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Growth, volatility and learning

By

Keith Blackburn and Ragchaasuren Galindev

Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, UK

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Growth, volatility and learning*

Keith Blackburn and Ragchaasuren Galindev
Centre for Growth and Business Cycles Research
School of Economic Studies, University of Manchester

Abstract

This paper presents a simple stochastic growth model in which productivity improvements are the result of both internal (deliberate) and external (serendipitous) learning behaviour. The model is used to illustrate how these different mechanisms of endogenous technological change can lead to different implications for the correlation between output growth and output variability.

Keywords: Business cycles; growth; volatility; learning.
JEL Classification: E32; O40.

1 Introduction

There is a growing body of literature on the potential interactions between the short-term (cyclical) and long-term (secular) fluctuations in output. This literature may be divided into two broad strands, reflecting two different approaches to the modelling of endogenous technological change.[1] According to one approach, technological change is the result of purposeful (internal) learning through deliberate actions which substitute for production activities. Under such circumstances, a recession has a positive effect on growth by reducing the opportunity cost of diverting resources away from manufacturing towards productivity improvements (e.g., Aghion and Saint-Paul

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Address for correspondence: Keith Blackburn, School of Economic Studies, University of Manchester, Manchester M13 9PL, England. **Tel:** 0161-275-3908. **Fax:** 0161-275-4928. **E-mail:** keith.blackburn@man.ac.uk.
[1] In addition to the references that follow, see de Hek (1999) and Jones *et al.* (1999) for two other recent contributions.

1998a, 1998b). According to the other approach, technological change is due to serendipitous (external) learning through non-deliberate actions which are complements to production activity. In this case a recession has a negative impact on growth by reducing factor employment through which expertise, knowledge and skills are acquired and disseminated (e.g., Martin and Rogers 1997, 2000). These conflicting results are often, though not always, associated with conflicting implications about the relationship between long-term growth and short-term volatility. *A priori*, there is no fundamental reason for presuming that this relationship should be of one particular sign - positive or negative - under one particular growth mechanism - internal learning or external learning. Equally, there is no compelling reason for presupposing that growth should occur on the basis of one mechanism alone. The results of existing models in which this is a feature are difficult to compare since the models are structurally very different from each other. In view of these considerations, we present an analysis of the issue which consolidates and extends the current literature by integrating both types of learning into a single, unifying framework. The relative importance of each type is captured conveniently by a flexible parameterisation of the process governing technological change which reduces to a process based solely on one type or the other under alternative limiting configurations of parameter values. We find that, in general, the correlation between the mean and variance of output growth is more likely to be positive (negative) if technological change is driven predominantly by internal (external) learning. Our results may be viewed as providing a rationale for the lack of robust evidence on this relationship.[2]

2 A simple stochastic growth model

The economy is populated by a unit mass of identical, infinitely-lived agents who produce and consume a single commodity. The decision problem for each agent is to maximise

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [\gamma_t \log(C_t) + \lambda \log(1 - L_t - H_t)], \quad \beta \in (0,1),\ \lambda > 0, \tag{1}$$

subject to

$$C_t = \Psi Z_t L_t^{\alpha}, \quad \Psi > 0,\ \alpha \in (0,1), \tag{2}$$

$$Z_{t+1} = \Omega Z_t H_t^{\phi} \overline{L}_t^{\theta}, \quad \Omega > 1,\ \phi, \theta > 0. \tag{3}$$

[2]See Kneller and Young (2001) for a review of the empirical evidence.

The agent derives (expected) utility in (1) from consumption, C_t, and leisure, $1 - L_t - H_t$, where L_t denotes time spent working (producing output) and H_t denotes time spent learning (improving productivity). The term γ_t is a positively-valued random variable (a preference, or taste, shock) which follows a stationary process with mean μ and variance σ^2. The budget constraint of the agent is given in (2) which equates consumption with output, where Z_t is a technological shift factor in the production function. Technology evolves according to (3) which encompasses both internal (deliberate) and external (non-deliberate) learning behaviour. The former is represented by H_t, the amount of time that the agent devotes intentionally to improving her own productive efficiency, while the latter is captured by $\overline{L}_t$, the aggregate level of employment which determines the extent of knowledge spillovers among agents and which each agent takes rationally as given. The relative importance of these two mechanisms of growth is measured by the relative magnitudes of the parameters ϕ and θ. Limiting cases of interest are obtained under the parameter configurations $\{\phi > 0, \theta = 0\}$ (purely internal learning) and $\{\phi = 0, \theta > 0\}$ (purely external learning).

The solution to the above problem is characterised by the following decision rules for L_t and H_t:[3]

$$L_t = l(\gamma_t) = \frac{\alpha\gamma_t}{\lambda + \phi B\mu + \alpha\gamma_t}, \tag{4}$$

$$H_t = h(\gamma_t) = \frac{\phi B\mu}{\lambda + \phi B\mu + \alpha\gamma_t}, \tag{5}$$

where $B = \frac{\beta}{1-\beta}$. Evidently, $l'(\cdot) > 0$ and $h'(\cdot) < 0$: intuitively, an increase in γ_t (a positive demand shock) leads agents to devote more time to working and less time to learning because of an increase in the marginal utility of consumption which raises the opportunity cost of productivity-improving activities.[4]

Substituting (4) and (5) into (2) and (3), and using the equilibrium condition $L_t = \overline{L}_t$, one can obtain expressions for the growth rates of technology and output between periods:

$$\frac{Z_{t+1}}{Z_t} = \frac{\Omega(\phi B\mu)^{\phi}\alpha^{\theta}\gamma_t^{\theta}}{(\lambda + \phi B\mu + \alpha\gamma_t)^{\phi+\theta}} \equiv z(\gamma_t), \tag{6}$$

$$\frac{C_{t+1}}{C_t} = \frac{\Omega(\phi B\mu)^{\phi}\alpha^{\theta}\gamma_t^{\theta-\alpha}\gamma_{t+1}^{\alpha}}{(\lambda + \phi B\mu + \alpha\gamma_t)^{\phi+\theta-\alpha}(\lambda + \phi B\mu + \alpha\gamma_{t+1})^{\alpha}} \equiv c(\gamma_t, \gamma_{t+1}). \tag{7}$$

[3] The derivation of the solution is available on request from the authors.

[4] For the opposite reasons, an expected positive shock in the future (i.e., an increase in $\mu = E_t(\gamma_{t+1})$) leads to a decrease in L_t and an increase in H_t.

As indicated in the existing literature, the effect of demand shocks on technology growth depends essentially on the underlying mechanism of technological change: if this mechanism is based solely on internal learning (i.e., $\{\phi > 0, \theta = 0\}$), then $z'(\cdot) < 0$, implying that technology growth is countercyclical; conversely, if the mechanism entails only external learning (i.e., $\{\phi = 0, \theta > 0\}$), then $z'(\cdot) > 0$ so that technology growth is pro-cyclical. These conflicting results are explained by the conflicting responses of H_t and L_t described above. Similar observations can be made about the growth rate of output, though the expression for this is a little more complicated. Since output in each period depends on the state of technology and the level of employment in that period, the growth rate of output from one period to the next is a function of the shocks in both of those periods. A positive γ_t shock causes an increase in L_t, a decrease in H_t and either an increase or decrease in Z_{t+1}. These effects mean that C_t increases, while C_{t+1} either increases or decreases, so that the growth rate of output may either rise or fall. In the limiting cases of interest, we have $c_1(\cdot) < 0$ for $\{\phi > 0, \theta = 0\}$, while $c_1(\cdot) \gtreqless 0$ for $\{\phi = 0, \theta \gtreqless \alpha\}$. By contrast, a positive γ_{t+1} shock has an unambiguously positive effect on output growth, $c_2(\cdot) > 0$, by virtue of its positive effect on L_{t+1} and hence C_{t+1}. The non-linear properties of $c(\cdot)$ are governed by similar considerations. Thus $c_{11}(\cdot) > 0$ for $\{\phi > 0, \theta = 0\}$, $c_{11}(\cdot) \lesseqgtr 0$ for $\{\phi = 0, \theta \gtreqless \alpha\}$ and $c_{22}(\cdot) < 0$ in all cases.

3 Growth and volatility

Our interest is in the correlation between the mean and variance of output growth. These moments may be approximated from (7) as

$$Mean\left(\frac{C_{t+1}}{C_t}\right) \simeq c(\mu,\mu) + \tfrac{1}{2}[c_{11}(\mu,\mu) + c_{22}(\mu,\mu)]\sigma^2, \tag{8}$$

$$Var\left(\frac{C_{t+1}}{C_t}\right) \simeq \{[c_1(\mu,\mu)]^2 + [c_2(\mu,\mu)]^2\}\sigma^2. \tag{9}$$

Trivially, $Var(\frac{C_{t+1}}{C_t})$ is positively related to σ^2, the variance of the shock. Less trivially, $Mean(\frac{C_{t+1}}{C_t})$ may be either positively or negatively related to σ^2 depending on the sign of $f(\phi,\theta,\alpha) \equiv c_{11}(\mu,\mu) + c_{22}(\mu,\mu)$ which reflects the curvature properties of $c(\cdot)$. This is merely an example of the well-known result that the expected value of a convex (concave) function of a variable is increased (decreased) by a mean-preserving spread of that variable. Some tedious calculus and algebra reveal that

$$
\begin{aligned}
&sgn f(\phi,\theta,\alpha) = \\
&sgn\{(\theta-\alpha)(\lambda+\phi B\mu)[(\theta-\alpha-1)(\lambda+\phi B\mu)-2\alpha(1+\phi)\mu] \\
&+\alpha^2\phi(1+\phi)\mu^2+\alpha(\lambda+\phi B\mu)[(\alpha-1)(\lambda+\phi B\mu)-2\alpha\mu]\}. \qquad (10)
\end{aligned}
$$

The key implications of (10) may be realised by considering, in turn, each of our two limiting scenarios, $\{\phi > 0, \theta = 0\}$ and $\{\phi = 0, \theta > 0\}$. In the case of the former, $f(\phi, 0, \alpha) > 0$, implying a positive correlation between $Mean(\frac{C_{t+1}}{C_t})$ and $Var(\frac{C_{t+1}}{C_t})$. In the case of latter, $f(0, \theta, \alpha) \lesseqgtr 0$, implying either a negative or positive correlation between $Mean(\frac{C_{t+1}}{C_t})$ and $Var(\frac{C_{t+1}}{C_t})$. These results demonstrate how the relationship between growth and volatility may depend on the underlying mechanism of growth, itself. Intuitively, since $\frac{Z_{t+1}}{Z_t}$ is convex (concave) in γ_t for $\{\phi > 0, \theta = 0\}$ ($\{\phi = 0, \theta > 0\}$), the rate at which internal (external) learning occurs is greater (lower) in the presence of positive shocks, or booms, than in the presence of negative shocks, or recessions; as such, the gain in learning during good times more (less) than compensates for the loss in learning during bad times so that, on average, technological progress increases (decreases) with a mean-preserving spread of the shock. The ambiguity of the second result is notable since it appears to run counter to this intuition. Indeed, the normal presumption is that growth and volatility are negatively correlated when there is only external learning which increases at a decreasing rate with the level of economic activity (i.e., when $\phi = 0$ and $\theta \in (0, 1)$). According to (10), a sufficient condition for $f(0, \theta, \alpha) < 0$ is $\theta > \alpha$ which ensures that $\frac{C_{t+1}}{C_t}$ is concave in γ_t. Failing this, $\frac{C_{t+1}}{C_t}$ is convex in γ_t and this may dominate the concavity in γ_{t+1} such that $f(0, \theta, \alpha) > 0$.

Given the above, it is natural to surmise that, in general, the relationship between $Mean(\frac{C_{t+1}}{C_t})$ and $Var(\frac{C_{t+1}}{C_t})$ will depend on the relative magnitudes of ϕ, θ and α. Consider, for example, a configuration of these parameters, $\{\phi^*, \theta^*, \alpha^*\}$, such that $f(\phi^*, \theta^*, \alpha^*) = 0$. Then one may conjecture that $f(\phi, \theta^*, \alpha^*) \gtreqless 0$ for $\phi \gtreqless \phi^*$, $f(\phi^*, \theta, \alpha^*) \gtreqless 0$ for $\theta \lesseqgtr \theta^*$ and $f(\phi^*, \theta^*, \alpha) \gtreqless 0$ for $\alpha \gtreqless \alpha^*$. This conjecture is supported by the results from a numerical analysis of (10). As a benchmark, we assign equal importance to the two types of learning ($\phi = \theta$) and set values for parameters which imply a zero correlation between growth and volatility ($f(\cdot) = 0$), together with an average annual growth rate of 2.5 percent and an average proportion of time spent on leisure of 65 percent. The benchmark values of the key parameters are $\{\phi^* = 0.30, \theta^* = 0.30, \alpha^* = 0.50\}$.[5] By varying each of these parameters in turn, holding all others constant, we verify the above conjecture and arrive

[5] The values of the other parameters are $\{\beta = 0.90, \lambda = 5.00, \Psi = 1.00, \Omega = 3.00, \mu = 1.00, \sigma^2 = 1.00\}$.

at the general conclusion that $f(\cdot) > 0$ ($f(\cdot) < 0$) is more likely for relatively high (low) values of ϕ, relatively low (high) values of θ and relatively high (low) values of α. Thus the model predicts that the correlation between growth and volatility may be either positive or negative according to whether technological change is driven predominantly by internal learning or external learning.

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