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## ECONOMIC FUNDAMENTALS AND SELF-FULFILLING CRISES: Some Evidence from Mexico<sup>†</sup>

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## Abstract

This paper considers a model of debt stabilisation under a fixed exchange rate in which a currency crisis can develop as the result of self-fulfilling speculation, following a bifurcation in the behaviour of economic fundamentals. Based on this theoretical framework and by exploiting the test developed by Jeanne (1997), this paper provides evidence that self-fulfilling speculation was at work in the 1994 Mexican crisis. In terms of fundamentals, we show that the critical variables in generating the Mexican crisis were the fast rise in US\$-denominated public debt (tesebonos), the appreciated real exchange rate and the small rises in unemployment and primary deficit.

*Keywords:* Currency crisis, Speculation, Multiple equilibria, 1994 Mexican Crisis. *JEL classification:* F31, F32, F33, F34.

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## 1. INTRODUCTION

The Mexican economic crisis in 1994 and the more recent Asian crisis, have revived the debate on speculative attacks and the subsequent collapse of exchange rate regimes. Traditionally, there have been two main approaches in explaining currency crises. The 'first generation' approach, based on the seminal work by Krugman (1979) and Flood and Garber (1984), views currency crises as the inevitable outcome of inconsistent economic policies or macroeconomic imbalances.<sup>1</sup> Although 'first generation' models dominated the 1980's, they seemed powerless in explaining the typical phenomena observed in the recent speculative crisis in Asia, Mexico and Europe. These were often characterised by sudden shifts in market expectations that resulted in high levels of capital flight, beyond those implied by economic fundamentals. This led to the 'second-generation' approach, which interprets currency crises as emerging from shifts in agents' expectations about the future path of economic policy even when macroeconomic policy is consistent with a fixed exchange rate policy.<sup>2</sup> This approach however relies critically on a motivation for such shifts in expectations. Sachs, Tornell and Velasco (1996a), for example, suggest that the 1994 Mexican crisis was largely the result of a self-fulfilling panic that occurred only after the government's gross foreign reserves and its debt reached some particular values.<sup>3</sup> Once the state of the economy entered this range of values, large shifts in expectations occurred causing panic and taking the economy to a new equilibrium.

This paper provides further support to the view that the 1994 Mexican crisis was determined not only by fundamentals but also by animal spirits and panic. As in Sachs et al (1996a), currency crises in this paper are explained within a system of multiple equilibria

<sup>&</sup>lt;sup>1</sup> For a survey see Agenor, Bhandari and Flood (1992).

<sup>&</sup>lt;sup>2</sup> See Obstfeld (1986, 1994, 1996), Jeanne (1995) and Sachs et al (1996).

and following a bifurcation in the fundamentals, though our methodology incorporates the technology of Jeanne (1997).<sup>4</sup> In particular, the net benefits from maintaining a peg are decomposed into those arising purely from fundamentals and those determined probabilistically, as the private sector is uncertain over the credibility of the policy maker. Animal spirits or panic do not arise arbitrarily in this model, but only when economic fundamentals enter some critical range indicating that the probability of opting out of the peg has become high. The use of this technology has the added advantage in that it allows us to apply the estimation procedure developed in Jeanne (1997) to test for self-fulfilling expectations.<sup>5</sup> The results in this paper suggest some evidence of self-fulfilling expectations and panic in the 1994 Mexican crisis. We find that the key variables in generating multiple equilibria and determining the devaluation probability were the large switch from peso to US\$-denominated debt, the real exchange rate and the small rises in unemployment and the primary deficit.<sup>6</sup>

In section 2, we present a theoretical model that focuses on unemployment and debt stabilisation. Section 3, presents how multiple equilibria can arise in such a model under rational expectations, based on a modified analysis of Jeanne (1997). Section 4, focuses on the 1994 Mexican crisis and segregates it into three distinct phases that justify our theoretical framework. Then using Jeanne's (1997) technology, we test for evidence of self-fulfilling speculation during the 1994 Mexican crisis. Section 5 concludes.

<sup>&</sup>lt;sup>3</sup> For other studies where panic is to blame for the cause of the Mexican Crisis, see Frankel and Schmukler (1996), Edwards and Savastano (1998) and Radelet and Sachs (1999).

<sup>&</sup>lt;sup>4</sup> By 'bifurcations' we refer to a state where although a system is characterised by fundamentals, for critical values of the parameters affecting these fundamentals small disturbances can lead to a change in the systems' dynamic behaviour, but also in this paper the devaluation probability (See Azariadis, 1993).

<sup>&</sup>lt;sup>5</sup> Jeanne (1997) chooses to test for the 1992-3 French franc devaluation, though in his concluding remarks he mentions that it would be interesting if this model was used to test for self-fulfilling speculation in other episodes such as of other EMS currencies or the Mexican peso.

## 2. The Model

We consider a small open economy with perfect capital mobility, flexible nominal variables and a rational private sector.<sup>7</sup> The policy maker shares the dilemma recently faced by some Latin American countries. As part of a stabilisation programme the government announces a fixed peg so as to boost its policy credibility. However, because the economy is running a large deficit there is a time inconsistency problem as the policy maker has an incentive to devalue the currency and reduce the cost of the country's debt. To capture this policy dilemma, we assume that the policy maker's loss function is positively related to unemployment and deficit growth,<sup>8</sup>

(1) 
$$L_t = u_t^2 + \psi \Delta d_t + \delta C_t,$$

where u is the unemployment rate,  $\Delta d_t$  is the growth in government debt proportional to nominal GDP,  $\psi$  is preference parameter for debt stability and  $C_t$  is an exogenous cost associated with abandoning the peg;  $\delta$  is a dummy that takes the value of 1 when the policy maker opts out and 0 when the peg is maintained. Assuming that seigniorage and capital formation and its returns are constant, the government's and central bank's consolidated budget constraint, as a proportion to GDP, is:

<sup>&</sup>lt;sup>6</sup> Although our empirical approach is different, our results are not inconsistent with those in Sachs et al (1996) and Calvo and Mendoza (1996) that identify the US-dollar-denominated debt as a major factor in the Mexican crisis.

<sup>&</sup>lt;sup>7</sup> The role of rational agents is discussed below.

<sup>&</sup>lt;sup>8</sup> As with most of the speculative crisis literature, the policy maker's loss function is focused on a very shortterm stabilisation policy. This combined with the fact that Mexico's direct concern during the period examined was the reduction in its debt as % of GDP explains the presence of the latter in equation (1). However, debt stability in the loss function has also been used in models dealing with the transition of EU countries into the EMU, where one of the main criteria for entry was that government borrowing should not exceed 3% GDP, (see Tabellini and Via 1989, Jensen and Jensen 1995).

(2) 
$$\frac{\Delta B_t + e_t (\Delta B_t^* - \Delta F_t^*)}{P_t Y_t} = \frac{(G_t - T_t)}{P_t Y_t} + \frac{i_t B_t}{P_t Y_t} + \frac{i_t^* e_t (B_t^* - F_t^*)}{P_t Y_t}$$

 $\Delta$  is the first difference operator;  $e_t$  is the nominal exchange rate,  $B_t$  and  $B_t^*$  are pesodenominated and US\$-denominated debt respectively;  $G_t - T_t$  is the primary deficit (government consumption - tax revenue);  $i_t$  and  $i_t^*$  are the nominal domestic and world interest rates respectively;  $F_t^*$  denotes official foreign reserves and  $(F_t^* - B_t^*)$  are the government's net foreign asset;  $P_t Y_t$  denotes nominal GDP. Letting,  $d_t = (B_t + e_t (B_t^* - F_t^*))/P_t Y_t$  be the non-monetary debt proportional to nominal GDP, the effect of a change in  $d_t$  can be approximated as,

(3) 
$$\Delta d_{t} = \frac{\Delta B_{t} + e_{t}(\Delta B_{t}^{*} - \Delta F_{t}^{*})}{P_{t}Y_{t}} - (\rho_{t} + \pi_{t})d_{t} - e_{t}(f_{t}^{*} - b_{t}^{*})\Delta e_{t}$$

where  $\pi_t = \Delta P_t / P_t$ ,  $\rho_t = \Delta Y_t / Y_t$  and  $\Delta e_t = \Delta e_t / e_t$  denote the inflation rate, the economic growth rate and a proportional depreciation of the nominal exchange rate respectively. Assuming constant economic growth  $(\rho = 0)$ , <sup>9</sup> substituting equation (2) into (3) and writing  $x_t = X_t / P_t Y_t$ , (for all variables other than e, i and  $i^*$ ), we obtain the effects of a change in debt as a proportion of GDP,

(4) 
$$\Delta d_t = g_t - \tau_t + (i_t - \pi_t)d_t + (i_t - (i_t^* + \Delta e_t))e_t(f_t^* - b_t^*).$$

Given perfect capital mobility and treating domestic and foreign assets as perfect substitutes, the domestic and foreign interest rates are linked by the uncovered interest parity,  $i_t = i_t^* + E_{t-1}\Delta e_t$  where  $E_{t-1}\Delta e_t$  is the private sector's expectation at t-1 of the change in the exchange rate. Assuming for simplicity that domestic and foreign prices are linked by the purchasing power parity and that foreign prices are normalised to unit,  $P_t = e_t$ , and  $\pi_t = \Delta e_t$ . Using the above information in equation (4) we obtain,

(5) 
$$\Delta d_t = \hat{g}_t - (\Delta e_t - E_{t-1}\Delta e_t)d_t - (\Delta e_t - E_{t-1}\Delta e_t)z_t^*,$$

where  $\hat{g}_t = g_t - \tau_t + i_t^* d_t$  is the fiscal deficit with no expected devaluations, (i.e. the primary deficit plus the debt service estimated at the world interest rate);  $z_t^* = e_t (f_t^* - b_t^*)$ are net foreign assets in terms of domestic value and as percentage of GDP. From equation (5) the government's temptation to devalue unexpectedly becomes more transparent. Surprise devaluations reduce the public debt requirement by reducing the real interest rate on the debt service and also by raising the real value of the net foreign assets. From the last term in equation (4), a devaluation is shown to raise the rate of return on foreign assets  $(i^* + \Delta e)$  in relation to opportunity cost of domestic borrowing (i), thus raising the value of foreign assets held by the Central Bank, though this effect depends on the net foreign position of the economy. Equation (5) shows that only unexpected depreciations can reduce the required public debt. This is because from the UIP expected depreciations increase the domestic interest rate, (i), in line with the depreciation, thus eliminating the benefits of an exchange rate depreciation. Reducing government debt however, is not the government's only objective, as unemployment must also be kept low. For simplicity we capture this by an expectations-augmented Phillips curve, which assuming instantaneous power parity we

<sup>&</sup>lt;sup>9</sup> Economic growth played no crucial role in the Mexican crisis, (see also Dornbusch and Werner 1994).

can write as:<sup>10</sup>.

(6) 
$$u_t = \overline{u} + \lambda u_{t-1} - \beta (\Delta e_t - E_{t-1} \Delta e_t) \qquad \beta > 0.$$

Rational agents fully anticipate exogenous fiscal policy, but unexpected devaluations can surprise the private sector and so reduce both unemployment and the real debt, (as indicated by equations 5 and 6). Devaluations however are not cost free. First, we have assumed a direct cost involved as  $\delta$  takes the value of 1 in the policy maker's loss function when a devaluation takes place.<sup>11</sup> Second, given uncertainty rational agents, at any time, assess the government's credibility by forming expectations of whether,  $\Delta e > 0$ . This feature is incorporated in the model, by assuming a reputation cost that is captured by a probability,  $q_{t-1}$ , which is estimated by the private sector at time t-1 that the policy maker will opt-out and devalue at time t. In particular, Jeanne (1997) decomposes the *net benefit* of maintaining the peg as:

(7) 
$$\overline{V_t} = V_t - \gamma q_{t-1},$$

where  $V_t$  is the gross benefit of maintaining the peg at time t and  $q_{t-1}$  is the probability evaluated by speculators at time t-1 that the policy maker will opt out in period t. This equation captures the idea that in any period the net benefit of maintaining the peg is not a function only of fundamentals, but also subject to the credibility of the policy maker. A lower credibility, (higher  $q_{t-1}$ ), reduces the net benefit of maintaining the peg. Using this

<sup>&</sup>lt;sup>10</sup> This equation is similar to that employed by Obstfeld (1994) and Jeanne (1997), however our assumptions imply that in unemployment should also be affected by fiscal variables. For simplicity and with no loss of generality, we suppress all exogenous fiscal effects into  $\bar{u}$ .

<sup>&</sup>lt;sup>11</sup> This opting-out cost can be interpreted in a number of ways but here we treat it as exogenously defined, and concentrate on a different type of source of cost that can be more important, that of credibility.

and the above assumptions we write the unemployment and deficit equations corresponding to a devaluation (denoted by d), or maintaining the fixed peg (denoted by f) as,

(8) 
$$u_t^d = \overline{u} + \lambda u_{t-1} - \beta (1 - q_{t-1}) \Delta e_t$$

(9)  $\Delta d_t^d = \hat{g}_t - (d_t + z_t^*)(1 - q_{t-1})\Delta e_t,$ 

(10) 
$$u_t^f = \overline{u} + \lambda u_{t-1} + \beta q_{t-1} \Delta e_t,$$

(11) 
$$\Delta d_t^f = \hat{g}_t + (d_t + z_t^*)q_{t-1}\Delta e_t.$$

From (8)-(11) we note that even when the policy maker chooses to maintain the peg in the next period the private sector, faced with a time inconsistency problem, still treats the policy maker's credibility with scepticism resulting in an inflation bias,  $\beta q_{t-1} \Delta e$ . Using the above information, the policy maker must assess the net benefit from maintaining the peg. Following Jeanne (1997), we assume that at any time t the policy maker may be in a "soft" mood with a probability  $\mu$ , in which case he maintains the peg only if  $\overline{V} > 0$ , or in a "tough" mood, with probability  $1 - \mu$ , where the policy maker maintains the peg whatever the cost. The net benefit of maintaining the peg is given by the welfare loss difference between a devaluation and that of maintaining the peg,  $\overline{V}_t = L_t^d - L_t^f$ ,

(12) 
$$\overline{V}_t = C_t - 2\beta(\overline{u} + \lambda u_{t-1})\Delta e_t - \psi(d_t + z_t^*)\Delta e_t + (\beta \Delta e_t)^2 - 2(\beta \Delta e_t)^2 q_{t-1}.$$

Using equation (7) we can write equation (12) as,

(13) 
$$\overline{V}_{t} = V_{t} - \gamma q_{t-1},$$
$$V_{t} = C_{t} - 2\beta(\overline{u} + \lambda u_{t-1})\Delta e_{t} - \psi(d_{t} + z_{t}^{*})\Delta e_{t} + (\beta \Delta e_{t})^{2},$$
$$\gamma = 2(\beta \Delta e)^{2}.$$

From equation (13), higher levels of  $\psi$ , unemployment rate and public debt indicate a worsening of fundamentals, thus reducing the benefit from maintaining the peg ( $V_t$ ). In addition to  $V_t$  that captures the effects of exogenous fundamentals, lack of credibility in the policy maker also generates a credibility cost, captured by  $\gamma q_{t-1}$ . The latter is associated with the probability of a devaluation and it is mainly the result of animal spirits since it is present regardless of whether the government opts out or maintains the peg.

Although higher levels of unemployment and public debt unambiguously deteriorate fundamentals, the role of the currency composition of debt is more complex. As equation (13) suggests, the higher is the peso-denominated debt the higher may be the benefits from devaluing the domestic currency; conversely the higher is the foreign currency denominated debt the higher is the benefit of maintaining the fixed peg, since  $z_t^* = e_t (f_t^* - b_t^*)$ . One therefore would expect an increase in foreign-currency debt to improve fundamentals, indicating a lower probability of devaluation, whereas higher peso-denominated debt to deteriorate fundamentals and increase the probability of devaluation. Indeed, the reason why many countries, choose to borrow in foreign currency, is to make the fixed peg more credible by reducing the temptation of devaluing.<sup>12</sup> However, unlike with other macro variables, the effect of the currency composition of debt on fundamentals and therefore on the probability of devaluation may be reversed once a crisis is imminent. First, just before a crisis interest rate differentials start rising, yet the substitution of foreign debt for domestic debt will worsen the public debt requirement if the domestic real interest

<sup>&</sup>lt;sup>12</sup> This is believed to have been the case in Brazil, Mexico, Korea, Turkey and other countries.

rate exceeds the foreign real interest rate corrected for real exchange depreciation.<sup>13</sup> This is thought to have been the problem in a number of countries that had attracted a large amount of foreign borrowing, including Mexico (see Anand and Van Wijnbergen, 1987, Sachs, Tornell and Velasco, 1996b, and Melike Altinkemer, 1996). Second, when a country's foreign currency-denominated debt exceeds its foreign reserves,  $(b_t^* > f_t^*)$ , devaluations result directly in further increases in the public debt requirement, and therefore a worsening of fundamentals (see Buiter, 1990). Perhaps more important however, is the reverse effect that foreign currency debt can have directly on the probability of a devaluation once the crisis is imminent. Substitution of domestic for foreign currency debt, just before a crisis, increases the probability of a bad financial collapse and makes it more difficult and costly for the government to rollover foreign currency debt. Moreover, the perception of conversion risk together with partial default of foreign currency debt, further increases the probability of devaluation. The 1997 Mexican crisis was indeed affected by all of the above three problems; real interests were rising, foreign currency debt was well exceeding the country's foreign reserves and the government was finding it hard to rollover debt, ending up illiquid.<sup>14</sup>

## 3. MULTIPLE EQUILIBRIA AND SPECULATIVE ATTACKS

The policy makers' net benefit has been estimated optimally, (based on the welfare function), but also by taking the private sector's expectations and so the credibility cost into account. Similarly, the private sector is acting rationally since given uncertainty over the

<sup>&</sup>lt;sup>13</sup> This can be seen by writing equation (4) as  $\Delta d_t = g_t - \tau_t + (i - \pi)b - (i^* - \pi + \Delta e_t)z_t^*$  where b is substituted for  $e_t b_t^*$  at the cost  $(i - \pi) > (i^* - \pi + \Delta e_t)$ .

<sup>&</sup>lt;sup>14</sup> We discuss further the effects of the composition of Mexican debt with our estimation results in section 4.

policy maker's actions it chooses to act in a fashion similar to those models where a problem of time inconsistency arises. In particular, the private sector expects that the benefit of maintaining the peg at time t+1 is equal to the fundamentals in the model. Denoting the fundamentals in the model by  $\phi_t$  we can write,  $E_tV_{t+1} = \phi_t$ . Therefore, any deviation from the expected fundamental must be due to a stochastic element,  $V_{t+1} - E_tV_{t+1} = V_{t+1} - \phi_t = \varepsilon_{t+1}$ , or  $V_{t+1} = \phi_t + \varepsilon_{t+1}$ , where  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ , indicating that the gross benefit from maintaining the peg depends on exogenous fundamentals plus random events. Since  $\phi$  represents all exogenous variables and expectations are rational, the probability of a devaluation at time t is equal to the probability of the government being 'soft' ( $\mu$ ) and that the net benefit of maintaining the peg is negative at time t+1,

(14) 
$$q_t = \mu \operatorname{Pr} ob[\overline{V}_{t+1} < 0].$$

From (13) for  $\overline{V}_{t+1} < 0$  we must have  $V_{t+1} < \gamma q_t$ , or  $\varepsilon_{t+1} < \gamma q_t - \phi_t$ , so we can write,

(15) 
$$q_t = \mu \operatorname{Pr} ob[\varepsilon_{t+1} < \gamma q_t - \phi_t].$$

Since  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$  equation (15) can be written as,

(16) 
$$q_t = \mu F(\gamma q_t - \phi_t)$$

where F(.) is the cumulative normal distribution function of f(.). Because both sides of equation (16) are increasing functions of q, it is possible to have more than one level of devaluation probability (q) that is consistent for any given fundamental  $\phi_t$ , giving rise to multiple equilibria. Jeanne (1997) shows, that multiple equilibria in this model arise only under certain parameter values that can be shown geometrically. Figure 1, plots equation



Figure 1. (a) Unique equilibrium; (b) Multiple equilibria

(16) for unique and multiple equilibria. The 45° line shows the left-hand side of equation (16) whilst the  $C_{\phi}$  curve gives the right-hand side of that equation.  $C_{\phi}$  is a plot of the  $F(\cdot)$ function perturbed by  $\mu$  and  $\phi_t$ , where  $\mu$  being a constant just scales this function, whilst  $\phi_t$  shifts the function. The points of intersections between the 45° line and the  $C_{\phi}$  curve are the points for which equation (16) holds. With  $C_{\phi} \equiv \mu F(x_t)$ , where  $x_t = \gamma q_t - \phi_t$ , the

gradient of 
$$C_{\phi}$$
 is,  $\frac{dC_{\phi}}{dq} = \frac{\partial C_{\phi}}{\partial F(x)} \frac{\partial F(x)}{\partial x} \frac{\partial x}{\partial q} = \mu \frac{\partial F(x)}{\partial x} \gamma$ . Since  $F(x_t)$  is the cumulative

normal distribution function,  $\frac{\partial F(x)}{\partial x} = f(x)$  is the probability density function and so,

(17) 
$$\frac{dC_{\phi}}{dq} = \mu \gamma f(x) \,.$$

Since f(x) is the standard *Gaussian* function, with the familiar bell shaped curve, it obtains its maximum at x = 0. Hence the slope attains its maximum value, (i.e. it is steepest) at,  $dC_{\phi}/dq = \mu\gamma f(0)$ . From the definition of x, f(x) = 0 when  $\gamma q_t = \phi_t$ . Thus, the slope of  $C_{\phi}$  attains its maximum at  $\mu\gamma f(0)$  at which point  $q = \phi/\gamma$ .

Unique equilibrium: If  $\mu\gamma f(0) < 1$ , then the slope of  $C_{\phi}$  is everywhere strictly smaller than 1 so that the steepest range of the  $C_{\phi}$  curve (at the point  $q = \phi/\gamma$ ), lies below the 45° line. In this case there is only one unique equilibrium for q, shown as  $q^*(\phi)$  in figure 1(a) and it is determined merely by fundamentals  $(\phi_t)$ .

Multiple equilibria: For multiple equilibria we require two conditions.

(18) 
$$\mu \gamma f(0) > 1 \Rightarrow \gamma > \frac{1}{\mu f(0)},$$

(19) 
$$\phi_t \in [\underline{\phi}, \overline{\phi}].$$

Equation (18) relates to the structural parameters in the model and implies that the slope of  $C_{\phi}$  is greater than one at its maximum value,  $q = \phi/\gamma$ , or that the  $C_{\phi}$  curve crosses the 45° line at more than one point. Equation (19) relates to the time-dependent fundamentals and implies that at time *t* these lie within a range of critical values,  $\phi_t \in [\phi, \overline{\phi}]$ , that correspond to the critical probabilities,  $\overline{q}$  and  $\underline{q}$ , that make the fixed peg vulnerable to self-fulfilling speculation. The critical values of  $\phi$  and  $\overline{\phi}$  can be found when  $\mu\gamma f(0) > 1$  and the value of  $\phi$  is such that the lower turning point of the  $C_{\phi}$  curve is below the 45° line and the upper turning point above the 45° line degree line, as shown in figure 1(b).<sup>15</sup>

Figure 2, plots the relationship between economic fundamentals and expected devaluation. The thick line shows the behaviour of expected devaluation as a function of economic fundamentals when  $\mu\gamma f(0) < 1$ , while the thinner line shows the expected devaluation corresponding to  $\mu\gamma f(0) > 1$ . The two vertical dotted lines,  $\phi$  and  $\phi$ , specify the interval  $[\phi, \phi]$ . This interval distinguishes between three different states,  $s_t = i$ , (i = 1, 2, 3) that the economy can alternate between when there are multiple equilibria. *State 1*, holds between points  $H, \overline{R}$ , *state 2* lies between points,  $\overline{R}, \underline{R}$  and *state 3* between  $L, \underline{R}$ . In *State 2* the relationship between economic fundamentals and expected devaluation is counterintuitive, because it implies that devaluation expectations increase with improvements



Figure 2. Devaluation probability and fundamentals

in economic fundamentals. As Jeanne (1997) shows, *State 2* has some undesirable properties that make it a less plausible candidate. We therefore expect that when fundamentals enter the region  $[\phi, \overline{\phi}]$ , such as point  $\phi$  in figure 2, the expected devaluation can take a maximum of three equilibria points,  $q_1(\phi), q_2(\phi)$  and  $q_3(\phi)$ , which make the exchange rate regime susceptible to self-fulfilling speculation. Self-fulfilling speculation or panic in figure 2, is shown by a jump from a low devaluation probability such as  $q_1(\phi)$  to a high one such a point  $q_2(\phi)$  or  $q_3(\phi)$ . This can occur only when economic fundamentals are high or low, (within the range  $[\phi, \overline{\phi}]$ ), but not too high or too low or else the benefit next period will always be positive or negative irrespective of expectations. For example, when

<sup>&</sup>lt;sup>15</sup> In the appendix we provide a more formal treatment of their derivation, (see also Jeanne 1997).

economic fundamentals are too good,  $\phi > \overline{\phi}$ , (to the right of point *L*), the devaluation probability is uniquely defined and lies close to zero. Similarly, when fundamentals are too bad,  $\phi < \phi$ , (to the left of point H), the devaluation probability is again uniquely defined and closer to  $\mu$ , the probability of the policy maker being soft.

So far, we have used Jeanne's (1997) model and modified its economic fundamentals so as to account for both deficit and unemployment. We have shown that a currency crisis in this model can emerge as the 'natural' outcome of a deterioration in economic fundamentals (fall in  $\phi$ ) but also as the result of animal spirits and self-fulfilling speculation. The latter occurs only for some *bifurcation* values of the fundamentals that result in some autonomous jump from a low to a high level of devaluation expectation. In what follows we use this modified model to test for self-fulfilling speculation in the 1994 Mexican crisis.

## 4. TESTING FOR SELF-FULFILLING SPECULATION IN THE 1994 MEXICAN CRISIS

Most economists now agree that the Mexican Peso crisis in December 1994 could not have been merely the result of economic policy mistakes but also of panic in the private sector. The latter however is not easily quantified. The model by Jeanne (1994), that explains crises as initiated by changes in fundamentals followed by a speculative attack appeals as an ideal candidate for the Mexican crisis.

For the purpose of this paper we can distinguish between three different phases in the Peso crisis that match our theoretical framework. *Phase 1*, of rather sound economic fundamentals (with  $\phi > \overline{\phi}$  and a low q); *phase 2*, characterised by exogenous political shocks combined with economic policy mistakes (shifts in  $\phi$ ) and signals to the private



Figure 3. (a) Mexican Debt; (b) Real Exchange Rate; (c) Unemployment rate; (d) Primary Deficit

sector of a softer policy stance (raising  $\mu$  and q); and finally *phase 3*, characterised by panic and speculative attack (as the economy enters the critical interval,  $\phi \in [\phi, \overline{\phi}]$ ).

*Phase 1:* The large fiscal deficit and the debt crisis of the 1980's placed debt stabilisation as Mexico's main priority. In 1988, Mexico pegged the peso to the dollar while later, in 1989, exchange rate policy moved to a crawling peg. Between 1990-1993 fiscal and anti-inflation policy were strict, Mexico's international competitiveness increased and with its membership in NAFTA both domestic and foreign investors were optimistic. This resulted in a large capital inflow, a large part of which consisted of portfolio investment. During this phase the Mexican economy was fundamentally sound.<sup>16</sup>

*Phase 2* enters with the end of 1993 and with a series of political shocks such as the rebellion of Chiapas in January 1994, the assassination of presidential candidate, D.Colosio, in March 1994, uncertainty prior and after the Congressional and Presidential election of August 1994, and the assassination of Ruiz Massieu who was to become the majority party ruling leader in the Lower House of Congress. Such political events placed the Mexican economy in a vulnerable position, which combined with economic policy mistakes made the speculative attack inevitable. The Central Bank intervened by raising the interest rates backed by a domestic credit expansion and a decline in foreign reserves while public debt had kept rising from US\$ 9.77bn in March 1993 to US\$ 25.88bn in October 1994. Figure 3, shows that within the first quarter of 1994 there was a deterioration of macroeconomic fundamentals. Both *cetes* (peso-denominated bonds) and *tesebonos* (US\$-denominated bonds) increased and unemployment had also started rising. The rate of growth in the primary deficit also increased slightly during that period. The

<sup>&</sup>lt;sup>16</sup> However, some economists believed that the peso was perhaps moderately overvalued, (see also Dornbusch and Werner 1994 and Sachs et al 1996).

latter was in response to the government's attempts to stimulate employment in the wake of the Chiapas uprising in early 1994 and in the run up to the presidential elections. Since the 1990s and as a result of the exchange rate based stabilisation program, there had been an appreciation in the real exchange rate, and although there was a small depreciation in June 1994, this was not enough to compensate for the loss in competitiveness that occurred during the previous years. At the same time, despite the government's commitment to the exchange rate stability, the Mexican government gave a signal of weakening its exchange rate policy and so its credibility, by announcing on 20 December 1994 a rise of the upper ceiling of the exchange rate by 13%. This was coupled with a sudden large switch from 3month peso-denominated government debt (Cetes) to 3-month dollar-denominated government debt (Tesobonos). The latter was an attempt by the government to reduce the cost of its public debt by an even higher expected depreciation of the peso.<sup>17</sup> This marked the first speculative attack and the beginning of *phase 3*. The peso depreciated immediately to the new ceiling while pessimism and panic started spreading among domestic and foreign investors. Some observers suggest that panic was initiated by domestic portfolio investors who had little trust in their own government and feared a large devaluation.<sup>18</sup> This almost simultaneously contaminated other financial markets and resulted in a sustained attack on the peso. In March 1995, the Mexican peso hit its lowest since the December crisis, it fell by almost 50% of its 1994 value.

This succinct history of the Mexican crisis seems to match closely the process we have described in the above model. Therefore, to test for self-fulfilling speculation in the 1994 Mexican crisis we use the Maximum Likelihood method developed by Jeanne (1997)

<sup>&</sup>lt;sup>17</sup> See also a theoretical interpretation of this in section 2 and footnote 11. <sup>18</sup> See also Frankel and Schmukler (1996).

to estimate speculative attacks on the French franc.<sup>19</sup> This is based on a test where the likelihood function is estimated as the product of the likelihood of the model prediction error and that of a state transition process so as to incorporate both required conditions for multiple equilibria given by equations (18) and (19). By normalising  $\gamma = 1$ , the first condition,  $\mu\gamma f(0) > 1$  is rewritten, as z > 1, where  $z \equiv \mu f(0) = \frac{\mu}{\sigma\sqrt{2\pi}}$ . The test for self-fulfilling speculation is therefore a test of the null hypothesis,  $H_0: z \le 1$ . This test is conducted using the likelihood ratio,  $LR = 2(\log L^* - \log L_{z=1}^*)$ , where  $L^*$  is the maximum of the unrestricted likelihood function L and  $L_{z=1}^*$  is the maximum of the likelihood function that z = 1. This is distributed as a chi-square with one degree of freedom (i.e.  $\Pr[\chi_1^2 > LR^*] = 1\%$ ). Rejection of the null hypothesis implies that we cannot accept the hypothesis of no self-fulfilling speculation.

The macroeconomic variables representing fundamentals are, peso-denominated and dollar-denominated debt, the real exchange rate, the unemployment rate and the primary deficit. We use monthly data for the sample period January 1993 to September 1995.<sup>20</sup> From our theoretical model, an increase in the primary deficit, or the unemployment rate, signify a deterioration of economic fundamentals, thus *a priori* we expect the coefficient on these variables to be negative. An increase in the real exchange rate corresponds to an increase in the external competitiveness of the economy hence an

<sup>&</sup>lt;sup>19</sup> This test was inspired by Dagsvic and Jovanovic (1994). For the reader's convenience a detailed analysis of this test is provided in the Appendix.

<sup>&</sup>lt;sup>20</sup> Data on unemployment were provided by the National Institute of Economic and Geographical Information (INEGI), while data on public accounts and the real exchange rate were obtained from the Banco de Mexico.

improvement in economic fundamentals, and so we expect its coefficient to be positive.<sup>21</sup> Perhaps more ambiguous is the expected sign on the currency composition of debt. The model suggests that *ceteris paribus* we should expect a negative sign on the peso denominated debt (cetes) and a positive sign on the dollar denominated debt (tesebonos), improvement in economic fundamentals, and so we expect its coefficient to be positive.<sup>22</sup> Perhaps more ambiguous is the expected sign on the currency composition of debt. The model suggests that *ceteris paribus* we should expect a negative sign on the positive.<sup>22</sup> Perhaps more ambiguous is the expected sign on the currency composition of debt. The model suggests that *ceteris paribus* we should expect a negative sign on the peso denominated debt (cetes) and a positive sign on the dollar denominated debt (tesebonos), though, as we have already discussed in Section 2, these effects may be reversed when a crisis is imminent or during a crisis.

We use the interest rate differential between peso-denominated Cetes and US\$denominated tesobonos, as a measure of the expected rate of devaluation. Agenor and Masson (1999) show that for the case of Mexico the latter provides a more comprehensive measure of expectations held by the private sector than economic surveys.<sup>23</sup> The resulting devaluation probability is shown in figure 4, as 'actual' probability. The plot shows a significant jump in devaluation expectations in November 1994, just prior to the onset of the crisis. The estimation results are given in Table 1. The marginal significance levels of the coefficients given in the third column are the P-values from the asymptotic likelihood ratio estimates. Figure 4, indicates a good fit suggesting that the model performed reasonably well showing a slight increase in the devaluation probability just prior to the actual devaluation.

<sup>&</sup>lt;sup>21</sup> The real exchange rate index is calculated as the ratio of the price of tradables to the price of non-tradables (base 1990), rather than the inverse ratio used by the IMF and that employed in Jeanne 1997.

 $<sup>^{22}</sup>$  The real exchange rate index is calculated as the ratio of the price of tradables to the price of non-tradables (base 1990), rather than the inverse ratio used by the IMF and that employed in Jeanne 1997.



Figure 4. Devaluation probability

The primary deficit, the real exchange rate and unemployment have the expected signs and they are all significant at the 5% level. Notice however that the signs on *cetes* and *tesebonos* are the opposite of what the fundamentals of the model suggest. This can be explained by the fact that the estimates in table 1 are mainly driven by the crisis period which may reverse the expected signs on these variables. As we have already shown, substitution of domestic for foreign currency debt can act as a credibility enhancing mechanism that is helpful in reducing devaluation expectations as well as inflation expectations. However, the effects of this strategy may be reversed when a crisis is

<sup>&</sup>lt;sup>23</sup> Note that Jeanne (1997) concerned with devaluations within the ERM uses the drift adjustment method developed in Svenson (1993) for measuring expected devaluations in target zones.

imminent.<sup>24</sup> Following the political developments and the assassination of Colosio before the crisis and the devaluation of the peso from 3.47 pesos to the dollar to 3.99, there was a massive capital flight, followed with a sharp increase in interest rates. At the same time the government was substituting cetes for tesebonos. By June 1994, the tesobonos outstanding already exceeded cetes (see figure 3a). This strategy however, in conjunction with the massive capital flight, meant that in the first half of 1995 foreign-currency obligations were considerably in excess of Mexican dollar reserves.<sup>25</sup> At the same time real domestic interest rates were high and rising. During that period further increases in foreign currency debt implied increases in the public debt requirement and a higher probability of a bad financial collapse. A large number of institutional holders of tesobonos wanted to withdraw rapidly from the market in the wake of devaluation announcements and this made it increasingly difficult for the government to rollover the stock of tesebonos. For many observers this is believed to have generated the panic that led to the crisis of December 1994.<sup>26</sup>

Figure 5, shows the behaviour of economic fundamentals. The upper and lower fundamental thresholds are  $\overline{\phi} = 0.41063$  and  $\underline{\phi} = 0.39559$  respectively.<sup>27</sup> Figure 5, indicates that economic fundamentals were declining since the beginning of 1994 (as described in *phase 2* above) and entered the critical zone of multiplicity (*phase 3*) in November 1994, just prior to the announcement of the depreciation of the peso by 13%.

<sup>&</sup>lt;sup>24</sup> Indeed, estimating the same model outside the crisis period, (ie. 1993:1-1994:12), produces all the expected signs in relation to the fundamentals. Primary deficit: -9.96167e-005 (0.46363); Cetes: -2.16537e-004 (0.18167); Tesebonos: 1.00836e-004 (0.00145); Real exchange rate: 9.26552e-005 (0.00877); Unemployment: -.00200 (0.43320), with  $\mu$ =0.55933 and Log L\*=91.25864. However, as Jeanne (1977) points out, the model does not explain satisfactorily the month-to-month fluctuations in the devaluation probability outside the episodes of crisis.

<sup>&</sup>lt;sup>25</sup> Foreign reserves fell from around \$30 billion in February to about \$11 billion in December 1994.

<sup>&</sup>lt;sup>26</sup> See also, Sachs, Tornell and Velasco, (1996b) Calvo and Mendoza (1996) and Melike Altinkemer, (1996).

<sup>&</sup>lt;sup>27</sup> For the theoretical estimation of these thresholds see in the Appendix A.



Figure 5. Macroeconomic Fundamentals ( $\phi$ )

The most important parameter in this test is z, the size of which indicates the possibility of bifurcation and multiple equilibria. The Likelihood Ratio equal to  $LR = 2(LogL^* - LogL_{z=1}^*) = 7.21$  is greater than the  $\chi^2$  critical value of 6.64 at the 1%. We can therefore reject the null hypothesis and conclude that at least a part of the speculative attack in 1994 Mexican crisis was self-fulfilling.

The estimated transition probabilities, shown in Table 2, indicate that if agents had high devaluation expectations prior to entering the zone of multiplicity, they were more likely to maintain such expectations once they entered the critical zone. There is however a chance -approximately 24%- that if they had a low devaluation expectations they would shift to a high devaluation expectation. The transition matrix suggests that the economy

Table 1.					
Variable	Coefficient	P-Value ( $\chi^2(1)$ )			
Constant	-0.00273				
Primary deficit (% GDP)	-0.00022	0.00686*			
Peso-denominated debt (% GDP)	0.00555	0.15653			
US\$-denominated debt (%GDP)	-0.00069	0.00782*			
Real exchange rate	0.00009	0.01239*			
Unemployment rate	-0.00415	0.00307*			
μ 0.80622					
z 1.09206					
Log L <sup>*</sup> 95.55655					
Log L <sup>*</sup> <sub>z=1</sub> 91.95138					
Estimation by Simplex. Sample: 1993:01-1995:09; * = significance at the 5% level.					

Table 2.		
Matrix Θ		

(0.7611	0	0.2389
0	1	0
0.0061	0	0.9939)

\_\_\_\_

was more likely to be in a crisis state than not or that agents had high devaluation expectations when the economy entered the critical zone.

## 5. CONCLUSION

This paper considers a debt stabilisation policy model under fixed exchange rates where self-fulfilling speculation can arise following a bifurcation in the fundamentals. Using the test developed by Jeanne (1997) to test for speculative attacks against the French Franc, we are able to test for self-fulfilling speculation in the 1994 Mexican crisis. The empirical results suggest that some self-fulfilling speculation was at work in the 1994 Mexican crisis. It is shown that a critical variable in generating the panic that led to the Mexican crisis was the sudden large-scale switch from peso-denominate debt (mainly cetes) to US\$-

denominated public debt (tesebonos) but also the small rises in unemployment and primary deficit as well the appreciated real exchange rate. Intuitively, our results provide support to the view that crises need not emerge purely because of adverse economic fundamentals, or purely because of irrationality or inherently unstable financial markets, but because of elements of both. For the case of Mexico, it appears, that political shocks followed by radical shifts in economic policy awoke animal spirits in the private sector. This, combined with a historically weak credibility in economic policy in Mexico, led to a speculative attack and panic that resulted in the 1994 Mexican crisis.

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#### APPENDIX

## A. Derivation of tangency conditions for $\phi$ and q:

The tangency condition for  $\phi$  and q can be written as

(A1) 
$$\mu \gamma f(\gamma \overline{q} - \overline{\phi}) = 1$$
 (with  $\overline{\phi} < \gamma \overline{q}$ )

(A2) 
$$\mu \gamma f(\gamma q - \phi) = 1$$
 (with  $\overline{\phi} > \gamma \overline{q}$ ).

From A1,

(A3) 
$$(\gamma \overline{q} - \overline{\phi}) = f^{-1} \left( \frac{1}{\mu \gamma} \right) > 0 \text{ since } \overline{\phi} < \gamma \overline{q}$$

where  $f^{-1}(\cdot)$  denotes the inverse function of  $f(\cdot)$  and here is shown to positive for  $\overline{\phi} < \gamma \overline{q}$ . Using,  $q_t = \mu F(\gamma q_t - \phi_t)$ , from equation (16), we can write

(A4) 
$$\overline{\phi} = \gamma \mu F(\gamma \overline{q} - \overline{\phi}) - f^{-1} \left(\frac{1}{\mu \gamma}\right),$$

and using (A3) we write,

(A5) 
$$\overline{\phi} = \gamma \mu F \left( f^{-1} \left( \frac{1}{\mu \gamma} \right) \right) - f^{-1} \left( \frac{1}{\mu \gamma} \right).$$

Since the inverse function of  $f(\cdot)$  is negative when  $\phi < \gamma q$ , the low tangency point,  $(\phi)$ , is

(A6) 
$$\underline{\phi} = \gamma \mu F \left( -f^{-1} \left( \frac{1}{\mu \gamma} \right) \right) + f^{-1} \left( \frac{1}{\mu \gamma} \right).$$

### **B.** Derivation of the Estimation test

The estimation procedure is based on Jeanne (1997) and inspired by Dagsvic and Jovanovic (1994). This is based on three equations and a matrix form:

(B1) 
$$\hat{q}_t = q_t + \eta_t, \qquad \eta_t \sim N(0, \sigma_\eta^2)$$

(B2) 
$$q_t = \mu F_\sigma (\gamma q_t - \phi_t),$$

(B3) 
$$\phi_t = c' v_t,$$

(B4) 
$$\Theta = \begin{bmatrix} \theta(1,1) & 0 & \theta(1,3) \\ 0 & 1 & 0 \\ \theta(3,1) & 0 & \theta(3,3) \end{bmatrix},$$

where  $\hat{q}_t$  is the actual probability of devaluation and  $\eta_t$  is a prediction error. Equation (B2) corresponds to equation (16) where  $F(\cdot)$  is the cumulative normal distribution with variance  $\sigma^2$  and  $\gamma$  normalised to unit. Equation (B3) is a linear specification of the fundamentals, where  $v_t$  is a matrix including a vector of ones and the relevant macroeconomic variables and c is a vector of coefficients. The state selection process follows a stochastic Markov process that is independent of economic fundamentals so that we can empirically distinguish animal spirits from the latter. This state transition, given in (B4), is captured by the transition matrix,  $\Theta = (\Theta(i, j))_{1 \le i, j \le 3}$ , where  $\Theta(i, j)$  are the probabilities of transition from state *i* to *j* when fundamentals lie in the range  $(\phi, \phi)$ . In matrix  $\Theta$ , the economy is constrained to be either in state 1 or 3, to capture the fact that state 2 is less plausible and tends to become dynamically unstable. Setting  $\gamma = 1$ , then from  $\mu \gamma f(0) > 1$  with  $f(0) = \frac{1}{\sigma \sqrt{2\pi}}$ , the condition for multiple equilibria can be written as,

 $z = \frac{\mu}{\sigma\sqrt{2\pi}} > 1$ . The parameters to be estimated are then  $z, c, \Theta, \mu$  and the sequence of

states  $\{s_t\}_{t=1}^{T}$  using the maximum likelihood estimation method, primarily because of the non-linear nature of the problem.<sup>28</sup> Given this and that the fundamental process is assumed to be independent of the state transition process, the probability of observing any sample value  $\hat{q}_i$  is,

(B5) 
$$f(\hat{q}_{t}) = \left(2\pi\sigma_{\eta}^{2}\right)^{\frac{1}{2}} \cdot e^{-\frac{1}{2\sigma_{\eta}^{2}}(\hat{q}_{t}-q)} \cdot \theta(s_{t-1},s_{t}),$$

where  $\theta(s_{t-1}, s_t)$ , is the probability of being in state  $s_t$  conditional on the state in the previous period. The likelihood function is therefore given by,

(B6) 
$$L = \prod_{t=1}^{T} (2\pi\sigma_{\eta}^{2})^{-\frac{1}{2}} \cdot e^{-\frac{1}{2\sigma_{\eta}^{2}}\eta_{t}^{2}} \cdot \prod_{t \in D} \theta(s_{t-1}, s_{t}) = (2\pi\sigma_{\eta}^{2})^{-\frac{T}{2}} \cdot e^{-\frac{1}{2\sigma_{\eta}^{2}}\prod_{t=1}^{T}\eta_{t}^{2}} \cdot \prod_{t \in D} \theta(s_{t-1}, s_{t})$$

<sup>&</sup>lt;sup>28</sup> Because we assumed that  $\eta$  is normal, then it follows that  $\hat{q}$  is also normally distributed, since this would then be a linear combination of a normal random variable.

where *D* denotes the time period when transition across states is possible, that is the time period  $D = \{t, \phi \in (\phi, \overline{\phi})\}$ . Taking logs, the log likelihood is

(B7) 
$$\log L = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma_{\eta}^{2}) - \frac{1}{2\sigma_{\eta}^{2}}\sum_{t=1}^{T}\eta_{t}^{2} + \sum_{t\in D}\log\theta(s_{t-1},s_{t})$$

We simplify this by concentrating  $\sigma_{\eta}$ ,  $\frac{\partial log L}{\partial \sigma_{\eta}} = -\frac{T}{\sigma_{\eta}} + \frac{1}{\sigma_{\eta}^{3}} \sum_{1}^{T} \eta_{t}^{2} = 0$ ,  $\Rightarrow \sigma_{\eta}^{2} = \frac{1}{T} \sum_{1}^{T} \eta_{t}^{2}$ .

Substituting this in the log likelihood function,

(B8) 
$$\log L = \overline{c} - \frac{T}{2} \log \left( \frac{1}{T} \sum_{1}^{T} \eta_t^2 \right) + \sum_{t \in D} \log \theta(\mathbf{s}_{t-1}, \mathbf{s}_t),$$

where  $\bar{c} = -\frac{T}{2}(1 + \log(2\pi))$  is a constant. Maximising this function over each state is equivalent to choosing the state that minimises the mean square of the prediction error at each period. This will yield the sequence of states  $\{s_t\}_{t=1}^T$ . We can then maximise,

(B9) 
$$\log L_s = \sum_{t \in D} \theta(s_{t-1}, s_t) \sum_{t \in D}^3 \theta(i, j) = 1$$
, subject to  $\sum_{t \in D}^3 \theta(i, j) = 1$ , which yields,

(B10) 
$$\hat{\theta}(\mathbf{i},\mathbf{j}) = \frac{\mathbf{n}(\mathbf{i},\mathbf{j})}{\mathbf{n}(\mathbf{i})}.$$

The estimated probability of a transition from state *i* to state *j*, is the sample proportion of the number of times the economy has jumped from state *i* to *j* to the number of times the economy was in state *i*. Once  $\hat{\theta}(i, j)$  has been determined, we can then maximise  $\log L$  over  $z, \mu$  and  $\gamma$ . Since the likelihood function is discontinuous, because of the occurrence of bifurcation, we use the *simplex algorithm* method used in Jeanne (1997). We can the test for the presence of self-fulfilling speculation using the likelihood ratio  $LR = 2 * (\log L^* - \log L_{z=1}^*)$ , under the null hypothesis  $H_0: z \le 1$ . This is distributed as a chi-square with one degree of freedom. Rejection of the null hypothesis therefore implies that we cannot accept the hypothesis of an absence of self-fulfilling speculation.