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### **Credit Risk, Excess Reserves and Monetary Policy: The Deposits Channel**

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# Credit Risk, Excess Reserves and Monetary Policy: The Deposits Channel\*

George J. Bratsiotis<sup>†</sup>

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## Abstract

This paper examines the role of the *precautionary demand for liquidity* and the *interest on reserves* as two potential determinants of the *deposits channel* that can help explain the role of monetary policy, particularly at the near zero-bound. At high levels of precautionary liquidity hoarding the optimal policy response of a Taylor rule is shown to indicate a zero weight on inflation. This result is explained by the effect that the demand for liquidity has on the deposit rate which determines the intertemporal choices of households. Similarly, through its effect on the deposits channel the interest on reserves can act as the main tool of monetary policy, that is shown to provide higher welfare gains in relation to a simple Taylor rule. This result is shown to hold at the zero-bound and it is independent of the precautionary demand for liquidity, or fiscal theory of the price level properties.

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*Keywords:* Deposits channel; zero-bound monetary policy; excess reserves; credit risk; balance sheet channel; interest on reserves; required reserve ratio; welfare; DSGE models.

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# 1 Introduction

In the aftermath of the 2007 financial crisis the US economy was characterized by a zero-bound policy rate, high excess reserves, a relatively lower policy emphasis on inflation and also the introduction and management of the interest on reserves. During that period bank deposits increased and deposit rates exceeded policy rates.<sup>1</sup> The deposits market may provide a significant channel in explaining the role of monetary policy but has so far received little attention. Dreschler, Savov and Schnabl (2017) provide empirical evidence that the deposits channel can account for the entire transmission of monetary policy through bank balance sheets. Their model identifies market power in the deposit markets driven by changes in the policy rate as a potential determinant of the deposits channel. Higher policy rates feed into the market power of banks and this widens the policy-deposit rate spread resulting in deposit outflows and a contraction of lending and economic activity and vice versa. In this paper, I propose two new potential determinants of the deposits channel that can help explain the role of monetary policy, particularly at the lower zero-bound period after 2007.<sup>2</sup> These are, (i) the precautionary demand for liquidity due to credit risk and (ii) the interest on reserves. These two factors may be more relevant determinants of the deposits channel for the period after 2007, as they both came into prominence just as the policy rate entered a long period of being fixed at the lower zero-bound, while the deposit and excess reserve levels kept rising and deposit rates exceeded the policy rate. Both precautionary liquidity hoarding and the interest on reserves are shown to be important factors in explaining determinacy conditions and the role of monetary policy, particularly at the zero-bound. The findings in this paper can shed some light on the zero-interest rate policy pursued in countries that experienced large excess reserves, (i.e. Japan, U.S. and the UK), but also on why the interest on reserves was given a new focus by the US Fed and other central banks during the zero-bound period.

At the centre of the Fed's policy focus during that period was the unprecedented levels of excess reserves and liquidity hoarding in the banking sector. Efforts by the Fed to boost economic activity through quantitative easing increased the level of excess reserves to even higher levels (see Figure 1). During that period banks appeared willing to hoard reserves rather than lend and offered deposit rates that exceeded the policy rate. The fall in loans

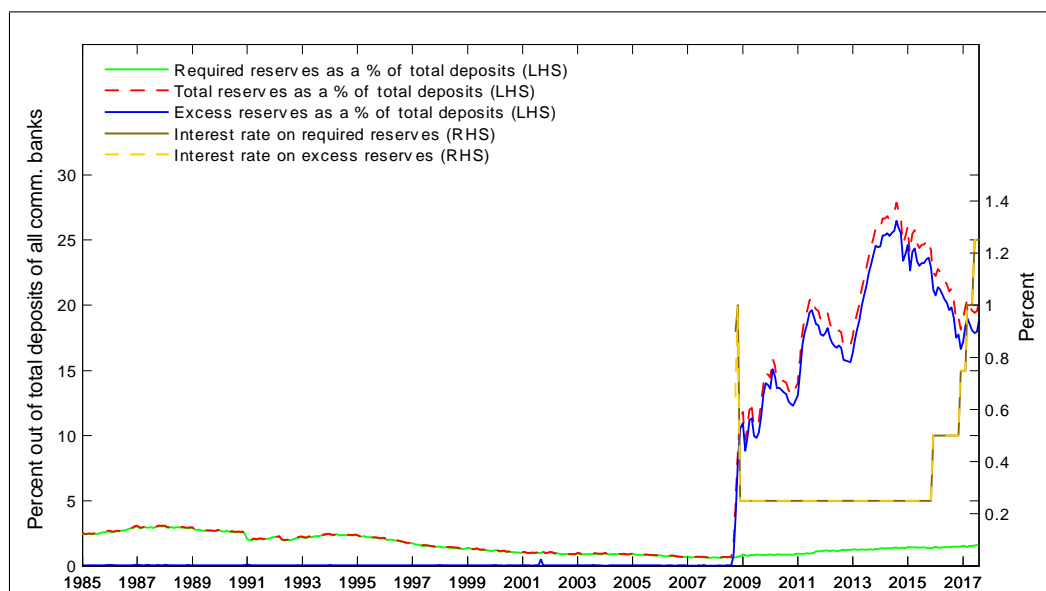
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<sup>1</sup>This was the experience of both the US, (Dreschler, Savovand and Schnal 2017), and the UK (McLeay and Thomas 2016).

<sup>2</sup>In this paper by 'deposits channel', I refer to how changes in the level of deposits and deposit rates can affect both directly (by affecting household decisions) and through the bank balance sheets (and loans) real economic activity and the business cycle.

was not just confined in the mortgage credit market that had just been shocked with the subprime mortgage crisis, but was also evident in the production sector, (Figures 2 a and b). The gradual quantitative easing between 2008-2013 contributed largely to the dramatic drop in the loan-to-reserve ratio, however during that period the loan-to-deposit ratio was also falling for all loans, including Commercial & Industrial loans. With the policy rate fixed at the lower zero-bound during that period, this was an indication that other factors, beyond monetary policy and quantitative easing, or confidence in the housing market, were contributing to the accumulation of excess reserves and the deposits channel.

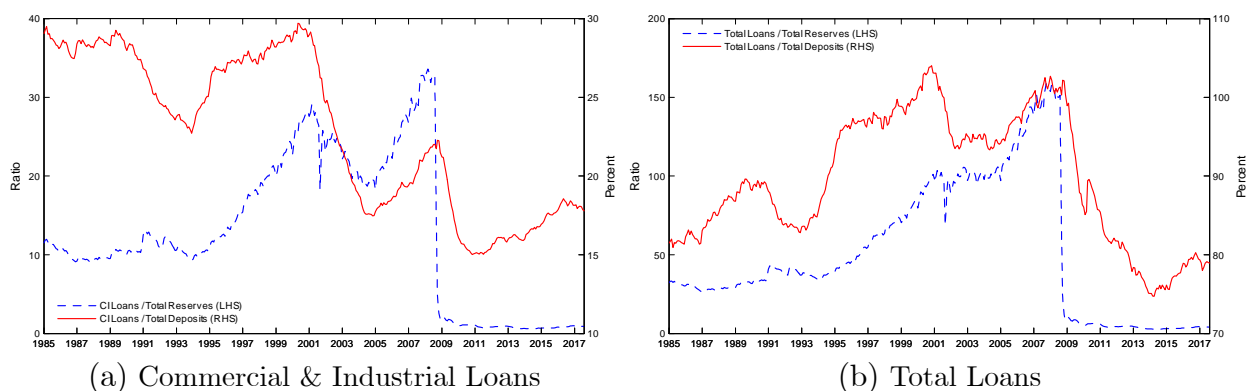
Figure 1: Interest on Reserves and Required and Excess Reserves



Source: <https://fred.stlouisfed.org/series>

On the supply side, the observed high levels of excess reserves have been attributed to precautionary reasons, or liquidity risk, that prompted banks to hoard their reserves and lend their surplus reserves to the inter-bank market at a high liquidity premium, which raised the interbank spread and reduced the volume of loans: a *bank balance sheet channel* effect, (i.e. Kashyap and Stein, 2000, Allen, Carletti and Gale, 2009, Agénor and El Aynaoui, 2010, Gertler and Kiyotaki, 2011, Acharya and Skeie 2011, Ashcraft, McAndrews and Skeie, 2011, Heider, Hoerova and Holthausen, 2015). On the demand side, the increased borrowing cost, combined with a drop in economic activity and the expectations of a looming recession, made households and firms also unwilling to demand loans. Thus, a fall in the demand for loans in the non-financial sector is also believed to

Figure 2: Loans as a Share of Deposits and Reserves



have contributed to the increase in excess reserves. This is a *firm balance sheet channel* that affects the cost of borrowing in relation to the firm’s net worth or collateral (Bernanke and Gertler 1995, Bernanke, Gertler and Gilchrist, 1999, Gertler and Kiyotaki, 2011, De Fiore and Tristani, 2011). Empirical evidence suggests that both the supply of loans and the demand for loans channels were operational during that period. The demand for credit channel was more important for the households, but the supply side channel was more important for firms and the production sector. The supply side also exacerbated the recession by the way that the decline in the value of bank liabilities also reduced the level of insurance and thus collateral such liabilities provided to their holders, (Quadrini, 2017). Overall, the impact of monetary policy is shown to be more significant through the credit supply channel than the demand for credit channel (Angelini, Nobili and Picillo 2011, Jiménez and Ongena 2012, Ciccarelli, Maddaloni, and Peydró, 2015).

Another factor that attracted much policy attention when excess reserves were high was the interest on reserves. In the aftermath of the financial crisis, when the US Fed was trying to manage liquidity at zero-bound policy rates, it placed much more emphasis on the interest on reserves than the required reserve ratio.<sup>3</sup> Figure 1, shows that following the introduction of interest on reserves in 2008, along with quantitative easing, excess reserves increased dramatically. However, even when the interest on reserves was reduced from 100 to 25 basis points to encourage bank lending, excess reserves continued to

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<sup>3</sup>Similar policies were adopted by the Bank of Japan, Bank of Canada, the ECB and other central banks in recent years.

accumulate and even after 2013, when the quantitative easing programme had finished, the interest on reserves did not appear to be the main determinant of the level of excess reserves, indicating that some underlying risk in the economy was driving excess reserves.

This paper examines the role of monetary policy in a model where both the credit supply and the demand for credit channels are operational, but where the credit supply side, and particularly the deposits channel and the balance sheet channel, are shown to be more important for monetary policy. I introduce a DSGE model where the credit risk lies in the non-financial production sector, but this risk can create a precautionary demand for liquidity in the deposits market that results in liquidity (reserve) hoarding in the financial sector. The latter decreases the supply of loans to firms through a change in the financial sector's balance sheet and raises the rate on liquid assets, the deposit rate here. Risk is also shown to increase the risk premium on the loan rate and thus the cost of borrowing of firms with respect to their collateral, generating a cost channel. Thus, credit risk affects the aggregate economy through both the supply and demand side of credit. Various policy scenarios are examined that are assessed in terms of welfare, where welfare is calculated based on a second order approximation of the household's utility function.

As in Drechsler, Savov and Schnab (2017), the deposits channel is shown to be crucial for the results in this paper, but the transmission mechanism, the research framework and the focus here are very different. I show that when the policy rate is fixed at the zero-bound, two other determinants of the deposits channel (other than how changes in the policy rate affect the monopolistic power of banks) are the precautionary demand for liquidity and the interest on reserves. These two determinants of the deposits channel can also explain how a unique equilibrium can be pinned down even at the zero-bound. It is shown that when banks hoard large amounts of excess reserves, the optimal response of the Taylor rule is to set a zero weight on the inflation rate, which implies a lower zero-bound policy rate. This is shown to be a *determinate* outcome despite the violation of the *Taylor principle*. The key to this result is the fact that at times of high perceived credit risk, the precautionary demand for liquidity becomes higher and this raises the deposit rate offered by banks in relation to the policy rate (and interbank rate), as they want to avoid deposit outflows. The deposit rate is the rate at which households discount consumption and other portfolio decisions. With a high demand for precautionary liquidity, the deposit rate can increase sufficiently to control the Euler equation and the output gap and hence inflation and provide *determinacy* even when the policy rate is at the zero-bound.

Within this framework the paper also examines the role of the interest on reserves. It is

shown that the use of the interest on reserves can facilitate monetary policy (Taylor rule) and increase welfare more than the required reserve ratio. This is because the latter is shown to have conflicting effects through the *balance sheet* and *deposits channels*, whereas the interest on reserves has complementary effects through these two channels. It is shown that unlike the required reserve ratio that acts as a ‘tax’ on banks and has a negative effect on the deposit rate, the interest on reserves acts as a ‘subsidy’ to banks and has a positive effect on the deposit rate, thus supporting the *deposits channel* identified in the paper.

More importantly, it is shown that through its effects on the deposits channel and the balance sheet channel, the interest on reserves can act as the main tool of monetary policy, producing higher welfare gains than a simple Taylor rule. In a recent paper, Hall and Reis (2016) use a model that relies on the properties of the *fiscal theory of the price level* to show that the interest on reserves can pin down a unique equilibrium of the price level and provide a simpler rule to the Taylor rule. Here I show that even in a conventional DSGE model of financial frictions, absent of *fiscal theory of the price level* properties, the interest on reserves can pin down a *determinant* equilibrium through its effects on the deposits channel and balance sheet channel.

The rest of the paper is organized as follows. Section 2, introduces the main framework and derives the decisions of deposit and loan banks. It also describes the aggregate equilibrium, the log-linearized system and the steady state properties of the model. Section 3 examines the role of precautionary reserves, the interest on reserves and the required reserve ratio for monetary policy, under various simple optimal policy rules and following financial and supply shocks. Section 4 concludes.

## 2 The Model

Consider a closed economy, dynamic stochastic general equilibrium (DSGE) model of financial intermediation with nominal frictions (sticky prices), financial frictions (risky loans to firms), a *cost channel*, a *balance sheet* channel and bank capital requirements. There is a representative household, a continuum of intermediate goods firms producing differentiated goods, a competitive final good firm, a competitive banking sector with a deposit and a lending bank and a central bank that can use the policy rate, the interest on reserves and the required reserve ratio as its policy tools.

Intermediate goods firms borrow from the lending bank to fund wage payments to households. Their production is subject to an idiosyncratic shock, which makes their

loan repayment risky requiring a fraction of their output as collateral. The deposit bank satisfies a required reserve ratio and then decides whether the rest of its deposits should be made available in the interbank market, or held as reserves at the central bank receiving the interest on reserves. The lending bank's funds are made of deposits (from the interbank market), bank capital and central bank liquidity and are subject to bank capital requirements. Bank capital (equity) offers a higher rate of return because of the bank's exposure to credit risk. The supply of bank equity is, for simplicity, fixed and determined by regulatory requirements. The loan rate is set by the lending bank based on the distribution of the idiosyncratic risk of firms. Taking the loan rate as given, each intermediate goods firm decides on the level of prices, employment and thus loans. There is a full transmission of risk from firms to the lending bank with bank equity holders absorbing the cost of default.

## 2.1 Households

The household maximizes,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} - v \frac{h_t^{1+\eta}}{1+\eta} \right), \quad (1)$$

where  $\sigma > 0$  is the intertemporal elasticity of substitution in consumption,  $v$  is a labour preference scale parameter,  $\eta$  is the inverse of the Frisch elasticity of labour supply,  $\mathbb{E}_t$  the expectations operator and  $\beta \in (0, 1)$  is the discount factor. Households enter period  $t$  with real cash holdings of  $m_t$ . They receive real wage income,  $w_t h_t$ , where  $w_t$  is the real wage and  $h$  is employment hours. They supply deposits,  $d_t$ , to the deposit bank and invest in bank equity,  $e_t$ , both defined in real terms. The household's remaining income is spent on a basket of consumption goods  $c_t$ , subject to the cash-in-advance constraint,  $c_t \leq m_t + w_t h_t - d_t - e_t$ . At the end of the period the household receives gross interest payments on deposits,  $R_t^d$ , bank capital holdings,  $(1 - \Phi_t(\varepsilon_t^*))R_t^e$ , in addition to the incurred fixed cost of equity issuing,  $\varrho^E$ , and aggregate real profits from firms and financial intermediaries,  $\sum \Pi_t^s$ ,  $s \in \{j, d\}$ .<sup>4</sup> The term  $\Phi_t(\varepsilon_t^*)$  denotes the probability of credit default (derived and explained below). In case of default, equity holders must absorb any incurred financial losses. The real value of cash carried over to period  $t + 1$

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<sup>4</sup>The inclusion of the fixed cost of equity issuing,  $\varrho^E$ , is mainly for adjusting the steady state value of  $R_t^e$ .



is,

$$m_{t+1} \frac{p_{t+1}}{p_t} = m_t + w_t h_t - d_t - e_t - c_t + R_t^d d_t + (1 - \Phi_t(\varepsilon_t^*)) (R_t^e + \varrho^e) e_t + \sum_{s \in \{j, d\}} \Pi_t^s, \quad (2)$$

where  $p_t$  denotes the price of the final good and  $\pi_t \equiv \frac{p_t}{p_{t-1}}$  is the gross inflation rate. With a positive deposit rate and taking wages and prices as given, the first order conditions with respect to  $c_t$ ,  $d_t$  and  $e_t$  are,

$$c_t^{-\sigma} = \beta \mathbb{E}_t R_t^d \frac{p_t}{p_{t+1}} c_{t+1}^{-\sigma}, \quad (3)$$

$$w_t = \nu h_t^\eta c_t^\sigma, \quad (4)$$

$$R_t^e = \frac{R_t^d}{1 - \Phi_t(\varepsilon_t^*)} - \varrho^E. \quad (5)$$

Equations (3) and (4) describe the household's Euler equation and labour supply respectively, while equation (5) is the arbitrage-free condition between the return on bank capital and the risk free deposit rate. Thus the deposit rate determines the intertemporal choices of consumers and also sets the benchmark rate for the equity rate.

## 2.2 Final Goods Firm

The competitive final good firm assembles all intermediate goods,  $y_{j,t}$ ,  $j \in (0, 1)$ , to produce a final output,  $y_t$ , which then sells at the price  $p_t$ . This is produced using a CES technology with Dixit-Stiglitz (1977) preferences,  $y_t = \left( \int_0^1 y_{j,t}^{\frac{\lambda_p - 1}{\lambda_p}} dj \right)^{\frac{\lambda_p}{\lambda_p - 1}}$ , where  $\lambda_p > 1$ , is the elasticity of substitution between differentiated intermediate goods. The demand for each intermediate good  $j$  is,  $y_{j,t} = y_t \left( \frac{p_{j,t}}{p_t} \right)^{-\lambda_p}$ , where  $p_{j,t}$ , is the price set by intermediate firm  $j$  and  $p_t = \left( \int_0^1 p_{j,t}^{1-\lambda_p} dj \right)^{\frac{1}{1-\lambda_p}}$  is the average price index.

## 2.3 Intermediate Goods Firms

The production of each firm  $j$  is,

$$y_{j,t} = z_{j,t} h_t, \quad (6)$$

where  $z_{j,t}$  captures the total productivity innovation experienced by firm  $j$ , which is subject to both an economy-wide supply shock,  $A_t$ , and an idiosyncratic shock,  $\varepsilon_{j,t}$ ,

$$z_{j,t} = A_t \varepsilon_{j,t}. \quad (7)$$

The economy-wide supply shock,  $A_t$ , follows an  $AR(1)$  process,  $\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A$ , where  $\epsilon_t^A$  is an i.i.d. shock, with standard deviation  $\sigma^A$  and mean  $A = 1$ . The idiosyncratic shock,  $\epsilon_{j,t}$ , is uniformly distributed over the interval  $(\underline{\epsilon}, \bar{\epsilon})$ , with a constant variance and a mean of unity, so that at the symmetric aggregate equilibrium level,  $z_t = A_t$ .<sup>5</sup> Each firm borrows loans to cover its working capital, in real terms<sup>6</sup>,

$$l_{j,t} = w_t h_t. \quad (8)$$

In a good state the firm repays the lending bank the full borrowing cost,  $R_t^l l_{j,t} = R_t^l w_t h_t$ , where  $R_t^l$  is the gross loan rate, as set in the financial contract between the bank and the firm (derived below). As production is risky, borrowing requires some collateral that the firm can pledge in the event of default. It is assumed that in the latter case the lender seizes a fraction  $\chi$  of the firm's output as collateral, (see also Agénor, Bratsiotis and Pfajfar, 2014). Default occurs when the real value of seizable collateral is less than the amount that needs to be repaid,  $\chi y_{j,t} < R_t^l l_{j,t}$ . Using eqs (6) and (8), the maximum *cut-off* value below which the firm defaults is,

$$\epsilon_{j,t}^* = \frac{R_t^l w_t}{\chi A_t}. \quad (9)$$

The cut-off value is a function of the cost of borrowing,  $R_t^l$ , the cost of labour,  $w_t$ , the share  $\chi$ , and the aggregate productivity shock,  $A_t$ .

Price setting is based on Calvo-type contracts, where  $\omega_p$  firms keep their prices fixed, while the rest,  $(1 - \omega_p)$  of firms adjust prices optimally by taking the loan rate, derived below, as given. Each firm  $j$  maximises,

$$\max_{P_{j,t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \lambda_{t+s} \{ \Pi_{t+s}^j \}$$

subject to,  $\Pi_t^j = \frac{P_{j,t}}{P_t} y_{j,t} - mc_t y_{j,t}$  and eqs (6-9), where from the firm's cost minimization problem real marginal cost is,

$$mc_t(R_t^l) = \frac{R_t^l w_t}{z_t}. \quad (10)$$

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<sup>5</sup>The assumption of the idiosyncratic shock  $\epsilon_{j,t}$  following a uniform distribution is to facilitate a tractable probability of default with no loss in generality.

<sup>6</sup>The assumption that all firms' funding is external is mainly for simplicity, but also because the paper focuses on economies with substantial credit frictions.

From the firm's maximization problem the new Keynesian Phillips curve equation is,<sup>7</sup>

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + k_p \widehat{mc}_t, \quad (11)$$

where  $k_p = (1 - \omega_p)(1 - \omega_p \beta) / \omega_p$ . Equation (11) is a standard new Keynesian Phillips curve with a cost channel,  $mc_t(R_t^l)$ , however, as shown below, the loan rate here is driven by both the probability of default and other key financial variables.

## 2.4 The Financial Sector

The financial sector is represented by two types of banks: a deposit bank and a lending bank.

### 2.4.1 The Deposit Bank

The deposit bank receives deposits from all households. It keeps a fraction of its deposits as total reserves,  $\tilde{r}_t$ , at the central bank for which it receives a gross interest on reserves,  $R_t^{ior}$ , and makes the rest of its deposits,  $(1 - \tilde{r}_t)d_t$ , available to the lending bank at the gross interbank rate  $R_t$ . The interbank rate,  $R_t$ , which is also the policy rate here, the required reserve ratio,  $\varsigma_t$ , and the interest rate on required and excess reserves,  $R_t^{ior}$ , are set by the central bank. The deposit bank's maximization problem is,

$$\begin{aligned} \max_{d_t, \tilde{r}_t} \Pi_t^d &= R_t^{ior} \tilde{r}_t d_t + R_t (1 - \tilde{r}_t) d_t - R_t^d d_t - G_t^{\tilde{r}}(\cdot), \\ s.t. \quad G_t^{\tilde{r}} &= \left[ (\psi_1 - \psi_3 \Phi_t(\varepsilon_t^*)) (\tilde{r}_t - \varsigma_t) + \frac{\psi_2}{2} (\tilde{r}_t - \varsigma_t)^2 \right] d_t, \end{aligned} \quad (12)$$

where the benefits and costs of keeping excess reserves,  $\tilde{r}_t - \varsigma_t$ , are captured by a convex function,  $G_t^{\tilde{r}}(\cdot)$ , where  $\psi_1 < 0$ , and  $\psi_2 > 0$ . With  $\psi_3 = 0$ ,  $G_t^{\tilde{r}}(\cdot)$  reduces to the cost function used in Glocker and Towbin (2012), where the first term  $\psi_1(\tilde{r}_t - \varsigma_t)$  captures any exogenous benefits of holding excess reserves, whereas the quadratic term,  $\psi_2(\cdot)$ , captures the fact that benefits of excess reserves start decreasing after some level.<sup>8</sup> In addition, an innovation here is that I allow the benefit of holding excess reserves to be endogenously determined by the degree of precautionary demand for liquidity. In particular, I assume that the underlying risk in the non-financial sector,  $\Phi_t(\varepsilon_t^*)$  can create

<sup>7</sup>Hats denote log-linearisations from steady state.

<sup>8</sup>Glocker and Towbin (2012) use the Bernanke, Gertler and Gilchrist (1999) framework to show that when the loss function of the central bank incorporates a financial stability objective, the use of reserve requirements can lead to non-negligible welfare improvements. Their policy analysis focuses mainly on the effects of the required reserve ratio, while the interest on reserves is assumed to be constant.

a demand for precautionary liquidity in the financial sector, where the higher is  $\psi_3$  the higher is the perceived credit risk by the financial sector and therefore the higher is the benefit of holding excess reserves.<sup>9</sup>

From its maximization problem, (12), the deposit bank sets the deposit rate ,

$$R_t^d = R_t - \tilde{r}_t(R_t - R_t^{ior}) - \left[ (\psi_1 - \psi_3 \Phi_t(\varepsilon_t^*)) (\tilde{r}_t - \varsigma_t) + \frac{\psi_2}{2} (\tilde{r}_t - \varsigma_t)^2 \right], \quad (13)$$

and the fraction of deposits it wants to keep as excess reserves,

$$\tilde{r}_t - \varsigma_t = -\frac{(\psi_1 - \psi_3 \Phi_t(\varepsilon_t^*))}{\psi_2} - \frac{(R_t - R_t^{ior})}{\psi_2} \quad (14)$$

With  $\psi_3 = 0$ , a fixed positive level of excess reserves measured by  $-\psi_1/\psi_2 > 0$ , ( $\psi_1 < 0$ ) is always held by the deposit bank, as in Glocker and Towbin (2012), and the only variable driving excess reserves endogenously is the opportunity cost of holding reserves, measured by the spread between the interbank rate and interest on reserves,  $R_t - R_t^{ior}$ . In addition here, excess reserves may also be driven by the desire of banks to hold liquidity for precautionary reasons. The higher is the perceived credit risk of default, (the higher is  $\psi_3$ ), the higher is the desired level of excess reserves. Similarly, for any given level of  $\psi_3$ , any shock that increases the actual probability of default in the credit markets  $\Phi_t(\varepsilon_t^*)$ , will result in a higher level of excess reserves held for precautionary reasons. This way the model can capture the dynamics of excess reserves due to both changes in the interest on reserves and liquidity hoarding for precautionary reasons.

In general, the deposit rate decreases the higher are the costs of holding excess reserves, including the opportunity cost of holding total reserves ( $R_t - R_t^{ior}$ ), and it increases with the benefits of holding excess reserves, and thus with the desire of banks to hold liquidity for precautionary reasons. A higher precautionary demand for reserves implies that banks care more about liquidity and as they want to avoid a deposit outflow they are prepared to pay a higher deposit rate, regardless of the policy rate. This is shown to be an important effect, because it implies that the deposit rate can deviate from the policy (interbank) rate. Similarly, for any given policy rate, a higher interest on reserves reduces the opportunity cost of holding reserves and raises the deposit rate . This is also shown to be an important effect in explaining the role of the interest on reserves in the

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<sup>9</sup>In Christiano, Motto and Rostagno (2010), the benefits of excess reserves are determined by an exogenous stochastic process. Agénor and El Aynaoui (2010) use an explicit demand for excess reserves but the liquidity risk is exogenous (captured by a fixed probability of a liquidity shock) and their focus is on the required reserve ratio. In Dressler and Kirsting (2015) excess reserves are also set optimally but there is no risk of default and monetary policy is conducted by exogenous money growth.

conduct of monetary policy through the deposits channel. I examine the latter in more detail in sections 3.2 and 3.4.

### 2.4.2 The Lending Bank

The lending bank raises  $(1 - \tilde{r}_t)d_t$  funds from the deposit bank at the inter-bank gross rate,  $R_t$ , and also issues regulatory bank capital,  $e_t$ , at the gross rate  $R_t^e$ .<sup>10</sup> The lending bank can also receive extra liquidity  $l_t^{cb}$  from the central bank which is remunerated also at the policy rate,  $R_t$ .<sup>11</sup> The lending bank's balance sheet in real terms is,

$$l_t = (1 - \tilde{r}_t)d_t + e_t + l_t^{cb}. \quad (15)$$

The lending rate is set at the beginning of the period, before firms engage in production activity and labour demand and pricing decisions. Given a competitive environment it is assumed that, on average, the bank breaks even such that the expected income from lending to firms is equal to the total costs of borrowing these funds (deposits, bank equity and liquidity from the central bank).<sup>12</sup> The lending bank's expected intra-period zero profit condition from lending to firm  $j$  is,

$$\int_{\varepsilon_{j,t}^*}^{\bar{\varepsilon}} R_t^l l_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} \chi y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = [(1 - \tilde{r}_t)d_t + l_t^{cb}] R_t + e_t R_t^e + \varrho_t^l l_{j,t}, \quad (16)$$

where  $f(\varepsilon_{j,t})$  is the probability density function of  $\varepsilon_{j,t}$ . The first part of the left hand side of (16) is the expected repayment to the bank in the non-default state, while the second part is the expected return to the bank in the default state, including the collateral commitment,  $\chi_t y_{j,t}$ , which forms part of the financial contract (Agénor, Bratsiotis and Pfajfar, 2014). The right hand side is the total cost of funds, where  $\varrho_t^l$  captures all other exogenous costs related to loans  $l_{j,t}$  (i.e. including transaction, monitoring costs and other loan-related innovations).  $\varrho_t^L$  follows an AR(1) process, with a mean value of  $\varrho^L$ ,  $\varrho_t^L = (1 - \rho^e)\varrho^L + \rho^e \varrho_{t-1}^L + \epsilon_t^e$ , where  $\epsilon_t^e$  is an i.i.d. normal random variable with zero mean and standard deviation  $\sigma_\rho$ . To derive the lending rate, substitute equations (6) and (7) into (16), together with  $R_t^l l_{j,t} = \chi A_t \varepsilon_{j,t}^* h_t$  (from 9), and  $(1 - \tilde{r}_t)d_t + l_t^{cb} = l_t - e_t$ , (from

<sup>10</sup>Since raising funds through equity is more costly for the bank ( $R_t^e > R_t$ ), bank equity is issued merely to satisfy regulatory bank capital requirements in this model.

<sup>11</sup>Introducing a liquidity injection,  $l_t^{cb}$ , is simply to allow the markets to clear at equilibrium, (see Ravenna and Walsh, 2006).

<sup>12</sup>This condition is employed purely for simplicity and clarity as the focus of this paper is not on the monopolistic power of banks. For the effects of monetary policy on the monopolistic behaviour of banks, through the deposits channel, see Dreschler, Savov and Schnal (2017).

15), to obtain,

$$R_t^l l_{j,t} = (l_t - e_t)R_t + e_t R_t^e + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} (\varepsilon_{j,t}^* - \varepsilon_{j,t}) \chi A_t h_t f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \varrho^L l_{j,t}$$

To find an explicit expression for the probability of default, I use the fact that  $\varepsilon_{j,t}$  follows a uniform distribution over the interval  $(\underline{\varepsilon}, \bar{\varepsilon})$ , with a probability density  $1/(\bar{\varepsilon} - \underline{\varepsilon})$  and a mean  $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2 = 1$ . Using this information and the definition  $\gamma_t = e_t/l_t$ , where  $\gamma_t$  is the bank capital-to-loan ratio, the loan rate is,<sup>13</sup>

$$R_t^l = R_t + \gamma_t(R_t^e - R_t) + \frac{\chi}{l_t/y_t} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \Phi_t^2(\varepsilon_t^*) + \varrho_t^L, \quad (17)$$

where  $y_t = A_t h_t$  and  $\Phi_t(\varepsilon_t^*) = \int_{\underline{\varepsilon}}^{\varepsilon_t^*} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ , is the probability of credit default.<sup>14</sup> From (17), the loan rate is shown to increase, the stricter is bank regulation, (capital-to-loan ratio)  $\gamma_t$ , the higher is bank equity spread,  $(R_t^e - R_t)$ , and the higher is the risk premium (third term in 17), which is a function of the probability of loan default  $\Phi_t(\varepsilon_t^*)$ , the loan-to-output value  $l_t/y_t$ , and the fraction of collateral,  $\chi$ ; it is also subject to the financial shock,  $\varrho_t^L$ .

Thus credit risk in the non-financial sector raises the loan rate spread and thus the cost of borrowing for firms. In the financial sector, depending on the value of  $\psi_3 \geq 0$ , this credit risk can also generate a precautionary demand for reserves. The interest on reserves and the required reserve ratio affect the loan rate through two channels: (i) through their effect on the deposit rate, (13), which in turn affects the equity rate,  $R_t^e$  (see 5) and (ii) through the balance sheet, as they affect total reserves which in turn determine the level of credit to firms, (see 14 and 15).

## 2.5 Monetary Policy

The monetary policy rate  $R_t$ , which is also the interbank rate in this model, is set according to a standard Taylor rule, where monetary policy responds to aggregate inflation

<sup>13</sup>The cut-off value,  $\varepsilon_{j,t}^*$ , depends on the state of the economy and it is identical across all firms. Similarly, real wages and the labour employed by each firm are identical and therefore the loan rate applies to all firms and so in what follows the subscript  $j$  is dropped.

<sup>14</sup>The default risk of the firm is fully transmitted to the bank. As the bank's total funds are provided to identical firms and the loan rate has been derived from a break even condition, it is implied that at equilibrium when the firm's default condition is satisfied the bank would also no longer break even (see also 16). Hence, the bank's default probability is approximated to the firm's default probability,  $\Phi_t(\varepsilon_t^*)$ , with no loss in generality.

and the output gap,

$$R_t = R^{(1-\phi)} R_{t-1}^\phi \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{x_t}{x} \right)^{\phi_y} \right]^{(1-\phi)}, \quad (18)$$

where,  $\phi \in (0, 1)$  is the degree of interest rate smoothing;  $\phi_y, \phi_\pi > 0$ , are policy coefficients and  $y_t/y_t^* \equiv x_t$  is the output gap. The policy rate has a direct effect on two important channels in this model, the *cost of borrowing* channel and the *deposits* channel. It works through the *cost of borrowing* channel by affecting the loan rate, (17), which affects the marginal cost, (10), and inflation, (11). It works through the *deposits channel* by affecting the deposit rate (13) which in turn affects, the household's intertemporal substitution of consumption, (3), but also the bank equity return (5), which in turn also affects the loan rate (17) and hence the cost of borrowing channel.<sup>15</sup>

Other than the policy rate, monetary policy can also be implemented by the required reserve ratio and the interest on reserve. Both these tools affect the deposits channel and the balance sheet channel and are examined in more detail in sections, 3.2 and 3.3 below.

## 2.6 The Log-linearized Aggregate Equilibrium

At the aggregate equilibrium where the goods market clears, and with no fixed capital investment or government intervention, aggregate demand is determined by aggregate consumption,  $y_t = c_t$ . On the production side equilibrium also requires that  $\int_0^1 \left( \frac{p_{j,t}}{p_t} \right)^{-\lambda_p} y_t = \int_0^1 \varepsilon_{j,t} A_t h_t$ . Using the distribution properties of the idiosyncratic shocks (that has an average mean of unity and at the symmetric equilibrium,  $\int_0^1 \varepsilon_{j,t} = 1$ ), aggregate equilibrium becomes  $y_t = A_t h_t / \Delta_t$ , where  $\Delta_t \equiv \int_0^1 \left( \frac{p_{j,t}}{p_t} \right)^{-\lambda_p}$  is the price dispersion index. Also, at the aggregate equilibrium we assume that  $l_t^{cb} = m_{t+1} \frac{p_{t+1}}{p_t} - m_t$ , and  $w_t h_t = (1 - \tilde{r}_t) d_t + e_t + l_t^{cb} = l_t$  and also that the financial markets clear.

The model is log-linearized around its non-stochastic, zero inflation, flexible price steady state. The flexible price level of output is,  $y_t^f = \left( \frac{z_t^{1+\eta}}{\vartheta_p R_t^{l,f}} \right)^{\frac{1}{\eta+\sigma}}$ , where  $\vartheta_p = \lambda_p / (\lambda_p - 1)$  is the price mark-up and  $R_t^{l,f}$  is the loan rate under flexible prices. The efficient level of output, free of both financial frictions and nominal rigidities and estimated at a constant policy rate, is  $y_t^* = \left( \frac{z_t^{1+\eta}}{\vartheta_p} \right)^{\frac{1}{\eta+\sigma}} > y_t^f$ .<sup>16</sup> Also, for the purpose of this paper, it is assumed

<sup>15</sup>Dreschler, Savov and Schnal (2017), have already shown that the policy rate can affect the deposits channel through the monopolistic power of banks, but this channel is relaxed here as this channel is not the focus of this paper.

<sup>16</sup>See Ravenna and Walsh (2006).

that the bank capital requirement constraint remains fixed, so that  $\gamma_t = \gamma$ .<sup>17</sup> The log-linearized versions of (3), (4), (5), (11) (13), (14), (15) and (17), can then be used to express the model in terms of the efficient output gap,  $\hat{x}_t = \hat{y}_t - \hat{y}_t^*$ , where  $\hat{y}_t^* = \frac{1+\eta}{\sigma+\eta}\hat{z}_t$ , the inflation rate,  $\hat{\pi}_t$ , the equity rate,  $\hat{R}_t^e$ , the deposit rate  $\hat{R}_t^d$ , total reserves,  $\hat{r}_t$ , loans,  $\hat{l}_t$ , the loan rate,  $\hat{R}_t^l$  and the default risk,  $\hat{\Phi}_t$ ,

$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \sigma^{-1} \left( \hat{R}_t^d - \mathbb{E}_t \hat{\pi}_{t+1} \right) + \hat{u}_t, \quad (19)$$

where  $\hat{u}_t \equiv ((1 + \eta)/(\sigma + \eta)) \left( \mathbb{E}_t \hat{A}_{t+1} - \hat{A}_t \right)$ ,

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + k_p (\eta + \sigma) \hat{x}_t + k_p \hat{R}_t^l, \quad (20)$$

$$\hat{R}_t^e = \hat{R}_t^d + \left( \frac{\Phi}{1 - \Phi} \right) \hat{\Phi}_t, \quad (21)$$

$$\begin{aligned} \hat{R}_t^d = & \frac{(1 - \tilde{r}) R}{R^d} \hat{R}_t + \frac{\tilde{r} R^{ior}}{R^d} \hat{R}_t^{ior} - \frac{\tilde{r} (R - R^{ior})}{R^d} \hat{r}_t + \frac{(\tilde{r} - \varsigma) \psi_3 \Phi}{R^d} \hat{\Phi}_t \\ & - \frac{[\psi_1 - \psi_3 \Phi + \psi_2 (\tilde{r} - \varsigma)] (\tilde{r} \hat{r}_t - \varsigma \hat{\varsigma}_t)}{R^d}, \end{aligned} \quad (22)$$

$$\hat{r}_t = \frac{\varsigma \hat{\varsigma}_t}{\tilde{r}} - \frac{1}{\psi_2 \tilde{r}} (R \hat{R}_t - R^{ior} \hat{R}_t^{ior}) + \frac{\psi_3 \Phi}{\psi_2 \tilde{r}} \hat{\Phi}_t, \quad (23)$$

$$\hat{l}_t = \hat{w}_t + \hat{h}_t = (\eta + \sigma) \hat{x}_t + \hat{y}_t, \quad (24)$$

$$\hat{R}_t^l = \frac{(1 - \gamma) R}{R^l} \hat{R}_t + \frac{\gamma R^e}{R^l} \hat{R}_t^e + \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \frac{\chi \Phi^2}{R^{ll}/y} (\hat{y}_t - \hat{l}_t + 2 \hat{\Phi}_t) + \hat{\varrho}_t^L, \quad (25)$$

$$\hat{\Phi}_t = \frac{\varepsilon^*}{\varepsilon^* - \underline{\varepsilon}} \left( \hat{R}_t^l + (\eta + \sigma) \hat{x}_t \right), \quad (26)$$

where  $\hat{R}_t$  is determined by the log-linearized policy rule, (18),  $\hat{\varrho}_t^L = \rho_\varrho \hat{\varrho}_{t-1}^L + \hat{\varepsilon}_t^\varrho$  and  $\hat{R}_t^{ior}$  and  $\hat{\varsigma}_t$  are defined below.

## 2.7 Steady State and Parameterization

The key steady state equations are,  $R = 1/\beta$ ,  $R^e = \frac{R^d}{1 - \Phi(\varepsilon^*)} - \varrho^E$ ,  $R^l = (1 - \gamma)R + \gamma R^e + \chi \frac{y}{l} \left( \frac{\bar{\varepsilon} - \underline{\varepsilon}}{2} \right) \Phi^2$ ,  $\Phi = \frac{\varepsilon^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ , where  $\varepsilon^* = \frac{\mu_\varepsilon}{\chi \vartheta_p}$ ,  $\tilde{r} = \varsigma - \frac{(\psi_1 - \psi_3 \Phi(\varepsilon^*))}{\psi_2} - \frac{(R - R^{ior})}{\psi_2}$ , and  $R^d = R - \tilde{r}(R - R^{ior}) - \left[ (\psi_1 - \psi_3 \Phi)(\tilde{r} - \varsigma) + \frac{\psi_2}{2} (\tilde{r} - \varsigma)^2 \right]$ . Table 1 presents the baseline parameter values of the model. Most of the parameter values follow largely the existing

<sup>17</sup>Changes in the bank capital constraints can be important for balance sheet effects and the supply side of credit. However, this is not the focus of this paper. Moreover, there is evidence that capital constraints could not explain the massive scale rise observed in liquidity hoarding by banks and their unwillingness to lend following the 2007 financial crisis (Ennis and Wolman, 2015).



literature, whereas other parameter values are chosen so that the steady state values match observed ratios for advanced economies. The idiosyncratic productivity shock's range is set to  $\underline{\varepsilon} = 0.80$  and  $\bar{\varepsilon} = 1.20$ , so that  $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2 = 1$  and the steady state fraction of collateral received by the bank is set to  $\chi = 97\%$ .<sup>18</sup> These values, together with a price mark-up of 20%, ( $\mu_p = 1.20$ ), generate a steady state credit risk of 3.04% and a loan rate of 7.5% (in annual terms). The required reserve ratio at the steady state is  $\varsigma = 2\%$ , which is the average ratio (usually between 1-3%) in OECD countries.<sup>19</sup> The initial steady state spread between the policy rate and interest on reserves is set to  $R - R^{ior} = 100$  bps (quarterly), and the values of  $\psi_1, \psi_2$  and  $\psi_3$  are chosen so that at the steady state with default probability,  $\Phi = 3\%$ , total excess reserves are,  $\tilde{r} - \varsigma \approx 2\%$  and total reserves are,  $\tilde{r} \approx 0.04$ . The elasticity measuring the exogenous benefits of holding excess reserves is set to a very small value,  $\psi_1 = -0.03$ , and for clarity of the net effects of  $\varsigma$  and  $R^{ior}$ , I set  $\psi_2 = 1$ .  $\psi_3$ , which captures the degree of precautionary demand for liquidity, is allowed to vary from almost zero in the baseline case, (i.e.  $\psi_3 = 0.001$ ) to much higher values that reflect higher levels of liquidity hoarding.

Table 1: Baseline Parameterization

Parameter	Value	Description
$\beta$	0.99	Discount Factor
$\sigma$	1.00	Intertemporal Substitution in Consumption
$\eta$	0.30	Inverse of the Frisch Elasticity of Labour Supply
$v$	1.00	Labour Preference Scale Parameter
$\lambda_p$	6.00	Elasticity of Demand - Intermediate Goods
$\omega_p$	0.80	Degree of Price Stickiness
$A$	1.00	Average Productivity Parameter
$\underline{\varepsilon}$	0.85	Idiosyncratic Risk: Lower Range
$\bar{\varepsilon}$	1.15	Idiosyncratic Risk: Upper Range
$\chi$	0.97	Proportion of Output Seized when Firms Default
$\theta$	0.08	Bank Capital Adequacy Ratio
$\varsigma$	0.02	Steady State Required Reserve Ratio
$\psi_1$	-0.03	Linear cost function parameter
$\psi_2$	1.00	Quadratic cost function parameter
$\psi_3$	0.001	Excess Reserves Responsiveness to Credit Risk
$\phi$	0.70	Persistence in Taylor Rule
$\phi_\pi$	1.50	Response of Policy Rate to Inflation Deviations
$\phi_x$	0.00	Response of Policy Rate to Output Gap Deviations

<sup>18</sup>The value of  $\chi = 0.97$  is justified in Agénor, Bratsiotis and Pfajfar, (2014).

<sup>19</sup>A 2% required reserve ratio also reflects the ratio proposed recently for the Euro Zone countries.

The bank capital ratio is  $\gamma = (\theta + \Phi) = 11\%$ , where  $\theta = 0.08$ , represents the fixed bank capital to loan ratio, as set by the Basel Accords.<sup>20</sup> Some of the key steady state values resulting from the above parameterization are (in annual terms),  $\Phi = 0.03$ ,  $R = 1.0404$ ,  $R^{ior} = 1.0001$ ,  $R^d = 1.0384$ ,  $R^e = 1.053$ ,  $R^l = 1.074$  and  $l/y = 0.81$ .<sup>21</sup> The persistence parameters and the standard deviations of the financial shock and the supply shock are calibrated in line with Benes and Kumhof (2015), where  $\rho_\rho = 0.87$  and  $\sigma_\rho = 0.11$ , while  $\rho_A = 0.92$  and  $\sigma_A = 0.024$ , respectively.<sup>22</sup>

### 3 Optimal Simple Policy Rules and Welfare

This section uses a numerical welfare analysis to determine the optimal responses of monetary policy rules following financial and supply shocks. The central bank's objective function is derived by a second order approximation around the efficient steady state of the household's expected utility function (1), where the consumer's welfare losses are expressed as a fraction of steady state consumption<sup>23</sup>,

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U}{U_{cc}} \right) = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\lambda_p}{k_p} \right) \hat{\pi}_t^2 + (\eta + \sigma) (\hat{x}_t)^2 \right] \quad (27)$$

where,  $\hat{x}_t = \hat{y}_t - \hat{y}_t^*$  is the welfare relevant output gap and  $\hat{y}_t^*$  is the log deviation of the efficient output from its steady-state. The average welfare loss per period is given by,  $\mathbb{L} = \frac{1}{2} [(\lambda_p/k_p) \text{var}(\hat{\pi}_t) + (\eta + \sigma) \text{var}(\hat{x}_t)]$ . The net welfare gain from the simple optimal policy rules considered are estimated based on the difference in consumer welfare losses between the baseline policy rule  $\mathbb{W}^B$  and the optimal policy rule  $\mathbb{W}_t^O$ ,

$$CE = \{1 - \exp [(1 - \beta) (\mathbb{W}_t^O - \mathbb{W}_t^B)]\} \times 100,$$

where the higher is  $CE$ , a consumption equivalent measure, the larger is the net welfare gain from the optimal policy.

<sup>20</sup>This partly mimics a Basel II type regulation, where borrowers' credit risk is taken into account in the determination of the overall bank capital ratio.

<sup>21</sup>For the steady state values of the equity and loans rates we also use,  $\rho^E = 0.028$  and  $\rho^L = 0.008$ .

<sup>22</sup>Christiano, Motto and Rostagno also report a riskiness shock of  $\sigma_\rho = 0.119$ .

<sup>23</sup>The derivation of the welfare loss function follows Ravenna and Walsh (2006), who also incorporate the monetary policy cost channel. In the efficient steady state, price mark ups and financial distortions are eliminated through appropriate subsidies.

### 3.1 Precautionary Liquidity and Optimal Taylor Rule Responses

This section examines the role of the precautionary demand for liquidity and the size of excess reserves for monetary policy. Table 2 reports the optimal policy responses of the Taylor rule (OTR, based on 18), and the net welfare gains, measured as deviations from the baseline policy and reported in terms of the consumption equivalent measure  $CE$ , following an adverse financial shock and an adverse supply shock. Monetary policy responses are examined under different levels of excess reserves ( $ER \equiv \tilde{r} - \varsigma$ ) that arise from assuming different degrees of precautionary liquidity hoarding,  $\psi_3$ , (see 14). The grid search range is,  $\phi_\pi = [0, 3]$ , and  $\phi_x = [0, 1]$ .

**Result 1:** *At low levels of precautionary liquidity and excess reserves, where the deposit rate is also low, the optimal response of the monetary policy rule is to maintain its focus on inflation. As precautionary liquidity rises to higher levels, raising the deposit rate, the optimal response of the Taylor rule is to reduce its relative weight on inflation. This result holds for both financial and supply shocks, although it is stronger for the former.*

Table 2: Welfare Effects: Excess Reserves and Optimal Policy Responses

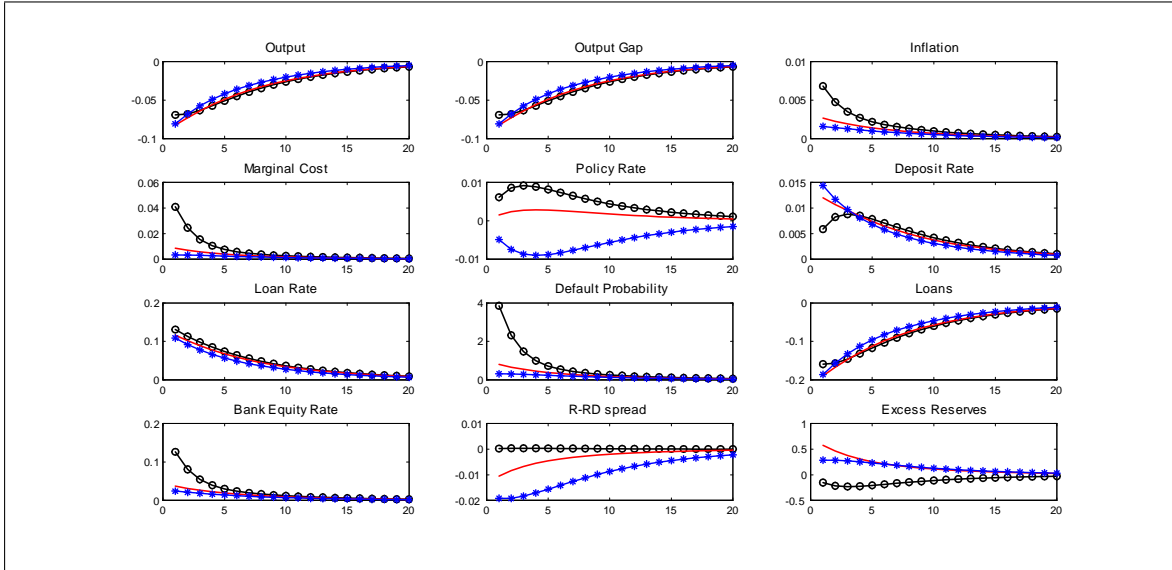
(a) Loan Rate Shock,				
Policy	$\psi_3=0.001, ER \approx 2\%$	$\psi_3=3.5, ER \approx 12\%$	$\psi_3=5.4, ER \approx 18\%$	$\psi_3=7.6, ER \approx 25\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_y = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02372$	$\phi_\pi = 1.90$ $\phi_x = 0.00$ $CE = 0.00001$	$\phi_\pi = 1.50$ $\phi_x = 0.30$ $CE = 0.00226$	$\phi_\pi = 0.00^*$ $\phi_x = 0.20$ $CE = 0.00344$
(b) Adverse Technology Shock				
Policy	$\psi_3=0.001, ER \approx 2\%$	$\psi_3=2.0, ER \approx 8\%$	$\psi_3=13.5, ER \approx 42\%$	$\psi_3=16.2, ER \approx 51\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_y = 0$ $CE = -$	$\phi_\pi = 1.5, \phi_x = 0$ $CE = -$
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00254$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00055$	$\phi_\pi = 2.70$ $\phi_x = 0.00$ $CE = 0.00000$	$\phi_\pi = 0.80^*$ $\phi_x = 1.00$ $CE = 0.00000$

\*Taylor Principle violated

Result 1 follows directly from the results in Table 2 and Figures 3 and 4 that plot these scenarios. Following financial and real shocks the optimal response of the policy rate

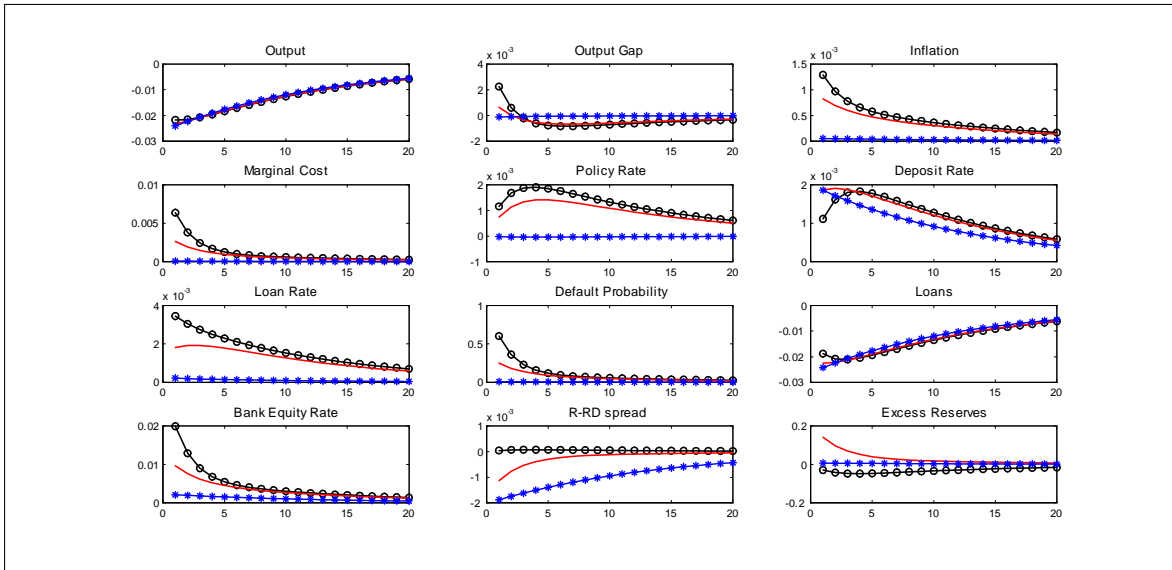
is shown to be largely determined by the level of excess reserves. When excess reserves are around low steady state levels, (i.e. 2-7%), the optimal policy response to a financial shock that reduces the output gap and increases inflation (due to the cost channel), is to increase the policy rate so as to curb inflation. However, when excess reserves are at higher levels, the optimal response of the policy rate is to decrease the relative weight on inflation with respect to the output gap. This result can be explained as follows.

Figure 3: Financial Shock: Excess Reserves and Optimal Policy Responses



Black(o) line: ER=2%; Red(solid) line: ER=12% and Blue(star) line: ER=25%.

Figure 4: Adverse Supply Shock: Excess Reserves and Optimal Policy Responses



Black(o) line: ER=2%; Red(solid) line: ER=8% and Blue(star) line: ER=51%.

When excess reserves are low, a financial shock that raises the cost of borrowing, or an adverse supply shock, increase the inflation rate requiring the policy rate to increase (see Black(o) line: ER=2%). However, when there is a high precautionary demand for liquidity, the deposits channel becomes stronger as the level of excess reserves and the deposit rate rise (see section 2.4.1). Higher excess reserves imply a reduced supply of loans (through the *balance sheet* channel - eq. 15), while the higher deposit rate reduces (through the households intertemporal substitution effect) the output gap and the inflation rate, which in turns eases inflationary pressures and allows the policy rate to fall, (see Figures 3 and 4). The fall in the policy rate also reduces the probability of default and the cost of borrowing, thus further reducing the inflation rate and the policy rate through the financial accelerator. In the case of the adverse supply shock, where both the output gap and inflation increase, it takes a much higher level of excess reserves for the monetary policy to abandon its anti-inflationary stance. In our example, it takes excess reserves of over 40% of total deposits for monetary policy to start decreasing its weight on inflation, but at such critical levels of excess reserves the welfare gains from such policy are shown to be very small.

**Proposition 1:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\psi_3 > 0$ , that is, there is a positive degree of precautionary demand for liquidity that raises the deposit rate.*

**Proof.** A formal proof of Proposition 1 is provided in the Appendix.

It is a well-established fact that in this general class of macro models a fixed interest rate results in *indeterminacy*, (see Woodford 2011). Here it is shown that when the policy rate is fixed at the zero-bound, a positive degree of precautionary demand for liquidity ( $\psi_3 > 0$ ) can result in *determinacy*. This is because with  $\psi_3 > 0$ , the deposit rate responds to credit risk which in turn affects both the cost channel and inflation (through the loan rate) but also the output gap (through marginal cost - eq 26). This implies that even in the absence of an active monetary policy, the economy can be self-disciplined through ‘prudent’ banking, in this case precautionary liquidity hoarding that decreases the supply of loans and raises the deposit rate in respond to credit risk to pin down a unique equilibrium. With the policy rate at the zero-bound, as the degree of precautionary demand for liquidity increases the deposit rate can increase above the policy rate and act as a ‘substitute’ for the policy rate (see *R-RD spread* in Figures 3 and 4).

**Result 2:** *At high levels of precautionary liquidity and excess reserves, the optimal weight on inflation in the Taylor rule can be shown to be less than unity, ( $\phi_\pi < 1$ ). This is a determinate equilibrium despite the violation of the Taylor Principle. Determinacy holds because the desire for excess liquidity drives the deposit rate above the policy rate, with the former controlling the behaviour of consumption (Euler equation) and inflation.*

This follows directly from Proposition 1, in combination with the results in Table 2, and Figures 3 and 4. In a conventional model where the deposit rate does not deviate from the policy rate, the Taylor Principle, ( $\phi_\pi > 1$ ), cannot be violated because the policy rate is also the rate that pins down the behaviour of consumption and aggregate demand. In this case a lower-zero bound interest rate would result in *indeterminacy*.

### 3.2 The Effects of the Interest on Reserves and the Required Reserve Ratio on the Deposits Channel

From (13) and (14) the required reserve ratio,  $\varsigma_t$ , and the interest on reserves,  $R_t^{ior}$ , are shown to affect the deposit channel, through both the level of total reserves and the deposit rate, though not in the same way. Both monetary policy tools have a positive effect on total reserves,  $\partial\tilde{r}_t/\partial\varsigma_t = 1 > 0$ , and  $\partial\tilde{r}_t/\partial R_t^{ior} = 1/\psi_2 > 0$ , ( $\psi_2 > 1$ ) respectively. Thus a higher required reserve ratio, or a higher interest on reserves, increases total reserves. This is a *balance sheet effect* which restricts the amount of credit liquidity available in the economy, (see 15). However, the effects that these two monetary policy tools have on the *deposits channel* through the deposit rate are in the opposite direction.

The effect of the required reserve ratio on the deposit rate is expected to be negative. In particular,  $\partial R_t^d/\partial\varsigma_t = \psi_1 - \psi_3\Phi_t(\varepsilon_t^*) - (R_t - R_t^{ior}) + (\tilde{r}_t - \varsigma_t)\psi_2$ , or using (14),  $\partial R_t^d/\partial\varsigma_t = -2(R_t - R_t^{ior}) \leq 0$ , assuming,  $R_t^{ior} \leq R_t$ . Thus, the higher is the opportunity cost of holding deposits as reserves, (receiving  $R_t^{ior}$ ), rather than making them available in the interbank market, (receiving  $R_t$ ), the larger is this negative effect. This result is consistent with studies that show that the required reserve ratio acts as ‘*a tax*’ on bank deposits, which then banks pass on to the depositor as a lower deposit rate, (see also Loungani and Rush, 1995, Agénor and El Aynaoui, 2010, Montoro and Moreno, 2011, Glocker and Towbin, 2012). Conversely, by lowering the required reserve ratio monetary policy can encourage a higher deposit rate, but since  $\partial\tilde{r}_t/\partial\varsigma_t > 0$ , this will imply a decrease in total reserves which will encourage some credit expansion through the *balance sheet channel*

(eq 15), and eventually increase the output gap and inflation. In this case, by freeing up some deposits and increasing borrowing the *balance sheet channel* mitigates the decrease in the output gap and inflation achieved by the higher deposit rate through the *deposits channel*.

In contrast, the effect of the interest on reserves on the deposit rate is positive. From (13),  $\partial R_t^d / \partial R_t^{ior} = \varsigma_t - \psi_1 / \psi_2 + \psi_3 \Phi_t(\varepsilon_t^*) / \psi_2 - (R_t - R_t^{ior}) / \psi_2 > 0$ , ( $\psi_1 < 0$  and  $\psi_2 > 0$ ) and thus a higher interest on reserves increases the deposit rate. This is because the interest on reserves acts as ‘a *subsidy*’ on bank deposits thus encouraging a higher deposit rate (see also Dutkowsky and VanHoose, 2013, Ireland 2014, Güntner 2015). Since, the effect of the interest on reserves on total reserves is also positive,  $\partial \tilde{r}_t / \partial R_t^{ior} > 0$ , a higher interest on reserves will also reduce the reserves available for credit. Thus here both the *deposits channel* (through the higher deposit rate) and the *balance sheet channel* (through higher excess reserves) work in a complementary way in reducing the size of borrowing, (the *cost channel*), the output gap and inflation. This, as we demonstrate next, implies that the use of the interest on reserves requires a lower intervention from the policy rate and it can deliver higher welfare gains than the required reserve ratio.

### 3.3 Optimal Policy Rule Responses with *Interest on Reserves* or *Required Reserve Ratio*

This section examines the optimal responses of the Taylor Rule, (as in section 3.1), but when the latter, (18), is facilitated by either a required reserve ratio rule, or an interest on reserves rule, as shown by (28) and (29) below. Some recent studies show that required reserve ratio rules that respond in a countercyclical fashion to macroprudential or financial variables, such as credit or credit spreads, can help promote financial and macro stability.<sup>24</sup> Accordingly, the following rule for the required reserve ratio is considered,

$$\varsigma_t = \varsigma \left( \frac{l_t}{l} \right)^{\mu_\varsigma^l}, \quad (28)$$

where the elasticity  $\mu_\varsigma^l$  measures the responsiveness of the required reserve ratio to credit expansions from its steady state. Similarly, the motivation for examining the interest rate on reserves in a similar fashion, comes from the fact that this monetary tool has been used by a number of central banks recently to facilitate their credit liquidity management

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<sup>24</sup>See Gray, (2011), Montoro and Moreno (2011), Glocker and Towbin, (2012), Chadha and Corrado, (2012), Mimir, Sunel and Taşkın, (2013), Agénor and Pereira da Silva (2014) and Mora (2014).



(including the US Fed and the ECB) and has attracted considerable research attention in the aftermath of the Great Recession.<sup>25</sup> Accordingly, the following rule for the interest on reserves is examined,<sup>26</sup>

$$R_t^{ior} = R^{ior} \left( \frac{l_t}{l} \right)^{\mu_{ior}^l}, \quad (29)$$

where  $\mu_{ior}^l$  measures the responsiveness of the interest on reserves to credit expansions from its steady state. The optimal policy parameters are grid-searched within the following ranges,  $\phi_\pi = [0, 3]$ ,  $\phi_x = [0, 1]$ , and  $\mu_\zeta^l, \mu_{ior}^l = [-3, 3]$ . Table 3 (a) and (b), summarise the optimal policy responses when the central bank uses a combination of an optimal Taylor rule (OTR) with, either the required reserve ratio (OTR+ $\zeta$ ), or the interest on reserves (OTR+ $R^{ior}$ ) as supporting monetary policy tools. The welfare effects, reported in consumption equivalent measures (CE), are given in terms of net increases from the baseline policy case (that is a simple Taylor rule with  $\phi_\pi = 1.5$  and  $\phi_x = 0$ ).

**Result 3:** *As supporting monetary policy tools, both the required reserve ratio and the interest on reserves can improve on the welfare outcomes achieved by a simple optimal Taylor rule. However, the interest on reserves can achieve higher welfare gains than the required reserve ratio because of the conflicting effect that the latter has through the balance sheet and deposits channels. This effect is independent of the precautionary demand for liquidity, although at higher excess reserves such net welfare gains can be amplified.*

This result follows from Table 3 and Figures 5 and 6. Table 3, shows that the use of either the required reserve ratio or the interest on reserves can improve on the welfare outcomes achieved by a simple optimal Taylor rule (reported in Table 2). However, the use of the interest on reserves is shown to provide higher welfare gains than the required reserve ratio. This is because of the conflicting effect that the required reserve ratio has on the deposit rate and the balance sheet channels, an effect which is independent of the level of liquidity hoarding by banks (see section 3.2).

<sup>25</sup>See Hall (2002), Goodfriend (2002, 2011), Curdia and Woodford (2011), Bech and Klee (2011), Dutkowskya and VanHoose (2011), Kashyap and Stein (2012), Ireland (2014), Cochrane (2014), Dressler and Kersting, (2015), Hall and Reis (2016).

<sup>26</sup>Note that both these policy tools could also respond to a number of other financial variables but the results are not expected to be very different. For example, allowing them to respond to the loan rate spread (i.e.  $\zeta_t = \left( \frac{R_t^l/R_t}{R^l/R} \right)^{\mu_\zeta^s}$  and  $R_t^{ior} = \left( \frac{R_t^l/R_t}{R^l/R} \right)^{\mu_{ior}^s}$ ), produces very similar results. These results can be made available upon request.

Table 3: Welfare Effects: Interest on Reserves and Required Reserve Ratio

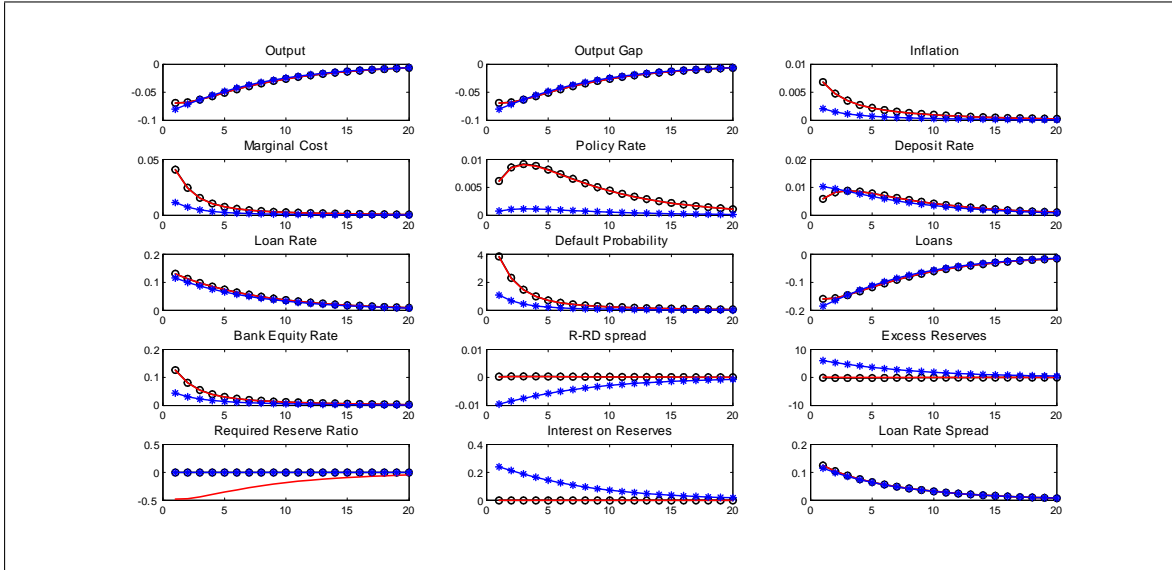
(a) Loan Rate Shock,				
Policy	$\psi_3=0.001, ER\approx 2.0\%$	$\psi_3=3.5, ER\approx 12\%$	$\psi_3=5.4, ER\approx 18\%$	$\psi_3=7.6, ER\approx 25\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$
OTR+ $\varsigma$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.02384$	$\phi_\pi = 1.90$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.00$ $CE = 0.00004$	$\phi_\pi = 3.00$ $\phi_x = 0.60$ $\mu_\varsigma^l = 3.00$ $CE = 0.00261$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.00866$
OTR+ $R_t^{ior}$	$\phi_\pi = 1.20$ $\phi_x = 0.0$ $\mu_{ior}^l = -1.30$ $CE = 0.02881$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -3.00$ $CE = 0.00924$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -2.30$ $CE = 0.01002$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_{ior}^l = -1.40$ $CE = 0.01044$
(b) Adverse Technology Shock				
Policy	$\psi_3=0.001, ER\approx 2\%$	$\psi_3=2.0, ER\approx 8\%$	$\psi_3=13.5, ER\approx 42\%$	$\psi_3=16.2, ER\approx 51\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$
OTR+ $\varsigma$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.00255$	$\phi_\pi = 3.0$ $\phi_x = 0.0$ $\mu_\varsigma^l = 3.0$ $CE = 0.00056$	$\phi_\pi = 2.70$ $\phi_x = 0.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.0000$	$\phi_\pi = 0.80^*$ $\phi_x = 1.00$ $\mu_\varsigma^l = 3.00$ $CE = 0.0000$
OTR+ $R_t^{ior}$	$\phi_\pi = 3.00$ $\phi_x = 0.20$ $\mu_{ior}^l = -1.90$ $CE = 0.00288$	$\phi_\pi = 3.00$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.80$ $CE = 0.00074$	$\phi_\pi = 3.0$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.20$ $CE = 0.0000$	$\phi_\pi = 0.00^*$ $\phi_x = 1.00$ $\mu_{ior}^l = -0.10$ $CE = 0.0000$

\*Taylor Principle violated

Figure 5, demonstrates this result for the case of a financial shock when liquidity hoarding is zero, ( $\psi_3=0$ , which implies excess reserves of about 2%).<sup>27</sup> The financial shock raises, the probability of default, the cost of borrowing and inflation and reduces the output gap. The optimal response of the required reserve ratio is shown to be an increase in the deposit rate but, as shown in section 3.2, this can only be achieved with a decrease in the required reserve ratio, which frees some reserves for loans. The interest on reserves is shown to stabilize inflation and the output gap more effectively through the deposits and balance sheet channels, by having a stronger effect on the deposit rate while simultaneously raising the level of excess reserves which reduces the volume of loans. This is shown to take the pressure off the policy rate, which does not have to rise as much as in the cases of the simple optimal Taylor rule (OTR), or the latter supported by the required reserve ratio (OTR+ $\varsigma$ ). A lower policy rate implies that the probability of default, the equity rate, the loan rate and hence marginal cost and inflation are now lower which in turn feed back to keeping the policy rate low.

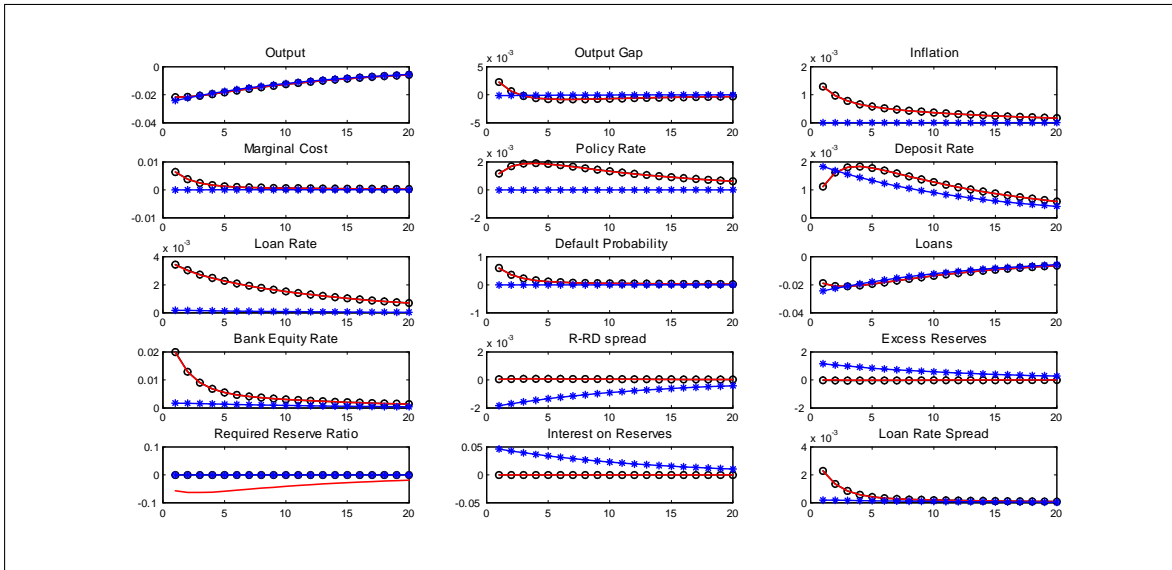
<sup>27</sup>The results for the optimal Taylor rule (Black(o) line=OTR), are given in Table 2 (a) and (b) respectively.

Figure 5: Financial Shock: Interest on Reserves and Required Reserve Ratio



Black(o) line=OTR; Red(solid) line=OTR+ $\varsigma$ ; Blue(star) line=OTR+ $R^{ior}$ , (all at ER=2%)

Figure 6: Adverse Supply Shock: Interest on Reserves and Required Reserve Ratio



Black(o) line=OTR; Red(solid) line=OTR+ $\varsigma$ ; Blue(star) line=OTR+ $R^{ior}$ , (all at ER=2%)

Similar results are shown in the case of an adverse technology shock, in Figure 6, where the combination of a stronger anti-inflationary policy, combined with a rise in the interest on reserves is shown to stabilize inflation and the output gap more effectively than the required reserve ratio. Although this effect is independent of the degree of precautionary demand for liquidity, Table 3 shows that the welfare gains from using the interest on reserves as a supporting monetary policy tool, can be amplified at higher levels of excess reserves, particularly for the case of financial shocks.

### 3.4 The Interest on Reserves as a Main Policy Tool.

Hall and Reis (2016) use a simple model of the fiscal theory of the price level to show that the interest rate on reserves (nominal or real) can pin down a unique equilibrium of the price level. In this section I show that even in a conventional DSGE model, the interest on reserves can fully replace the Taylor rule and still provide a unique equilibrium. As in Hall and Reis (2016), the focus is on the interest on reserves in a model of credit frictions, however the innovation here does not rely on the properties of the fiscal theory of the price level, but on how the interest on reserves affects the deposits channel when it responds to the source of credit friction, that is credit risk here. I demonstrate this with two different type of policy rules: (i) the interest on reserves responds to deviations of the level of excess reserves from their steady state,

$$R_t^{ior} = R^{ior} \left( \frac{\tilde{r}_t - \varsigma_t}{\tilde{r} - \varsigma} \right)^{\mu_{ior}^{ER}}, \quad (30)$$

where  $\mu_{ior}^{ER}$  captures the policy responsiveness to excess reserves, and (ii) the interest on reserves responds directly to deviations of the level of credit risk from its steady state,

$$R_t^{ior} = R^{ior} \left( \frac{\Phi_t}{\bar{\Phi}} \right)^{\mu_{ior}^{\Phi}}, \quad (31)$$

where  $\mu_{ior}^{\Phi}$  captures the policy responsiveness to credit risk. Both (30) and (31) imply that the interest on reserves responds to changes in the level of credit riskiness in the economy, the former through the effect that credit risk has on the level of excess reserves, whereas the latter directly. Tables 4 and 5 summarise the results.

**Proposition 2:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\mu_{ior}^{\Phi} > 0$ , that is, the interest on reserves can affect the deposit channel in response to changes in credit risk, when the latter affects the cost channel and*

the output gap. This result is independent of the level of liquidity hoarding in the banking system, ( $\psi_3 \geq 0$ ) and fiscal theory of the price level properties.

**Proof.** A formal proof of Proposition 2 is provided in the *Appendix*.

Table 4: Interest on Reserves at the Zero-Bound: Responses to Excess Reserves

(a) Loan Rate Shock,				
Policy	$\psi_3=0, ER \approx 0\%$	$\psi_3=2.0, ER \approx 6\%$	$\psi_3=4.0, ER \approx 12\%$	$\psi_3=10, ER \approx 30\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = N/A$ $CE = N/A$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.10$ $CE = 0.00518$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.10$ $CE = 0.00011$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = -0.18$ $CE = 0.00025$
(b) Adverse Supply Shock				
Policy	$\psi_3=0, ER \approx 0.0\%$	$\psi_3=2.0, ER \approx 6\%$	$\psi_3=8.3, ER \approx 25.0\%$	$\psi_3=13.5, ER \approx 40.0\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_y = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = N/A$ $CE = N/A$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.08$ $CE = 0.00095$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.27$ $CE = 0.0000$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^{ER} = 0.42$ $CE = 0.0000$

N/A=Indeterminacy. Values of  $\psi_1 = 0$  and  $R_t = R_t^{ior}$  are used so that  $\psi_3=0$  implies  $ER \approx 0\%$ .

Table 5: Interest on Reserves at the Zero-Bound: Responses to Risk

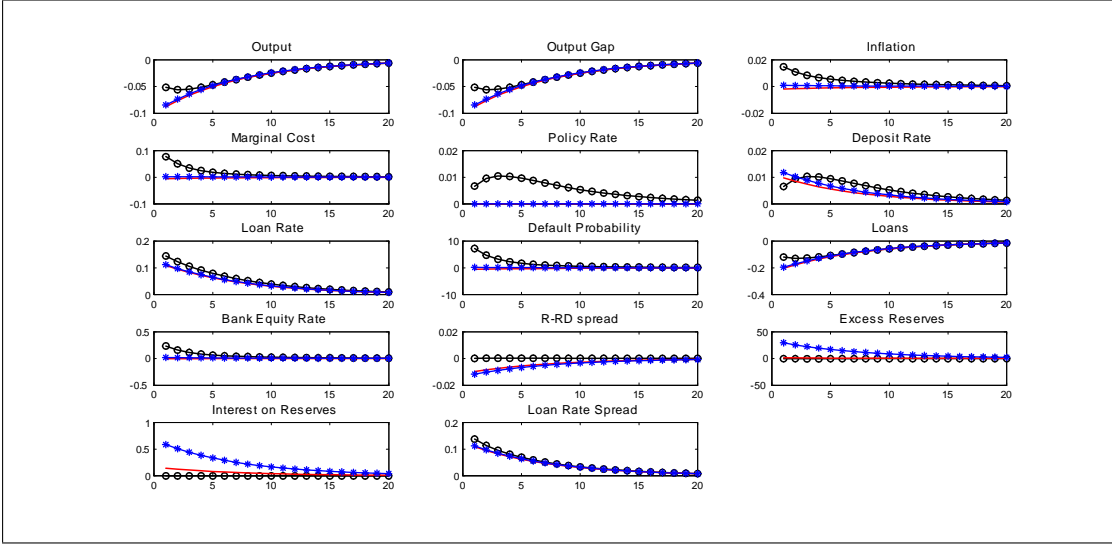
(a) Loan Rate Shock,				
Policy	$\psi_3=0, ER \approx 0\%$	$\psi_3=2.0, ER \approx 6\%$	$\psi_3=4.0, ER \approx 12\%$	$\psi_3=10, ER \approx 30\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 2.76$ $CE = 0.02666$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 0.64$ $CE = 0.00744$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 0.29$ $CE = 0.00113$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = -0.11$ $CE = 0.00025$
(b) Adverse Technology Shock				
Policy	$\psi_3=0, ER \approx 0\%$	$\psi_3=2.0, ER \approx 6\%$	$\psi_3=8.3, ER \approx 25.0\%$	$\psi_3=13.5, ER \approx 40.0\%$
Baseline	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_x = 0$	$\phi_\pi = 1.5, \phi_y = 0$	$\phi_\pi = 1.5, \phi_x = 0$
$R_t^{ior}$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = -3.00$ $CE = 0.00263$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.00095$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.0000$	$\phi_\pi = 0, \phi_x = 0$ $\mu_{ior}^\Phi = 3.00$ $CE = 0.0000$

Values of  $\psi_1 = 0$  and  $R = R^{ior}$  are used so that  $\psi_3=0$  implies  $ER \approx 0\%$ .

**Result 4:** *When the policy rate is at the zero-bound, the interest on reserves can act as a single monetary policy tool, through its effects on the deposit and balance sheet channels, producing higher welfare gains than a simple Taylor rule. However, such net welfare gains diminish the higher is the steady state level of excess reserves.*

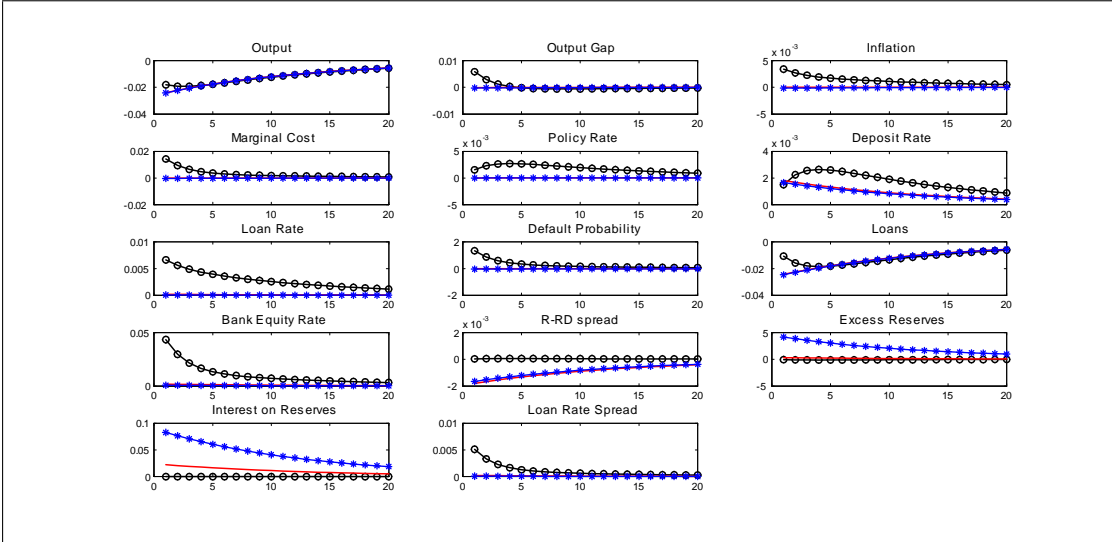
Result 4, follows from Proposition 2, Tables 4 and 5 and Figures 7 and 8 that plot the optimal responses of the interest on reserves rules (eqs. 30 and 31), at the zero-bound,  $\widehat{R}_t = 0$ . The intuition here is that the source of credit friction, that is credit risk, affects the output gap and inflation. We have already shown that the interest on reserves affects the deposits channel. Therefore by responding to the level of credit riskiness in the economy, the interest on reserves can pin down inflation and the output gap, through its effects on the deposits channel and subsequently the balance sheet channel. This is demonstrated in Figures 7 and 8, where following a financial shock, the optimal response of the interest on reserves is to increase, resulting in higher excess reserves, a higher deposit rate and a fall in loans. Thus through the deposits channel and balance sheet channel, the higher interest on reserves reduces the output gap, the loan spread and the cost of borrowing and thus inflation. Tables (4) and (5) also suggest show that the net welfare gains from using the interest on reserves as a single monetary policy tool diminish the higher is the existing steady state level of excess reserves.

Figure 7: Financial Shock: Interest on Reserves at the Zero Bound



Black(o) line=Baseline (ER=0%); Red(solid) line=eq(30),  $\mu_{ior}^{ER}=0.10$ ,  $\psi_3=2.0$ , (ER=6%); Blue(star) line=eq(31),  $\mu_{ior}^{\Phi}=2.76$ ,  $\psi_3=0$ , (ER=0%). All estimated with  $\psi_1=0$ ,  $R_t=R_t^{ior}$ .

Figure 8: Adverse Supply Shock: Interest on Reserves at the Zero-Bound



Black(o) line=Baseline (ER=0%); Red(solid) line=eq(30),  $\mu_{ior}^{ER}=0.08$ ,  $\psi_3=2.0$ , (ER=6%); Blue(star) line=eq(31),  $\mu_{ior}^{\Phi}=-3.00$ ,  $\psi_3=0$ , (ER=0%). All estimated with  $\psi_1=0$ ,  $R_t=R_t^{ior}$ .

### 3.5 The Role of the Interest on Reserves Spread, $(R_t - R_t^{ior})$

In some of the above analysis we have assumed a positive steady state spread between the policy rate and the interest on reserves,  $R > R^{ior}$ . In practice however, both the US Fed and the Bank of England are currently setting the interest rate on reserves equal to the policy rate. Moreover, there is evidence that the federal funds rate regularly traded below the interest on reserves (see also Goodfriend 2015). The  $R_t - R_t^{ior}$  spread determines the opportunity cost of holding reserves. The lower is the interest on reserves in relation to the policy rate the higher is the ‘tax on holding reserves’ (Ireland, 2015). Conversely, the smaller is the  $R_t - R_t^{ior}$  spread, the higher is expected to be the level of excess reserves.

Indeed, Table 6 shows that the steady state level of excess reserves increases as the  $R - R^{ior}$  spread moves from 100bps to 0 bps. Here, with no precautionary demand for liquidity, or any other factors affecting the level of excess reserves, a rise in the spread by 100 basis points is shown to result in 1% increase in excess reserves. This, as Table 6 shows, discourages lending which can result in some welfare gains when central banks are concerned with inflation and output stability, as the optimal inflation policy response,  $(\phi_\pi = 3.00)$ , is supported with some stricter liquidity management. However, this does not necessarily imply that setting the interest on reserves equal to the policy rate is the best central banks could do with this policy tool.

Table 6: Welfare Effects: The role of the Interest on Reserves Spread

R- $R^{ior}$ =	100bps (ER=2.0%)	70bps (ER=2.3%)	50bps (ER=2.5%)	R= $R^{ior}$ (ER=3.0%)
(a) Loan Rate Shock				
Baseline	$\phi_\pi = 1.5, \phi_x = 0, CE = -$			
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02372$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02399$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.02418$	$\phi_\pi = 3.0$ $\phi_x = 0.00$ $CE = 0.02464$
(b) Adverse Supply Shock				
OTR	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00254$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00258$	$\phi_\pi = 3.00$ $\phi_x = 0.00$ $CE = 0.00260$	$\phi_\pi = 3.0$ $\phi_x = 0.00$ $CE = 0.00267$

Table 7 uses the same assumptions as Table 6, but allows the interest on reserves to deviate from  $R_t^{ior} = R_t$  and respond optimally to either excess reserves (eq. 30) or directly to credit risk (eq 31). It is shown that using the interest on reserves as an independent policy tool to facilitate monetary policy can increase welfare gains beyond those of simply setting,  $R_t^{ior} = R_t$ .



Table 7: Welfare Effects: Interest on Reserves as an Independent Policy Tool

(a) Loan Rate Shock				
Baseline	$\phi_\pi = 1.5,$	$\phi_x = 0,$	$R=R^{ior}(\text{ER}=3.0\%),$	$\psi_3 = 0, \quad CE = -$
OTR	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	-	$CE = 0.02464$
OTR+ $R_t^{ior}$	(eq 30)	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^{ER} = 0.10 \quad CE = 0.02589$
OTR+ $R_t^{ior}$	(eq 31)	$\phi_\pi = 0.00,$	$\phi_x = 1.00$	$\mu_{ior}^\Phi = 3.00 \quad CE = 0.03989$
(b) Adverse Supply Shock				
OTR	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	-	$CE = 0.00267$
OTR+ $R_t^{ior}$	(eq 30)	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^{ER} = 0.10 \quad CE = 0.00274$
OTR+ $R_t^{ior}$	(eq 31)	$\phi_\pi = 3.00,$	$\phi_x = 0.00$	$\mu_{ior}^\Phi = 3.00 \quad CE = 0.00301$

**Result 5:** *Setting the interest on reserves equal to policy rate eliminates the tax on holding reserves. This encourages excess reserves which can increase welfare gain in times of liquidity management. However, such welfare gains are outperformed by allowing the interest on reserves to act as an independent policy tool to facilitate the policy rate.*

This result, which follows from Tables 6 and 7, is due to the effects that the interest on reserves has through the deposits and balance sheet channels, already explained in sections 3.2 and 3.3.

## 4 Concluding Discussion

This paper emphasizes the importance of the deposits channel in explaining the role of monetary policy, particularly during periods when the policy rate is fixed at the lower zero-bound. Dreschler, Savov and Schnabl (2017) show that during conventional monetary policy times the deposits channel can be affected by the way that changes in the policy rate affect the monopolistic behaviour of banks. In this paper I relax this mechanism and examine other potential determinants of the deposits channel that can be operational even in periods of non-conventional monetary policy. I identify two new potential factors that are shown to crucially affect the role of monetary policy through the deposits channel: the precautionary demand for liquidity and the interest on reserves.

It is shown that the higher is the precautionary demand for liquidity in the financial sector, the lower is the optimal relative weight that central banks should attach on inflation in the Taylor rule. When credit riskiness is perceived by banks to be very high, then the precautionary demand for liquidity can self-discipline the financial sector through liquidity hoarding which increases excess reserves and raises the deposit rate above the policy rate. The latter effect is shown to pin down the inflation rate and provide a unique

equilibrium even when the policy rate is at the zero-bound. In practice, the extent to which the precautionary demand for liquidity can effect the deposits channel depends of course on how the financial sector perceives credit riskiness, because as the model indicates, even when the actual credit risk,  $\Phi_t(\varepsilon_t^*)$  is high, if banks undermine such risk (i.e. place a low  $\psi_3$ ) then the deposits channel, and the subsequent balance sheet channel, may be too weak to be effective.

A more active role for monetary policy, particularly at the zero-bound, is to try and affect the deposits channel through the interest on reserves. When banks become unresponsive to the policy rate that sets the benchmark for their transactions at the interbank market, they may become more responsive to the rate offered on the reserves they are sitting on. It is shown that through its effect on the deposits channel, the interest on reserves can act as a single monetary policy tool that can provide determinacy and a higher welfare gain than a simple Taylor rule. This result is supportive of the recent findings in Hall and Reis (2016) that a payment on reserves rule is sufficient to stabilise the price level and it is simpler to implement than a Taylor rule. However, here the results are established within a conventional new Keynesian macroeconomic framework and are shown to be independent of fiscal debt, the degree of liquidity hoarding, or the type of shocks affecting the economy (financial or real shocks).

In practice, the implementation of the interest on reserves depends on the rule considered. Responding to excess reserves (as one of the rules considered here), or total reserves, as in Hall and Reis (2016), should be easier to implement in practice than a Taylor rule that requires the calculation of output gaps and natural real interest rates. However, if the interest on reserves responds to the level of credit riskiness in the economy, (a rule shown to perform particularly well in this paper), then the latter may be harder to calculate and so the level of excess reserves may be preferred as a proxy to perceived risk. The findings also question whether the current practice of setting the interest on reserves equal to the policy rate, (i.e. US Fed, Bank of England), undermine the full potential of the interest on reserves as an independent policy tool.

The paper also shows that the required reserve ratio can affect the deposits channel but its effectiveness is weaker, because of the conflicting effects it has on the balance sheet channel, that overall makes the interest on reserves a more welfare-enhancing monetary policy tool that also implies less interest rate intervention.

In general, the findings in this paper may help explain the zero-interest rate policy observed in countries that experienced large excess reserves. It can also justify why the interest on reserves, rather than required reserves, became the focus of monetary

policy during the zero-bound period in a number of countries. Overall, the paper invites more research on the determinants of the deposits channel for explaining the role and effectiveness of monetary policy, particularly in times of unconventional monetary policy, but also on the potential role of the interest on reserves as a monetary policy tool.

## References

- [1] Allen, F., Carletti, E. and Gale, D., 2009. Interbank market liquidity and central bank intervention. *Journal of Monetary Economics*, 56(5), pp.639-652.
- [2] Agénor, P.R., Bratsiotis, G.J., and Pfajfar, D., 2014. Credit Frictions, Collateral, And The Cyclical Behavior Of The Finance Premium. *Macroeconomic Dynamics*, 18(05), pp.985-997.
- [3] Agénor, P.R., and Pereira da Silva, L.A., 2014. Macroprudential regulation and the monetary transmission mechanism. *Journal of Financial Stability*, 13, 44-63.
- [4] Agénor, P.R. and El Aynaoui, K., 2010. Excess liquidity, bank pricing rules, and monetary policy. *Journal of Banking & Finance*, 34(5), pp.923-933.
- [5] Angelini, P., Nobili, A., and Picillo, C., 2011. The interbank market after August 2007: what has changed, and why?. *Journal of Money, Credit and Banking*, 43(5), 923-958.
- [6] Ashcraft, A., McAndrews, J., & Skeie, D., 2011. Precautionary reserves and the interbank market. *Journal of Money, Credit and Banking*, 43(s2), 311-348.
- [7] Bech, M. L., and Klee, E., 2011. The mechanics of a graceful exit: Interest on reserves and segmentation in the federal funds market. *Journal of Monetary Economics* 58.5, 415-431.
- [8] Benes, J., and Kumhof. M., 2015. Risky bank lending and countercyclical capital buffers. *Journal of Economic Dynamics and Control*, 58, 58-80.
- [9] Bernanke, B.S. and Gertler, M., 1995. Inside the black box: the credit channel of monetary policy transmission (No. w5146). National bureau of economic research.
- [10] Bernanke, B. S., Gertler, M. and Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1, 1341-1393.
- [11] Board of Governors of the Federal Reserve System, 2008. Press Release, October 6, 2008, (<http://www.federalreserve.gov/newsevents/press/monetary/20081006a.htm>).
- [12] Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3), 383-398.

- [13]Chadha, J. S., and Corrado, L., 2012. Macro-prudential policy on liquidity: What does a DSGE model tell us? *Journal of Economics and Business*, 64(1), 37-62.
- [14]Ciccarelli, M., Maddaloni, A., & Peydró, J. L., 2015. Trusting the bankers: A new look at the credit channel of monetary policy. *Review of Economic Dynamics*, 18(4), 979-1002.
- [15]Christiano, L. J., Motto, R., & Rostagno, M., 2010. Financial factors in economic fluctuations. ECB Working Paper No. 1192.
- [16]Chochrane, John H., 2014. Monetary policy with interest on reserves. *Journal of Economic Dynamics and Control* 49: 74-108.
- [17]Cúrdia, V., and Woodford, M., 2011. The central-bank balance sheet as an instrument of monetary policy. *Journal of Monetary Economics*, 58(1), 54-79.
- [18]De Fiore, F., Tristani, O., 2011. Credit and the natural rate of interest. *Journal of Money Credit and Banking* 43(2-3), 407-440.
- [19]Dixit, A.K., and Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. *The American Economic Review*, 67, 297-308.
- [20]Drechsler, I., Savov, A. and Schnabl, P., 2017. The deposits channel of monetary policy. *The Quarterly Journal of Economics*, 132(4), pp.1819-1876.
- [21]Dressler, S.J., and Kersting, E.K., 2015. Excess reserves and economic activity. *Journal of Economic Dynamics and Control*, 52, pp.17-31.
- [22]Dutkowsky, D.H., and VanHoose, D. D., 2011. Interest on bank reserves and optimal sweeping. *Journal of Banking and Finance*, 35(9), 2491-2497.
- [23]Dutkowsky, D.H. and VanHoose, D. D., 2013. Interest on reserves, unregulated interest on demand deposits, and optimal sweeping. *Journal of Macroeconomics*, 38, pp.192-202.
- [24]Ennis, H. M., and Wolman, A. L., 2015. Large excess reserves in the United States: A view from the cross-section of banks. *International Journal of Central Banking*, 11(1), 251-289.
- [25]Gertler, M., and Kiyotaki, N., 2011. Financial Intermediation and Credit Policy in Business Cycle Analysis, in B. M. Friedman and M. Woodford, (eds), *Handbook of Monetary Economics*, Elsevier, New York NY.
- [26]Glocker, C., and Towbin, P., 2012. Reserve Requirements for Price and Financial Stability: When Are They Effective? *International Journal of Central Banking*, 8(1), 65-113.
- [27]Del Giovane, P., Eramo, G., and Nobili, A., 2011. Disentangling demand and supply in credit developments: a survey-based analysis for Italy. *Journal of Banking &*

- finance, 35(10), 2719-2732.
- [28]Goodfriend, M., 2002. Interest on reserves and monetary policy. Federal Reserve Bank of New York Economic Policy Review, 8(1), 13-29.
- [29]Goodfriend, M., 2011. Central banking in the credit turmoil: An assessment of Federal Reserve practice. Journal of Monetary Economics, 58(1), pp.1-12.
- [30]Goodfriend, M., 2015. The Fed Should Fix the Interest on Reserves Floor. Prepared for the Shadow Open Market Committee Meeting, New York, New York, March (Vol. 20).
- [31]Gray, S. 2011. Central bank balances and reserve requirements. 1-36. International Monetary Fund, no. 11-36.
- [32]Güntner, J. H. F., 2015. The federal funds market, excess reserves, and unconventional monetary policy. Journal of Economic Dynamics and Control, 53, 225-250.
- [33]Hall, R. E., 2002. Controlling the price level. Contributions to Macroeconomics, 2(1), 1038–1059. Article 5
- [34]Hall, R. E., and Reis, R., 2016. Achieving price stability by manipulating the central bank’s payment on reserves. National Bureau of Economic Research, No. w22761, (2016).
- [35]Heider, F., Hoerova, M. and Holthausen, C., 2015. Liquidity hoarding and interbank market rates: The role of counterparty risk. Journal of Financial Economics, 118(2), pp.336-354.
- [36]Ireland, P. N., 2014. The macroeconomic effects of interest on reserves. Macroeconomic Dynamics 18(06), 1271-1312.
- [37]Kashyap, A.K. and Stein, J.C., 2000. What do a million observations on banks say about the transmission of monetary policy?. American Economic Review, pp.407-428.
- [38]Kashyap, A. K., and Stein, J. C., 2012. The Optimal Conduct of Monetary Policy with Interest on Reserve. American Economic Journal: Macroeconomics, 4(1), 266-282.
- [39]Loungani, P., and Rush, M., 1995. The effect of changes in reserve requirements on investment and GNP. Journal of Money, Credit and Banking, 27(2), 511-526.
- [40]McLeay, M., and Thomas, R., 2016. UK broad money growth in the Long Expansion 1992–2007: what can it tell us about the role of money?. The UK Economy in the Long Expansion and its Aftermath, 131.
- [41]Mimir, Y., Sunel, E. and Taşkın, T., 2013. Required reserves as a credit policy tool. The BE Journal of Macroeconomics, 13(1), pp.823-880.

- [42] Montoro, C., and Moreno, R., 2011. The use of reserve requirements as a policy instrument in Latin America. *BIS Quarterly Review* (March), 53-65.
- [43] Mora, N., 2014. Reason for reserve? Reserve requirements and credit." *Journal of Money, Credit and Banking* 46.2-3: 469-501.
- [44] Quadrini, V., 2017. Bank liabilities channel. *Journal of Monetary Economics*.
- [45] Ravenna, F., and Walsh, C. E., 2006. Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53(2), 199-216.
- [46] Taylor, John B., 2008. "Monetary Policy and the State of the Economy", Testimony before the Committee on Financial Services, U.S. House of Representatives, February 26, 2008.
- [47] Woodford, M., 2010. Financial intermediation and macroeconomic analysis. *The Journal of Economic Perspectives*, 24(4), pp.21-44.
- [48] Woodford, M., 2011. *Interest and prices: Foundations of a theory of monetary policy*. Princeton University Press.

## 5 APPENDIX

**Proposition 1:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\psi_3 > 0$ , that is, there is a positive degree of precautionary demand for liquidity that raises the deposit rate.*

### Proof of Proposition 1:

Consider a simplified version of the lower zero bound case where the policy rate is fixed  $\widehat{R}_t = 0$ , around its steady state value of  $R = 1/\beta$ . We also assume that the required reserve ratio and the interest on reserves are also fixed,  $\widehat{R}_t^{ior} = \widehat{\zeta}_t = 0$ , as assumed in Table 2. These assumptions imply that the model can be reduced to three log-linearized equations,

$$\begin{aligned}\widehat{x}_t &= \mathbb{E}_t \widehat{x}_{t+1} - \sigma^{-1} \left( \widehat{R}_t^d - \mathbb{E}_t \widehat{\pi}_{t+1} \right) + \widehat{u}_t, \\ \widehat{\pi}_t &= \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p (\eta + \sigma + \Gamma) \widehat{x}_t, \\ \widehat{R}_t^d &= B \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) (\eta + \sigma + \Gamma) \widehat{x}_t.\end{aligned}$$

Substituting  $\widehat{R}_t^d$  into  $\widehat{x}_t$  we can write the above equations into a 2 x 2 vector system,

$$\begin{bmatrix} \mathbb{E}_t \widehat{x}_{t+1} \\ \mathbb{E}_t \widehat{\pi}_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ \widehat{\pi}_t \end{bmatrix} + u_t, \quad \text{where, } A = \begin{bmatrix} 1 + B \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \frac{\eta + \sigma + \Gamma}{\sigma} \right) & 0 \\ 0 & 1/\beta \end{bmatrix},$$

where,

$$B = \frac{\psi_3 \Phi (-\psi_1 + \psi_3 \Phi - (R - R^{ior}))}{\psi_2 R^d},$$

$\psi_1 < 0$ ,  $\psi_2 > 0$  and  $\psi_3 \geq 0$  and,

$$\Gamma = (\eta + \sigma) \left( \frac{\left( B + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} + \frac{(\varepsilon^* + \underline{\varepsilon}) \chi \Phi}{2l/y}}{R^l - \left( B + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} - \frac{\varepsilon^* \chi \Phi}{l/y}} \right).$$

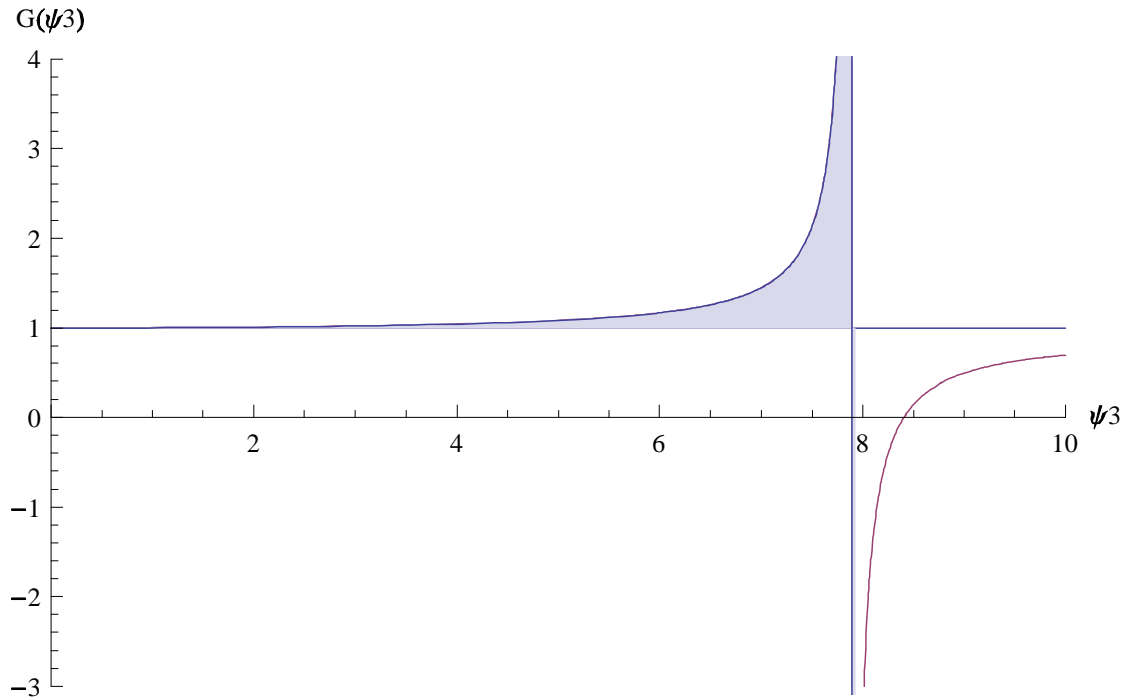
It follows that the necessary condition for a *determinate* equilibrium is,

$$G(\psi_3) = 1 + \frac{B(1-\beta)(\eta + \sigma + \Gamma)}{\beta\sigma} \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} > 1.$$

From this and the definition of  $B$  and  $\Gamma$  above, it is shown that when the policy rate is fixed at the zero-bound, a necessary condition for stability is that,  $\psi_3 > 0$ . Eliminating liquidity hoarding for precautionary reasons, that is setting  $\psi_3 = 0$ , results in  $B = 0$  and the determinacy condition is not met,  $G(\psi_3 = 0) = 1$ . This implies that in the absence

of an active policy rate, ( $\widehat{R}_t = 0$ ), which could affect the deposit rate, setting  $\psi_3 = 0$ , entirely eliminates the endogenous response of the deposit rate to the output gap and inflation (see  $\widehat{R}_t^d$  above) and the model becomes *indeterminate* as in the standard case. Table 9, shows the determinacy region (shaded area above unity) for different values of  $\psi_3$ , at the zero-bound, keeping all other parameters constant at their baseline level (see Table 1).

Figure 9: Determinacy Region for Different Values of  $\psi_3$



**Proposition 2:** *When the policy rate is fixed at the zero-bound a necessary condition for determinacy is that  $\mu_{ior}^\Phi > 0$ , that is, the interest on reserves can affect the deposit channel in response to changes in credit risk, when the latter affects the cost channel and the output gap. This result is independent of the level of liquidity hoarding in the banking system, ( $\psi_3 \geq 0$ ) and fiscal theory of the price level properties.*

**Proof of Proposition 2:**

Consider a simplified version of the zero-bound case where the policy rate is fixed  $\widehat{R}_t = 0$ , around its steady state value of  $R = 1/\beta$ . We also assume that the required reserve ratio is also fixed,  $\widehat{\zeta}_t = 0$ , as employed in section 3.4, These assumptions imply



the model can be reduced to the following three equations,

$$\begin{aligned}\hat{x}_t &= \mathbb{E}_t \hat{x}_{t+1} - \sigma^{-1} \left( \hat{R}_t^d - \mathbb{E}_t \hat{\pi}_{t+1} \right) + \hat{u}_t, \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + k_p \left( \eta + \sigma + \tilde{\Gamma} \right) \hat{x}_t, \\ \hat{R}_t^d &= \tilde{B} \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \eta + \sigma + \tilde{\Gamma} \right) \hat{x}_t.\end{aligned}$$

Substituting  $\hat{R}_t^d$  into  $\hat{x}_t$  we can write the above equations into a 2 x 2 vector system,

$$\begin{bmatrix} \mathbb{E}_t \hat{x}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ \hat{\pi}_t \end{bmatrix} + u_t, \quad \text{where, } A = \begin{bmatrix} 1 + \tilde{B} \left( \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \left( \frac{\eta + \sigma + \tilde{\Gamma}}{\sigma} \right) & 0 \\ 0 & 1/\beta \end{bmatrix},$$

where,

$$\tilde{B} = \frac{\psi_3 \Phi (-\psi_1 + \psi_3 \Phi - (R - R^{ior})) + \mu_{ior}^\Phi R^{ior} (-\psi_1 + \psi_2 \varsigma + \psi_3 \Phi - (R - R^{ior}))}{\psi_2 R^d},$$

where  $\psi_1 < 0$ ,  $\psi_2 > 0$  and  $\psi_3 \geq 0$  and,

$$\tilde{\Gamma} = (\eta + \sigma) \left( \frac{\left( \tilde{B} + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} + \frac{(\varepsilon^* + \underline{\varepsilon}) \chi \Phi}{2l/y}}{R^l - \left( \tilde{B} + \frac{\Phi}{1-\Phi} \right) \frac{\gamma R^e \varepsilon^*}{\bar{\varepsilon} - \varepsilon^*} - \frac{\varepsilon^* \chi \Phi}{l/y}} \right).$$

It follows that the necessary condition for a *determinate* equilibrium is,

$$\tilde{G}(\mu_{ior}^\Phi) = 1 + \frac{\tilde{B}(1 - \beta)(\eta + \sigma + \tilde{\Gamma})}{\beta \sigma} \frac{\varepsilon^*}{\bar{\varepsilon} - \underline{\varepsilon}} > 1.$$

It follows that if the policy rate is fixed at the zero-bound,  $\hat{R}_t = 0$ , and there is no precautionary liquidity hoarding in the banking system, ( $\psi_3 = 0$ ), then a necessary condition for a *determinate* equilibrium is  $\mu_{ior}^\Phi > 0$ . With  $\psi_3 = 0$ , setting  $\mu_{ior}^\Phi = 0$  results in  $\tilde{B} = 0$  hence the condition for a unique equilibrium cannot be satisfied as  $\tilde{G}(\mu_{ior}^\Phi = 0) = 1$ . Table 10, shows the determinacy region for different values of  $\mu_{ior}^\Phi$  at the zero bound  $\hat{R}_t = 0$ , when  $\psi_1 = \psi_3 = 0$  and  $ER = 0\%$ .

Figure 10: Determinacy Region for Different Values of mu-phi

