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# **Credit Risk, Excess Reserves and Monetary Policy: The Deposits Channel**

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# Credit Risk, Excess Reserves and Monetary Policy: The Deposits Channel\*

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## Abstract

This paper examines the role of *precautionary liquidity* (reserves) and the *interest on reserves* as two potential determinants of the *deposits channel* that can help explain the role of monetary policy, particularly at the near zero-bound. Through the *deposits channel* either of these two determinants can explain a number of effects including, (i) zero-bound optimal policy rates, (ii) a negative deposit rate spread, but also (iii) determinacy at the lower-zero bound. Similarly, through its effects on the *deposits channel* the interest on reserves can act as the main tool of monetary policy, that is shown to provide higher welfare gains than a simple Taylor rule. This result is shown to hold at the zero-bound and it is independent of precautionary liquidity, or the fiscal theory of the price level.

*JEL Classification Numbers:* E31, E32, E44, E52, E50, G28

*Keywords:* Deposits channel; zero-bound monetary policy; excess reserves; credit risk; balance sheet channel; interest on reserves; required reserve ratio; welfare; DSGE models.

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# 1 Introduction

In the aftermath of the 2007 financial crisis, and particularly during the period 2008-2015, the US economy was characterized by very high excess reserves, a zero-bound policy rate, with a relatively lower focus on inflation, and the introduction and management of the interest on reserves. During this period there were large reductions in the loan-to-deposit ratio and deposit rates exceeded the policy rate.<sup>1</sup> The deposits market may provide a significant channel in explaining the role of monetary policy at the time, but so far this channel has received little attention.<sup>2</sup> Drechsler, Savov and Schnabl (2017) provide empirical evidence that in general the deposits channel can account for the entire transmission of monetary policy through bank balance sheets. They identify market power in the deposit markets driven by changes in the policy rate as the potential determinant of the deposits channel. Higher policy rates feed into the market power of banks which do not pass this increase on to deposit rates. This increases the policy-deposit rate spread resulting in deposit outflows and a contraction of loans and economic activity and vice versa. This mechanism however, may fall short of explaining the lower zero-bound period 2008-2015, when despite the very low policy rates, with even negative deposit rate spreads, and despite the loss in their market power, banks channeled their increased deposits into excess reserves, thus reducing loans and economic activity. In this paper, I propose two new potential determinants of the deposits channel that can help explain the role of monetary policy, particularly at the lower zero-bound period. These are, (i) *precautionary liquidity* due to credit risk and (ii) the *interest on reserves*. These two factors are shown to be more relevant determinants of the *deposits channel* for the lower zero bound period, as they both came into prominence just as the policy rate became ineffective. Through the *deposits channel* each of these factors, (precautionary liquidity, or the interest on reserves) can explain, (a) very low, or zero, optimal weights on inflation in the Taylor rule, (b) a negative deposit rate spread, and significantly (c) determinacy at the lower-zero bound. The theoretical findings in this paper can shed some light on the zero-interest rate policy pursued in countries that experienced large excess reserves, (i.e. Japan, U.S. and the UK), but also on why the interest on reserves was given a new focus by the US Fed and other central banks during the zero-bound period.

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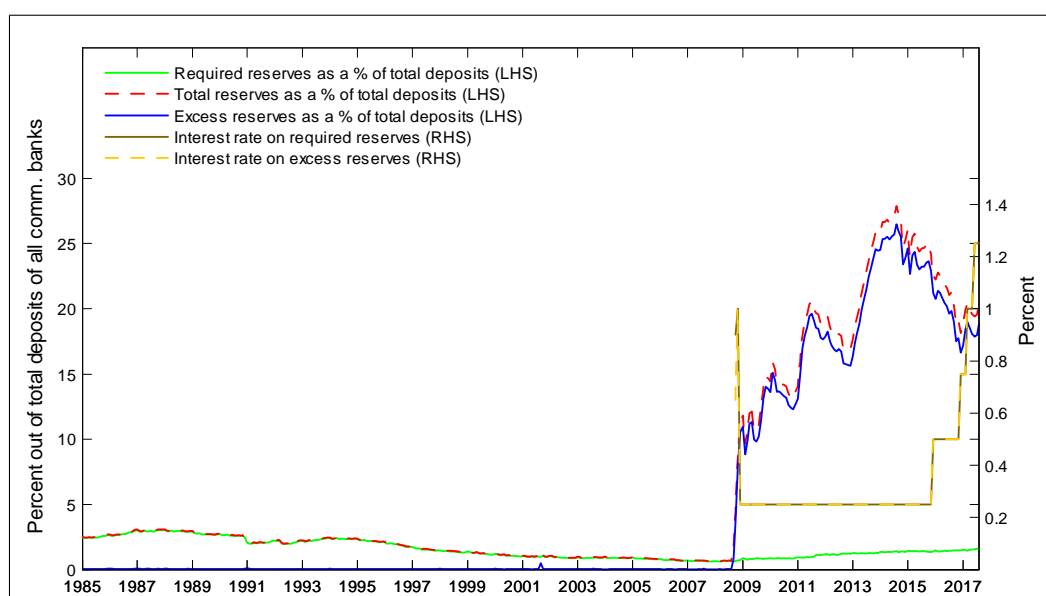
<sup>1</sup>This was the experience in bank deposits of both the US, (see also Drechsler, Savov and Schnabl 2017), and the UK (see McLeay and Thomas 2016). Other liquid forms of money, such as the 12-month CDs, also experienced negative deposit spread during this period.

<sup>2</sup>By the ‘deposits channel’, this paper refers to how changes in the level of deposits and deposit rates can affect real economic activity and the business cycle, both directly (by affecting household decisions) and through the bank balance sheets (and loans)

## 2 Background and Related Literature

At the centre of the Fed’s policy focus following the 2007 financial crisis was the unprecedented levels of excess reserves and liquidity hoarding in the banking sector. The gradual quantitative easing between 2008-2013 contributed largely to the increased level of excess reserves (see Figure 1) and the dramatic drop in the loan-to-reserve ratio, (Figures 2 a and b). However, the accumulation of excess reserves was not only the outcome of quantitative easing. There is sufficient evidence to suggest that during that period banks appeared more willing to hoard reserves rather than lend (see below). This is also clear from the loan-to-deposit ratio shown in Figures 2 a and b to be falling for all loans, including Commercial & Industrial loans. Indeed, the fall in loans was not just confined in the mortgage credit market, that had just been shocked with the subprime mortgage crisis, but was also evident in the production sector. With the policy rate fixed at the lower zero-bound during that period, this was an indication that other factors, beyond monetary policy or the large-scale quantitative easing, or confidence in the housing market, were contributing to the accumulation of excess reserves and the deposits channel.

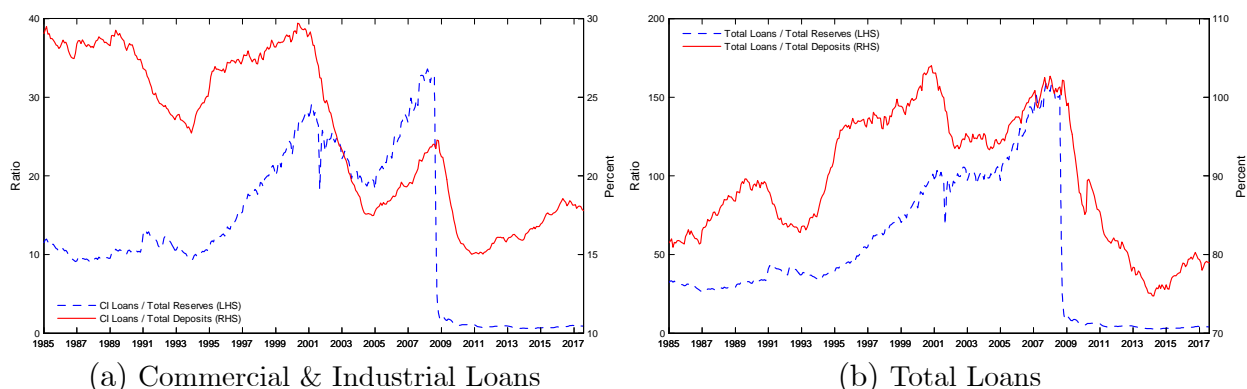
Figure 1: Interest on Reserves and Required and Excess Reserves



Source: <https://fred.stlouisfed.org/series>

On the supply side, the observed high levels of excess reserves during that period have been attributed to *precautionary reasons*, or *liquidity risk*, that prompted banks to hoard their reserves and lend their surplus reserves to the inter-bank market at a high liquidity

Figure 2: Loans as a Share of Deposits and Reserves



premium, which raised the interbank spread and reduced the volume of loans: a *bank balance sheet channel* effect, (i.e. Kashyap and Stein, 2000, Allen, Carletti and Gale, 2009, Gertler and Kiyotaki, 2011, Acharya and Skeie 2011, Ashcraft, McAndrews and Skeie, 2011, Bianchi and Bigio, 2014, Heider, Hoerova and Holthausen, 2015). On the demand side, the increased borrowing cost, combined with a drop in economic activity and the expectations of a looming recession, made households and firms also unwilling to demand loans. Thus, a fall in the demand for loans in the non-financial sector is also believed to have contributed to the increase in excess reserves. This is a *firm balance sheet channel* that affects the cost of borrowing in relation to the firm’s net worth or collateral (Bernanke and Gertler 1995, Bernanke, Gertler and Gilchrist, 1999, Gertler and Kiyotaki, 2011, De Fiore and Tristani, 2011). Empirical evidence suggests that both the supply of loans and the demand for loans channels were operational during that period. The demand for credit channel was more important for the households, but the supply side channel was more important for firms and the production sector.<sup>3</sup> Overall, the impact of monetary policy was shown to be more significant through precautionary liquidity and the credit supply channel in general, than the demand for credit channel (see Angelini, Nobili and Picillo 2011, Jiménez and Ongena 2012, Bianchi and Bigio, 2014, Ciccarelli, Maddaloni, and Peydró, 2015).

<sup>3</sup>The supply side also exacerbated the recession by the way that the decline in the value of bank liabilities also reduced the level of insurance and thus collateral such liabilities provided to their holders, (Quadrini, 2017).

The second factor that attracted much policy attention when excess reserves were high was the interest on reserves. In the aftermath of the financial crisis, when the US Fed was trying to manage liquidity at zero-bound policy rates, it placed much emphasis on the interest on reserves and very little on the required reserve ratio, which remained around its steady state level throughout that period (see Figure 1).<sup>4</sup> Figure 1, shows that following the introduction of interest on reserves in 2008, along with quantitative easing, excess reserves increased dramatically. However, even when the interest on reserves was reduced from 100 to 25 basis points to encourage bank lending, excess reserves continued to accumulate and even after 2013, when the quantitative easing programme had been completed, excess reserves kept rising and remained very high. Throughout that period, although the interest on reserves did not appear to be the main determinant of the level of excess reserves the Fed continued to rely on it as one of its key monetary policy tools.

### **3 This paper: Innovations and Main Results**

This paper focuses on the above observations and particularly on the fact that during the period 2008-2015 the US economy was characterized by,<sup>5</sup> (1) increased precautionary reserves and loan hoarding; (2) a persistent drop in loans, initially from the supply side and the balance sheet channel, but later also from the demand side as the recession started to set in; (3) a negative deposit rate spread, as the deposit rate exceeded the policy rate, (4) a zero-bound policy rate and overall a relatively lower focus on inflation and (5) emphasis on a new monetary policy tool, the interest on reserves, which was preferred over the required reserve ratio. Although the recent literature provides plenty of support for (1) and (2) (see above), there has been little theoretical rationale for facts (3), (4) and (5), particularly in combination with observations (1) and (2). This paper attempts to bridge this gap.

The starting point is the existing evidence on (1) and (2) (see above literature), which indicates that the accumulation of excess reserves was not purely the outcome of the quantity easing programme, but also of precautionary reserves and persistent loan hoarding. To examine the latter effect, I introduce a DSGE model where both the credit supply and the demand for credit channels are operational, but where the credit supply side, and particularly the deposits channel, is shown to be more important for monetary policy, particularly at the zero-bound. The credit risk lies in the non-financial production sector,

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<sup>4</sup>Similar policies were adopted by the Bank of England, Bank of Japan, Bank of Canada, the ECB and other central banks in recent years.

<sup>5</sup>These observations were shared by other advanced economies including the UK.

but this risk can create an accumulation of precautionary liquidity (reserve hoarding) in the financial sector (deposits market). This is in contrast to the bulk of the above literature where the risk lies in the interbank sector and it is usually modelled as random bank withdrawals, (i.e. Ashcraft, McAndrews and Skeie, 2011, Bianchi and Bigio, 2014).<sup>6</sup> For generality, I model the risk in the production of intermediate goods, although the results are expected to be robust when the risk lies anywhere in the non-financial sector, including the housing market. The key point is that when the financial sector perceives an underlying risk in the non-financial sector, excess reserves rise and this decreases the supply of loans to firms in the non-financial sector through a change in the financial sector's balance sheet, while at the same time the rate offered on liquid assets, the deposit rate here, rises relatively to the policy rate. Risk is also shown to increase the risk premium on the loan rate and thus the cost of borrowing of firms, generating a cost channel. Various policy scenaria are examined that are assessed in terms of welfare, where welfare is calculated based on a second order approximation of the household's utility function.

The results show that through the deposits channel, both precautionary liquidity and the interest on reserves are key determinants of monetary policy. It is shown that when banks hoard large amounts of excess reserves in response to a potential risk in the non-financial sector, the optimal response of the Taylor rule is to set a zero weight on the inflation rate, which implies a lower zero-bound policy rate. This is shown to be a *determinate* outcome despite the violation of the *Taylor principle*. The key to this result is the fact that at times of high perceived credit risk, precautionary reserves rise but so does the deposit rate offered by banks in relation to the policy rate (and interbank rate), as banks want to avoid deposit outflows. The deposit rate is the rate at which households discount consumption and other portfolio decisions. With a high precautionary liquidity, the deposit rate can increase sufficiently to control the Euler equation and the output gap and hence inflation and provide *determinacy* even when the policy rate is at the zero-bound. This case also implies a negative deposit rate spread. Intuitively, through accumulated precautionary liquidity and a reduction in lending, the markets self-discipline the macro economy, reducing the output gap and inflation.

Within this framework the paper also examines the role of the interest on reserves. It is shown that the use of the interest on reserves can facilitate monetary policy (Taylor rule) and increase welfare more than the required reserve ratio. This is because the latter

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<sup>6</sup>In reality, unlike the Great Depression in the 1930's, where cash withdrawals increased dramatically, pushing up the currency ratio to very high levels, the latter ratio even fell during the Great Recession, pointing to no substantial evidence of large bank withdrawals during the recent crisis.

is shown to have conflicting effects through the *deposits channel*, whereas the interest on reserves has complementary effects through this channel.

More importantly, it is shown that through the *deposits channel* the interest on reserves can act as the main tool of monetary policy, even at the zero-bound, producing higher welfare gains than a simple Taylor rule. Recently, Cochrane (2014) and Hall and Reis (2016) show that in models that share the properties of the *fiscal theory of the price level*, the interest on reserves can determine inflation (or the price level). Here, I show that even in a conventional DSGE model of financial frictions, absent of theory of the price level properties, the interest on reserves can pin down a *determinant* equilibrium through its effects on the *deposits channel*.

The rest of the paper is organized as follows. Section 4, introduces the main framework and derives the decisions of households, firms and banks. It also describes the aggregate equilibrium, the log-linearized system and the steady state properties of the model. Section 5 examines the role of precautionary reserves, the interest on reserves and the required reserve ratio for monetary policy, under various simple optimal policy rules and following financial and supply shocks. Section 6 concludes.

## 4 The Model

Consider a closed economy dynamic stochastic general equilibrium (DSGE) model of financial intermediation with nominal frictions (sticky prices), financial frictions (risky loans to firms), a *cost channel*, a *balance sheet* channel and bank capital requirements. There is a representative household, a continuum of intermediate goods firms producing differentiated goods, a competitive final good firm, a competitive bank with two branches, a deposit and a lending branch, and a central bank that can use the policy rate, the interest on reserves and the required reserve ratio as its policy tools.

Intermediate goods firms borrow from the lending bank to fund wage payments to households. Their production is subject to an idiosyncratic shock, which makes their loan repayment risky requiring a fraction of their output as collateral. The deposit bank satisfies a required reserve ratio and then decides whether the rest of its deposits should be made available in the interbank market, or held as reserves at the central bank receiving the interest on reserves. The lending bank's funds are made of deposits (from the interbank market), bank capital and central bank liquidity and are subject to bank capital requirements. Bank capital (equity) offers a higher rate of return than the deposit rate because of the bank's exposure to credit risk. The supply of bank equity is,



for simplicity, fixed and determined by regulatory requirements. The loan rate is set by the lending bank based on the distribution of the idiosyncratic risk of firms. Taking the loan rate as given, each intermediate goods firm decides on the level of prices, employment and thus loans. There is a full transmission of risk from firms to the lending bank with bank equity holders absorbing the cost of default.

## 4.1 Households

Households maximize their expected lifetime utility,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} - v \frac{h_t^{1+\eta}}{1+\eta} \right), \quad (1)$$

where  $\sigma > 0$  is the intertemporal elasticity of substitution in consumption,  $v$  is a labour preference scale parameter,  $\eta$  is the inverse of the Frisch elasticity of labour supply,  $\mathbb{E}_t$  the expectations operator and  $\beta \in (0, 1)$  is the discount factor. Households enter period  $t$  with real cash holdings of  $m_t$ . They receive real wage income,  $w_t h_t$ , where  $w_t$  is the real wage and  $h$  is employment hours. They supply deposits,  $d_t$ , to the deposit bank and invest in bank equity,  $e_t$ , both defined in real terms. Their remaining income is spent on a basket of consumption goods  $c_t$ , subject to the cash-in-advance constraint,  $c_t \leq m_t + w_t h_t - d_t - e_t$ . At the end of the period households receive gross interest payments on deposits,  $R_t^d$ , and on bank capital holdings,  $(1 - \Phi_t(\varepsilon_t^*))R_t^e$ , (net of the incurred fixed cost of equity issuing,  $\varrho^E$ ), as well as aggregate real profits from firms and financial intermediaries,  $\sum \Pi_t^s$ ,  $s \in \{j, d\}$ .<sup>7</sup> The term  $\Phi_t(\varepsilon_t^*)$  denotes the probability of credit default in the non-financial sector (derived and explained below). In case of default, equity holders must absorb any incurred financial losses. The real value of cash carried over to period  $t + 1$  is,

$$m_{t+1} \frac{p_{t+1}}{p_t} = m_t + w_t h_t - d_t - e_t - c_t + R_t^d d_t + (1 - \Phi_t(\varepsilon_t^*))(R_t^e + \varrho^e) e_t + \sum_{s \in \{j, d\}} \Pi_t^s, \quad (2)$$

where  $p_t$  denotes the price of the final good and  $\pi_t \equiv \frac{p_t}{p_{t-1}}$  is the gross inflation rate. With a positive deposit rate and taking wages and prices as given, the first order conditions with respect to  $c_t$ ,  $d_t$  and  $e_t$  are,

$$c_t^{-\sigma} = \beta \mathbb{E}_t R_t^d \frac{p_t}{p_{t+1}} c_{t+1}^{-\sigma}, \quad (3)$$

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<sup>7</sup>The inclusion of the fixed cost of equity issuing,  $\varrho^E$  is mainly for adjusting the steady state value of  $R_t^e$ .

$$w_t = v h_t^\eta c_t^\sigma, \quad (4)$$

$$R_t^e = \frac{R_t^d}{1 - \Phi_t(\varepsilon_t^*)} - \rho^E. \quad (5)$$

Equations (3) and (4) describe the household's Euler equation and labour supply respectively, while equation (5) is the arbitrage-free condition between the return on bank capital and the risk free deposit rate. Thus the deposit rate determines the intertemporal choices of consumers and also sets the benchmark rate for the equity rate.

## 4.2 Final Goods Firm

The competitive final good firm assembles all intermediate goods,  $y_{j,t}$ ,  $j \in (0, 1)$ , to produce a final output,  $y_t$ , which then sells at the price  $p_t$ . This is produced using a CES technology with Dixit-Stiglitz (1977) preferences,  $y_t = \left( \int_0^1 y_{j,t}^{\frac{\lambda_p-1}{\lambda_p}} dj \right)^{\frac{\lambda_p}{\lambda_p-1}}$ , where  $\lambda_p > 1$ , is the elasticity of substitution between differentiated intermediate goods. The demand for each intermediate good  $j$  is,  $y_{j,t} = y_t \left( \frac{p_{j,t}}{p_t} \right)^{-\lambda_p}$ , where  $p_{j,t}$ , is the price set by intermediate firm  $j$  and  $p_t = \left( \int_0^1 p_{j,t}^{1-\lambda_p} dj \right)^{\frac{1}{1-\lambda_p}}$  is the average price index.

## 4.3 Intermediate Goods Firms

The production of each firm  $j$  is,

$$y_{j,t} = z_{j,t} h_t, \quad (6)$$

where  $z_{j,t}$  captures the total productivity innovation experienced by firm  $j$ , which is subject to both an economy-wide supply shock,  $A_t$ , and an idiosyncratic shock,  $\varepsilon_{j,t}$ ,

$$z_{j,t} = A_t \varepsilon_{j,t}. \quad (7)$$

The economy-wide supply shock,  $A_t$ , follows an  $AR(1)$  process,  $\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A$ , where  $\epsilon_t^A$  is an i.i.d. shock, with standard deviation  $\sigma^A$  and mean  $A = 1$ . The idiosyncratic shock,  $\varepsilon_{j,t}$ , is uniformly distributed over the interval  $(\underline{\varepsilon}, \bar{\varepsilon})$ , with a constant variance and a mean of unity, so that at the symmetric aggregate equilibrium,  $z_t = A_t$ .<sup>8</sup> Each firm borrows loans to cover its working capital, in real terms<sup>9</sup>,

$$l_{j,t} = w_t h_t. \quad (8)$$

<sup>8</sup>The assumption of the idiosyncratic shock  $\varepsilon_{j,t}$  following a uniform distribution is to facilitate a tractable probability of default with no loss in generality.

<sup>9</sup>The assumption that all firms' funding is external is mainly for simplicity, but also because the paper focuses on economies with substantial credit frictions.

In a good state the firm repays the lending bank the full borrowing cost,  $R_t^l l_{j,t} = R_t^l w_t h_t$ , where  $R_t^l$  is the gross loan rate, as set in the financial contract between the bank and the firm (derived below). As production is risky, borrowing requires some collateral that the firm can pledge in the event of default. It is assumed that in the latter case the lender seizes a fraction  $\chi$  of the firm's output as collateral.<sup>10</sup> Default occurs when the real value of seizable collateral is less than the amount that needs to be repaid,  $\chi y_{j,t} < R_t^l l_{j,t}$ . Using eqs (6) and (8), the maximum *cut-off* value below which the firm defaults is,

$$\varepsilon_{j,t}^* = \frac{R_t^l w_t}{\chi A_t}. \quad (9)$$

The cut-off value is a function of the cost of borrowing,  $R_t^l$ , the cost of labour,  $w_t$ , the share  $\chi$ , and the aggregate productivity shock,  $A_t$ .

Price setting is based on Calvo-type contracts, where  $\omega_p$  firms keep their prices fixed, while the rest  $(1 - \omega_p)$  of firms adjust prices optimally by taking the loan rate, (derived below), as given. Each firm  $j$  maximizes,

$$\max_{P_{j,t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \lambda_{t+s} \{ \Pi_{t+s}^j \}$$

subject to,  $\Pi_t^j = \frac{P_{j,t}}{P_t} y_{j,t} - mc_t y_{j,t}$  and eqs (6-9), where from the firm's cost minimization problem real marginal cost is,

$$mc_t(R_t^l) = \frac{R_t^l w_t}{z_t}. \quad (10)$$

From the firm's maximization problem the new Keynesian Phillips curve equation is,<sup>11</sup>

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p \widehat{mc}_t, \quad (11)$$

where  $k_p = (1 - \omega_p)(1 - \omega_p \beta) / \omega_p$ . Equation (11) is a standard new Keynesian Phillips curve with a cost channel,  $mc_t(R_t^l)$ , however, as shown below, the loan rate here is driven by both the probability of default and other key financial variables.

#### 4.4 The Financial Sector

The financial sector is represented by a competitive bank with two branch-banks: a deposit bank and a lending bank.

<sup>10</sup>See also Agénor, Bratsiotis and Pfajfar (2014).

<sup>11</sup>Hats denote log-linearizations from steady state.

#### 4.4.1 The Deposit Bank

The deposit bank receives deposits from all households; it keeps a fraction of its deposits as total reserves,  $\tilde{r}_t$ , at the central bank for which it receives a gross interest on reserves,  $R_t^{ior}$ , and makes the rest of its deposits,  $(1 - \tilde{r}_t)d_t$ , available to the lending bank at the gross interbank rate  $R_t$ . The interbank rate,  $R_t$ , which is also the policy rate here, the required reserve ratio,  $\varsigma_t$ , and the interest rate on required and excess reserves,  $R_t^{ior}$ , are set by the central bank. The deposit bank's maximization problem is,

$$\begin{aligned} \max_{d_t, \tilde{r}_t} \Pi_t^d &= R_t^{ior} \tilde{r}_t d_t + R_t (1 - \tilde{r}_t) d_t - R_t^d d_t - G_t^{\tilde{r}}(\cdot), \\ \text{s.t.} \quad G_t^{\tilde{r}} &= \left[ (\psi_1 - \Theta_t)(\tilde{r}_t - \varsigma_t) + \frac{\psi_2}{2} (\tilde{r}_t - \varsigma_t)^2 \right] d_t, \end{aligned} \quad (12)$$

where the benefits and costs of keeping excess reserves,  $\tilde{r}_t - \varsigma_t$ , are captured by the cost function,  $G_t^{\tilde{r}}(\cdot)$ , where  $\psi_1 < 0$ , and  $\psi_2 > 0$ . With  $\Theta_t = 0$ ,  $G_t^{\tilde{r}}(\cdot)$  reduces to the convex cost function used in Glocker and Towbin (2012), where the quadratic term,  $\psi_2(\cdot)$ , captures the cost of maintaining reserves, where the first term  $\psi_1(\tilde{r}_t - \varsigma_t)$  captures any exogenous benefits of holding excess reserves<sup>12</sup>. An innovation here is that I endogenize the latter. In particular, I allow the benefit of holding excess reserves to be increasing linearly with the probability,  $\Theta_t$ , that at any point in time the bank will require a higher fraction of excess reserves  $(\tilde{r}_t - \varsigma_t)d_t$ . I consider two main factors that can increase  $\Theta_t$ : a potential risk in the non-financial sector,  $\Phi_t(\varepsilon_t^*)$  where risky bank loans are ultimately used, and the announcement of some news, or other exogenous shocks,  $\theta_t$ ,

$$\Theta_t = \psi_3 \Phi_t(\varepsilon_t^*) + \theta_t, \quad (13)$$

where  $\psi_3 \geq 0$  is a pass-through elasticity discussed below. Thus,  $\Theta_t$  is driven both endogenously, by the degree which banks perceive a potential credit risk in the non-financial sector, and exogenously through a stochastic process.<sup>13</sup>

<sup>12</sup>Glocker and Towbin (2012) use the Bernanke, Gertler and Gilchrist (1999) framework to show that when the loss function of the central bank incorporates a financial stability objective, the use of reserve requirements can lead to non-negligible welfare improvements. Their policy analysis focuses mainly on the effects of the required reserve ratio, while the interest on reserves is assumed to be constant.

<sup>13</sup>In Christiano, Motto and Rostagno (2010), the relative usefulness of excess reserves is determined purely by an exogenous stochastic process. In Dressler and Kirsting (2015) excess reserves are chosen in a process where banks draw an idiosyncratic lending cost that can be high or low and this determines whether loans are made or alternatively excess reserves increase. Thus, although in their model firms also borrow for their working capital, the risk is modelled implicitly by banks and so there is no transmission of risk from the non-financial sector to banks; also monetary policy is conducted purely by exogenous money growth. They show that increases in the real loan rate increases the deposit rate and this can overturn the effects of money injections.

From its maximization problem, (12), the deposit bank sets the deposit rate,

$$R_t^d = R_t - \tilde{r}_t(R_t - R_t^{ior}) - \left[ (\psi_1 - \Theta_t)(\tilde{r}_t - \varsigma_t) + \frac{\psi_2}{2}(\tilde{r}_t - \varsigma_t)^2 \right], \quad (14)$$

and the fraction of deposits it wants to keep as excess reserves,

$$\tilde{r}_t - \varsigma_t = - \left( \frac{\psi_1 - \Theta_t}{\psi_2} \right) - \left( \frac{R_t - R_t^{ior}}{\psi_2} \right) \quad (15)$$

With  $\Theta_t = 0$ , a fixed positive level of excess reserves measured by  $-\psi_1/\psi_2 > 0$ , ( $\psi_1 < 0$ ) is held by the deposit bank, as in Glocker and Towbin (2012), and the only variable driving excess reserves endogenously is the opportunity cost of holding reserves, measured by the spread between the interbank rate and interest on reserves,  $R_t - R_t^{ior}$ .

More importantly here, excess reserves increase with the probability  $\Theta_t$  that the bank will require precautionary reserves, either because of a potential risk in the non-financial sector,  $\Phi_t(\varepsilon_t^*)$ , or because of exogenous shocks,  $\theta_t$ . Obviously the size of the pass through elasticity,  $\psi_3$ , matters here. A high  $\psi_3$  implies that banks perceive a high risk coming from creditors in the non-financial markets, increasing their desired level of excess reserves. If  $\psi_3 = 0$ , then even when there is an underlying risk in the credit markets, the financial sector does not respond to it, leaving excess reserves unaffected.

From (14) the deposit rate decreases the higher are the costs of holding excess reserves, including the opportunity cost of holding total reserves ( $R_t - R_t^{ior}$ ), and it increases with the benefits of holding excess reserves, and thus with the desire of banks to hold liquidity for precautionary reasons. This is because when banks care about liquidity they are prepared to pay a higher deposit rate in relation to the policy rate, because they want to avoid deposit outflows. This is shown to be an important effect for the results in this paper, because it implies that the deposit rate can deviate from the policy (interbank) rate. Similarly, for any given policy rate, a higher interest on reserves reduces the opportunity cost of holding reserves and raises the deposit rate. This is also shown to be an important effect in explaining the role of the interest on reserves in the conduct of monetary policy through the deposits channel. These effects are examined in more detail in sections 5.3 and 5.4.

#### 4.4.2 The Lending Bank

The lending bank raises  $(1 - \tilde{r}_t)d_t$  funds from the deposit bank at the inter-bank gross rate,  $R_t$ , and also issues regulatory bank capital,  $e_t$ , at the gross rate  $R_t^e$ .<sup>14</sup> The lending bank can also receive extra liquidity  $l_t^{cb}$  from the central bank which is remunerated also at the policy rate,  $R_t$ .<sup>15</sup> The lending bank's balance sheet in real terms is,

$$l_t = (1 - \tilde{r}_t)d_t + e_t + l_t^{cb}. \quad (16)$$

The lending rate is set at the beginning of the period, before firms engage in production activity and pricing decisions. Given a competitive banking environment it is assumed that, on average, the bank breaks even such that the expected income from lending to firms is equal to the total costs of borrowing these funds (deposits, bank equity and liquidity from the central bank) to firm  $j$ .<sup>16</sup> The lending bank's expected intra-period zero profit condition from lending to firm  $j$  is,

$$\int_{\underline{\varepsilon}^*}^{\bar{\varepsilon}} R_t^l l_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} \chi y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = [(1 - \tilde{r}_t)d_{j,t} + l_{j,t}^{cb}] R_t + e_{j,t} R_t^e + \varrho_t^l l_{j,t}, \quad (17)$$

where  $f(\varepsilon_{j,t})$  is the probability density function of  $\varepsilon_{j,t}$ . The first part of the left hand side of (17) is the expected repayment to the bank in the non-default state, while the second part is the expected return to the bank in the default state, including the collateral commitment,  $\chi_t y_{j,t}$ , which forms part of the financial contract. The right hand side is the total cost of funds, where  $\varrho_t^l$  captures all other exogenous costs related to loans  $l_{j,t}$  (i.e. including transaction, monitoring costs and other loan-related innovations).  $\varrho_t^l$  follows an AR(1) process,  $\varrho_t^l = (1 - \rho^e)\varrho^l + \rho^e \varrho_{t-1}^l + \epsilon_t^e$ , where  $\varrho^l$  is its mean value, and  $\epsilon_t^e$  is an i.i.d. normal random variable with zero mean and standard deviation  $\sigma_\rho$ . Substitute (6) and (7) into (17), together with  $R_t^l l_{j,t} = \chi A_t \varepsilon_{j,t}^* h_t$  (from 9), to obtain,

$$R_t^l l_{j,t} = \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} (\varepsilon_{j,t}^* - \varepsilon_{j,t}) \chi A_t h_t f(\varepsilon_{j,t}) d\varepsilon_{j,t} + [(1 - \tilde{r}_t)d_{j,t} + l_{j,t}^{cb}] R_t + e_{j,t} R_t^e + \varrho_t^l l_{j,t}$$

To find an explicit expression for the probability of default, I use the fact that  $\varepsilon_{j,t}$  follows a uniform distribution over the interval  $(\underline{\varepsilon}, \bar{\varepsilon})$ , with a probability density  $1/(\bar{\varepsilon} - \underline{\varepsilon})$  and a

<sup>14</sup>Since raising funds through equity is more costly for the bank ( $R_t^e > R_t$ ), bank equity is issued merely to satisfy regulatory bank capital requirements in this model.

<sup>15</sup>Introducing a liquidity injection,  $l_t^{cb}$ , is simply to allow the markets to clear at equilibrium, (see Ravenna and Walsh, 2006).

<sup>16</sup>This condition is employed purely for simplicity and clarity as the focus of this paper is not on the monopolistic power of banks, (Dreschler, Savov and Schnabl, 2017).

mean  $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2 = 1$ . Using this and the definitions  $(1 - \tilde{r}_t)d_t + l_t^{cb} = l_t - e_t$ , (from 16), and  $\gamma_t = e_t/l_t$ , where  $\gamma_t$  is the bank capital-to-loan ratio, the loan rate is,<sup>17</sup>

$$R_t^l = R_t + \gamma_t(R_t^e - R_t) + \frac{\chi}{l_t/y_t} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \Phi_t^2(\varepsilon_t^*) + \varrho_t^L, \quad (18)$$

where  $y_t = A_t h_t$  and  $\Phi_t(\varepsilon_t^*) = \int_{\underline{\varepsilon}}^{\varepsilon_t^*} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ , is the probability of credit default.<sup>18</sup> From (18), the loan rate is shown to increase, the stricter is bank regulation, (capital-to-loan ratio)  $\gamma_t$ , the higher is bank equity spread,  $(R_t^e - R_t)$ , and the higher is the risk premium (third term in 18), which is a function of the probability of loan default  $\Phi_t(\varepsilon_t^*)$ , the loan-to-output value  $l_t/y_t$ , and the fraction of collateral,  $\chi$ ; it is also subject to the financial shock,  $\varrho_t^L$ .

Thus credit risk in the non-financial sector raises the loan rate spread and thus the cost of borrowing for firms. In the financial sector, depending on the value of  $\psi_3 \geq 0$ , this credit risk can also generate an accumulation of precautionary reserves. The interest on reserves and the required reserve ratio affect the loan rate through two channels: (i) through the balance sheet, as they affect total reserves that determine the level of credit to firms, (15 and 16), and (ii) through their effect on the deposit rate, (14), which in turn also affects the equity rate,  $R_t^e$  (5).

## 4.5 Monetary Policy

The policy interest rate,  $R_t$ , is set according to a conventional Taylor rule,

$$R_t = R^{(1-\phi)} R_{t-1}^\phi \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{x_t}{x} \right)^{\phi_y} \right]^{(1-\phi)}, \quad (19)$$

where,  $\phi \in (0, 1)$  is the degree of interest rate smoothing;  $\phi_y, \phi_\pi > 0$ , are policy coefficients and  $y_t/y_t^* \equiv x_t$  is the output gap. The policy rate has a direct effect on two important channels in this model, the *cost of borrowing channel* and the *deposits channel*. It works through the *cost channel* by affecting the loan rate, (18), which affects the marginal cost, (10), and inflation, (11). It works through the *deposits channel* by affecting the deposit rate (14), which in turn affects the household's intertemporal substitution of consumption,

<sup>17</sup>The cut-off value,  $\varepsilon_{j,t}^*$ , depends on the state of the economy and at equilibrium is identical across all firms. Similarly, real wages and the labour employed by each firm are identical and therefore the loan rate applies to all firms and so in what follows the subscript  $j$  is dropped.

<sup>18</sup>The default risk of the firm is fully transmitted to the bank. As the bank's total funds are provided to identical firms and the loan rate has been derived from a break even condition, it is implied that at equilibrium when the firm's default condition is satisfied the bank would also no longer break even (see also 17). Hence, with no loss in generality, the bank's default probability is approximated to the firm's default probability,  $\Phi_t(\varepsilon_t^*)$ ,

(3), but also the bank equity return (5), which in turn also affects the loan rate (18) and hence the cost channel.<sup>19</sup> Monetary policy can also be supported by the required reserve ratio and the interest on reserves. Both these tools affect the deposits channel and the balance sheet channel; they are examined in more detail in sections 5.3 and 5.3.1.

## 4.6 The Log-linearized Aggregate Equilibrium

At the aggregate equilibrium and with no fixed capital investment or government intervention, aggregate demand is determined by aggregate consumption,  $y_t = c_t$ . On the production side equilibrium also requires that  $\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\lambda_p} y_t = \int_0^1 \varepsilon_{j,t} A_t h_t$ . Using the distribution properties of the idiosyncratic shocks (that has an average mean of unity and at the symmetric equilibrium,  $\int_0^1 \varepsilon_{j,t} = 1$ ), aggregate equilibrium becomes  $y_t = A_t h_t / \Delta_t$ , where  $\Delta_t \equiv \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\lambda_p}$  is the price dispersion index. At the aggregate equilibrium we also assume that  $l_t^{cb} = m_{t+1} \frac{p_{t+1}}{p_t} - m_t$ , and  $w_t h_t = (1 - \tilde{r}_t) d_t + e_t + l_t^{cb} = l_t$  and that the financial markets clear.

The model is log-linearized around its non-stochastic, zero inflation, flexible price steady state. The flexible price level of output is,  $y_t^f = \left(\frac{z_t^{1+\eta}}{\vartheta_p R_t^{l,f}}\right)^{\frac{1}{\eta+\sigma}}$ , where  $\vartheta_p = \lambda_p / (\lambda_p - 1)$  is the price mark-up and  $R_t^{l,f}$  is the loan rate under flexible prices. The efficient level of output, free of both financial frictions and nominal rigidities and estimated at a constant policy rate, is  $y_t^* = \left(\frac{z_t^{1+\eta}}{\vartheta_p}\right)^{\frac{1}{\eta+\sigma}} > y_t^f$ .<sup>20</sup> Also, for the purpose of this paper, it is assumed that the bank capital requirement constraint remains fixed, so that  $\gamma_t = \gamma$ .<sup>21</sup> The log-linearized versions of (3), (4), (5), (11) (14), (15), (16) and (18), can then be used to express the model in terms of the efficient output gap,  $\hat{x}_t = \hat{y}_t - \hat{y}_t^*$ , where  $\hat{y}_t^* = \frac{1+\eta}{\sigma+\eta} \hat{z}_t$ , the inflation rate,  $\hat{\pi}_t$ , the equity rate,  $\hat{R}_t^e$ , the deposit rate  $\hat{R}_t^d$ , total reserves,  $\hat{r}_t$ , loans,  $\hat{l}_t$ , the loan rate,  $\hat{R}_t^l$  and the default risk,  $\hat{\Phi}_t$ ,

$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \sigma^{-1} \left( \hat{R}_t^d - \mathbb{E}_t \hat{\pi}_{t+1} \right) + \hat{u}_t, \quad (20)$$

where  $\hat{u}_t \equiv ((1 + \eta) / (\sigma + \eta)) \left( \mathbb{E}_t \hat{A}_{t+1} - \hat{A}_t \right)$ ,

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + k_p (\eta + \sigma) \hat{x}_t + k_p \hat{R}_t^l, \quad (21)$$

<sup>19</sup>Dreschler, Savov and Schnal (2017), show that the policy rate can affect the deposits channel through the monopolistic power of banks. This channel is relaxed here as it is not the focus of this paper.

<sup>20</sup>See Ravenna and Walsh (2006).

<sup>21</sup>Changes in the bank capital constraints can be important for balance sheet effects and the supply side of credit, though this is not the focus of this paper. Moreover, there is evidence that bank capital constraints could not explain the massive scale rise observed in liquidity hoarding by banks and their unwillingness to lend following the 2007 financial crisis (Ennis and Wolman, 2015).



$$\widehat{R}_t^e = \widehat{R}_t^d + \left( \frac{\Phi}{1 - \Phi} \right) \widehat{\Phi}_t, \quad (22)$$

$$\begin{aligned} \widehat{R}_t^d = & \frac{(1 - \widetilde{r})R}{R^d} \widehat{R}_t + \frac{\widetilde{r}R^{ior}}{R^d} \widehat{R}_t^{ior} - \frac{\widetilde{r}(R - R^{ior})}{R^d} \widehat{r}_t + \frac{(\widetilde{r} - \varsigma)}{R^d} \Theta \widehat{\Theta}_t \\ & - \frac{[\psi_1 - \Theta + \psi_2(\widetilde{r} - \varsigma)](\widetilde{r}\widehat{r}_t - \varsigma\widehat{\varsigma}_t)}{R^d}, \end{aligned} \quad (23)$$

$$\widehat{r}_t = \frac{\varsigma}{\widetilde{r}} \widehat{\varsigma}_t + \frac{\Theta}{\psi_2 \widetilde{r}} \widehat{\Theta}_t, - \frac{1}{\psi_2 \widetilde{r}} (R \widehat{R}_t - R^{ior} \widehat{R}_t^{ior}) \quad (24)$$

$$\Theta \widehat{\Theta}_t = \psi_3 \Phi \widehat{\Phi}_t + \widehat{\theta}_t \quad (25)$$

$$\widehat{l}_t = \widehat{w}_t + \widehat{h}_t = (\eta + \sigma) \widehat{x}_t + \widehat{y}_t, \quad (26)$$

$$\widehat{R}_t^l = \frac{(1 - \gamma)R}{R^l} \widehat{R}_t + \frac{\gamma R^e}{R^l} \widehat{R}_t^e + \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \frac{\chi \Phi^2}{R^l l / y} (\widehat{y}_t - \widehat{l}_t + 2\widehat{\Phi}_t) + \widehat{\varrho}_t^L, \quad (27)$$

$$\widehat{\Phi}_t = \frac{\varepsilon^*}{\varepsilon^* - \underline{\varepsilon}} \left( \widehat{R}_t^l + (\eta + \sigma) \widehat{x}_t \right), \quad (28)$$

where  $\widehat{R}_t$  is determined by the log-linearized Taylor rule, (19);  $\widehat{\varrho}_t^L = \rho_\varrho \widehat{\varrho}_{t-1}^L + \widehat{\varepsilon}_t^\varrho$ ,  $\widehat{\theta}_t = \rho_\theta \widehat{\theta}_{t-1} + \widehat{\varepsilon}_t^\theta$  and  $\widehat{R}_t^{ior}$  and  $\widehat{\varsigma}_t$  are defined below.

## 4.7 Steady State and Parameterization

The key steady state equations are,  $R = 1/\beta$ ,  $R^e = \frac{R^d}{1 - \Phi(\varepsilon^*)} - \varrho^E$ ,  $R^l = (1 - \gamma)R + \gamma R^e + \chi \frac{y}{l} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \Phi^2$ ,  $\Phi = \frac{\varepsilon^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ , where  $\varepsilon^* = \frac{\mu_\varepsilon}{\chi \varrho_p}$ ,  $\widetilde{r} = \varsigma - \frac{(\psi_1 - \psi_3 \Phi(\varepsilon^*))}{\psi_2} - \frac{(R - R^{ior})}{\psi_2}$ , and  $R^d = R - \widetilde{r}(R - R^{ior}) - \left[ (\psi_1 - \psi_3 \Phi)(\widetilde{r} - \varsigma) + \frac{\psi_2}{2}(\widetilde{r} - \varsigma)^2 \right]$ . Table 1 presents the baseline parameter values of the model. Most of the parameter values follow largely the existing literature, whereas the rest are chosen so that the steady state values match observed ratios for advanced economies. The idiosyncratic productivity shock's range is set to  $\underline{\varepsilon} = 0.80$  and  $\bar{\varepsilon} = 1.20$ , so that  $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2 = 1$  and the steady state fraction of collateral received by the bank is set to  $\chi = 97\%$ .<sup>22</sup> These values, together with a price mark-up of 20%, ( $\mu_p = 1.20$ ), generate a steady state credit risk of 3.04% and a loan rate of 7.5% (in annual terms). The required reserve ratio at the steady state is  $\varsigma = 2\%$ , which is the average ratio (usually between 1-3%) in OECD countries.<sup>23</sup> The baseline steady state spread between the policy rate and interest on reserves is set to  $R - R^{ior} = 100$  bps (quarterly), and the values of  $\psi_1$ ,  $\psi_2$  and  $\psi_3$  are chosen so that at the steady state with default probability,  $\Phi = 3\%$ , total excess reserves are,  $\widetilde{r} - \varsigma \approx 2\%$  and total reserves are,

<sup>22</sup>The value of  $\chi = 0.97$  is justified in Agénor, Bratsiotis and Pfajfar, (2014).

<sup>23</sup>A 2% required reserve ratio also reflects the ratio proposed recently for the Euro Zone countries.

$\tilde{r} \approx 0.04$ . The elasticity measuring the exogenous benefits of holding excess reserves is set to a very small value,  $\psi_1 = -0.03$ , and for tractability of the net effects of  $\varsigma$  and  $R^{ior}$ , I set  $\psi_2 = 1$ . The parameter  $\psi_3$ , that regulates the degree of precautionary liquidity, is allowed to vary from almost zero in the baseline case, (i.e.  $\psi_3 = 0.001$ ) to much higher values that reflect a high perceived risk. The bank capital ratio is  $\gamma = (\bar{\gamma} + \Phi) = 11\%$ , where  $\bar{\gamma} = 0.08$ , represents the fixed bank capital to loan ratio, as set by the Basel Accords.<sup>24</sup> Some of the key steady state values resulting from the above parameterization are (in annual terms),  $\Phi = 0.03$ ,  $R = 1.0404$ ,  $R^{ior} = 1.0001$ ,  $R^d = 1.0384$ ,  $R^e = 1.053$ ,  $R^l = 1.074$  and  $l/y = 0.81$ .<sup>25</sup>

Table 1: Baseline Parameterization

Parameter	Value	Description
$\beta$	0.99	Discount Factor
$\sigma$	1.00	Intertemporal Substitution in Consumption
$\eta$	0.30	Inverse of the Frisch Elasticity of Labour Supply
$v$	1.00	Labour Preference Scale Parameter
$\lambda_p$	6.00	Elasticity of Demand - Intermediate Goods
$\omega_p$	0.80	Degree of Price Stickiness
$A$	1.00	Average Productivity Parameter
$\underline{\varepsilon}$	0.85	Idiosyncratic Risk: Lower Range
$\bar{\varepsilon}$	1.15	Idiosyncratic Risk: Upper Range
$\chi$	0.97	Proportion of Output Seized when Firms Default
$\bar{\gamma}$	0.08	Bank Capital Adequacy Ratio
$\varsigma$	0.02	Steady State Required Reserve Ratio
$\psi_1$	-0.03	Linear cost function parameter
$\psi_2$	1.00	Quadratic cost function parameter
$\psi_3$	0.001	Excess Reserves Responsiveness to Credit Risk
$\phi$	0.70	Persistence in Taylor Rule
$\phi_\pi$	1.50	Response of Policy Rate to Inflation Deviations
$\phi_x$	0.00	Response of Policy Rate to Output Gap Deviations

The persistence parameters and the standard deviations of the financial shock and the supply shock are calibrated in line with Benes and Kumhof (2015), where  $\rho_\varrho = 0.87$  and  $\sigma_\varrho = 0.11$ , while  $\rho_A = 0.92$  and  $\sigma_A = 0.024$ , respectively.<sup>26</sup>

<sup>24</sup>This partly mimics a Basel II type regulation, where borrowers' credit risk,  $\Phi$ , is taken into account in the determination of the overall bank capital ratio.

<sup>25</sup>For the steady state values of the equity and loans rates we also use,  $\varrho^E = 0.028$  and  $\varrho^L = 0.008$ .

<sup>26</sup>Christiano, Motto and Rostagno also report a riskiness shock of  $\sigma_\varrho = 0.119$ .

## 5 Optimal Simple Policy Rules and Welfare

This section uses a numerical welfare analysis to determine the optimal responses of monetary policy rules following financial and supply shocks. The central bank's objective function is derived by a second order approximation around the efficient steady state of the household's expected utility function (1), where the consumer's welfare losses are expressed as a fraction of steady state consumption<sup>27</sup>,

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U}{U_{cc}} \right) = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\lambda_p}{k_p} \right) \hat{\pi}_t^2 + (\eta + \sigma) (\hat{x}_t)^2 \right] \quad (29)$$

where,  $\hat{x}_t = \hat{y}_t - \hat{y}_t^*$  is the welfare relevant output gap and  $\hat{y}_t^*$  is the log deviation of the efficient output from its steady-state. The average welfare loss per period is given by,  $\mathbb{L} = \frac{1}{2} [(\lambda_p/k_p) \text{var}(\hat{\pi}_t) + (\eta + \sigma) \text{var}(\hat{x}_t)]$ . The net welfare gain from the simple optimal policy rules considered are estimated based on the difference in consumer welfare losses between the baseline policy rule  $\mathbb{W}^B$  and the optimal policy rule  $\mathbb{W}_t^O$ ,

$$CE = \{1 - \exp [(1 - \beta) (\mathbb{W}_t^O - \mathbb{W}_t^B)]\} \times 100,$$

where the higher is  $CE$ , a consumption equivalent measure, the larger is the net welfare gain from the optimal policy.

### 5.1 Precautionary Liquidity and Optimal Taylor Rule Responses

This section examines the role of the precautionary liquidity and the size of excess reserves for monetary policy. Table 2 reports the optimal policy responses of the Taylor rule (OTR, based on 19), and the net welfare gains, measured as deviations from the baseline policy and reported in terms of the consumption equivalent measure  $CE$ , following an adverse financial shock and an adverse supply shock. Monetary policy responses are examined under different levels of steady state excess reserves ( $ER \equiv \tilde{r} - \varsigma$ ) that arise from assuming different degrees of responsiveness to credit risk,  $\psi_3$ , (see 15). For all results reported in this section the interest on reserves and the required reserve ratio are kept constant at their steady state values.<sup>28</sup>

<sup>27</sup>The derivation of the welfare loss function follows Ravenna and Walsh (2006), who also incorporate the monetary policy cost channel. In the efficient steady state, price mark ups and financial distortions are eliminated through appropriate subsidies.

<sup>28</sup>The grid search range is,  $\phi_\pi = [0, 3]$ , and  $\phi_x = [0, 1]$ .

Table 2: Welfare Effects: Excess Reserves and Optimal Policy Responses

(a) Loan Rate Shock,				
Policy	$\psi_3=0.001$ , ER $\approx$ 2%	$\psi_3=3.5$ , ER $\approx$ 12%	$\psi_3=5.4$ , ER $\approx$ 18%	$\psi_3=7.6$ , ER $\approx$ 25%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_y = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02372$	$\phi_\pi = 1.90$ $\phi_x = 0.00$ $CE = 0.00001$	$\phi_\pi = 1.50$ $\phi_x = 0.30$ $CE = 0.00226$	$\phi_\pi = 0.00^*$ $\phi_x = 0.20$ $CE = 0.00344$
(b) Adverse Technology Shock				
Policy	$\psi_3=0.001$ , ER $\approx$ 2%	$\psi_3=2.0$ , ER $\approx$ 8%	$\psi_3=13.5$ , ER $\approx$ 42%	$\psi_3=16.2$ , ER $\approx$ 51%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_y = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00254$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00055$	$\phi_\pi = 2.70$ $\phi_x = 0.00$ $CE = 0.00000$	$\phi_\pi = 0.80^*$ $\phi_x = 1.00$ $CE = 0.00000$
(c) News Shock				
Policy	$\psi_N=0.001$ , ER $\approx$ 2%	$\psi_N=3.5$ , ER $\approx$ 8%	$\psi_N=13.5$ , ER $\approx$ 42%	$\psi_N=30.2$ , ER $\approx$ 51%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_y = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.60$ $CE = 0.00000$	$\phi_\pi = 3.00$ $\phi_x = 0.60$ $CE = 0.00000$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00000$	$\phi_\pi = 0.80^*$ $\phi_x = 1.00$ $CE = 0.00000$

\*Taylor principle violated

**Result 1:** *At low levels of precautionary liquidity and thus excess reserves the optimal response of the Taylor rule is to maintain its focus on inflation. As precautionary liquidity rises to higher levels, raising the deposit rate, the optimal response of the Taylor rule is to reduce its relative weight on inflation. This result holds for both financial and supply shocks, although it is stronger for the former.*

Result 1 follows directly from Table 2 and Figures 3 and 4. Following financial and real shocks the optimal response of the policy rate is shown to be largely determined by the level of excess reserves. When excess reserves are around low steady state levels, (i.e. 2-8% of total deposits), the optimal policy response to a financial shock that reduces the output gap and increases inflation (due to the cost channel), is to increase the policy rate so as to curb inflation. However, as excess reserves rise, the optimal response of the policy rate is to decrease the relative weight on inflation with respect to the output gap. This result can be explained as follows. At low levels of excess reserves, a financial shock that raises the cost of borrowing, or an adverse supply shock, increase the inflation

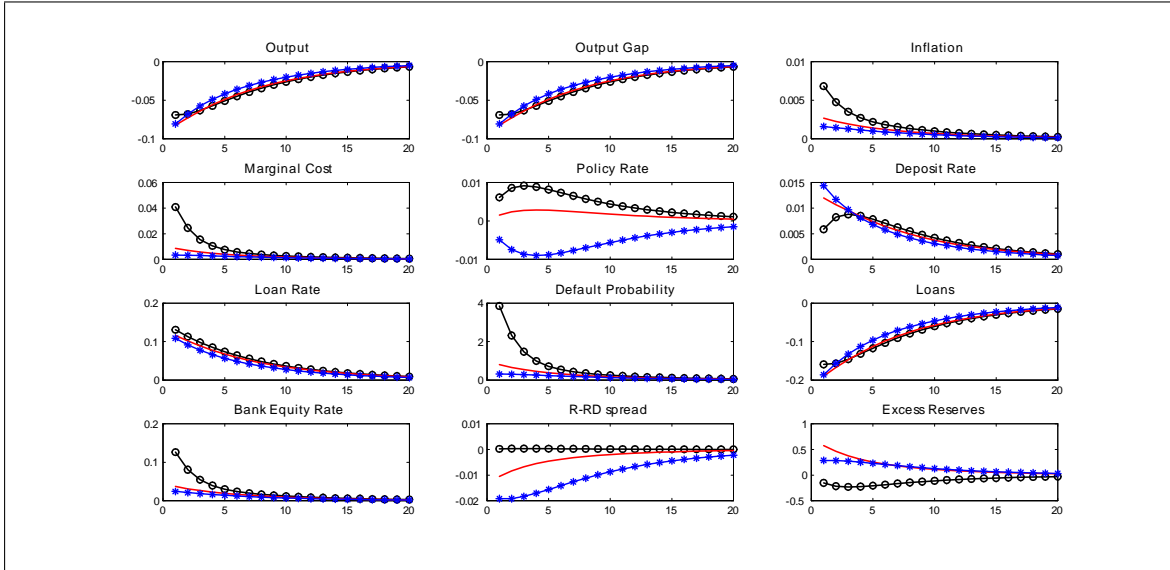
rate requiring the policy rate to increase (see Black(o) line: ER=2% of total deposits). However, when precautionary liquidity is high the deposits channel becomes stronger, as both the proportion of deposits held as excess reserves and the deposit rate rise. Higher excess reserves imply a reduced supply of loans (through the *balance sheet* channel - eq. 16), while the higher deposit rate reduces (through the households intertemporal substitution effect) the output gap and the inflation rate, which also eases inflationary pressures and allows the policy rate to decrease, (see Figures 3 and 4). The fall in the policy rate also reduces the probability of default and the cost of borrowing, thus further reducing the inflation rate and the policy rate through the financial accelerator. In the case of an adverse supply shock, where both the output gap and inflation increase, it takes a much higher level of excess reserves for the monetary policy to abandon its anti-inflationary stance. In this example, it takes excess reserves of over 40% of total deposits for monetary policy to start reducing its weight on inflation, but at such critical levels of excess reserves the welfare gains from such policy are shown to be very small.

**Proposition 1:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\psi_3 > 0$ , that is, there is a positive degree of precautionary liquidity that raises the deposit rate above the policy rate.*

**Proof.** A formal proof of Proposition 1 is provided in the Appendix.

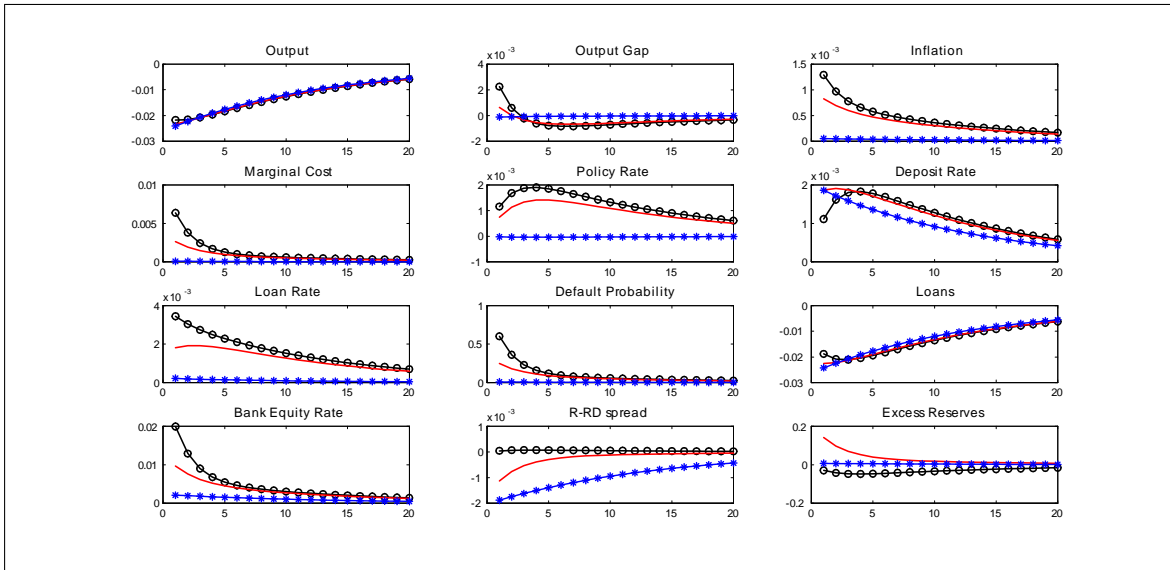
It is a well-established fact that in this general class of macro models a fixed interest rate results in *indeterminacy*, (see Woodford 2011). Here it is shown that when the policy rate is fixed at the zero-bound, a positive degree of precautionary liquidity ( $\psi_3 > 0$ ) can result in *determinacy*. This is because with  $\psi_3 > 0$ , the deposit rate responds to credit risk which affects both the cost channel and inflation (through the loan rate) but also the output gap (through marginal cost - eq 28). With the policy rate at the zero-bound, as the degree of precautionary liquidity increases the deposit rate can increase above the policy rate, thus acting as a ‘substitute’ for the policy rate (see *R-RD spread* in Figures 3 and 4). This implies that even in the absence of an active monetary policy, the economy can be self-disciplined through ‘prudent’ banking, in the form of precautionary liquidity hoarding that decreases the supply of loans and raises the deposit rate in response to credit riskiness, which help pin down a unique equilibrium.

Figure 3: Financial Shock: Excess Reserves and Optimal Policy Responses



Black(o) line: ER=2%; Red(solid) line: ER=12% and Blue(star) line: ER=25%.

Figure 4: Adverse Supply Shock: Excess Reserves and Optimal Policy Responses



Black(o) line: ER=2%; Red(solid) line: ER=8% and Blue(star) line: ER=51%.

**Result 2:** *At high steady state levels of precautionary liquidity and excess reserves, the optimal weight on inflation in the Taylor rule can be shown to be less than unity, ( $\phi_\pi < 1$ ). This is a determinate equilibrium despite the violation of the Taylor principle. Determinacy holds because the desire for excess liquidity drives the deposit rate above the policy rate, with the former controlling the behavior of consumption (Euler equation) and inflation.*

This follows directly from Proposition 1, in combination with the results in Table 2, and Figures 3 and 4. In a conventional model where the deposit rate does not deviate from the policy rate, the *Taylor principle* cannot be violated because the policy rate is also the rate that pins down the behavior of consumption and aggregate demand. In this conventional case a lower-zero bound interest rate would result in *indeterminacy*.

## 5.2 Excess Reserves Shocks ( $\hat{\theta}_t$ ) and Monetary Policy

This section considers the role of monetary policy when excess reserves increase in response to a stochastic shock  $\hat{\theta}_t$ , i.e. due to news or other exogenous shocks.<sup>29</sup> We assume that  $\psi_3 = 0$ , and keep the interest on reserves and the required reserve ratio constant at their steady state values. Here it is shown that the optimal response of the Taylor rule is,  $\phi_\pi = 3.0$ , and  $\phi_x = 0.60$ , implying a net improvement of  $CE = 0.00029$ , (from the baseline case with  $\phi_\pi = 1.5$ , and  $\phi_x = 0$ ). Figure 5 plots the baseline and optimal responses of the Taylor rule to an increase in  $\hat{\theta}_t$ , assuming,  $\rho_\theta = 0.80$  and  $\sigma_\theta = 0.01$ . Unlike a loan rate, or a negative aggregate supply shock, that increase inflation through credit frictions and the cost channel, a stochastic increase in excess reserves causes, loans, the output gap and hence inflation to decrease but the deposit rate to increase, given the sudden increase in the demand for reserves. Since excess reserves shocks move the inflation rate and the deposit rate in opposite directions, the policy rate is required to intervene and stabilize inflation. In Figure 5 the optimal response of the Taylor rule is shown to suggest a reduction of the policy rate, below the baseline case, so as to stabilize loans, the output gap and inflation.

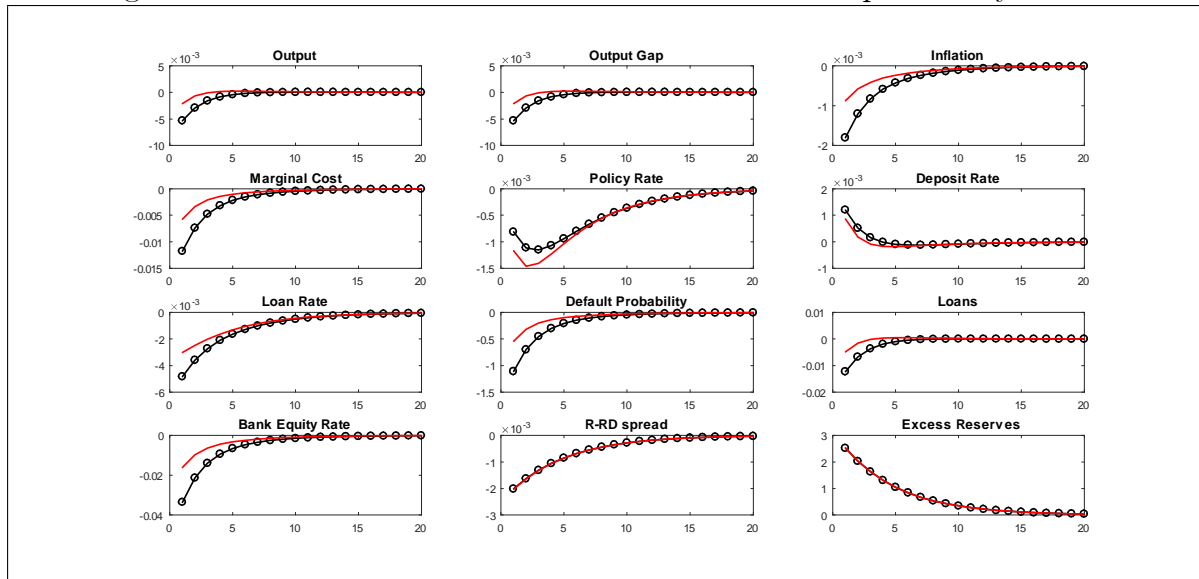
**Result 3:** *Following a stochastic increase in excess reserves, the optimal response in the Taylor rule is to keep a relative high weight on inflation. This is because such a shock*

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<sup>29</sup>Examples of this case would be the January 2000 dot com bubble crash, or the September 2001 terrorist attack.

causes the inflation rate and the deposit rate to move in opposite directions thus requiring the intervention of the policy rate.

Figure 5: An Excess Reserves Shock: Benchmark and Optimal Taylor Rule



Black(o) line: Benchmark case; Red(solid) line: Optimal Taylor.

### 5.3 The Effects of the Interest on Reserves and the Required Reserve Ratio on the Deposits Channel

From (14) and (15) the required reserve ratio,  $\zeta_t$ , and the interest on reserves,  $R_t^{ior}$ , are shown to affect the *deposits channel* through both the *balance sheet effect* (total reserves) and the *deposit rate effect*, though not in the same way. Both monetary policy tools have a positive effect on total reserves,  $\partial \tilde{r}_t / \partial \zeta_t = 1 > 0$ , and  $\partial \tilde{r}_t / \partial R_t^{ior} = 1/\psi_2 > 0$ , ( $\psi_2 > 1$ ) respectively. Thus a higher required reserve ratio, or a higher interest on reserves, increase total reserves and reduces loans available for loans. This is a *balance sheet effect* which restricts the amount of credit available in the economy, (see 16) and has a negative impact on the output gap and inflation. However, the effects that these two monetary policy tools have on the *deposits channel* through the deposit rate work in opposite directions.

The effect of the required reserve ratio on the deposit rate is expected to be negative. In particular,  $\partial R_t^d / \partial \zeta_t = \psi_1 - \psi_3 \Phi_t(\varepsilon_t^*) - (R_t - R_t^{ior}) + (\tilde{r}_t - \zeta_t)\psi_2$ , or using (15),  $\partial R_t^d / \partial \zeta_t = -2(R_t - R_t^{ior}) \leq 0$ , assuming,  $R_t^{ior} \leq R_t$ . Thus, the higher is the opportunity cost of holding deposits as reserves, (receiving  $R_t^{ior}$ ), rather than making them available in the



interbank market, (receiving  $R_t$ ), the larger is this negative effect. This result is consistent with studies that show that the required reserve ratio acts as ‘a tax’ on bank deposits, which then banks pass on to the depositor as a lower deposit rate.<sup>30</sup> Thus a higher required reserve ratio lowers the deposit rate, with the latter effect mitigating the balance sheet effect of the former, by encouraging a higher output gap and inflation.

In contrast, the effect of the interest on reserves on the deposit rate is positive. From (14),  $\partial R_t^d / \partial R_t^{ior} = \varsigma_t - \psi_1 / \psi_2 + \psi_3 \Phi_t(\varepsilon_t^*) / \psi_2 - (R_t - R_t^{ior}) / \psi_2 > 0$ , ( $\psi_1 < 0$  and  $\psi_2 > 0$ ) and thus a higher interest on reserves increases the deposit rate. This is because the interest on reserves acts as ‘a subsidy’ on bank deposits thus encouraging a higher deposit rate.<sup>31</sup> Since, the effect of the interest on reserves on total reserves is also positive,  $\partial \tilde{r}_t / \partial R_t^{ior} > 0$ , a higher interest on reserves will also reduce the reserves available for credit. Thus here both the effects of the *deposits channel* (i.e. through higher excess reserves and a higher deposit rate) work in a complementary way in reducing the size of borrowing, (the *cost channel*), the output gap and inflation. This, as shown next, implies that the use of the interest on reserves requires a lower intervention from the policy rate and it can deliver higher welfare gains than the required reserve ratio.

### 5.3.1 Optimal Policy Responses with *Interest on Reserves* or *Required Reserve Ratio*

This section examines the optimal responses of the Taylor rule, (19), when the latter is facilitated by either a required reserve ratio rule or an interest on reserves rule, as shown by (30) and (31) below. Some recent studies show that required reserve ratio rules that respond in a countercyclical fashion to macroprudential or financial variables, such as credit or credit spreads, can help promote financial and macro stability.<sup>32</sup> Accordingly, the following rule for the required reserve ratio is considered,

$$\varsigma_t = \varsigma \left( \frac{l_t}{l} \right)^{\mu_\varsigma^l}, \quad (30)$$

where the elasticity  $\mu_\varsigma^l$  measures the responsiveness of the required reserve ratio to credit deviations from its steady state. The motivation for examining the interest rate on reserves in a similar fashion, comes from the fact that this monetary tool has recently been

<sup>30</sup>See also Loungani and Rush, 1995, Agénor and El Aynaoui, 2010, Montoro and Moreno, 2011, Glocker and Towbin, 2012.

<sup>31</sup>See also Dutkowsky and VanHoose, 2013, Ireland 2014, and Güntner 2015.

<sup>32</sup>See Gray, (2011), Montoro and Moreno (2011), Glocker and Towbin, (2012), Chadha and Corrado, (2012), Mimir, Sunel and Taşkın, (2013), Agénor and Pereira da Silva (2014) and Mora (2014).

used by a number of central banks (including the US Fed and the ECB) to facilitate their credit liquidity management and has attracted much research attention in the aftermath of the Great Recession.<sup>33</sup> Accordingly, the following rule for the interest on reserves is examined,<sup>34</sup>

$$R_t^{ior} = R^{ior} \left( \frac{l_t}{l} \right)^{\mu_{ior}^l}, \quad (31)$$

where  $\mu_{ior}^l$  measures the responsiveness of the interest on reserves to credit deviations from its steady state.<sup>35</sup> Tables 3 (a) and (b), summarize the optimal policy responses when the central bank uses a combination of an optimal Taylor rule (OTR) with, either the required reserve ratio (OTR+ $\varsigma$ ), or the interest on reserves (OTR+ $R^{ior}$ ) as supporting monetary policy tools. The welfare effects, reported in consumption equivalent measures (CE), are given in terms of net gains from the baseline policy case (that is a simple Taylor rule with  $\phi_\pi = 1.5$  and  $\phi_x = 0$ ).

**Result 4:** *As supporting monetary policy tools, both the required reserve ratio and the interest on reserves can improve on the welfare outcomes achieved by just a simple optimal Taylor rule. However, the interest on reserves can achieve higher welfare gains than the required reserve ratio because of the conflicting effect that the latter has through the deposits channel (i.e. the balance sheet and the deposit rate effects). This result is independent of precautionary liquidity.*

Result 4 follows from section 5.3, Tables 2 and 3, and Figures 6 and 7. Figure 6, demonstrates this result for the case of a financial shock when liquidity hoarding is zero, ( $\psi_3=0$ , which implies excess reserves of about 2% of total deposits).<sup>36</sup> The financial shock raises, the probability of default, the cost of borrowing and inflation and reduces the output gap. The optimal response of the required reserve ratio is to cause a rise in the deposit rate but, as shown in section 5.3, this can only be achieved with a decrease in the required reserve ratio which frees some reserves for loans through the balance sheet.

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<sup>33</sup>See Hall (2002), Goodfriend (2002, 2011), Curdia and Woodford (2011), Bech and Klee (2011), Dutkowskya and VanHoose (2011), Kashyap and Stein (2012), Ireland (2014), Cochrane (2014), Dressler and Kersting, (2015), Hall and Reis (2016).

<sup>34</sup>Note that both these policy tools could also respond to a number of other financial variables but the results are not expected to be very different. For example, allowing them to respond to the loan rate spread (i.e.  $\varsigma_t = \left( \frac{R_t^l/R_t}{R^l/R} \right)^{\mu_\varsigma^s}$  and  $R_t^{ior} = \left( \frac{R_t^l/R_t}{R^l/R} \right)^{\mu_{ior}^s}$ ), produces very similar results. These results can be made available upon request.

<sup>35</sup>The optimal policy parameters are grid-searched within the following ranges,  $\phi_\pi = [0, 3]$ ,  $\phi_x = [0, 1]$ , and  $\mu_\varsigma^l, \mu_{ior}^l = [-3, 3]$ .

<sup>36</sup>The results for the optimal Taylor rule (Black(o) line=OTR), are given in Table 2 (a) and (b) respectively.

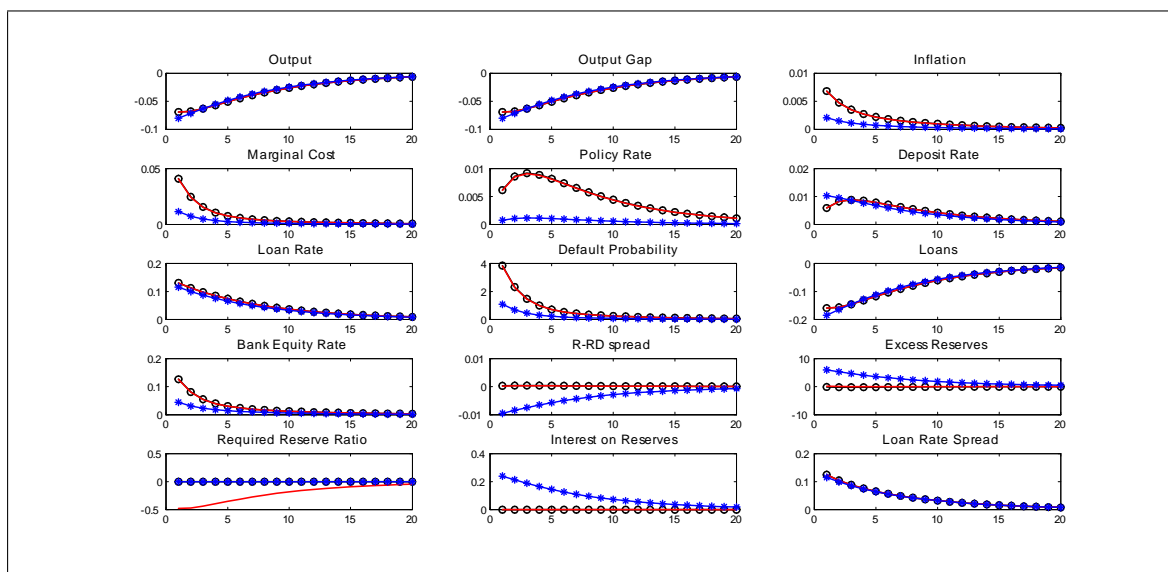
Table 3: Welfare Effects: Interest on Reserves and Required Reserve Ratio

(a) Loan Rate Shock,				
Policy	$\psi_3=0.001, ER\approx 2.0\%$	$\psi_3=3.5, ER\approx 12\%$	$\psi_3=5.4, ER\approx 18\%$	$\psi_3=7.6, ER\approx 25\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$
OTR+ $\varsigma$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.02384$	$\phi_\pi = 1.90$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.00$ $CE = 0.00004$	$\phi_\pi = 3.00$ $\phi_x = 0.60$ $\mu_\varsigma^l = 3.00$ $CE = 0.00261$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.00866$
OTR+ $R_t^{ior}$	$\phi_\pi = 1.20$ $\phi_x = 0.0$ $\mu_{ior}^l = -1.30$ $CE = 0.02881$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -3.00$ $CE = 0.00924$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -2.30$ $CE = 0.01002$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_{ior}^l = -1.40$ $CE = 0.01044$
(b) Adverse Technology Shock				
Policy	$\psi_3=0.001, ER\approx 2\%$	$\psi_3=2.0, ER\approx 8\%$	$\psi_3=13.5, ER\approx 42\%$	$\psi_3=16.2, ER\approx 51\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$
OTR+ $\varsigma$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.00255$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.00056$	$\phi_\pi = 2.70$ $\phi_x = 0.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.0000$	$\phi_\pi = 0.80^*$ $\phi_x = 1.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.0000$
OTR+ $R_t^{ior}$	$\phi_\pi = 3.00$ $\phi_x = 0.20$ $\mu_{ior}^l = -1.90$ $CE = 0.00288$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.80$ $CE = 0.00074$	$\phi_\pi = 3.0$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.20$ $CE = 0.0000$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.10$ $CE = 0.0000$

\*Taylor principle violated

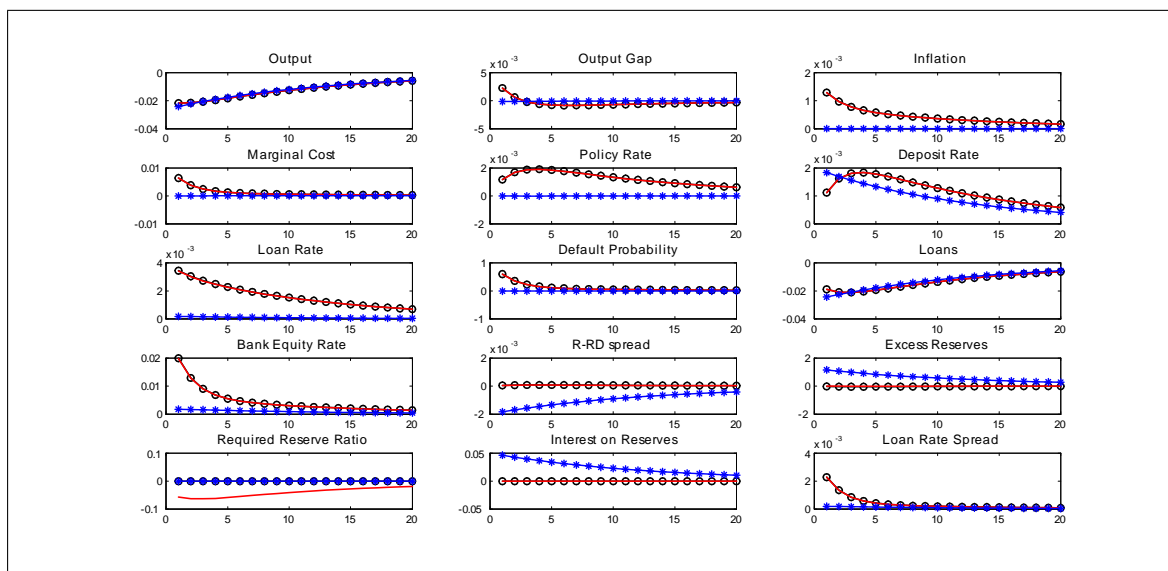
The interest on reserves is shown to stabilize inflation and the output gap more effectively through the deposits and balance sheet channels, by having a stronger effect on the deposit rate, while simultaneously raising the level of excess reserves and reducing loans. This takes the pressure off the policy rate, which does not have to be as aggressive as in the cases of the simple optimal Taylor rule (OTR), or the latter supported by the required reserve ratio (OTR+ $\varsigma$ ). A lower policy rate implies that the default probability, the bank equity rate, the loan rate and hence marginal cost and inflation are lower, which in turn support a low policy rate. Similar results are shown in the case of an adverse technology shock, in Figure 7, where the combination of a strong response to inflation, combined with a rise in the interest on reserves is shown to stabilize inflation and the output gap more effectively than the required reserve ratio. This result is shown to be independent of the degree of precautionary liquidity.

Figure 6: Financial Shock: Interest on Reserves and Required Reserve Ratio



Black(o) line=OTR; Red(solid) line=OTR+ $\varsigma$ ; Blue(star) line=OTR+ $R^{ior}$ , (all at ER=2%)

Figure 7: Adverse Supply Shock: Interest on Reserves and Required Reserve Ratio



Black(o) line=OTR; Red(solid) line=OTR+ $\varsigma$ ; Blue(star) line=OTR+ $R^{ior}$ , (all at ER=2%)

## 5.4 The Interest on Reserves as a Main Policy Tool.

Cochrane (2014) indicates that conventional theories cannot explain how at times of large balance sheets the interest on reserves can determine inflation, but shows that this is possible with the backing of fiscal policy and the fiscal theory of the price level. In a similar spirit, Hall and Reis (2016) use a simple model of the fiscal theory of the price level to show that an interest payment on reserves can pin down a unique equilibrium of the price level based on arbitrage between only two periods. In this section, I show that even in a conventional DSGE model, free of fiscal theory price level properties, the interest on reserves can provide a unique equilibrium and fully replace the Taylor rule. As in Hall and Reis (2016), the focus is on the interest on reserves in a model of credit frictions, however the innovation here does not rely on the properties of the fiscal theory of the price level, but on how the interest on reserves affects the deposits channel when the former responds to the source of credit frictions, that is, the level of credit riskiness in this model. I demonstrate this with two different type of policy rules: (i) the interest on reserves responds to deviations of excess reserves from their steady state level,

$$R_t^{ior} = R^{ior} \left( \frac{\tilde{r}_t - \varsigma_t}{\tilde{r} - \varsigma} \right)^{\mu_{ior}^{ER}}, \quad (32)$$

where  $\mu_{ior}^{ER}$  measures the policy responsiveness to excess reserves and (ii) the interest on reserves responds directly to deviations of credit risk from its steady state level,

$$R_t^{ior} = R^{ior} \left( \frac{\Phi_t}{\Phi} \right)^{\mu_{ior}^{\Phi}}, \quad (33)$$

where  $\mu_{ior}^{\Phi}$  measures the policy responsiveness to credit risk. Both (32) and (33) imply that the interest on reserves responds to changes in the level of credit riskiness in the economy, the former through the effect that credit risk has on the level of excess reserves, whereas the latter directly. Tables 4 and 5 summarize the results.

**Proposition 2:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\mu_{ior}^{\Phi} > 0$ , that is, the interest on reserves can affect the deposit channel in response to changes in the credit riskiness in the economy, when the latter affects the output gap and the cost channel (inflation). This result is independent of the level of precautionary liquidity in the banking system, ( $\psi_3 \geq 0$ ), or the fiscal theory of the price level.*

**Proof.** A formal proof of Proposition 2 is provided in the *Appendix*.

Table 4: Interest on Reserves at the Zero-Bound: Responses to Excess Reserves

(a) Loan Rate Shock,				
Policy	$\psi_3=0$ , ER $\approx$ 0%	$\psi_3=2.0$ , ER $\approx$ 6%	$\psi_3=4.0$ , ER $\approx$ 12%	$\psi_3=10$ , ER $\approx$ 30%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = N/A$ $CE = N/A$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.10$ $CE = 0.00518$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.10$ $CE = 0.00011$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = -0.18$ $CE = 0.00025$
(b) Adverse Supply Shock				
Policy	$\psi_3=0$ , ER $\approx$ 0.0%	$\psi_3=2.0$ , ER $\approx$ 6%	$\psi_3=8.3$ , ER $\approx$ 25.0%	$\psi_3=13.5$ , ER $\approx$ 40.0%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_y = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = N/A$ $CE = N/A$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.08$ $CE = 0.00095$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.27$ $CE = 0.0000$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.42$ $CE = 0.0000$

N/A=Indeterminacy. Values of  $\psi_1 = 0$  and  $R = R^{ior}$  are used so that  $\psi_3=0$  implies ER $\approx$ 0%.

Table 5: Interest on Reserves at the Zero-Bound: Responses to Credit Risk

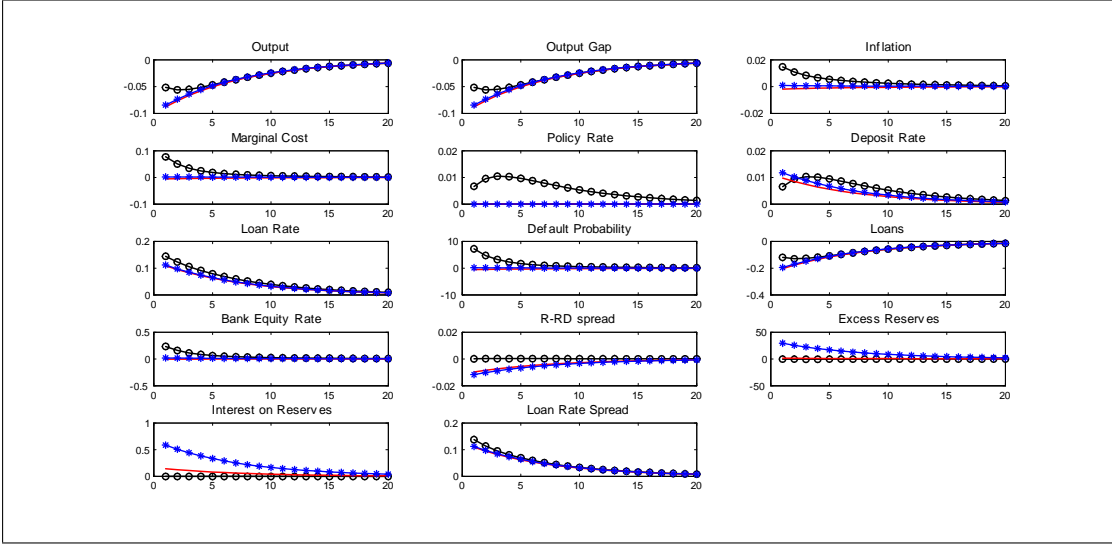
(a) Loan Rate Shock,				
Policy	$\psi_3=0$ , ER $\approx$ 0%	$\psi_3=2.0$ , ER $\approx$ 6%	$\psi_3=4.0$ , ER $\approx$ 12%	$\psi_3=10$ , ER $\approx$ 30%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 2.76$ $CE = 0.02666$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 0.64$ $CE = 0.00744$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 0.29$ $CE = 0.00113$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = -0.11$ $CE = 0.00025$
(b) Adverse Technology Shock				
Policy	$\psi_3=0$ , ER $\approx$ 0%	$\psi_3=2.0$ , ER $\approx$ 6%	$\psi_3=8.3$ , ER $\approx$ 25.0%	$\psi_3=13.5$ , ER $\approx$ 40.0%
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = -3.00$ $CE = 0.00263$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.00095$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.0000$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.0000$

Values of  $\psi_1 = 0$  and  $R = R^{ior}$  are used so that  $\psi_3=0$  implies ER $\approx$ 0%.

**Result 5:** *When the policy rate is at the zero-bound the interest on reserves can act as a single monetary policy tool, through its effects on the deposits channels, producing higher welfare gains than a simple Taylor rule.*

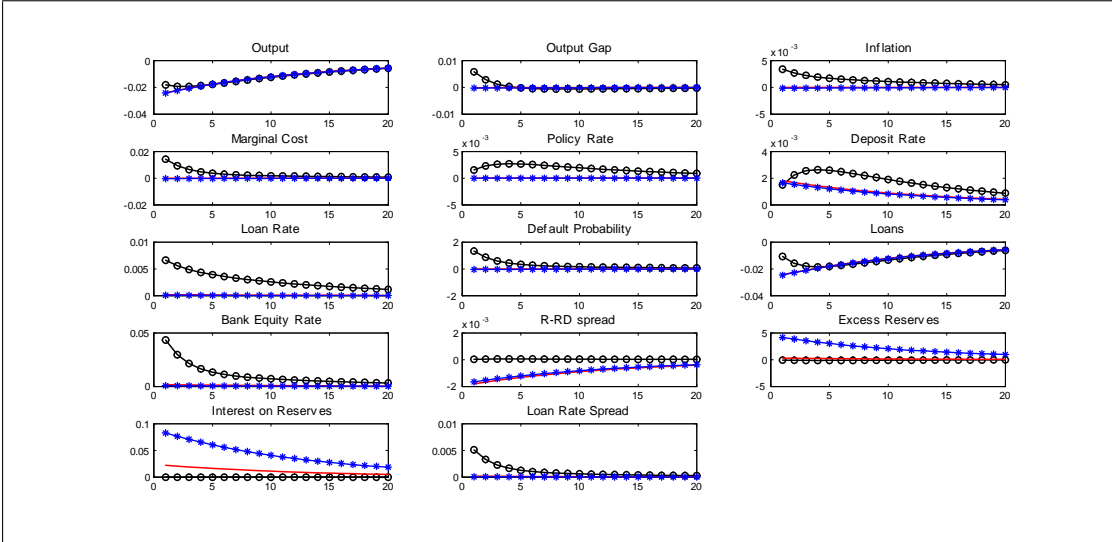
Result 5, follows from Proposition 2, Tables 4 and 5 and Figures 7 and 8 that plot the optimal responses of the interest on reserves rules (eqs. 32 and 33), at the zero-bound,  $\widehat{R}_t = 0$ . The intuition here is that the source of credit friction, that is the level of credit riskiness in the economy, affects the output gap and inflation (through the cost channel). Thus, by responding to the level of credit riskiness in the economy, the interest on reserves can pin down inflation and the output gap, through the deposits channel, that is through its effects on the balance sheets of banks and the deposit rate. This is demonstrated in Figures 7 and 8, where following a financial shock that raises inflation, the optimal response of the interest on reserves is to increase, resulting in higher excess reserves, a fall in loans and a higher deposit rate, which reduce the output gap, the loan spread and the cost of borrowing and thus inflation. Tables (4) and (5) also show that there are net welfare gains to be made from using the interest on reserves as a single monetary policy tool over the simple Taylor rule.

Figure 8: Financial Shock: Interest on Reserves at the Zero Bound



Black(o) line=Baseline (ER=0%); Red(solid) line=eq(32),  $\mu_{ior}^{ER}=0.10$ ,  $\psi_3=2.0$ , (ER=6%); Blue(star) line=eq(33),  $\mu_{ior}^{\Phi}=2.76$ ,  $\psi_3=0$ , (ER=0%). All estimated with  $\psi_1=0$ ,  $R_t=R_t^{ior}$ .

Figure 9: Adverse Supply Shock: Interest on Reserves at the Zero-Bound



Black(o) line=Baseline (ER=0%); Red(solid) line=eq(32),  $\mu_{ior}^{ER}=0.08$ ,  $\psi_3=2.0$ , (ER=6%); Blue(star) line=eq(33),  $\mu_{ior}^{\Phi}=-3.00$ ,  $\psi_3=0$ , (ER=0%). All estimated with  $\psi_1=0$ ,  $R_t=R_t^{ior}$ .



## 5.5 The Role of the Interest on Reserves Spread, $(R_t - R_t^{ior})$

In much of the above analysis we have assumed a positive steady state spread between the policy rate and the interest on reserves,  $R > R^{ior}$ . In practice however, since its introduction in 2008, the US Fed has been setting the interest rate on reserves equal to the policy rate.<sup>37</sup> Moreover, there is evidence that the federal funds rate regularly traded below the interest on reserves (Goodfriend 2015). The  $R_t - R_t^{ior}$  spread determines the opportunity cost of holding reserves. The lower is the interest on reserves in relation to the policy rate the higher is the ‘tax on holding reserves’ (Ireland, 2015). Conversely, the smaller is the  $R_t - R_t^{ior}$  spread, the higher is expected to be the level of excess reserves. Indeed, Table 6 shows that the steady state level of excess reserves increases as the  $R - R^{ior}$  spread is reduced from 100bps to 0 bps. Here, with no precautionary liquidity, ( $\psi_3=0.001$ , and ER=2.0%) or other factors affecting the level of excess reserves, a rise in the spread by 100 basis points is shown to result in an 1% increase in excess reserves at the steady state. This tends to restrict lending, which as Table 6 shows, can result in some small welfare gains when policy is concerned with inflation. However, this does not necessarily imply that setting the interest on reserves equal to the policy rate is the best central banks could do with this policy tool. Table 7 is based on exactly the same parameterization as Table 6, but allows the interest on reserves to deviate from  $R_t^{ior} = R_t$  and respond optimally to either excess reserves (eq. 32) or directly to credit risk (eq 33). It is shown that using the interest on reserves as an independent policy tool to facilitate monetary policy can increase welfare gains beyond those of a simple Taylor Rule, or simply setting,  $R_t^{ior} = R_t$ .

Table 6: Welfare Effects: The role of the Interest on Reserves Spread

R- $R^{ior}$ =	100bps (ER=2.0%)	70bps (ER=2.3%)	50bps (ER=2.5%)	R= $R^{ior}$ (ER=3.0%)
(a) Loan Rate Shock				
Baseline	$\phi_\pi = 1.5, \phi_x = 0, CE = -$			
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02372$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02399$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02418$	$\phi_\pi = 3.0$ $\phi_x = 0.00$ $CE = 0.02464$
(b) Adverse Supply Shock				
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00254$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00258$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00260$	$\phi_\pi = 3.0$ $\phi_x = 0.00$ $CE = 0.00267$

<sup>37</sup>This has also been the practice of the Bank of England.

Table 7: Welfare Effects: Interest on Reserves as an Independent Policy Tool

(a) Loan Rate Shock				
Baseline	$\phi_\pi = 1.5,$	$\phi_x = 0,$	$R=R^{ior}(\text{ER}=3.0\%),$	$\psi_3 = 0, \quad CE = -$
OTR	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	-	$CE = 0.02464$
OTR+ $R_t^{ior}$	(eq 32) $\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^{ER} = 0.10$	$CE = 0.02589$
OTR+ $R_t^{ior}$	(eq 33) $\phi_\pi = 0.00,$	$\phi_x = 1.00$	$\mu_{ior}^\Phi = 3.00$	$CE = 0.03989$
(b) Adverse Supply Shock				
OTR	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	-	$CE = 0.00267$
OTR+ $R_t^{ior}$	(eq 32) $\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^{ER} = 0.10$	$CE = 0.00274$
OTR+ $R_t^{ior}$	(eq 33) $\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^\Phi = 3.00$	$CE = 0.00301$

**Result 6:** *Setting the interest on reserves equal to policy rate eliminates the tax on holding reserves. This encourages excess reserves which can increase welfare gain in times of liquidity management. However, such welfare gains are outperformed by allowing the interest on reserves to act as an independent policy tool to facilitate the policy rate.*

This result, which follows from Tables 6 and 7, is due to the effects that the interest on reserves can have through the deposits channel, as already explained in sections 5.3 and 5.3.1.

## 6 Concluding Discussion

This paper emphasizes the importance of the deposits channel in explaining the role of monetary policy, particularly during periods when the policy rate is fixed at the lower zero-bound. Two new potential factors that can determine monetary policy through the deposits channel are identified: *precautionary liquidity* in times of high perceived risk and the *interest on reserves*.

It is shown that high levels of excess reserves in the financial sector, due to a potential credit risk in the real sector, imply a lower optimal relative weight on inflation in the Taylor rule. The intuition of this is that when the financial sector perceives credit riskiness to be high, precautionary liquidity raises excess reserves and restricts loans to the real sector, while the deposit rate (the rate offered on liquid assets) that affects intertemporal household decisions, can rise above the policy rate. Both these effects of the deposits channel act as a self-discipline device in the banking system that is shown to pin down the inflation rate and provide a unique equilibrium even when the policy rate is at the zero-bound. In practice, the extent to which precautionary liquidity can effect the deposits channel depends on how the financial sector perceives credit riskiness, because as the

model indicates even when the actual credit risk,  $\Phi_t(\varepsilon_t^*)$  is high, if banks undermine such risk (implying a low pass-through elasticity  $\psi_3$ ), then the deposits channel may be too weak to determine a unique equilibrium without the intervention of the policy rate.

A more active role for monetary policy, particularly at the zero-bound, is to try and affect the deposits channel through the interest on reserves. When banks become unresponsive to the policy rate that sets the benchmark for their transactions at the interbank market, they may become more responsive to the rate offered on the reserves they are sitting on. This paper shows that through its effect on the deposits channel, the interest on reserves can act as a single monetary policy tool that can provide determinacy but also a higher welfare gain than a simple Taylor rule. This result is supportive of the recent findings in Cochrane (2014) and Hall and Reis (2016) that a payment on reserves rule is sufficient to stabilize the price level and it is simpler to implement than a Taylor rule. However, here these results are established within a conventional new Keynesian macroeconomic framework and are shown to be independent of the fiscal theory of the price level, or the degree of liquidity hoarding.

In practice, the implementation of an interest on reserves policy depends on the rules considered. Responding to excess reserves (as one of the rules considered here), or total reserves, as in Hall and Reis (2016), should be easier to implement in practice than a Taylor rule that requires the calculation of output gaps and natural real interest rates. However, if the interest on reserves responds to the level of credit riskiness in the economy, (a rule shown to perform particularly well in this paper), then the latter may be harder to calculate and so the level of excess reserves may be preferred as a proxy to perceived risk. The findings also question whether the current practice of setting the interest on reserves equal to the policy rate, (i.e. US Fed, Bank of England), undermine the full potential of the interest on reserves as an independent policy tool. The paper also shows that the required reserve ratio can affect the deposits channel, but its effectiveness is weaker because of the conflicting effects it has through this channel that overall makes the interest on reserves a more welfare-enhancing monetary policy tool that also requires less interest rate intervention.

In general, the findings in this paper may shed some light on the zero-interest rate policy observed in countries that experienced large excess reserves. It can also justify why the interest on reserves, rather than required reserves, became the focus of monetary policy during the zero-bound period in a number of countries. Overall, the paper invites more research on the determinants of the deposits channel for explaining the role and effectiveness of monetary policy, particularly in times of unconventional monetary policy,

but also on the potential role of the interest on reserves as an independent monetary policy tool.

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## 7 APPENDIX

**Proposition 1:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\psi_3 > 0$ , that is, there is a positive degree of precautionary liquidity that raises the deposit rate above the policy rate.*

### Proof of Proposition 1:

Consider a simplified version of the lower zero bound case where the policy rate is fixed around its steady state value of  $R = 1/\beta$  (hence  $\widehat{R}_t = 0$ ). We also assume  $\theta_t = 0$  hence  $\Theta_t = \psi_3 \Phi_t(\varepsilon_t^*)$  and the required reserve ratio and the interest on reserves are also fixed, so that  $\widehat{R}_t^{ior} = \widehat{\zeta}_t = 0$ , as assumed in Table 2. These assumptions imply that the model can be reduced to three log-linearized equations,

$$\begin{aligned}\widehat{x}_t &= \mathbb{E}_t \widehat{x}_{t+1} - \sigma^{-1} \left( \widehat{R}_t^d - \mathbb{E}_t \widehat{\pi}_{t+1} \right) + \widehat{u}_t, \\ \widehat{\pi}_t &= \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p (\eta + \sigma + \Gamma) \widehat{x}_t, \\ \widehat{R}_t^d &= B \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) (\eta + \sigma + \Gamma) \widehat{x}_t.\end{aligned}$$

Substituting  $\widehat{R}_t^d$  into  $\widehat{x}_t$  we can write the above equations into a 2 x 2 vector system,

$$\begin{bmatrix} \mathbb{E}_t \widehat{x}_{t+1} \\ \mathbb{E}_t \widehat{\pi}_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ \widehat{\pi}_t \end{bmatrix} + u_t, \quad \text{where, } A = \begin{bmatrix} 1 + B \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \frac{\eta + \sigma + \Gamma}{\sigma} \right) & 0 \\ 0 & 1/\beta \end{bmatrix},$$

where,

$$B = \frac{\psi_3 \Phi (-\psi_1 + \psi_3 \Phi - (R - R^{ior}))}{\psi_2 R^d},$$

$\psi_1 < 0$ ,  $\psi_2 > 0$  and  $\psi_3 \geq 0$  and,

$$\Gamma = (\eta + \sigma) \left( \frac{(B + \frac{\Phi}{1-\Phi}) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} + \frac{(\varepsilon^* + \underline{\varepsilon}) \chi \Phi}{2l/y}}{R^l - (B + \frac{\Phi}{1-\Phi}) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} - \frac{\varepsilon^* \chi \Phi}{l/y}} \right),$$

where  $R^e = \frac{R^d}{1-\Phi(\varepsilon^*)} - \varrho^E$ , and  $R^l = (1-\gamma)R + \gamma R^e + \chi \frac{y}{l} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \Phi^2$ . It follows that a necessary condition for a *determinate* equilibrium is,

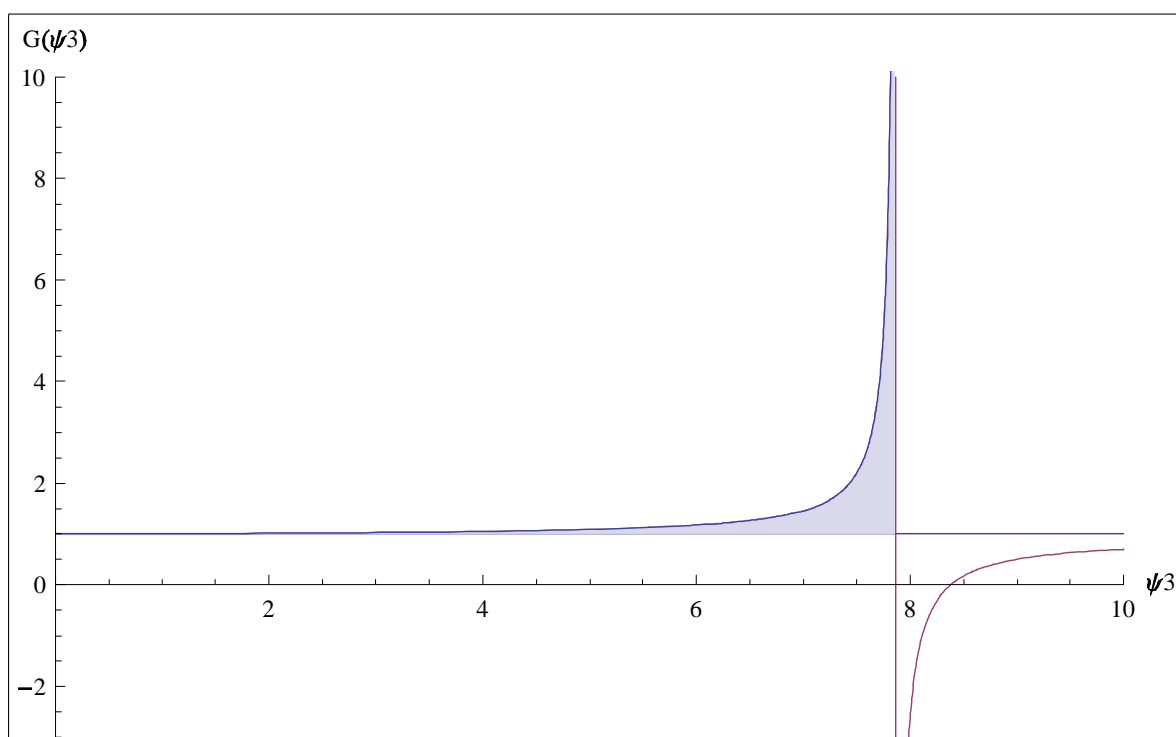
$$G(\psi_3) = 1 + \frac{B(1-\beta)(\eta + \sigma + \Gamma)}{\beta\sigma} \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} > 1.$$

From this and the definition of  $B$  and  $\Gamma$  above, it is shown that when the policy rate is fixed at the zero-bound, a necessary condition for determinacy is that,  $\psi_3 > 0$ . Eliminating



precautionary liquidity, that is setting  $\psi_3 = 0$ , results in  $B = 0$  and the determinacy condition is not met, as  $G(\psi_3 = 0) = 1$ . This implies that in the absence of an active policy rate, ( $\widehat{R}_t = 0$ ), which could affect the deposit rate, setting  $\psi_3 = 0$ , entirely eliminates the endogenous response of the deposit rate to the output gap and inflation (see  $\widehat{R}_t^d$  above) and the model becomes *indeterminate* as in the standard case. Figure 10, shows the determinacy region (shaded area above unity) for different values of  $\psi_3$ , at the zero-bound and keeping all other parameters constant at their baseline level (see Table 1).

Figure 10: Determinacy Region for Different Values of  $\psi_3$



Based on this example, it is shown that any value,  $0 < \psi_3 \lesssim 7.88$  can provide determinacy. The discontinuity above this value occurs because as reserves rise above a certain level, (here approximately above 30% of total deposits), then in the absence of active monetary policy (i.e. at the zero bound and with no intervention of any other monetary policy tool), further rises in  $\psi_3$  can increase  $B$  to a level beyond which  $\Gamma$  turns negative, thus reducing the deposit rate and the effectiveness of the deposit channel. This also reduces  $G(\psi_3)$  to below unity thus resulting in indeterminacy.

**Proposition 2:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\mu_{ior}^{\Phi} > 0$ , that is, the interest on reserves can affect the deposit channel in response to changes in the credit riskiness in the economy, when the latter affects the output gap and the cost channel (inflation). This result is independent of the level of precautionary liquidity in the banking system, ( $\psi_3 \geq 0$ ), or the fiscal theory of the price level.*

**Proof of Proposition 2:**

Consider a simplified version of the zero-bound case where the policy rate is fixed  $\widehat{R}_t = 0$ , around its steady state value of  $R = 1/\beta$ , as before, but here we only assume that the required reserve ratio is fixed,  $\widehat{\varsigma}_t = 0$ , and allow the interest on reserves to vary, as employed in section 5.4. These assumptions imply the model can be reduced to the following three equations,

$$\begin{aligned}\widehat{x}_t &= \mathbb{E}_t \widehat{x}_{t+1} - \sigma^{-1} \left( \widehat{R}_t^d - \mathbb{E}_t \widehat{\pi}_{t+1} \right) + \widehat{u}_t, \\ \widehat{\pi}_t &= \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p \left( \eta + \sigma + \widetilde{\Gamma} \right) \widehat{x}_t, \\ \widehat{R}_t^d &= \widetilde{B} \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \eta + \sigma + \widetilde{\Gamma} \right) \widehat{x}_t.\end{aligned}$$

Substituting  $\widehat{R}_t^d$  into  $\widehat{x}_t$  we can write the above equations into a 2 x 2 vector system,

$$\begin{bmatrix} \mathbb{E}_t \widehat{x}_{t+1} \\ \mathbb{E}_t \widehat{\pi}_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ \widehat{\pi}_t \end{bmatrix} + u_t, \quad \text{where, } A = \begin{bmatrix} 1 + \widetilde{B} \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \frac{\eta + \sigma + \widetilde{\Gamma}}{\sigma} \right) & 0 \\ 0 & 1/\beta \end{bmatrix},$$

where,

$$\widetilde{B} = \frac{\psi_3 \Phi (-\psi_1 + \psi_3 \Phi - (R - R^{ior})) + \mu_{ior}^{\Phi} R^{ior} (-\psi_1 + \psi_2 \varsigma + \psi_3 \Phi - (R - R^{ior}))}{\psi_2 R^d},$$

where  $\psi_1 < 0$ ,  $\psi_2 > 0$  and  $\psi_3 \geq 0$  and,

$$\widetilde{\Gamma} = (\eta + \sigma) \left( \frac{\left( \widetilde{B} + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} + \frac{(\varepsilon^* + \underline{\varepsilon}) \chi \Phi}{2l/y}}{R^l - \left( \widetilde{B} + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} - \frac{\varepsilon^* \chi \Phi}{l/y}} \right).$$

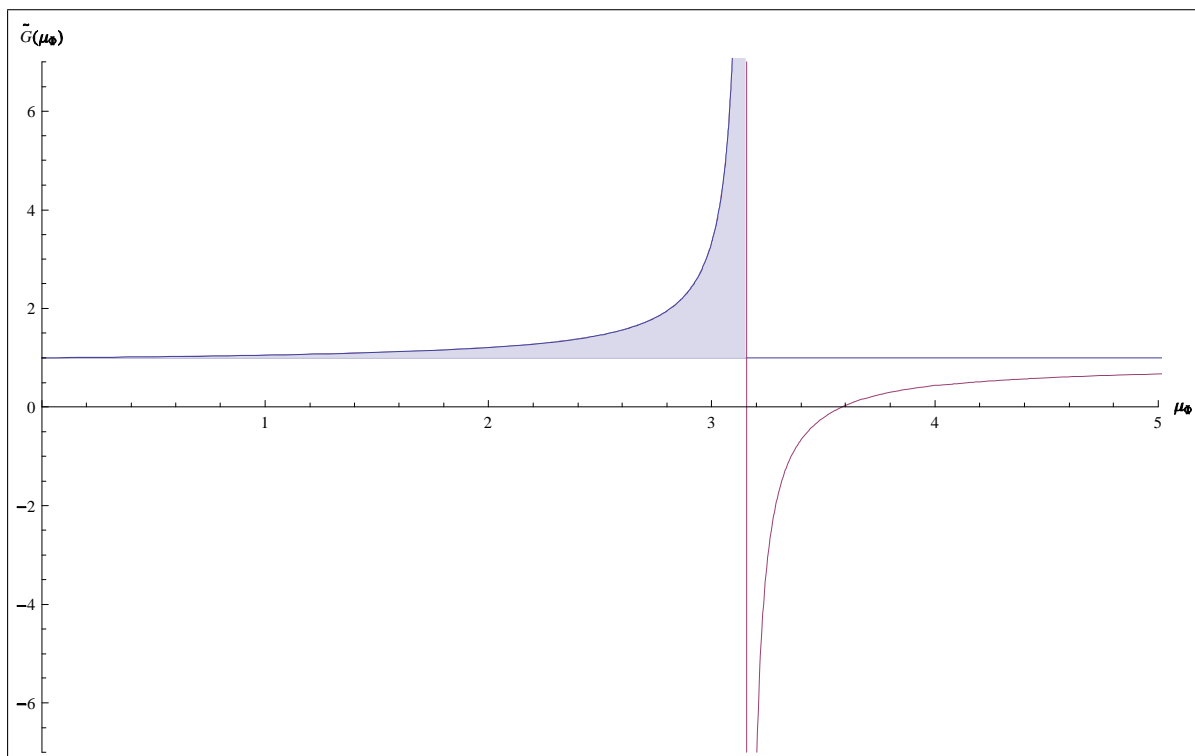
It follows that a necessary condition for a *determinate* equilibrium is,

$$\widetilde{G}(\mu_{ior}^{\Phi}) = 1 + \frac{\widetilde{B}(1 - \beta)(\eta + \sigma + \widetilde{\Gamma})}{\beta \sigma} \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} > 1.$$

It follows that if the policy rate is fixed at the zero-bound,  $\widehat{R}_t = 0$ , and there is no

precautionary liquidity in the banking system, ( $\psi_3 = 0$ ), then a necessary condition for a *determinate* equilibrium is  $\mu_{ior}^\Phi > 0$ . With  $\psi_3 = 0$ , setting  $\mu_{ior}^\Phi = 0$  results in  $\tilde{B} = 0$ , hence the condition for a unique equilibrium cannot be satisfied as  $\tilde{G}(\mu_{ior}^\Phi = 0) = 1$ . Figure 11, shows the determinacy region for different values of  $\mu_{ior}^\Phi$  at the zero bound  $\hat{R}_t = 0$ , when  $R = R^{ior} = 1/\beta$ ,  $\psi_1 = \psi_3 = 0$  and (steady state)  $ER = 0\%$ .

Figure 11: Determinacy Region for Different Values of mu-phi



Based on this example, it is shown that any value,  $0 < \mu_{ior}^\Phi \lesssim 3.1$  can provide determinacy. The discontinuity above this value occurs because setting very high values of  $\mu_{ior}^\Phi$  can increase  $\tilde{B}$  to a level beyond which  $\tilde{\Gamma}$  turns negative and this reduces the deposit rate and the effectiveness of the deposit channel. This also reduces  $\tilde{G}(\mu_{ior}^\Phi)$  to below unity thus resulting in indeterminacy.