Health and Knowledge Externalities: Implications for Growth and Public Policy

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Abstract

Interactions between knowledge and health are studied in a three-period over- 
lapping generations model with health persistence. Reproductive agents face a 
non-zero probability of death in adulthood. In addition to working, adults al- 
llocate time to child rearing. Growth dynamics are shown to depend in critical 
ways on the externalities associated with knowledge and health. Depending on 
the strength of these externalities, the best policy to improve education out- 
comes may be to spend relatively more on children’s health. Trade-offs between 
education and health spending can be internalized by setting the optimal com- 
position of expenditure so as to maximize the growth rate. With an endogenous 
adult survival rate, multiple growth paths may emerge. A reallocation of public 
spending from education to health may shift the economy from a low-growth 
equilibrium to a high-growth path.

JEL Classification Numbers: O41, H54, I18

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1 Introduction

A large strand of the literature on economic growth focuses on human capital, which is often defined to include mainly education (or knowledge), health, and entrepreneurship or more generally abilities. In particular, a key premise of the literature is that education and good health enhance worker productivity and promote growth. Investments in health, in particular, can influence the pace of economic growth via their effects on a variety of health outcomes and health-related factors, including labor market participation and labor productivity, life expectancy, savings, and fertility decisions. Conversely, poor health may impede not only physical strength but also mental abilities, incentives to invest in education, and the ability to provide child care; as a result, it may not only be a cause of persistent poverty, but also an outcome of poverty. There is much evidence to support this two-way causality; Lorentzen et al. (2008), for instance, found a bidirectional link between life expectancy and income.

An extensive analytical and empirical literature has also focused on the possible interactions between some components of human capital, especially education and health, and how they affect growth. Benos and Zotou (2014) for instance, using meta-regression analysis, found that the growth effect of education is not homogeneous across studies, but varies according to several factors, including differences in the data used to measure education and model specification. More importantly, they point out that heterogeneity may be due to the fact that education is conditional on health outcomes, and that these effects vary across countries. Conversely, increasing education levels—above and beyond their effect on income—can also improve health outcomes. This implies that, in general, education, health and growth are all determined simultaneously, as documented empirically by Finlay (2007) in a cross-country study.

At the same time, there is significant evidence suggesting that late life health is the outcome of a cumulative process of exposure to health risks in childhood, especially infectious diseases in the first years of life. By determining health outcomes later in life, health in childhood may therefore play a critical role in the determination of health and socioeconomic status in adulthood (see Case et al. (2005) and Smith (2009)). There is therefore health persistence, which represents an important source
of dynamics in a growth context. The link between childhood health and health in adulthood can operate in the opposite direction as well. Indeed, there is evidence suggesting that cognitive and physical impairments of children may begin in utero, due to inadequate nutrition and poor health of the mother—illustrated most dramatically through mother-to-child transmission of HIV. According to estimates reported by Bloom and Canning (2005), for instance, an estimated 30 million infants are born each year in developing countries with impaired growth due to poor nutrition during fetal life. More generally, the health of parents may also affect the health of their children, after they are born, to the extent that it determines their physical and mental ability to provide child care.

This chapter examines the interactions between education and health, as two key components of human capital, and their impact on economic growth, in an overlapping generations (OLG) model. In the model, education and health outcomes are jointly determined, taking into account the possible externalities briefly referred to earlier. At the same time, the key difference between these components is that education (or knowledge) can be accumulated without bounds, whereas health status cannot. In addition, the paper accounts for the fact that (as noted earlier) parents’ health affects directly the health of their children (intergenerational transmission), and that health outcomes in childhood may affect health outcomes in adulthood (intragenerational transmission). As a result, health status displays persistence, as in Osang and Sarkar (2008), de la Croix and Licandro (2013), and Agénor (2015) for instance.

Another issue that the chapter addresses relates to the fact that the provision of education or health services, while complementary to each other at the microeconomic level,  

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1 See Case et al. (2002), Case et al. (2005), Paxson and Schady (2007), Smith (2009), and surveys by Behrman (2009) and Currie (2009), and Bleakley (2010). Agénor et al. (2014) discuss the recent literature on both issues from a gender perspective.

2 See for instance the results of Powdthavee and Vignoles (2008) for Britain.

3 Tang and Zhang (2007) develop an OLG model with education and health but do not account for externalities. Tamura (2006) an Ricci and Zachariadis (2013) develop OLG models where schooling exerts external effects on health, in the form of a negative effect on adult mortality in the first case and a positive effect on longevity in the second. In the model of Hazan and Zoabi (2006), health is, in addition to education, an input in the production of human capital. However, these contributions do not fully examine bidirectional effects, and the role of public policy, as is done here. Finally, Agénor and Neanidis (2011) do account for these effects but health is not stationary.

4 The requirement that health status be stationary is consistent with the specification in Osang and Sarkar (2008) and Agénor (2015).
level, requires the use of public resources. At the macroeconomic level, there is therefore
an inherent potential trade-off between education and health services. Understanding
the nature of these trade-offs, and the role that externalities may play, is thus critical
for public policy.

The remainder of the chapter is organized as follows. Section 2 provides a discus-
sion of the interactions between education and health. Section 3 presents a 3-period
OLG model that captures the key linkages between education and health as well as
the persistence in health, that is, the impact of health status in childhood on health
outcomes in adulthood. In the model both components of human capital, very much
like conventional economic goods, require a variety of inputs to be produced. Section 4
solves for the optimal household decision rules and derives the balanced growth path.
Section 5 studies the impact of public policy on education and health outcomes, as
well as economic growth. Section 6 endogenizes the adult survival rate and considers
the extent to which multiple growth paths may emerge. The issue of whether an in-
crease in public spending in health or education may allow a country to escape from
a low-growth equilibrium is also addressed. The last section of the paper offers some
concluding remarks.

2 Background

As briefly noted earlier, health and education are largely interlinked in their contri-
bution to growth because they both contribute to human capital accumulation. This
section provides a more detailed review of the recent evidence on the two-way interac-
tions between health and education. The causal link from health to education is first
discussed and the reverse link is taken up next.

2.1 Impact of Health on Education

It is now well recognized that health can have a sizable effect on education and the
accumulation of knowledge. Indeed, good health and nutrition are essential prereq-

5This section draws in part on Groot and van den Brink (2007), Agénor (2012, Chapter 3), and
6See Bleakley (2010b) for an overview of the evidence on the impact of health and education.
uisites for effective learning by children (see Glewwe and Miguel (2008)). In a study based on Ecuadorian data, Paxson and Schady (2007) found that health measures such as height for age and weight for age are positively related to language development (a measure of cognitive ability). When healthier children do better in school, this in turn promotes health-related knowledge (see Behrman (2009)).

Improving the health of individuals also increases the effectiveness of education, as noted for instance by Galor and Mayer-Foulkes (2004). In Bangladesh, the Food for Education program, which provided a free monthly ration of food grains to poor families in rural areas if their children attended school, was highly successful in increasing school enrollment (particularly for girls), promoting attendance, and reducing dropout rates (see Ahmed and Arends-Kuenning (2006)). In a study focusing on rural Guatemala, Maluccio et al. (2009) found that improving nutrition during early childhood has a substantial impact on adult educational outcomes. In Tanzania, the use of insecticide-treated bed nets reduced the incidence of malaria and increased attendance rates in schools (Bundy et al. (2006)). In western Kenya, deworming treatment improved primary school participation by 9.3 percent, with an estimated 0.14 additional years of education per pupil treated (see Miguel and Kremer (2004)). McCarthy et al. (2000) found that malaria morbidity, viewed as a proxy for the overall incidence of malaria among children, has a negative effect on secondary enrollment ratios. Bleakley (2007) found that deworming of children in the American South had an effect on their educational achievements while in school, whereas Bloom et al. (2005) found that children vaccinated against a range of diseases (including measles, polio, and tuberculosis) as infants in the Philippines performed better in language and IQ scores at the age of ten, compared to unvaccinated children—even within similar social groups. Thus, early vaccination appears to have a significant effect on subsequent learning outcomes.

Ampaabeng and Tan (2013): examine the role of early childhood health in human capital accumulation in Ghana. They find that differences in intelligence test scores can be robustly explained by the differential impact of a famine that occurred in 1983 in different parts of the country and the impacts are most severe for children under two years of age during the famine.

Bundy et al. (2006), in their overview of experience on the content and consequences
of school health programs (which include for instance treatment for intestinal worm infections), emphasized that these programs can raise productivity in adult life not only through higher levels of cognitive ability but also through their effect on school participation and years of schooling attained. At a more aggregate level, the cross-country regressions of Baldacci et al. (2004) show that health outcomes (as proxied by the under-five child mortality rate) have a statistically significant effect on school enrollment rates.

Conversely, inadequate nutrition, which often takes the form of deficiencies in micronutrients, reduces the ability to learn and study. Zinc deficiency, in particular, impairs brain and motor functions. Poor nutritional status can therefore adversely affect children’s cognitive development, and this may translate into poor educational attainment, as documented in Behrman (1996, 2009), Miguel (2005), Schultz (2005), and Bundy et al. (2006). Poor health, in the form of respiratory infections for instance, is also an important underlying factor for low school enrollment, absenteeism, and high dropout rates.

Another channel through which health can improve education outcomes and spur growth is through higher life expectancy and changes in time allocation within households. Increases in life expectancy tend to raise the incentive to invest in education (in addition to increasing the propensity to save, as discussed later) because the returns to schooling are expected to accrue over longer periods. Thus, at the individual level, to the extent that spending on health increases planning horizons, it may also raise the returns (as measured by the discounted present value of wages) of greater expenditure on education. In a study of Sri Lanka between the period 1946 and 1953, Jayachandran and Lleras-Muney (2009) found that a reduction in maternal mortality risk increases female life expectancy and female literacy. In a study of Brazil, Soares (2006) also found that higher longevity is associated with improved schooling outcomes. These results are both consistent with the view that longer life expectancy encourages investment in education.

The evidence also suggests that intrafamily allocations regarding school and work time of children tend to be adjusted in the face of disease within the family; in turn, these adjustments may influence education outcomes and thus the rate of economic
growth. As discussed by Corrigan et al. (2005), for instance, when parents become ill, children may be pulled out of school to care for them, take on other responsibilities in the household, or work to support their siblings. Hamoudi and Birdsall (2004) provided evidence that AIDS reduced schooling rates in sub-Saharan Africa. These results are consistent with the view that the risk that children may be infected by AIDS tends to deter parents from investing in their education, as argued by Bell et al. (2006). Put differently, an environment where there is great uncertainty about child survival may create a precautionary demand for children, with less education being provided to each of them. In turn, weaker education outcomes may hamper economic growth, as illustrated by Arndt (2006) in his study of AIDS and growth in Mozambique.

Health in childhood may affect health and income in adulthood through education.\footnote{See Gertler and Zeitlin (1996, 2002) Mayer-Foulkes (2005), Miguel (2005), and surveys by Behrman (1996) and Currie (2009).} Pain, fatigue, and malnutrition—in addition to being a primary cause of child mortality, as documented by Pelletier et al. (2003)—can reduce the ability to concentrate and to learn. Illness can crowd out other activities that might be beneficial to child development. Some health conditions, such as attention-deficit hyperactivity disorder or deafness for instance, can also have a direct, negative impact on cognitive or verbal ability, respectively. Studies have shown indeed that education levels in adulthood are to a large extent already determined during childhood. Measures of child development, such as cognitive and verbal ability, predict measures of education outcomes in adulthood, such as earnings and employment (see Currie (2000)).\footnote{At the same time, child development may be also related to a child’s socioeconomic background (see Taylor et al. (2004)). If so then children from disadvantaged families may fall behind early in life and may be unable to catch up later.} Salm and Schunk (2008) found that gaps in child development between socioeconomic groups can be explained by differences in child health. In a study of German data, they found that 18.4 percent of the gap in cognitive ability and 64.8 percent of that in verbal ability between children of college educated parents and less educated parents can be attributed to poor initial health conditions.\footnote{See also Oreopoulos et al. (2008), who found in a study for Canada that poor infant health is a strong predictor of future education outcomes.}
2.2 Impact of Education on Health

A significant body of research (at both the micro and macro levels) has also shown that higher education levels can improve health outcomes. The positive effect of education on health (just like the effect of health on education) works partly through income; but there are other channels as well.

Several studies have found that where mothers are better educated (and presumably more aware of health risks to their children), infant mortality rates are lower.\textsuperscript{10} Better-educated women tend, on average, to have more knowledge about health risks. For developing countries in general, Smith and Haddad (2000) estimated that improvements in female secondary school enrollment rates are responsible for 43 percent of the 15.5-percentage-point reduction in the child underweight rate recorded during the period 1970-95. For sub-Saharan Africa, it has been estimated that five additional years of education for women could reduce infant mortality rates by up to 40 percent (see Summers (1994)). In the cross-section regressions for developing countries reported by McGuire (2006), average years of female schooling have a statistically significant impact on under-five mortality rates. In Niger alone, researchers have found that infant mortality rates are lower by 30 percent when mothers have a primary education level, and by 50 percent when they have completed secondary education. Paxson and Schady (2007), in a study of Ecuador, found that the cognitive development of children aged 3 to 6 years is positively associated with the level of education of their mother.\textsuperscript{11} Of course, third factors could be at play as well; more educated women earn more and are more likely to live in urban areas, where access to health facilities, or nutritional supplements, is easier. But in many instances the positive effect of education on health persists even after controlling for location and other factors. Indeed, Wagstaff and Claeson (2004) found that an increase in female education reduces infant mortality and raises the survival rate for children, even after controlling for income effects.\textsuperscript{12}


\textsuperscript{11}Evidence that education affects health outcomes is also available for industrial countries; see for instance Cutler and Lleras-Muney (2006) and Altindag et al. (2011) for the United States.

\textsuperscript{12}As pointed out by Kohler and Soldo (2004), in practice, there are two potential channels that may relate parent’s education to children’s health and offsprings’ late life health outcomes: if it is the father’s education that is a stronger predictor of the child’s health, then this may indicate that
Aslam et al. (2012) studied the relationship between parental schooling and child health outcomes (height and weight) in Pakistan. They considered several mechanisms through which parental schooling may promote better child health: educated parents’ greater household income, exposure to media, literacy, labor market participation, health knowledge, and the extent of maternal empowerment within the home. They found that while father’s education is positively associated with the immunization decision, mother’s education is more critically associated with longer term health outcomes.\textsuperscript{13} There is also evidence suggesting that better educated individuals are more able to adopt healthy lifestyles and inspire their children to follow the same type of behavior (see Grossman and Kaestner (1997), Silles (2009), and Mullahy and Robert (2010)). For instance, Cutler and Lleras-Muney (2010) found that, controlling for several factors, better educated people in the United Kingdom and the United States are less likely to be obese, less likely to smoke, and less likely to be heavy drinkers. This is consistent with the evidence reviewed by Cutler and Lleras-Muney (2008), which suggests that increasing levels of education lead to different thinking and decision-making patterns.

Conversely, a low level of education may also lead to maternal malnutrition, with dire consequences for children. Inadequate intakes of nutrients during pregnancy have been found to have irreversible effects on children’s brain development, as noted earlier.\textsuperscript{14} Inadequate diets may have adverse effects on mental health as well (and therefore the ability to raise children), as argued in a report by the Mental Health Foundation (2006).

The foregoing discussion suggests that the causality between health and education can go both ways, and that taking into account these interactions is essential to study their joint effect on economic growth. Some of the evidence reported to earlier can education operates through economic circumstances (because fathers may be those who were the primary suppliers of economic resources in the family). If, in contrast, health status is determined to a large extent by the mother’s education, then this may suggest that education determines health through knowledge about health care and health behavior that are essential determinants of children’s health outcomes.

\textsuperscript{13}The gender dimension of the interactions between education and health is further discussed in the concluding remarks.

\textsuperscript{14}Research at the National institute of Health in the United States, for instance, has shown that the children of mothers who ate food with little omega-3 fatty acids had a lower IQ than children who did.
indeed be interpreted from the perspective of bidirectional causality. The results of Kohler and Soldo (2004) for instance, who found in a study of Mexico that individuals with low levels of education have higher mortality rates than better-educated individuals, may also be due to the fact that the level of education varies positively with health status. The next section presents a formal analysis of the interactions between education and health and their impact on economic growth.

3 The Model

Consider an OLG economy where a single good is produced and individuals live (at most) for three periods: childhood, adulthood and old age. They accumulate knowledge in the first period, supply labor in the second, and retire in the third. The good can be either consumed in the period it is produced or stored to yield capital at the beginning of the following period. Each individual is endowed with one unit of time in each period of life. Schooling is mandatory; children therefore devote all their time to schooling and depend on their parents for consumption. In middle age, individuals allocate their time between child rearing and the labor market. In old age, all time is devoted leisure. The only source of income is therefore wages in the second period of life, which serves to finance consumption in adulthood and old age. Savings can be held only in the form of physical capital. Agents have no other endowments, except for an initial stock of physical capital, $K_0$ at time $t = 0$, which is held by an initial generation of retirees.

Reproduction is asexual. In adulthood each individual bears $n \geq 1$ children, who are born with the same innate abilities. Keeping children healthy and fostering their education involves a cost, in terms of the parent’s time. Reproduction is asexual. In adulthood each individual bears $n \geq 1$ children, who are born with the same innate abilities. Keeping children healthy and fostering their education involves a cost, in terms of the parent’s time. Children mature safely into adulthood. At the end of the second period of life, there is a non-zero probability of dying. For children, education and health status depend on the time parents allocate to rearing their offspring, on the parent’s level of education or health, as well as access to public services. Health status in adulthood depends solely on the individual’s health in childhood. There is therefore state dependence in health outcomes. This specification is consistent with the evidence discussed earlier, according to which children who

\footnote{For simplicity, the direct cost of schooling and the cost of keeping children healthy (medicines, and so on) are abstracted from.}
experience poor health have on average significantly poorer health as adults.

In addition to individuals, the economy is populated by firms and an infinitely-lived government. Firms produce marketed goods using private capital and effective labor. The government spends on education, health, and some unproductive services. All government services are provided free of charge. Only the wage income of adults is subject to taxation. The government cannot borrow and therefore must run a balanced budget in each period. Finally, all markets clear and there are no debts or bequests between generations.

### 3.1 Individuals

At the beginning of their adult life in \( t \), each individual born at \( t - 1 \) bears 1 children. Population is thus constant. Raising a child involves a time cost; each parent devotes \( \varepsilon^R_t \in (0, 1) \) units of time to that activity, namely for home schooling and to take care of the child’s health (breast feeding, taking children to medical facilities for vaccines, and so on). Adults also allocate time, in proportion \( \varepsilon^W_t \), to working. The individual’s time constraint is thus

\[
\varepsilon^W_t + \varepsilon^R_t = 1. \tag{1}
\]

By implication, although access to “out of home” health and education services \textit{per se} are free, child rearing involves a cost in terms of foregone wage income and consumption.

Assuming that consumption of children is subsumed in their parent’s consumption, an individual’s expected lifetime utility at the beginning of period \( t \) is specified as

\[
U = \ln c_t^{t-1} + \eta_E \ln e_t^C + \eta_H \ln h_t^C + p \frac{\ln c_{t+1}^{t-1}}{1 + \rho}, \tag{2}
\]

where \( c_{t+j} \) denotes consumption in period \( t + j \), with \( j = 0, 1 \), \( \rho > 0 \) the discount rate, and \( p \in (0, 1) \) the probability of survival from adulthood to old age, which is taken as constant for the moment. Children’s education, \( e_t^C \), and health, \( h_t^C \), matter to parents. Coefficients \( \eta_E \) and \( \eta_H \) are both positive and measure the individual’s relative preference for children’s education and health, respectively.\(^{16}\)

\(^{16}\)If parents care equally about the health and education of their child, \( \eta_E = \eta_H \).
The period-specific budget constraints are

$$c^t_{-1} + s_t = (1 - \tau)a_t\varepsilon_t^W w_t,$$

where $a_t$ is individual labor productivity, $w_t$ the wage rate, $s_t$ saving, $r_{t+1}$ the rental rate of capital, and $\tau \in (0, 1)$ the tax rate. Equation (4) indicates that individuals consume at period $t + 1$ with probability $p$.\(^{17}\)

Combining these two equations yields the consolidated budget constraint

$$c^t_{-1} + \frac{pc^t_{-1}}{1 + r_{t+1}} = (1 - \tau)a_t\varepsilon_t^W w_t.$$  

Each individual maximizes (2), subject to (5), with respect to $c^t_{-1}$, $c^t_{t+1}$ and $\varepsilon^R_t$, with $\varepsilon^W_t$ solved for residually from (1). In a second step, parents allocate rearing time between education and health, in fixed proportions $\chi \in (0, 1)$ and $1 - \chi$, respectively. Thus, along the lines suggested by Guryan et al. (2008), time spent with children is an investment in their education and health outcomes.

### 3.2 Firms

There is a continuum of identical firms, indexed by $i \in (0, 1)$. They produce a single nonstorable good, which is used either for consumption or investment. Production requires the use of effective labor and physical capital, which firms rent from the currently old agents.

Assuming a Cobb-Douglas technology, the production function of firm $i$ takes therefore the form

$$Y^i_t = (A_t\varepsilon_t^{W,i}N^i_t)^{\beta}(\frac{K^i_t}{N})^{1-\beta},$$

where $K^i_t$ denotes the firm-specific stock of physical capital, $A_t$ average, economy-wide labor productivity (which is the same for all firms), $N^i_t$ the number of adult workers employed by firm $i$, $\varepsilon_t^{W,i}$ the time allocated by each individual to work at firm $i$, and

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\(^{17}\)Alternatively, it could be assumed that the saving left by individuals who do not survive to old age is confiscated by the government, which transfers them in lump-sum fashion to surviving members of the same cohort. The effective rate of return to saving would thus be $(1 + r_{t+1})/p$, which would yield an equation similar to (4). See Agénor (2012, Chapter 3) for a simple derivation.
\( \alpha, \beta \in (0, 1) \). Thus, production exhibits constant returns to scale in firm-specific inputs, effective labor \( A_t \varepsilon_t^{W,i} N_t^i \) and capital \( K_t^i \). However, there is a population externality; the greater the size of the adult population at \( t, N \), the lower the productivity of each firm’s capital stock. This congestion effect reflects the possibility that if more workers use fixed physical assets (such as roads, for instance) it becomes more difficult for each firm to use them (due to traffic jams, for instance). The magnitude of this congestion effect is measured by the parameter \( \phi_N \geq 0 \).

Markets for both physical capital and labor are competitive. Each firm’s objective is to maximize profits, \( \Pi^i_t \), with respect to labor services and capital, taking \( A_t \) and input prices as given:

\[
\max_{N_t^i, K_t^i} \Pi^i_t = Y_t^i - r_t K_t^i - w_t A_t \varepsilon_t^{W,i} N_t^i.
\]

Profit maximization yields

\[
w_t = \beta Y_t^i / A_t \varepsilon_t^{W,i} N_t^i, \quad r_t = (1 - \beta) Y_t^i / K_t^i,
\]

which implies that inputs are paid at their marginal product.

Given that all firms are identical, in a symmetric equilibrium \( N_t^i = N_t = \bar{N} \) and \( K_t^i = K_t, \forall i \). Thus, these conditions become

\[
w_t = \beta Y_t / A_t \varepsilon_t^{W,i} \bar{N}, \quad r_t = (1 - \beta) Y_t / K_t.
\]

Average productivity is given by

\[
A_t = E_t h_t,
\]

where \( E_t \) is the average stock of knowledge and \( h_t \) average adult health status. Thus, both education and health affect individual productivity. For tractability, a simple multiplicative form is used.\(^{18}\)

Because the number of firms is normalized to 1, aggregate output is given by, using (9),

\[
Y_t = \int_0^1 Y_t^i = \bar{N}^{\beta - \phi_N(1 - \beta)} \left( \frac{E_t}{K_t} \right)^{\beta} h_t^\beta K_t.
\]

\(^{18}\)A more general specification would be to set \( A_t = E_t^\chi H_t^{1-\chi} \), where \( \chi \in (0, 1) \).
To eliminate the scale effect associated with population requires setting $\beta - \phi_N (1 - \beta) = 0$, or equivalently $\phi_N = \beta / (1 - \beta)$. By implication, using (1),

$$Y_t = (1 - \varepsilon_t^R)^{\beta} x_t^\beta h_t^\beta K_t,$$

(10)

where $x_t = E_t / K_t$ is the knowledge-physical capital ratio (or, for short, the knowledge-capital ratio). Given that, as shown later, both $e_t$ and $h_t$, as well as $\varepsilon_t^R$, are constant in the steady state, the model is linear in the physical capital stock and exhibits therefore endogenous growth.

3.3 Schooling

The schooling technology depends on several inputs. First, it depends on the time allocated to education in childhood; as noted earlier, children must allocate all of their time to education, and this is normalized to unity. Second, knowledge accumulation is affected by the time allocated by parents to child rearing. As noted earlier, a sequential process is considered: parents determine first the total amount of time allocated to rearing their children, $\varepsilon_t^R$, and then subdivide that time into a fraction $\chi \in (0, 1)$ allocated to home schooling and $1 - \chi$ allocated to health care.

Third, knowledge accumulation depends on government spending on education, $G_t^E$, per child. Given that each individual has only one child, the total number of children is simply equal to the adult population, $N$. Fourth, it depends on the level of education of the parent. Because individuals are identical within a generation, parental education is taken to be equal to the average stock of knowledge of the current generation. Finally, to capture the health externality discussed earlier, schooling depends on how healthy the individual is in childhood.

Normalizing the adult population to unity, knowledge acquired in childhood, $e_t^C$, is thus given by

$$e_t^C = (G_t^E)^{\nu_3} E_t^{1 - \nu_1} (\chi \varepsilon_t^R)^{\nu_2} (h_t^C)^{\nu_3},$$

(11)

where $\nu_1 \in (0, 1)$ and $\nu_2, \nu_3 > 0$. For tractability, constant returns to scale are imposed on the education technology with respect to public spending per child and average parental knowledge. In addition, as in Hazan and Zoabi (2006) for instance, education and health are complements in the production of knowledge ($\partial^2 e_t^C / \partial E_t \partial h_t^C > 0$).
In adulthood, individuals do not engage in additional learning.\footnote{This assumption is consistent with the evidence for Sub-Saharan Africa for instance, which suggests that only 6.8 percent of youth engage in tertiary education, compared to a world average of 30 percent (United Nations (2016, p. 46)).} Assuming for simplicity no depreciation and full persistence in learning, the knowledge that each individual has in the second period of life, $e_{t+1}$, is therefore

$$e_{t+1} = e_t^C.$$ \hfill (12)

Substituting (11) in (12) yields

$$e_{t+1} = \left( \frac{G_t^E}{E_t} \right)^{\mu_1} E_t (\chi \varepsilon_t^R)^{\mu_2} (h_t^C)^{\mu_3}, \hfill (13)$$

Thus, because a parent’s education affects his children’s learning ability, there is serial dependence in knowledge. In addition, knowledge in adulthood also depends on health status in childhood.

### 3.4 Health Status

The health status of a child, $h_t^C$, depends on the amount of time allocated by each parent to rearing them, the average parent’s level of education and health, $E_t$ and $h_t$, respectively, and government expenditure on health, $G_t^H$, per child. This last effect captures for instance the impact of public spending on children’s nutrition, which reduces their vulnerability to disease and improves their health. Thus, health status in childhood is given by

$$h_t^C = \left( \frac{G_t^H}{K_t^{\phi_H}} \right)^{\theta_1} [1 - \chi] \varepsilon_t^R \left[ E_t^{\theta_2} h_t^{\theta_3} \right]^{\theta_4}, \hfill (14)$$

where all coefficients are positive. The externality associated with parental education is thus captured by $\theta_3$, whereas the external (intergenerational) effect associated with parental health is captured by $\theta_4$. With $\theta_4 < 1$, parental health exerts a diminishing marginal effect on a child’s health. In addition, the supply of public health services is congested by the stock of physical capital, with a congestion parameter $\phi_H > 0$.\footnote{See Osang and Sarkar (2008) and Agénor (2015). Of course, a similar argument could apply for the production of education services in (11). However, unlike health, knowledge does grow without bounds and the specification adopted in that equation is sufficient to ensure that.} As before, this congestion effect could represent the effect of an intensive use of a fixed
stock of public physical assets (such as roads or electricity) to produce goods, which makes it more difficult to access health facilities. Alternatively, the scaling of $G^H_t$ by $K^{PH}_t$ can be viewed as capturing the fact that greater economic activity (as proxied by the capital stock) has potentially adverse effects on children’s well being (as a result of air pollution for instance), which in turn mitigates the benefits of public spending on health.\footnote{Activity in this case could of course be measured by the level of final output, but given the linear relationship between $Y_t$ and $K_t$ implied by (10) the use of the latter is mainly a matter of convenience.}

Equation (15) can be rewritten as

$$h^C_t = \left(\frac{G^H_t}{K_t}\right)^{\theta_1}[(1 - \chi)\varepsilon_t^{R_t}]^{\theta_2} \left(\frac{E_t}{K_t}\right)^{\theta_3} h^C_{t-1} + \phi_{Ht} \theta_1.$$ 

To ensure that health status is stationary (as shown later), the restriction $\phi_{Ht} = (\theta_1 + \theta_3)/\theta_1$ is imposed. The above expression therefore becomes

$$h^C_t = \left(\frac{G^H_t}{K_t}\right)^{\theta_1}[(1 - \chi)\varepsilon_t^{R_t}]^{\theta_2} \left(\frac{E_t}{K_t}\right)^{\theta_3} h^C_{t-1}. \tag{15}$$

To capture the idea (discussed in the introduction) that cognitive deficits in early life may be impossible to reverse, and that health does not deteriorate over time, the health status of adults is assumed to depend only on their health status in childhood:

$$h_{t+1} = h^C_t. \tag{16}$$

Substituting (15) in (16) yields

$$h_{t+1} = \left(\frac{G^H_t}{K_t}\right)^{\theta_1}[(1 - \chi)\varepsilon_t^{R_t}]^{\theta_2} \left(\frac{E_t}{K_t}\right)^{\theta_3} h^C_t. \tag{17}$$

In the steady state, the public health spending-capital ratio, time allocated to child rearing, and the knowledge-capital ratio are all constant; health status is thus stationary as well. In contrast to knowledge, it does not grow without bounds. This is the fundamental difference, alluded to earlier, between education and health as sources of human capital.

### 3.5 Government

The government taxes only adults at the constant rate $\tau \in (0, 1)$ and spends a total of $G^E_t$ on education, $G^H_t$ on health, and $G^I_t$ on other (unproductive) items. It cannot
issue bonds and must therefore run a balanced budget:

$$G_t = G_t^E + G_t^H + G_t^U = \bar{N}\tau A_t \varepsilon_t w_t,$$  \hspace{1cm} (18)

Shares of spending are constant fractions of revenues:

$$G_t^h = v_h \bar{N}\tau A_t \varepsilon_t w_t, \quad h = E, H, U$$ \hspace{1cm} (19)

where $v_h \in (0, 1)$. Combining (18) and (19) therefore yields

$$\sum v_h = 1. \hspace{2cm} (20)$$

In sum, the model captures the possible bidirectional externalities associated with health and education, discussed in the previous section, through the parameters $\nu_3$ and $\theta_3$. If $\nu_3 = 0$, health generates no benefit in terms of childhood education, whereas if $\theta_3 = 0$ knowledge has no benefit in terms of health outcomes. Through the parameter $\theta_4$, the model captures also intergenerational persistence in health.

### 3.6 Market Clearing and Equilibrium

Given the assumption of full depreciation of the stock of physical capital, the asset market-clearing condition requires tomorrow’s capital stock to be equal to today’s aggregate savings:

$$K_{t+1} = \bar{N}s_t.$$ \hspace{2cm} (21)

The following definition may therefore be proposed:

**Definition 1.** A competitive equilibrium for this economy is a sequence of prices \(\{w_t, r_t\}_{t=0}^{\infty}\), income and time allocations \(\{c_t^{-1}, c_{t+1}^{-1}, s_t, \varepsilon_t^R\}_{t=0}^{\infty}\), physical capital stock \(\{K_{t+1}\}_{t=0}^{\infty}\), knowledge stock \(\{E_{t+1}\}_{t=0}^{\infty}\), health status of children and adults \(\{h^C_t, h_t\}_{t=0}^{\infty}\), a constant tax rate $\tau$ and constant spending shares $v_E, v_H$ such that, given the initial stocks $K_0, E_0 > 0$, individuals maximize utility, firms maximize profits, markets clear, and the government budget is balanced.

In equilibrium, individual productivity must also be equal to the economy-wide average productivity, so that $a_t = A_t$, and similarly for knowledge, so that $e_t = E_t$.

The following definition characterizes the balanced growth path:

**Definition 2.** A balanced growth equilibrium is a competitive equilibrium in which $c_t^{-1}, c_{t+1}^{-1}, Y_t, w_t, E_t,$ and $K_t$ all grow at the constant endogenous rate $1 + \gamma$, the rate of return on private capital $r_t$ is constant, and health status is constant.
4 Steady-State Growth

The solution of the individual’s maximization problem is provided in the Appendix. It shows that in equilibrium,
\[
\tilde{e}^R = \frac{(\eta_E\nu_2 + \eta_H\theta_2)(1 - \sigma)}{1 + (\eta_E\nu_2 + \eta_H\theta_2)(1 - \sigma)}, \quad \tilde{e}^W = 1 - \tilde{e}^R,
\]
where \(\sigma\) is the marginal propensity to save, defined as
\[
\sigma = \frac{p}{(1 + \rho) + p} < 1,
\]
(22)

From these solutions, it can be shown that an increase in the survival probability, \(p\), increases the savings rate, \(s\), lowers time allocated to child rearing, \(\tilde{e}^R\), and raises time allocated to market work, \(\tilde{e}^W\). The first result is fairly standard and consistent with the empirical evidence on longevity and fertility.\(^{22}\) Through a life-cycle effect, a higher adult survival rate dictates a need for higher savings to finance consumption in old age, and thereby has a positive effect, \(ceteris paribus\), on savings in adulthood. At the same time, an increase in the survival rate leads, \(ceteris paribus\), to less total time allocated to caring for children, as in Zhang and Zhang (2005), for instance. Thus, parents also increase their savings by allocating more time to market work.

The dynamic system driving the economy is also derived in the Appendix, in terms of two variables, health status in adulthood, \(h_t\), and the knowledge-capital ratio, \(x_t = E_t/K_t\). Specifically, the model can be condensed into a first-order linear difference equation system in \(\hat{h}_{t+1} = \ln h_{t+1}\) and \(\hat{x}_{t+1} = \ln x_{t+1}\) which (ignoring constant terms) can be written as
\[
\begin{bmatrix}
\hat{x}_{t+1} \\
\hat{h}_{t+1}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{x}_t \\
\hat{h}_t
\end{bmatrix},
\]
(24)
where
\[
a_{11} = 1 - \beta + \phi_1,
\]
\[
a_{12} = \phi_2 - \beta,
\]
\[
a_{21} = \beta\theta_1 + \theta_3 > 0, \quad a_{22} = \beta\theta_1 + \theta_4 > 0,
\]
\(^{22}\)See for instance Blackburn and Cipriani (2002) and Zhang and Zhang (2005).
and
\[
\phi_1 = -(1 - \beta)(\nu_1 + \theta_1 \nu_3) + (\theta_1 + \theta_3)\nu_3 \geq 0,
\]
\[
\phi_2 = \beta(\nu_1 + \theta_1 \nu_3) + \theta_4 \nu_3 > 0.
\]

As also shown in the Appendix, the balanced-growth rate of output per worker is given by
\[
1 + \gamma = \beta \sigma (1 - \tau) (1 - \hat{\varepsilon})^\beta \hat{x}^\beta \hat{h}^\beta,
\]
where \(\hat{x}\) and \(\hat{h}\) are the steady-state values of \(x_t\) and \(h_t\), which are solutions of the system
\[
\hat{x} = \begin{cases} 
\frac{\Lambda_1}{\beta \sigma (1 - \tau)} \left( \hat{\varepsilon}^R \nu_2 + \theta_2 \nu_3 (1 - \hat{\varepsilon}^R) \beta (\nu_1 + \theta_1 \nu_3) - 1 \right) \hat{h}^{\phi_2 - \beta} 
\end{cases}^{1/(\beta - \phi_1)},
\]
\[
\hat{h} = [\Lambda_2 (1 - \hat{\varepsilon}^R)^{\beta \theta_1} \left( \hat{\varepsilon}^R \right)^{\theta_2 \nu_3} x^{\beta \theta_1 + \theta_3} ]^{1/[1 - (\beta \theta_1 + \theta_4)]},
\]
where
\[
\Lambda_1 = (\nu_1^\gamma)^{\nu_2} (\nu_1^\gamma)^{\nu_3} (1 - \chi)^{\nu_2 \nu_3},
\]
\[
\Lambda_2 = (\nu_1^\gamma)^{\nu_1} (1 - \chi)^{\nu_2},
\]
In what follows it will be assumed that \(\theta_4 \in (0, 1)\) is not too large, to ensure that \(1 - (\beta \theta_1 + \theta_4) > 0\). But to make further progress, alternative cases regarding the externality parameters \(\theta_3\) and \(\nu_3\) must be considered.

**Case 1.** If there are no externalities of any sort, that is, \(\theta_3 = \nu_3 = 0\), then \(\phi_1 = -(1 - \beta)\nu_1 < 0\), so that \(\beta - \phi_1 > 0\) and, given that \(\nu_1 \in (0, 1)\), \(\phi_2 - \beta = \beta (\nu_1 - 1) < 0\).

**Case 2.** If there is only an education externality for health, that is, \(\nu_3 = 0\) and \(\theta_3 > 0\), then again \(\phi_1 = -(1 - \beta)\nu_1 < 0\), \(\beta - \phi_1 > 0\), and \(\phi_2 - \beta = \beta (\nu_1 - 1) < 0\).

**Case 3.** If there is only a health externality for education, that is, \(\nu_3 > 0\) and \(\theta_3 = 0\), then \(\phi_1 = -(1 - \beta)(\nu_1 + \theta_1 \nu_3) + \theta_1 \nu_3 \beta (\nu_1 + \theta_1 \nu_3) - \nu_1 \geq 0\), \(\beta - \phi_1 = -\beta [(\nu_1 + \theta_1 \nu_3) - 1] + \nu_1 \geq 0\), and \(\phi_2 - \beta = \beta [(\nu_1 + \theta_1 \nu_3) - 1] + \theta_4 \nu_3 \geq 0\).

**Case 4.** If both types of externalities are present, that is, \(\nu_3, \theta_3 > 0\), then for \(\beta - \phi_1 > 0\), it must be that \((\theta_1 + \theta_3) \nu_3 < \beta + (1 - \beta)(\nu_1 + \theta_1 \nu_3)\), whereas for \(\phi_2 - \beta < 0\) it must be that \(\nu_3 < \beta (1 - \nu_1)/(\beta \theta_1 + \theta_4)\).

\(^{23}\)Using \(\theta_1 = 0.55\), as in Osang and Sarkar (2008, Table 4) for instance, and a standard value of \(\beta = 0.65\), this condition implies that \(\theta_4\) cannot be higher than 0.64.
In what follows, the focus will be on the two opposite cases 1 and 4, assuming in the latter that the combination of \( \nu_3 \) and \( \theta_3 \) is such that both restrictions are satisfied.\(^{24}\)

To establish the signs of \( a_{11} \) and \( a_{12} \), note that in Case 1,

\[
a_{11} = 1 - \beta + \phi_1 = (1 - \beta)(1 - \nu_1) > 0,
\]

\[
a_{12} = \phi_2 - \beta = \beta(\nu_1 - 1) < 0,
\]

whereas in Case 4, given that again \( \beta - \phi_1 > 0 \), then \(-\beta + \phi_1 < 0\), and thus, \( a_{11} = 1 - \beta + \phi_1 \geq 0 \). And given that \( \phi_2 - \beta < 0 \) then \( a_{12} < 0 \).\(^{25}\)

Equations (26) and (27) define the steady-state relationships between \( x_t \) and \( h_t \). In both Cases 1 and 4, \( \beta - \phi_1 > 0 \) and \( \phi_2 - \beta < 0 \). Equation (26) defines a convex curve depicted as XX in Figure 1, whose slope is negative and given by \( (\phi_2 - \beta)/(\beta - \phi_1) < 0 \). Similarly, Equation (27) defines a curve depicted as HH, whose slope is positive and given by \( [1 - (\beta\theta + \theta_4)]/(\beta\theta_1 + \theta_3) > 0 \).\(^{26}\) It is immediately clear from the shape of these curves that there is a unique equilibrium, located at point \( E \). The knowledge-physical capital ratio and health status in adulthood are thus both constant in the steady state. As shown in the Appendix, for empirically plausible values of the parameters \( \beta \), \( \theta_4 \), \( \nu_1 \) and \( \theta_1 \), the equilibrium is stable in Case 1 (where there are no externalities, or, more generally, when these externalities are weak), as well as in Case 4.\(^{27}\)

Before studying the effects of public policy in this setting, it is worth noting the conflicting effects of time allocated to child rearing, \( \bar{\varepsilon}^R \), on the steady-state solutions (26) and (27), and thus on the steady-state growth rate (25). On the one hand, increased time devoted to child rearing improves health and education, which raise productivity, but on the other it reduces time allocated to market work. This trade-off

\(^{24}\)Note that Case 2 is qualitatively very similar to Case 1. An exhaustive analysis of all cases would require a numerical calibration.

\(^{25}\)There are also intermediate cases, where one type of externality is high and the other low, which are ignored for the moment to facilitate the exposition of the graphical analysis.

\(^{26}\)Curve HH can be either concave or convex, depending on whether \( [1 - (\beta\theta_1 + \theta_4)]/(\beta\theta_1 + \theta_3) \geq 1 \). For illustrative purposes, it is shown as concave in Figure 1. The difference between Cases 1 and 4, of course, is that the slopes of the two curves would different, depending on the values of \( \nu_3 \) and \( \theta_3 \). However, this difference is inconsequential for a qualitative analysis.

\(^{27}\)Note that if \( \phi_2 = \beta \) then \( a_{12} = 0 \) and system (24) is recursive; the dynamics are in terms of \( \hat{x}_t \) only. Then stability requires \( a_{11} = 1 - \beta + \phi_1 < 1 \), or \( \phi_1 < \beta \). If \( \nu_3 = 0 \), then this condition becomes \( \beta(\nu_1 - 1) < 1 \) which is always satisfied.
becomes clear when considering, as is done later, the case of an endogenous survival rate.

5 Public Policy

Equations (22), (23), as well as (25)-(27) can be used to study the impact of changes in the shares of government spending on time allocation and growth, assuming that these increases are either budget neutral and financed by a cut in unproductive spending \((dv_h + dv_U = 0, h = E, H)\) or instead by a cut in the other component of productive spending \((dv_E + dv_H = 0)\).\(^{28}\) In the latter case, there is a trade-off between the two components of expenditure, which can be internalized by solving for the growth-maximizing share of one of them. These issues are considered in turn, with a focus on steady-state effects rather than transitional dynamics.

5.1 Changes in Government Spending

Consider first a budget-neutral increase in public spending in education, financed by a cut in unproductive expenditure \((dv_E + dv_U = 0)\). The results are illustrated in Figure 2. Curve \(XX\) shifts to the right, whereas curve \(HH\) does not change. The equilibrium moves from point \(E\) to point \(E'\), implying that the outcome is both an improvement in health status and a higher knowledge-physical capital ratio. Consequently, the steady-state growth rate, as can be inferred from (25), increases unambiguously. The stronger the externalities associated with education and health, the stronger these effects are.

Consider now a budget-neutral increase in health spending, again financed by a cut in unproductive expenditure \((dv_H + dv_U = 0)\). The results are illustrated in Figure 3. Curve \(XX\) shifts to the right, whereas curve \(HH\) shifts down. However, there are now two cases to consider, depending on the magnitude of the shift in these curves. In both cases, while health status always improves, the net effect on education outcomes is ambiguous.

Indeed, Scenario A depicts the case where \(XX\) shifts strongly, to \(X'X'\), relatively

\(^{28}\) A variety of other experiments could also be conducted, such as for instance a change in parental time allocated between the health and education needs of their children, that is, a change in \(\chi\). These experiments are left to the interested reader.
to $HH$. The new equilibrium is at $E'$, characterized (as in the case of higher spending on education), by an improvement in both health status and education. However, the figure also illustrates the case where $XX$ shifts relatively little, to $X''X''$, so that the new equilibrium is at $E''$. In this scenario, which is likely to occur (in particular) when the externality of health for education is low, health status improves but education outcomes deteriorate. Similarly, Scenario B corresponds to the case where $HH$ can either shift a little (to $HH'$, so that the new equilibrium is at $E'$) or a lot (to $HH''$, so that the new equilibrium is at $E''$). In the first scenario, both education and health outcomes improve, whereas in the second the only benefits are in terms of health status. When the effects on education outcomes are negative, the net effect on growth is ambiguous—even if changes in government spending are financed by cuts in unproductive spending. As shown in the Appendix, outcome $E''$ corresponds to the case where the health externality for education is weak, whereas outcome $E'$ corresponds to the case where that externality is sufficiently strong. In addition, outcomes $C$ in Scenario A, and outcomes $C'$ and $C''$ in Scenario B, correspond to the case where $XX$ does not change at all, which is what occurs when there is no health externality for education, that is, $\nu_3 = 0$.

Intuitively, the reason why the effect on the human-physical capital ratio is ambiguous when health spending is increased is as follows. The direct effect of higher spending on health is an improvement in health status and productivity, which raises output, and therefore government spending across the board. This has a positive effect on both education and health outcomes. However, at the same time the increase in income raises savings and investment, and therefore the stock of physical capital as well. The latter effect dominates, so that the human-physical capital ratio falls. By contrast, when the health externality for education is sufficiently strong, the increase in knowledge dominates, and the human-physical capital ratio increases. Thus, whether the net effect on the steady-state growth rate of output is negative or positive cannot be determined a priori. But the stronger the direct effect of health spending on health status, or the stronger the health externality, the more likely it is that an increase in government spending on health will lead to higher growth.
5.2 Growth-Maximizing Policy

However, when there is a trade-off in spending, that is, when $d v_E + d v_H = 0$, the results are not clear cut. Indeed, from (25)-(27), the growth-maximizing share of government spending on education is given by setting $d \ln (1 + \gamma) / dv_E = 0$, that is,

$$\frac{d \ln (1 + \gamma)}{dv_E} \bigg|_{dv_E+dv_H=0} = \beta \frac{d \ln \tilde{x}}{dv_E} \bigg|_{dv_E+dv_H=0} + \beta \frac{d \ln \tilde{h}}{dv_E} \bigg|_{dv_E+dv_H=0} = 0.$$

As shown in the Appendix, it can be established that the growth-maximizing share of spending on education is given by

$$v^*_E = \frac{b_{13}(1 - b_{21})}{(b_{13} + b_{14})(1 - b_{21}) + b_{24}(1 - b_{12})} < 1,$$

where, in both Cases 1 and 4,

$$b_{12} = -\frac{\phi_2 - \beta}{\beta - \phi_1} > 0, \quad b_{13} = \frac{\nu_1}{\beta - \phi_1} > 0, \quad b_{14} = \frac{\theta_1 \nu_3}{\beta - \phi_1} > 0,$$

$$b_{21} = -\frac{\beta \theta_1 + \theta_3}{1 - (\beta \theta_1 + \theta_4)} < 0, \quad b_{24} = \frac{\theta_4}{1 - (\beta \theta_1 + \theta_4)} > 0,$$

which imply that

$$1 - b_{12} = \frac{\phi_2 - \phi_1}{\beta - \phi_1}, \quad b_{13} + b_{14} = \frac{\nu_1 + \theta_1 \nu_3}{\beta - \phi_1} > 0, \quad 1 - b_{21} = \frac{1 + \theta_3 - \theta_4}{1 - (\beta \theta_1 + \theta_4)} > 0.$$

Formula (30) is quite complicated in general. To make further progress in assessing the role of externalities, a simple numerical exercise can be performed. Parameter values are set at $\beta = 0.65$ (a standard value), $\theta_4 = 0.6$ (to ensure that $1 - (\beta \theta_1 + \theta_4) > 0$), $\nu_1 = 0.55$ (as in Osang and Sarkar (2008)), $\theta_1 = 0.55$ (for symmetry), and $\nu_3 = \theta_3 = 0$ initially. Thus, as implied by (30), with no externalities of any sort, $v^*_E = 0.421$. Using the same values, with $\nu_3 = 0$ and $\theta_3 = 0.4$ yields $v^*_E = 0.593$, whereas with $\nu_3 = 0.4$ and $\theta_3 = 0$ the result is $v^*_E = 0.296$. With $\nu_3 = \theta_3 = 0.4$, then $v^*_E = 0.457$. More generally, Figure 4 illustrates how the optimal share of spending on education varies when $\nu_3$ and $\theta_3$ vary between 0 and 1. Thus, when only the externality of education for health is positive, the larger the share of spending on education should be (or the lower the share of health spending should be). Conversely, the stronger the health externality for education, the lower should be the share of spending on education. When the externalities are equally strong, the effect is not exactly symmetric; it is still optimal to spend a bit more on education.
6 Endogenous Survival Rate

In the foregoing analysis the survival rate, \( p \), was assumed exogenous. Suppose now, as in Tang and Zhang (2007), Osang and Sarkar (2008), and Agénor (2015) for instance, that life expectancy is endogenous and related directly to health status.\(^{29}\) To capture this link, one approach is to relate the survival rate directly to the individual’s own health status. In solving their optimization problem, parents would then internalize the implications of their time allocation decisions.

An alternative, simpler approach is to assume that the survival probability of any particular individual depends on average health status in the economy—which, in equilibrium, is of course the same for all individuals. Thus, when choosing their consumption and time allocation, agents continue to take \( p \) as given and the solutions derived earlier continue to apply.

Suppose then that the adult survival rate is a piece-wise function defined as

\[
p_t = \begin{cases} 
  p_m < p_M \text{ for } h_t < h_L, \\
  f(h_t) \text{ for } h_L \leq h_t < h_H, \\
  p_M < 1 \text{ for } h_t \geq h_H, 
\end{cases}
\]

(31)

where \( f' > 0 \) and \( f'' < 0 \). Thus, if health status is below \( h_L \), the likelihood of surviving to old age is \( p_m \). In the context of poor countries, this could reflect the fact that at first, improvements in health status do not translate into higher survival rates. As health status improves above that threshold, the relationship between \( p_t \) and \( h_t \) is positive and concave over the range \((h_L, h_H)\).\(^{30}\) It becomes constant again at \( p_M \) and \( p_m < p_M < 1 \) for values of health status above \( h_H \). Put differently, beyond a certain point, further changes in health status have no effect on the probability to survive—perhaps reflecting the fact that there always remains a risk of accidental death.\(^{31}\)

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\(^{29}\)Some other contributions which focus on knowledge accumulation, such as Blackburn and Cipriani (2002), Cervellati and Sunde (2005), Castelló-Climent and Rafael Doménech (2008) for instance, have assumed that life expectancy is related to education. This can be justified given that, as noted earlier, improved knowledge can lead for instance to changes in lifestyle that may lead to better health outcomes. In the present setting, a more general approach, of course, would be to consider jointly education and health status as determinants of life expectancy. However, this would complicate significantly the analysis and would detract from the main contribution of this paper.

\(^{30}\)A simple functional form for \( f \) could be the exponential function, that is, \( p_t = 1 - 1/\exp(h_t) \).

\(^{31}\)As noted in Agénon (2015), in the model health status can be interpreted as a broad measure of health, such as the body mass index (BMI). From that perspective, the thresholds \( h_L \) and \( h_H \)
The implication of this analysis is as follows. Consider first the case where initially health status is at or below $h_L$, so that $p$ is constant. Suppose that an ambitious increase in spending on health, financed by a cut in unproductive spending, leads to an improvement in health status (despite having ambiguous effects on education outcomes) and that this increase is large enough to move the economy into the intermediate range $(h_L, h_H)$. The survival rate therefore increases, which raises the savings rate and time allocated to market work, thereby promoting growth. However, in this setting rearing time is productive; it benefits both education and health outcomes. A reduction in rearing time may therefore have adverse effects on these variables, despite higher government spending. Moreover, these effects may be magnified if externalities are strong. As a result, the net effect on growth can be ambiguous—even if the increase in public spending on health is offset by a cut in unproductive spending.$^{32}$ Conversely, it is also possible that the net effect on growth is positive; a health subsidy can help move the economy from a low-growth equilibrium to an equilibrium with a higher saving rate, higher life expectancy, and faster growth. If the direct effect on health status is positive, a strong externality of health on education would increase the likelihood of a transition from stagnation to growth. This result is thus consistent with those of Tang and Zhang (2007), albeit in a model where health and education externalities are not accounted for, and Hazan and Zoabi (2006), who emphasize (as is the case here) the importance of a sufficiently high degree of complementarity between health and education in the production of knowledge.$^{33}$ In addition, even if an increase in spending on health is financed by a cut in education, it is still possible for the net effect on growth to be positive if the health externality for education is strong.

Finally, it is worth noting that, with $p$ constant, the values of $\nu_2$ and $\theta_2$ (which measure the effect rearing time on education and health outcomes, respectively) do

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$^{32}$Based on the previous discussion, if the increase is public spending on health is financed by a cut in spending on education, the possibility of an adverse effect on growth would be magnified.

$^{33}$Hazan and Zoabi (2006), however, focus on private expenditure on health and education, not public spending.
not matter for stability, as shown in the Appendix. However, when \( p \) is endogenously related to health status, \( h \), it affects the savings rate and thus time allocated to child rearing, \( \bar{R} \), which therefore becomes endogenous—and so does time allocated to market work. Consequently, the stability conditions discussed in the Appendix would be more complicated. In addition, this would affect the slopes of \( XX \) and \( HH \) in Figures 1 to 3, as well as the impact of changes in government spending on growth, as discussed earlier. Finally, with an endogenous survival rate and time allocation, the solution of the growth-maximizing problem would also become highly nonlinear and establishing an explicit formula, as in (30), would not be feasible.

7 Concluding Remarks

Education and health are two important dimensions of human capital. The purpose of this paper was to review the evidence on the interactions between these two dimensions, present an endogenous growth model that captures them, and study the impact of public policy in that setting. A key feature of the model is that health is distinct from knowledge as a source of human capital because it cannot grow without bounds. In addition, as suggested by the evidence, causality can go both ways: policies that impact educational attainment could have a large effect on health outcomes, and vice versa. The model also accounts for the fact that there is substantial evidence to suggest that parental health affects the health of children at birth, and that health in late life is the outcome of a cumulative process of exposure to health risks in childhood.

The analysis showed that growth dynamics depend in critical ways on the externalities associated with knowledge and health. Depending on the strength of these externalities, the best policy to improve education outcomes may actually be to spend relatively more on children’s health. Trade-offs between education and health spending can be internalized by setting the optimal composition of expenditure so as to maximize the growth rate. All else equal, the stronger the health (education) externality in education (health), the smaller (larger) the share of spending on education should be. With an endogenous adult survival rate, multiple growth paths may emerge. A reallocation of public spending from education to health may shift the economy from
a low-growth equilibrium to a high-growth path. However, if the time allocation effect associated with an endogenous increase in the survival probability—a reduction in time allocated to child rearing, which on the one hand leads to an increase in time allocated to market work, but on the other may adversely affect education and health outcomes, and thus productivity—it is possible (theoretically at least) that an increase in government spending may have an adverse effect on growth.

The analysis presented in this paper can be extended in several directions. First, at the empirical level, it would be useful to conduct a detailed cross-country econometric analysis of a simultaneous determination of schooling or education levels, health outcomes and economic growth. This will allow an assessment of the magnitude of the externalities associated with education and health, and provide some of the key parameters needed for a full-blown calibration of the model, in order to study numerically its properties.

Second, the fertility rate could be endogenized, to assess how changes in health outcomes can affect the decision to have children. Based on the results in Agénor (2015), one can infer what happens in that case: an increase in the survival rate (due to an improvement in health status, itself related to higher spending on health, as discussed earlier), would reduce the fertility rate and total time allocated to child rearing. The effect on unit rearing time, however, is likely to be ambiguous. Intuitively, the reduction in the fertility rate allows parents to allocate more time to each of them to improve their health—even though total time devoted to child rearing falls—in effect, substituting quality to quantity. Because changes in rearing time have persistent effects on health and education, they would also alter in significant ways the dynamics of the economy and the possibility of multiple equilibria.

Finally, the analysis could be extended by introducing a gender dimension. This would allow, in particular, to study how the level of knowledge of each parent (which may differ due to discrimination, both at home and in the market place, against women) affects education and health outcomes for their children. For instance, Breierova and Duflo (2004), in a study of Indonesia, found that female and male education seem equally important factors in reducing child mortality. However, in a study of Pakistan, Aslam et al. (2012) found that while father’s education is positively associated with
the immunization decision, mother’s education is more critically associated with longer term health outcomes. Accounting for a gender dimension would help to consider how a broader set of policies can affect education and health outcomes, as well as, ultimately, economic growth.
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Figure 1
Balanced Growth Equilibrium
Figure 2
Increase in Government Spending on Education
Figure 3
Increase in Government Spending on Health

Scenario A

Scenario B
Figure 4
Externalities and Growth-Maximizing Share of Spending on Education

\[\text{Diagram showing a 3D surface plot with axes labeled } \nu_3 \text{ and } \theta_3.\]