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# Monetary Policy with Sectoral Trade-offs

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#### Abstract

We formulate a two-sector New Keynesian economy that features sectoral heterogeneity along three main dimensions: price stickiness, consumption goods durability, and the inter-sectoral trade of input materials. The combination of these factors deeply affects inter-sectoral and intra-sectoral stabilization. In this context, we examine the welfare properties of simple rules that adjust the policy rate in response to the output gap and alternative measures of final goods price inflation. Aggregating durable and non-durable goods prices depending on the relative frequency of sectoral price-setting may induce a severe bias. Due to factor demand linkages, the cost of production in one sector is influenced by price-setting in the other sector of the economy. As a result, measures of aggregate inflation that weigh sectoral price dynamics based on the relative degree of price rigidity do not allow the central bank to keep track of the effective speeds of sectoral price adjustment.

JEL classification: E23, E32, E52

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### 1 Introduction

Along with major differences in the time span over which they yield consumer utility, durable and non-durable consumption goods are characterized by deep peculiarities in their production and price-setting. Such structural traits are paramount to the monetary transmission mechanism (Barsky et al., 2007) and need to be accounted for when designing realistic multi-sector economies. From a normative viewpoint, the literature available to date has extensively reported that sectoral heterogeneity presents the central bank with a nontrivial trade-off. With a single instrument, the policy maker cannot replicate the frictionless equilibrium allocation in each sector of the economy. This principle applies whenever sectoral discrepancies concern at least one of the following characteristics: price rigidity (Aoki, 2001), durability of different consumption goods (Erceg and Levin, 2006), inter-sectoral trade of input materials (Huang and Liu, 2005; Petrella and Santoro, 2011). All these factors are widely recognized to be major determinants of the relative price of goods produced by different sectors, which in turn exerts a strong influence on aggregate inflation (Reis and Watson, 2010). Therefore, drawing predictions based on single-sector models fails to reflect the underlying sources of aggregate inflation dynamics. The present study addresses these issues from a normative perspective, integrating the main sources of sectoral heterogeneity into a two-sector New Keynesian economy.

In the economy under examination the monetary authority cannot attain the Pareto optimal allocation consistent with the full stabilization of sectoral productions and inflation rates, even when distortions in the labor market (i.e., imperfect labor mobility) and the goods market (i.e., monopolistic competition) are removed. Thus, we turn our attention to policy strategies capable of attaining second best outcomes. To this end, we derive an appropriate welfare metric through a quadratic approximation of households' utility and assume that the central bank pursues its policy under timeless perspective commitment (Woodford, 1999, 2003). In doing so, the policy maker needs to account for some distinctive features that affect both inter-sectoral and intra-sectoral stabilization. As for the first aspect, Erceg and Levin (2006) showed that durables are much more interest rate-sensitive than non-durables, even though they feature a relatively lower expenditure share of households' consumption. 

This property exacerbates the trade-off entailed in stabilizing real activity in the two sectors, as

<sup>&</sup>lt;sup>1</sup>Bouakez et al. (2014, 2009) have shown that heterogeneity in price rigidity is a crucial factor to understand why sectoral inflation rates do not feature analogous responses to monetary policy shocks (see also Galesi and Rachedi, 2016), while the degree of durability has important implications for explaining sectoral output responses.

<sup>&</sup>lt;sup>2</sup>Though other forms of sectoral heterogeneity could be envisaged, we focus on the most pervasive ones that have been explored by the literature available to date.

<sup>&</sup>lt;sup>3</sup>Petrella and Santoro (2012) report substantial heterogeneity in sectoral inflation dynamics, with variations in the income share of input materials traded among sectors representing a key driver.

<sup>&</sup>lt;sup>4</sup>The pronounced magnitude of durables' response depends on two inherent features of this type of good: first, the demand for durables is for a stock, so that changes in the stock demand translate into much larger fluctuations in the flow demand for newly produced goods; second, the presence of sectoral price rigidities mitigates the role that changes in the relative price of durables play in insulating the durables sector from shocks.

compared with models featuring two non-durable goods. In the present work we show that factor demand linkages also play a major role in shaping the behavior of the relative price. In fact, the cross-industry flow of input materials is responsible for magnifying negative sectoral co-movement in the face of monetary policy innovations. As for intra-sectoral stabilization, it is important to recall that intermediate goods reduce the slope of the sectoral production schedules, as compared with models that neglect the presence of input materials (Petrella et al., 2014). This property limits the pass-through from the sectoral marginal costs to the respective rates of inflation. As a result, the central bank may attach greater importance to limiting fluctuations in the sectoral production gaps, as compared with models that disregard the role of input materials.

We assess the capability of simple interest-rate feedback rules to mimic the optimal policy benchmark. One obvious advantage of these policy functions is to abstract from the stringent informational requirements of the rule under timeless perspective. Moreover, while the model-consistent welfare criterion involves sector-specific variables, we assume that the policy rate is adjusted in response to broad measures of real activity and prices. A major problem we are confronted with when designing optimal interest rate rules for multi-sector economies is to find the most appropriate inflation rate to target (see, e.g., Huang and Liu, 2005). To this end, the model lends itself to account for three options: (i) aggregate inflation, according to which the sectoral inflation rates are aggregated depending on the relative size of each sector; (ii) sticky-price inflation, which weighs sectoral price dynamics depending on both the relative size and the relative degree of rigidity in price-setting of each sector; (iii) a measure of aggregate inflation that removes fluctuations in the price of oil from changes in the general price level. It is important to stress that sticky-price inflation is typically believed to be the most appropriate variable to monitor relative price changes in multi-sector environments that feature heterogeneous speeds of sectoral price adjustment (Woodford, 2003, pp. 435-443). Also policy makers display increasing interest in this type of measures. For instance, the Atlanta FED regularly publishes its Sticky Price Index, which sorts the components of the consumer price index (CPI) into either flexible or sticky (slow to change) categories, based on the frequency of their price adjustment. Also Eusepi, Hobijn, and Tambalotti (2011) have recently constructed a cost-of-nominal-distortions index (CONDI), whose weights depend on sectoral price stickiness.

The analysis of the interest-rate feedback rules delivers one key finding: targeting sticky-price inflation produces a higher loss of social welfare, as compared with reacting to aggregate inflation or to a measure of price changes that excludes oil price dynamics. This result crucially rests on the presence of inter-sectoral production linkages and the cross-industry externalities they generate in terms of price-setting. Even if durable goods prices are assumed to be relatively more flexible, they

inherit considerable extrinsic persistence through the influence of non-durable goods prices on their marginal cost of production. For the same reason, price stickiness in the non-durable goods sector is partly attenuated through the inter-sectoral input-output channel. As a result, aggregating durable and non-durable inflation depending on the relative frequency of sectoral price-setting may induce a severe bias, as the central bank does not properly keep track of the effective speeds of sectoral price adjustment. A similar issue emerges when taking the perspective of a delegated central banker that balances the stabilization of the output gap with that of an aggregate measure of price changes. In this case, weighing sectoral inflations based on their relative degree of price rigidity does not allow the central banker to take into account the intra-sectoral stabilization trade-off in the durable goods sector. In fact, while the dual mandate implicitly attributes a sizeable weight to stabilizing the gross production of durables, their inflation rate is virtually neglected by measures of sticky-price inflation, despite their marginal cost depends on the cost borne to buy input materials from the non-durable goods sector.

The remainder of the paper is laid out as follows: Section 2 introduces the theoretical setting; Section 3 reports the calibration of the model economy; Section 4 discusses the implementation of the optimal monetary policy; Section 5 examines the stabilization properties of alternative regimes for monetary policy-making. Section 6 concludes.

## 2 The Model

We develop a New Keynesian DSGE model with two sectors that produce durable and non-durable goods, respectively. The model economy is populated by a large number of infinitely-lived households. Each of them is endowed with one unit of time and derives utility from consuming of durable goods, non-durable goods and leisure.

The production technology of both sectors employs labor, input materials and oil. The two sectors are connected through factor demand linkages.<sup>5</sup> In this respect, the inter-sectoral flows of intermediate goods are determined by the input-output matrix of the economy, which is based on the implicit assumption that – once goods enter the production process – they only last a single period, vanishing within the assembled good. Despite appearing somewhat counterintuitive, this principle also applies to durables being used as input materials.<sup>6</sup> One example should help us clarifying this point. According to the Standard Industry Classification adopted by the Bureau of Economic Analysis,

<sup>&</sup>lt;sup>5</sup>Throughout the paper we will refer to 'factor demand linkages' as indicating cross-industry flows of input materials. Should a specific feature of the model economy be essentially determined by the use of intermediate goods in the production process (i.e., inter-sectoral relationships are not essential), we will explicitly refer to 'input materials'.

<sup>&</sup>lt;sup>6</sup>By contrast, goods that are repeatedly employed in the production process (i.e., investment inputs) are recorded in the Capital Flow Table.

group 322 identifies the establishments that are primarily engaged in manufacturing glass containers for commercial packing and bottling. These are key primary inputs for the food and beverage sector (group 20), which produces non-durable goods. Another typical example is group 3411 in the durable goods sector, which includes the producers of metal cans. These are among the main suppliers of firms included in group 203 (i.e., canned foods), whose final product is regarded as a non-durable good.

Inter-sectoral production linkages do not only represent a salient feature of multi-sector economies, one that has a crucial role for understanding the transmission of shocks to the economy (Holly and Petrella, 2012). In fact, they should also be seen as essential building blocks of business cycle models that aim at generating realistic degrees of sectoral output volatility and co-movement.<sup>7</sup> In connection with the specific context we examine, it is well known that sticky-price models incorporating sectoral heterogeneity in price stickiness – usually in the form of sticky non-durable goods prices and flexible prices of durables – cannot generate positive sectoral co-movement in the face of monetary policy innovations (Barsky et al., 2007). Inter-sectoral production linkages have been successfully introduced as a remedy to this lack of co-movement (Bouakez et al., 2011; Sudo, 2012; Di Pace, 2011).

#### 2.1 Consumers

Households derive their income from supplying labor to the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. Their consumption preferences are defined over  $H_t$  – a composite of non-durable goods  $(C_t^n)$  and an 'effective' stock of durable goods  $(\mathfrak{D}_t)$  – as well as labor,  $L_t$ . They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(H_t, L_t\right),\tag{1}$$

where  $H_t = (C_t^n)^{\mu_n} \mathfrak{D}_t^{\mu_d}$ ,  $\mu_n$  and  $\mu_d$  denote the expenditure shares on non-durable and durable goods (so that  $\mu_n + \mu_d = 1$ ) and  $\beta$  is the discount factor. We assume that the representative household's period utility function takes the form:

$$U(H_t, L_t) = \frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v}; \quad \varrho > 0$$
 (2)

<sup>&</sup>lt;sup>7</sup>Horvath (1998, 2000) and Carvalho (2009) show that cross-industry flows of input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between sectors, as originally hinted by Long and Plosser (1983). Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of sectoral economic activity. See also Hornstein and Praschnik (1997).

where  $\sigma$  is the inverse of the intertemporal elasticity of substitution and v is the inverse of the Frisch elasticity of labor supply. Durable goods are accumulated according to the following law of motion:

$$D_t = C_t^d + (1 - \delta) D_{t-1}, \tag{3}$$

where  $\delta$  is the depreciation factor. The effective stock of durables scales the effect of a quadratic cost of adjustment (see, e.g., Bernanke, 1985):<sup>8</sup>

$$\mathfrak{D}_t = D_t - \frac{\Xi}{2} \frac{(D_t - D_{t-1})^2}{D}, \quad \Xi \ge 0, \tag{4}$$

where D denotes the steady state stock of durable consumption goods.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{n,d\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{n,d\}} W_t^i L_t^i + \sum_{i=\{n,d\}} \Psi_t^i - T_t,$$
(5)

where  $B_t$  denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate  $R_t$ ,  $W_t^i$  denotes the nominal wage rate in sector  $i = \{n, d\}$  and  $T_t$  denotes a lump-sum tax paid to the government. The term  $\Psi_t^n + \Psi_t^d$  captures the nominal flow of dividends from both sectors of production.

We assume that labor can be either supplied to sector n or sector d, according to a CES aggregator:

$$L_t = \left[\phi^{-\frac{1}{\lambda}} \left(L_t^n\right)^{\frac{1+\lambda}{\lambda}} + (1-\phi)^{-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}},\tag{6}$$

where  $\lambda$  denotes the elasticity of substitution in labor supply, and  $\phi$  is the steady state ratio of labor supply in the non-durable goods sector over total labor supply (i.e.,  $\phi = L^n/L$ ). This functional form conveniently allows us to account for different degrees of labor mobility between sectors, depending on  $\lambda$ . To see this we report the equilibrium relationship that governs inter-sectoral mobility. This

<sup>&</sup>lt;sup>8</sup>Including an adjustment cost of the stock of durables allows us to obtain results in line with the empirical evidence on the behavior of durable consumption over the business cycle. King and Thomas (2006) show how the partial adjustment mechanism helps accounting for the aggregate effects of discrete and occasional changes in durables consumption.

<sup>&</sup>lt;sup>9</sup>The available evidence suggests that labor and capital are not perfectly mobile across sectors. Davis and Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Bouakez, Cardia, and Ruge-Murcia (2011) report evidence suggesting that perfect labor mobility across sectors, with its implication that sectoral nominal wages are the same (at the margin), is an imperfect characterization of the data.

can be obtained from the first order conditions with respect to  $L_t^n$  and  $L_t^d$ :<sup>10</sup>

$$\frac{W_t^n}{W_t^d} = \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\lambda}} \left(\frac{L_t^n}{L_t^d}\right)^{\frac{1}{\lambda}},\tag{7}$$

For  $\lambda=0$  labor is prevented from moving across sectors. For  $\lambda\to\infty$  workers devote all time to the sector paying the highest wage. Hence, at the margin, all sectors pay the same hourly wage and perfect labor mobility is attained. For  $\lambda<\infty$  hours worked are not perfect substitutes. An interpretation of this is that workers have a preference for diversity of labor and would prefer working closer to an equal number of hours in each sector, even in the presence of wage discrepancies. This assumption also reflects the common observation that human capital tends to be sector-specific in the short run (see, e.g., Matsuyama, 1992; Wacziarg and Wallack, 2004). In this respect, the CES aggregator in (6) implies that labor market frictions are neutralized in the steady state, so that the inefficiency associated with sectoral wage discrepancies is only temporary.

#### 2.2 Producers

The production side of the economy consists of two distinct sectors producing durable (sector d) and non-durable goods (sector n). Each sector is composed of a continuum of firms producing differentiated products. Let  $Y_t^n$  ( $Y_t^d$ ) denote gross output of the non-durable (durable) goods sector:

$$Y_t^i = \left[ \int_0^1 \left( Y_{ft}^i \right)^{\frac{\varepsilon_t^i - 1}{\varepsilon_t^i}} df \right]^{\frac{\varepsilon_t^i}{\varepsilon_t^i - 1}}, \qquad i = \{n, d\}$$
 (8)

where  $\varepsilon_t^i$  denotes the time-varying elasticity of substitution between differentiated goods in the production composite of sector  $i = \{n, d\}$ . Each production composite is produced in the 'aggregator' sector operating under perfect competition. The f<sup>th</sup> firm in sector i faces the following demand schedule:

$$Y_{ft}^i = \left(\frac{P_{ft}^i}{P_t^i}\right)^{-\varepsilon_t^i} Y_t^i, \qquad i = \{n, d\}$$

$$(9)$$

 $<sup>^{10}{\</sup>rm The}$  first order conditions from consumers' optimization are available in Appendix A.

<sup>&</sup>lt;sup>11</sup>Horvath (2000) motivates a similar specification based on the desire to capture some degree of sector-specificity to labor while not deviating from the representative consumer/worker assumption. In a similar vein, we conveniently employ this mechanism to allow for imperfect labor mobility between sectors.

where  $P_t^i$  is the price of the composite good in the i<sup>th</sup> sector. From (8) and (9) the relationship between the firm-specific and the sector-specific price is:

$$P_t^i = \left[ \int_0^1 \left( P_{ft}^i \right)^{1 - \varepsilon_t^i} df \right]^{\frac{1}{1 - \varepsilon_t^i}}, \qquad i = \{n, d\}.$$
 (10)

The production technology of a generic firm f in sector i employs input materials produced by both sectors in the economy, labor and oil:<sup>12</sup>

$$Y_{ft}^{i} = Z_{t}^{i} \left[ \frac{\left(M_{ft}^{ni}\right)^{\gamma_{ni}} \left(M_{ft}^{di}\right)^{\gamma_{di}}}{\gamma_{ni}^{\gamma_{ni}} \gamma_{di}^{\gamma_{di}}} \right]^{\alpha_{Mi}} \left(L_{ft}^{i}\right)^{\alpha_{Li}} \left(O_{ft}^{i}\right)^{\alpha_{Oi}}$$

$$(11)$$

$$\alpha_{Mi} + \alpha_{Li} + \alpha_{Oi} = 1, \qquad i = \{n, d\}$$

where  $Z_t^i$   $(i=\{n,d\})$  is a sector-specific productivity shock,  $L_{ft}^i$  denotes the number of hours worked in the  $f^{th}$  firm of sector i,  $M_{ft}^{ji}$   $(j=\{n,d\})$  denotes material inputs produced in sector j and supplied to firm f in sector i,  $O_{ft}^i$  is the amount of oil employed in the production of the same firm f. Moreover,  $\gamma_{ij}$   $(i,j=\{n,d\})$  denotes the generic element of the  $(2\times 2)$  input-output matrix and corresponds to the steady state share of total intermediate goods used in the production of sector j and supplied by sector i.<sup>13</sup>

Material inputs are combined according to a CES aggregator:

$$M_{ft}^{ji} = \left[ \int_0^1 \left( M_{kf,t}^{ji} \right)^{\left( \varepsilon_t^j - 1 \right) / \varepsilon_t^j} dk \right]^{\varepsilon_t^j / \left( \varepsilon_t^j - 1 \right)}, \tag{12}$$

where  $\left\{M_{kf,t}^{ji}\right\}_{k\in[0,1]}$  is a sequence of intermediate inputs produced in sector j by firm k, which are

<sup>&</sup>lt;sup>12</sup>The production technology does not feature physical capital. There are both practical and technical reasons for this choice. First, capital only accounts for 16% of the total cost of production, followed by labor (34%) and intermediate goods (50%) (source: Dale Jorgenson's data on input expenditures by US industries). Second, from a normative viewpoint Woodford (2003) and Schmitt-Grohe and Uribe (2007) show that the key policy prescriptions for the New Keynesian framework are not affected by the presence of capital in the production technology. Third, as it will be clear from Section 4, in the present setting we are able to report a compact welfare criterion that allows for a clear understanding of the stabilization trade-offs involved by the two-sector economy. By contrast, in the presence of capital accumulation, the welfare loss would not retain the same properties (see Edge, 2003 for the derivation of a utility-based welfare function in a model with endogenous capital accumulation). Fourth, in connection with the specific two-sector framework we examine, the transmission channel embodied by sectoral production linkages is not qualitatively affected by the presence of physical capital, as it has been discussed by Sudo (2012). All in all, including capital in the production technology would not alter the key transmission mechanisms of this framework, while rendering the normative analysis more convoluted.

<sup>&</sup>lt;sup>13</sup>The input-output matrix is normalized, so that the elements of each column sum up to one:  $\sum_{j=\{n,d\}} \gamma_{jn} = 1$  (and  $\sum_{j=\{n,d\}} \gamma_{jd} = 1$ ).

employed in the production process of firm f in sector i.

Firms in both sectors set prices given the demand functions reported in (9). They are also assumed to be able to adjust their price with probability  $1 - \theta_i$  in each period. When they are able to do so, they set the price that maximizes expected profits:

$$\max_{P_{ft}^{i}} E_{t} \sum_{s=0}^{\infty} (\beta \theta_{i})^{s} \Omega_{t+s} \left[ P_{ft+s}^{i} (1+\tau_{i}) - M C_{ft+s}^{i} \right] Y_{ft+s}^{i}, \qquad i = \{n, d\}$$
(13)

where  $\Omega_{t+s}$  is the stochastic discount factor consistent with households' maximizing behavior,  $\tau_i$  is a subsidy to producers in sector i, while  $MC_{fs}^i$  denotes the marginal cost of production of firm f in sector i. The optimal pricing choice, given the sequence  $\{P_t^n, P_t^d, Y_t^n, Y_t^d\}$ , reads as:

$$\overline{P}_{ft}^{i} = \frac{\varepsilon_t^{i}}{\left(\varepsilon_t^{i} - 1\right)\left(1 + \tau_i\right)} \frac{E_t \sum_{s=0}^{\infty} (\beta \theta_i)^s \Omega_{t+s} M C_{ft+s}^{i} Y_{ft+s}^{i}}{E_t \sum_{s=0}^{\infty} (\beta \theta_i)^s \Omega_{t+s} Y_{ft+s}^{i}}, \qquad i = \{n, d\}.$$

$$(14)$$

Note that assuming time-varying elasticities of substitution translates into sectoral cost-push shocks that allow us to account for sector-specific shift parameters in the supply schedules.

In every period each firm solves a cost minimization problem to meet demand at its stated price.

The first order conditions from this problem result in the following relationships:

$$MC_{ft}^{i}Y_{ft}^{i} = \frac{W_{t}^{i}L_{ft}^{i}}{\alpha_{Li}} = \frac{S_{t}O_{ft}^{i}}{\alpha_{Oi}} = \frac{P_{t}^{n}M_{ft}^{ni}}{\alpha_{Mi}\gamma_{ni}} = \frac{P_{t}^{d}M_{ft}^{di}}{\alpha_{Mi}\gamma_{di}}, \qquad i = \{n, d\}.$$
(15)

where  $S_t$  is the nominal price of imported oil in domestic currency. It is useful to express the sectoral real marginal cost as a function of the relative price and the real wage prevailing in each sector  $i, j = \{n, d\}, i \neq j$ :

$$\frac{MC_t^i}{P_t^i} = \frac{\left(Q_t^i\right)^{\gamma_{ji}\alpha_{Mi}} \left(RW_t^i\right)^{\alpha_{Li}} \left(S_t^i\right)^{\alpha_{Oi}}}{\alpha_{Mi}^{\alpha_{Mi}} \alpha_{Li}^{\alpha_{Ci}} \alpha_{Oi}^{\alpha_{Oi}} Z_t^i},\tag{16}$$

where  $S_t^i = S_t/P_t^i$  denotes the price of oil relative to that of goods produced in sector i,  $RW_t^i = W_t^i/P_t^i$  is the real wage in sector i and  $Q_t^i$  denotes the price of sector i relative to that of sector j. Since  $Q_t^n = (Q_t^d)^{-1}$ , in what follows we normalize so as to have a single relative price  $Q_t = P_t^n/P_t^d$ . Equation (16) makes it clear that the relative price exerts a direct effect on the real marginal cost of each sector, whose magnitude depends on the size of the cross-industry flows of input materials. Specifically, for

the i<sup>th</sup> sector the absolute impact of  $Q_t$  on  $MC_t^i/P_t^i$  is related to the importance of the other sector as input supplier, i.e. on the magnitude of the off-diagonal elements in the input-output matrix ( $\gamma_{nd}$  and  $\gamma_{dn}$ ). This is a distinctive feature of the framework we deal with, one that has crucial implications for monetary policy-making. By contrast, in traditional multi-sector models without factor demand linkages the relative price primarily affects the extent to which consumers substitute durables for non-durables.

#### 2.3 Market Clearing

The production allocation at the sectoral level is such that

$$P_t^i Y_t^i = P_t^i \left( C_t^i + M_t^{in} + M_t^{id} \right) + S_t O_t^i, \qquad i = \{n, d\}$$
(17)

Equation (17) implies that sectoral gross production in nominal terms includes the value of consumption and intermediate goods employed in either sector, as well as the value of imported oil.<sup>14</sup> Moreover, oil market clearing requires that the sum of oil imported by both sectors equals its supply:

$$O_t^n + O_t^d = O_t,$$

where  $O_t$  is exogenous<sup>15</sup> and described by an autoregressive process of order one (see Bodenstein *et al.*, 2008).

#### 2.4 The Government and the Monetary Authority

The government serves two purposes in the economy. First, it taxes households and provides subsidies to firms, so as to eliminate distortions arising from monopolistic competition in the markets for both classes of consumption goods. This task is pursued via lump-sum taxes that maintain a balanced fiscal budget. Second, the government delegates monetary policy to an independent central bank. In this respect, we initially assume that the short-term nominal interest rate is used as the instrument of monetary policy and the policy maker is able to pre-commit to a time-invariant rule. We then explore the properties of interest rate rules whose reaction coefficients to output and inflation are

<sup>&</sup>lt;sup>14</sup>Implicitly, part of the domestic production is exported so as to cover the cost of the imported oil (i.e.,  $P_t^i E_t^i = S_t O_t^i$ , where  $E_t^i$  denotes the exports of sector i). This structure is similar to the one envisaged by Blanchard and Galí (2007), though they also allow for a portion of imported oil to be used for consumption purposes.

<sup>&</sup>lt;sup>15</sup>As a result, the oil price is determined by the intersection between a vertical supply schedule and an endogenous demand. This choice reflects a large consensus on the determinants of fluctuations in the price of oil. In fact, since Kilian (2009), the empirical literature has highlighted that oil supply shocks only account for a small fraction of the fluctuations in oil prices.

computed so as to minimize a quadratic welfare function consistent with consumers' utility, as well as the implications of delegating monetary policy to a central banker with different preferences for output and inflation stabilization, as compared with the general public.

## 3 Solution and Calibration

To solve the model, we log-linearize behavioral equations and resource constraints around the non-stochastic steady state and take the percentage deviation from their counterparts under flexible prices. The difference between log-variables under sticky prices and their linearized steady state is denoted by the symbol "^", while we use "\*" to denote percent deviations of variables in the efficient equilibrium (i.e., flexible prices and constant elasticities of substitution) from the corresponding steady state value. Finally, we use "~" to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.<sup>16</sup>

The model is calibrated at a quarterly frequency. We set  $\beta = 0.993$  and  $\sigma = 2$ . The inverse of the Frisch elasticity of labor supply, v, is set to 3, while  $\lambda = 1$ , so as to account for limited labor mobility between sectors. As for the parameters characterizing the production technologies and the consumption expenditure shares, we rely on Bouakez, Cardia, and Ruge-Murcia (2011) and Sudo (2012). The expenditure share on non-durable goods,  $\mu_n$ , is set to 0.682. As for the income share of input materials in the two sectors, we set  $\alpha_{Mn} = 0.746$  and  $\alpha_{Md} = 0.581$ . In line with Bodenstein, Erceg, and Guerrieri (2008), we set the income share of oil at 2% in both sectors. The entries of the input-output matrix are set in line with the input-use table of the US economy for the year 2005:  $\gamma_{nn} = 0.887$  and  $\gamma_{nd} = 0.490$ . These values imply a positive net flow of input materials from the non-durable goods sector to the durable goods sector. The depreciation rate of the stock of durables is assumed to be 2.5\%, while  $\Xi = 600$ , as in Erceg and Levin (2006). We assume that sectoral elasticities of substitution have a steady state value equal to 11. At different stages of the analysis we allow for both symmetric and asymmetric degrees of nominal rigidity across sectors. In the symmetric case we set  $\theta_n = \theta_d = 0.75$  (i.e., an average duration of four quarters). In the case of asymmetric price stickiness we set  $\theta_n = 0.75$  and  $\theta_d = 0.25$  (i.e., an average duration of 4 months). These values imply that durable prices are relatively more flexible, as suggested by Bils and Klenow (2004a) and Nakamura and Steinsson (2008b). This view is also supported by Bouakez, Cardia, and Ruge-Murcia (2009), who construct and estimate a six-sector DSGE model of the US economy, reporting that

<sup>&</sup>lt;sup>16</sup>The steady state conditions are reported in Appendix B. We omit the time subscript to denote variables in the steady state. Appendix C presents the economy under flexible prices.

<sup>&</sup>lt;sup>17</sup>This value is based on value-added data at the industry level (mining and utilities), as well as data for imports of oil, gas, coal and electricity. In reality, there are some minor differences in the sectoral usage of oil for production purposes. However, the our qualitative results are virtually unaffected by allowing for mild heterogeneity across sectors.

the null hypothesis of price flexibility cannot be rejected for durable manufacturing, construction, agriculture and mining.

As discussed above, the linearized system features two sector-specific technology shocks,  $z_t^n$  and  $z_t^d$ . The cost-push shocks,  $\eta_t^n$  and  $\eta_t^d$ , are reduced-form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves. As for the supply of oil, this is denoted by  $o_t$ . Exogenous variables are assumed to follow a first-order stationary VAR with iid innovations and, unless we state otherwise, diagonal covariance matrix. We set the parameters capturing the persistence and variance of the productivity growth stochastic processes so that  $\rho^{z^n} = \rho^{z^d} = 0.95$  and  $\sigma^{z^n} = \sigma^{z^d} = 0.01$ , respectively. These values are consistent with the empirical evidence showing that technology shocks are generally small, but highly persistent (see Cooley and Prescott, 1995; Huang and Liu, 2005). As for the cost-push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2009), assuming that these are purely transitory, with  $\sigma^{\eta^n} = \sigma^{\eta^d} = 0.02$ . Finally, we impose  $\rho^o = 0.8$  and  $\sigma^o = 0.02$  in the AR(1) process for the oil shock, in line with the transitory component of oil supply in Bodenstein, Erceg, and Guerrieri (2008). This calibration implies that: i) on impact, oil supply shocks account for roughly about 9% of the forecast error variance of the real oil price; ii) oil prices are substantially more volatile than inflation.<sup>18</sup>

# 4 Monetary Policy

In the present context the central bank cannot attain the Pareto optimal allocation consistent with the full stabilization of inflation and the output gap in both sectors, even when distortions in the labor market (i.e., imperfect labor mobility) and the goods market (i.e., monopolistic competition) are removed. Thus, we turn our attention to policy strategies capable of attaining second best outcomes. We first explore equilibrium dynamics under the assumption that the policy maker can credibly commit to a rule derived from the minimization of a utility-based welfare loss function. The optimal policy consists of maximizing the conditional expectation of intertemporal household utility, subject to private sector's behavioral equations and resource constraints. A 'timeless perspective' approach is pursued (Woodford, 1999, 2003). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past. <sup>19</sup> We then consider interest rate rules whose reaction

<sup>&</sup>lt;sup>18</sup>The first point is broadly consistent with the results reported by Juvenal and Petrella (2015). As for the second point, taking quarterly data from 1986 onwards, the standard deviation of the (HP-filtered) real oil price is about 20 times more volatile than inflation (computed from the GDP deflator). For the same series, in the model we get a factor of 17.

<sup>&</sup>lt;sup>19</sup>We focus on timeless perspective policy-making, as in Petrella, Rossi, and Santoro (2014) we show this policy regime can hardly be outperform by discretion when using a model-consistent welfare criterion.

coefficients to aggregate activity and alternative measures of overall price inflation are computed so as to minimize the loss of social welfare.

#### 4.1 The Welfare Criterion

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.<sup>20</sup> Our procedure follows the standard analysis of Woodford (2003), adapted to account for the different sources of sectoral heterogeneity captured by our model. The resulting intertemporal social loss function reads as:

$$SW_{0} \approx -\frac{U_{H}(H)H}{2}\Theta E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\frac{\sigma-1}{\Theta}\left(\mu_{n}\widetilde{c}_{t}^{n}+\mu_{d}\widetilde{d}_{t}\right)^{2}\right.$$

$$\left.+\varsigma\left[\varpi\left(\pi_{t}^{n}\right)^{2}+\left(1-\varpi\right)\left(\pi_{t}^{d}\right)^{2}\right]+\left(1+v\right)\left[\omega\widetilde{c}_{t}^{n}+\left(1-\omega\right)\widetilde{c}_{t}^{d}\right]^{2}\right.$$

$$\left.+S\left(\widetilde{d}_{t}-\widetilde{d}_{t-1}\right)^{2}\right\}+\text{t.i.p.}+O\left(\|\xi\|^{3}\right),$$

$$(18)$$

where:

$$S \equiv \mu_d \Theta^{-1} \Xi + (1 - \delta) (1 - \omega) \delta^{-2}, \tag{19}$$

$$\Theta \equiv \frac{\mu_n \left[1 - \beta \left(1 - \delta\right)\right] + \mu_d \delta}{1 - \beta \left(1 - \delta\right)},\tag{20}$$

$$\varpi \equiv \phi \varepsilon^n \left( \kappa_n \varsigma \right)^{-1}, \tag{21}$$

$$\omega \equiv \frac{\mu_n \left[ 1 - \beta \left( 1 - \delta \right) \right]}{\mu_n \left[ 1 - \beta \left( 1 - \delta \right) \right] + \mu_d \delta},\tag{22}$$

$$\varsigma \equiv \phi \varepsilon^n \left(\kappa_n\right)^{-1} + \left(1 - \phi\right) \varepsilon^d \left(\kappa_d\right)^{-1},\tag{23}$$

$$\kappa_i \equiv \frac{(1 - \beta \theta_i) (1 - \theta_i)}{\theta_i}, \ i = \{n, d\}, \tag{24}$$

<sup>&</sup>lt;sup>20</sup>We assume that the shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions. Appendix F reports the derivation of the quadratic welfare function.

and t.i.p. collects the terms independent of policy stabilization, whereas  $O(\|\xi\|^3)$  summarizes all terms of third order or higher.

The welfare criterion (18) balances, along with fluctuations in aggregate consumption (or, equivalently, value added), sectoral inflation variability and a term that reflects a preference to smooth the accumulation of the stock of durable goods.<sup>21</sup> It is worth noting that – under specific assumptions about the model parameters – the welfare criterion nests popular specifications in the New Keynesian literature. When labor is the only production input (i.e.,  $\alpha_{Ln} = \alpha_{Ld} = 1$ ) the loss function reduces to that obtained in traditional two-sector models where consumption and gross output are equalized. Furthermore, setting  $\delta = 1$  and  $\Xi = 0$  returns the case considered by Woodford (2003, pp. 435-443).<sup>22</sup>

The weights of the time-varying terms in (18) can be interpreted as follows: (i)  $\varsigma$  indexes the total degree of nominal stickiness in the economy and it is inversely related to both  $\kappa_d$  and  $\kappa_n$ ; (ii)  $\varpi$  accounts for the relative degree of price stickiness in the non-durable goods sector; (iii)  $\omega$  is the relative weight of non-durable consumption over total consumption when durable goods are reported as a flow. This is an inverse function of  $\Theta$ . In turn, the latter depends on the degree of durability of goods produced in sector d. For  $\delta = 0$  it reduces to  $\mu_n$ , whereas for  $\delta = 1$  it equals one. Therefore, as the degree of durability increases, the weight attached to the non-durable consumption gap increases with respect to that attached to the durable term. Notice also that the relative importance of sectorspecific inflation variability depends on the steady-state ratio of labor supplied to the non-durable goods sector to the total labor force  $(\phi)$ . To parse the quantitative impact of sectoral heterogeneity on the policy targets, Table 1 reports the weights of different stabilization objectives in (18), under alternative settings. As we shall see in the next section, factor demand linkages have major effects on the transmission of shocks to the economy. However, their impact on the welfare criterion appears limited. Analogous considerations apply to consumption goods durability. By contrast, imposing different degrees of sectoral price rigidity has considerable influence on the parameterization of the welfare metric.

#### Insert Table 1 here

<sup>&</sup>lt;sup>21</sup> Further details on the linear approximation of this term are available in the technical appendix. Assuming durables accumulation smoothing as a stabilization objective should help at counteracting the amplification effect of changes in the stock demand of durables on the flow demand of newly produced durable goods. However, as discussed by Erceg and Levin (2006) this term makes a relatively minor contribution to the overall loss. To see this, consider that  $S\Delta \tilde{d}_t^2 \simeq \left[\mu_d \Theta^{-1} \Xi \delta^2 + (1-\delta) (1-\omega)\right] \left(\tilde{c}_t^d\right)^2$ . As a matter of fact, the first term in the square brackets is relatively small, as even large values of  $\Xi$  are inevitably counteracted by the factor  $\mu_d \Theta^{-1} \delta^2$ .

<sup>&</sup>lt;sup>22</sup>Needless to say, eliminating structural asymmetries between sectors and assuming perfectly correlated shocks would render the two-sector model observationally equivalent to the standard (one sector) New Keynesian model. In this case the loss function would reduce to the familiar benchmark  $\mathcal{SW}_0 \approx -\frac{U_H(H)H}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[(\sigma+v)\tilde{y}_t^2+\varepsilon(\kappa)^{-1}\pi_t^2\right]+\text{t.i.p.}+O\left(\|\xi\|^3\right)$ , where sectoral superscripts have been removed to denote the aggregate variables and the economy-wide coefficients.

#### 4.2 Heterogeneity and Sectoral Stabilization

As the model accounts for several frictions and distortions (e.g., relative price distortions, price-setting frictions, imperfect labor mobility, inter-sectoral input-output interactions), it is important to examine how each of them affects sectoral stabilization under the optimal monetary policy. To this end, Figure 1 reports the loss of welfare under alternative benchmark models, as a function of the degree of labor mobility.<sup>23</sup> Specifically, the upper panels of Figure 1 consider a situation with symmetric price stickiness, while the remaining panels account for the presence of relatively more flexible prices for durables.

Petrella and Santoro (2011) have shown that even under perfect labor mobility (i.e.,  $\lambda \to \infty$ ) the central bank may not attain the Pareto optimal allocation consistent with the full stabilization of both sectors (i.e., manufacturing and services), unless the technology shock buffeting one sector equals the other one, scaled by the ratio between the sectoral income shares of input materials. This condition is invariant to the presence of goods with different durability. However, no full stabilization can be attained in the present setting, unless a rather restrictive set of assumptions are made, as stated by *Proposition 1*.

**Proposition 1** In the model with sticky prices and perfect labor mobility across sectors, there exists no monetary policy that can attain the Pareto optimal allocation unless the following condition is met:<sup>24</sup>

$$\frac{z_t^d - \alpha_{Od} s_t^{*d}}{z_t^n - \alpha_{On} s_t^{*n}} = \frac{\alpha_{Ld}}{\alpha_{Ln}}.$$
(25)

Proof. See Appendix C.

#### 4.2.1 Perfectly Correlated Sectoral Shocks

The left-hand panels of Figure 1 consider situations with perfectly correlated sectoral shocks. As predicted by *Proposition 1* a symmetric production structure always ensures full stabilization under perfect labor mobility, even in the presence of durability. Moreover, full stabilization is attainable even at low values of  $\lambda$  in the presence of no durables (i.e.,  $\delta = 1$ ). Otherwise, durability amplifies the loss of welfare in the presence of limited labor mobility, even if the relative price remains at its steady state level by virtue of  $z_t^n = z_t^d$ ,  $\forall t.^{25}$  It should be noted that, under limited labor mobility, the loss

 $<sup>^{23}</sup>$ We temporarily rule out the oil shock and the sectoral cost-push shocks.

<sup>&</sup>lt;sup>24</sup> Allowing for imperfect labor mobility would only constrain further the ability of the monetary authority to neutralize exogenous perturbations.

<sup>&</sup>lt;sup>25</sup>This situation is close to that considered by Erceg and Levin (2006), as they assume partially correlated sectoral innovations.

of welfare is attenuated when input materials are employed by symmetric production technologies, as compared with the case of  $\alpha_{Mn} = \alpha_{Md} = 0$ . The mitigation induced by intermediate goods on the trade-off is more evident at low values of  $\lambda$ : the underlying intuition is that in the model with input materials firms have the chance to adjust the mix of their production inputs even if labor cannot move across sectors. This option is a priori ruled out when input materials are not employed in the production process or they feature asymmetric income shares, in which cases a higher loss of social welfare obtains.

#### Insert Figure 1 here

Finally, we should note that the absolute value of the loss is consistently lower in the presence of more flexible prices for durables, as we should expect on *a priori* grounds. As we shall see in Section 4.2.2, this property carries over to the case of uncorrelated shocks.

#### 4.2.2 Uncorrelated Sectoral Shocks

We now focus on the right-hand panels of Figure 1, where we have assumed uncorrelated technology disturbances. As we know from *Proposition 1* full stabilization can never be attained in this case, even if the two sectors feature the same production structure and labor can move between sectors so as to offset discrepancies between the nominal wage rates. Nevertheless it is worth noting that, while higher labor mobility tends to offset the welfare discrepancies among different model economies under symmetric frequencies of price-setting, asymmetric degrees of price rigidity induce quantitatively important welfare gaps that seem to be insulated from changes in  $\lambda$ . In fact, asymmetric price stickiness exacerbates the relative price distortion, and more so when we introduce factor demand linkages in the two-sector economy with both durables and non-durables. Furthermore, in the baseline calibration with  $\alpha_{Mn} \neq \alpha_{Md}$  the loss of welfare is substantially higher, as compared with the alternative scenarios.<sup>26</sup>

In this setting durability tends to attenuate the loss of welfare, no matter whether input materials are employed in the same proportion across different sectors or they are excluded from the set of production inputs. To provide some intuition on this result we assume, without loss of generality,  $\Xi = 0$  and re-write the Euler equation for durable consumption in a more compact form by applying

<sup>&</sup>lt;sup>26</sup>In fact, asymmetric production technologies always imply the highest loss of welfare, regardless of the correlation structure between sectoral shocks.

repeated forward substitution:

$$\frac{U_{C_t^n}}{Q_t} = \sum_{j=0}^{\infty} (1 - \delta)^j \beta^j E_t \left[ U_{D_{t+j}} \right], \tag{26}$$

where  $U_{C_t^n}$  ( $U_{D_t}$ ) is the marginal utility with respect to non-durable (durable) consumption. Barsky, House, and Kimball (2007) note that in the case of durables with low depreciation rates, the right-hand side of (26) is heavily influenced by the marginal utilities of durable service flows in the distant future. When shocks hitting the economy are temporary, the forward-looking terms do not deviate from their steady-state values, and so even significant variation in the first few terms only have a small impact on the present value. This means that the present value is close-to-invariant, even in the face of substantial temporary movements in  $U_{D_t}$ .<sup>27</sup> Given that the right hand-side of (26) remains fairly constant, any variation in the relative price instantly impacts on the marginal utility of non-durable consumption: this is exactly what happens under  $\delta = 0.025$ . Assuming uncorrelated technology disturbances induces substantial volatility in  $Q_t$ , so that  $U_{C_t^n}$  fluctuates accordingly, especially in the case of asymmetric price stickiness. Thus, from a policy-making standpoint stabilizing the relative price is equivalent to stabilizing non-durable expenditure and vice versa. Otherwise, for  $\delta \to 1$  the stock-flow ratio for durables increases and both  $U_{C_t^n}$  and  $U_{D_t}$  vary in the face of movements in  $Q_t$ , so that it is impossible for the policy maker to jointly stabilize the marginal utilities of different consumption goods and their relative price.<sup>28</sup>

#### 4.3 Impulse-response Analysis

According to Erceg and Levin (2006) the policy maker faces a particularly severe sectoral stabilization trade-off in the presence of durables, even under the full commitment optimal policy. In light of the evidence presented so far, embedding sectoral linkages in a two-sector economy with durable and non-durable goods has non-negligible normative implications. Therefore, it seems important to isolate the contribution of factor demand linkages to the transmission of shocks under the optimal policy. To this end, we compare our baseline setting with a model that neglects the presence of input materials. Figure 2 reports equilibrium dynamics following a one-standard-deviation technology shock in the non-durable goods sector, under different assumptions about the production structure.<sup>29</sup> All variables

<sup>&</sup>lt;sup>27</sup>This approximation is equivalent to saying that the demand for durable goods displays an almost infinite elasticity of intertemporal substitution. Even a small drop in the relative price of durables today relative to tomorrow would cause people to delay their purchases.

<sup>&</sup>lt;sup>28</sup>Otherwise, the relative price gap can always be closed in the presence of perfectly correlated sectoral shocks and no durability.

<sup>&</sup>lt;sup>29</sup>The responses to sectoral innovations in the durable goods sector are reported in Appendix G.

but the interest rate are reported in percentage deviation from their frictionless level. Furthermore, symmetric nominal rigidity is assumed, with  $\theta_n = \theta_d = 0.75.^{30}$ 

#### Insert Figure 2 here

A positive technology shock in the non-durable goods sector renders the production of these goods relatively cheaper. However, under sticky prices  $P_t^n$  is prevented from reaching the level consistent with prices being set in a frictionless fashion. This determines a drop in the consumption gap of nondurables. Also  $Q_t$  lies above the level that would prevail under flexible prices, so that the consumption gap of durables increases. Therefore, a sector-specific technology shock faces the policy maker with the problem of stabilizing diverging consumption gaps in the two sectors. Faced with divergent responses of the production gaps in the two sectors, Erceg and Levin (2006) suggest that keeping the consumption of non-durables at potential requires a "sharp and persistent fall" in the real interest rate. By contrast, a sharp rise in the policy instrument is required to close the consumption gap of durable goods. As a result, the nominal rate of interest initially rises to stabilize durables expenditure, and gradually declines to accommodate the stabilization of non-durable expenditure. In our model with input materials monetary policy-making features the same type of response, though both the initial contraction and the subsequent expansion are wider, as compared with the benchmark economy with no sectoral linkages. In fact, factor demand linkages exacerbate the inter-sectoral stabilization tradeoff that otherwise operates under the traditional demand channel, as indicated by higher reactiveness of the relative price gap. In turn, the cross-industry flow of input materials amplifies the response of non-durable consumption under flexible prices, thus inducing a greater drop in their consumption gap. At the same time, sectoral linkages are responsible for amplifying the response of the durable consumption gap.

#### Insert Figure 3 here

Figure 3 reports equilibrium dynamics following a cost-push shock in the non-durables sector. A distinctive feature of the model with factor demand linkages is that a rise in the relative price of non-durables counteracts the deflationary effect in the durable sector that otherwise operates through the conventional demand channel. Concurrently, factor demand linkages are responsible for inducing a contraction of both the durable and non-durable production gap, while in the benchmark model the sectoral production gaps display negative co-movement. In light of this, in the presence of sectoral linkages the central bank pursues a (weakly) contractionary policy, initially accompanied by a negative

<sup>&</sup>lt;sup>30</sup>As in Strum (2009) we opt for this choice to prevent the central bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). In the next section we draw various policy implications in the presence of asymmetric degrees of sectoral price rigidity.

real rate of interest. Such a policy response is justified by the fact that changes in the relative price are channeled through the sectoral marginal costs and act as an endogenous attenuator of deflationary pressures in the sector that is not directly hit by the cost-shifter (see (16)).

#### Insert Figure 4 here

Figure 4 graphs the effects of a drop in the supply of oil. We set the shock so as to induce a one percentage point increase in the oil price (on impact). As in the face of a sector-specific technology shock, the policy maker is faced with the problem of stabilizing diverging sectoral consumption and production gaps. In fact, equilibrium dynamics in this case is close to the one we would observe in the face of a negative technology shock in the non-durable sector, with the central bank opting for a particularly restrictive policy stance in the model with factor demand linkages. It is also important to note that combining sectoral inflations based on the size of each sector returns an index of general price inflation that is one order of magnitude less volatile than oil price changes. This factor is explicitly taken into account in the next section, where we build a measure of domestically generated inflation that excludes the price of oil from a measure of general price level, so as avoid that the monetary policy stance accounts for price movements that are ever too volatile.

# 5 Alternative Monetary Policy Regimes

This section aims at examining the performance of alternative policy regimes with respect to policy-making under timeless perspective. We first examine a family of interest rate rules that react to aggregate measures of real activity and inflation. We then allow for the possibility that monetary policy is delegated to a central banker with different preferences for output and inflation stabilization, as compared with the public, whose welfare criterion is represented by (18). In both cases, we emphasize the importance of accounting for sectoral heterogeneity and production linkages when aggregating the sectoral rates of inflation into a measure of general price changes.

#### 5.1 Interest Rate Rules

We turn our attention to a family of simple monetary policy rules akin to those examined by Taylor (1993), Giannoni and Woodford (2003), Schmitt-Grohe and Uribe (2007) and Leith, Moldovan, and Rossi (2012). A number of empirical contributions (see, among others, Taylor, 1993, Clarida *et al.*, 2000, Lubik and Schorfeide, 2004) have shown that these policy functions capture, *prima facie*, the behavior of various central banks in the OECD countries. The aim of this section is to evaluate how

these rules may mimic the optimal policy, while abstracting from its stringent informational requirements.<sup>31</sup> As such, our analysis provides us with a direct understanding of how sectoral heterogeneity affects the implementability of simple instrument rules that target aggregate measures of real activity and inflation. In this respect, finding the most appropriate inflation rate to target is an issue of concern when dealing with sectoral heterogeneity (see, e.g., Huang and Liu, 2005). We will consider three alternative measures: (i) aggregate inflation, according to which the sectoral inflation rates are aggregated depending on the relative size of each sector; (ii) sticky-price inflation, which weighs the sectoral rates depending on both the relative size and the relative degree of rigidity in price-setting of each sector; (iii) domestically generated inflation, which excludes fluctuations in the price of oil – the only imported good in the economy – from changes in the general price level. The first definition may be regarded as a model-consistent counterpart of inflation as measured by the growth rate of the output deflator.<sup>32</sup> As for sticky-price inflation, this is conceived to capture factors that affect relative price changes, whose importance for sectoral stabilization has been recognized by a number of authors (see, e.g., Aoki, 2001; Woodford, 2003; Eusepi et al., 2011). Finally, domestically generated inflation allows the central bank to isolate the behavior of general prices without distraction from spikes in volatile oil prices.<sup>33</sup>

We examine the welfare properties of a rule that responds to contemporaneous measures of inflation and output gap, as well as a rule characterized by the possibility of adjusting the policy rate with some gradualism:

$$i_t = \phi_\pi \pi_t^i + \phi_u \widetilde{y}_t, \tag{27}$$

$$i_t = \rho i_{t-1} + (1 - \rho) \left( \phi_\pi \pi_t^i + \phi_y \widetilde{y}_t \right), \quad \pi_t^i = \left\{ \pi_t^{agg}, \pi_t^{sticky}, \pi_t^{dg} \right\}$$
 (28)

where

$$\pi_t^{agg} = \varkappa \pi_t^n + (1 - \varkappa) \pi_t^d \tag{29}$$

$$\pi_t^{sticky} = \varpi \pi_t^n + (1 - \varpi) \pi_t^d \tag{30}$$

$$\pi_t^{dg} = \frac{\varkappa}{1 - \alpha_{On}} \left( \pi_t^n - \alpha_{On} \pi_t^O \right) + \frac{1 - \varkappa}{1 - \alpha_{Od}} \left( \pi_t^d - \alpha_{Od} \pi_t^O \right)$$
(31)

<sup>&</sup>lt;sup>31</sup>Erceg and Levin (2006) follow an analogous line of reasoning, studying the stabilization properties of targeting rules that, despite the fact the welfare criterion involves sector-specific variables, do not consider sector-specific output gaps and inflation rates.

<sup>&</sup>lt;sup>32</sup>It is important to stress that, using the model-consistent definition of CPI inflation (i.e.,  $\pi_t^{CPI} = \mu_n \pi_t^n + \mu_d \pi_t^d$ ), would produce very similar results to those obtained with the output deflator.

<sup>&</sup>lt;sup>33</sup>In practice, this is generally accomplished by removing oil price dynamics from CPI/PCE inflation measures. In our model oil only enters as a production input. Therefore, we opt for the concept of domestically generated inflation (see, e.g., Buiter, 1998), which allows us to remove oil prices from an overall index of price changes.

where  $\varkappa \equiv Y^n/\left(Y^n + Y^d\right)$  and  $\pi_t^O$  denotes oil price inflation.<sup>34</sup> Note that we employ  $\varpi$  as the weight attached to the rate of inflation of non-durables, in line with the factor that balances the volatility of sectoral inflation dynamics in the welfare criterion (18). This convolution of parameters also depends on the sectoral elasticities of substitution, as in the cost-of-nominal-distortions index (CONDI) elaborated by Eusepi, Hobijn, and Tambalotti (2011). However, as both elasticities are calibrated at the same value, sectoral inflations are weighted according to the inverse of the slope of the sector-specific New Keynesian Phillips curve.<sup>35</sup>

We determine the reaction coefficients so that  $\rho \in [0,1]$ ,  $\phi_{\pi} \in [1,5]$  and  $\phi_{y} \in [0,5]$ .<sup>36</sup> Thus, we search for the best combination of the response coefficients to minimize the unconditional welfare measure in the decentralized equilibrium of the model economy. Table 2 reports these values, together with the difference between the loss under the optimized rules and that under the optimal policy benchmark. All losses are computed as a percentage of steady-state consumption.

#### Insert Table 2 here

Under a contemporaneous data rule  $\phi_{\pi}$  consistently hits the upper bound of its support.<sup>37</sup> Most notably, while in a model without input materials it is desirable to set  $\phi_y = 0$  when all shocks are at play,<sup>38</sup> in the economy under examination it is recommendable to adjust the rate of interest in response to movements in the output gap, no matter the upper bound we impose on inflation responses. This result is crucially driven by the interplay between durables and inter-sectoral production linkages. Compared with models where consumers' utility only depends on non-durable consumption, durable goods introduce additional volatility in the system. Moreover, factor demand linkages magnify the response of real activity to sectoral cost-shifters, as it has been shown in the previous section. Due to this additional source of volatility, the policy maker needs to adjust the policy rate in response to  $\tilde{y}_t$ . To explain why this is possible in the present setting, we need to start from the observation that increasing the income share of intermediate goods inevitably reduces the slope of the New Keynesian Phillips curve. This means that the pass-through from the real marginal cost to the rate of inflation is attenuated, relative to the scenario with no input materials. In fact, the real marginal cost of sector i is a homogeneous function of degree  $1 - \alpha_{Mi} (1 - \gamma_{ji}) < 1$ , with  $i, j = \{n, d\}$  and  $i \neq j$ . Such an

 $<sup>^{34}</sup>$  Appendix H reports the analytical steps to derive  $\pi_t^{dg}$  (see also Buiter, 1998).

<sup>&</sup>lt;sup>35</sup>In light of this property, the resulting sticky-price inflation measure is in line with the inflation rate consistent with the Sticky Price Index published by the Atlanta FED.

<sup>&</sup>lt;sup>36</sup>The range of variation for  $\phi_{\pi}$  and  $\phi_{y}$  is selected so as to retain the property of implementability for the selected policy rule, avoiding to allow for unreasonably high response coefficients to inflation and the output gap (see Schmitt-Grohe and Uribe, 2007).

<sup>&</sup>lt;sup>37</sup>This is the case no matter the support we consider for the response to the rate of inflation. In fact, it is possible to show that raising the upper bound of this value – even to implausibly high values – would marginally improve the loss of social welfare. Additional evidence on this point is available, upon request, from the authors.

<sup>&</sup>lt;sup>38</sup>This can be checked in Table I1, reported in Appendix I.

attenuation effect reflects the presence of strategic complementarities stemming from the input-output interactions that take place in each sector. As shown by Basu (1995), this type of interactions have the potential to turn small price-setting frictions into considerable degrees of real rigidity. From a policy viewpoint, this flattening of the sectoral supply schedules allows the central bank to adjust the policy rate in response to fluctuations in real activity, as variations in the demand of each sector produce lower effects on prices.

Compared with reacting to either aggregate or domestically generated inflation, targeting the sticky-price rate of inflation produces a higher loss of social welfare under both the contemporaneous data and the inertial rule. This result appears rather surprising, as the literature on optimal monetary policy has posed strong emphasis on the importance of accounting for the relative degree of price stickiness when aggregating sectoral prices.<sup>39</sup> Moreover, a number of central banks are paying increasing attention to developing measures of general price inflation that should account for different frequencies of sectoral price adjustment, so as to capture different aspects of the inflation process.<sup>40</sup> To dig deeper on this result, we perform an exercise aimed at understanding how the weights attached to the sectoral rates of inflation should optimally be assigned to minimize the loss of social welfare. To this end, we compute the loss of welfare under two rules: one in which  $\phi_y$  is set at the value consistent with aggregate inflation targeting, and one in which it equals the value under sticky-price inflation targeting.<sup>41</sup> In both cases we set  $\phi_{\pi}$  at its upper bound, while varying the weight attached to durable goods inflation,  $\omega_D$ , over the unit interval.

#### Insert Figure 5 here

According to Figure 5, under both rules excess loss is minimized at a value of  $\omega_D$  that is remarkably close to the level consistent with targeting aggregate inflation. The intuition for this result rests on the effect exerted by the price of a sectoral good on the cost of producing goods in the other sector. In fact, even if they are adjusted at shorter time-intervals, durable goods prices inherit considerable persistence from the price of non-durable goods through factor demand linkages. For analogous reasons, price stickiness in the non-durable goods sector is partly attenuated due to sectoral complementarities in price-setting. Therefore, cross-industry interactions in price-setting need to be carefully accounted for by the central bank when computing measures of (final goods) price inflation that should keep

<sup>&</sup>lt;sup>39</sup>In fact, Woodford (2003) shows that targeting sticky-price inflation produces a lower loss of welfare than targeting aggregate inflation in a single-sector model with technology shocks. Appendix I supports this prescription for a two-sector model economy without factor demand linkages.

<sup>&</sup>lt;sup>40</sup>For instance, Bryan and Meyer (2010) show that sticky prices appear to incorporate expectations about future inflation to a greater degree than prices that change on a frequent basis, while flexible prices respond more powerfully to economic conditions.

<sup>&</sup>lt;sup>41</sup>This exercise only considers the two measures of aggregate price changes that can be retrieved as a weighted averages of sectoral inflations.

track of changes in the relative price. In the specific setting under examination, the measure of sticky-price inflation that is consistent with the welfare-theoretic function only accounts for intrinsic inflation persistence, while disregarding inter-sectoral input-output interactions as a source of extrinsic persistence. As a result, even if the durable goods sector features a higher frequency of price-setting (see, e.g., Bils and Klenow, 2004b and Nakamura and Steinsson, 2008a), sticky-price inflation targeting attaches "too much" importance to non-durable goods inflation. This fact is indirectly confirmed by Bouakez, Cardia, and Ruge-Murcia (2014), who estimate sectoral Calvo parameters in line with the results of Nakamura and Steinsson (2008a), implying that available non-structural estimates only capture intrinsic inflation persistence at the sectoral level, while disregarding extrinsic persistence inherited through production costs and, in turn, input materials.

#### 5.2 Delegation

There are various reasons why monetary policy in practice might depart from the minimization of a rather convoluted welfare metric, such as (18). For instance, various parameters and components of the loss function might not be known with certainty and/or the public may opt for delegating monetary policy to an independent central banker, whose preferences for alternative stabilization objectives differ from those of the public (see, e.g., Rogoff, 1985). The advantages of a simple mandate are evident in terms of communication and accountability (see Nunes et al., 2015). Under these circumstances, the policy rule would depend on the objective function effectively faced by the policy maker. The next step in the analysis aims at understanding which of the inflation rates (29)-(31) should be accounted for by a policy maker whose welfare criterion only weights fluctuations in the output gap against those in a given measure of general price inflation, according with the following welfare function:

$$L_t^D = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \pi_t^i \right)^2 + \varphi^y \left( \widetilde{y}_t \right)^2 \right]. \tag{32}$$

To this end, we assume that the weight attached to output stabilization,  $\varphi^y$ , may vary over a finite support, with the weight attached to the quadratic inflation term set to one. Concurrently, the loss of welfare welfare is evaluated through the model-consistent metric, (18).

#### Insert Figure 6 here

As indicated by Figure 6 attaching a small, yet non zero, weight to output stabilization achieves the lowest welfare loss. Furthermore, note that sticky-price inflation targeting dominates aggregate inflation targeting at  $\varphi^y \approx 0$ . This can be explained upon examining the weights attached to different stabilization objectives in (18): according to Table 1, the model-consistent welfare criterion induces the

central bank to pursue a strongly anti-inflationary policy, with the weights attached to the quadratic terms of sectoral inflations depending on the relative degree of price rigidity and no concern for sectoral inflation co-movement.<sup>42</sup> This feature is captured by the exercise portrayed in Figure 6: at low values of  $\varphi^y$  the loss of the delegated central banker comes closer to mimic (18) by considering a measure of sticky-price inflation, which attaches little or no weight to the cross-term involving sectoral inflations. By contrast, when the policy maker receives a dual mandate that stresses both inflation and output stabilization, weighting sectoral inflations based on the size of each sector is preferable, and more so at intermediate values of  $\varphi^y$ .<sup>43</sup> Once again, the intuition for this result depends on the 'effective' degree of sectoral price stickiness and how this is weighed by alternative indices of general price inflation. In fact, balancing sticky-price inflation volatility with output gap volatility does not allow the central banker to tackle the intra-sectoral trade-off in the durable goods. This is because, by virtue of  $\varphi^y > 0$ , the dual mandate implicitly attributes a sizeable weight to the stabilization of durable goods production, while durable goods inflation is close to be neglected, despite their cost of production inherits a certain degree of extrinsic persistence through the cost of input materials supplied by the non-durable goods sector.

Finally, it is worth noting that delegating monetary policy to a central banker concerned with stabilizing domestically generated inflation results into a higher loss of welfare, as compared with targeting either aggregate or sticky-price inflation. This is because the trade-off captured by a targeting rule involving domestically generated inflation and output volatility compares, de facto, a measure of inflation that is insulated from swings in the price of oil with a measure aggregate real activity that is instead affected by oil price dynamics. Not surprisingly, such a policy setting turns out to be suboptimal with respect to both alternatives.

#### 6 Conclusions

We have presented a two-sector New Keynesian economy that features sectoral heterogeneity along three main dimensions: price stickiness, consumption goods durability, inter-sectoral trade of input materials. Such distinctive traits of modern industrialized economies display non-trivial interactions that limit both inter-sectoral and intra-sectoral stabilization. In this context we examine the potential of some simple interest-rate feedback rules to mimic the optimal policy under timeless perspective commitment. A clear advantage of these policy functions is to abstract from the stringent informational

<sup>&</sup>lt;sup>42</sup>Table 1 highlights how the weight on the quadratic terms involving the sectoral rates of inflation is substantially larger than the overall weight attached to the quadratic terms involving real activity.

<sup>&</sup>lt;sup>43</sup>Not surprisingly, as  $\varphi^y$  increases and overcomes the weight attached to inflation variability, the losses associated with aggregate and sticky-price inflation tend to converge to the same level. Eventually, as  $\varphi^y \to \infty$  the losses converge to 1.67.

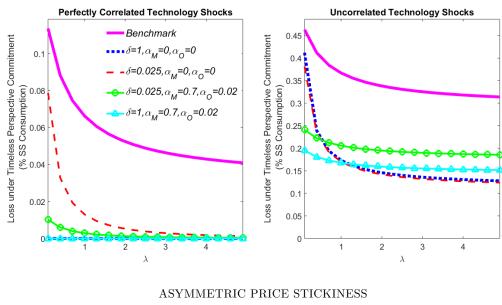
requirements implied by the welfare-theoretic loss function, which involves sector-specific variables. To this end, we assume that the policy rate is adjusted in response to aggregate measures of real activity and inflation. Asymmetric degrees of price rigidity and inter-sectoral production linkages do matter when it comes to aggregating sectoral inflation rates into an overall index of price inflation. Even if non-durable goods prices are assumed to be relatively stickier, durable goods inflation inherits considerable extrinsic persistence through the intermediate input channel.

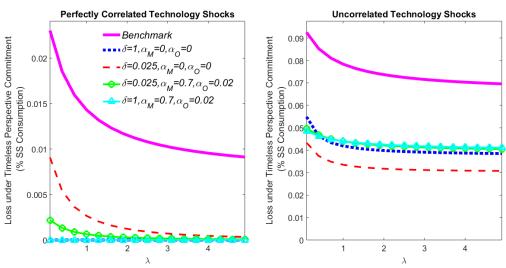
The present work carries a key implication for the computation of appriopriate measures of aggregate inflation. A policy maker that neglects sectoral production linkages may incur in a severe bias when trying to compute broad measures of inflation that should keep track of relative price changes, aggregating sectoral inflations based on the relative size of each sector and the relative degree of price rigidity. In fact, the available estimates of sectoral price rigidity only account for a measure of notional stickiness in price-setting, while disregarding inherited stickiness through production costs, which are in turn affected by the price of input materals.

# Figures and Tables

#### FIGURE 1: WELFARE LOSS UNDER VARYING DEGREES OF LABOR MOBILITY

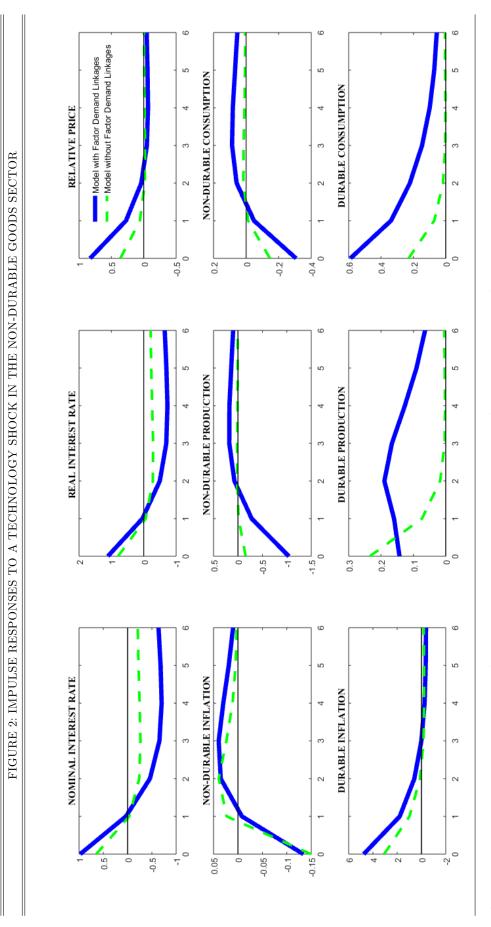
#### SYMMETRIC PRICE STICKINESS



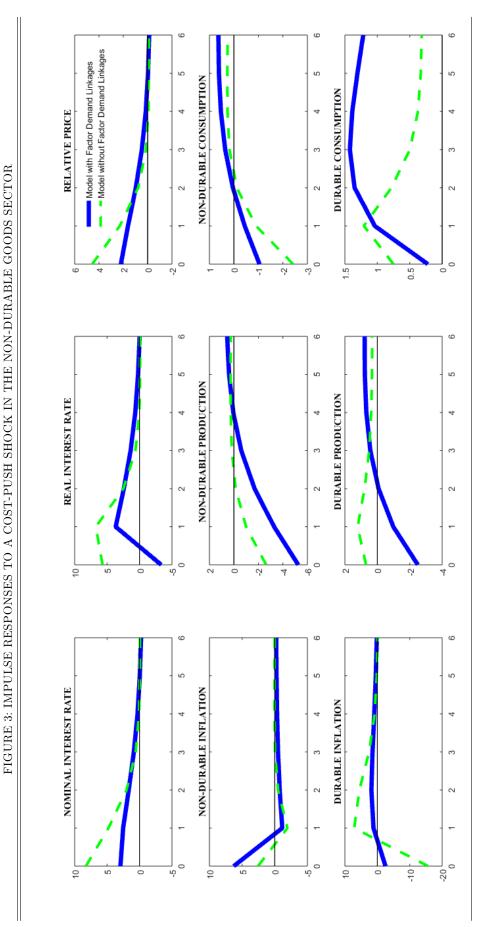


Notes: We report the loss of welfare under timeless perspective, computed as a percentage of steady state aggregate consumption (multiplied by 100) for various model economies and conditional on different shock configurations. The squared (blue) line refers to a model without input materials and two non-durable consumption goods; the dotted (green) line refers to a model with input materials employed symmetrically in the production technologies of the two sectors; the dashed (red) line refers to a model without input materials; the triangled (turquoise) line refers to a model with input materials employed symmetrically in the production technologies of the two sectors and two non-durable consumption goods; the continuous (purple) line refers to the baseline calibration. The left-hand panel reports the loss of welfare under perfectly correlated technology shocks, while in the right-hand panel we consider uncorrelated disturbances. In both cases we rule out cost-push shocks.

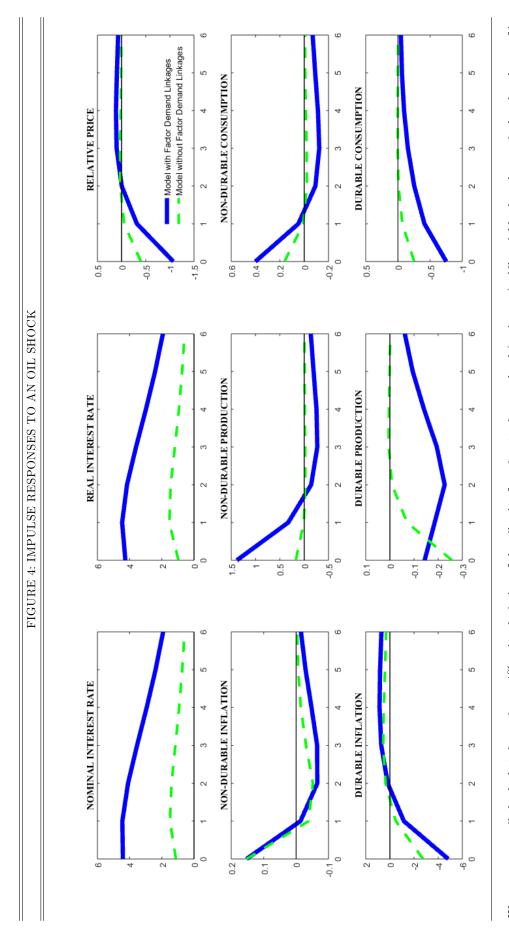
.



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

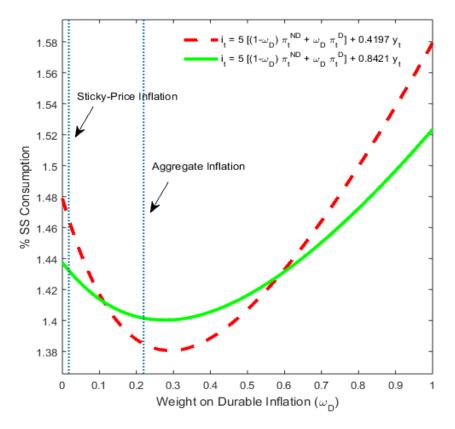


Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

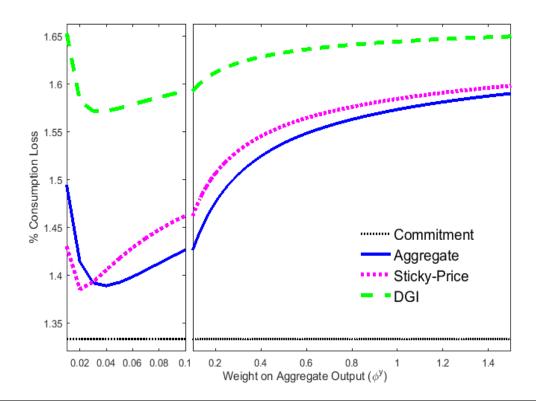


are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of Notes: We generate an oil shock that determines a 1% point deviation of the oil price from its steady-state level (on impact). All variables but the nominal and real rate of interest good are equivalent.

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Notes: Figure 5 shows the excess loss of welfare (relative to the policy under timeless perspective) under two different rules: one in which the response to the output gap is consistent with that under aggregate inflation targeting (red-dashed line), and one in which  $\phi_y$  equals the value consistent with targeting sticky-price inflation (green-continuous line). In both cases  $\phi_{\pi}$  is set at the upper bound, as indicated by the computation reported in Table 2, while the weight attached to the sectoral rates of inflation varies over the unit interval. The vertical (dotted) lines denote the weights we have used to compute domestically generated and aggregate inflation in the exercise reported in Table 2. All losses are expressed as a percentage of steady state consumption.



Notes: Figure 6 portrays the loss of welfare as a percentage of steady-state consumption, conditional on different values of the delegated central banker's preference for output stabilization. The weight attached to inflation stabilization is set to one. We consider three possible inflation targets: aggregate, sticky-price and domestically generated inflation.

TABLE 1: RELATIVE WEIGHTS IN THE LOSS FUNCTION							
Symmetric Price Stickiness							
Parameterization	S	Θ	$\overline{\omega}$	ω	ς		
$\delta = 1$ , no FDL	0	1	0.6594	0.6594	129.1475		
$\delta = 1$ , FDL	0	1	0.7366	0.6594	129.1475		
$\delta = 0.025$ , no FDL	667.934	0.924	0.7136	0.7136	129.1475		
$\delta = 0.025,  \mathrm{FDL}$	667.934	0.924	0.7630	0.7136	129.1475		

Asymmetric Price Stickiness						
Parameterization	S	Θ	$\varpi$	$\omega$	ς	
$\delta = 1$ , no FDL	0	1	0.9809	0.6594	86.8197	
$\delta = 1$ , FDL	0	1	0.9867	0.6594	96.4138	
$\delta = 0.025$ , no FDL	667.934	0.9240	0.9851	0.7136	93.5580	
$\delta = 0.025,  \mathrm{FDL}$	667.934	0.9240	0.9884	0.7136	99.6957	

Notes: Table 1 reports the weights associated with the stabilization objectives in the loss function (18), depending on different model economies nested in the framework under examination. In the case with asymmetric price stickiness the average duration of the price of non-durables set at 4 quarters, whereas we reduce the duration of durable prices to 1.3 quarters.

TABLE 2: INTEREST RATE RULES				
Contemporaneous Data Rule				
Inflation Targeting	$\phi_{\pi}$	$\phi_{y}$	$L^R$	
Aggregate	5	0.4197	0.0487	
Sticky-price	5	0.8421	0.0981	
DGI	5	0.5543	0.0479	

Inertial Rule						
Inflation Targeting	ρ	$\phi_{\pi}$	$\phi_y$	$L^R$		
Aggregate	0.5932	5	0.3650	0.0384		
Sticky-price	0.7999	4.9701	0.7718	0.0479		
DGI	0.5517	4.8532	0.4906	0.0398		

Notes: Table 2 reports – conditional on the realization of all shocks – the reaction coefficients under the contemporaneous data rule and the inertial rule. The parameters  $\rho$ ,  $\phi_{\pi}$  and  $\phi_{y}$  are computed so as to minimize the loss of social welfare (18). The table also reports  $L^{R}$ , which denotes the log-deviation of the loss under the optimal rule and the loss under timeless perspective. All losses are expressed as a percentage of steady state consumption. The average duration of the price of non-durables is set at 4 quarters, while durable prices are re-set every 1.3 quarters. The loss under timeless perspective equals 1.3326.

# APPENDIX A: First Order Conditions from Households' Utility Maximization

Maximizing (1) subject to (3), (4), (5), and (6) leads to a set of first-order conditions that can be re-arranged to obtain:

$$\mu_n H_t^{1-\sigma} (C_t^n)^{-1} = \beta R_t E_t \left[ \frac{\mu_n H_{t+1}^{1-\sigma} (C_{t+1}^n)^{-1}}{\Pi_{t+1}^n} \right], \tag{33a}$$

$$\frac{\mu_n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} = E_t \left\{ \beta \left( 1 - \delta \right) \mu_n \frac{H_{t+1}^{1-\sigma} P_{t+1}^d}{C_{t+1}^n P_{t+1}^n} + \right. \tag{33b}$$

$$+ \frac{\mu_{d}H_{t}^{1-\sigma}}{\mathfrak{D}_{t}\left[1 - \frac{\Xi}{D}\left(D_{t} - D_{t-1}\right)\right]^{-1}} + \beta \frac{\Xi}{D} \frac{\mu_{d}H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1}\left(D_{t+1} - D_{t}\right)^{-1}} \right\},\,$$

$$W_t^n \frac{\mu_n H_t^{1-\sigma} (C_t^n)^{-1}}{P_t^n} = \varrho \phi^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} (L_t^n)^{\frac{1}{\lambda}},$$
(33c)

$$W_t^d \frac{\mu_n H_t^{1-\sigma} (C_t^n)^{-1}}{P_t^n} = \varrho (1-\phi)^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} \left( L_t^d \right)^{\frac{1}{\lambda}}.$$
 (33d)

## APPENDIX B: Some Useful Steady State Relationships

As in the competitive equilibrium, real wage in each sector equals the marginal product of labor. Thus, we can derive the following relationship between the production of non-durables and that of durables in the steady state:

$$\frac{Y^n}{Y^d} = \frac{\alpha_{Ld}\phi}{\alpha_{Ln} (1 - \phi)} Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumption can be derived from the Euler conditions:

$$\frac{C^n}{C^d} = \left(1 - \beta \left(1 - \delta\right)\right) \frac{\mu_n}{\mu_d} \frac{1}{\delta} Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\begin{split} \frac{M^{nn}}{Y^n} &= \alpha_{Mn} \gamma_{nn}, \\ \frac{M^{nd}}{Y^n} &= \frac{\alpha_{Ln}}{\alpha_{Ld}} \frac{1 - \phi}{\phi} \alpha_{Md} \gamma_{nd}, \\ \frac{S^n O^n}{Y^n} &= \alpha_{On}, \\ \frac{C^n}{Y^n} &= 1 - \frac{M^{nn}}{Y^n} - \frac{M^{nd}}{Y^n} - \frac{S^n O^n}{Y^n}. \end{split}$$

Analogously, for the durable goods sector:

$$\frac{M^{dn}}{Y^d} = \frac{\alpha_{Ld}}{\alpha_{Ln}} \frac{\phi}{1 - \phi} \alpha_{Mn} \gamma_{dn},$$

$$\frac{M^{dd}}{Y^d} = \alpha_{Md} \gamma_{dd},$$

$$\frac{S^d O^d}{Y^d} = \alpha_{Od},$$

$$\frac{C^d}{Y^d} = 1 - \frac{M^{dn}}{Y^d} - \frac{M^{dd}}{Y^d} - \frac{S^d O^d}{Y^d}.$$

These conditions prove to be crucial in the second-order approximation of consumers' utility to eliminate the linear terms. Moreover, they allow us to derive the steady state ratio of labor supply in the non-durable goods sector over the total labor supply  $(\phi)$ . To this end, we take the ratio between the following equations:

$$C^{n} = (1 - \beta (1 - \delta)) \frac{\mu_{n}}{\mu_{d}} \frac{1}{\delta} Q^{-1} C^{d},$$

$$Y^{n} = \frac{\alpha_{Ld} \phi}{\alpha_{Ln} (1 - \phi)} Q^{-1} Y^{d}.$$

Thus:

$$\frac{C^n}{Y^n} = (1 - \beta (1 - \delta)) \frac{\alpha_{Ln} (1 - \phi)}{\alpha_{Ld} \phi} \frac{\mu_n}{\mu_d} \frac{1}{\delta} \frac{C^d}{Y^d}$$

which returns a coherent value of  $\phi$ .

#### The Relative Price in the Steady State

We start from the steady-state condition for the marginal cost in the non-durable goods sector:

$$MC^{n} = \bar{\phi}_{n} \left[ (P^{n})^{\gamma_{nn}} \left( P^{d} \right)^{\gamma_{dn}} \right]^{\alpha_{Mn}} (W^{n})^{\alpha_{Ln}} S^{\alpha_{On}}.$$

where  $\bar{\phi}_n = \frac{1}{\alpha_{Mn}^{\alpha_{Mn}} \alpha_{Ln}^{\alpha_{Ln}} \alpha_{On}^{\alpha_{On}}}$ . As in the steady state the production subsidies neutralize distortions due to imperfect competition:

$$P^n = MC^n$$
.

After some trivial manipulations it can be shown that:

$$\bar{\phi}_n Q^{-\alpha_{M_n} \gamma_{dn}} \left( R W^n \right)^{\alpha_{L_n}} \left( S^n \right)^{\alpha_{O_n}} = 1. \tag{34}$$

Analogously, for the durable goods sector:

$$\bar{\phi}_d Q^{\alpha_{Md}\gamma_{nd}} \left(RW^d\right)^{\alpha_{Ld}} \left(S^d\right)^{\alpha_{Od}} = 1. \tag{35}$$

Using the fact that  $S^n/S^d = 1/Q$ :

$$\frac{\left(\bar{\phi}_{n}\right)^{\frac{1}{\alpha_{On}}}}{\left(\bar{\phi}_{d}\right)^{\frac{1}{\alpha_{Od}}}}\frac{Q^{-\frac{\alpha_{Mn}\gamma_{dn}}{\alpha_{On}}}\left(RW^{n}\right)^{\frac{\alpha_{Ln}}{\alpha_{On}}}}{Q^{\frac{\alpha_{Md}\gamma_{nd}}{\alpha_{Od}}+1}\left(RW^{d}\right)^{\frac{\alpha_{Ld}}{\alpha_{Od}}}=1$$

Moreover, as in the steady state  $W^n = W^d = W$ ,  $RW^d = RW^nQ$ . This allows us to find a closed form expression for Q.

# APPENDIX C: Relative Price in the Efficient Equilibrium with Perfect labor Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labor market. On the labor market this condition, obtained for  $\lambda \to \infty$ , ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{n^*} = W_t^{d^*} = W_t^*. (36)$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{n^*} = MC_t^{n^*} P_t^{d^*} = MC_t^{d^*}. (37)$$

Conditions (36) and (37) imply that:

$$\bar{\phi}_{n} \frac{(Q_{t}^{*})^{-\alpha_{Mn}\gamma_{dn}} (W_{t}^{*})^{\alpha_{Ln}} \left(\frac{1}{P_{t}^{*n}}\right)^{\alpha_{Ln}} (S_{t}^{*n})^{\alpha_{On}}}{Z_{t}^{n}} = 1, \tag{38}$$

$$\bar{\phi}_d \frac{\left(Q_t^*\right)^{\alpha_{Md}\gamma_{nd}} \left(W_t^*\right)^{\alpha_{Ld}} \left(\frac{1}{P_t^{*d}}\right)^{\alpha_{Ld}} \left(S_t^{*d}\right)^{\alpha_{Od}}}{Z_t^d} = 1 \tag{39}$$

We then use both conditions to eliminate  $W_t^*$ :

$$(Q_t^*)^{\frac{\alpha_{Mn}\gamma_{dn}}{\alpha_{Ln}} + \frac{\alpha_{Md}\gamma_{nd}}{\alpha_{Ld}} + 1} = \frac{\left(\bar{\phi}_n \frac{(S_t^{*n})^{\alpha_{On}}}{Z_t^n}\right)^{\frac{1}{\alpha_{Ln}}}}{\left(\bar{\phi}_d \frac{(S_t^{*d})^{\alpha_{Od}}}{Z_t^d}\right)^{\frac{1}{\alpha_{Ld}}}}$$

#### **Proof of Proposition 1**

Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is,  $\widetilde{rmc}_t^n = \widetilde{rmc}_t^d = 0$ ,  $\forall t$ . It follows from the pricing conditions that  $\pi_t^n = \pi_t^d = 0$ ,  $\forall t$ . The relative price evolves as:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.$$

Since we also have that  $\Delta \widetilde{q}_t = 0$ , the equation above implies that  $\pi_t^n - \pi_t^d = \Delta q_t^*$ . From the analysis above:

$$q_t^* = \frac{1}{1+\varkappa} \left[ \frac{1}{\alpha_{Id}} z_t^d - \frac{1}{\alpha_{In}} z_t^n + \frac{\alpha_{On}}{\alpha_{In}} \left( s_t - p_t^{*n} \right) - \frac{\alpha_{Od}}{\alpha_{Id}} \left( s_t - p_t^{*d} \right) \right],$$

Therefore, it cannot be that  $\pi_t^n = \pi_t^d = 0$ , unless  $\Delta q_t^* = 0$ , which translates into:

$$\frac{z_t^d - \alpha_{Od} s_t^{*d}}{z_t^n - \alpha_{On} s_t^{*n}} = \frac{\alpha_{Ld}}{\alpha_{Ln}}.$$

### APPENDIX D: Equilibrium Dynamics in the Efficient Equilibrium

This appendix details the linearized system in the efficient equilibrium:

$$c_t^{d*} = \frac{1}{\delta} d_t^* - \frac{1 - \delta}{\delta} d_{t-1}^*, \tag{40}$$

$$c_t^{n*} = \frac{1}{\gamma} r_t^* + E_t c_{t+1}^{n*} + \frac{(1-\sigma)\mu_d}{\gamma} E_t \Delta d_{t+1}^*$$
(41)

$$c_{t}^{n*} = \frac{1}{\mu_{n} (1 - \sigma)} \left\{ \left[ 1 - \mu_{d} (1 - \sigma) \right] d_{t}^{*} + \frac{1}{1 - \beta (1 - \delta)} \left[ (\mu_{n} (1 - \sigma) - 1) c_{t}^{n*} + \mu_{d} (1 - \sigma) d_{t}^{*} - q_{t}^{*} \right] + \frac{(1 - \delta) \beta}{\left[ 1 - \beta (1 - \delta) \right]} \left[ (\mu_{n} (1 - \sigma) - 1) E_{t} c_{t+1}^{n*} + \mu_{d} (1 - \sigma) E_{t} d_{t+1}^{*} - E_{t} q_{t+1}^{*} \right] +$$

$$+\Xi \left(d_{t}^{*}-d_{t-1}^{*}\right)-\beta\Xi \left(E_{t}d_{t+1}^{*}-d_{t}^{*}\right)\right\} \tag{42}$$

$$rw_t^{n*} = -\gamma c_t^{n*} - (1 - \sigma) \mu_d d_t^* + \left(\vartheta \phi + \frac{1}{\lambda}\right) l_t^{n*} + \vartheta (1 - \phi) l_t^{d*}, \tag{43}$$

$$l_t^{n*} = \lambda \left( r w_t^{n*} - r w_t^{d*} + q_t^* \right) + l_t^{d*},$$
 (44)

$$y_t^{n*} = z_t^n + \alpha_{Mn}\gamma_{nn}m_t^{nn*} + \alpha_{Mn}\gamma_{dn}m_t^{dn*} + \alpha_{Ln}l_t^{n*} + \alpha_{On}o_t^{n*}, \tag{45}$$

$$y_t^{d*} = z_t^d + \alpha_{Md}\gamma_{nd}m_t^{nd*} + \alpha_{Md}\gamma_{dd}m_t^{dd*} + \alpha_{Ld}l_t^{d*} + \alpha_{Od}o_t^{d*}, \tag{46}$$

$$y_t^{n*} = \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} m_t^{nn*} + \frac{M^{nd}}{Y^n} m_t^{nd*} + \frac{S^n O^n}{Y^n} (s_t^{n*} + o_t^{n*}),$$

$$(47)$$

$$y_t^{d*} = \frac{C^d}{Y^d} c_t^{d*} + \frac{M^{dn}}{Y^d} m_t^{dn*} + \frac{M^{dd}}{Y^d} m_t^{dd*} + \frac{S^d O^d}{Y^d} \left( s_t^{d*} + o_t^{d*} \right), \tag{48}$$

$$0 = rw_t^{n*} + l_t^{n*} - y_t^{n*}, (49)$$

$$0 = rw_t^{d*} + l_t^{d*} - y_t^{d*}, (50)$$

$$0 = m_t^{nn*} - y_t^{n*}, (51)$$

$$0 = m_t^{nd*} + q_t^* - y_t^{d*}, (52)$$

$$0 = m_t^{dn*} - q_t^* - y_t^{n*}, (53)$$

$$0 = m_t^{dd*} - y_t^{d*}, (54)$$

$$0 = s_t^{n*} + o_t^{n*} - y_t^{n*}, (55)$$

$$0 = s_t^{d*} + o_t^{d*} - y_t^{d*} (56)$$

$$q_t^* = s_t^{d*} - s_t^{n*} (57)$$

$$o_t = \frac{O^n}{O}o_t^{n*} + \frac{O^d}{O}o_t^{d*} \tag{58}$$

$$z_t^n = \rho^{z^n} z_{t-1}^n + u_t^{z^n}, \quad u_t^{z^n} \stackrel{i.i.d.}{\sim} (0, \sigma^{z^n})$$
(59)

$$z_{t}^{n} = \rho^{z^{n}} z_{t-1}^{n} + u_{t}^{z^{n}}, \quad u_{t}^{z^{n}} \stackrel{i.i.d.}{\sim} (0, \sigma^{z^{n}})$$

$$z_{t}^{d} = \rho^{z^{d}} z_{t-1}^{d} + u_{t}^{z^{d}}, \quad u_{t}^{z^{d}} \stackrel{i.i.d.}{\sim} (0, \sigma^{z^{d}})$$
(60)

$$o_t = \rho^o o_{t-1} + u_t^o, \quad u_t^o \stackrel{i.i.d.}{\sim} (0, \sigma^o)$$

$$(61)$$

where  $\vartheta \equiv \left(v - \frac{1}{\lambda}\right)$  and  $\gamma \equiv \left(1 - \sigma\right)\mu_n - 1$ .

## APPENDIX E: Log-linear Economy

Here we report the log-linear economy in extensive form:

$$\widetilde{c}_t^n = \frac{1}{\gamma} \left( \widehat{r}_t - E_t \pi_{t+1}^n - r r_t^* \right) + E_t \widetilde{c}_{t+1}^n + \frac{(1-\sigma)\mu_d}{\gamma} E_t \Delta \widetilde{d}_{t+1}, \tag{62}$$

$$\widetilde{c}_{t}^{n} = \frac{1}{\mu_{n} (1 - \sigma)} \left\{ \left[ 1 - \mu_{d} (1 - \sigma) \right] \widetilde{d}_{t} + \frac{1}{1 - \beta (1 - \delta)} \left[ (\mu_{n} (1 - \sigma) - 1) \widetilde{c}_{t}^{n} + \mu_{d} (1 - \sigma) \widetilde{d}_{t} - \widetilde{q}_{t} \right] + \frac{(1 - \delta) \beta}{\left[ 1 - \beta (1 - \delta) \right]} \left[ (\mu_{n} (1 - \sigma) - 1) E_{t} \widetilde{c}_{t+1}^{n} + \mu_{d} (1 - \sigma) E_{t} \widetilde{d}_{t+1} - E_{t} \widetilde{q}_{t+1} \right] +$$
(63)

$$+\Xi\left(\widetilde{d}_{t}-\widetilde{d}_{t-1}\right)-\beta\Xi\left(E_{t}\widetilde{d}_{t+1}-\widetilde{d}_{t}\right)\right\}$$

$$\widetilde{c}_t^d = \frac{1}{\delta} \widetilde{d}_t - \frac{1 - \delta}{\delta} \widetilde{d}_{t-1}, \tag{64}$$

$$\widetilde{rw}_{t}^{n} = -\gamma \widetilde{c}_{t}^{n} - (1 - \sigma) \mu_{d} \widetilde{d}_{t} + \vartheta (1 - \phi) \widetilde{l}_{t}^{d} + \left(\vartheta \phi + \frac{1}{\lambda}\right) \widetilde{l}_{t}^{n}, \tag{65}$$

$$\widetilde{l}_t^n = \lambda \left( \widetilde{rw}_t^n - \widetilde{rw}_t^d + \widetilde{q}_t \right) + \widetilde{l}_t^d, \tag{66}$$

$$\pi_t^n = \beta E_t \pi_{t+1}^n + \frac{(1 - \beta \theta_n) (1 - \theta_n)}{\theta_n} \widetilde{rmc}_t^n + \eta_t^n, \tag{67}$$

$$\pi_t^d = \beta E_t \pi_{t+1}^d + \frac{(1 - \beta \theta_d)(1 - \theta_d)}{\theta_d} \widetilde{rmc}_t^d + \eta_t^d, \tag{68}$$

$$\widetilde{y}_{t}^{n} = \alpha_{Mn} \gamma_{nn} \widetilde{m}_{t}^{nn} + \alpha_{Mn} \gamma_{dn} \widetilde{m}_{t}^{dn} + \alpha_{Ln} \widetilde{l}_{t}^{n} + \alpha_{On} \widetilde{o}_{t}^{n}, \tag{69}$$

$$\widetilde{y}_t^d = \alpha_{Md}\gamma_{nd}\widetilde{m}_t^{nd} + \alpha_{Md}\gamma_{dd}\widetilde{m}_t^{dd} + \alpha_{Ld}\widetilde{l}_t^d + \alpha_{Od}\widetilde{o}_t^d, \tag{70}$$

$$\widetilde{y}_t^n = \frac{C^n}{Y^n} \widetilde{c}_t^n + \frac{M^{nn}}{Y^n} \widetilde{m}_t^{nn} + \frac{M^{nd}}{Y^n} \widetilde{m}_t^{nd} + \frac{S^n O^n}{Y^n} \left( \widetilde{s}_t^n + \widetilde{o}_t^n \right), \tag{71}$$

$$\widetilde{y}_t^d = \frac{C^d}{Y^d} \widetilde{c}_t^d + \frac{M^{dn}}{Y^d} \widetilde{m}_t^{dn} + \frac{M^{dd}}{Y^d} \widetilde{m}_t^{dd} + \frac{S^d O^d}{Y^d} \left( \widetilde{s}_t^d + \widetilde{o}_t^d \right), \tag{72}$$

$$\widetilde{rmc}_t^n = \widetilde{rw}_t^n + \widetilde{l}_t^n - \widetilde{y}_t^n, \tag{73}$$

$$\widetilde{rmc}_t^d = \widetilde{rw}_t^d + \widetilde{l}_t^d - \widetilde{y}_t^d, \tag{74}$$

$$\widetilde{rmc}_t^n = \widetilde{m}_t^{nn} - \widetilde{y}_t^n, \tag{75}$$

$$\widetilde{rmc}_t^d = \widetilde{m}_t^{nd} + \widetilde{q}_t - \widetilde{y}_t^d, \tag{76}$$

$$\widetilde{rmc}_t^n = \widetilde{m}_t^{dn} - \widetilde{q}_t - \widetilde{y}_t^n, \tag{77}$$

$$\widetilde{rmc_t^d} = \widetilde{m}_t^{dd} - \widetilde{y}_t^d, \tag{78}$$

$$\widetilde{rmc}_t^n = \widetilde{s}_t^n + \widetilde{o}_t^n - \widetilde{y}_t^n \tag{79}$$

$$\widetilde{rmc}^d = \widetilde{s}_t^d + \widetilde{o}_t^d - \widetilde{y}_t^d \tag{80}$$

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^* \tag{81}$$

$$0 = \frac{O^n}{O}\tilde{o}_t^n + \frac{O^d}{O}\tilde{o}_t^d \tag{82}$$

$$\widetilde{q}_{t} = \widetilde{s}_{t}^{d} - \widetilde{s}_{t}^{n}$$

$$\eta_{t}^{n} \stackrel{i.i.d.}{\sim} (0, \sigma^{\eta^{n}})$$

$$\eta_{t}^{d} \stackrel{i.i.d.}{\sim} (0, \sigma^{\eta^{d}})$$
(83)

where  $\gamma = (1 - \sigma) \mu_n - 1$  and

$$\frac{O^{n}}{O} = \frac{\alpha_{On}\alpha_{Ld}\phi}{\alpha_{Ld}\phi\alpha_{On} + \alpha_{Ln}(1-\phi)\alpha_{Od}},$$

$$\frac{O^{d}}{O} = \frac{\alpha_{Ln}(1-\phi)\alpha_{Od}}{\alpha_{Ld}\phi\alpha_{On} + \alpha_{Ln}(1-\phi)\alpha_{Od}}.$$

### APPENDIX F: Second-order Approximation of the Utility Function

Following Woodford (2003), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_{t} = U\left(C_{t}^{n}, D_{t}\right) - V\left(L_{t}\right).$$

We start from a second-order approximation of the utility from consumption of durable and non-durable goods:

$$U(C_{t}^{n}, D_{t}) \approx U(C^{n}, D) + U_{C^{n}}(C^{n}, D) (C_{t}^{n} - C^{n}) + \frac{1}{2} U_{C^{n}C^{n}} (C^{n}, D) (C_{t}^{n} - C^{n})^{2}$$

$$+ U_{D}(C^{n}, D) (D_{t} - D) + \frac{1}{2} U_{DD} (C^{n}, D) (D_{t} - D)^{2} + \frac{1}{2} \Xi U_{D} (C^{n}, D) (D_{t} - D_{t-1})^{2}$$

$$+ U_{C^{n}D} (C^{n}, D) (C_{t}^{n} - C^{n}) (D_{t} - D) + O(\|\xi\|^{3}),$$

$$(85)$$

where  $O\left(\|\xi\|^3\right)$  summarizes all terms of third order or higher. Notice that:

$$\begin{split} &U_{D}\left(C^{n},D\right) &= \left(\mu_{d}C^{n}/\mu_{n}D\right)U_{C^{n}}\left(C^{n},D\right),\\ &U_{C^{n}C^{n}}\left(C^{n},D\right) &= \left[\mu_{n}\left(1-\sigma\right)-1\right]\left(C^{n}\right)^{-1}U_{C^{n}}\left(C^{n},D\right),\\ &U_{DD}\left(C^{n},D\right) &= \left[\mu_{d}\left(1-\sigma\right)-1\right]\left(\mu_{d}C^{n}/\mu_{n}D\right)U_{C^{n}}\left(C^{n},D\right),\\ &U_{C^{n}D}\left(C^{n},D\right) &= \mu_{d}\left(1-\sigma\right)D^{-1}U_{C^{n}}\left(C^{n},D\right). \end{split}$$

As  $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2} (\widehat{c}_t^n)^2$ , where  $\widehat{c}_t^n = \log \left( \frac{C_t^n}{C^n} \right)$  is the log-deviation from steady state under sticky prices, we obtain:

$$U(C_{t}^{n}, D_{t}) \approx U(C^{n}, D) + U_{C^{n}}(C^{n}, D) C^{n} \left[ \widehat{c}_{t}^{n} + \frac{1}{2} (\widehat{c}_{t}^{n})^{2} \right] +$$

$$+ \frac{1}{2} \left[ \mu_{n} (1 - \sigma) - 1 \right] U_{C^{n}}(C^{n}, D) C \left[ \widehat{c}_{t}^{n} + \frac{1}{2} (\widehat{c}_{t}^{n})^{2} \right]^{2} +$$

$$+ U_{D}(C^{n}, D) D \left( \widehat{d}_{t} + \frac{1}{2} \widehat{d}_{t}^{2} \right) + \frac{1}{2} \left[ \mu_{d} (1 - \sigma) - 1 \right] U_{D}(C^{n}, D) D \left( \widehat{d}_{t} + \frac{1}{2} \widehat{d}_{t}^{2} \right)^{2} +$$

$$+ \frac{1}{2} \Xi U_{D}(C^{n}, D) D \left( \widehat{d}_{t} - \widehat{d}_{t-1} \right)^{2} +$$

$$+ \mu_{d} (1 - \sigma) U_{C^{n}}(C^{n}, D) C^{n} \left[ \widehat{c}_{t}^{n} + \frac{1}{2} (\widehat{c}_{t}^{n})^{2} \right] \left( \widehat{d}_{t} + \frac{1}{2} \widehat{d}_{t}^{2} \right) + \text{t.i.p.} + O \left( \|\xi\|^{3} \right),$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second-order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = (1 - \delta) D_{t-1} + C_t^d.$$

For a general function F(Y,X) the second-order Taylor approximation can be written as:

$$F(Y,X) \approx F_Y(Y,X) Yy + F_X(Y,X) Xx + \frac{1}{2} (F_X(Y,X) X + F_{XX}(Y,X) X^2) x^2 + \frac{1}{2} (F_Y(Y,X) Y + F_{YY}(Y,X) Y^2) y^2 + F_{YX}(Y,X) YXxy.$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log \left[ (1 - \delta) D_{t-1} + C_t^d \right].$$

Therefore:

$$F_{D} = \frac{(1-\delta)}{(1-\delta)D + C^{d}} = \frac{(1-\delta)}{(1-\delta)D + \delta D} = \frac{(1-\delta)}{D},$$

$$F_{C^{d}} = \frac{1}{(1-\delta)D + C^{d}} = \frac{1}{D},$$

$$F_{DD} = -\frac{(1-\delta)^{2}}{[(1-\delta)D + C^{d}]^{2}} = -\frac{(1-\delta)^{2}}{D^{2}},$$

$$F_{C^{d}C^{d}} = -\frac{1}{[(1-\delta)D + C^{d}]^{2}} = -\frac{1}{D^{2}},$$

$$F_{DC^{d}} = -\frac{1-\delta}{[(1-\delta)D + C^{d}]^{2}} = -\frac{1-\delta}{D^{2}}.$$

Considering that in the steady state  $C^d = \delta D$ :

$$\widehat{d}_{t} \approx \frac{(1-\delta)}{D}D\widehat{d}_{t-1} + \frac{1}{D}\delta D\widehat{c}_{t}^{d} + \\
+ \frac{1}{2}\left[\frac{(1-\delta)}{D}D - \frac{(1-\delta)^{2}}{D^{2}}D^{2}\right]\widehat{d}_{t-1}^{2} + \\
+ \frac{1}{2}\left(\frac{1}{D}D - \frac{1}{D^{2}}D^{2}\right)\left(\widehat{c}_{t}^{d}\right)^{2} - \frac{1-\delta}{D^{2}}\widehat{d}_{t-1}\widehat{x}_{t} \\
\approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_{t}^{d} + \frac{(1-\delta)\delta}{2}\widehat{d}_{t-1}^{2} + \frac{(1-\delta)\delta}{2}\left(\widehat{c}_{t}^{d}\right)^{2} - \frac{(1-\delta)\delta}{2}\widehat{c}_{t}^{d}\widehat{d}_{t-1} \\
\approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_{t}^{d} + \frac{(1-\delta)\delta}{2}\left(\widehat{d}_{t-1} - \widehat{c}_{t}^{d}\right)^{2}.$$

Thus:

$$\widehat{d}_t \approx (1 - \delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \psi_t,\tag{86}$$

where:

$$\widehat{\psi}_{t} = \frac{(1-\delta)\delta}{2} \left(\widehat{c}_{t}^{d} - \widetilde{d}_{t-1}\right)^{2}$$
$$= \frac{(1-\delta)}{2\delta} \left(\widehat{d}_{t} - \widetilde{d}_{t-1}\right)^{2}.$$

Now, let us iterate backward (86), to obtain:

$$\sum_{t=0}^{\infty} \beta^{t} \widehat{d}_{t} = \frac{1}{1-\beta (1-\delta)} d_{0} + \sum_{t=0}^{\infty} \beta^{t} \left[ \frac{\delta}{1-\beta (1-\delta)} \widehat{c}_{t}^{d} + \frac{1}{1-\beta (1-\delta)} \widehat{\psi}_{t} \right].$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function. The next step is to derive a second-order approximation for labor disutility. Recall that:

$$\widehat{l}_t = \phi \widehat{l}_t^n + (1 - \phi) \widehat{l}_t^d.$$

Therefore the second-order approximation reads:

$$V(L_t) \approx V_L(L) L \left[ \phi \hat{l}_t^n + (1 - \phi) \hat{l}_t^d + \frac{\phi (1 + 2v\phi)}{2} (\hat{l}_t^n)^2 + \frac{(1 - \phi) [1 + 2v (1 - \phi)]}{2} (\hat{l}_t^d)^2 \right] +$$

$$+ \text{t.i.p.} + O(\|\xi\|^3).$$

After these preliminary steps, we need to find an expression for  $\hat{l}_t^n$  and  $\hat{l}_t^d$ . Given the definition of the marginal cost, in equilibrium we get:

$$L_{t}^{n} = \frac{\alpha_{Ln}MC_{t}^{n}}{W_{t}^{n}} \int_{0}^{1} Y_{jt}^{n} dj = \alpha_{Ln} \frac{Q_{t}^{-\alpha_{Mn}\gamma_{dn}} (RW_{t}^{n})^{\alpha_{Ln}-1} (S_{t}^{n})^{\alpha_{On}}}{Z_{t}^{n}} Y_{t}^{n} \int_{0}^{1} \left(\frac{P_{jt}^{n}}{P_{t}^{n}}\right)^{-\varepsilon_{t}^{n}} dj,$$

$$L_{t}^{d} = \frac{\alpha_{Ld}MC_{t}^{d}}{W_{t}^{d}} \int_{0}^{1} Y_{kt}^{d} dk = \alpha_{Ld} \frac{Q_{t}^{\alpha_{Md}\gamma_{nd}} (RW_{t}^{d})^{\alpha_{Ld}-1} (S_{t}^{d})^{\alpha_{Od}}}{Z_{t}^{d}} Y_{t}^{d} \int_{0}^{1} \left(\frac{P_{kt}^{d}}{P_{t}^{d}}\right)^{-\varepsilon_{t}^{d}} dk.$$

Thus, we can report the linear approximation of the expressions above:

$$\widehat{l}_t^n = -\alpha_{Mn}\gamma_{dn}\widehat{q}_t + (\alpha_{Ln} - 1)\widehat{rw}_t^n + \alpha_{On}\widehat{s}_t^n - z_t^n + \widehat{y}_t^n + X_{nt}, 
\widehat{l}_t^d = \alpha_{Md}\gamma_{nd}\widehat{q}_t + (\alpha_{Ld} - 1)\widehat{rw}_t^d + \alpha_{Od}\widehat{s}_t^d - z_t^d + \widehat{y}_t^d + X_{dt},$$

where:

$$X_{nt} = \log \left[ \int_{0}^{1} \left( \frac{P_{jt}^{n}}{P_{t}^{n}} \right)^{-\varepsilon_{t}^{n}} dj \right] \qquad X_{dt} = \log \left[ \int_{0}^{1} \left( \frac{P_{kt}^{d}}{P_{t}^{d}} \right)^{-\varepsilon_{t}^{d}} dk \right]$$
(87)

If we set  $\hat{p}_{jt}^n$  to be the log-deviation of  $\frac{P_{jt}^n}{P_t^n}$  from its steady state, which means that a second-order Taylor expansion of  $\int_{-\infty}^{\infty} \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj$  reads as:

$$\int_{0}^{1} \left(\frac{P_{jt}^{n}}{P_{t}^{n}}\right)^{-\varepsilon_{t}^{n}} dj \approx \int_{0}^{1} \left[1 - \varepsilon^{n} \widehat{p}_{jt}^{n} - \varepsilon^{n} \widehat{p}_{jt}^{n} \widehat{\varepsilon}_{t}^{n} + \frac{1}{2} (\varepsilon^{n})^{2} (\widehat{p}_{jt}^{n})^{2}\right] dj + O\left(\|\xi\|^{3}\right)$$

$$= 1 - \varepsilon^{n} \mathbf{E}_{i} \widehat{p}_{jt}^{n} - \varepsilon^{n} \mathbf{E}_{i} \widehat{p}_{jt}^{n} \widehat{\varepsilon}_{t}^{n} + \frac{1}{2} (\varepsilon^{n})^{2} \mathbf{E}_{i} (\widehat{p}_{jt}^{n})^{2} + O\left(\|\xi\|^{3}\right),$$

where  $\mathbf{E}_{i}\widehat{p}_{jt}^{n} \equiv \int_{0}^{1} \widehat{p}_{jt}^{n} dj$  and  $\mathbf{E}_{i} \left(\widehat{p}_{jt}^{n}\right)^{2} \equiv \int_{0}^{1} \left(\widehat{p}_{jt}^{n}\right)^{2} dj$ . At this stage, we need an expression for  $\mathbf{E}_{i}\widehat{p}_{jt}^{n}$ .

$$P_t^n = \left[ \int_0^1 \left( P_{jt}^n \right)^{1 - \varepsilon_t^n} dj \right]^{\frac{1}{1 - \varepsilon_t^n}},$$

which can be re-arranged as:

$$1 \equiv \int\limits_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} \approx 1 + (1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n + \frac{1}{2}\left(1-\varepsilon^n\right)^2\left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceding equations yields:

$$0 = \int_{0}^{1} \left[ \left( 1 - \varepsilon^{n} \right) \widehat{p}_{jt}^{n} - \varepsilon^{n} \widehat{p}_{jt}^{n} \widehat{\varepsilon}_{t}^{n} + \frac{1}{2} \left( 1 - \varepsilon^{n} \right)^{2} \left( \widehat{p}_{jt}^{n} \right)^{2} \right] dj + O\left( \left\| \xi \right\|^{3} \right),$$

which reduces to:

$$\mathbf{E}_{i}\widehat{p}_{jt}^{n} = \frac{\varepsilon^{n} - 1}{2}\mathbf{E}_{i}\left(\widehat{p}_{jt}^{n}\right)^{2} + O\left(\left\|\xi\right\|^{3}\right).$$

Thus:

$$\int_{0}^{1} \left(\frac{P_{jt}^{n}}{P_{t}^{n}}\right)^{-\varepsilon_{t}^{n}} dj = 1 + \frac{\varepsilon^{n}}{2} \mathbf{E}_{i} \left(\widehat{p}_{jt}^{n}\right)^{2} + O\left(\|\xi\|^{3}\right).$$

Now, notice that:

$$\mathbf{E}_{i} (\widehat{p}_{jt}^{n})^{2} = \mathbf{E}_{i} \left[ (p_{jt}^{n})^{2} - 2p_{jt}^{n} p_{t}^{n} + (p_{t}^{n})^{2} \right] + O(\|\xi\|^{3}),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of  $p_t^n = \int_0^1 p_{jt}^n dj$ , as this term is multiplied by other first-order terms each time it appears. With this, we have a second-order approximation:

$$\mathbf{E}_{i}\left(\widehat{p}_{jt}^{n}\right)^{2} \equiv var_{j}p_{jt}^{n}.$$

Therefore, the second-order approximation can be represented as:

$$X_{nt} = \frac{\varepsilon^n}{2} var_j p_{jt}^n + O\left(\left\|\xi\right\|^3\right).$$

Analogous steps in the sector producing durable goods lead us to:

$$X_{dt} = \frac{\varepsilon^d}{2} var_k p_{kt}^d + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$var_{j}p_{jt}^{n} = \theta_{n}var_{j}p_{jt-1}^{n} + \frac{\theta_{n}}{1 - \theta_{n}}(\pi_{t}^{n})^{2} + O(\|\xi\|^{3}),$$
  
$$var_{k}p_{kt}^{d} = \theta_{d}var_{k}p_{kt-1}^{d} + \frac{\theta_{d}}{1 - \theta_{d}}(\pi_{t}^{d})^{2} + O(\|\xi\|^{3}).$$

Iterating these expressions forward leads to:

$$\sum_{t=0}^{\infty} \beta^{t} var_{j} p_{jt}^{n} = (\kappa_{n})^{-1} \sum_{t=0}^{\infty} \beta^{t} (\pi_{t}^{n})^{2} + \text{t.i.p.} + O(\|\xi\|^{3}),$$
 (88)

$$\sum_{t=0}^{\infty} \beta^t var_k p_{kt}^d = (\kappa_d)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{89}$$

where

$$\kappa_n = \frac{(1 - \beta \theta_n) (1 - \theta_n)}{\theta_n},$$

$$\kappa_d = \frac{(1 - \beta \theta_d) (1 - \theta_d)}{\theta_d}.$$

After these preliminary steps, we can write  $W_t$  as:

$$\mathcal{W}_{t} \approx U_{C^{n}}(C^{n}, D) C^{n} \left\{ \widehat{c}_{t}^{n} + \frac{1}{2} \left[ \mu_{n} (1 - \sigma) \right] (\widehat{c}_{t}^{n})^{2} + (\mu_{d}/\mu_{n}) \widehat{d}_{t} + \frac{1}{2} \left[ \mu_{d} (1 - \sigma) \right] (\mu_{d}/\mu_{n}) \widehat{d}_{t}^{2} + \mu_{d} (1 - \sigma) \widehat{c}_{t}^{n} \widehat{d}_{t} + \frac{1}{2} \Xi \left( \mu_{d}/\mu_{n} \right) \left( \widehat{d}_{t} - \widehat{d}_{t-1} \right)^{2} \right\} + \\ -V_{L}(L) L \left\{ \phi \widehat{l}_{t}^{n} + (1 - \phi) \widehat{l}_{t}^{d} + \right. \\ \left. + \left( \frac{1 + v}{2} \right) \left[ \phi^{2} \left( \widehat{l}_{t}^{n} \right)^{2} + (1 - \phi) \left( \widehat{l}_{t}^{d} \right)^{2} + 2\phi (1 - \phi) \widehat{l}_{t}^{n} \widehat{l}_{t}^{d} \right] \right\} + \\ + \text{t.i.p.} + O \left( \|\xi\|^{3} \right).$$

We now consider the linear terms in  $W_t$ , which are collected under  $\mathcal{L}W_t$ :

$$\begin{split} \mathcal{L}\mathcal{W}_{t} &= \frac{U_{C^{n}}\left(C^{n},D\right)C^{n}}{\mu_{n}}\left\{\mu_{n}\widehat{c}_{t}^{n} + \mu_{d}\widehat{d}_{t}\right\} + \\ &-V_{L}\left(L\right)L\left\{\phi\left(-\alpha_{Mn}\gamma_{dn}\widehat{q}_{t} + \left(\alpha_{Ln} - 1\right)\widehat{rw}_{t}^{n} + \alpha_{On}\widehat{s}_{t}^{n} + \widehat{y}_{t}^{n}\right) + \\ &+ \left(1 - \phi\right)\left(\alpha_{Md}\gamma_{nd}\widehat{q}_{t} + \left(\alpha_{Ld} - 1\right)\widehat{rw}_{t}^{d} + \alpha_{Od}\widehat{s}_{t}^{d} + \widehat{y}_{t}^{d}\right)\right\} + \\ &+ \text{t.i.p.} + O\left(\left\|\xi\right\|^{2}\right). \end{split}$$

We then substitute for the real wages from marginal cost expressions, so as to get:

$$\mathcal{LW}_{t} = \frac{U_{C^{n}}\left(C^{n}, D\right)C^{n}}{\mu_{n}} \left\{ \mu_{n} \widehat{c}_{t}^{n} + \mu_{d} \widehat{d}_{t} \right\} +$$

$$- \frac{V_{L}\left(L\right)L\phi}{\alpha_{Ln}} \left( \widehat{y}_{t}^{n} - \alpha_{Mn}\gamma_{nn} \widehat{m}_{t}^{nn} - \alpha_{Mn}\gamma_{dn} \widehat{m}_{t}^{dn} - \alpha_{On} \widehat{o}_{t}^{n} \right) +$$

$$- \frac{V_{L}\left(L\right)L\left(1 - \phi\right)}{\alpha_{Ld}} \left( \widehat{y}_{t}^{d} - \alpha_{Md}\gamma_{nd} \widehat{m}_{t}^{nd} - \alpha_{Md}\gamma_{dd} \widehat{m}_{t}^{dd} - \alpha_{Od} \widehat{o}_{t}^{d} \right) +$$

$$+ \text{t.i.p.} + O\left( \|\xi\|^{2} \right).$$

After iterating forward and substituting the second-order approximation for the accumulation equation

of durables we get:

$$\mathcal{L}W_{t} = U_{C^{n}}(C^{n}, D) C^{n} \left\{ \widehat{c}_{t}^{n} + \frac{\delta}{1 - \beta (1 - \delta)} \frac{\mu_{d}}{\mu_{n}} \widehat{c}_{t}^{d} + \frac{1}{1 - \beta (1 - \delta)} \mu_{d} \psi_{t} \right\} +$$

$$-V_{L}(L) L \left\{ \frac{\phi}{\alpha_{Ln}} \left( \widehat{y}_{t}^{n} - \alpha_{Mn} \gamma_{nn} \widehat{m}_{t}^{nn} - \alpha_{Mn} \gamma_{dn} \widehat{m}_{t}^{dn} - \alpha_{On} \widehat{o}_{t}^{n} \right) +$$

$$+ \frac{(1 - \phi)}{\alpha_{Ld}} \left( \widehat{y}_{t}^{d} - \alpha_{Md} \gamma_{nd} \widehat{m}_{t}^{nd} - \alpha_{Md} \gamma_{dd} \widehat{m}_{t}^{dd} - \alpha_{Od} \widehat{o}_{t}^{d} \right) \right\} +$$

$$+ \text{t.i.p.} + O \left( \|\xi\|^{2} \right). \tag{90}$$

We now employ the following steady-state relationships:

$$V_{L^{n}}(L^{n}) L^{n} = \phi V_{L}(L) L$$

$$V_{L^{d}}(L^{d}) L^{d} = (1 - \phi) V_{L} L(L)$$

$$\frac{-V_{L^{n}}(L^{n})}{U_{C^{n}}(C^{n})} = \frac{Y^{n} \alpha_{Ln}}{L^{n}}$$

$$\frac{-V_{L^{d}}(L^{d})}{U_{C^{n}}(C^{n})} = \frac{Y^{d} \alpha_{Ld}}{L^{d}Q}$$

$$U_{H}(H) H = \frac{U_{C^{n}}(C^{n}, D) C^{n}}{\mu_{n}}$$

So as to get:

$$\mathcal{LW}_{t} = U_{H}(H)H \begin{cases} \mu_{n}\widehat{c}_{t}^{n} + \frac{\delta\mu_{d}}{1-\beta(1-\delta)}\widehat{c}_{t}^{d} \\ -\frac{Y^{n}}{C^{n}}\mu_{n}\left(\widehat{y}_{t}^{n} - \alpha_{Mn}\gamma_{nn}\widehat{m}_{t}^{nn} - \alpha_{Mn}\gamma_{dn}\widehat{m}_{t}^{dn} - \alpha_{On}\widehat{o}_{t}^{n}\right) + \\ -\frac{Y^{d}}{C^{d}}\frac{C^{c}}{C^{n}}\mu_{n}\left(\widehat{y}_{t}^{d} - \alpha_{Md}\gamma_{nd}\widehat{m}_{t}^{nd} - \alpha_{Md}\gamma_{dd}\widehat{m}_{t}^{dd} - \alpha_{Od}\widehat{o}_{t}^{d}\right) + \\ +\text{t.i.p.} + O\left(\|\xi\|^{2}\right). \end{cases}$$

where the term  $\frac{1}{1-\beta(1-\delta)}\mu_d\psi_t$  has been included among the non-linear elements of  $\mathcal{W}_t$ . It is now possible to show that  $\mathcal{LW}_t = 0$ , using the following steady-state relationships:

$$C^{n} = (1 - \beta (1 - \delta)) \frac{\mu_{n}}{\mu_{d}} \frac{1}{\delta} C^{d},$$

$$Y^{n} = \frac{\alpha_{Ld} \phi}{\alpha_{Ln} (1 - \phi)} Y^{d},$$

as well as the linearized conditions that express the gross production in the two sectors.

After dropping the linear terms in  $W_t$  we are left with:

$$\sum_{t=0}^{\infty} \beta^{t} \mathcal{W}_{t} \approx U_{H}(H) H \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{1-\sigma}{2} \left( \mu_{n} \widehat{c}_{t}^{n} + \mu_{d} \widehat{d}_{t} \right)^{2} + \frac{1}{1-\beta (1-\delta)} \mu_{d} \widehat{\psi}_{t} + \frac{\mu_{d}}{2} \Xi \left( \widehat{d}_{t} - \widehat{d}_{t-1} \right)^{2} + \frac{\Theta}{2} \left[ \phi \varepsilon^{n} (\kappa_{n})^{-1} (\pi_{t}^{n})^{2} + (1-\phi) \varepsilon^{d} (\kappa_{d})^{-1} (\pi_{t}^{d})^{2} \right] + \left[ -\left(\frac{1+v}{2}\right) \Theta^{-1} \left[ \mu_{n} \widehat{c}_{t}^{n} + \frac{\delta \mu_{d}}{1-\beta (1-\delta)} \widehat{c}_{t}^{d} \right]^{2} \right\} +$$

$$+ \text{t.i.p.} + O\left( \|\xi\|^{3} \right),$$

where

$$\Theta = \left(\frac{C^n}{Y^n}\right)^{-1} \frac{\alpha_{Ln}\mu_n}{\phi} = \frac{\mu_n \left[1 - \beta \left(1 - \delta\right)\right] + \mu_d \delta}{1 - \beta \left(1 - \delta\right)}.$$

We next consider the deviation of social welfare from its Pareto-optimal level:

$$\sum_{t=0}^{\infty} \beta^{t} \widetilde{W}_{t} = \sum_{t=0}^{\infty} \beta^{t} \left( W_{t} - W_{t}^{*} \right) \approx$$

$$-\frac{U_{H} \left( H \right) H}{2} \Theta \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{\sigma - 1}{\Theta} \left( \mu_{n} \widetilde{c}_{t}^{n} + \mu_{d} \widetilde{d}_{t} \right)^{2} + \right.$$

$$+ \left[ \mu_{d} \Theta^{-1} \Xi + \left( 1 - \delta \right) \left( 1 - \omega \right) \delta^{-2} \right] \left( \widetilde{d}_{t} - \widetilde{d}_{t-1} \right)^{2} +$$

$$+ \varsigma \left[ \varpi \left( \pi_{t}^{n} \right)^{2} + \left( 1 - \varpi \right) \left( \pi_{t}^{d} \right)^{2} \right] + \left( 1 + v \right) \left[ \omega \widetilde{c}_{t}^{n} + \left( 1 - \omega \right) \widetilde{c}_{t}^{d} \right]^{2} \right\} + \text{t.i.p.} + O \left( \| \xi \|^{3} \right),$$

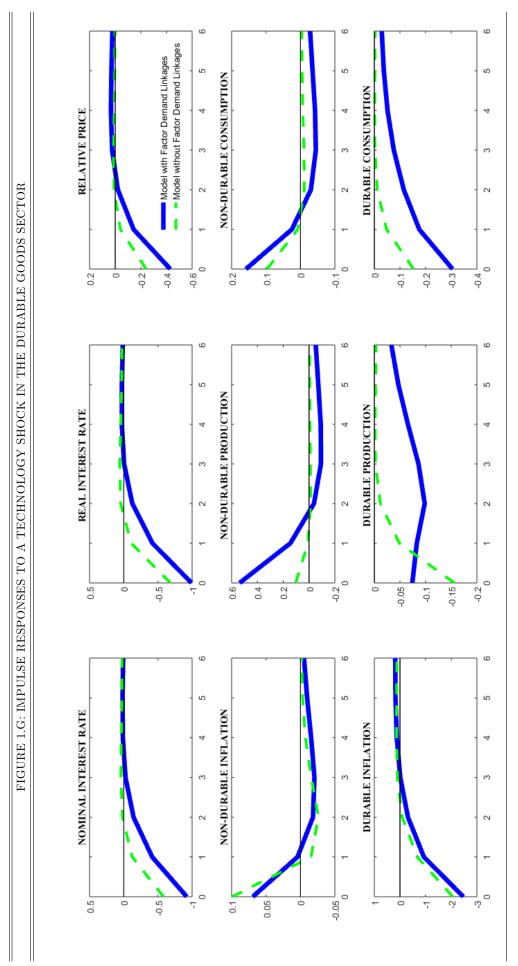
where the following notation has been introduced:

$$\omega = \frac{\mu_n \left[1 - \beta \left(1 - \delta\right)\right]}{\mu_n \left[1 - \beta \left(1 - \delta\right)\right] + \mu_d \delta},$$

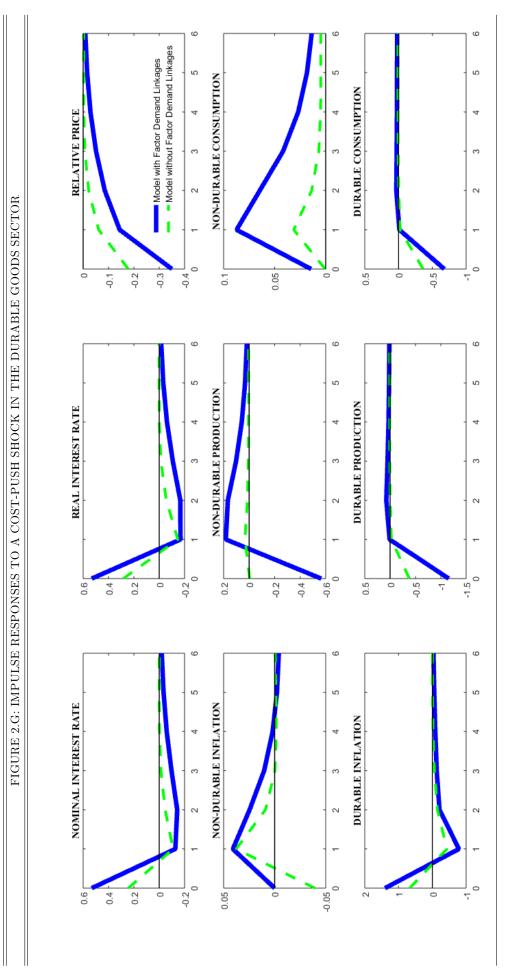
$$\varpi = \frac{\phi \varepsilon^n \left(\kappa_n\right)^{-1}}{\varsigma},$$

$$\varsigma = \phi \frac{\varepsilon^n}{\kappa_n} + \left(1 - \phi\right) \frac{\varepsilon^d}{\kappa_d}.$$

APPENDIX G: Impulse-responses to Shocks in the Durable Goods Sector



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.



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#### APPENDIX H: Domestically Generated Inflation

In policy environments it is customary to build measures of core inflation that remove fluctuations in the oil price from changes in the general price level (typically CPI/PCE inflation rates). However, in our model oil only enters as an imported production input. Therefore, to obtain equation (31) in the manuscript we appeal to the concept of domestically generated inflation, which represents a viable option to exclude the price of imported goods from a given measure of aggregate inflation. Specifically, domestically generated inflation,  $\pi_t^{dg}$ , is defined as the contribution of domestic factor price inflation to an overall rate of inflation. As a first accounting identity, we define domestically-generated gross output inflation as a weighted average of domestically generated inflation at the sectoral level:

$$\pi_t^{dg} = \kappa \pi_t^{n,dg} + (1 - \kappa) \pi_t^{d,dg} \tag{91}$$

where  $\pi_t^{i,dg}$  (i=n,d) denotes domestically generated inflation in sector and  $\varkappa \equiv Y^n/\left(Y^n+Y^d\right)$  accounts for the relative size of the non-durable goods sector. In turn, to disentangle the effect of the price inflation of imported and domestic input materials, one can define the inflation rate the i<sup>th</sup> sector as

$$\pi_t^i = \gamma_i \pi_t^{i,m} + (1 - \gamma_i) \pi_t^{i,dg}, \quad i = n, d.$$
 (92)

where  $\pi_t^{i,m}$  (i=n,d) denotes the price inflation rate of imported input materials by sector i and  $\gamma_i$  is the cost share of imported input materials in the production of sector i. Given that oil is the only imported good by both sectors (i.e.,  $\pi_t^{n,m} = \pi_t^{d,m}$ ), it is natural to set  $\gamma_i = \alpha_{O_i}$  and  $\pi_t^{i,m}$  can be replaced with  $\pi_t^O$ . Thus, rearranging terms in the Equation (33b) one obtains the following definition for the domestically generated inflation rate at the sectoral level:

$$\pi_t^{i,dg} = \frac{1}{1 - \alpha_{O_i}} \pi_t^i - \frac{\alpha_{O_i}}{1 - \alpha_{O_i}} \pi_t^O, \quad i = n, d.$$
(93)

Once we isolate the domestically generated part of sectoral inflation in each of the two sectors, we obtain  $\pi_t^{dg}$  as

$$\pi_t^{dg} = \varkappa \left( \frac{1}{1 - \alpha_{O_n}} \pi_t^n - \frac{\alpha_{O_n}}{1 - \alpha_{O_n}} \pi_t^O \right) + (1 - \varkappa) \left( \frac{1}{1 - \alpha_{O_d}} \pi_t^d - \frac{\alpha_{O_d}}{1 - \alpha_{O_d}} \pi_t^O \right) \tag{94}$$

# APPENDIX I: Optimal Rules in Models without Factor Demand Linkages

The present appendix reports further results from the implementation of a contemporaneous data rule and an inertial rule. Table I1 reports the coefficients of the optimal rule and the associated loss within a model economy without factor demand linkages. Moreover, as we restrict our focus to the comparison between aggregate and sticky-price inflation, we also eliminate oil from the production technology, without loss of generality. As a preliminary check, we can note that the standard result reported by Woodford (2003), according to which targeting sticky-price inflation is preferable to focusing on aggregate inflation, is confirmed in the model without factor demand linkages, conditional on the economy being perturbated by technology shocks only.

Notably, in a model where input materials are disregarded but sectors remain fundamentally asymmetric, policy inertia is only accepted when the policy maker targets sticky-price inflation and the economy is perturbated by all the three types of shock. This is due to various structural and exogenous factors inducing sectoral asymmetry and contributing to shift the relative price, even in the absence of factor demand linkages. Under these circumstances, when targeting aggregate inflation the policy maker seeks to attach the highest possible response to general price changes, while featuring no reaction to the output gap. By contrast, sticky-price inflation targeting needs to be complemented by a certain degree of inertia. How to explain this finding? When the policy maker targets  $\pi_t^{sticky}$ , inflation in the durable goods sector receives a rather low weight, both because this is relatively smaller and because it features lower price rigidity: this allows the central bank to adopt a certain degree of policy inertia, so as deal with the intersectoral stabilization trade-off that emerges due to various factors inducing sectors to co-move negatively. By contrast, when targeting a measure of general price inflation that merely considers the relative size of each sector, durable goods inflation is necessarily overweighed: to balance this bias, interest rate inertia is rejected, so as to avoid attributing too much importance to the durable sector, whose reaction in the face of both cost-push and technology shocks typically calls for a persistent response of the policy instrument (see Erceg and Levin, 2006). This intuition is confirmed by comparing Table I1 with the welfare outcomes of a perfectly symmetric model where neither input materials nor oil are employed (i.e.,  $\alpha_{Li} = \alpha_{Li} = 1$ ,  $i = \{n, d\}$ ), consumption goods feature the same expenditure share (i.e.,  $\mu_n = \mu_d$ ), both sectors produce a non-durable good (i.e.,  $\delta = 1$ ), labor is perfectly mobile across sectors (i.e.,  $\lambda \to \infty$ ) and sectoral shocks are perfectly correlated. As predicted by Schmitt-Grohe and Uribe (2007), interest rate inertia is accepted in this case. In fact,  $\rho = 0.73$  and  $\phi_{\pi} = \phi_{\eta} = 5$  conditional on a technology shock only, while  $\rho = 0.25$  and  $\phi_{\pi} = 5, \ \phi_{\nu} = 0, \ \text{conditional on all shocks}.$ 

TABLE I1: OPTIMAL INTEREST RATE RULES - NO FACTOR DEMAND LINKAGES											
	Technology shock					Technology shock					
	$\phi_\pi$	$\phi_y$	$L^R$	$L^T$		ho	$\phi_\pi$	$\phi_y$	$L^R$	$L^T$	
Aggregate	3.2940	5	0.0022	0.0072		0	3.2940	5	0.0022	0.0072	
Sticky-price	5	5	0.0020	0.0072		0	5	5	0.0020	0.0072	
	All shocks					All shocks					
	$\phi_{\pi}$	$\phi_y$	$L^R$	$L^T$		ρ	$\phi_{\pi}$	$\phi_y$	$L^R$	$L^T$	
Aggregate	5	0	0.1676	0.5095		0	5	0	0.1676	0.5095	
Sticky-price	5	0	0.2353	0.5095		0.5416	4.9919	0	0.1196	0.5095	

Notes: Table II reports – conditional on different shock configurations – the reaction coefficients under the contemporaneous data rule and the inertial rule in the absence of factor demand linkages and oil (i.e.,  $\alpha_{Mi} = \alpha_{Oi} = 0$ ,  $i = \{n, d\}$ ). The parameters  $\rho$ ,  $\phi_{\pi}$  and  $\phi_{y}$  are computed so as to minimize the loss of social welfare (Eq. 18). The table also reports the loss under timeless perspective,  $L^{T}$ , as well as  $L^{R}$ , which denotes the log-deviation of the loss

under the optimal rule and the loss under timeless perspective. All losses are expressed as a percentage of steady state consumption. The average duration of the price of non-durables is set at 4 quarters, while durable prices are re-set every 1.3 quarters.

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