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Human capital, Demographic Transition and Economic Growth

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Abstract

This paper extends the literature on economic growth and demographic change by developing a neo-classical model of endogenous growth in which both economic and demographic outcomes are jointly determined. The key point in this model is the endogenisation of child mortality rate by linking it to parents' human capital, defined in a broad sense to include both education and health. The numerical simulation of this model confirms that as economic development takes place there will be a decline in child mortality rate followed by similar trend in fertility rate, hence, population growth rate.

Keywords: Demographic Transition, Human Capital, Economic Growth, Mortality, Fertility.

JEL classification: J13, O41

1 Introduction

There is a large body of literature that studies the role of human capital in the process of economic development. This literature has been inspired by two main observations. The first is that human capital, like physical capital, is an important factor input which can be accumulated over time to increase the economy's productive potential (Locus, 1988, Barro, 1991 and 1996, Barro and Sala-i-Martin, 1995, Mankiw, Romer and Weil, 1992 and Romer, 1989). The second is that human capital accumulation is intimately linked to other development phenomena such as income distribution and demographic transition (Eicher and Garcia-Penalosa, 1999, and Galor and Moav, 1999). The latter of these is concerned with the relationship between population change and economic development, a theme which occupied the attention of many early economists, and which continues to attract widespread interest.

Demographic transition theory has gained increasing support for its ability to provide rigorous theoretical explanations of a variety of real world phenomena (Kirk, 1996). According to this theory demographic changes accompany the process of economic development and are completed in three main stages. The first stage is a Malthusian regime, characterised by stagnation and underdevelopment along with high fertility and high mortality. Following this is the transition (pre-modernised) regime, involving an acceleration of technological progress and an increase in per capita income, accompanied by a decline in mortality and then a decline in fertility as well. The final stage is the modernised regime associated with developed economies with high per capita incomes and low fertility and mortality rates (Galor and Weil, 1999).

In this paper I develop a two-sector neo-classical model of endogenous growth for the purpose of studying the relationship between demographic transition and human capital accumulation. This model has two distinctions from other models in the literature. The first is that the term human capital is used in a broad sense to include both education and health, rather than linking it only to the former as in most other analyses. There is a large body of work which indicates that health human capital is as equally important as education human capital in the process of growth and development. Mushkin (1962) argues that investment in health raises the effectiveness of individuals in society both as producers and consumers. Knowles and Owen (1995, 1997) obtain empirical results which indicate that health is a major determinant of per capita output. Bloom et. al. (2001) find that an increase in life expectancy by one year leads to a 4% increase in output, while Barro and Sala-i-Martin (1995) suggest that the growth rate of output could increase by as much as 1.4 % points per year for each one standard-deviation increase in life expectancy. Another notable observation is the relationship between health and education (Mushkin 1962). On the one hand, better health services provided by well trained personnel and certain types of health programmes (e.g. in personal hygiene and sanitation) depend crucially on the level of education. On the other hand, better health status tends to reduce the number of days lost in schooling because of ill health, tends to reduce the cost of education per effective labour force member due to death of children of school age, and tends to increase life expectancy which increase the rate of return to investment in education. Grossman (1973) finds that schooling affects positively and significantly current health, while past health is an essential determinant of the years of formal schooling. Rivera and Currais (1999) stress that sustainable growth is not achievable by any country unless its labour force possesses minimum levels of both health and education. They document three factors to support their hypothesis that, in addition to education, health is essential for improved labour productivity and increased economic growth. These factors are (i) investment in health reduces incapacity, debility and the number of days workers are off sick, (ii) better health reduces the rate of depreciation of human capital and (iii) higher investment in health reduces the cost of treatment, and therefore, internalises the negative externalities associated with longevity.

The second distinguishing feature of this model is the endogenisation of mortality. In the seminal paper of Becker, Murphy, and Tamura (1990), both human capital and fertility are endogenised but not mortality. More recently Zhang et. al. (2001), Weil et. al. (2000) and Ehrlich and Lui (1991) have studied the effects of mortality on long-run growth in overlapping generations models where agents face a probability of surviving to old-age. In all of these models, however, this probability is treated as exogenous. Blackburn and Cipriani (2001) overcome this shortcoming by modelling an agent's life expectancy or probability of survival as a function of the stock of human capital (defined in the narrow sense) he inherits from his parent. In another paper Blackburn and Cipriani (1998) endogenise child mortality by linking it to private and public expenditures on health, though there is no human capital in their model. By contrast the model presented below links mortality directly to human capital which contributes to both longevity and production. By doing so, the model simultaneously accounts for the decline in both fertility and mortality, together with the growth of investment in human capital and growth of per capita income, all of which are striking aspects of the development process (Weil et. al. 2000).

The paper proceeds as follows. The model is presented in section 2 and solved in section 3. The steady state properties of the model are studied in section 4. Section 5 turns to an analysis of the transitional dynamics. Section 6 concludes.

2 The model

I consider an artificial economy in which there is an endogenous population of infinitely-lived dynasties comprised of altruistic households. Households accumulate both human capital h and physical capital k to produce and consume a single good y according to a Cobb-Douglas production technology with constant returns to scale. That is,

$$y = Ak^{\alpha}(lh)^{1-\alpha} \qquad \text{with } 0 < \alpha, l < 1 \tag{1}$$

where l denotes time spent on work.

The specification of the human capital technology follows the formulation suggested by Lucas (1988) where changes in the level of human capital depend on the amount of time not spent working. In this model, this time includes investment in both education and health. Normalising total time available to one, it follows that

$$\dot{h} = B(1-l)h \tag{2}$$

The size of a dynasty is equal to the population of the economy and changes according to

$$\dot{N} = (n-m)N \tag{3}$$

where n and m denote, respectively, the rates of fertility and (child) mortality so that ($n - m$) represents the number of surviving children. A key feature of the model is the endogenisation of mortality by linking it to changes in parent's human capital h. This is done through the following mortality function:

$$m = M\mu(h) \qquad \text{with } \mu'(h) < 0,\ \ \mu(0) = \bar{\mu},\ \lim_{h\to\infty}\mu(h) = \underline{\mu} \text{ and } 0 < M < 1 \tag{4}$$

where M is a parameter that represents exogenous determinants of mortality such as disease, sanitation and the state of environment. Equation (4) shows an inverse relationship between the mortality rate and the level of parents' human capital[1], subject to upper and lower bounds.

The dynasty's budget constraint can be expressed as

$$\dot{k} = Ak^{\alpha}(lh)^{1-\alpha} - \left[(1+q)(n-m)+\delta\right]k - c \tag{5}$$

where c is per capita consumption, δ is the depreciation rate of physical capital and q is the cost of rearing children. Following (Becker, 1991) it is assumed that rearing children is costlier the higher is the level of development, which captures the opportunity cost of parental time. Accordingly, the cost of child rearing is expressed as a proportion of per capita capital $(n-m)qk$.

Altruistic parents maximise their utility by deciding optimally their fertility, consumption and the allocation of their time between work and investment in their human capital. I use a modified version of the utility function developed by Barro and Becker (1989)[2].

[1] This link between mortality and human capital has been used by others (e.g. Blackburn and Cipriani (1998), Zhang et. al., (2001), and Blackburn and Cipriani (2001)).
[2] See Barro and Sala-i-Martin (1995) for more discussion.

The utility of an adult is a function of his own consumption, own number of children and the utility of each mature child. The utility of the next generation enters additively with a multiplicative discount factor that is negatively correlated with the number of offspring. The recursive structure of the utility function enables us to re-write this function as a discounted sum of the adult's own utility and the utility of all future generations. In the case of logarithmic preferences, this utility function is given by

$$U = \int_0^\infty \exp(-\rho t)\left[\log(c) + \phi \log(n-m) + \varphi \log(N)\right] dt \quad \text{with} \quad \rho, \phi, \varphi > 0, \tag{6}$$

where ρ is the discount rate, and φ and ϕ are the utility weights on a dynasty's size and children respectively. Equation (6) shows that the dynasty's utility function depends on the levels of consumption and numbers of children of all descendants in the same family.

The decision problem of a household is to maximise (6) subject to (2), (4) and (5). The first order conditions are[3]

$$\frac{1}{c} = \Lambda_1 \tag{7}$$

$$\Lambda_1 (1-\alpha) A \left(\frac{lh}{k}\right)^{-\alpha} = \Lambda_2 B \tag{8}$$

$$\frac{\phi}{n-m} - \Lambda_1 k(1+q) + \Lambda_3 N = 0 \tag{9}$$

[3] These conditions are derived from the Hamiltonian $H = \exp^{-\rho t}\left[\log(c)+\phi\log(n-M\mu(h))+\varphi\log(N)\right]+ \lambda_1\left[Ak^{\alpha}(lh)^{1-\alpha}-[(1+q)(n-M\mu(h))+\delta]k-c\right]+\lambda_2[B(1-l)h]+\lambda_3[(n-M\mu(h))N]$,
where $\exp^{-\rho t}\lambda_i=\Lambda_i$ $(i=1,2,3)$, which is maximised with respect *to c, l, n, k, h,* and *N*

$$\dot{\Lambda}_1 = \Lambda_1 \left[\rho - \alpha A \left(\frac{lh}{k} \right)^{1-\alpha} + \delta + (1+q)(n-m) \right] \tag{10}$$

$$\dot{\Lambda}_2 = \frac{\phi M \mu'(h)}{n-m} - \Lambda_1 \left[(1-\alpha) A \left(\frac{lh}{k} \right)^{-\alpha} l - (1+q) M \mu'(h) k \right] + \Lambda_2 \left(\rho - B(1-l) \right)$$

$$+ \Lambda_3 M \mu'(h) N \tag{11}$$

$$\dot{\Lambda}_3 = \Lambda_3 (\rho - n + m) - \frac{\varphi}{N} \tag{12}$$

where Λ_1, Λ_2, and Λ_3 are the costate variables associated with *k*, *h* and *N* respectively.

Equation (7) gives the marginal utility of consumption. Equations (8) and (9) are the static optimality conditions for l and *n* respectively. Equations (10), (11) and (12) are the dynamic optimality conditions relating to *k*, *h* and *N* respectively. The transversality conditions for this optimisation problem are

$$\lim_{t \to \infty} \exp(-\rho t) \Lambda_1(t) k(t) = \lim_{t \to \infty} \exp(-\rho t) \Lambda_2(t) h(t) = \lim_{t \to \infty} \exp(-\rho t) \Lambda_3(t) N(t) = 0 \tag{13}$$

3 Solution of the model

The equilibrium behaviour of the household is defined by the Hamiltonian system given above together with equations (2), (3) and (5). To study this equilibrium I follow the current practice of transforming variables so as to render them stationary in the steady state. This is done by dividing by the stock of per capita capital k and defining the new variables $\eta = \frac{lh}{k}$ (the ratio of human to physical capital) and $\gamma = \frac{c}{k}$ (the ratio of consumption to capital) both of which are constant in the steady state. The equilibrium may then be computed as follows.

Equation (7) gives

$$-\frac{\dot{c}}{c}=\frac{\dot{\Lambda}_1}{\Lambda_1} \tag{14}$$

From equations (10) and (14),

$$\frac{\dot{c}}{c}=-\rho+\alpha A\eta^{1-\alpha}-(1+q)(n-m)-\delta \tag{15}$$

By making use of equation (5), the growth rate of physical capital is given by

$$\frac{\dot{k}}{k}=A\eta^{1-\alpha}-(1+q)(n-m)-\delta-\gamma \tag{16}$$

Given that $\frac{\dot{\gamma}}{\gamma}=\frac{\dot{c}}{c}-\frac{\dot{k}}{k}$, combining equations (15) and (16) yield

$$\frac{\dot{\gamma}}{\gamma}=\gamma-\rho-(1-\alpha)Az \tag{17}$$

where $z=\eta^{1-\alpha}$. In turn, combining equations (7) and (9) delivers

$$\frac{\phi}{n-m}=\frac{1+q}{\gamma}-\Lambda_3 N \tag{18}$$

Defining $\pi=\Lambda_3 N$, and using equations (3) and (12) yields $\dot{\pi}=\rho\pi-\varphi$ which represents an unstable process. The general solution to this differential equation is $\pi(t)=\varphi/\rho+\left[\pi(0)-\varphi/\rho\right]\exp(\rho t)$. Thus, $\exp(-\rho t)\pi(t)=\exp(-\rho t)(\varphi/\rho)+\pi(0)-(\varphi/\rho)$, which would violate the transversality condition associated with N in equation (13) unless $\pi(0)=\varphi/\rho$. Hence $\pi(t)=\varphi/\rho$ for all t. Given this, together with the previous observations, equation (18) implies

$$n=M\mu(h)+\frac{\rho\phi\gamma}{(1+q)\rho-\varphi\gamma} \tag{19}$$

This equation expresses an important property in the demography literature regarding the relationship between changes in fertility and mortality along the development process. This property holds that mortality decline is a fundamental cause of fertility decline (Mason, 1997 and Kirk, 1996). The equation also shows that fertility is positively related to φ and ϕ (the utility weights on dynasty size and children) and negatively related to ρ and q (the rate of time preference and the child rearing cost). Fertility is also positively related to γ (the consumption-capital ratio).

Multiplying both sides of equation (9) by $\mu'(h)$ and using equations (8) and (11), it follows that

$$\frac{\dot{\Lambda}_2}{\Lambda_2} = \rho - B \tag{20}$$

Differentiating equation (8) with respect to time gives

$$\frac{\dot{\Lambda}_2}{\Lambda_2} = \frac{\dot{\Lambda}_1}{\Lambda_1} - \alpha\frac{\dot{\eta}}{\eta} = -\frac{\dot{c}}{c} - \alpha\frac{\dot{\eta}}{\eta} \tag{21}$$

Substituting equations (20) and (15) in equation (21), and making use of equation (19),

$$\frac{\dot{\eta}}{\eta} = \frac{1}{\alpha}\left[B - \alpha A\eta^{1-\alpha} + \frac{(1+q)\rho\phi\gamma}{(1+q)\rho - \varphi\gamma} + \delta\right] \tag{22}$$

Recall that $z = \eta^{1-\alpha}$. Then equation (22) can be rewritten as follows

$$\frac{\dot{z}}{z} = \frac{1-\alpha}{\alpha}\left[B + \delta - \alpha Az + \frac{(1+q)\rho\phi\gamma}{(1+q)\rho - \varphi\gamma}\right] \tag{23}$$

Given that $\eta = \frac{lh}{k}$, then

$$\frac{\dot{l}}{l} = \frac{\dot{\eta}}{\eta} - \frac{\dot{h}}{h} + \frac{\dot{k}}{k} \tag{24}$$

Substituting equations (2), (16) and (22) in equation (24), and making use of equation (19)

$$\frac{\dot{l}}{l} = \frac{1-\alpha}{\alpha}\left[B + \delta + \frac{(1+q)\rho\phi\gamma}{(1+q)\rho - \varphi\gamma}\right] + Bl - \gamma \tag{25}$$

The general equilibrium of the model is now given by equations (2), (17), (19), (23) and (25). These are five equations in five variables, h, z, γ, l and n.

4 The steady State

In the steady state $\dot{\gamma} = \dot{z} = \dot{l} = 0$ so $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = g^*$ (the steady state balanced growth rate). In addition, $\mu(h) = \underline{\mu}$ (i.e. mortality rate is at its lower limit). Let x^* denote the steady state value of x. Then the system of equations (2), (17), (19), (23), and (25) can be written in the steady state as

$$\gamma^* - \rho - (1-\alpha)Az^* = 0 \tag{26}$$

$$B + \delta - \alpha Az^* + \frac{(1+q)\rho\phi\gamma^*}{(1+q)\rho - \varphi\gamma^*} = 0 \tag{27}$$

$$\frac{1-\alpha}{\alpha}\left[B + \delta + \frac{(1+q)\rho\phi\gamma^*}{(1+q)\rho - \varphi\gamma^*}\right] + Bl^* - \gamma^* = 0 \tag{28}$$

$$g^* = B(1 - l^*) \tag{29}$$

$$n^* = M\underline{\mu} + \frac{\rho\phi\gamma^*}{(1+q)\rho - \varphi\gamma^*} \tag{30}$$

Equations (26), (27) and (28) imply

$$l^* = \frac{\rho}{B} \tag{31}$$

Equation (31) shows that the steady-state value of work time depends negatively on the productivity of the human capital sector and positively on the time preference rate. Equations (29) and (31) yield

$$g^* = B - \rho$$

Equation (32) shows that the steady state growth rate depends positively on the productivity of the human capital sector and negatively on the time preference rate. The other variables (γ^*, z^* and n^*) are determined from equations (26), (27) and (30). Total differentiation of these equations allows us to study the effects of parameter changes on these variables. Details of the computations are outlined in an appendix and the results are summarised in the following table.

Table (1) Comparative Statistics

	γ^*	z^*	n^*
A	±	±	±
B	+	+	+
q	−	−	−
φ	+	+	+
ϕ	+	+	+
ρ	±	±	±
M	0	0	+

As one would expect, an increase in the utility weights on the number of children ϕ or family size φ has a positive effect on fertility (n^*), together with a positive effect on z^* and γ^*. By contrast, an increase in the cost of child rearing q has a negative effect on n^*, z^* and γ^*. The effect of an exogenous increase in mortality M is to raise fertility, as is the effect of an increase in efficiency of the human capital production B. The effects of changes in the technology of production A and the time preference ρ are generally ambiguous.

5 Transitional Dynamics

The transitional dynamics of the model can be studied by means of phase diagrams in (z, γ) and (γ, l) spaces. To derive the $\dot{\gamma} = 0$, $\dot{z} = 0$ and $\dot{l} = 0$ curves, equations (17), (23) and (25) are linearised around the steady state by using a Taylor series expansion. The linear approximation of the model is given by

$$\dot{\gamma} \cong \gamma^*(\gamma - \gamma^*) - \gamma^*(1-\alpha)A(z - z^*) \tag{33}$$

$$\dot{z} \cong \left(\frac{1-\alpha}{\alpha}\right)\frac{(1+q)^2\rho^2\phi}{\left[(1+q)\rho - \varphi\gamma^*\right]^2} z^*(\gamma - \gamma^*) - (1-\alpha)Az^*(z - z^*) \tag{34}$$

$$\dot{l} \cong \left[\left(\frac{1-\alpha}{\alpha}\right)\frac{(1+q)^2\rho^2\phi}{\left[(1+q)\rho - \varphi\gamma^*\right]^2} - 1\right] l^*(\gamma - \gamma^*) + Bl^*(l - l^*) \tag{35}$$

These equations can be written in state-space form where the determinant of the transition matrix is given by

$$Det = B\gamma^*(1-\alpha)Az^*\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{(1+q)^2\rho^2\phi}{\left[(1+q)\rho - \varphi\gamma^*\right]^2} - 1\right] \tag{36}$$

Since the determinant must be negative to secure saddle path stability, the term in square brackets must be negative.

The slope of the locus $\dot{\gamma} = 0$ is $\left.\frac{d\gamma}{dz}\right|_{\dot{\gamma}=0} = (1-\alpha)A > 0$, and the slope of the locus $\dot{z} = 0$ is

$\left.\frac{d\gamma}{dz}\right|_{\dot{z}=0} = \frac{\alpha A[(1+q)\rho - \varphi\gamma]^2}{(1+q)^2\rho^2\phi} > 0$. Because of the above observation of saddle path stability, it must be true that $\left.\frac{d\gamma}{dz}\right|_{\dot{z}=0} > \left.\frac{d\gamma}{dz}\right|_{\dot{\gamma}=0}$ which means that the $\dot{\gamma}=0$ line is less positively sloped than the $\dot{z}=0$ line. Finally, the slope of the $\dot{l}=0$ line is given by

$$\left.\frac{d\gamma}{dl}\right|_{\dot{l}=0} = -\frac{B}{\left(\frac{1-\alpha}{\alpha}\right)\frac{(1+q)^2\rho^2\phi}{\left[(1+q)\rho-\varphi\gamma^*\right]^2}-1} > 0 .$$

The phase diagrams are plotted in Figure (1). Typically, one characterises a developing country as having an abundance of human capital relative to physical capital. This implies that the initial value of z, z_0, would be relatively high, so that $z_0 > z^*$. Consequently, z will follows a monotonically downward trend as development takes place towards z^*. Starting from anywhere to the right of z^* the economy will progress along the saddle path SS until it reaches the steady state. Similar movements are observed in other variable so that all are positively correlated. It is also true that γ, n and m are positively correlated, implying that fertility and mortality both decline with the level of development.

The above results are illustrated in Figure (2) by using a calibrated version of the model and applying the time elimination method (Mulligan and Sala-i-Martin, 1991). To conduct these calibration it is necessary to specify a form for the mortality function. I choose the following form:

$$m = M\mu(h) = M\left(\underline{\mu}+\frac{\overline{\mu}-\underline{\mu}}{1+h^{\theta}}\right) \qquad \text{with } 0<\theta<1 \qquad (37)$$

This specification has the same properties of the mortality function in equation (4). The results of the simulations show declining paths for (γ, l, n, m). The downward trend in l shows that people work less and invest more in their health and education as development takes place. The downward trends in both n and m show that both fertility and mortality are negatively related to the level of development. Accordingly, the model is capable of providing time series behaviour for fertility, mortality, education and health which fit the stylised facts of demographic transition.

6 Conclusion:

Examining the joint determination of economic and demographic changes has been the purpose of this paper. The model features transitional dynamics towards a steady state equilibrium, where the economy develops along a balanced, endogenous growth bath. The model delivers sensible results about the evolution of an economy in which fertility, mortality, human capital accumulation, consumption and savings are simultaneously determined in dynamic general equilibrium of intertemporally optimising dynastic families. These results accord with empirical observations about demographic changes placing emphasis on the role of human capital as an important factor in both stimulating growth and reducing mortality.

Naturally, the model could be extended in many ways. One possibility is to allow mortality rates to be age dependant. A notable feature of demographic transition is that infant mortality rates tend to decline first (at low levels of development), followed by a decline in adult mortality rates (at higher levels of development). Modelling this process would be an avenue of research worth pursuing.

Appendix Comparative Statistics

Totally differentiating and then rearranging equations (26), (27) and (30) yield:

$$d\gamma^* - (1-\alpha)Adz^* = d\rho + (1-\alpha)z^* dA \tag{B1}$$

$$\frac{(1+q)^2\rho^2\phi}{[(1+q)\rho-\varphi\gamma^*]^2}d\gamma^* - \alpha Adz^* = -dB - d\delta + \alpha z^* dA + \frac{\varphi\rho\phi\gamma^{*2}}{[(1+q)\rho-\varphi\gamma^*]^2}dq$$
$$+\frac{\varphi(1+q)\phi\gamma^{*2}}{[(1+q)\rho-\varphi\gamma^*]^2}d\rho - \frac{(1+q)\rho\gamma^*}{[(1+q)\rho-\varphi\gamma^*]^2}d\phi - \frac{(1+q)\rho\phi\gamma^{*2}}{[(1+q)\rho-\varphi\gamma^*]^2}d\varphi \tag{B2}$$

$$\frac{(1+q)\rho^2\phi}{[(1+q)\rho-\varphi\gamma^*]^2}d\gamma^* - dn^* = -\underline{\mu}dM + \frac{\rho^2\phi\gamma^*}{[(1+q)\rho-\varphi\gamma^*]^2}dq + \frac{\varphi\phi\gamma^{*2}}{[(1+q)\rho-\varphi\gamma^*]^2}d\rho$$
$$-\frac{\rho\gamma^*}{(1+q)\rho-\varphi\gamma^*}d\phi - \frac{\rho\phi\gamma^{*2}}{[(1+q)\rho-\varphi\gamma^*]^2}d\varphi \tag{B3}$$

In matrix form these equations can be rewritten as

$$\begin{bmatrix} 1 & -(1-\alpha)A & 0 \\ \dfrac{(1+q)^2\rho^2\phi}{[(1+q)\rho-\varphi\gamma^*]^2} & -\alpha A & 0 \\ \dfrac{(1+q)\rho^2\phi}{[(1+q)\rho-\varphi\gamma^*]^2} & 0 & -1 \end{bmatrix} \begin{bmatrix} d\gamma^* \\ dz^* \\ dn^* \end{bmatrix} = \begin{bmatrix} b_{1A}dA + b_{1\rho}d\rho \\ b_{2A}A - b_{2B}dB - b_{2\delta}d\delta + b_{2\rho}d\rho + b_{2q}dq - b_{2\phi}d\phi - b_{2\varphi}d\varphi \\ b_{3\rho}d\rho + b_{3q}dq - b_{3\phi}d\phi - b_{3\varphi}d\varphi - b_{3M}dM \end{bmatrix}$$

The effects of parameter changes on (γ^*, z^*, and n^*) reported in Table (1) are then derived by applying Cramer's Rule.

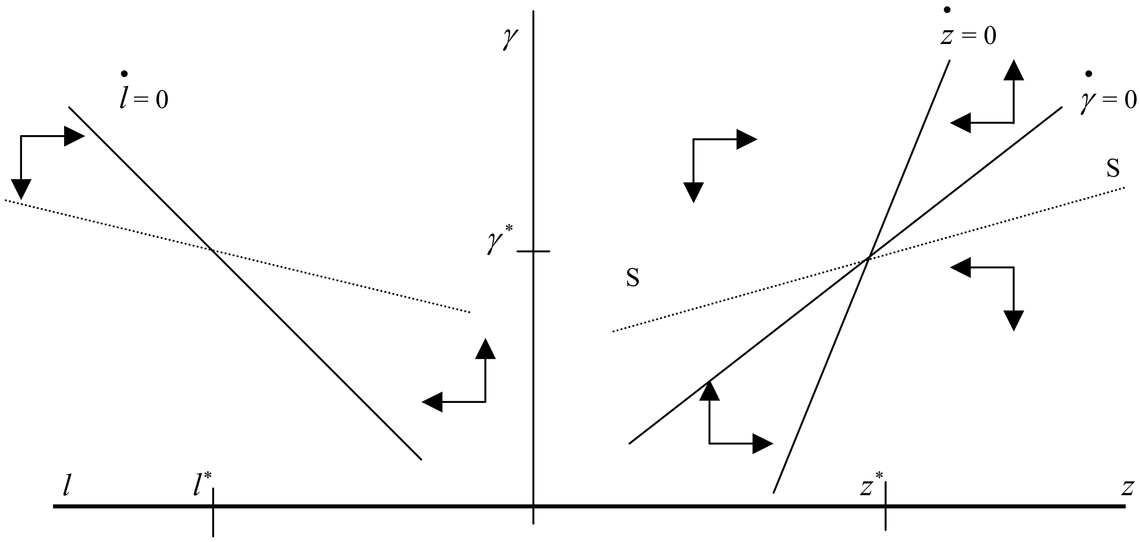
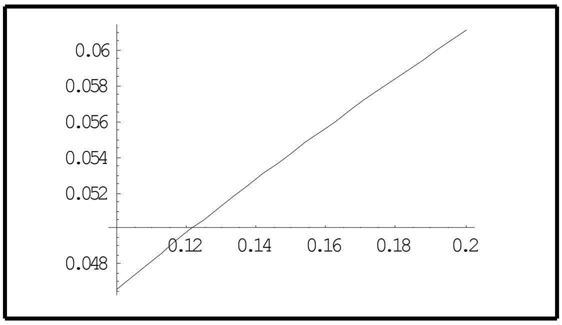
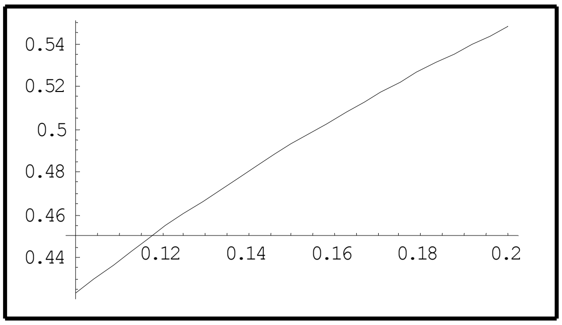


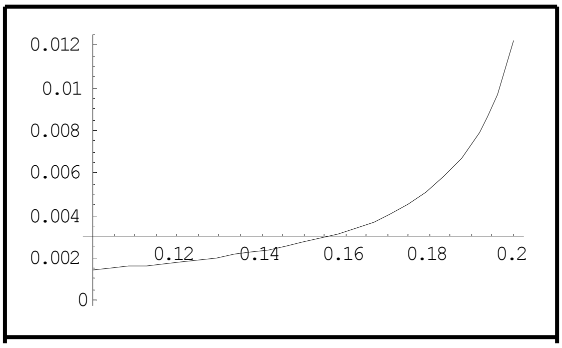
Figure (1) Perfect Foresight Equilibrium



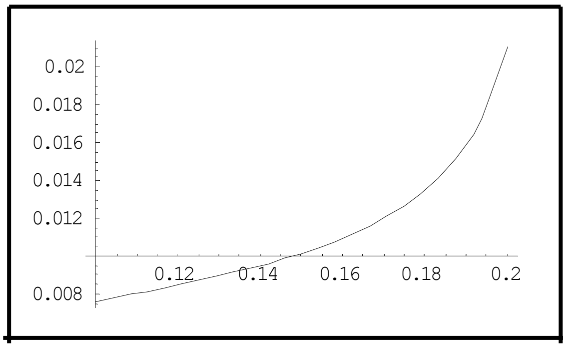
A: γ(z)



B: l(z)



C: m(z)

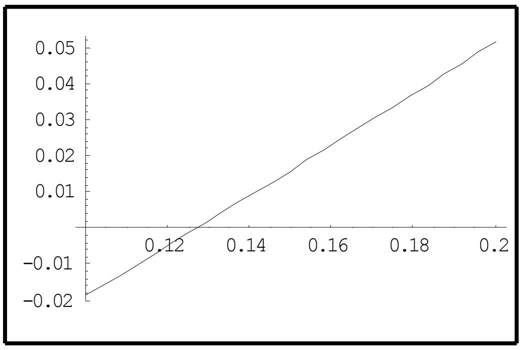


D: n(z)

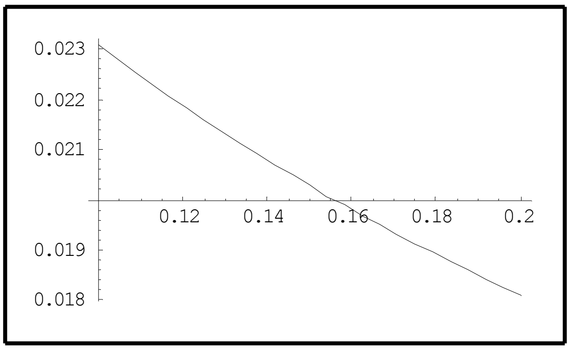
Figure (2): Transitional Dynamics

Table (2) Parameter values:

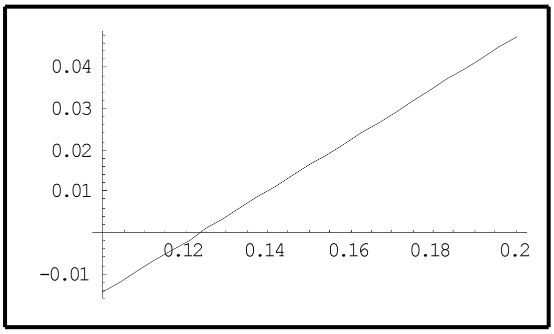
α	δ	ϕ	θ	ρ	φ	A	B	M	q	$\overline{\mu}$	$\underline{\mu}$
0.75	0.05	0.2	0.8	0.02	0.2	0.9	0.04	$9*10^{-6}$	1	0.3	0.003



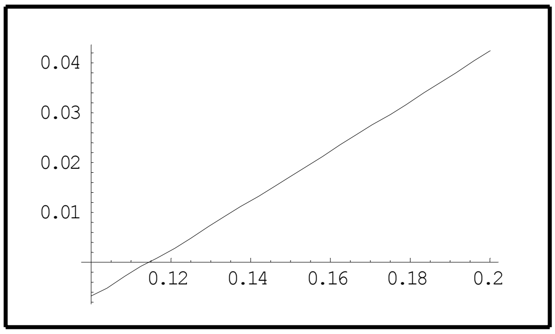
A: $g_k(z)$



B: $g_h(z)$



C: $g_c(z)$



D: $g_y(z)$

Figure (3): Dynamics of Growth Rates

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