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### **Growth, Uncertainty and Finance.**

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# Growth, Uncertainty and Finance\*

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## Abstract

We study the effects of uncertainty on long-run growth in two model economies, where households fund risky investment projects of entrepreneurs in the presence of financial market imperfections. Imperfections in the first model are due to asymmetric information which is resolved through costly state verification. In this case, some entrepreneurs may decide at the outset not to borrow and not to run projects. Imperfections in the second model are due to incomplete enforceability of loan contracts. In this case, all entrepreneurs are willing to borrow, but some of them may choose not to run projects, preferring to abscond with their loans, instead. We show that, in both cases, an increase in uncertainty increases the rate of interest on loans which increases the number of entrepreneurs who abstain from running projects. This reduces capital accumulation and growth. We also show that financial market frictions have similar effects, and that the effects of uncertainty disappear when these frictions are absent.

## 1 Introduction

There is a strong presumption that the growth of an economy depends importantly on the risks, or uncertainties, that pervade individual decision making. From uncertainty about incomes to uncertainty about lifetimes, the probabilistic nature of the economic environment can have significant implications for the incentives and opportunities of individuals to engage in growth-promoting activities, such as investment, invention and innovation. The precise way in which uncertainty may impact on growth has already been

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studied in a number of analyses. In this paper we identify another possible channel that has not, to our knowledge, been fully revealed before.

Early interest in the relationship between growth and uncertainty can be found in much of the pioneering work on optimal savings and investment behaviour in stochastic maximising models (e.g., Brock and Mirman 1972; Levhari and Srinivasan 1969; Mirman 1971; and Rothschild and Stiglitz 1971; Sandmo 1970). Amongst other results, this research revealed how an increase in uncertainty about incomes could have ambiguous effects on capital accumulation depending on individuals' attitudes towards risk. These attitudes - reflected in the curvature of the utility function - determine the extent to which individuals' decisions are dominated by precautionary motives which induce agents to save more in times of greater uncertainty.<sup>1</sup> The same conclusion has been reached in more recent analyses based on stochastic endogenous growth models. Examples include Smith (1996), who considers an overlapping generations economy with shocks to productivity, and de Hek (1999), who adopts a representative agent framework with random returns to knowledge accumulation. In both cases an increase in uncertainty - measured as a mean-preserving spread in the distribution of the shock - is more likely to raise (reduce) long-run growth the higher (lower) is the degree of relative risk aversion, or, equivalently, the lower (higher) is the elasticity of intertemporal substitution. Other recent investigations have associated uncertainty with a more aggregative measure of volatility (e.g., the variance of output growth) which has been shown to have a positive or negative effect on average output growth depending on the underlying mechanism of technological change and the underlying source of stochastic fluctuations (e.g., Aghion and Saint-Paul 1998a, 1998b; Blackburn and Galindev 2003; Blackburn and Pelloni 2004; Martin and Rogers 1997, 2000). This body of research includes several empirical studies, the most recent of which suggest that the relationship between growth and volatility is negative (e.g., Kneller and Young, 2001; Martin and Rogers, 2000; Ramey and Ramey, 1995).<sup>2</sup>

Our particular interest in this paper is the way in which uncertainty may

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<sup>1</sup>On the other hand, greater uncertainty about the returns on savings may cause savings to fall. From a different perspective, it has also been shown how greater uncertainty about the returns on investment projects may deter firms from investing due to irreversibility effects (e.g., Dixit and Pindyck 1994).

<sup>2</sup>Strictly speaking, volatility and uncertainty are two different concepts that refer to two different phenomena: by the former is meant fluctuations in a variable, while by the latter is meant unpredictability of these fluctuations. Of course, to the extent that the two phenomena usually go hand-in-hand, it is common practice to use the concepts interchangeably. In some models (e.g., Aizenman and Marion 1993; Hopenhayn and Muniagurria 1996) the source of uncertainty is volatility of government policy, rather than shocks to technology or preferences.

impact on growth as a result of financial market imperfections. There is now a substantial volume of research devoted towards understanding and verifying the existence of linkages between the real and financial development of economies.<sup>3</sup> Broadly speaking, this research shows how improvements in financial arrangements between borrowers and lenders can foster economic growth by expanding the opportunities for channelling a larger fraction of savings into investment and for undertaking more productive types of investment.<sup>4</sup> These opportunities may arise in a number of ways, including a greater pooling of risks, a higher quality of information, a lower cost of monitoring and a lower cost of transactions. That such possibilities exist must be a consequence of the inefficient functioning of the financial system to begin with. To many observers, it is precisely the misallocation of resources due to financial market imperfections that poses one of the greatest obstacles to real economic development.<sup>5</sup> In the analysis that follows we show how such imperfections can also provide scope for uncertainty to affect long-run growth.

The basic framework that we use for our analysis describes an overlapping generations economy in which homogeneous households (lenders) fund risky investment projects of heterogeneous entrepreneurs (borrowers) by drawing up mutually-agreeable loan contracts. The outcome of a project is capital and the accumulation of capital is the engine of growth in the economy. Contracts are entered into prior to the realisation of project outcomes, implying uncertainty for both lenders and borrowers. Acceptance of a contract entails a commitment by a household to make a loan out of its current income and an obligation by an entrepreneur to repay this loan out of the future proceeds from capital production. The amount of loan repayment is given by the predetermined contractual interest rate.

Against the above background, we study the effects of uncertainty on growth in two distinct, but related, models of financial market imperfections. The first model is one in which imperfections arise due to asymmetric information between borrowers and lenders: only the former can directly observe the outcomes of projects, while the latter must spend resources on acquiring such information. This raises the possibility that a borrower may try to

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<sup>3</sup>Both theoretically and empirically, this research has grown extensively over the past decade. Rather than single out just a few contributions, we refer the reader to the surveys of Capasso (2004), Driffill (2003) and Levine (1997).

<sup>4</sup>At the same time, since wealthier economies are more able to afford the costs of making such improvements, then financial development may not only influence, but may also be influenced by, the level of real development.

<sup>5</sup>This view was espoused most forcefully in the early contributions of McKinnon (1973) and Shaw (1973).

avoid honouring his debt obligations by claiming falsely that he is bankrupt. The solution to this problem entails costly state verification on the part of lenders (e.g., Diamond 1984; Gale and Hellwig 1985; Townsend 1979). In this model some entrepreneurs may choose at the outset not to take on loans and not to run projects. The second model is one in which imperfections arise because of lack of enforceability of loan contracts: borrowers have an opportunity of reneging on debt payments by simply absconding with their loans (e.g., Banerjee and Newman 1993; Blackburn and Bose 2002; Galor and Zeira 1993). Doing so means that these funds must be consumed, rather than invested in capital production. In this model all entrepreneurs choose to take on loans but not all of them choose to run projects.<sup>6</sup>

We show that, in both models, an increase in uncertainty about project outcomes leads to an increase in the contractual interest rate and an increase in the number of entrepreneurs who abstain from running projects. Capital accumulation and growth are reduced as a result. We also show that, in both models, an increase in the extent of financial market frictions has similar effects, and that the effects of uncertainty disappear when such frictions are absent. These findings may be related to the contrasting experiences of economies that differ in their exposure to uncertainty and their vulnerability to inefficiencies. Such differences are most notable between developed and developing countries, the latter of which display much greater levels of volatility and much lower levels of efficient finance. Our analysis indicates how these features may well impair growth prospects and how they may help to explain cross-country differences in per capita incomes.

In Section 2 we describe the general environment in which economic activity takes place. Section 3 presents the model of asymmetric information, while Section 4 contains the model of imperfect enforcement. Concluding remarks are contained in Section 5.

## 2 The Basic Set-up

Time is discrete and indexed by  $t = 0, 1, \dots, \infty$ . There is a countable infinite number of two-period-lived agents belonging to overlapping generations of dynastic families. Each generation is divided at birth into two groups of market participants - households (or lenders) and entrepreneurs (or borrow-

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<sup>6</sup>We note that, in practice, both asymmetric information and imperfect enforcement (together with other types of financial market imperfection) may well co-exist. This is a feature of the static partial equilibrium models of Agenor and Aizenman (1998a,b) and Aizenman and Powell (2003). We study the two types of imperfection separately because of differences in other modelling aspects to which they give rise.

ers). To fix ideas, we normalise the total population to 2, assume equal sized groups of mass 1 and unite newly-born lenders with newly-born borrowers in randomly-matched pairs.<sup>7</sup> All agents are risk neutral, deriving linear utility from their old-age consumption of final output. Output is produced by a unit mass of firms using labour (hired from households) and capital (rented from entrepreneurs). One unit of labour is supplied by each young household, while varying amounts of capital are provided by old entrepreneurs. Capital production entails the operation of a risky project when an entrepreneur is young and without any means of self-finance. For a project to be taken on, an entrepreneur must acquire a loan from a household with a promise to repay this loan in the future under the terms set by a non-renegotiable financial contract. We begin to formalise this environment with reference to the circumstances surrounding agents of generation  $t$ .

The representative firm engaged in final manufacturing combines  $n_t$  units of labour with  $k_t$  units of capital to produce  $y_t$  units of output according to

$$y_t = A(n_t K_t)^\alpha k_t^{1-\alpha}, \quad A > 0, \alpha \in (0, 1). \quad (1)$$

where  $K_t$  denotes the aggregate stock of capital.<sup>8</sup> Labour is hired at the competitively-determined wage rate  $w_t$ , while capital is rented at the competitively-determined rental rate  $r_t$ . Profit maximisation implies  $w_t = \alpha A n_t^{\alpha-1} K_t^\alpha k_t^{1-\alpha}$  and  $r_t = (1 - \alpha) A n_t^\alpha K_t^\alpha k_t^{-\alpha}$ . In equilibrium, where  $n_t = 1$  and  $k_t = K_t$ , these conditions become

$$w_t = \alpha A k_t, \quad (2)$$

$$r_t = r = (1 - \alpha) A. \quad (3)$$

An entrepreneur who undertakes a risky project is able to produce  $\kappa_{t+1}$  units of capital with  $l_t$  units of loans in accordance with

$$\kappa_{t+1} = B(1 + \beta_t)l_t, \quad B > 0. \quad (4)$$

The term  $\beta_t$  is a random variable, the value of which is realised by an entrepreneur subsequent to the acquisition of a loan. For simplicity, we assume

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<sup>7</sup>The assumption of a one-to-one matching between lenders and borrowers is a convenient abstraction that has been exploited by others (e.g., Bencivenga and Smith 1993; Blackburn *et al.* 2005; Bose and Cothren 1997). It may be justified by appealing to the existence of search costs which prohibit the break-up of any initial lender-borrower pairing. The model could be reformulated without the assumption to obtain the same results as those derived below.

<sup>8</sup>Thus we allow for an aggregate externality in the production of goods, as in many types of endogenous growth model (e.g., Romer (1986)).

that  $\beta_t$  is uniformly distributed over the interval  $[-b, b]$  with probability density function  $f(\beta_t) = \frac{1}{2b}$  ( $b < 1$ ). On completion of a project, the entrepreneur earns an income of  $r\kappa_{t+1}$  by renting his capital to final goods producers. Out of this income, he is obliged to repay the loan that was used to finance the project to begin with. Denoting by  $R_{t+1}$  the rate of interest on loans, the amount of loan repayment is  $(1 + R_{t+1})l_t$ . An entrepreneur who makes this repayment is left with a net income of  $rB(1 + \beta_t)l_t - (1 + R_{t+1})l_t$ . At the same time, capital market imperfections provide an opportunity for borrowers to avoid honouring their debt obligations. The precise nature of these imperfections is specified later.

Each young household earns a wage income of  $w_t$  from supplying its unit labour endowment to final goods firms. In addition to financing a risky project, a household can save its income safely by accessing its own storage technology that converts one unit of output at time  $t$  into  $1 + \rho$  ( $\rho > 0$ ) units of output at time  $t + 1$ . This technology functions as a type of riskless asset that pays a fixed rate of return with certainty. Like entrepreneurs, households are uninformed about the outcomes of projects until loans have been made and values of  $\beta_t$  have been realised. Prior to these events, households set the interest rate on loans, subject to their uncertainty and subject to the frictions in the capital market.

An equilibrium loan contract is a financial arrangement between an entrepreneur and a household that allows borrowing and lending to take place on mutually-agreeable terms. These terms are such as to maximise the entrepreneur's expected utility, subject to the household's individual rationality (participation) constraint. This constraint states that the household's expected income from lending is no less than its certain income from storage. In equilibrium the constraint holds with equality.

Our primary interest is in the effects of uncertainty, or volatility, on growth through changes in the financial arrangements between agents. An increase in uncertainty is measured by a mean-preserving spread of  $\beta_t$ . This corresponds simply to an increase in the bounds on this variable, as determined by the distributional support parameter  $b$ . We shall also be interested in the extent to which growth is affected by capital market imperfections, a precise measure of which will be given subsequently. Both the interpretation of  $\beta_t$  and the nature of financial frictions are different in the two models that we consider. These differences, together with other features, are reflected in different propagation mechanisms through which market uncertainty and market imperfections impact on growth. We now present each model in turn.

### 3 Model I: Project Financing With Asymmetric Information

The first scenario that we consider is one in which  $\beta_t$  represents a technology shock that is identically and independently distributed across entrepreneurs. Information is symmetric *ex ante* in the sense that, prior to drawing up loan contracts, all agents are equally uninformed about the realisations of this shock. By contrast, information is asymmetric *ex post* in the sense that, subsequent to agreeing on loan contracts, only entrepreneurs are able to directly observe the shock. This informational asymmetry is the source of financial market imperfections. The implication is that an entrepreneur may try to default on his loan repayment by claiming falsely that he is bankrupt due to a bad realisation of  $\beta_t$ . The solution to this problem involves costly state verification, whereby a lender spends resources on investigating a borrower whenever bankruptcy is declared with the view to observing and claiming all of the borrower's income (e.g., Diamond 1984; Gale and Hellwig 1985; Townsend 1979).<sup>9</sup> It is plausible to imagine that more resources must be spent in the case of larger scale operations and more complicated projects. For the purposes of the present analysis, we capture this conveniently by making the simple assumption that the cost of verification is  $cl_t$  ( $c > 0$ ), being proportional to size of the loan (e.g., Azariadis and Chakraborty 1999). A measure of the extent of capital market imperfections (i.e., the cost to lenders of acquiring information) is then given by  $c$ . Similarly, we assume that the operation of a project requires entrepreneurial effort of  $\epsilon l_t$  ( $\epsilon > 0$ ), being also proportional to the size of the project and yielding linear disutility. Entrepreneurs are differentiated according to differences in  $\epsilon$  which we assume to be uniformly distributed over the interval  $[0, e]$  with probability density function  $h(\epsilon) = \frac{1}{e}$  ( $e > 0$ ). Thus  $\int_x^z h(\cdot) d\epsilon = \frac{z-x}{e}$  provides a measure of entrepreneurs for whom  $\epsilon \in (x, z)$ . Depending on the value of  $\epsilon$ , which is public information, an entrepreneur decides whether or not to produce capital by taking on a loan and running a project. If not, then the entrepreneur remains inactive and receives zero payoff.

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<sup>9</sup>As in some other analyses (e.g., Agenor and Aizenman 1998a,b; Aizenman and Powell 2003; Gertler and Gilchrist 1993), one could also think of lenders as incurring costs of contract enforcement in the sense of having to spend resources (such as legal fees) on seizing the incomes (and any collateral) of bankrupt borrowers. We study contract enforcement from a different perspective in the second of our models.



### 3.1 Equilibrium Loan Contracts

Given the above, we may write the utility of a non-bankrupt capital producer as

$$u_t = rB(1 + \beta_t)l_t - (1 + R_{t+1})l_t - \epsilon l_t, \quad (5)$$

and the utility of a bankrupt capital producer as

$$u_t = -\epsilon l_t. \quad (6)$$

A borrower realises the value of  $\beta_t$  subsequent to his acquisition of a loan. The borrower is bankrupt whenever  $rB(1 + \beta_t) < 1 + R_{t+1}$ . We may infer from this that, for any given  $R_{t+1}$ , there is a critical value of  $\beta_t$  below which bankruptcy occurs and above which bankruptcy does not occur. This is the value  $\widehat{\beta}_t$  that satisfies

$$rB(1 + \widehat{\beta}_t) = 1 + R_{t+1}. \quad (7)$$

Naturally,  $\widehat{\beta}_t$  is increasing in  $R_{t+1}$ : *ceteris paribus*, the higher is the interest rate on loans, the more productive must be a borrower if he is to be able to make his loan repayment. The *ex post* utility of a borrower is therefore given by either (5) or (6) depending on whether  $\beta_t \in [\widehat{\beta}_t, b]$  or  $\beta_t \in [-b, \widehat{\beta}_t)$ . It follows that, prior to observing  $\beta_t$ , an entrepreneur's expected utility from running a project is

$$E(u_t) = \int_{\widehat{\beta}_t}^b [rB(1 + \beta_t)l_t - (1 + R_{t+1})l_t] f(\beta_t) d\beta_t - \epsilon l_t. \quad (8)$$

A project will be taken on by the entrepreneur if the above expression is non-negative.

Households make loans in the knowledge that borrowers may declare bankruptcy. If so, then the borrowers' claims are verified and households appropriate all of the proceeds from capital production, less the costs of verification. Bankruptcy is declared only if  $\beta_t \in [-b, \widehat{\beta}_t)$ , in which case a household's return per unit of loan is  $rB(1 + \beta_t) - c$ . Alternatively, if  $\beta_t \in [\widehat{\beta}_t, b]$ , then the household is paid back in full, earning a return of  $1 + R_{t+1}$  per unit of loan. As indicated earlier, an equilibrium loan contract is one in which, prior to the realisation of  $\beta_t$ , a household's expected return from lending is equal to its safe return from storage. That is,

$$1 + \rho = \int_{\widehat{\beta}_t}^b (1 + R_{t+1}) f(\beta_t) d\beta_t + \int_{-b}^{\widehat{\beta}_t} [rB(1 + \beta_t) - c] f(\beta_t) d\beta_t \quad (9)$$

For any given  $\widehat{\beta}_t$ , this expression determines the contractual interest rate,  $R_{t+1}$ .

Given (9), we may re-write (8) as

$$\begin{aligned} E(u_t) &= \int_{-b}^b rB (1 + \beta_t) l_t f(\beta_t) d\beta_t - \int_{-b}^{\hat{\beta}_t} c l_t f(\beta_t) d\beta_t - (1 + \rho) l_t - \epsilon l_t \\ &= \left[ rB - \left( \frac{\hat{\beta}_t + b}{2b} \right) c - (1 + \rho) - \epsilon \right] l_t. \end{aligned} \quad (10)$$

Accordingly, the condition for an entrepreneur to take on a loan and run a project is that  $[\cdot] \geq 0$  in (10), or  $rB - \left( \frac{\hat{\beta}_t + b}{2b} \right) c - (1 + \rho) \geq \epsilon$ . This defines a critical value of  $\epsilon$ , denoted  $\hat{\epsilon}$ , that satisfies

$$\hat{\epsilon} = rB - \left( \frac{\hat{\beta}_t + b}{2b} \right) c - (1 + \rho). \quad (11)$$

Capital production is undertaken by any entrepreneur for whom  $\epsilon \in [0, \hat{\epsilon}]$ , and is not undertaken by any entrepreneur for whom  $\epsilon \in (\hat{\epsilon}, e]$ . The total population of capital producers is therefore  $\int_0^{\hat{\epsilon}} h(\epsilon) d\epsilon = \frac{\hat{\epsilon}}{e}$ .<sup>10</sup> The expression in (11) implies that  $\hat{\epsilon}$  is a decreasing function of  $\left( \frac{\hat{\beta}_t + b}{2b} \right) c$ , the expected verification cost per unit of loan, which is passed on by lenders to borrowers. This cost is higher the higher is  $c$  (meaning that more resources must be spent in the event of verification) and the higher is  $\hat{\beta}_t$  (meaning that verification is more likely). An increase in either of these terms will therefore reduce the population of capital producers.

Subtraction of (9) from (7) yields

$$R_{t+1} - \rho = \int_{-b}^{\hat{\beta}_t} [rB(1 + \hat{\beta}_t) - rB(1 + \beta_t) + c] f(\beta_t) d\beta_t \quad (12)$$

This expression may be interpreted as showing the interest rate spread between risky and riskless assets. The size of spread depends on how much a lender expects to lose when a borrower goes bankrupt and is unable to make his full loan repayment (i.e., when  $\beta_t \in [-b, \hat{\beta}_t)$ ). To be sure, observe from (7) that the first integral term on the right-hand-side of (12) is equal to  $\int_{-b}^{\hat{\beta}_t} (1 + R_{t+1}) f(\cdot) d\beta_t$  which measures the expected amount of non-repayment

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<sup>10</sup>If  $\hat{\epsilon} < 0$  ( $\hat{\epsilon} > e$ ), then no (every) entrepreneur produces capital. Whilst this is perfectly feasible, it is possible to rule out such an outcome by imposing appropriate restrictions on parameter values. This follows from the fact that the lowest (highest) value of  $\hat{\epsilon}$  is  $rB - c - (1 + \rho)$  ( $rB - (1 + \rho)$ ), corresponding to the case in which  $\hat{\beta}_t = b$  ( $\hat{\beta}_t = -b$ ). It turns out that  $\epsilon \geq 0$  by virtue of the restriction  $rB - c - (1 + \rho) \geq 0$  that we appeal to later in solving the optimal contracting problem.

per unit of loan as a result of bankruptcy. Conversely, the second and third integral terms on the right-hand-side of (12) give the expected amount of income per unit of loan that is claimed from a bankrupt borrower, net of verification costs. Accordingly, (12) implies that the contractual interest rate on loans is set as a simple mark-up over lenders' safe return from storage, where the size of mark-up is equal to the expected net income lost due to bankruptcy. This mark-up rule may be simplified to

$$R_{t+1} = \rho + \frac{rB(\widehat{\beta}_t + b)^2}{4b} + \left( \frac{\widehat{\beta}_t + b}{2b} \right) c. \quad (13)$$

As above, there is a positive relationship between  $R_{t+1}$  and  $\widehat{\beta}_t$ : *ceteris paribus*, lenders set a higher contractual interest rate the more likely it is that bankruptcy will occur.

The expressions in (7) and (12) define two independent relationships between  $R_{t+1}$  and  $\widehat{\beta}_t$ . Together, these relationships form a simultaneous system, the solution to which is unique and feasible under the parameter restriction  $1 + \rho + c < rB < \frac{1+\rho}{1-b}$ . This solution is given by<sup>11</sup>

$$\widehat{\beta}^* = b - \frac{c + \sqrt{4rBb[rB - (1 + \rho) - c] + c^2}}{rB} \equiv \beta(b, c), \quad (14)$$

$$R^* = \rho + \frac{rB[\beta(b, c) + b]^2}{4b} + \left[ \frac{\beta(b, c) + b}{2b} \right] c \equiv R(b, c). \quad (15)$$

From (11), we also have

$$\widehat{\epsilon}^* = rB - \left[ \frac{\beta(b, c) + b}{2b} \right] c - (1 + \rho) \equiv \epsilon(b, c). \quad (16)$$

The two key parameters in the model are  $b$  and  $c$ . The former, which determines the spread of the distribution of  $\beta_t$ , provides a measure of uncertainty. The latter, which is the cost of verification per unit of loan, acts as an indicator of capital market frictions. The result in (14) implies that  $\beta_b(\cdot) > 0$  and  $\beta_c(\cdot) > 0$ . The result in (15) reveals similarly that  $R_b(\cdot) > 0$  and  $R_c(\cdot) > 0$ . And the result in (16) shows that  $\epsilon_b(\cdot) < 0$  and  $\epsilon_c(\cdot) < 0$ . In words, the greater is the degree of uncertainty and/or the greater is the extent of capital market imperfections, the higher is a borrower's chances of going bankrupt, the higher is the interest rate on loans and the lower is the number of entrepreneurs who take on loans. The effects of uncertainty are due to the fact that the loan repayment is a non-linear (specifically, concave) function

<sup>11</sup>Details of the derivations can be found in Appendix A.

of  $\beta_t$ . To be sure, recall that the repayment (per unit of loan) is  $rB(1 + \beta_t)$  if  $\beta_t \in [-b, \widehat{\beta}^*)$ , and  $1 + R^*$  if  $\beta_t \in [\widehat{\beta}^*, b]$ . The expected repayment is therefore reduced by a mean-preserving spread of  $\beta_t$ . Lenders compensate for this by charging a higher interest rate on loans which increases the likelihood that bankruptcy will occur and so raises the expected verification cost. Since this reduces the expected profits from capital production, fewer entrepreneurs have an incentive to engage in this activity. The effects of financial market imperfections operate in a similar way. An increase in  $c$  increases the expected verification cost which raises the contractual interest rate and makes bankruptcy more likely. Faced with lower expected returns from borrowing, fewer entrepreneurs find it profitable to take on loans.

It is instructive to consider what outcomes would transpire in an environment of perfect capital markets. This is the case in which  $c = 0$ , meaning that lenders are able to costlessly verify the bankruptcy claims of borrowers. Under such circumstances, the expression in (16) becomes independent of  $b$ , implying that the degree of uncertainty is irrelevant for entrepreneurial decisions about whether or not to take on loans: the number of entrepreneurs who do take on loans is simply fixed. It is straightforward to verify that, compared to the above, the equilibrium values of  $R_{t+1}$  and  $\widehat{\beta}_t$  are lower, while the equilibrium value of  $\epsilon$  is higher.

## 3.2 Growth

The foregoing analysis reveals how certain aspects of borrowing and lending are influenced by conditions of uncertainty and costly state verification in financial markets. In general, the greater is the degree of uncertainty and/or the more costly is verification, the higher is the interest rate on borrowing and the fewer is the number of borrowers. In what follows we study the implications of these results for capital accumulation and growth.

Capital is produced by entrepreneurs who choose to take on loans and run projects. These are entrepreneurs for whom  $\epsilon \in [0, \widehat{\epsilon}^*]$ . The expected amount of capital produced by each of them is given from (4) as  $E(\kappa_{t+1}) = \int_{-b}^b B(1 + \beta_t) l_t f(\beta_t) d\beta_t = B l_t$ . Since  $\beta_t$  is identically and independently distributed across projects, we may appeal to the law of large numbers to deduce an expression for the total amount of capital produced from all projects: that is  $k_{t+1} = \int_0^{\widehat{\epsilon}^*} B l_t h(\epsilon) d\epsilon = \left(\frac{B\widehat{\epsilon}^*}{e}\right) l_t$ . The size of loan to each capital producer is equal to the wage of the household with which he is paired. Thus  $l_t = w_t$ , where  $w_t$  is determined according to (2). It follows that the process governing aggregate capital accumulation is  $k_{t+1} = \left(\frac{\alpha AB}{e}\right) \widehat{\epsilon}^* k_t$ . Given this, then the

equilibrium growth rate of capital,  $g^* = \frac{k_{t+1}}{k_t}$ , is determined as

$$g^* = \left( \frac{\alpha AB}{e} \right) \epsilon(b, c) = g(b, c). \quad (17)$$

The result in (17) implies that  $g_b(\cdot) < 0$  and  $g_c(\cdot) < 0$ : growth is reduced by an increase in the degree of uncertainty and/or an increase in the costs of verification. As we have seen, each of these events leads to a higher contractual interest rate, a higher expected verification cost and a lower number of entrepreneurs who run projects. With fewer capital producers, capital production is reduced and, with it, so too is growth.<sup>12</sup>

In the absence of financial market frictions ( $c = 0$ ), growth is unaffected by uncertainty. This follows from our earlier result that, when verification is costless,  $\hat{\epsilon}^*$  is independent of  $b$ , implying that uncertainty is irrelevant for an entrepreneur's decision about whether or not to run a project. Compared to the case in which verification is costly, there is a greater number capital producers which is reflected in a higher growth rate of capital.

## 4 Model II: Project Financing With Imperfect Enforcement

The second scenario that we consider is one in which  $\beta_t$  represents an idiosyncratic characteristic - the technical proficiency or human capital - of an entrepreneur with which he is randomly endowed at birth. That is, we assume that entrepreneurs are differentiated according to their technical capabilities (skills, expertise and knowledge) and are distributed according to the distribution of  $\beta_t$ . Thus  $\int_x^z f(\cdot) d\beta_t = \frac{z-x}{2b}$  provides a measure of entrepreneurs with  $\beta_t \in (x, z)$ . Information is symmetric both *ex ante* and *ex post* in the sense that all agents - borrowers and lenders - are equally informed (uninformed) about the realisations of  $\beta_t$  after (before) loans have been made. The source of capital market imperfections is a weakness in the enforceability of loan contracts, implying the possibility that debt payments may be reneged upon. Specifically, we imagine that a borrower has an opportunity to abscond with his loan and default on his debt payment by fleeing from his current location and consuming his illegal income in hiding elsewhere (e.g., Banerjee

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<sup>12</sup>The effect of verification costs on growth is also studied by Azariadis and Chakraborty (1999) in a similar model to ours. In their case, however, the effect hinges on the assumption that lenders make provision for these costs in advance of giving loans to entrepreneurs, all of whom run projects. An increase in costs reduces capital accumulation by reducing the total amount of loans available.

and Newman 1993; Blackburn and Bose 2002; Galor and Zeira 1993). We suppose that a borrower is able to do this only by foregoing capital production (which we treat as being essentially immobile or prohibitively costly to re-locate) and, instead, operating a storage technology that converts one unit of output (i.e., loans) at time  $t$  into  $B_0(1 + \beta_t)$  ( $B_0 > 0$ ) units of output at time  $t + 1$ . For simplicity, we assume that both activities require the same amount of entrepreneurial effort which we normalise to zero. By way of simplifying the analysis further and making it comparable to the previous framework, we also assume that capital production and storage deliver the same expected yield of consumable income,  $rB = B_0$ . A borrower who defaults faces a probability,  $p \in (0, 1)$ , of evading detection and a probability,  $1 - p$ , of being apprehended. In the event of the former, the borrower retains the full amount of  $B_0(1 + \beta_t)l_t$  as income, leaving his creditors with nothing. In the event of the latter, the borrower earns zero income, having all of  $B_0(1 + \beta_t)l_t$  seized by his creditors as the maximum punishment for his misdemeanor. A measure of the extent of capital market frictions (i.e., imperfect contract enforceability) is given by  $p$ .<sup>13</sup>

## 4.1 Equilibrium Loan Contracts

Given the above, we may write entrepreneurial utility in the case of non-defaulting as

$$u_t = rB(1 + \beta_t)l_t - (1 + R_{t+1})l_t, \quad (18)$$

and entrepreneurial utility in the case of defaulting as

$$u_t = \begin{cases} rB(1 + \beta_t)l_t, & \text{with prob. } p, \\ 0, & \text{with prob. } 1 - p. \end{cases} \quad (19)$$

As before, an entrepreneur realises the value of  $\beta_t$  subsequent to his acquisition of a loan. Once this realisation occurs, the entrepreneur chooses whether or not to default on the loan. The condition for an entrepreneur to default is that his expected utility from doing so is no less than his expected utility from not doing so. From (18) and (19), this condition is established as

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<sup>13</sup>Other authors have studied imperfect enforcement in terms of lenders' potential to seize borrowers' incomes in the event of going bankrupt on projects (e.g., Agenor and Aizenman 1998a,b; Aizenman and Powell 2003). Specifically, a lender is able to seize only a fraction,  $x \in (0, 1)$ , of a borrower's income and must incur a cost,  $z$ , of doing this. In our case imperfect enforcement refers to the opportunity for a borrower to take flight with his loan and not to actually run any project. For simplicity, we set  $x = 1$  (since the possibility of unrecouped defaulter income is already captured by  $p$ , the probability of successful default) and  $z = 0$  (since the cost of recoupment can be analysed in much the same way as the cost of verification that we studied in the previous model).

$prB(1 + \beta_t) \geq rB(1 + \beta_t) - (1 + R_{t+1})$ , or  $(1 - p)rB(1 + \beta_t) \leq 1 + R_{t+1}$ . We may deduce from this that, for any given  $R_{t+1}$ , there is a maximum (critical) value of  $\beta_t$  below which defaulting occurs and above which defaulting does not occur. This is the value  $\widehat{\beta}_t$  that satisfies

$$(1 - p)rB(1 + \widehat{\beta}_t) = 1 + R_{t+1}. \quad (20)$$

Evidently,  $\widehat{\beta}_t$  is increasing in  $R_{t+1}$ : *ceteris paribus*, an entrepreneur is more likely to default the higher is the interest on loans that he would otherwise have to pay. *Ex ante*, the probability of defaulting is  $\int_{-\widehat{\beta}_t}^{\widehat{\beta}_t} f(\cdot) d\beta_t = \frac{\widehat{\beta}_t + b}{2b}$  which is equal to the mass of entrepreneurs who actually do default. *Ex post*, the income from borrowing is either (18) or (19) depending on whether  $\beta_t \in (\widehat{\beta}_t, b]$  or  $\beta_t \in [-b, \widehat{\beta}_t]$ .<sup>14</sup> It follows that, prior to observing  $\beta_t$ , an entrepreneur's expected utility from acquiring a loan is

$$\begin{aligned} E(u_t) &= \int_{\widehat{\beta}_t}^b [rB(1 + \beta_t)l_t - (1 + R_{t+1})l_t] f(\beta_t) d\beta_t \\ &\quad + p \int_{-b}^{\widehat{\beta}_t} rB(1 + \beta_t)l_t f(\beta_t) d\beta_t. \end{aligned} \quad (21)$$

The entrepreneur will take on a loan if the above expression is non-negative.

A household that lends makes contingent claims on the two alternative income streams of an entrepreneur - that is, the income from capital production if the entrepreneur does not default, and the income from illegal storage if the entrepreneur defaults but is caught. In the case of the former,  $\beta_t \in (\widehat{\beta}_t, b]$  and the return per unit of loan is  $1 + R_{t+1}$ . In the case of the latter,  $\beta_t \in [-b, \widehat{\beta}_t]$  and the (expected) return per unit of loan is  $(1 - p)rB(1 + \beta_t)$ . As in the previous model, an equilibrium loan contract is one in which, prior to the realisation of  $\beta_t$ , a household's expected return from lending is equal to its certain return from storage. That is,

$$1 + \rho = \int_{\widehat{\beta}_t}^b (1 + R_{t+1}) f(\beta_t) d\beta_t + (1 - p) \int_{-b}^{\widehat{\beta}_t} rB(1 + \beta_t) f(\beta_t) d\beta_t. \quad (22)$$

For any given  $\widehat{\beta}_t$ , this expression determines the contractual interest rate,  $R_{t+1}$ .

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<sup>14</sup>Note that an entrepreneur would always default before he would allow himself to go bankrupt on a project. As in the previous model, bankruptcy occurs if  $rB(1 + \beta_t) < 1 + R_{t+1}$ , or if  $\beta_t < \widehat{\beta}_t$ , where  $\widehat{\beta}_t$  satisfies (7). Under such circumstances, the entrepreneur cedes the entire amount  $rB(1 + \beta_t)$  to his creditors. But since the value of  $\widehat{\beta}_t$  in (7) is lower than the value of  $\widehat{\beta}_t$  in (20), any realisation of  $\beta_t$  that is less than the latter will cause the entrepreneur to default, irrespective of whether he is bankrupt.

By virtue of (22), it is possible to re-write (21) as

$$\begin{aligned} E(u_t) &= \int_{-b}^b rB(1 + \beta_t)l_t f(\cdot) d\beta_t - (1 + \rho)l_t \\ &= [rB - (1 + \rho)]l_t \end{aligned} \quad (23)$$

Thus, assuming that  $rB > 1 + \rho$ , an entrepreneur will always want to borrow. Subtraction of (22) from (20) yields

$$R_{t+1} - \rho = \int_{-b}^{\widehat{\beta}_t} [(1 - p)rB(1 + \widehat{\beta}_t) - (1 - p)rB(1 + \beta_t)] f(\beta_t) d\beta_t \quad (24)$$

Like before, this expression may be interpreted as showing the interest rate spread between risky and riskless assets. In the present model, the size of spread depends on how much a lender expects to lose when a borrower reneges on his debt obligation (i.e., when  $\beta_t \in [-b, \widehat{\beta}_t]$ ). To see this, observe from (20) that the first integral term on the right-hand-side of (24) is equal to  $\int_{-b}^{\widehat{\beta}_t} (1 + R_{t+1}) f(\cdot) d\beta_t$  which measures the expected amount of repayment per unit of loan that goes missing through reneging. Conversely, the second integral term on the right-hand-side of (24) gives the expected amount of income per unit of loan that is extracted from a reneger who is caught. Accordingly, (24) implies that the contractual interest rate on loans is set as a simple mark-up over lenders' safe return from storage, where the size of mark-up is equal to the expected net income lost due to defaulting. A more precise expression for this mark-up rule is

$$R_{t+1} = \rho + \frac{(1 - p)rB(\widehat{\beta}_t + b)^2}{4b}. \quad (25)$$

As above, there is a positive relationship between  $R_{t+1}$  and  $\widehat{\beta}_t$ : *ceteris paribus*, lenders set a higher interest rate on loans the greater is the likelihood that defaulting will occur.

Together, the expressions in (20) and (25) form a simultaneous system in which  $R_{t+1}$  and  $\widehat{\beta}_t$  are determined jointly. Under the parameter restriction  $1 + \rho < (1 - p)rB < \frac{1 + \rho}{1 - b}$ , there exists a unique feasible solution to this system, as given by<sup>15</sup>

$$\widehat{\beta}^* = b - 2\sqrt{\frac{b[(1 - p)rB - (1 + \rho)]}{(1 - p)rB}} \equiv \beta(b, p), \quad (26)$$

$$R^* = \rho + \frac{(1 - p)rB[\beta(b, p) + b]^2}{4b} \equiv R(b, p). \quad (27)$$

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<sup>15</sup>Details of the derivations can be found in Appendix B.



To complete the picture, let  $m_t$  denote the mass of entrepreneurs who default (or the probability that any single entrepreneur will default). Given the above, we may re-write our earlier expression for this as

$$m^* = \frac{\beta(b, p) + b}{2b} \equiv m(b, p). \quad (28)$$

The two key parameters in this model are  $b$  (the support on the distribution of  $\beta_t$ ) and  $p$  (the probability of successful default due to imperfect enforceability of loan contracts). Like the previous model, the former provides a measure of uncertainty, while the latter acts as an indicator of financial market frictions. The results in (26), (27) and (28) imply the following:  $\beta_b(\cdot) > 0$  and  $\beta_p(\cdot) > 0$ ;  $R_b(\cdot) > 0$  and  $R_p(\cdot) > 0$ ; and  $m_b(\cdot) > 0$  and  $m_p(\cdot) > 0$ . Thus the greater is the degree of uncertainty and/or the greater is the imperfectness of capital markets the more likely will a borrower default, the greater will be the incidence of defaulting and the higher will be the contractual interest rate. The effects of uncertainty are again due to the fact that a household's income from lending is a concave function of  $\beta_t$ , being equal to  $(1-p)rB(1+\beta_t)$  per unit of loan if  $\beta_t \in [-b, \widehat{\beta}^*]$ , and  $1+R^*$  per unit of loan if  $\beta_t \in (\widehat{\beta}^*, b]$ . The expected value of income from lending is therefore reduced by a mean-preserving spread of  $\beta_t$ . In response to this, lenders set a higher contractual interest rate which increases the probability that defaulting will occur. The effects of capital market imperfections operate in a similar fashion. An increase in  $p$  makes defaulting more attractive and therefore more likely, reducing the expected income of lenders who respond by raising the cost of borrowing.

Capital market imperfections vanish when financial contracts are fully enforceable. This corresponds to the case in which  $p = 0$ , implying that an entrepreneur who defaults is sure to be caught so that defaulting will never take place. Under such circumstances, uncertainty has no effect on the decisions of entrepreneurs, all of whom run projects (produce capital) irrespective of the value of  $b$ . Given this, then the condition in (20) is understood to be a simple bankruptcy condition, where bankruptcy occurs (does not occur) for entrepreneurs with  $\beta_t \in [-b, \widehat{\beta}^*]$  ( $\beta_t \in (\widehat{\beta}^*, b]$ ), and the term  $\int_{-b}^{\widehat{\beta}^*} rB(1+\beta_t)f(\beta_t)d\beta_t$  in (22) is understood to be a household's expected return per unit of loan from seizing the income of a bankrupt capital producer. Essentially, the model becomes the same as the previous framework when verification is costless.<sup>16</sup> It is straightforward to show that the equilibrium values of  $R_{t+1}$ ,  $\widehat{\beta}_t$  and  $m_t$  are all lower than those established above.

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<sup>16</sup>With  $p = c = 0$ , (14) and (26) deliver the same value of  $\widehat{\beta}^*$ , while (15) and (27) yield the same value of  $R^*$ .

## 4.2 Growth

The foregoing analysis demonstrates how the contractual arrangements between borrowers and lenders is influenced by the uncertainty and enforceability in financial markets. In general, the greater is the degree of uncertainty and/or the weaker is the power of enforceability the higher is the interest rate on loans and the higher is the incidence of defaulting. We now study the implications of these results for capital accumulation and growth.

Capital is produced by each entrepreneur who does not default on his loan - that is, each entrepreneur for whom  $\beta_t \in (\widehat{\beta}^*, b]$ , where  $\widehat{\beta}^*$  is determined in (26). The technology used to produce capital is given in (4). It follows that capital production in the economy as a whole is  $k_{t+1} = \int_{\widehat{\beta}^*}^b B(1+\beta_t)l_t f(\cdot) d\beta_t$ . Each entrepreneur receives a size of loan equal to the wage of the household with which he is partnered. Thus  $l_t = w_t$ , where  $w_t$  is given in (2). Accordingly, we may write the process governing aggregate capital accumulation as  $k_{t+1} = \int_{\widehat{\beta}^*}^b B(1+\beta_t)\alpha A k_t f(\beta_t) d\beta_t$ . From this, the equilibrium growth rate of capital,  $g = \frac{k_{t+1}}{k_t}$ , may be deduced as

$$g^* = \alpha AB \left\{ \frac{b - \beta(b, p)}{2b} + \frac{b^2 - [\beta(b, p)]^2}{4b} \right\} \equiv g(b, p). \quad (29)$$

The result in (29) implies that  $g_b(\cdot) < 0$  and  $g_p(\cdot) < 0$ : that is, growth is reduced by an increase in financial market uncertainty and/or an increase in financial market frictions. As we have established above, an increase in uncertainty leads to an increase in the contractual interest rate and an increase in the population of entrepreneurs who default. Equivalently, there is a reduction in the mass of capital producers, implying a reduction in aggregate capital accumulation and hence a reduction in growth. As we have also established, a weakening in the enforceability of contracts leads similarly to a higher interest rate on loans and a higher incidence of defaulting. With fewer entrepreneurs producing capital, capital accumulation falls and, with it, so does growth.

In the absence of financial market imperfections ( $p = 0$ ), the growth rate is simply  $\alpha AB$  since defaulting does not occur. Rather, as indicated earlier, all entrepreneurs produce capital and either pay back loans or go bankrupt. Compared to the above, the growth rate in this case is higher and is independent of  $b$ . Thus, with perfect capital markets, uncertainty has no effect on growth.

## 5 Conclusions

This paper has been concerned with the relationship between growth, uncertainty and finance. We have studied this within the context of two distinct, but related, models of financial market imperfections. In both models growth occurs endogenously through the accumulation of capital which is produced from risky projects that require external funding. This funding is made available through mutually-agreeable financial arrangements between borrowers (entrepreneurs) and lenders (households), the latter of whom may alternatively invest in a riskless asset that pays a safe rate of return. The terms of these arrangements, or contracts, are determined prior to the realisation of project outcomes, specifying the interest rate at which loans must be repayed conditional on the uncertainty and imperfections that exist in the market.

In the first model imperfections arise due to asymmetric information between borrowers and lenders: only the former are able to directly observe the outcomes of projects, while the latter must spend resources on acquiring this information. The incentive problem to which this gives rise is solved by a contract that involves costly state verification. Given this contract, not all entrepreneurs find it profitable to borrow and run projects.

In the second model imperfections arise because of lack of enforceability of contracts: borrowers have an opportunity of defaulting on debt repayments by absconding with loans. Doing so means that loans must be consumed, rather than invested in projects. Under the resulting contract, all entrepreneurs find it profitable to borrow, but not all of them choose to run projects.

Both models yield similar implications. The contractual interest rate on loans is set as a mark-up over the risk-free rate of return that lenders' could earn from their alternative safe investment opportunity. The size of this mark-up depends on the amount of income that lenders expect to lose as a result of non-repayment of loans - an event that is more likely the higher is the degree of uncertainty and the greater is the extent of financial market frictions. Accordingly, an increase in uncertainty, an increase in verification costs or a decrease in contract enforceability will each give rise to a higher contractual interest rate. The effect of this is to reduce the number of entrepreneurs who undertake projects, either by weakening the incentives to take on loans at the outset, or by strengthening the incentives to abscond with loans. With fewer projects being operated, capital production and growth are reduced. Significantly, the effects of uncertainty depend critically on the existence of financial market imperfections: absent such imperfections and these effects disappear entirely.

## Appendix A: Solution of Model I

The results in (14), (15) and (16) are derived as follows. Combining (7) and (13) yields the quadratic equation

$$0 = rB\widehat{\beta}_t^2 - 2(rBb - c)\widehat{\beta}_t - [4rBb - rBb^2 - 4b(1 + \rho) - 2bc]. \quad (\text{A1})$$

Hence

$$\widehat{\beta}_t = b - \frac{c}{rB} \pm \frac{\sqrt{4rBb[rB - (1 + \rho) - c] + c^2}}{rB}. \quad (\text{A2})$$

A sufficient condition for ruling out complex roots is that  $rB \geq 1 + \rho + c$ . Given this, together with the fact that  $\widehat{\beta}_t \leq b$ , the only possible solution to (A2) is that given by  $\widehat{\beta}^*$  in (14). The restriction  $rB \leq \frac{1+\rho}{1-b}$  ensures that  $\widehat{\beta}_t \geq -b$  as well. Substitution of (14) into (13) gives the expression for  $R^*$  in (15), while substitution of (14) into (11) gives the expression for  $\widehat{\epsilon}^*$  in (16).

The properties of the functions  $\beta(\cdot)$ ,  $R(\cdot)$  and  $\epsilon(\cdot)$  are verified as follows. From (14), we find that

$$\beta_b(\cdot) = 1 - \frac{2[rB - (1 + \rho) - c]}{\sqrt{4rBb[rB - (1 + \rho) - c] + c^2}}, \quad (\text{A3})$$

$$\beta_c(\cdot) = -\frac{1}{rB} \left\{ 1 + \frac{c - 2rBb}{\sqrt{4rBb[rB - (1 + \rho) - c] + c^2}} \right\}. \quad (\text{A4})$$

From (15), we have

$$R_b(\cdot) = \frac{2rB(\widehat{\beta}^* + b)[b - \widehat{\beta}^* + 2b\beta_b(\cdot)] + c[b\beta_b(\cdot) - \widehat{\beta}^*]}{4b^2}, \quad (\text{A5})$$

$$R_c(\cdot) = \frac{rB(\widehat{\beta}^* + b)\beta_c(\cdot) + \widehat{\beta}^* + b + c\beta_c(\cdot)}{2b}. \quad (\text{A6})$$

And from (16), we obtain

$$\epsilon_b(\cdot) = -\frac{c[b\beta_b(\cdot) - \widehat{\beta}^*]}{2b^2}, \quad (\text{A7})$$

$$\epsilon_c(\cdot) = -\frac{\widehat{\beta}^* + b + c\beta_c(\cdot)}{2b}. \quad (\text{A8})$$

Under the above parameter restrictions, it is deduced that  $\beta_b(\cdot) > 0$  and  $\beta_c(\cdot) > 0$ ,  $R_b(\cdot) > 0$  and  $R_c(\cdot) > 0$ , and  $\epsilon_b(\cdot) < 0$  and  $\epsilon_c(\cdot) < 0$ .

The properties of the function  $g(\cdot)$  in (17) follow straightforwardly from the fact that

$$g_b(\cdot) = \left(\frac{\alpha AB}{e}\right) \epsilon_b(\cdot), \quad (\text{A9})$$

$$g_c(\cdot) = \left(\frac{\alpha AB}{e}\right) \epsilon_c(\cdot). \quad (\text{A10})$$

Thus  $g_b(\cdot) < 0$  and  $g_c(\cdot) < 0$ .

## Appendix B: Solution of Model II

The results in (26), (27) and (28) are derived as follows. Combining (20) and (25) gives the quadratic equation

$$0 = (1-p)rB\widehat{\beta}_t^2 - 2(1-p)rBb\widehat{\beta}_t - [4(1-p)rBb - (1-p)rBb^2 - 4b(1+\rho)]. \quad (\text{B1})$$

Hence

$$\widehat{\beta}_t = b \pm 2\sqrt{\frac{b[(1-p)rB - (1+\rho)]}{(1-p)rB}}. \quad (\text{B2})$$

Since  $\widehat{\beta}_t \leq b$ , then the only possible solution is that given by  $\widehat{\beta}^*$  in (26). The restriction  $(1-p)rB > 1+\rho$  rules out complex roots, while the restriction  $(1-p)rB \leq \frac{1+\rho}{1-b}$  ensures that  $\widehat{\beta}_t \geq -b$ . Substitution of (26) into (25) gives the expression for  $R^*$  in (27), while substitution of (26) into  $\frac{\widehat{\beta}_t+b}{2b}$  gives the expression for  $m^*$  in (28).

The properties of the functions  $\beta(\cdot)$ ,  $R(\cdot)$  and  $m(\cdot)$  are verified as follows. From (26), we find that

$$\beta_b(\cdot) = 1 - \sqrt{\frac{(1-p)rB - (1+\rho)}{(1-p)rBb}}, \quad (\text{B3})$$

$$\beta_p(\cdot) = \frac{(1+\rho)}{(1-p)^2} \sqrt{\frac{b(1-p)}{rB[(1-p)rB - (1+\rho)]}}. \quad (\text{B4})$$

From (27), we have

$$R_b(\cdot) = \frac{(1-p)rB(\widehat{\beta}^* + b)[b - \widehat{\beta}^* + 2b\beta_b(\cdot)]}{4b^2}, \quad (\text{B5})$$

$$R_p(\cdot) = \frac{rB(\widehat{\beta}^* + b)[2(1-p)\beta_p(\cdot) - \widehat{\beta}^* - b]}{4b}. \quad (\text{B6})$$

And from (28), we obtain

$$m_b(\cdot) = \frac{b\beta_b(\cdot) - \widehat{\beta}^*}{2b^2}, \quad (\text{B7})$$

$$m_p(\cdot) = \frac{\beta_p(\cdot)}{2b}. \quad (\text{B8})$$

Under the above parameter restrictions, it is deduced that  $\beta_b(\cdot) > 0$  and  $\beta_p(\cdot) > 0$ ,  $R_b(\cdot) > 0$  and  $R_p(\cdot) > 0$ , and  $m_b(\cdot) > 0$  and  $m_p(\cdot) > 0$ .

To derive the properties of the function  $g(\cdot)$  in (29), we compute

$$g_b(\cdot) = \frac{\alpha AB}{2b^2} \left\{ \widehat{\beta}^* - b\beta_b(\cdot) + \frac{b^2 + \widehat{\beta}^* [\widehat{\beta}^* - 2b\beta_b(\cdot)]}{2} \right\}, \quad (\text{B9})$$

$$g_p(\cdot) = -\frac{\alpha AB(1 + \widehat{\beta}^*)\beta_p(\cdot)}{2b}. \quad (\text{B10})$$

These expressions yield  $g_b(\cdot) < 0$  and  $g_p(\cdot) < 0$ .

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