or a decrease in taxation would be equal. However, in such a scenario the model would abstract from a potentially very important channel through which policy variability affects the economic environment. The reason for assuming that individuals receive lump-sum transfers is to avoid such a situation and allow proportional taxation to affect the equilibrium solution for learning e_t. An alternative way to achieve this, without resorting to lump-sum transfers, would be to assume that the government raises revenues through lump-sum taxation. The reason I have avoided this is because the way through which proportional or lump-sum taxation affect equilibrium decisions is quite different and, therefore, such an approach would undermine the comparability of this model with the one presented in section 3.

optimality condition for H_{t+1}, equating the marginal cost and the marginal benefit of a higher human capital stock. This marginal benefit can be decomposed into the expected discounted value of the additional knowledge that can be gained in the future and the expected discounted value of the extra future output production, both as a result of the higher human capital stock.

4.2 General Equilibrium

The analytical solution to this model begins by assuming that since the government follows a balanced budget constraint, it divides its total revenues $\tau_t Y_t$ between productive spending and the provision of lump-sum transfers. Denoting the constant fraction of total revenues allocated to the production of public goods and services by $\alpha \in (0,1)$, it follows that

$$G_t = \alpha \tau_t Y_t, \tag{33}$$
$$T_t = (1-\alpha)\tau_t Y_t. \tag{34}$$

Given (1) and (33), the tax rate is equal to[8]

$$\tau_t = \frac{\gamma_t}{\alpha}. \tag{35}$$

Multiplying both sides of (32) with H_{t+1} yields

$$\psi_t H_{t+1} = \beta(1-\lambda)E_t[\psi_{t+1}H_{t+2}] + \beta\Phi E_t[(1-\tau_{t+1})\xi_{t+1}H_{t+1}]. \tag{36}$$

Using equation (1), we can write the resources constraint $Y_t = C_t + G_t$ as

$$C_t = (1-\gamma_t)Y_t. \tag{37}$$

[8] An additional restriction to this model is $\tilde{\gamma} + \sigma < \alpha$. This ensures that that the after-tax return to output production is strictly positive.

Taking account of (26), (30), (35) and (37), the second expectations term on the right hand side of (36) can be written as

$$E_t[(1-\tau_{t+1})\xi_{t+1}AH_{t+1}] = \frac{1}{\alpha}E_t\left(\frac{\alpha-\gamma_{t+1}}{1-\gamma_{t+1}}\right). \tag{38}$$

Given the properties of the probability distribution specified for the random variable γ_{t+1}, one can write (38) as

$$\frac{1}{\alpha}E_t\left(\frac{\alpha-\gamma_{t+1}}{1-\gamma_{t+1}}\right) = \frac{1}{2\alpha}\left(\frac{\alpha-\tilde{\gamma}-\sigma}{1-\tilde{\gamma}-\sigma}+\frac{\alpha-\tilde{\gamma}+\sigma}{1-\tilde{\gamma}+\sigma}\right) = \Omega. \tag{39}$$

Taking account that the shocks are i.i.d. through time, we can substitute (39) back in (36) to get

$$\psi_t H_{t+1} = \beta(1-\lambda)E_t[\psi_{t+1}H_{t+2}] + \beta\Omega. \tag{40}$$

Equation (40) is an expectations difference equation which can be solved with the method of repeated substitution. Imposing the transversality condition on human capital $\lim_{\tau\to\infty}[\beta(1-\lambda)]^{\tau}E_t[\psi_{t+\tau}H_{t+\tau+1}] = 0$, we can obtain the solution to (40) as

$$\psi_t H_{t+1} = \frac{\beta\Omega}{1-\beta(1-\lambda)}, \tag{41}$$

since $\beta(1-\lambda) < 1$ by assumption.[9]

Equation (31) can be written as

$$\delta = \frac{(1-\lambda)\psi_t H_{t+1}}{e_t}. \tag{42}$$

[9] Once more, the solution in (41) can be verified by direct substitution of the result back in equation (40).

Substituting (39) and (41) in (42) and rearranging yields the optimal solution for learning as

$$\tilde{e} = \frac{\beta(1-\lambda)}{2\alpha\delta[1-\beta(1-\lambda)]}\left(\frac{\alpha-\tilde{\gamma}-\sigma}{1-\tilde{\gamma}-\sigma}+\frac{\alpha-\tilde{\gamma}+\sigma}{1-\tilde{\gamma}+\sigma}\right)=e(\sigma). \tag{43}$$

Notice that, in equilibrium, the optimal solution for learning is time invariant. This is because the benefits from devoting more time in education in the current period are reaped in the future as the higher level of human capital stimulates future output production. Consequently, when deciding their learning activities, households form expectations for the future benefits of these decisions. Eventually, the optimal solution for learning is time invariant because the probability distribution of the random policy variable generates constant mean and variance.

Given the result in (43), it is easy to check that $\partial\tilde{e}/\partial\sigma < 0$, i.e., an increase in policy variability leads to a decrease in the amount of time that households spend accumulating human capital. Intuition for this result can be gained by further inspection of (38). As argued previously, the expectation term in this expression represents, partially, the benefits from accumulating human capital resulting from an increase in future output production as can be seen from equation (36). Evidently, the term inside brackets is decreasing in γ_{t+1} as more public goods require higher income taxation which lead to a decrease in the return from output production. Additionally, this term is concave in γ_{t+1}. Nevertheless, what matters for the households' decisions is the *expected* value of the term inside brackets which is decreasing in a mean-preserving spread on the distribution of the random policy variable as a result of the concavity. In this model, rather than resorting to precautionary investment in human capital, households respond to future income uncertainty, generated by higher policy variability, with a decrease in the resources they devote for this type of investment.

4.3 Growth

To obtain the growth rate of the economy, combine equations (1), (26), (27) and (43) to get

$$\frac{Y_{t+1}}{Y_t} = \Lambda A^{\lambda} \left[e(\sigma)\right]^{1-\lambda} (\gamma_t)^{\lambda}, \tag{44}$$

where $e(\sigma)$ is given in (43). Like the previous model, equation (44) gives the actual (or temporary) growth rate of output which depends on different realisations of γ_t. Clearly, a temporary increase (decrease) in γ_t causes a temporary increase (decrease) in output growth as it corresponds to higher (lower) amounts of productive goods and services that enhance the quality of education.

The trend growth rate can be obtained by taking into account the properties of the probability distribution of γ_t and substituting (43) in (44). Eventually, one can derive the following

$$Mean\left(\frac{Y_{t+1}}{Y_t}\right) = \frac{\Lambda A^{\lambda}[\beta(1-\lambda)]^{1-\lambda}}{2\{2\alpha\delta[1-\beta(1-\lambda)]\}^{1-\lambda}}\left(\frac{\alpha-\tilde{\gamma}-\sigma}{1-\tilde{\gamma}-\sigma}+\frac{\alpha-\tilde{\gamma}+\sigma}{1-\tilde{\gamma}+\sigma}\right)^{1-\lambda}\left[(\tilde{\gamma}-\sigma)^{\lambda}+(\tilde{\gamma}+\sigma)^{\lambda}\right] \equiv \tilde{\mathrm{M}} \tag{45}$$

Given the above, we can establish that $\partial\tilde{\mathrm{M}}/\partial\sigma < 0$, i.e., policy variability affects trend growth negatively. There are two distinct channels through which an increase in policy variability impinges on the long-run rate of output growth. Nevertheless their resulting effects move at the same direction. The first one is through the time that households spend accumulating human capital and it is negative for the reasons outlined previously. The second one is also negative and derives through the way that the presence of productive government spending affects the statistical properties of the actual growth rate. As we can see from (44), the actual growth rate is concave in γ_t, therefore a mean-preserving spread in the distribution of the random variable (i.e., an increase in σ) will also have a negative effect on trend growth through this channel.

5 Conclusions

In this paper I have presented two models of endogenous growth in which policy variability emerges as a result of randomness in the level of productive spending provided by the government. Contrary to existing analyses, I have shown that even when policy variables affect investment decisions in either physical or human capital, an increase in policy variability need not necessarily be related with trend output growth in a positive way because of precautionary motives that induce more of these two types of investment. In the first model, the sign of the relationship between policy variability and growth depends on the relative contribution of public inputs in the economy's production technology. In the second model, policy variability has always a negative effect on trend growth not only as a direct result of the provision of public spending in education but also because the private sector's response to higher uncertainty is to reduce rather than increase the resources spend for activities that increase human capital.

In the models I presented, I have substituted generality for rigour and analytical tractability. However, the analysis on this paper should not be viewed as claiming general conclusions but rather as a first step on identifying new mechanisms on how policy variability can be linked to trend growth and on how these mechanisms can be qualified once we consider the different sources through which ongoing growth is feasible, as those are identified in the literature. A worth pursuing extension to this analysis is to consider a model that includes both physical and human capital, with productive spending affecting both sectors of the economy, with more general assumptions about preferences and technologies and with more general stochastic processes for the policy variable. Evidently, it is not possible to get closed-form solutions in such a framework, nevertheless this extended and more general model could be simulated numerically as to give an idea on the issue of how policy variability affects output growth in the long-run under more general assumptions. I leave such considerations for future research.

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