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The effect of nominal shock uncertainty on output growth

By

Elena Andreou[‡], Alessandra Pelloni[*] and Marianne Sensier[†]

[‡]Tilburg University and University of Cyprus

[*]University of Rome, Tor Vergata, Italy

[†]Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, UK

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Elena Andreou,
Tilburg University and University of Cyprus,

Alessandra Pelloni,
University of Rome, Tor Vergata,

and

Marianne Sensier,
Centre for Growth and Business Cycle Research,
University of Manchester
http://www.ses.man.ac.uk/cgbcr/

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Address for correspondence:
Elena Andreou
Econometrics and Finance Groups
CentER, Office B907
Tilburg University
P.O. Box 90153
5000 LE, Tilburg
The Netherlands.

email: e.andreou@uvt.nl

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ABSTRACT

This paper tests some empirical implications of a theoretical model which proposes that the relationship between growth and its uncertainty depends fundamentally on whether the stochastic shocks causing fluctuations are real or nominal and on the presence of nominal rigidities in the economy. Shock uncertainty associated with cyclical variation is captured by a dynamic conditional variance model that estimates the time-varying, unpredictable volatility of nominal and real shocks and their effects on growth. In the context of a bivariate GARCH-in-Mean model we test the empirical conditional mean and variance relationships of nominal money and production growth rates in the G7 countries. We find that growth uncertainty has an insignificant effect but nominal money shock uncertainty exerts a negative and significant influence on growth for some of the G7. This is considered as supportive empirical evidence of the theoretical model predictions particularly on the link between nominal shock uncertainty and output growth. Another implication of the theoretical model that gains empirical support is that an increase in the average rate of money growth has a positive effect on the average output growth rate.

JEL classification: C32, E32, O42.

Keywords: growth uncertainty, learning-by-doing, monetary uncertainty, multivariate GARCH-in-mean, nominal rigidity.

1. Introduction

The question of how business cycle fluctuations affect long-term growth has been the subject of much recent debate. Our theoretical analysis suggests that the relationship between growth and its uncertainty depends on whether the source of stochastic fluctuations is real or nominal and on the presence of nominal rigidities in the economy.[1] These results are established in the context of a stochastic monetary model in which endogenous growth occurs through learning-by-doing and the two cases of a competive labour market and of nominal wage setting are considered. We empirically investigate the linkages between money and output growth and their uncertainties using time-series data for the G7 in a system that allows growth rates and uncertainties to interact for each country – namely in a Multivariate GARCH-in-Mean (M-GARCH-M) model (see for instance, Bollerslev, 1990 and Bolleslev et al., 1994). The money and output growth dynamic equations are a function of their time-varying conditional innovation variances that represent the uncertainty factors. This model has also been adopted by Edler (2003), Grier and Perry (1996, 2000) to study the effects of inflation uncertainty. The link between the theoretical model and the econometric specification as well as some of the advantages of the latter are discussed. To give a preview of our results, we find a significant, negative relationship between output growth and nominal money shock uncertainty for some of the G7 and a significant positive relationship between output growth and nominal money growth average for most of the G7. These results are consistent with the theoretical predictions of our model. Although the link between growth and output uncertainty turns out to be insignificant, the sign of this relationship is positive in most countries, as predicted by the model.

Since the seminal contribution by Nelson and Plosser (1982), it has become customary to treat most macroeconomic time series as containing stochastic trends. Until lately, these trends were typically associated with the occurrence of exogenous technology shocks that follow a unit root process, a perspective exemplified in early real business cycle models. Such a perspective is based on the traditional dichotomy between business cycles and growth. Fatas (2000) argues however that to explain the persistence of output fluctuations a model is needed where progress is endogenous. Indeed a key implication of stochastic endogenous growth theory is that any temporary disturbance can have a permanent effect on output so long as it changes the amount of resources on which productivity improvements depend, so that stochastic trends are generated not by some arbitrary, exogenous impulse process, but rather by endogenous responses of technology to changes in the current state of the economy. In models following Schumpeter (1942) recessions are events that have a positive impact on growth as they reduce the opportunity cost of diverting resources away from manufacturing towards technological

[1] Throughout the paper we use the terms volatility, variability and uncertainty interchangeably to define the conditional standard deviation of a variable. For instance, growth uncertainty is equivalent to the volatility/variability of the innovation of output growth rate conditional on its mean dynamic behavior and that of other variables which is estimated by a parametric GARCH type model, the details of which are discussed in the empirical section 3.

improvements (e.g Caballero and Hammour, 1994 Aghion and Saint-Paul 1998). Therefore, the overall relationship between short-term volatility and long-term growth is likely to be positive and the size of the recessions is positively correlated with average growth (Helpman and Trajtenberg, 1998). More recently, Francois and Lloyd-Ellis (2002) unify the Schumpeterian theory of creative destruction with Shleifer's theory on 'animal spirits' to develop a parsimonious model of endogenous cyclical growth, according to which growth and volatility are negatively related across economies.

In models in the spirit of Arrow (1962), revived by Romer (1986) where growth takes the form of learning-by-doing, recessions are episodes that have a negative effect on growth (Blackburn, 1999, Pelloni, 1997, Martin and Rogers, 2000). Taking account of optimal savings de Hek (1999) shows that under learning-by-doing volatility can have a negative or positive effect on growth depending on the elasticity of the marginal utility of consumption. In particular the effect will be negative if the elasticity of the marginal utility of consumption is lower than one. Similarly Jones et al. (1999), in the context of a human capital accumulation model, find that lower variability implies lower growth for levels of risk aversion equal or higher than one. All these are purely real models, which thereby only analyse the effects of real uncertainty. Dotsey and Sarte (2000) study instead the influence of money volatility on growth in a convex model where production is linear in capital, money is introduced through a cash-in-advance constraint and there is perfect price flexibility. They derive a positive effect of money volatility on growth and a negative effect of average money growth on output growth.

In our model, where utility is logarithmic in consumption, an increase in the volatility of preferences leads, through precautionary savings, to an increase in the rate of growth. Coming to nominal volatility, in the case of perfectly flexible prices, money shocks have no real effects in the short or in the long run. However when nominal wages are (optimally) set one period in advance, an increase in the volatility of money growth will reduce the rate of income growth, due to the fact that the production function is concave in labor. Our basic result is therefore that it is important to isolate the source of volatility before one can answer to the question whether volatility is good or bad for growth. Another result we derive is that average money growth has a positive effect on average income growth, with nominal wage setting, if the variance of money growth does not change. In fact given this variance an increase in expected money growth means a decrease in the standardized size of expectational errors agents make when setting the wage: the related loss in efficiency is also reduced.

The significance and sign of the relationship between output growth and its uncertainty has also been the subject of many empirical papers. Some studies find a negative effect between output uncertainty and growth based on cross-section (e.g. Martin and Rogers, 2000) and panel (e.g. Ramey and Ramey, 1995) approaches, or a positive effect (e.g. Grier and Tullock, 1989) and sometimes even a zero one (e.g. Dawson and Stephenson, 1997). Evidence from the few time series investigations of individual countries is also mixed with positive correlation in some (Caporale and McKiernan, 1996), negative in others (Peel and Speight, 1998) and even statistically insignificant in others (Grier and Perry, 2000, Peel and Speight, 1998). More recent empirical studies attempt to provide explanations for this mixed evidence: Imbs (2002) offers a disaggregated sectoral analysis on growth and uncertainty while Kroft and Lloyd-Ellis (2002) decompose cross-sectional aggregate volatility and analyze its effects on growth.

There are also many studies that link growth with inflation and/or money growth and their uncertainties (see Temple, 2000 for an overview). The empirical evidence is extensive and although many studies find that economic activity is negatively related to the mean and/or the variance of inflation and money growth (e.g. Aizenman and Marion, 1993, Davis and Kanago, 1996, Grier and Perry, 2000, Judson and Orphanides, 1996, Kormendi and Meguire, 1985, Makin, 1982), few studies find a positive relationship (e.g. Coulson and Robins, 1985) and others argue that the cross-section relationship is not robust (Levine and Renelt, 1992, Levine and Zevros, 1993).

Some studies emphasize that it is difficult to separate the effects of inflation/money average and variance on growth given the high correlation between the two variables (Temple, 2000, Dotsey and Sarte, 2000). Our econometric analysis deals with this problem using a model that disentangles and jointly estimates the partial effects and causalities of the conditional means and innovation volatilities of growth and nominal money. It is however important to note that we do not examine inflation and its uncertainty because we expect them to be correlated with both nominal and real shocks in the economy. Instead we focus on money growth shocks and their uncertainty. This is also directly linked to our theoretical model according to which inflation is endogenous. We estimate uncertainty of output and money growth jointly from the variances of innovations of Vector Autogressive (VAR) specifications.

Cross-section and panel-data approaches have been traditionally preferred to time series methods to study long-run growth because it is argued that findings in time series studies are dominated by high frequency data variations without any implications for the long run rate of growth (e.g. Temple, 1999). However, as Durlauf and Quah (1999) show in a recent, comprehensive empirical review of cross-country growth empirics, there are advantages and disadvantages in all three econometric approaches. In our econometric model the cyclical fluctuations and uncertainties of shocks are captured by a nonlinear dynamic heteroskedastic model. The empirical bivariate GARCH-M model focuses on the effects of nominal and output volatilities on their growth rates by modelling output growth as a function of lagged output growth, lagged monetary growth and the conditional variances of these two series that represent the uncertainty factors. We model monetary growth in an analogous fashion.

There are various arguments for complementing the empirical evidence on volatility, growth and the role of nominal shock uncertainty over the business cycle with time series specifications, our example here being the Multivariate-GARCH in Mean (M-GARCH-M) model. A brief outline of these arguments is given below and more details can be found in section 3:

(i) The variance of economic series may exhibit a time-varying behavior due to dynamic heteroskedasticity captured by ARCH-type estimators (e.g. Engle, 1983). This specification of uncertainty also allows us to assess the persistence of shocks to the volatility due to their autoregressive type structure as well as their feedback in the conditional mean due to the joint estimation. Moreover, in this context the notion of uncertainty of shocks relates to the innovation volatility once the conditional mean between variables has been modelled. This measure of uncertainty relates to the variance of innovations and is closer to the notion of uncertainty relating to the unpredictable innovation of a variable (e.g. Cuikerman and Meltzer, 1986) as well as the one used in studies that specify volatility based on the time series cyclical dimension of the data (similar to Ramey and Ramey, 1995) rather than on the cross-section deviations (e.g. Martin and Rogers, 2000).

(ii) The temporal aggregation of economic variables influences the information and causality relationships. In the context of linear multivariate time series models temporal aggregation implies loss of information regarding Granger-causality, impulse response analysis, exogeneity, measures of persistence, forecasting (e.g. Marcellino, 1999, Sims, 1971, interalia). With reference to growth empirics, Levine and Zevros (1993) point out that temporal aggregation of variables over long horizons involved in a cross-section approach makes it very difficult to interpret estimated coefficients based on data that are averaged over decades during which business cycles, policy changes and economic instabilities have influenced economic growth. This argument becomes even more relevant when studying the variations of money shocks. In addition, ten-year averages of money growth smooth out the fluctuations in series and therefore under-estimate the variance of variables. More precisely the temporal aggregation results in Drost and Nijman (1993) and Meddahi and Renault (2002) show that if a (flow or stock) variable, say at monthly frequency, has a dynamic volatility structure e.g. follows a GARCH process, then temporal aggregation over long horizons will in the limit imply that all the GARCH effects will eventually disappear and the process will instead have a constant variance. Hence aggregation of such processes could mean losing information at higher sampling frequencies such as monthly that are also of interest e.g. to policy makers such as Central Banks. Moreover, temporal aggregation of multivariate time series processes may imply different lags and correlations signs at different sampling frequencies. For instance, Saint Paul (1993) presents related evidence whereby low frequency movements in output greater than 16 years are positively related to average growth, whereas high frequency ones namely between 2-4 years have a negative effect on growth.

(iii) Our time-series model provides a framework to examine the causality-in-mean and in-variance hypotheses (Granger, 1988) as opposed to the traditional cross-country regressions that mainly focus on *contemporaneous* correlations between growth and various explanatory variables and do not provide information regarding the direction of causality. This is directly related to our theoretical propositions that provide testable hypotheses regarding the direction of causality of the uncertainty of real and nominal shocks on growth.

(iv) The M-GARCH-M model allows us to disentangle the empirical effects of the average and the variance of money growth on output growth by jointly estimating a system of dynamic conditional moments. In addition, we consider the innovation uncertainty of other shocks jointly with that of output and conditional on other explanatory variables. In this way we address an important issue that is left unsolved by many existing studies, as mentioned above. The analysis also shows that the causality of nominal money shock uncertainty on output growth is an additional channel of uncertainty. Omitting the money variance from the study of the overall effects of growth uncertainty on growth will lead to a bias in the estimated coefficients. This may help explain some of the mixed empirical evidence of output uncertainty on growth mentioned above.

Summarizing, the econometric specification examines the empirical evidence for the following questions: Does nominal shock uncertainty adversely affect output growth? Similarly, does growth uncertainty affect the growth rate? Does average money growth affect the average output growth? Which of these two sources of uncertainty exerts a relatively more significant effect on growth in the G7? Empirical answers to these questions are presented for each country separately as well as jointly for the G7 group using individual and multiple significance hypothesis tests.

The structure of the paper is as follows: The next section describes the theoretical model. Section 3 presents the empirical GARCH-M model and Section 4 explains the testable hypotheses derived from the theoretical model. Section 5 details the empirical results for the G7 countries. Section 6 concludes the paper.

2. The Theoretical Analysis

In this section we present a stochastic monetary model, in which long-run growth is sustained by learning-by-doing. The model we use augments the one in Blackburn and Pelloni (2001) by introducing an intermediate goods sector characterized by imperfect competition between firms. The case of perfect competition obtains when the elasticity of substitution between these goods goes to infinity. This extension seemed an advisable theoretical refinement in the light of the difficulty of reconciling perfect competition and learning-by-doing analysed by Dasgupta and Stiglitz (1988). In Blackburn and Pelloni (2001) fiscal shocks are also considered but here we focus on monetary and preference shocks only as the variability of fiscal shocks is found to be too low for the purposes of our empirical analysis. The model predicts that growth will be positively affected by the volatility in real (preference) shocks and negatively affected by the volatility in nominal shocks.

2.1. Firms

The description of the productive sector is drawn from work on monopolistic competition by Dixit and Stiglitz (1977), developed in macroeconomic models by Blanchard and Kiyotaki (1987) and by Benhabib and Farmer (1994)) among others. There is a continuum of intermediate goods Y(i) where $i \in$ (0,1). Final output, which can be consumed or invested, is given by

$$Y_t = \left(\int_0^1 Y_{it}{}^{\sigma} di \right)^{1/\sigma} \tag{1}$$

where $\sigma \in (0,1)$. Note that (1) displays constant returns to scale. The final good sector is competitive.

First order conditions for profit maximization imply demand functions for intermediate goods given by:

$$P_{it} = P_t \left(\frac{Y_{it}}{Y_t} \right)^{\sigma - 1} \tag{2}$$

where P_t is the price of the final good , which can be consumed or invested and has a depreciation rate of 100%,[2] and P_{it} is the price of the ith intermediate good.

The technology for producing an intermediate commodity is Cobb-Douglas:

$$Y_{it} = \overline{K_t}^{1-\psi} N_{it}{}^{\alpha} K_{it}{}^{\psi}, \quad \alpha, \psi \in (0,1). \tag{3}$$

where N_{it} is labor, K_{it} capital and $\overline{K_t}$ is the economy-wide average capital. The assumption of imperfect competition allows us to consider the case of increasing returns at the firm level, which obtain when $\alpha + \psi \geq 1$. Romer (1986), working in a perfect competition framework, has revived the Arrovian hypothesis of 'learning-by-doing through investing' as a rationale for increasing returns to capital. As alluded to before Dasgupta and Stiglitz (1988) notice however that learning-by-doing is consistent with perfect competition only on the implausible condition that not even a fraction of the accumulation of knowledge can be appropriated at the firm level. Assuming imperfect competition is therefore preferable.

Labor and capital are hired from households at the real wage rate W_t / P_{it} and real rental rate R_t, respectively, where W_t is the nominal wage. Profit maximisation implies:

$$\frac{W_t}{P_{it}} = \alpha\sigma \overline{K}_t^{\psi} N_{it}^{\alpha-1} K_{it}^{\psi} = \frac{\alpha\sigma Y_{it}}{N_{it}}, \tag{4}$$

$$R_t = \phi \overline{K}_t^{\psi} N_{it}^{\alpha} K_{it}^{\psi-1} = \frac{\psi\sigma Y_{it}}{K_{it}}. \tag{5}$$

[2] As usual, this hypothesis is needed for obtaining a closed form solution.

To keep things simple we assume every intermediate commodity is produced with the same technology and we focus on a symmetric equilibrium. This means that $Y_{it} = Y_t, K_{it} = \overline{K}_t, N_{it} = N_t, P_{it} = P_t, \forall i$ so that (4) and (5) become:

$$\frac{W_t}{P_t} = \alpha\sigma N_t^{\alpha-1} K_t = \frac{\alpha\sigma Y_t}{N_t}, \qquad (6)$$

$$R_t = \psi\sigma N_t^{\alpha} = \frac{\psi\sigma Y_t}{K_t}. \qquad (7)$$

2.2. Households

We assume a constant population normalised to one of identical, immortal households. At time t, the representative household wants to maximize

$$E_t U = \sum_{s=0}^{\infty} \beta^{t+s} E_t \left[\gamma_{t+s} \log(C_{t+s}) + \theta \log\left(\frac{M_{t+s-1}\phi_{t+s}}{P_{t+s}} \right) - \lambda L_{t+s} \right], \quad \beta \in (0,1), \theta, \lambda > 0 \quad (8)$$

where E_t denotes expectations, C_t consumption, and L_t labor, $L_t \in [0,1]$ and γ_t represents a preference shock, at time t. The quantity M_{t-1} denotes beginning-of-period t (i.e., end-of-period) nominal cash balances which are increased by a proportional stochastic monetary transfer, ϕ_t.[3] Money supply, M_t, is then given by: $M_t = M_{t-1}\phi_t$. We assume that both disturbances $\{\gamma_t, \phi_t\}$ are governed by independent, stationary stochastic processes with constant means and constant variances. Moreover the shocks are assumed to have bounded positive supports. The bounds on employment are then always respected (i.e. we do not have corner solutions). The unconditional expected values and variances of the disturbances are denoted, respectively, by $\{\mu_\gamma, \mu_\phi\}$ and $\{\sigma_\gamma^2, \sigma_\phi^2\}$.

The budget constraint at time t for the household is given by

$$C_t + \frac{M_t}{P_t} + A_{t+1} = \frac{W_t}{P_t} L_t + \frac{M_{t-1}\phi_t}{P_t} + R_t A_t + \Pi_t, \qquad (9)$$

where A_t is real assets and Π_t the firms' profits.

Each agent maximises the expected value of utility in (8) subject to the sequence of budget constraints in (9). Agents are assumed to know the values of all parameters, the current and past values of all variables and the probability distributions of all shocks. Households choose consumption, money balances and asset holdings according to the following necessary conditions:

$$\frac{\gamma_t}{C_t} = \beta E_t \left(\frac{\gamma_{t+1} R_{t+1}}{C_{t+1}} \right), \qquad (10)$$

[3] The assumption that monetary transfers are proportional (rather than lump-sum) made for tractability is not new (e.g. Benassy 1995).

$$\frac{\gamma_t}{P_t C_t} = \beta \frac{E_t(\theta_{t+1})}{M_t} + \beta E_t\left(\frac{\gamma_{t+1}\phi_{t+1}}{P_{t+1}C_{t+1}}\right), \qquad (11)$$

We consider two different assumptions for the labor market: perfect competition between workers with wage flexibility and wage setting by unions. Under the first assumption a further optimising condition is:

$$\lambda P_t C_t = \gamma_t W_t \qquad (12)$$

Under the second assumption monopolistic unions choose a nominal wage at which households supply whatever labor is demanded by firms. We assume that wage setting takes place prior to the realisation of shocks on the basis of one-period contracts, as in the early contracting models of Gray (1976) and Fischer (1977). As in more recent models (e.g. Gali 1999) we suppose however that the contract wage is chosen so as to maximise households' expected utility, rather than to satisfy some *ad hoc* criterion. In other words workers at time *t*-1 choose the wage W_t so as to maximize (8) given the sequence of budget constraints in (9) and labor demand, as expressed in (6). The optimal wage is then found to satisfy

$$\lambda E_{t-1}(N_t) = \alpha W_t E_{t-1}\left(\frac{\gamma_t N_t}{P_t C_t}\right). \qquad (13)$$

The equilibrium behavior of the household is characterised completely by the first-order conditions in (10), either (11) or (13), the budget constraint in (9) and the transversality conditions,

$$\lim{}_{\tau\to\infty} \beta^\tau E_t((\gamma_{t+\tau}M_{t+\tau-1}\phi_{t+\tau})/P_{t+\tau}C_{t+\tau}) = \lim{}_{\tau\to\infty} \beta^\tau E_t(\gamma_{t+\tau}A_{t+\tau+1}/C_{t+\tau}) = 0 .$$

2.4 General Equilibrium

The general equilibrium solution is computed by combining the optimising conditions obtained so far with the market clearing conditions $C_t + K_{t+1} = Y_t$ (for goods), $K_t = A_t$ (for capital), and $N_t = L_t$ (for labor) plus the already assumed one that money supply equals money demand.

If we substitute the expression for the interest rate in terms of income and capital from (5) in (10) and recall that $C_t + K_{t+1} = Y_t$ we are able to write (10) as

$$\frac{\gamma_t K_{t+1}}{C_t} = \sigma\beta\psi\mu_\gamma + \sigma\beta\psi E_t\left(\frac{\gamma_{t+1}K_{t+2}}{C_{t+1}}\right), \qquad (14)$$

this defines a stochastic expectations difference equation. Considering the transversality condition $\lim{}_{\tau\to\infty} \beta^\tau E_t(\gamma_{t+\tau}A_{t+\tau+1}/C_{t+\tau}) = 0$ its solution is given by:

$$K_{t+1} = \frac{a\mu_\gamma}{(1-a)\gamma_t + a\mu_\gamma} Y_t, \qquad (15)$$

where $a \equiv \sigma\beta\psi$. Given $C_t + K_{t+1} = Y_t$ (15) implies

$$C_t = \frac{(1-a)\gamma_t}{(1-a)\gamma_t + a\mu_\gamma} Y_t, \tag{16}$$

Given $H_t = H_{t-1}\phi_t$ and $M_t = H_t$ (11) becomes

$$\frac{\gamma_t M_t}{P_t C_t} = \beta\theta + \beta E_t\left(\frac{\gamma_{t+1} M_{t+1}}{P_{t+1} C_{t+1}}\right). \tag{17}$$

solving (17) using the other transversality condition

$$\lim{}_{\tau\to\infty} \beta^\tau E_t((\gamma_{t+\tau} M_{t+\tau-1}\phi_{t+\tau})/P_{t+\tau}C_{t+\tau}) = 0\,.$$

and substituting in for consumption its expression in terms of income given by (16) we have:

$$\frac{M_t}{P_t} = \frac{(1-a)\beta\theta}{(1-\beta)[(1-a)\gamma_t + a\mu_\gamma]} Y_t, \tag{18}$$

According to (15), (16) and (18), for a given level of output, consumption increases, investment decreases and money demand decreases with higher realisations of the preference shock, γ_t. These responses are non-linear since the average output shares of consumption, capital and cash balances are influenced by the variances of the shocks: an increase in the volatility of preference shocks causes a fall in the average share of consumption, but a rise in the average shares of investment and money demand.

If the labor market is competitive we have:

$$N_t = \frac{\sigma\alpha[(1-a)\gamma_t + a\mu_\gamma]}{\lambda(1-a)}. \tag{19}$$

This is obtained by substituting in (12) for consumption its expression in terms of income given by (16) and then using the second equality in (6).

With one-period wage contracts, substituting in (18) the expression for income in terms of labor and the real wage given by (6) we get:

$$N_t = \frac{\alpha\sigma(1-\beta)\left((1-a)\gamma_t + \mu_\gamma\right)}{(1-a)\beta W_t} M_t \tag{20}$$

while the optimum wage is found to be:

$$W_t = \frac{\lambda(1-\beta)E_{t-1}(M_t)}{\alpha\beta\theta} \qquad (21)^4$$

Substituting this expression for the wage in (20) the equilibrium level of employment with contracts is found to be

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[4] If we combine (6), (13) and (16) we find: $E_{t-1}N_t = \dfrac{\alpha^2\mu_\gamma\sigma}{\lambda(1-a)}$.

By (20) taking expectations we get $E_{t-1}N_t = \dfrac{\alpha\sigma(1-\beta)\mu_\gamma}{(1-a)\beta W_t} E_{t-1}M_t$. Equating the two expressions for $E_{t-1}N_t$ we get the optimal wage.

$$N_t = \frac{\alpha^2\sigma[(1-a)\gamma_t + a\mu_\gamma]\phi_t}{\lambda(1-a)\mu_\phi}. \quad (22)$$

Notice the positive relationship between both shocks and employment, consumption, capital and output.

2.5 Growth and Cycles

To study the linkages between the cyclical and secular properties of aggregate fluctuations, we solve for the growth rate of output, from which the growth rates of other non-stationary variables (consumption and capital) may be inferred.

If the labor market is competitive, using (3), (15) and (19) we get:

$$\Delta Y_t := \frac{Y_{t+1}}{Y_t} = \frac{a\mu_\gamma}{(1-a)\gamma_t + a\mu_\gamma}\left[\frac{\sigma\alpha[(1-a)\gamma_{t+1} + a\mu_\gamma]}{\lambda(1-a)}\right]^\alpha. \quad (23)$$

Notice the rate of growth is concave in the current realization of the shock: this is due to decreasing marginal productivity of labor: labor increases linearly but output does not. The rate of growth is however convex in the lagged realization of the shock. This is due to saving behavior: from (15) we see that the propensity to save is a convex function of the preference shock. This is transmitted linearly to production, given the constant marginal productivity of capital.

Using standard approximation theorems we have:

$$E(\Delta Y) := E\left(\frac{Y_{t+1}}{Y_t}\right) \cong A\left(1 + \frac{(1-a)^2[\alpha(\alpha-1)+2]}{2\mu_\gamma^{\ 2}}\sigma_\gamma^2\right) \quad (24)$$

$$\text{var}(\Delta Y) := Var\left(\frac{Y_{t+1}}{Y_t}\right) \cong \frac{A^2(1-a)^2\left(\alpha^2+1\right)}{\mu_\gamma^{\ 2}}\sigma_\gamma^2 \quad (25)$$

where $A = a\left(\frac{\sigma\alpha\mu_\gamma}{\lambda(1-a)}\right)^\alpha$.

We then have that both the mean and the variance of the rate of growth of output are increasing in the variance of the preference shock. The first effect means that the positive effect of this variance on the rate of growth through the precautionary saving channel more than offsets the negative effect though the employment channel.

Let us now consider the economy with contracts. We have, using (3), (15) and (22)

$$\Delta Y_t := \frac{Y_{t+1}}{Y_t} = \frac{a\mu_\gamma}{(1-a)\gamma_t + a\mu_\gamma}\left[\frac{\sigma\alpha^2[(1-a)\gamma_{t+1} + a\mu_\gamma]\phi_{t+1}}{\lambda(1-a)\mu_\phi}\right]^\alpha. \quad (26)$$

The growth rate of output, ΔY_t, is now dependent on the realisations of both real and nominal shocks. The mean and variance of this growth rate are approximated, respectively, by

$$E(\mathit{\Delta} Y) := E\left(\frac{Y_{t+1}}{Y_t}\right) \cong \alpha^{\alpha} A\left(1+\frac{(1-a)^2[\alpha(\alpha-1)+2]}{2\mu_{\gamma}^{2}}\sigma_{\gamma}^{2}+\frac{\alpha(\alpha-1)}{2\mu_{\phi}^{2}}\sigma_{\phi}^{2}\right) \tag{27}$$

$$\mathrm{var}(\mathit{\Delta} Y) := Var\left(\frac{Y_{t+1}}{Y_t}\right) \cong \left(\alpha^{\alpha} A\right)^2\left(\frac{A^2(1-a)^2\left(\alpha^2+1\right)}{\mu_{\gamma}^{2}}\sigma_{\gamma}^{2}+\frac{\alpha^2}{\mu_{\phi}^{2}}\sigma_{\phi}^{2}\right) \tag{28}$$

First notice that with zero variance in money growth we obtain no effects of average money growth on output growth. Money superneutrality under certainty is in fact expected when, as in our model, the utility function is additively separable in consumption, money and labor (see Wang and Yip, 1992). For a given variance of money growth, an increase in average money growth leads to higher output growth because it means an improvement in the quality of information available to the agents when they choose the nominal wage and consequently a reduction in the distortion arising from the fact the wage is not perfectly flexible. In general average growth falls while its cyclical volatility rises with an increase in the variance of the monetary growth shock. This type of disturbance impacts on growth through its (linear) effect on employment, of which output is a concave function by virtue of diminishing returns to labor.[5] Under such circumstances, one is confronted with a negative, not positive, correlation between long-term growth and short-term volatility so that smoother cyclical fluctuations are associated with steeper, not flatter, secular trends. The fact that the average and the variance of money growth have opposite effects on output growth, together with the fact that in reality the two tend to be highly correlated, may provide a partial rationale for some of the inconclusive results in empirical literature of growth and inflation. For the purposes of the empirical analysis, as will be clear below, it is convenient to substitute in equation (24) (or in eq. (27)) the expression for σ_{γ}^{2} in terms of $\mathrm{var}(\Delta Y)$ obtained from equation (25) (or eq. (28)). We obtain (24′) and (27′), the first pertaining to an economy with a perfectly competitive labor market the second pertaining to an economy with contracts:

$$E(\mathit{\Delta} Y) \cong A\left(1+\frac{[\alpha(\alpha-1)+2]}{2A^2(\alpha^2+1)}\mathrm{var}\left(\Delta Y\right)\right) \tag{24′}$$

$$E(\mathit{\Delta} Y) \cong \alpha^{\alpha} A+\frac{[\alpha(\alpha-1)+2]}{2\alpha^{\alpha} A^3(\alpha^2+1)}\mathrm{var}\left(\Delta Y\right)+\left[-\frac{\alpha^{\alpha+2}[\alpha(\alpha-1)+2]}{A(\alpha^2+1)}+\alpha^{\alpha+1}A(\alpha-1)\right]\frac{\sigma_{\phi}^{2}}{2\mu_{\phi}^{2}} \tag{27′}$$

Summing up, the model predicts that real shocks uncertainty will have a positive effect on growth whereas the monetary shock uncertainty will have a negative effect (or no effect) depending on the structure of the labor market. Moreover, the average money growth will have a positive effect on average output growth. These theoretical propositions constitute empirically testable hypotheses as demonstrated in the context of the empirical models presented below. It is important to note that equations (24′) and

(27′) summarize the relationships between the static average money and growth and their uncertainties. Equations (23) and (26) shows the dynamic output growth process. From equation (26) we find the direction and sign of causality between nominal and real uncertainties and growth when nominal wages are fixed.

3. The Empirical Analysis

In this section we present the details of the econometric model, its connection with the theoretical model and some of its advantages. The empirical results follow in section 4. The assumptions of the theoretical model in section 2 fit the group of developed and industrialized countries. Hence the empirical analysis focuses on the G7 that also represents a group of relatively homogenous economies. For the purposes of the empirical specification we consider the theoretical relations in (24′) and (27′) that explicitly substitute the variances of shocks into the expected output growth parameterization. These equations refer to perfectly and imperfectly competitive labor markets, respectively, and show that the uncertainty of real shocks has a positive effect on growth whereas the uncertainty of nominal shocks has no effect (in eq. (24′)) or negative effect (in eq. (27′)) on growth.

The multivariate Generalised AutoRegressive Conditional Heteroscedastic in Mean (GARCH-M) model presents the framework for evaluating the empirical evidence of the above theoretical propositions. The set of hypotheses tested are detailed in the following section. The relationship between money and output and their uncertainties is modelled by a bivariate GARCH-M(1,1) with constant conditional correlation in the spirit of Bollerslev (1990) given by:

$$\Delta M_t = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta M_{t-i} + \sum_{i=1}^{q} \beta_{2i} \Delta Y_{t-i} + \beta_3 \sigma^2_{\Delta M,t} + \beta_4 \sigma^2_{\Delta Y,t} + \varepsilon_t \tag{29}$$

$$\sigma^2_{\Delta M,t} = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 \sigma^2_{\Delta M,t-1} \tag{30}$$

$$\Delta Y_t = \beta_5 + \sum_{i=1}^{p} \beta_{6i} \Delta Y_{t-i} + \sum_{i=1}^{q} \beta_{7i} \Delta M_{t-i} + \beta_8 \sigma^2_{\Delta M,t} + \beta_9 \sigma^2_{\Delta Y,t} + v_t \tag{31}$$

$$\sigma^2_{\Delta Y,t} = \alpha_3 + \alpha_4 v^2_{t-1} + \alpha_5 \sigma^2_{\Delta Y,t-1} \tag{32}$$

$$COV_t = \rho_{\varepsilon v} \sigma_{\Delta M,t} \sigma_{\Delta Y,t} \tag{33}$$

Equation (29) describes the conditional mean of nominal money growth, ΔM_t, as a function of the past history of both money and real output growth, ΔY_t, and their conditional variances given by $\sigma^2_{\Delta M,t}$ and $\sigma^2_{\Delta Y,t}$, respectively, which are estimated by equations (30) and (32). Equation (30) represents the conditional variance of nominal money growth as the parametric measure of money uncertainty that

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[5] In Dotsey and Sarte (2000) a rise in the variability of money growth increases the average level of savings since less real money balances are held on average as their returns become more uncertain.

affects the conditional mean equations of money (29) as well as output (31). Note that it represents the conditional innovation volatility estimate once the conditional mean dynamics of all variables have been accounted for. This is captured by the Vector Autoregressive (VAR) model of money and output which is equivalent to equations (29) and (31) if we exclude the uncertainty variables, $\sigma^2_{\Delta M,t}$ and $\sigma^2_{\Delta Y,t}$. Most importantly it captures the time-varying behavior of uncertainty as shown by the autoregressive structure of $\sigma^2_{\Delta M,t}$ in equation (30). Moreover, it provides a framework to establish the statistical significance of volatility in the model and via its estimated coefficients to assess the persistence of the uncertainty of nominal shocks. Equation (31) describes the conditional mean of real output growth as a function of lags of output and money growth and their conditional variances and is the empirical counterpart of equations (24′) and (27′). Equation (32) is the conditional innovation variance of output growth (similar to equation (30)) and finally equation (33) specifies the constant conditional covariance between ε_t and ν_t. It is assumed that the two error terms, ε_t and ν_t, are jointly conditionally normal with zero means and conditional variance given by equations (30) and (32).[6] The above system of equations allows for the feedback relationship between the two variables and models jointly both the conditional mean and variance (or linear and nonlinear) dynamics which are estimated simultaneously using Maximum Likelihood methods. In the context of equation (31) we examine the empirical support of the theoretical propositions regarding the effects of nominal money and output growth uncertainty, $\sigma^2_{\Delta M,t}$ and $\sigma^2_{\Delta Y,t}$ respectively, on growth ΔY_t. In order to explain the difference in the notation between the empirical and theoretical models we note that in former specification the time series processes of money and output growth, denoted by ΔM_t and ΔY_t respectively, are governed by dynamics in the conditional mean and variances, $E_{t-1}(\Delta M_t)$, $E_{t-1}(\Delta Y_t)$ and $\sigma^2_{\Delta M,t}$, $\sigma^2_{\Delta Y,t}$, respectively, and parameterized using the bivariate GARCH-M model above. Note that the respective theoretical processes for money and output (μ_ϕ and $E(\Delta Y)$, respectively) as well as their uncertainties which are denoted by σ^2_ϕ and $\mathrm{var}(\Delta Y)$ in equations (27) and (28), are defined as independent, time-invariant processes (for purposes of analytical tractability). It is acknowledged that the fluctuations in output growth are associated with relatively short run business cycle movements. This is one of the aspects of time-series model that complements cross-section and panel studies on growth and uncertainty. Annual changes of growth in relation to output uncertainty are also examined in Saint-Paul (1993). It is acknowledged that other nonlinear time-series models have also been proposed to study growth. In Markov Switching models (Hamilton, 1989) other sources of non-constant variance are related to the heterogeneity of volatility across states of the economy. Similarly, threshold type processes have been proposed to capture non-linear mean

[6] Inflation is endogenous in our model. Hence we focus on money and output growth that closely match the theoretical model predictions.

relationships (e.g. Potter, 1995, Terasvirta and Anderson, 1992). In relation to cross-country regressions Durlauf and Johnson (1995) and Hansen (2000) consider Markov Switching and Threshold models, respectively.

The GARCH-M model is also adopted in Edler (2003) and Grier and Perry (2000) to study the relationship between US growth, inflation and their volatilities, as well as by Peel and Speight (1998) who estimate univariate quadratic-ARCH (and other non-linear) models for disaggregated industrial production series for US, UK, Germany and Japan.[7] The parametric measure of volatility implied by the GARCH specifications captures a measure consistent with the theoretical notion of uncertainty as the variance of the unpredictable innovation of a variable (e.g. Cuikerman and Meltzer, 1986), instead of simply calculating the unconditional standard deviations of money and output growth.[8] The GARCH specification also estimates the time varying behavior of volatilities which is consistent with the time-series dynamics of the data, modelled by Kim (1993) in a regime-switching context. Moreover, the simultaneous parametric specification and estimation of GARCH-M models allows not only the examination of the statistical significance of volatility itself but also its effect on the conditional mean of the variables of interest. This approach is more efficient for estimation and testing than any two-step OLS approach that involves estimated regressors (Pagan, 1984) especially in the presence of dynamics in the conditional variance. Moreover, equations (29) and (31) allow for a linear autoregressive structure similar to the VAR equations augmented by the variance-in-mean captured by $\sigma^2_{\Delta Y,t}$ and $\sigma^2_{\Delta M,t}$. In this context the Granger causality relationship between money and output growth rates is enlarged to allow for the dimension of the volatilities of shocks pertaining to those variables.

The stationarity and dependence properties of the volatility equations (30) and (32) offer an alternative way of interpreting the effects of shocks in the uncertainty of output and nominal money growth rates. The model allows us to examine three different useful aspects of volatilities: (i) If growth uncertainty follows a GARCH process then we can evaluate if the variance of growth or other economic variables have a significant temporal component. (ii) If the GARCH output dynamic coefficients, e.g. $(\alpha_1 + \alpha_2)$ in equation (30), are statistically significant and close to unity then we have an Integrated GARCH process (IGARCH) and shocks in output uncertainty are expected to have a significant and permanent effect on the variance of money growth. (iii) If in addition the relationship between the mean and volatility is captured by a GARCH-in-Mean process then we can evaluate the significance of

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[7] The variables are assumed to be stationary a hypothesis that is empirically examined prior the estimation of the model using unit root tests which leads us to consider the first differences of the above series.

[8] Note that although we have a measure of conditional innovation uncertainty we do not consider the Levine-Renelt conditioning information set as Ramey and Ramey (1995) since we follow a time-series approach and some of those variables are either not available at the monthly frequency or do not exhibit any temporal variation for studying in a time series context. Although additional explanatory variables can augment our conditional mean equations at this stage we choose to focus an empirical model as close as possible to the theoretical model by considering a bivariate model of five simultaneous equations for each country and joint hypotheses tests for all the G7 countries.

uncertainty on the average of output growth. This is due to the fact that the variance enters the conditional mean growth equation and its partial correlation with output can be subsequently examined in the presence of other uncertainty factors as well as average factors. Hence this model provides a context to disentangle the mean and variance effects of say nominal money on output growth by modelling all the conditional moments and estimating their interactions simultaneously.

4. Empirical tests of the model's predictions

This section presents the testable hypotheses relating to the theoretical predictions of the model analysed in section 2 regarding the effects and sources of uncertainty on growth for the G7 countries using the GARCH-M model discussed in section 3. Money growth, ΔM_t, is measured by the rate of growth of the narrow nominal money supply and output growth, ΔY_t, by the index of production (IOP) growth rates.[9] The model is estimated using monthly, seasonally adjusted data for the G7 countries over the maximum sample 1960 to 2000.[10] The choice of monthly sampling frequency reflects the objective to estimate conditional variances from short-run cyclical dynamics and to avoid loss of information due to temporal aggregation (as discussed in the introduction). The monthly difference of production with lags of up to a year is an attempt to capture both short- and relatively long-run growth effects. The G7 represents the group of homogenous countries that more closely correspond to the theoretical assumptions of the model. The details of the data sources, definitions, descriptions and samples are summarized in the Appendix, Table A1.

We estimate the empirical model in equations (29)-(33) for each country and our objective is to examine the support of the following theoretical model propositions using hypothesis tests for both the individual, country-specific and multiple, G7-group cases.

- Hypothesis (i) examines whether nominal money uncertainty is time varying as modeled by the GARCH equation (30) (where H_0: $\alpha_1 = \alpha_2 = 0$). If the sum of GARCH coefficients is close to unity then a shock in money uncertainty will have a persistent effect which means that they will not die out exponentially.
- Hypothesis (ii) tests whether the growth uncertainty specified in equation (32) provides a time varying measure of the growth variability (H_0: $\alpha_4 = \alpha_5 = 0$).
- Hypotheses (iii) (H_0: $\beta_3 = 0$) and (iv) (H_0: $\beta_4 = 0$) examine the significance of money and output uncertainty, respectively, in the money equation (29).

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[9] We also investigated some interest rate series as an alternative to money supply, but we could not always get consistent short-term interest rates across the G7 and for a long enough sample. Similarly, the choice of narrow instead of broad money measures is due to the fact that the former measure is relatively more comparable across countries. The IOP is used as a measure of monthly output due to the unavailability of the GDP at monthly frequency.

- Hypotheses (v) (H$_0$: β_8=0) and (vi) (H$_0$: β_9=0) examine the effects of money and growth uncertainty, respectively, in the output equation (31). The alternative hypotheses, H$_1$: β_8<0 and H$_1$: $\beta_9 > 0$, derive their signs from the theoretical predictions (see equations 24′ and 27′).
- Hypothesis (vii) (H$_0$: β_{7i}=0, i=1,….,q**)** examines the effects of money growth on output growth in equation (31). The alternative hypothesis derived from the theoretical model suggests that the overall effect will be positive, after controlling for its variance effects and if there are wage contracts (see equation 27′).

With respect to hypothesis (v) we emphasize that the effect of growth uncertainty on average growth might also be mixed in terms of sign and explanatory power depending on the significance and direction of real and nominal shocks particularly when the money variance is an omitted variable in the empirical regression of growth. This result may also be useful in explaining some of the existing conflicting empirical results on growth and uncertainty. Indeed there are numerous empirical papers, mentioned in the introduction, that find different signs for β_9 and only a few empirical results that study the effect of money uncertainty on growth. Moreover, the model allows us to disentangle the mean and variance effects of nominal money shock on growth by estimating all the moments jointly and testing their partial effects on average growth.

5. Empirical Results

In this section we discuss the empirical support of the above hypotheses using the bivariate GARCH-M models for the G7 countries. Table 1 presents the summary results for each country with the corresponding hypothesis tested and estimated GARCH-M coefficients and t-statistics. The detailed estimation and misspecification results for equations (29)-(33) for each country can be found in the Appendix (Tables A3-A9).[11] The estimation utilises the Berndt et al. (1974) numerical optimisation algorithm to calculate the maximum likelihood estimates of the parameters in (29)-(33) with the RATS program of Trevor (1994). The general-to-specific procedure is adopted for specifying the significant lags.[12]

First we investigate the hypotheses regarding the significance of the conditional volatility estimates for money and production growth since they represent the building blocks of the theoretical and

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[10] This sample period refers to Canada, Japan and the US. For European countries, France, Italy and Germany, the sample ends in 1998 that marks the era before the EMU. For the UK the sample period commences in 1972 due to M0 data unavailability.

[11] In the Appendix we present the detailed empirical results for each country including the linear and non-linear model estimation results and the Ljung-Box and McLeod-Li tests of residual dependence. We show that moving to the non-linear specification takes account of linear and non-linear dynamics that are apparent in these series. A discussion of these issues is also deferred to the Appendix.

[12] Various outliers exist for different countries (these are identified as greater than $\pm$ 2.5 standard deviations of the data) and for ease of estimation with GARCH models as well as avoiding spurious ARCH effects, these events are interpolated from each series. For the exact changes to the data refer to the Appendix, Table A2.

empirical paradigms. The results of hypotheses (i) and (ii) in Table 1 present strong evidence as to the significance of the parameters governing the estimated conditional variances in all countries and in both series. The estimations unfold another interesting aspect beyond the linear dynamic relationship between these macroeconomic variables, namely the dynamics in their conditional variances. Evidence of ARCH-type heteroscedasticity for the US money growth rate is also provided by Kim (1993) and Serletis (1995) and for the output growth by Edler (2003), Grier and Perry (2000) and by Peel and Speight (1998) for the US and UK production. It is worth noting that Kim (1993) considers a Markov-switching variance model and a time-varying parameter model for US monetary growth uncertainty whereas Serletis (1995) provides evidence of non-linear dynamics in the US money velocity measures. Our empirical findings provide further evidence regarding the effects of shocks in the nominal uncertainty as measured by the volatility persistence of money. In the GARCH equation (30) of money growth the persistence coefficient $(\alpha_1 + \alpha_2)$ is relatively high for Canada and the US which implies that shocks in nominal money uncertainty have a persistent effect in these countries compared with the remaining of the G7.[13] On the other hand, the countries characterised by significant and persistent volatility dynamics in output growth, $(\alpha_4 + \alpha_5)$, are Japan and the three European countries (E3), France, Italy and the UK. In the remaining of the G7 i.e. Canada, Germany and the US output shocks have a less persistent effect in the volatility of growth.

The effects of nominal money shock variability, $\sigma^2_{\Delta M,t}$, and growth uncertainty, $\sigma^2_{\Delta Y,t}$, on money growth are shown in equation (29) (and tested via hypotheses (iii) and (iv)) and on output growth are shown in equation (31) (and examined by hypotheses (v) and (vi)). As mentioned earlier, the latter is particularly interesting given the theoretical proposition that nominal shock uncertainty exerts a negative effect on growth. We investigate this hypothesis using two statistical procedures. First, we test each individual hypothesis for each country separately at a given level of significance, α. Second, we combine these k=1,..,7 individual hypotheses and apply a multiple test of significance based on a Bonferroni procedure. In this context we view each of the G7 as an alternative sample realization that yields individual statistics used to examine the empirical support for the global null hypothesis made up of the intersection of the individual null hypotheses such that $H_0^g=\{H_0^k, k=1,\ldots,7\}$. Appropriate methods are adopted to adjust the significance level to the multiple hypothesis test and a sequential test is performed to examine the sources of rejection, discussed below. If no empirical support is found for any of the H_0^k at

[13] Diebold (1986), Lamourex and Lastrapes (1990) present empirical evidence that volatility persistence may be the spurious effect to structural breaks or outliers in the sample. However, in the present analysis the estimated persistence effects are not due to outliers since these have been removed from the data before the estimation as shown in Table A2.

the adjusted significance level then we conclude that there is no empirical support for the global null hypothesis for the G7 group.[14]

Following the individual hypothesis test approach we find (at the 5% significance level) that nominal shock uncertainty has a negative but insignificant effect on monetary growth except in the UK (shown in Table 1, hypothesis (iii)). Similarly output uncertainty has an insignificant effect on money growth (shown by hypothesis (iv)). Therefore the G7 empirical results suggest that real shock uncertainty has an insignificant effect on money growth and so does nominal shock uncertainty except in the UK narrow money growth. We now turn to examine the empirical support of the theoretical prediction that nominal uncertainty has a negative effect on output growth. Table 1 (hypothesis (v)) reports evidence against the null hypothesis (β_8=0) for two of the G7 countries, namely Canada and Germany and weak evidence for the UK. The estimated coefficients for the first two countries show that there is a negative average elasticity of 0.2 between nominal money growth uncertainty and output growth. Moreover the overall sign of nominal money uncertainty (β_8) is negative in four of the G7 countries. The exceptions to this result are France, Italy and Japan where the estimated money uncertainty variable is not only positive and insignificant but also has a low coefficient in the first two countries. One possible explanation is that these two European economies had a very high degree of wage indexation until the second half of the 1980's (see for instance Bruno and Sachs, 1985 and OECD, 1984). In fact, in Italy wage indexation was present until the mid 1990s (Manacorda, 2002). Gray (1976) first noticed that wage indexation reduces the output effects of money shocks. In fact an economy with indexed wages reacts to nominal disturbances in a similar way to one in which nominal wages are flexible. This evidence is consistent with the theoretical results in section 2 according to which in the presence of nominal wage flexibility we expect no effects of nominal volatility on growth. Finally we examine hypothesis (vi) that addresses the effects of output uncertainty on growth. Following the single hypothesis test approach, output uncertainty turns out to be positive for four of the G7 but it is insignificant for all except Canada. The last hypothesis refers to the effect of money growth on output growth. The joint F-test for zero restrictions on the lagged coefficients of ΔM_{t-i}, β_{7i}, present strong evidence against the null hypothesis for all countries except the US and UK. In the remaining of the G7 there is a positive effect of nominal money growth on output and according to the empirical model an increase in money growth induces on average a 27% increase on output growth, once we control for monetary uncertainty.

In the Bonferroni procedure the multiple test of the global hypothesis H_0^g has an asymptotic bound to the significance levels of α^k=0.007 and 0.001 (given α=5% and 1%, respectively).[15] Hochberg

[14] The Bonferroni procedure is valid even if the alternative individual statistics of the hypotheses H_0^k are not strictly independent (see for instance, Gourieroux and Monfort, 1995).

[15] The critical values for a two-sided test are t*= 2.65 and 3.25 whereas for the one-sided test 2.45 and 2.98, at α^k=0.007 and 0.001, respectively.

(1988) and Rom (1990) interalia, suggest a modified Bonferroni approach following a sequentially rejective procedure according to which one starts by examining the largest p-value, p(m), of the individual hypotheses, H_0^k. If $p(m) \leq \alpha^k$ then all hypotheses are rejected. If not, then one can not reject H_0^g and goes on to compare the next largest p-value, p(m-1), with an adjusted confidence interval (e.g. Rom, 1990) based on the reduction of the sample size. If that is not rejected the above procedure is implemented in a sequential manner. Following the multiple significance test approach the empirical results show that the global null hypothesis for (iii) and (v) gain no support for the G7 group (using the 1% adjusted significance level). The multiple test results for the global hypothesis (iii) suggest that there is a significant negative effect of money shock uncertainty on money growth since the maximum t-value, $t(m)=-3.64>t^*=2.45$. Similarly the global hypothesis (v) results suggest that nominal money growth uncertainty exerts a negative effect on growth where $t(m)=-3.13>t^*$. It is interesting that we do not find evidence against the null hypotheses (iv) and (vi) using the Bonferroni procedure (and the two-sided critical values).

Summarising the empirical analysis we derive the following results. First, there is strong evidence of significant conditional heteroskedasticity effects in the time series behavior of monthly production and nominal money growth rates during the period 1960-2000. Shocks to nominal money growth uncertainty have a persistent effect in Canada and the US whereas shocks to output uncertainty are relatively more persistent in Japan and the E3 (France, Italy and the UK). The above two empirical results relating to the structure of volatility of nominal shocks and output growth are useful for impulse response and policy analysis. Second, the growth uncertainty has an insignificant effect on growth in the G7 except in Canada for which our theoretical prediction is empirically supported. Third, there is a negative and significant effect of nominal money shock uncertainty in the G7 countries when examined using the Bonferroni inequality for a multiple hypothesis test. Following the individual hypothesis test approach we find that nominal money shocks uncertainty exerts a significance influence on growth in Canada and Germany. A possible explanation of the insignificant and non-negative effects of nominal money uncertainty in the growth equation in France and Italy can be the wage indexation experience of these economies. Last but not least, the empirical analysis also presents evidence that average money growth has a positive effect on the average output growth for the majority of the G7. Further support for this result is provided by the G7 multiple significance test.

Finally, we have examined the sensitivity of the above results using other measures of money aggregates for which we find that similar results apply especially with respect to nominal money shock uncertainty. Moreover, for some of the G7 we expand the information set by including some relevant explanatory variables in the conditional mean equations such as short-run interest rates and find that the above results still hold. Last but not least, we re-estimate the model for different sub-samples as they relate to different inflation and monetary regimes and find that most of the above results also apply. It is worth noting that overall the results found for Canada in Table 1 (that provides the strongest empirical support of our theoretical model) are robust to different data transformations, lag selection methods and sub-samples analysis.[16] Similarly for the US we re-estimate the model in equations (29)-(33) for the period post the Volcker tightening regime by considering the sub-sample 1982-2000 and find that the results in Table 1 still hold.

6. Concluding Remarks.

The paper contributes to the analysis of the relationship between growth and its volatility by examining how the short-run nominal money uncertainty affects growth. The theoretical model predicts that the variability of nominal shocks has a negative effect on growth whereas the variability of real shocks yields a positive effect. Moreover, the average money growth has a positive effect on growth if there are wage contracts and after controlling for the money uncertainty effect. We examine the empirical significance of nominal money shock and output growth uncertainties by estimating simultaneously the effects of the dynamic volatilities of monthly money and output growth for the G7 countries in the conditional means of the money and output growth rate equations in the context of a bivariate GARCH-M model.

The empirical results show that the volatility of money and output growth rates exhibit statistically significant time-series effects in all the G7 using monthly data. The estimated conditional variance coefficients disclose that shocks in monetary uncertainty have a persistent effect in the Canada and the US. Similarly the estimated conditional variance of growth shows that shocks have a persistent character in the uncertainty of growth for the three European countries (E3) namely France, Italy and the UK. Such evidence is useful for impulse response and policy analysis. In addressing the effects of nominal money shock uncertainty on growth the Bonferroni multiple hypothesis test approach demonstrates a significant negative relationship in the G7. Furthermore, the empirical multiple test results show that output growth uncertainty has on average an insignificant effect on growth for most of the G7 except for Canada. This paper shows that it can be instructive to use an approach that separates nominal and growth uncertainties to understand how these relate to long-run growth.

Two possible explanations can be advanced in understanding the heterogeneity in the empirical results relating to the effect of nominal money uncertainty in the G7 as regards France and Italy. First, the theory suggests that in the presence of nominal wage indexation, growth is immune to the nominal shock uncertainty. The empirical analysis shows that this is the case for France and Italy which are countries that until recently experienced wage indexation policies. Second, the heterogeneity of the results regarding the significance of the variability of nominal shocks in the G7 countries could also be related to

[16] For instance in Canada the uncertainty of M1 growth has a significant negative effect in industrial production growth in all monthly sub-samples covering a decade since 1960s except in the 1990-2001 sub-sample. An explanation for this result is found in Siklos and Barton (2001) who argue that this is due to the changing financial structure. The Bank of Canada explains that the primary reason for lower-than-expected increases in broad monetary aggregates in recent years has been the shift of investors to equity, bond and mortgage funds.

the different types and degrees of labor market rigidities. These are possible avenues for future research. From a policy perspective, our analysis has been kept simple by assuming that monetary growth is determined by an exogenous stochastic process. The implications are that nominal monetary growth has a positive effect on output growth whereas nominal money shock uncertainty exerts a negative influence on growth. We find strong supportive evidence for the former hypothesis and weaker evidence for the latter for most of the G7 over the last four decades.

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Table 1: Summary results of the estimated Bivariate GARCH-M model for the money and output growth in the G7

Hypotheses:	Parameter Restrictions	Canada	France	Germany	Italy	Japan	UK	US
(i) Money volatility in eq. (30)	α_1=0	0.123 (3.96)	0.388 (4.56)	0.272 (3.92)	0.429 (4.35)	0.393 (3.86)	0.399 (4.24)	0.182 (3.57)
	α_2=0	0.832 (20.9)	0.127 (1.41)	0.213 (1.32)	0.339 (3.69)	0.385 (4.20)	0.220 (2.84)	0.732 (10.19)
(ii) Output volatility in eq. (32)	α_4=0	0.133 (2.43)	0.063 (2.47)	0.295 (3.53)	0.093 (3.75)	0.058 (2.09)	0.037 (2.06)	0.319 (5.01)
	α_5=0	0.434 (2.45)	0.901 (22.17)	0.124 (1.04)	0.678 (3.61)	0.895 (15.78)	0.963 (47.40)	0.211 (1.56)
(iii) Money volatility in eq. (29)	β_3=0	-0.080 (-1.11)	-0.042 (-0.57)	-0.024 (-0.20)	-0.051 (-0.52)	-0.064 (-0.75)	-0.818 (-3.64)	-0.145 (-0.55)
(iv) Output volatility in eq. (29)	β_4=0	0.193 (0.90)	0.010 (0.17)	-0.067 (-1.27)	0.016 (0.35)	-0.132 (-1.56)	0.024 (0.47)	-0.0004 (-0.01)
(v) Money volatility in eq. (31)	β_8=0	-0.227 (-3.13)	0.105 (0.73)	-0.220 (-1.98)	0.016 (0.10)	0.070 (0.88)	-0.419 (-1.44)	-0.321 (-1.03)
(vi) Output volatility in eq. (31)	β_9=0	0.697 (1.96)	-0.089 (-0.78)	0.012 (0.35)	-0.078 (-0.63)	0.054 (0.38)	0.015 (0.14)	-0.004 (-0.03)
(vii) Money growth in eq. (31)	β_{7i}=0	0.097 [0.003]	0.346 [0.000]	0.251 [0.000]	0.514 [0.001]	0.183 [0.000]	-0.033 [0.068]	-

Notes:

1. In each case we report the estimated parameters of the M-GARCH-M in equations (29)-(33) and the corresponding t-statistic in the parenthesis with the round brackets. The square brackets in the last row for hypothesis (vii) refer to p-values (details of which are discussed in note 5 below).
2. The results in Table 1 summarise the hypotheses that relate to the empirical specification in equations (29) to (33) and the predictions of the theoretical model discussed in section 4. The detailed estimation results can be found in the Appendix which present the full estimated linear and non-linear GARCH-M models for each country and some misspecification test results. The first column of Table 1 lists the hypotheses tested and discussed in notes 3-5 below.
3. For the hypotheses (i) and (ii) we report the volatility estimated parameters for each GARCH equation (30) and (32), respectively. The estimation results show that in all cases the estimated volatility parameters for both output and money growth rates are highly significant and their persistence coefficient is close to unity that implies that shocks to volatility have a slow decay.
4. For hypotheses (iii) and (iv) we examine the effects of money and growth uncertainties given by $\sigma^2_{\Delta M_t}$ and $\sigma^2_{\Delta Y_t}$, respectively, in the money growth equation (29). Similarly, for hypotheses (v) and (vi) we examine the effects of money and growth uncertainties in the output growth equation (31). The null hypothesis (v) of zero restrictions is tested versus the alternative of $\beta_8<0$ as suggested by the theoretical model.
5. For hypothesis (vii) we test the effects of money growth dynamics in the output growth equation (31) by imposing the joint zero restriction on all the β_{7i}\`s. This is tested using an LM test and the p-value from the chi-square test is reported in the square parenthesis. The reported estimated coefficient reflects the sum of the lagged money growth coefficients in equation (31) which according to the theoretical model it is expected to be positive. When no results are reported this is due to the general-to-specific variable choice selection procedure.

Appendix.

Table A1: G7 Data Definitions and Descriptions

Country	Money	Output
Canada	OECD M1 money supply	IFS industrial production index (excluding construction)
France	OECD M1 money supply (1960-1977 from OECD Historical Statistics)	IFS industrial production index (excluding construction)
Germany	OECD M1 money supply	OECD index of industrial production (excluding construction)
Italy	OECD M1 money supply (1964-1980 from OECD Historical Statistics)	IFS industrial production index (excluding construction)
Japan	OECD M1 money supply	IFS industrial production index (excluding construction)
UK	M0 monetary base from ONS (AVAE)	IFS industrial production index (excluding construction)
US	FRED M1 money stock (M1SL)	FRED industrial production index (excluding construction, INDPRO)

Data Sources:
OECD – Organisation for Economic Co-operation and Development;
IFS – International Financial Statistics from the IMF;
ONS – UK Office for National Statistics;
FRED – Federal Reserve Economic Data (http://research.stlouisfed.org/fred/).

Table A2: G7 Outliers Removed

Country	Money	Output
Canada	none	1982m12; 1984m6; 1984m12
France	1968m5; 1977m12, 1995m12, 1999m1	1963m3; 1968m5-6
Germany	1964m1; 1964m12; 1965m12; 1966m12; 1967m11-12; 1968m11-12; 1990m6, 12	1984m6
Italy	1984m2; 1988m2; 1992m2; 1996m2;	1969m10-12; 1972m12
Japan	1963m1; 1972m10	none
UK	1971m2-3; 1999m10-11	1972m2; 1974m1-3; 1978m4; 1979m1
US	none	1974m12

Note: The above outlier observations are removed by interpolation of the data.

Estimation and misspecification test results of bivariate GARCH-in-Mean models for the G7.

The empirical results of the hypotheses discussed in the paper are summarized in Table 1. The details of the estimation and misspecification test results for the bivariate GARCH-M models applied to each of the G7 countries are presented in Tables A1-A7 in this Appendix with the corresponding discussion given below. In addition in this Appendix we compare the proposed non-linear model with the linear dynamic model benchmark.

The estimation and test results for each of the G7 countries are reported in the respective Tables A1-A7 which are organised as follows. Each Table refers to one of the G7. In each table the first column represents the unknown coefficients defined in the GARCH-M equations (29)-(33). The Vector Autoregressive (VAR) is nested in this model defined by equations (29) and (31). The second column presents the OLS estimation results from the linear equations for money and output growth and the third column presents the Maximum Likelihood Estimates (MLEs) of the bivariate GARCH-M model. For these models the estimates are reported with the corresponding t-statistics in brackets next to the estimates. Misspecification testing of these models involves assessing the temporal dependence properties of the residuals that are assumed to be i.i.d. which is also useful for the choice of the lag length in the autoregressive structure of the conditional mean equations. We report the p-values for the Ljung-Box and McLeod-Li Portmanteau tests which test linear and second-order correlation in the residuals, respectively. The general-to-specific procedure is also adopted for specifying the significant lags along with the above misspecification tests applied to the linear equations of each country. It is worth mentioning that in most of the G7 the long lags selected using the above statistical procedures is also an attempt to capture the relatively longer run dynamics in the growth process.

The misspecification test results show that for the linear model the residuals of the money growth equation in all G7 (except France) suffer from ARCH effects given the rejection of the null hypothesis in the McLeod-Li tests. Similar evidence is provided by the residuals of the VAR equation for the production growth rate which also exhibit second order correlation in all G7 except Italy and the UK. The estimated bivariate GARCH-M models in the third column of each Table A1-A7 aim to capture the dynamic heteroskedasticity found in the linear models and introduce the GARCH series as influence variables in the conditional mean of production and money growth. In terms of misspecification analysis the GARCH-M models capture the first- and second-order dynamics in the data as shown by both of these Portmanteau test results. In comparing the linear and non-linear models (in each Table A1-A7) we observe that the lagged effects of money and production are in some cases reduced or turn out to be insignificant following the estimation and introduction of the contemporaneous variance estimates in the conditional mean. This result relates to the presence of ARCH and its relation to misspefication of the conditional mean and its parsimony also addressed in Lumsdaine and Ng (1999).

The estimation of the bivariate GARCH-M model in equations (29)-(33) provide the framework to study the joint nonlinear time-series behavior of money and output and examine the number of hypotheses described in Section 3 and presented in Table 1.

Estimation and Misspecification Test Results of Linear Regression and Bivariate GARCH(1,1)-in-Mean Model for each of the G7.

Table A3: Linear and GARCH-M Results for Canada

Canada	**Linear**	**GARCH-M**
	1961:02 - 2000:12	**1961:03 - 2000:12**
Eq. (29): β_0	0.6341 (7.36)	0.5191 (1.77)
M_{t-1}	-0.0715 (-1.60)	
M_{t-3}	0.0839 (1.88)	0.0994 (1.87)
M_{t-6}	0.1026 (2.31)	0.1016 (2.02)
M_{t-9}	0.1146 (2.53)	0.1374 (2.94)
M_{t-12}	-0.1535 (-3.37)	-0.1073 (-2.47)
Y_{t-8}	-0.0831 (-1.65)	-0.1230 (-2.63)
Y_{t-9}	-0.0783 (-1.56)	-0.1135 (-2.22)
Y_{t-11}	-0.1413 (-2.82)	-0.1320 (-2.42)
Y_{t-12}	0.1340 (2.65)	
$\sigma^2_{\Delta M_t}$		-0.0801 (-1.11)
$\sigma^2_{\Delta Y_t}$		0.1931 (0.90)
Eq. (30): α_0		0.0842 (2.32)
$\sigma^2_{\Delta M_{t-1}}$		0.8318 (20.91)
ε^2_{t-1}		0.1231 (3.96)
McLeod-Li	0.000	0.587
Ljung-Box	0.948	0.810
Eq. (31): Θ_0	0.2022 (2.84)	-0.2089 (-0.48)
Y_{t-1}	-0.1728 (-3.88)	-0.2091 (-4.33)
Y_{t-3}	0.1199 (2.68)	0.1214 (2.59)
Y_{t-4}	0.1281 (2.80)	0.0931 (2.06)
Y_{t-5}	0.0881 (1.95)	0.0842 (1.93)
Y_{t-8}	0.1339 (3.01)	0.0999 (2.36)
Y_{t-12}	-0.1025 (-2.32)	-0.1191 (-2.94)
M_{t-2}	0.0744 (1.90)	0.0876 (2.40)
M_{t-7}	0.0734 (1.85)	0.0977 (2.74)
M_{t-12}	-0.0866 (-2.14)	-0.0889 (-2.21)
$\sigma^2_{\Delta M_t}$		-0.2274 (-3.13)
$\sigma^2_{\Delta Y_t}$		0.6970 (1.96)
Eq. (32): α_3		0.5139 (2.49)
$\sigma^2_{\Delta Y_{t-1}}$		0.4339 (2.45)
v^2_{t-1}		0.1328 (2.43)
Eq. (33): $\rho_{\varepsilon v}$		-0.0087 (-0.17)
McLeod-Li	0.009	0.133
Ljung-Box	0.978	0.935

Table A4: Linear and GARCH-M Results for France

France	Linear	GARCH-M
	1961:02 - 1998:12	1961:03 - 1998:12
Eq. (29): β_0	0.2149 (2.78)	0.1721 (1.23)
M_{t-1}	-0.2532 (-5.49)	-0.1772 (-3.51)
M_{t-2}	-0.0678 (-1.41)	
M_{t-3}	0.2699 (5.60)	0.2517 (6.42)
M_{t-4}	0.1879 (4.00)	0.1173 (2.61)
M_{t-5}	0.1003 (2.02)	0.0803 (1.84)
M_{t-6}	0.1883 (3.87)	0.1899 (4.69)
M_{t-9}	0.0995 (2.13)	0.0756 (2.07)
M_{t-11}	0.1145 (2.58)	0.1291 (2.92)
Y_{t-2}	-0.0733 (-2.30)	-0.0740 (-2.75)
Y_{t-4}	0.1036 (3.27)	0.1046 (3.24)
Y_{t-8}	-0.0558 (-1.75)	
Y_{t-12}	-0.0582 (-1.89)	-0.0659 (-2.51)
$\sigma^2_{\Delta M_t}$		-0.0424 (-0.57)
$\sigma^2_{\Delta Y_t}$		0.0102 (0.17)
Eq. (30): α_0		0.7027 (7.01)
$\sigma^2_{\Delta M_{t-1}}$		0.1269 (1.41)
ε^2_{t-1}		0.3879 (4.56)
McLeod-Li	0.022	0.997
Ljung-Box	0.996	0.351
Eq. (31): Θ_0	0.1030 (1.11)	0.3437 (1.49)
Y_{t-1}	-0.3385 (-7.32)	-0.3664 (-7.52)
Y_{t-2}	-0.0780 (-1.70)	-0.1203 (-2.37)
Y_{t-5}	0.0938 (2.04)	
Y_{t-6}	0.1775 (3.92)	0.1517 (3.48)
M_{t-1}	0.2184 (3.66)	0.2126 (3.78)
M_{t-8}	0.1088 (1.84)	0.1333 (2.28)
M_{t-9}	-0.0911 (-1.52)	
$\sigma^2_{\Delta M_t}$		-0.1052 (-0.73)
$\sigma^2_{\Delta Y_t}$		-0.0887 (-0.78)
Eq. (32): α_3		0.0634 (1.52)
$\sigma^2_{\Delta Y_{t-1}}$		0.9013 (22.17)
v^2_{t-1}		0.0630 (2.47)
Eq. (33): $\rho_{\varepsilon v}$		-0.0672 (-1.28)
McLeod-Li	0.000	0.233
Ljung-Box	0.995	0.872

Table A5: Linear and GARCH-M Results for Germany

Germany	Linear	MGARCH-M
	1961:02 - 1998:12	**1961:03 - 1998:12**
Eq. (29): β_0	0.3343 (4.85)	0.6005 (3.72)
M_{t-3}	0.0897 (1.99)	0.0842 (1.99)
M_{t-5}	0.0788 (1.76)	0.0736 (1.70)
M_{t-6}	0.0860 (1.91)	0.1017 (2.47)
M_{t-12}	0.2768 (6.06)	0.1773 (4.50)
Y_{t-6}	-0.0733 (-2.54)	
Y_{t-7}	-0.1089 (-3.61)	-0.0843 (-2.99)
Y_{t-8}	-0.0678 (-2.40)	-0.0778 (-2.72)
Y_{t-12}	0.0448 (1.68)	
$\sigma^2_{\Delta M_t}$		-0.0244 (-0.20)
$\sigma^2_{\Delta Y_t}$		-0.0666 (-1.27)
Eq. (30): α_0		0.4420 (3.46)
$\sigma^2_{\Delta M_{t-1}}$		0.2129 (1.32)
ε^2_{t-1}		0.2722 (3.92)
McLeod-Li	0.000	0.118
Ljung-Box	0.721	0.386
Eq. (31): Θ_0	-0.0249 (-0.26)	0.2154 (1.34)
Y_{t-1}	-0.3542 (-7.71)	-0.3100 (-6.02)
Y_{t-2}	-0.1295 (-2.64)	-0.0918 (-1.97)
Y_{t-3}	0.1051 (2.25)	0.0919 (2.11)
Y_{t-5}	0.0705 (1.51)	
Y_{t-6}	0.0938 (2.04)	0.1321 (3.66)
Y_{t-8}	0.0892 (2.05)	0.0825 (2.03)
Y_{t-12}	0.0922 (2.15)	
M_{t-5}	0.1536 (2.14)	
M_{t-6}	0.2139 (2.98)	0.2509 (3.44)
$\sigma^2_{\Delta M_t}$		-0.2204 (-1.98)
$\sigma^2_{\Delta Y_t}$		0.0117 (0.35)
Eq. (32): α_3		1.8770 (6.00)
$\sigma^2_{\Delta Y_{t-1}}$		0.1238 (1.04)
v^2_{t-1}		0.2953 (3.53)
Eq. (33): $\rho_{\varepsilon v}$		-0.0055 (-0.10)
McLeod-Li	0.000	0.995
Ljung-Box	0.765	0.877

Table A6: Linear and GARCH-M Results for Italy

Italy	Linear	MGARCH-M
	1965:02 - 1998:12	1965:03 - 1998:12
Eq. (29): β_0	0.3621 (3.87)	0.2361 (1.09)
M_{t-2}	0.1067 (2.23)	0.1252 (2.61)
M_{t-3}	0.1455 (3.00)	0.1159 (2.28)
M_{t-6}	0.1077 (2.12)	0.1612 (3.71)
M_{t-8}	0.1101 (2.22)	0.1561 (3.91)
M_{t-9}	0.1689 (3.36)	0.1586 (3.46)
Y_{t-9}	-0.0484 (-2.49)	-0.0506 (-3.17)
$\sigma^2_{\Delta M_t}$		-0.0514 (-0.52)
$\sigma^2_{\Delta Y_t}$		0.0155 (0.35)
Eq. (30): α_0		0.2294 (3.83)
$\sigma^2_{\Delta M_{t-1}}$		0.3391 (3.69)
ε^2_{t-1}		0.4294 (4.35)
McLeod-Li	0.000	0.059
Ljung-Box	0.931	0.670
Eq. (31): Θ_0	-0.0945 (-0.53)	0.1740 (0.31)
Y_{t-1}	-0.4192 (-8.59)	-0.4131 (-7.13)
Y_{t-2}	-0.1406 (-2.89)	-0.1670 (-3.26)
Y_{t-4}	0.0961 (2.12)	0.0965 (2.49)
Y_{t-7}	-0.1110 (-2.47)	-0.1303 (-3.16)
M_{t-1}	0.2491 (2.31)	0.2711 (2.45)
M_{t-5}	0.2246 (2.03)	0.2429 (2.16)
$\sigma^2_{\Delta M_t}$		0.0159 (0.10)
$\sigma^2_{\Delta Y_t}$		-0.0776 (-0.63)
Eq. (32): α_3		6.9718 (7.66)
$\sigma^2_{\Delta Y_{t-1}}$		0.6780 (3.61)
v^2_{t-1}		0.0927 (3.75)
Eq. (33): $\rho_{\varepsilon v}$		0.0422 (0.77)
McLeod-Li	0.728	0.999
Ljung-Box	0.937	0.987

Table A7: Linear and GARCH-M Results for Japan

Japan	Linear	MGARCH-M
	1961:02 - 2000:12	**1961:03 – 2000:12**
Eq. (29): β_0	0.2568 (3.71)	0.5156 (3.48)
M_{t-3}	0.1789 (4.01)	0.2146 (4.46)
M_{t-5}	0.1514 (3.63)	0.1477 (4.23)
M_{t-6}	0.1786 (4.04)	0.1698 (4.27)
M_{t-9}	0.1354 (3.07)	0.1583 (4.26)
Y_{t-1}	0.0757 (2.73)	
$\sigma^2_{\Delta M_t}$		-0.0639 (-0.75)
$\sigma^2_{\Delta Y_t}$		-0.1318 (-1.56)
Eq. (30): α_0		0.2197 (4.21)
$\sigma^2_{\Delta M_{t-1}}$		0.3931 (3.86)
ε^2_{t-1}		0.3849 (4.20)
McLeod-Li	0.000	0.146
Ljung-Box	0.784	0.859
Eq. (31): Θ_0	-0.0069 (-0.07)	-0.0862 (-0.33)
Y_{t-1}	-0.2676 (-5.91)	-0.2469 (-5.31)
Y_{t-2}	0.0825 (1.76)	0.1022 (1.97)
Y_{t-3}	0.3436 (7.40)	0.3633 (7.34)
Y_{t-4}	0.1160 (2.44)	0.1005 (1.96)
Y_{t-5}	0.1105 (2.36)	0.0922 (1.84)
Y_{t-6}	0.0954 (2.04)	0.0915 (1.78)
Y_{t-9}	0.1008 (2.25)	0.1058 (2.08)
Y_{t-10}	-0.1209 (-2.81)	-0.1050 (-2.48)
M_{t-1}	0.1733 (2.89)	0.1833 (2.86)
M_{t-8}	0.0860 (1.44)	
$\sigma^2_{\Delta M_t}$		0.0703 (0.88)
$\sigma^2_{\Delta Y_t}$		0.0539 (0.38)
Eq. (32): α_3		0.0799 (1.30)
$\sigma^2_{\Delta Y_{t-1}}$		0.8949 (15.78)
v^2_{t-1}		0.0579 (2.09)
Eq. (33): $\rho_{\varepsilon v}$		-0.6180 (-1.26)
McLeod-Li	0.007	0.186
Ljung-Box	0.990	0.877

Table A8: Linear and GARCH-M Results for UK

UK	Linear	MGARCH-M
	1972:05 - 2000:12	**1972:06 - 2000:12**
Eq. (29): β_0	0.1876 (2.95)	0.4143 (4.34)
M_{t-1}	-0.1131 (-2.11)	-0.1642 (-3.14)
M_{t-2}	0.0899 (1.68)	
M_{t-3}	0.1945 (3.65)	0.1457 (3.94)
M_{t-4}	0.1182 (2.15)	
M_{t-5}	0.0915 (1.69)	0.1585 (4.35)
M_{t-6}	0.1053 (1.97)	0.2106 (6.20)
M_{t-7}	0.1171 (2.20)	0.1973 (5.53)
M_{t-8}	0.1667 (3.14)	0.1402 (4.09)
M_{t-12}	-0.0997 (-1.90)	-0.0919 (-2.94)
Y_{t-5}	0.0368 (1.42)	
$\sigma^2_{\Delta M_t}$		-0.8182 (-3.64)
$\sigma^2_{\Delta Y_t}$		0.0237 (0.47)
Eq. (30): α_0		0.2447 (6.46)
$\sigma^2_{\Delta M_{t-1}}$		0.2203 (2.84)
ε^2_{t-1}		0.3992 (4.24)
McLeod-Li	0.000	0.888
Ljung-Box	0.968	0.885
Eq. (31): Θ_0	0.1701 (1.82)	0.2156 (1.70)
Y_{t-1}	-0.1127 (-2.16)	-0.1803 (-3.35)
Y_{t-5}	0.1575 (3.07)	0.1134 (2.22)
Y_{t-7}	0.0713 (1.38)	
Y_{t-8}	0.0875 (1.68)	0.0965 (1.75)
Y_{t-9}	-0.0764 (-1.48)	
M_{t-7}	0.2314 (2.32)	0.1679 (1.75)
M_{t-9}	-0.3444 (-3.47)	-0.2011 (-1.93)
$\sigma^2_{\Delta M_t}$		-0.4193 (-1.44)
$\sigma^2_{\Delta Y_t}$		0.0148 (0.14)
Eq. (32): α_3		-0.0022 (-0.40)
$\sigma^2_{\Delta Y_{t-1}}$		0.9626 (47.40)
v^2_{t-1}		0.0365 (2.06)
Eq. (33): $\rho_{\varepsilon v}$		-0.0246 (-0.45)
McLeod-Li	0.808	0.005
Ljung-Box	0.662	0.832

Table A9: Linear and GARCH-M Results for US

US	Linear	MGARCH-M
	1961:02 - 2000:12	1961:03 - 2000:12
Eq.(29): β_0	0.1137 (3.42)	0.1036 (2.13)
M_{t-1}	0.4099 (9.23)	0.4270 (9.49)
M_{t-2}	-0.0899 (-1.85)	
M_{t-3}	0.1749 (3.87)	0.1684 (3.86)
M_{t-5}	0.0732 (1.62)	0.1047 (2.47)
M_{t-6}	0.0735 (1.63)	
M_{t-8}	0.0845 (1.87)	
M_{t-9}	0.1011 (2.24)	0.1369 (3.25)
Y_{t-2}	-0.0427 (-1.70)	
Y_{t-3}	-0.0733 (-2.91)	-0.0539 (-2.62)
Y_{t-9}	-0.0354 (-1.44)	
Y_{t-11}	-0.0546 (-2.15)	-0.0569 (-2.65)
Y_{t-12}	0.0657 (2.62)	0.0542 (2.38)
$\sigma^2_{\Delta M_t}$		-0.1451 (-0.55)
$\sigma^2_{\Delta Y_t}$		-0.0004 (-0.01)
Eq. (30): α_0		0.0153 (2.36)
$\sigma^2_{\Delta M_{t-1}}$		0.7319 (10.19)
ε^2_{t-1}		0.1815 (3.57)
McLeod-Li	0.000	0.207
Ljung-Box	0.995	0.989
Eq. (31): Θ_0	0.1469 (2.80)	0.3451 (3.37)
Y_{t-1}	0.2608 (5.73)	0.1823 (2.98)
Y_{t-2}	0.1459 (3.15)	
Y_{t-3}	0.0885 (1.97)	0.1189 (2.79)
Y_{t-9}	0.0612 (1.43)	
Y_{t-12}	-0.0712 (-1.68)	-0.0994 (-2.47)
M_{t-2}	0.1889 (2.70)	
M_{t-11}	-0.1676 (-2.35)	
$\sigma^2_{\Delta M_t}$		-0.3208 (-1.03)
$\sigma^2_{\Delta Y_t}$		-0.0042 (-0.03)
Eq. (32): α_3		0.2539 (4.29)
$\sigma^2_{\Delta Y_{t-1}}$		0.2110 (1.56)
v^2_{t-1}		0.3189 (5.01)
Eq. (33): $\rho_{\varepsilon v}$		-0.0611 (-1.21)
McLeod-Li	0.000	0.154
Ljung-Box	0.532	0.110