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# Monetary Policy Rules, Real Rigidity and Endogenous Persistence

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# Monetary Policy Rules, Real Rigidity and Endogenous Persistence

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Abstract
The bulk of literature on real rigidity attempts to identify sources of real rigidity in market imperfections while assuming that the money supply is exogenously set. This paper shows that monetary policy preferences affect the responsiveness of marginal cost to output and through this channel they are shown to determine (i) the degree of real rigidity and (ii) the degree of endogenous persistence. We find that substantial levels of real rigidity and persistence can be generated using plausible monetary parameter values and without relying on additional sources of real rigidity.

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# 1. Introduction

In the last few years New Keynesian economics has shown that small nominal frictions may not be substantial to explain large fluctuation in aggregate activity, unless they are combined with real rigidity. Real rigidities alone cannot impede full nominal flexibility but as Ball and Romer (1990), Romer (2001) and the subsequent literature shows, the combination of small nominal frictions with real rigidities are capable of generating output variations similar to those observed in actual data. This is because real rigidities feed endogenously into the degree of nominal rigidity -or nominal persistence in dynamic models- thus amplifying and prolonging the real effects of any type of shock.

An already vast existing literature, has shown that real rigidity can be found in imperfections in the product, labour, capital and financial markets.[1] A predicament however frequently associated with this literature is whether the size of the real rigidity required to produce substantial nominal and real persistence, can be obtained for realistic parameter values in such models. Taking for example the two potential sources of real rigidity explored in Ball and Romer (1990), namely asymmetric demand and efficiency wages. Both of these sources are capable of generating real rigidities, but as Ball and Romer (1990) points out real rigidity in the former type of models is determined by the "sharpness of the asymmetry in demand, a parameter for which we do not know realistic values". Similarly, Kiley (1997) argues that efficiency-wage models may not be an appropriate source of real rigidity because although they can generate acyclical real wages they do not necessarily imply a lower sensitivity of marginal cost to output or increased nominal persistence. From the empirical point of view, as sources of real rigidities are usually identified in small imperfections and in specific markets in the economy, it is difficult to assess their wider macroeconomic impact.

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[1] For a review of the related literature see Romer (2001), Benassi, Chirco and Colombo, (1994) and Blanchard and Fischer (1989).

In this paper we focus on the role of endogenous monetary policy as a new potential source of real rigidity and macro persistence. Whether policy rules can affect the degree of persistence is of course a well-established result that can be traced among others to Taylor (1980) and has been extensively examined more recently in dynamic general equilibrium models including, King and Watson (1996), Yun (1996), Rotemberg (1996), Rotemberg and Woodford (1997), Gavin and Kydland (1999), Ireland (2001), etc. However, this literature is not concerned with real rigidities, it rather focuses on the various effects of endogenous money supply on the degree of persistence. This is in contrast to the bulk of the New Keynesian literature on real rigidities, that explicitly focus on how imperfections in the economy can generate real rigidity, but conventionally assumes an exogenous money supply. Some interesting examples of this literature include, Woglom (1982), Mankiw (1985), Ball and Romer (1990), Hairault and Portier (1993), Benassi, Chirco and Colombo (1994), Kiley (1997, 1998), Jeanne (1998), Devereux and Yetman (2002) etc.

This paper attempts to provide a link between the two existing literatures. By focusing on the role of endogenous monetary policy, we examine whether and to what extent monetary policy rules can affect the degree of real rigidity and through this very channel the degree of endogenous persistence. Most research on real rigidity starts out to identify sources of real rigidity by assuming that aggregate demand is exogenously set. However, monetary policy rules, used widely by central banks, are set as a function of prices and output. We show that monetary policy preferences affect the slope of the aggregate demand curve. At the firm level this is shown to affect the responsiveness of marginal cost to changes in output. This results in monetary policy endogenously affecting the degrees of both real rigidity and nominal persistence. More importantly, we show that plausible values of monetary policy parameters can attain high degrees of real rigidity and nominal persistence without relying on any market imperfections or other exogenous sources of real rigidity as assumed previously.

The rest of the paper is organised as follows. In section 2, we present a microfounded model from which we derive the product demand, labour supply and optimal price decisions in each sector. Section 3, shows how monetary policy rules can endogenously affect these decisions and the degree of real rigidity. Section 4, extends the model to account for the effects of real rigidity in a Calvo-type price staggering model. In this section we examine how monetary policy rules, through their effect on real rigidities, can amplify persistence effects. Both sections 3 and 4 are supported with numerical simulations. Section 5 briefly discusses the implications of the model for the size of real rigidity required to explain actual observed persistence and section 6 concludes.

## 2. The Model

We consider a simple economy consisting of a continuum of imperfectly competitive firms, indexed by $j$, each producing a slightly different commodity and distributed uniformly over the interval $[0,1]$.[2] The economy is populated by many identical households, indexed by $i$, and distributed also uniformly over the interval $[0,1]$. Each typical household, consumes goods from all firms, receives a monetary transfer in the beginning of each period, supplies $L_i$ units of labour in a firm, at the competitive wage rate, and receives an equal share of profits from all sectors. The money supply is set by the central bank according to a monetary policy rule.

### 2.1 Monetary Policy Rule

The central bank follows a simple monetary policy rule, mimicking a Taylor rule, where the money supply is set in response to deviations in consumer price index and output from their target levels respectively.[3]

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[2] As we show below the assumption of imperfectly competitive firms play no crucial role in this model, though it does provide a rationale for price setting.
[3] Although it is certainly true that central bank manage stabilisation policy through short-term interest rates, the use of this particular monetary -rather than interest rate policy rule- is used merely for expositional convenience with no loss in generality (see also Bratsiotis and Martin, 1999). We expect the implications of our results to be robust with a Taylor rule.

$$(1) \qquad M_t = \exp(\tilde{M}_t) \left( \frac{P_t}{P^T} \right)^{-\phi} \left( \frac{Y_t}{Y^T} \right)^{-\psi} M_{t-1} \qquad\qquad 0 \le \phi, \psi \le \infty$$

$\tilde{M}_t$ is an exogenous component of money supply. $M_{t-1}$ and $M_t$ are the initial and current levels of money stock respectively. $P^T$ and $Y^T$ are the target or desired levels of price and output respectively. The higher is $\phi$ the higher is the weight that the policy maker attached on price stability, whereas a $\psi$ implies a higher concern for output stability. When $\phi = \psi = 0$, monetary policy is set completely exogenously, through $\tilde{M}_t$.

Equilibrium in the money markets is reached when the desired level of total money held by all representative agents, is equal to the total money stock supplied by the central bank, $\int_{i=0}^{1} M_{i,t} di = M_t$.

## 2.2 Households and Product Demand

As with most of the recent literature (see Blanchard and Kiyotaki, 1987) we assume households derive utility from the consumption of goods from all industries and from holding real money balances, but their utility decreases with the amount of labour they supply,

$$(2) \qquad U_{i,t} = C_{i,t}^\gamma \left( \frac{M_{i,t}}{P} \right)^{1-\gamma} - L_{i,t}^\delta, \quad \gamma \in (0,1), \quad \delta = (\eta+1)/\eta > 0, \quad \eta > 0.$$

$$(3) \qquad \text{where} \quad C_{i,t} = \left( \int_{j=0}^{1} C_{ji,t}^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)}, \qquad\qquad \sigma > 1.$$

$$(4) \qquad P_t = \left( \int_{j=0}^{1} P_{j,t}^{(1-\sigma)} dj \right)^{1/(1-\sigma)}$$

$C_{ji}$ and $C_i$ are the consumption of each product $j$ and the total consumption basket of a typical household $i$. $L_i$ are the units of labour supplied by each worker in their firm. $\delta - 1$ is the marginal disutility of labour and $\eta$ measures the labour supply elasticity, while $\sigma$ is the elasticity of substitution between consumption goods in a typical household's utility.[4] For simplicity, all consumption goods enter utility symmetrically. The representative household located in industry $j$ maximises utility by taking prices and wages as given and subject to the following budget constraint:

$$(5) \qquad P_t C_{i,t} + M_{i,t} = W_{i,t} L_{i,t} + m M_{i,t-1} + \int_{j=0}^{1} V_{ji,t} \equiv I_{i,t}$$

$M_{i0}$ denotes the initial money holdings of the typical household $i$ in each industry, while $M_i$ is the amount of money the household desires to hold. The initial money transfers of each household grow, by the end of the period, at the rate $m = \dfrac{M_t}{M_{t-1}}$, which is determined by the response of the money supply rule in equation (1). $W_i$ is the hourly wage rate earned by each agent and $V_{ji}$ is the share of profits from each firm $j$ distributed to each typical household $i$. From the maximisation problem described by equation (2)-(5), the typical household, chooses the desired levels of consumption for each commodity $j$ and desired money balances,

$$(6) \qquad C_{ji,t} = \frac{I_{i,t} - M_{i,t}}{P} \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma}$$

$$(7) \qquad M_{i,t} = (1 - \gamma) I_{i,t}$$

Substituting equation (7) into (6) and aggregating over all households in the economy using $C_{j,t} = \int_{i=0}^{1} C_{ji,t} di$, we obtain the total consumption of each product $j$,

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[4] The implications of these parameters are examined in more detail later.

$$
(8) \qquad C_{j,t} = \frac{\gamma \int_{i=0}^{1} M_{i,t}\, di}{(1-\gamma)P_t}\left(\frac{P_{j,t}}{P_t}\right)^{-\sigma}
$$

Equilibrium in the goods markets implies, $C_j = Y_j$. Using this and corresponding money market equilibrium condition defined above, we obtain the total demand for each product $j$:

$$
(9) \qquad Y_{j,t} = \frac{\gamma}{1-\gamma}\left(\frac{M_t}{P_t}\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \qquad\qquad \sigma > 1.
$$

Equation (9) represents the conventional product demand function in a monopolistic model of differentiated goods, with unit income elasticity of demand.

### 2.3 Exogenous Monetary Policy and Real Rigidity

Each firm $j$ produces output according to the labour-based production function,

$$
(10) \qquad Y_{j,t} = L_{j,t}^{\alpha}, \qquad\qquad \alpha < 1,
$$

and chooses its optimal price based on a standard profit maximisation function,

$$
(11) \qquad V_{j,t} = P_{j,t}Y_{j,t} - W_{j,t}L_{j,t}
$$

Using equation (9), (10) and (11), the optimal real price is set as a mark-up over real marginal cost:

$$
(12) \qquad \frac{P_{j,t}^{*}}{P_t} = \mu_j\, MC_{j,t}, \qquad MC_{j,t} = \frac{W_{j,t}}{P_t\alpha}Y_{j,t}^{(1-\alpha)/\alpha}, \qquad \mu_j = \frac{\sigma}{\sigma-1} > 1,
$$

6

where $\mu_j$ and $MC_j$ denote the price mark-up and the real marginal cost of firm $j$ respectively. Given price and wages, household derive their optimal labour supply so to maximise their indirect utility. Substituting equations (6) and (7) into (2) we derive the indirect utility of the representative household,

$$(13) \qquad U_{i,t} = \tilde{\gamma} \frac{I_{i,t}}{P_t} - L_{i,t}^{\delta}, \qquad\qquad \tilde{\gamma} = (1-\gamma)^{1-\gamma}\gamma^{\gamma}$$

Given this and equation (5), representative households in each sector choose their optimal levels of labour supply which we express in terms of real wages,

$$(14) \qquad \frac{W_{j,t}}{P_t} = \frac{\theta}{\tilde{\gamma}} Y_{j,t}^{1/\alpha\eta}$$

From equation (14), the output response of the competitive real wage is shown to be determined by the returns to scale ($\alpha$) and the market power of the firm ($\eta$). Substituting equation (14) into $MC$, using equations (9), (10) and (12) we obtain,

$$(15) \qquad MC_{j,t} = \frac{\theta}{\alpha\tilde{\gamma}} \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma b} (Y_t)^b$$

where, $Y_t \equiv \int_{j=0}^{1} Y_{j,t} dj = \frac{\gamma}{1-\gamma}\left(\frac{M}{P}\right)$ is the aggregate demand based on equation (9).

The parameter $b = \dfrac{1+\eta(1-\alpha)}{\alpha\eta} > 0$ determines the elasticity of the real marginal cost to output.[5] The lower is $b$ the more acyclical becomes marginal cost to changes in output and so the flatter becomes the MC curve and the higher is real rigidity.

From equations (15) and (12) we derive the optimal real price equation:

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[5] Having assumed decreasing returns to scale ($\alpha<1$), $b$ is strictly positive.

$$(16) \qquad \frac{P_{j,t}}{P_t} = \left( \frac{\theta \mu_j}{\alpha \tilde{\gamma}} \right)^{\rho/b} (Y_t)^{\rho}$$

where $\rho = (\sigma + 1/b)^{-1} \geq 0$, measures inversely the degree of *real rigidity,* as defined by Ball and Romer (1990). In particular, with a unit income elasticity of demand, $(\partial \log(Y)/\partial \log(M/P) = 1)$, we obtain $d \log(Pi/P)/d \log(M/P) = \rho$. From the definition of $\rho$, real rigidity is shown to be decreasing the higher is the output response of the real marginal cost ($b$) and the more competitive is the products market ($\sigma$). Intuitively this is because for higher values of $b$ and lower values of $\sigma$, the MC curve becomes steeper, raising the additional profits from adjusting prices.

**Table 1** provides estimates for the marginal costs elasticity and real rigidity for different utility and production parameters. Note that by assuming an exogenous money supply ($\phi = \psi = 0$), constant returns to scale ($\alpha = 1$) and using the empirically supported values of $\eta = 0.15$ and $\sigma = 7.7$, used in Ball and Romer (1990), this model also generates a real rigidity of only 0.127. [6] This value is denoted by a star (*) in Table 1, and forms the baseline case in both Ball and Romer (1990) and in our model. Having assumed perfectly competitive labour markets, **Table 1**, confirms the predicament emerging in this literature (see Ball and Romer 1990, Romer 2001) that high degrees of real rigidity can only be achieved for unrealistically high elasticities of the labour supply ($\eta$) and strongly competitive market structure ($\sigma$), which is not supported by empirical evidence. An additional observation here is that the lower are the returns to scale ($\alpha$) the higher becomes the elasticity of the marginal cost ($b$) and the lower is real rigidity. This is true for variations in $\eta$ rather than $\sigma$, since with labour being the only input in the production it is combinations of the labour market elasticity and returns to scale that are important for the elasticity of the marginal cost.

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[6] Note that a mark-up value of 0.15, as assumed in Romer and Ball (1990), also implies that $\sigma = \varepsilon = 7.7$ in this model, however since in this model we are not concerned with menu costs our analysis suppresses the role of the mark-up.

**Table 1. Exogenous Money supply and Real Rigidity**

| | | $\phi = \psi = 0$ | | | | | |
|---|---|---|---|---|---|---|---|
| *Labour (η) and product (σ) market competitiveness* | | *Output elasticity of MC (b)* | | | *Real Rigidity (ρ)* | | |
| | | α=1.0 | α=0.6 | α=0.2 | α=1.0 | α=0.6 | α=0.2 |
| (σ=5.0) | η=0.15 | 6.666 | 11.77 | 37.33 | 0.194 | 0.196 | 0.198 |
| | *η=0.7* | 1.428 | 3.047 | 11.14 | 0.175 | 0.187 | 0.196 |
| | *η=2.0* | 0.500 | 1.500 | 6.500 | 0.142 | 0.176 | 0.194 |
| | *η=25.0* | 0.040 | 0.733 | 4.200 | 0.033 | 0.157 | 0.192 |
| | *η=100.0* | 0.010 | 0.683 | 4.050 | 0.009 | 0.154 | 0.190 |
| | | | | | | | |
| (η=0.15) | σ=1.5 | 6.667 | 11.77 | 37.33 | 0.606 | 0.630 | 0.654 |
| | *σ=7.7* | 6.667* | 11.77 | 37.33 | 0.127* | 0.128 | 0.129 |
| | *σ=15.0* | 6.667 | 11.77 | 37.33 | 0.066 | 0.066 | 0.066 |
| | *σ=30.0* | 6.667 | 11.77 | 37.33 | 0.033 | 0.033 | 0.033 |
| | *σ=100.0* | 6.667 | 11.77 | 37.33 | 0.009 | 0.009 | 0.009 |

# 3. Endogenous monetary policy and real rigidity

Monetary policy preferences affect the slope of the AD curve. We can show this formally by substituting equation (1) into the aggregate demand -as shown in equation (15)- to obtain,

$$
(17) \qquad Y_t = \left( \frac{\gamma \bar{M}_t}{1-\gamma} \right)^{\frac{1}{1+\psi}} P_t^{-\left( \frac{1+\phi}{1+\psi} \right)}
$$

where $\bar{M}_t = \exp(\tilde{M}_t) Y^{T^\psi} P^{T^\phi} M_{t-1}$ denotes the exogenous level of the money stock. Equation (17) shows the familiar negative relationship between aggregate price and output to be determined also by the monetary policy parameters, $\phi$ and $\psi$. The more responsive is the aggregate price level to money (measured by $\phi$) the flatter is the AD curve. This is depicted in **figure 1**. A higher weight on price targeting, ($\phi$), makes demand more sensitive the aggregate price level and the aggregate demand flatter,
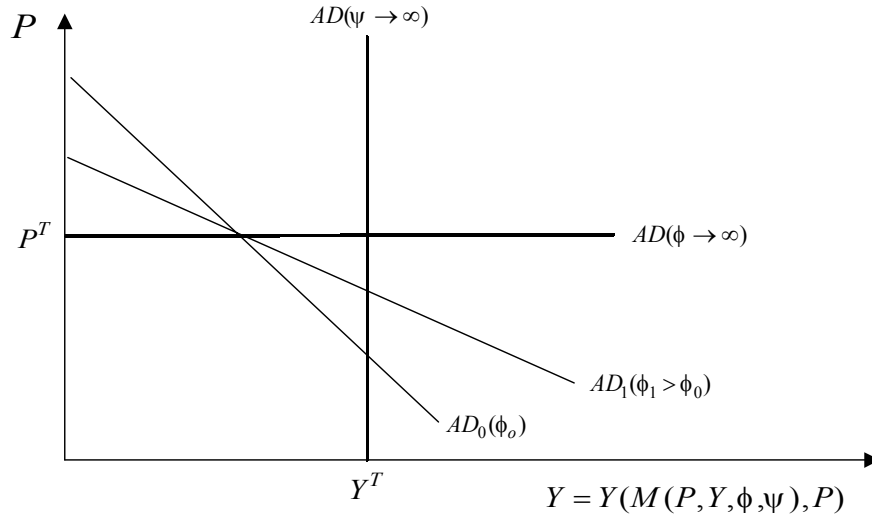
**Figure 1. Policy preferences and the slope of aggregate demand**

whereas a higher weight on output targeting, ($\psi$), makes the aggregate demand less responsive to aggregate price and the AD curve steeper. Strict price targeting implies a horizontal aggregate demand at the price target ($P^T$) whereas strict output targeting implies a vertical aggregate demand curve at the output target ($Y^T$), (see also Taylor, 1999)

The parameters of the monetary policy rule also affect the supply side of the economy. This become more transparent if we express the optimal price of the firm (equation 16), as a function of the aggregate price level and exogenous money. Using aggregate demand, as shown in equation (15) and assuming, initially, an exogenous money supply equation (16) can be written as, [7]

(18) $$P_{j,t}^* = \left( \frac{\delta \mu_j}{\alpha \tilde{\gamma}} \right)^{\rho/b} \left( \frac{\gamma M_t}{1-\gamma} \right)^{\rho} P_t^{1-\rho} ,$$

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[7] See also Romer (2001, p.285), Blanchard and Fischer (1989, p.385)

The lower is $\rho$ in equation (18) the higher is real rigidity. With $\rho \to 0$ prices are fixed and acyclical. Conversely, as $\rho \to \infty$ the optimal price responds fully to changes in aggregate demand and so real rigidity is fully eliminated.

The implications of monetary policy at the firm level can be shown by substituting equation (17) into the real marginal cost, equation (15)

$$(19) \qquad MC_{j,t} = \frac{\delta}{\alpha \tilde{\gamma}} \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma b} \left( \frac{\gamma \bar{M}_t}{1 - \gamma} \right)^{\frac{b}{1+\psi}} P_t^{-b \left( \frac{1+\phi}{1+\psi} \right)}$$

The more responsive is the aggregate price level in relation to exogenous money the higher is the cost of any producer from not adjusting their price (see also Blanchard and Fischer, 1989). Here, a higher weight on price targeting, ($\phi$), makes the aggregate price level more responsive to any level of money stock (a flatter AD) and this is shown to make the real marginal cost more sensitive to changes in the aggregate price level in equation (19). In effect the higher is $\phi$, the more responsive become price adjustments to any shock that affect the aggregate price level. A higher weight on output targeting ($\psi$) is shown to make the real marginal cost less responsive to total aggregate demand, (i.e. to both exogenous money and the aggregate price level). thus raising real rigidity. The latter effect can be seen more clearly by focusing explicitly on output stability (i.e. $\phi$=0), in which case equation (16) or (19) are now replaced by

$$(20) \qquad MC_{j,t} = \frac{\delta}{\alpha \tilde{\gamma}} \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma b} \left( \frac{\gamma \bar{M}_t}{1 - \gamma \, P_t} \right)^{b/(1+\psi)} .$$

From equation (20) it is clear that the higher is the value of output stability ($\psi$) the lower is the marginal cost response to aggregate demand. In effect, this reduces the costs from not adjusting to output variations, resulting in higher real rigidity.

Substituting equation (19) into the optimal real price, equation (12), we obtain,

$$(21) \qquad P_{j,t}^* = \left(\frac{\delta\mu_j}{\alpha\tilde{\gamma}}\right)^{\rho/b} \left(\frac{\gamma \bar{M}_t}{1-\gamma}\right)^{\hat{\rho}} P_t^{1-\hat{\rho}(1+\phi)}$$

Comparing equations (18) and (21), we observe that monetary policy preferences affect endogenously the degree of real rigidity, $\hat{\rho} = \dfrac{\rho}{1+\psi} \geq 0$. Expressing equation (21) in logs,[8]

$$(22) \qquad p_{j,t}^* = c + \hat{\rho}\left(\log(\gamma/(1-\gamma)) + \bar{m}_t\right) + \left(1 - \hat{\rho}(1+\phi)\right)p_t,$$

where $x = Log(X)$ and $c = \dfrac{\rho}{b}log(\delta\mu_j/\alpha\tilde{\gamma})$ is a constant determined by factors specific to the market structure, such as the degree of competitiveness in the product and labour markets and the mark-up, all of which are assumed to be fixed in this model. Real rigidity is now measured by the parameter $\hat{\rho}$, which reaches its maximum when $\hat{\rho} \to 0$ resulting in $p_{j,t}^* - p_t = c$, in which case the optimal real price does not respond to changes in aggregate demand but it is fixed at the level determined by the constant $c$. From equations (19) - (22) a higher preference for output stability (a higher $\psi$) is shown reduce the marginal cost and this lowers the response of the optimal price to the aggregate price level. If the central bank pursues strict output targeting, ($\psi \to \infty$), then the aggregate demand becomes completely inelastic, and the real marginal cost become insensitive to aggregate prices (flat), $\hat{\rho} \to 0$ and equation (22) results in $p_{j,t}^* - p_t = c$. Price targeting is shown to make price setting more sensitive to changes in the aggregate price level. This is because under price targeting a central bank is prepared to reduce real aggregate demand by

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[8] Note that because the monetary policy rule we consider here targets the aggregate levels of both output and prices, equation (21) is not homogeneous of degree one in output (i.e. in both $\bar{M}$ and P). The latter is true when the monetary rule targets only output (i.e. setting $\phi=0$).

whatever amount is required in order to eliminate price deviations from its price target, ($P^T$). This makes marginal costs very sensitive to changes in aggregate prices and as $\hat{\rho}$ becomes very large, optimal real prices become perfectly elastic to the price target. Table 2 below, provides numerical examples of the effects of different monetary policy parameters on real rigidity. We discuss these in more details in section 4.

## 4. Monetary policy and endogenous persistence

We have so far shown that by affecting the aggregate demand elasticity of marginal cost, monetary policy can endogenously affect the degrees of real rigidity. In the absence of nominal frictions of course, and in the symmetric equilibrium where $P_i^* = P$, real rigidities play no important role. In what follows we introduce nominal stickiness that enables real rigidity to be internalised into nominal persistence. We show that through their affect on real rigidity, monetary policy parameters can have an amplifying effect on macroeconomic persistence.

Consider, following Calvo (1983), an infrequent price setting where in every period the price of each firm has a fixed probability *(1-q)*, of remaining fixed at the previous period's price and a fixed probability *q* of being adjusted. In the absence of relative price friction (*q=1*) and given symmetry, each firm *j* will choose its optimal price ($p_{j,t}^*$) based on the symmetric Nash-equilibrium price obtained by setting $p_{j,t}^* = p_t$ in equation (17), or equation (22) with endogenous money supply. However, in the presence of infrequent price adjustments each firm setting a new price at time t, will chose the price contract that maximizes the present discounted value of expected future profits. Denoting the new price rule as $\hat{P}$, the firms now maximizes,

(23)
$$\sum_{s=0}^{\infty}((1-q)\beta)^s\{\hat{P}_{j,t}Y_{j,t+s}[\hat{P}_{j,t}]-W_{j,t+s}L_{j,t+s}[Y_{j,t+s}]\}$$

where $\beta$ is the firm's discount factor and $E$ is the expectations operator using the informational set available at time $t$.[9]  From equation (23), and denoting $W_{j,t+s}L'_{j,t+s}[Y_{j,t+s}]\equiv MC_{j,t+s}$ all firms adjusting their price at time $t$ will choose the same new optimal price,

(24)
$$\hat{P}_{j,t}=\frac{\mu_j\sum_{s=0}^{\infty}((1-q)\beta)^s\{MC_{j,t+s}Y_{j,t+s}P_{t+s}\}}{\sum_{s=0}^{\infty}((1-q)\beta)^s\{Y_{j,t+s}\}}$$

Log-linearising equation (24) and using the definition $p_{j,t}^*=\log(\mu_j)+mc_{j,t}+p_t$ from the log of equation (12) we obtain,[10]

(25)
$$\hat{p}_{j,t}=[1-(1-q)\beta]\sum_{s=0}^{\infty}((1-q)\beta)^s\{p_{j,t+s}^*\}$$

$$=[1-(1-q)\beta]p_t^*+(1-q)\beta E_t\hat{p}_{t+1}$$

From equation (25) the new price chosen by all firms adjusting prices at time $t$ is shown to be forward-looking. This equation explains the sources of persistence arising from *nominal* rather than *real rigidity*. *1/q* represents the expected time between price adjustments and so the higher is the probability of the price not being renewed in future periods (the lower is q) the higher will be the *nominal rigidity*.[11]

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[9] Note that since uncertainty plays no important role in this model, the assumption of a constant discount factor is an appropriate simplification.

[10] Because sectors are identical, apart from the timing of their price adjustment, the subscript j is dropped on the optimal price.

[11] The expected time of the price being fixed can be calculated as follows:

$$q+q(1-q)2+q(1-q)^2 3...=q\sum_{s=0}^{\infty}(1-q)^s(1+s)=1/q \quad \cdot$$

The aggregate price is given by the sum of all prices still in force and given by

(26) $\qquad p_t = q\hat{p}_t + (1-q)p_{t-1}$

Using (26) we substitute $\hat{p}$ out of (25) to obtain,

(27) $\quad p_t = \dfrac{1-q}{1+(1-q)^2\beta}\left(p_{t-1} + \beta E_t p_{t+1}\right) + \dfrac{q(1-(1-q)\beta)}{1+(1-q)^2\beta}\, p_t^*$

Finally, substituting $p_t^*$, equation (22), into equation (27) we obtain

(28) $\qquad p_t = \theta\left(p_{t-1} + \beta E_t p_{t+1}\right) + \theta k x_t$

where $\quad x_t = c + \dfrac{\hat{\rho}}{1+\phi}(\log(\gamma/(1-\gamma)) + \bar{m}_t); \quad \theta = \left(1+\beta+\hat{\rho}(1+\phi)k\right)^{-1} > 0; \qquad$ and

$k = q(\frac{1}{1-q} - \beta) > 0$. Normalising for simplicity the initial level of the money stock

$(M_{t-1})$ to unity and solving the second order difference equation in (27), we obtain,

(29) $\qquad p_t = \bar{p} + \lambda_1 p_{t-1} + \dfrac{(1-\lambda_1)}{1+\phi}\tilde{m}_t$

where, $\quad \bar{p} = \dfrac{\hat{q}\lambda_1 c}{1-\beta\lambda_1} + \dfrac{(1-\lambda_1)}{1+\phi}(\log(\gamma/(1-\gamma)) + \phi p^T + \psi y^T) \quad$ is a constant and

$\lambda_1 = \dfrac{1-\sqrt{1-4\beta\theta^2}}{2\beta\theta}$ is the stable root of the dynamic equation (14) which measures the

degree of nominal persistence. Writing equation (29) in terms of first difference in log

deviations, (i.e. $\pi_t = p_t - p_{t-1}$) we obtain,

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(30) $$\pi_t = \lambda_1 \pi_{t-1} + \frac{(1-\lambda_1)}{1+\phi} \Delta \tilde{m}_t,$$

and using equation (30) and the log of equation (18) and re-arranging, price stickiness is shown to generate the following output persistence,

(31) $$\Delta y_t = \lambda_1 \Delta y_{t-1} + \frac{\lambda_1}{1+\psi}(\Delta \tilde{m}_t - \Delta \tilde{m}_{t-1})$$

Equations (30) and (31) explain how monetary policy rules affect endogenously macro persistence. The closer is $\lambda_1$ to unity, the slower is the speed of price adjustment and so the more prolonged are the effects of demand shocks.[12] Conversely, as $\lambda_1 \to 0$, persistence is eliminated and prices respond fully to shocks. In this model endogenous persistence is generated through the monetary policy parameters. For any value of $0 < q < 1$, a higher $\psi$ results in a higher $\hat{\rho}$ and a higher $\theta$ and from equations (30) and (31) this is shown to increase macroeconomic persistence ($\lambda_1$). As $\psi \to \infty$, real rigidity approaches its maximum, ($\hat{\rho} \to 0$), $\theta \to (1+\beta)^{-1}$ and $\lambda_1 \to 1$ and so persistence reaches its maximum value. Intuitively, very strict output targeting results in a constant economic growth $\Delta y_t = \Delta y_{t-1}$. The higher is $\phi$, the higher becomes the response of optimal prices to the aggregate price level and this reduces macroeconomic persistence. As $\phi \to \infty$, $\lambda_1 \to 1$.[13] Intuitively, very strict price targeting results in zero inflation, $Log(P_t) - Log(P_{t-1}) \equiv \pi_t = 0$.

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[12] Here for simplicity we have allowed for exogenous aggregate demand shocks, through $\tilde{M}$, the dynamic implications however of our model are expected to be true for any type of shocks. As Romer (2001) suggests, one need not focus on monetary disturbances. The important issue here is how real rigidity can affect endogenously the degree of nominal persistence by prolonging the effects of any type of shocks.

[13] Note that as $\phi \to \infty$, $\lambda_1 \to 1$ before $\theta \to 1$.

**Table 2. Monetary Policy, Real Rigidity and Endogenous Persistence**

| Base Case* (Ball and Romer, 1990): $\eta=0.15$, $\sigma=7.7$, $\alpha=1$, $(\beta=0.8)$. | | | | |
|---|---|---|---|---|
| Monetary Policy Parameters $(\phi, \psi)$ | Real Rigidity $(\hat{\rho})$ | Nominal Persistence $(\lambda_1)$ | | |
| | | $q=0.8$ | $q=0.5$ | $q=0.3$ |
| Base case* : $\phi=\psi=0^*$ | $\hat{\rho}=\rho=0.127^*$ | 0.562 | 0.818 | 0.917 |
| For output: ($\phi=0$)  $\psi=0.2$ | 0.106 | 0.594 | 0.838 | 0.927 |
| $\psi=0.6$ | 0.079 | 0.644 | 0.865 | 0.942 |
| $\psi=1.5$ | 0.050 | 0.714 | 0.901 | 0.960 |
| $\psi=2.0$ | 0.042 | 0.740 | 0.913 | 0.965 |
| $\psi=15.0$ | 0.007 | 0.910 | 0.978 | 0.992 |
| For price: ($\psi=0$)  $\phi=0.2$ | - | 0.562 | 0.797 | 0.905 |
| $\phi=0.6$ | - | 0.475 | 0.761 | 0.883 |
| $\phi=1.5$ | - | 0.391 | 0.697 | 0.843 |
| $\phi=2.0$ | - | 0.357 | 0.669 | 0.825 |
| $\phi=15.0$ | - | 0.116 | 0.366 | 0.581 |
| Policy mix:  $\phi=1.2$, $\psi=0.5$ | 0.084 | 0.491 | 0.772 | 0.890 |
| $\phi=0.5$, $\psi=1.2$ | 0.057 | 0.629 | 0.857 | 0.938 |
| $\phi=0.85$, $\psi=1.2$ | 0.057 | 0.593 | 0.837 | 0.927 |
| $\phi=1.0$, $\psi=2.5$ | 0.036 | 0.659 | 0.873 | 0.946 |

**Table 2**, provides numerical simulations that demonstrate how different monetary policy preferences affect endogenously the degree of real rigidity persistence. For better transparency we use again as our benchmark, the exogenous money supply baseline case (denoted by a star)- used by Ball and Romer (1990), where $\rho=0.127$. This value is independent of the monetary policy parameters and so its persistence depends on the exogenous assumptions about nominal rigidity (q). As the model suggests, higher values of $\psi$ increase the degree of real rigidity. For example for the same baseline values a modest weight on output, of $\psi=1.5$, is shown to produce $\hat{\rho}=0.050$ which is more than double the real rigidity shown in the baseline case. Similarly, as the central bank increases $\psi$, real rigidity becomes higher and endogenous persistence increases, whereas a higher weight on price targeting is
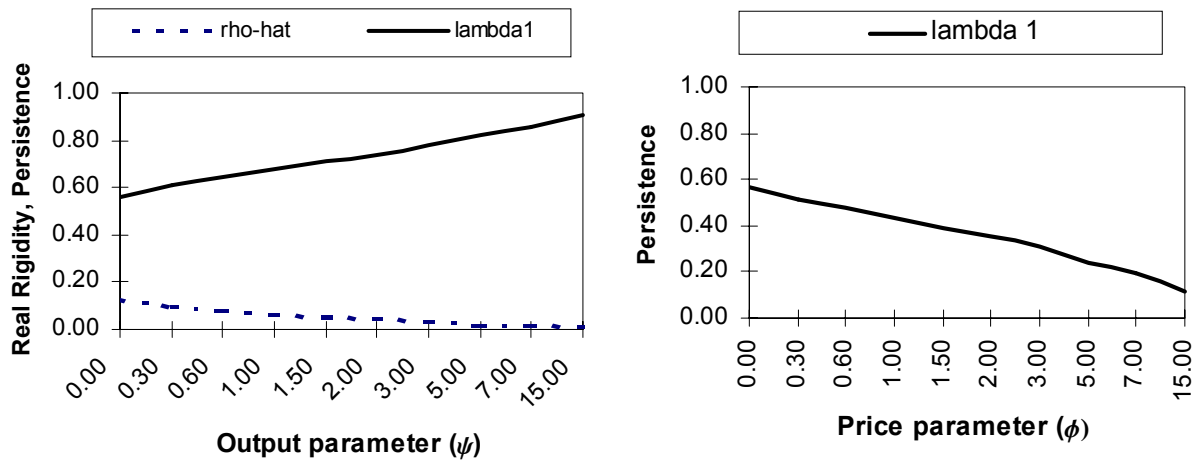
**Figure 2. Output and price preferences, real rigidity and persistence**

shown to reduce directly nominal persistence as it makes optimal price more responsive to the aggregate price level. For example, assuming that the average duration of price fixity is just above 3/4 quarters (i.e. q=0.8), then a typical value of $\psi$=1.5 (a value frequently used in central bank model simulations) is capable of raising persistence from $\lambda_1$=0.562 in the baseline case to a substantial $\lambda_1$=0.714. The opposite is true for policy preferences aiming at price stability that reduce persistence. A graphical presentation of the relationship between, the monetary policy parameters, real rigidity and persistence, is provided in **Figure 2**.

The intuition of our result is the following. The more determined is the central bank to maintain price stability, the faster the aggregate price level will converge to its target and so the higher will be the cost to the firm from not adjusting prices, following a shock. As a result price targeting makes price setting more sensitive to changes in aggregate price and this reduces persistence. Output targeting on the other hand implies smaller costs from not adjusting prices, since even large price changes will fail to ensure significant output changes when the central bank pursues an output target. As a result, output targeting make price setting less sensitive to changes in

output and this raises real rigidity and amplifies persistence. This way, monetary policy preferences are shown to determine the degree of real rigidity and persistence.

Finally, notice that our results are independent of any imperfections in the economy. Although for price setting behaviour it is important to look at imperfectly competitive firms, relaxing the assumption of monopolistic profits ($\mu=0$), does not affect the degree of real rigidity. Also, because the price elasticity of demand in this model is constant and equal to $\sigma$. This implies that real rigidity in this models is independent of both the mark-up and other assumptions about the marginal revenue curve of the firm, as shown in models where the source of rigidity is asymmetric product demand due to imperfect information, (Ball and Romer 1990).

## 5. Are real and nominal rigidities substitutes?

The results in this paper are consistent with the suggestion by Ball and Romer (1990) that higher degrees of real rigidity, regardless of the source, will magnify nominal rigidities and persistence. In this model, we showed that monetary policy itself can act as a potential source of real rigidity that will crucially determine the degree of persistence. However, a close examination of the theoretical model in conjunction with Table 2, suggest that that nominal persistence may act as a substitute for real rigidity and so we do not require high levels of both to produce substantial persistence. This observation is consistent with Jeanne (1998) who claims that even for plausible levels of real rigidity a small degree of nominal rigidity is sufficient to produce economic fluctuations as persistence as those observed in the data.

In this model, because real rigidity and persistence are shown to be sensitive to monetary policy preferences, the substitution between nominal and real rigidity appears to be even stronger. In particular, the substitution in this models seems to take place between the weight of output preferences ($\psi$) and the average expected time of prices being fixed ($1/q$). For any value $0 < q < 1$, the higher is $\psi$ and $1/q$ (i.e. the lower is $q$) the higher becomes $\lambda_1$. However it is also true that the higher is $\psi$, the

smaller becomes the rate at which increases in 1/q generate higher persistence. In Table 2 for example, the higher is nominal rigidity (the lower is q) the smaller is the contribution of $\psi$ (and so of real rigidity $\hat{\rho}$) to the degree of persistence ($\lambda_1$). Note that for $q = 1$ there is no persistence whereas for $q = 0$ nominal rigidity substitutes fully for real rigidity, as $\lambda_1 \rightarrow 1$, independently of the degree of real rigidity and the monetary policy parameters.

The implications of this substitutability may have some important empirical considerations. It may help explain why some OECD countries, placing a higher concern on price than output stability, may still exhibit a high level of macroeconomic persistence, without the need of additional sources of real rigidities. For example for some typical values of $\phi = 1.2$ and $\psi = 0.5$, and a reasonable value of $q = 0.5$, our baseline case is capable of generating an output persistence of $\lambda_1 = 0.772$. This is almost identical to that produced by Jeanne (1998) for the US, $\lambda_1^{US} = 0.769$, using an AR(2) process, as suggested by Cohrane (1988). For the same value of $q$ a relatively lower preference on price than output stability, ($\phi = 0.5$ and $\psi = 1.2$), generates a persistence of $\lambda_1 = 0.857$, which is almost identical to that produced in the same study for Italy ($\lambda_1^{IT} = 0.852$). Furthermore, policy weights of $\phi = 0.85$ and $\psi = 1.2$, combined with a lower frequency of price adjustment, $q = 0.3$, provide an output persistence of $\lambda_1 = 0.927$, which is identical to that produced for France ($\lambda_1^{FR} = 0.927$), a country with a higher price stability than Italy but a lower wage synchronization.

## 6. Concluding comments

This paper suggests that monetary policy preferences may provide a substantial source of real rigidity. Building on the result by Ball and Romer (1990), namely that the degree of real rigidity can crucially determine the degree of nominal frictions, this papers shows that monetary policy preferences affect the degree of real rigidity and

through this channel they endogenously determine the degree of nominal and real persistence.

Real marginal costs are believed to be a significant and quantitatively important determinant of inflation and output dynamics. In this paper we show the policy rules aiming at output stability, make the aggregate demand steeper, resulting in a more acyclical marginal cost (a flatter marginal cost curve), a higher real rigidity and a higher persistence. Conversely, a higher preference for price stability makes individual price setting more sensitive to the aggregate price level and this raises the responsiveness of the marginal cost to cyclical movements, resulting in lower persistence. Interestingly, this paper shows that for very reasonable values of the monetary policy parameters we can replicate high degrees of real rigidity and persistence as those observed in real data. More important perhaps is the fact that these can be obtained without relying on any imperfections in the economy or additional sources of real rigidity that are confined to a specific market, as previously assumed in the literature.

Our model also supports the view taken by Jeanne (1988), that nominal rigidity can act as substitute for real rigidity indicating that even for moderate values of real rigidity a small degree of nominal rigidity is sufficient to produce economic fluctuations as persistent as those observed in the data. Although in general, our paper focuses on the theoretical aspects of the relationship between monetary policy, real rigidity and the macroeconomic persistence, there are clear empirical implications emerging from our results that may help us understand better the full implications of monetary policy rules.

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