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Imperfectly Competitive Cycles with Keynesian and Walrasian Features

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Abstract

We consider a multi–sector overlapping generations model with oligopolistic firms in the output markets and wage–setting trade unions in the labour markets. A coordination problem between firms creates multiple temporary equilibria which are either Walrasian or of the Keynesian unemployment type. There exist many deterministic and stochastic equilibrium cycles fluctuating between Keynesian recession and Walrasian boom periods with arbitrarily long phases in each regime. The cycles are in accordance with certain empirical regularities. Money is neutral and superneutral, but appropriate countercyclical fiscal policies stabilize the cycles in a textbook Keynesian way.

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1 Introduction

We consider an economy with imperfect competition due to the presence of trade unions in labour market and oligopolistic firms in output markets. The strategic interaction amongst firms produces a novel coordination problem and multiple equilibria; within a dynamic macroeconomic model, this multiplicity leads to endogenous business cycles fluctuating between regimes of Keynesian unemployment and Walrasian full employment. The endogenous cycles are of particular interest compared to existing literature in that they emerge under relatively orthodox fundamentals, they generate stylized facts relating to asymmetry and procyclical vacancies, amongst others, and they may be stabilized by fiscal policies in a standard Keynesian way.

Firms interact by demanding labour at wages previously set by trade unions and producing output (under constant returns) which is sold, à la Cournot, at its market–clearing price. If aggregate labour demand exceeds available supply at the given wage, firms are rationed.[1] An individual firm's labour demand will cease to affect aggregate employment, and hence aggregate output or its price, if aggregate labour demand of rival firms by itself exceeds available supply. Thus at high levels of aggregate output, attempts by one firm to change its output by changing its labour demand produce a Walrasian rather than Cournotian price response. In a static, single–sector model we show (Propositions 1 and 2) that there is a range of wages, bounded above by the Walrasian equilibrium wage, within which this strategic interaction amongst firms produces two equilibria, one with low output and Keynesian unemployment (Cournotian response), and one with full employment output and excess vacancies (Walrasian response). Assuming a sunspot signal ("recession" or "boom") resolves the coordination problem, a trade union wishing to maximize worker surplus will choose wages leading to Keynesian unemployment in a recession and Walrasian equilibrium in a boom (Proposition 3).

We then embed this in a fairly orthodox overlapping generations model (constant returns production functions, gross substitutes utility functions) with a large num-

[1] In a number of models which analyse such a given wage Cournot scenario (most explicitly in d'Aspremont, Dos Santos Ferreira and Gérard–Varet (1989, 1995), Kaas (1998), Schultz (1992)), it is simply assumed to be infeasible for firms to offer such labour demands. Here we assume instead, as seems at least reasonable with given wages, that excess demands/supplies are rationed according to some well–behaved rationing mechanism.

ber of sectors, each with its own output, set of firms, sector–specific labour and trade union. With a deterministic or stochastic sunspot evolution (boom or recession in each period), we show that under laissez–faire, there exists a multiplicity of deterministic and stochastic rational expectations equilibria with endogenous cycling between boom periods of Walrasian full employment and recession periods of Keynesian unemployment (Theorem 1). The cycles typically exhibit the empirically plausible asymmetry properties of steepness, deepness and sharpness (see McQueen and Thorley (1993), Sichel (1993)), and co–movements of procyclical vacancies, real wages and inflation and countercyclical markups. Moreover, traditional Keynesian countercyclical fiscal policies (e.g. positive government expenditure in recessions but not in booms) can generate traditional responses, i.e. unit multipliers when the budget is balanced by lump–sum taxes in each period (Theorem 2) which become greater than one with unbalanced budgets at the beginning of a recession, balanced by lump–sum taxes at the end (Theorem 3).

The literature contains a number of models which generate multiple equilibria and/or endogenous business cycles from varying imperfect competition assumptions (see Silvestre (1995) for a relevant survey). Closest to our model are the overlapping generations models of d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1995), Jacobsen (2000) and Rivard (1994),[2] in all of which the cycles require some gross complementarity in demand, or increasing returns to labour.[3] A novelty of our paper is that we need no such assumptions, the cycles emerging from our model of union–firm interaction in sectoral product and labour markets which needs nothing "unusual" in fundamentals.[4] Because of its multiple temporary equilibria, our

[2]Also close to our model in the sense that they embody similar firm coordination problems but with wage–setting firms rather than trade unions are Gaygisiz and Madden (2002) where the labour market is spatially differentiated and multiple steady–states emerge, and the static model of Kaas and Madden (2000a) where the labour market is homogeneous but involuntary unemployment results despite the absence of trade unions.

[3]Gross complement assumptions are required in Jacobsen (2000), as they are for the perfectly competitive cycles of Benhabib and Day (1982) and Grandmont (1985). Rivard (1994) assumes increasing returns, d'Aspremont et al. (1995) assume either increasing returns or gross complementarity.

[4]In the wider multiple equilibrium literature, features of other models which create multiplicities are also absent from our paper, including (again) the increasing returns of Kiyotaki (1988), Manning (1990, 1992), Rivard (1994), the differing elasticities of consumption and investment demand of Gali (1994) and the strategic complementarity in the entry of new firms of Chatterjee, Cooper, and Ravikumar (1993).

model differs technically from most of the existing literature in that the backward (and forward) dynamics of our model follows a set–valued difference equation (i.e., the right hand side is a correspondence rather than a function), where the selection from the correspondence is dictated by the sunspot series. A large set (an infinity) of deterministic cycles emerges which are all locally determinate (stable in the backward perfect foresight dynamics), as well as nearby stochastic equilibria.[5] Our fluctuations are thus very much dependent on "animal spirits". However, a further important novelty of the paper compared with existing endogenous business cycle models is that it is unique in generating, in particular, the asymmetry and procyclical vacancy "stylized facts".

Although money is neutral and superneutral – as in other models of perfect and imperfect competition with rational expectations but without nominal rigidities – our Keynesian fiscal policy results stand in contrast to other models of the literature in which policy responses are of a more Walrasian nature. Dixon (1987), Mankiw (1988) and Bénassy (1995) consider general equilibrium models of imperfect competition and find positive balanced–budget multipliers less than one. In their models, the positive effect on output follows from a stimulation of labour supply since a higher tax burden causes a lower demand for leisure, provided that leisure is a normal good.[6] Jacobsen and Schultz (1994) consider an overlapping generations model with an imperfectly competitive labour market and show that fiscal policy can only affect output when the public and the private demand elasticities differ. If these elasticities are equal, there is full crowding out since price changes completely offset the increase in aggregate demand. In our model, a higher (lump–sum) tax burden has no effect on labour supply, and prices and wages are unaffected by the fiscal policy during recessions. Therefore, our model has unit multipliers and reproduces the policy result of the seminal article of Hart (1982) in a dynamic general equilibrium model.

The paper is organized as follows. Section 2 presents the static, single–sector model which is embedded in Section 3 in a dynamic multi–sector general equilibrium model.

[5] Azariadis and Smith (1998) consider a growth model with asymmetric information in the credit market which exhibits also multiple equilibria, set–valued dynamics and an infinity of locally stable cycles fluctuating between two regimes.

[6] Similar labour supply effects are also at work in the Walrasian OLG economies of Grandmont (1986) and Aloi, Jacobsen, and Lloyd-Braga (2000) who show that appropriate monetary and fiscal policy rules can stabilize endogenous fluctuations.

Section 4 studies endogenous business cycles under laissez–faire, and Section 5 contains our policy results. Section 6 concludes. All proofs not included in the text are contained in Appendix A.

2 The static single–sector model

We consider a single sector of an economy in which N price–setting firms produce a homogenous output good from inputs of sector–specific labour. Firms have constant returns production technologies $y^j = \ell^j$, $j = 1, \ldots, N$. Output demand is uniformly elastic with elasticity $\rho > 1$, and the inverse output demand function is denoted $p(y^1 + \ldots + y^N)$. There is a continuum $[0, L]$ of workers, each endowed with one indivisible unit of labour. Workers supply labour only if the wage exceeds the common reservation wage w^r. They are represented by a wage–setting trade union who aims to maximize the workers' surplus $\ell(w - w^r)$, where $\ell \leq L$ denotes sector employment and w denotes the sector wage. In this section, the output demand and labour supply functions are exogenously given as described above, but they are derived in the next section from the preferences of consumers in a multi–sector dynamic general equilibrium model.

We consider a two–stage game between the trade union and the firms which is the equivalent of the conventional "right–to–manage model" for an imperfectly competitive output market.[7] After the trade union sets the sector wage w at stage I, firms decide employment at stage II by signalling labour demands (or job offers) $J^1, \ldots, J^N$ to the market. Whenever sector labour demand $\sum_{j=1}^N J^j$ is not equal to sector labour supply L, employment levels $\ell^1, \ldots, \ell^N$ are determined by rationing. Our rationing mechanism satisfies various standard and desirable properties from the rationing literature (see e.g. Bénassy (1982)). First, rationing is frictionless, so only one side of the market is rationed, or $\sum_{j=1}^N \ell^j = \min(\sum_{j=1}^N J^j, L)$. Thus, if $L > \sum_{j=1}^N J^j$, some workers are unemployed and the firms' employment levels are $\ell^j = J^j$. On the other hand, if there is excess demand for labour, aggregate employment is L and we assume that this is allocated amongst firms via some continuous

[7] We focus on a monopolistic union for simplicity, but the model may be generalized to wage bargaining at stage I between the trade union and an employers' federation interested in maximizing the expected profits of its members.

function $c : \mathbb{R}^{N-1} \to \mathbb{R}_+$ which defines employment constraints for firms so that

$$\ell^j = \min \left(J^j, c((J^k)_{k \neq j}) \right) \ , \ j = 1, \ldots, N. \tag{1}$$

Hence we are assuming that the rationing mechanism is also continuous, symmetric (in that employment constraint functions are identical and symmetric for all firms) and non–manipulable (the employment constraint of each firm does not depend on its own labour demand).[8] An important consequence of these assumptions (see Lemma A.1 in the Appendix) is that if $N-1$ firms signal the same labour demand (J say) then the remaining firm gets at least the N-th share of labour supply plus the workers who do not find employment at the other firms. That is, the employment constraint facing the remaining firm is

$$c(J, \ldots, J) = \max \left(\frac{L}{N}, L - (N-1)J \right) \ . \tag{2}$$

From the labour allocated to firms, firms produce outputs $y^j = \ell^j$, and the sector output is then sold at the market clearing price $p(y^1 + \ldots + y^N)$.[9]

Hence in the stage II game, firms simultaneously demand labour $(J^j)_{j=1,\ldots,N}$ and make profits

$$\pi^j = p(\ell^1 + \cdots + \ell^N)\ell^j - w\ell^j \ , \ j = 1, \ldots, N,$$

where employment levels $(\ell^j)_{j=1,\ldots,N}$ follow from (1). An important feature of this game is that it possesses, for a certain range of wages, multiple Nash equilibria. Consider first an unemployment equilibrium which corresponds to a Cournot–Nash equilibrium in which firms directly compete in employment/output ignoring the employment constraint. Straightforward calculations show that the Cournot–Nash labour demand is a uniformly elastic function of the sector wage w satisfying

$$\ell^{CN}(w) \equiv \left(\frac{w}{w^{CN}} \right)^{-\rho} L \quad , \text{ where } w^{CN} \equiv \frac{\rho N - 1}{\rho N} p(L) \ .$$

[8]The uniform rationing scheme (see Bénassy (1982, Appendix J)) is an example of a rationing scheme satisfying our assumptions. Non–manipulability is required to rule out unlimited overbidding and to guarantee existence of equilibrium, and symmetry is imposed for the sake of convenience.

[9]This price would also be set by firms if firms would set prices simultaneously at a subsequent stage III. Unlike the result of Kreps and Scheinkman (1983), this result does not depend on the way consumers' demand is rationed at asymmetric prices (efficient, proportional etc.) and ensures that the Bertrand price setting produces an essentially Cournot outcome whenever demand is uniformly elastic (see Madden (1998)).

Whenever $w > w^{CN}$, the Cournot–Nash labour demand leads to unemployment, $\ell^{CN}(w) < L$. We refer to w^{CN} as the Cournot–Nash labour market clearing wage. Moreover, the Cournot–Nash labour demands are also Nash equilibrium strategies of the original stage II game in job offers $J^1, \ldots, J^N$ if $w > w^{CN}$. To see this, suppose that firms $2, \ldots, N$ offer jobs $J^2 = \ldots = J^N = J = \ell^{CN}(w)/N < L/N$. Hence, according to (2), the employment constraint of firm 1 is $c = L - (N-1)J$. If firm 1 deviates from the Cournot–Nash labour demand to some J^1, its employment would be $\ell^1 = \min(J^1, c)$, whereas employment levels of all other firms would remain the same. Hence, firm 1 would make strictly lower profit. This implies

Proposition 1: If $w > w^{CN} \geq w^r$, there exists a Nash equilibrium of the stage II game with unemployment in which $J^j = \ell^j = \ell^{CN}(w)/N$, $j = 1, \ldots, N$. The output price is $p = \frac{\rho N}{\rho N - 1} w$.

The Cournot–Nash equilibrium with unemployment is not the only equilibrium of the stage II game after the trade union sets some wage $w > w^{CN}$. There may also be an equilibrium in which firms create excess vacancies. Let $w^{WE} \equiv p(L)$ denote the Walrasian wage which is the wage that would prevail if firms were price takers and if the sector output and labour markets were in equilibrium. Obviously, the Walrasian wage exceeds the Cournot–Nash market clearing wage: $w^{WE} > w^{CN}$. If $w \leq w^{WE}$, there is an equilibrium of the stage II game in which firms create an excess demand for labour. Specifically suppose that $J^j = J \geq L/(N-1)$, $j = 1, \ldots, N$, so that the labour demand from any $(N-1)$ firms exceeds the labour supply at w. Now the employment/output decision of any one firm (say firm 1) has no effect on aggregate employment and output which remain at their full employment levels. Thus, the employment decision of firm 1 does not alter the resulting output price which remains $p(L)$, and therefore firm 1 effectively behaves as a price taker. As a result, firm 1 is willing to expand its demand for labour provided that the wage is less than or equal to the Walrasian wage w^{WE}. (2) implies that the employment constraint of firm 1 is $c = L/N$ so that its employment is $\ell^1 = \min(J^1, c)$. Hence, any $J^1 \geq c$ is a best response to $J^2 = \ldots = J^N = J \geq L/(N-1)$. In particular, $J^1 = J$ is a best response. This proves[10]

[10] We thank Harald Uhlig for pointing out the possibility of excess vacancy equilibria if the rationing scheme is manipulable (e.g. proportional rationing) and if job offers incur some positive costs, thereby ruling out unlimited overbidding (see Kaas and Madden (2002)).

Proposition 2: If $w^r \leq w \leq w^{WE}$, there is a Nash equilibrium of the stage II game with excess vacancies in which $J^j = J \geq L/(N-1)$ and $\ell^j = L/N$, $j = 1, \ldots, N$. The output price is $p(L) = w^{WE}$.

The relation between the sector wage w and sector employment in the stage II equilibria of Propositions 1 and 2 is illustrated in Figure 1. For wages in the interval $(w^{CN}, w^{WE}]$ there exist two Nash equilibria, one with unemployment and one with full employment and excess vacancies. We assume that firms coordinate on one of these equilibria in accordance with an exogenous (sunspot) state S. More specifically, in a "boom" state $S = B$ we assume that firms expect tight labour market conditions leading to excess demand for labour and full employment for all $w \leq w^{WE}$, such that the equilibrium of Proposition 2 emerges. In a "recession" state $S = R$, firms expect slack labour market conditions producing involuntary unemployment for all $w > w^{CN}$. Obviously, for all wages $w > w^{WE}$ an unemployment equilibrium follows and for all wages $w \leq w^{CN}$ a full employment equilibrium results, irrespective of the state R or B. This behaviour of firms leads to two eventual employment functions $\ell^B(w)$ and $\ell^R(w)$ which are

$$\ell^B(w) = \begin{cases} \ell^{CN}(w) & , \ w > w^{WE}, \\ L & , \ w \leq w^{WE}, \end{cases} \qquad \ell^R(w) = \begin{cases} \ell^{CN}(w) & , \ w > w^{CN}, \\ L & , \ w \leq w^{CN}. \end{cases}$$

The sectoral trade union observes the sunspot state S and anticipates the labour demand behaviour of firms which gives rise to $\ell^S(w)$, $S = R, B$. Hence the trade union sets the wage w to maximize $(w - w^r)\ell^S(w)$. The solution to this problem is stated in the following Proposition.[11]

Proposition 3:

(a) If $S = R$, the union's optimal sector wage is $w = \dfrac{\rho}{\rho - 1} w^r$, sector employment is $\ell^{CN}(w) < L$ and the sector output price is $p = \dfrac{\rho N}{\rho N - 1} w$, provided that $w > w^{CN}$.

[11] The equilibrium derived in this Proposition is a subgame perfect Nash equilibrium of the two–stage game for the two selections of stage II equilibria described by $\ell^B(w)$ and $\ell^R(w)$. Of course there are many other possible subgame perfect equilibria based on other selections, but these two selection rules are the only ones for which the selection is independent of the sector wage.

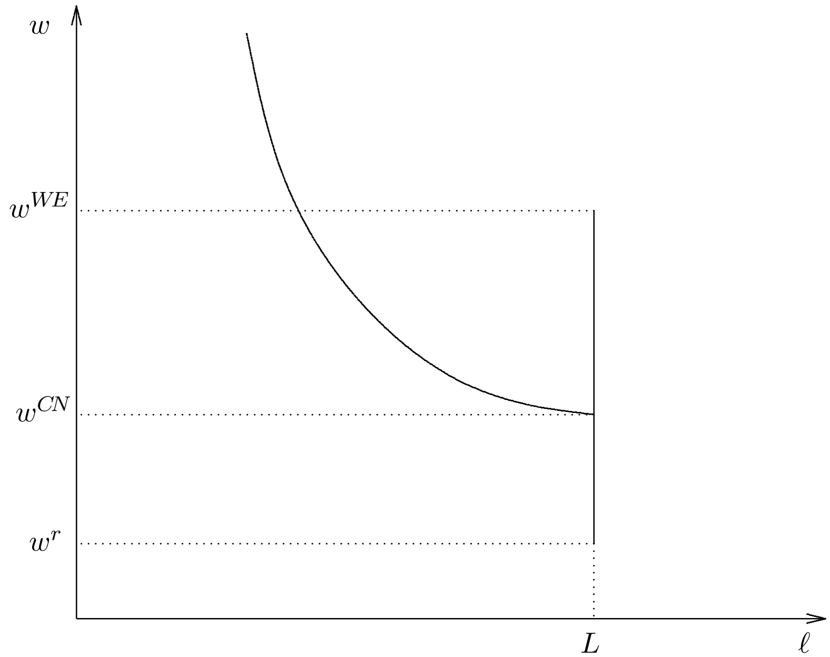

Figure 1: Sector employment and the sector wage.

(b) If $S = B$, the union's optimal sector wage is $w = w^{WE}$, sector employment is $\ell = L$ and the sector output price is $p = w$, provided that $w^{WE} \geq zw^{r}$ where $z \in (1, \rho/(\rho - 1))$ (to be defined in the proof).

In a recession state, the union demands a markup over the workers' reservation wage and there is unemployment, provided that the reservation wage is not too low. If $w^{r} \leq (\rho - 1)w^{CN}/\rho$, there would be full employment at the Cournot–Nash market clearing wage $w = w^{CN}$. This case is not relevant for the subsequent analysis, so we neglect it for the ease of exposition.

In a boom state, and if the reservation wage is not too high, the union sets the Walrasian wage, anticipating that this is the highest wage at which firms create excess vacancies and full employment.[12] If $w^{WE} < zw^{r}$, the equilibrium would be as specified in (a) involving unemployment, but again this case is not relevant for the subsequent analysis.

[12] One may wonder why firms do not overbid the union wage when they are on the long side of the market, so as to attract more workers. It is easy to see, however, that they would not do so at the Walrasian wage at which they get zero profits, since any overbidding would incur losses.

3 The dynamic multi–sector model

The economic environment

We consider now a dynamic general equilibrium model in discrete time $t = 0, \ldots, N$ comprising a continuum of sectors $s \in [0, 1]$, each of which being a copy of the representative sector of the previous section. There are three types of goods: labour, output and fiat money. In every sector s and in every period t, N firms produce the sector output from inputs of sector–specific labour under unit constant returns to scale, and there is a continuum $[0, L]$ of workers represented by a single utilitarian trade union. The sector price, wage, employment and output are denoted p_{st}, w_{st}, ℓ_{st}, y_{st}.

Workers (consumers) are described by two–period lived overlapping generations who supply labour in the first period of their life and who consume in both lifetime periods. Their labour endowment possesses specific attributes which allows them to be employed only in one sector, but not elsewhere in the economy.[13] However, young consumers receive profit income from various sectors (a negligible fraction coming from any one sector) and they consume output goods of all sectors. To finance future consumption, consumers save part of their income as fiat money. Preferences of consumers are represented by the utility function

$$u(C_t, C_{t+1}) - b\ell_t \ ,$$

$$C_t = \Big(\int_0^1 c_{st}^{(\rho-1)/\rho} ds \Big)^{\rho/(\rho-1)} \ , \ C_{t+1} = \Big(\int_0^1 c_{s,t+1}^{(\rho-1)/\rho} ds \Big)^{\rho/(\rho-1)} \ ,$$

where c_{st} and $c_{s,t+1}$ denote consumption of sector s output in period t, $t+1$ respectively, $\rho > 1$ is the elasticity of substitution between sector outputs, $\ell_t \in \{0, 1\}$ is labour supply, and $b \geq 0$ is disutility of work. u is assumed to be twice differentiable, strictly quasi–concave, strictly monotone, homogenous of degree one, and such that indifference curves do not cut the axes.

There is a government that consumes output goods, levies a lump–sum tax on young consumers, pays a nominal interest rate on money holdings (or taxes money

[13] One can think of the workers being endowed with sector–specific skills. If workers were employed in some other sector, their productivity would be considerably lower so that they cannot be paid a wage above their reservation wage. Indeed, empirical evidence suggests that industry–specificity contributes more to wage profiles than firm–specificity (see Neal (1995) and Parent (2000)).

holdings), and finances its deficit by seignorage. We denote by g_t the consumption index of the government[14] and by τ_t the real aggregate lump–sum tax. Then the government's period budget constraint is

$$M_t = M_{t-1} I_t + P_t g_t - P_t \tau_t \quad ,$$

where M_t denotes the money stock at the end of period t and I_t denotes the nominal gross interest rate. $P_t = \Big(\int_0^1 p_{st}^{1-\rho} ds \Big)^{1/(1-\rho)}$ is the aggregate price index.

From consumption demand of young and old consumers and of the government we obtain the aggregate demand identity (a detailed derivation is contained in the Appendix)

$$Y_t = \frac{M_t}{P_t \Big(1 - c\Big(\frac{P_{t+1}}{P_t I_{t+1}} \Big) \Big)} + \tau_t \ . \tag{3}$$

Here, $c(.)$ denotes the propensity to consume of young consumers as a function of the real interest rate. We assume that intertemporal consumption goods are gross substitutes, so that $c' \geq 0$ (savings are non–decreasing in the expected real interest rate). Sector output demands are the familiar uniformly elastic functions

$$y_{st} = \Big(\frac{p_{st}}{P_t} \Big)^{-\rho} Y_t \quad .$$

A sectoral labour market is in equilibrium if $y_{st} = \ell_{st} = L$ which implies that $p_{st} = P_t (L/Y_t)^{-1/\rho}$. Hence, the Walrasian sector wage ($w_{st} = p_{st}$) is the same for all sectors and satisfies

$$w_t^{WE} = P_t \Big(\frac{L}{Y_t} \Big)^{-1/\rho} \quad .$$

Similarly, the Cournot–Nash labour market clearing wage satisfies

$$w_t^{CN} = \frac{\rho N - 1}{\rho N} w_t^{WE} \quad .$$

Finally, from consumers' preferences we derive the reservation wage of workers (details are again in the Appendix) which is

$$w_t^r = b P_t \omega \Big(\frac{P_{t+1}}{P_t I_{t+1}} \Big) \ , \tag{4}$$

where $\omega(\theta) = u(c(\theta), (1 - c(\theta))/\theta)^{-1}$ is a strictly increasing function of the inverse of the real interest rate, θ. Hence, the reservation wage (and thereby union wage

[14]Specifically, we assume that $g_t = (\int_0^1 g_{st}^{(\rho-1)/\rho} ds)^{\rho/(\rho-1)}$, so that substitution elasticities (and hence demand elasticities) of the government and of consumers are the same. Therefore, there are no elasticity effects of fiscal policy unlike Jacobsen and Schultz (1994).

demands) go up when inflation rises. The Appendix also shows that a utilitarian sectoral trade union aims to maximize the workers' surplus $\ell_{st}(w_{st} - w_t^r)$.

Temporary equilibrium

We are now ready to formulate the temporary equilibrium of this economy in a period t which is defined as an equilibrium of prices, wages, employment and output given expectations about future prices and interest rates and a realization of the sunspot state S_t. We impose, for simplicity, the assumption that all sectors coordinate on the same sunspot state B or R, thus focusing on economy–wide booms and recessions and ignoring intersectoral fluctuations. Proposition 3 then implies that unions in all sectors set the same wage $w_{st} = w_t$ and that employment, output and prices in all sectors coincide: $\ell_{st} = \ell_t$, $y_{st} = Y_t = \ell_t$, and $p_{st} = P_t$.

Suppose that state R prevails in period t. Then from Proposition 3 (a) and (4), a temporary equilibrium (P_t, w_t, Y_t) with unemployment satisfies the aggregate demand identity (3) and

$$w_t = \frac{P_t b \rho}{\rho - 1} \omega\Big(\frac{P_{t+1}}{P_t I_{t+1}}\Big), \ P_t = \frac{\rho N}{\rho N - 1} w_t \ , \ Y_t < L. \tag{5}$$

In the terminology of the literature on disequilibrium macroeconomics (see e.g. Bénassy (1982)), such an equilibrium is of the Keynesian unemployment type since there is an "excess supply" on the labour and on the output market (more precisely, at the prevailing prices firms would want to sell more output and households would want to work more).

If state B prevails in period t, the condition $w_t^{WE} \geq z w_t^r$, which is required for a full employment equilibrium $Y_t = L$, turns out to be

$$z b \omega\Big(\frac{P_{t+1}}{P_t I_{t+1}}\Big) \leq 1 \ . \tag{6}$$

Hence, in this case a temporary equilibrium (P_t, w_t, Y_t) is a Walrasian equilibrium satisfying (3), (6), $Y_t = L$, and $w_t = P_t$. It is worth stressing that this equilibrium involves excess vacancies, so that the labour market is, strictly speaking, in disequilibrium. But since prices, wages, output and employment are Walrasian, we refer to this equilibrium as a Walrasian equilibrium. As mentioned before, there can also be an unemployment equilibrium if $S_t = B$ as there can be a full employment equilibrium if $S_t = R$. Such equilibria do not change the set of intertemporal equilibria, however, and are therefore neglected.[15]

Intertemporal equilibrium

Suppose now that the sequences of policy parameters (I_t), (g_t), (τ_t), an initial stock of money M_0, and a sequence of states (S_t) are given. We are interested in the set of intertemporal equilibria with perfect foresight. Denoting real balances by $m_t = M_t/P_t$, the government's budget constraint is

$$m_{t+1} - g_{t+1} + \tau_{t+1} = m_t \frac{I_{t+1} P_t}{P_{t+1}} \ . \tag{7}$$

Using this identity, the aggregate demand equation (3) can be rewritten as

$$Y_t = \frac{m_t}{1 - c\Big(\frac{m_t}{m_{t+1} - g_{t+1} + \tau_{t+1}}\Big)} + \tau_t \ . \tag{8}$$

If $S_t = R$, (5) implies that the real interest rate is a constant:

$$\frac{I_{t+1} P_t}{P_{t+1}} = \alpha \equiv \frac{1}{\omega^{-1}((\rho N - 1)(\rho - 1)/(bN\rho^2))} \ .$$

From (7), a Keynesian unemployment equilibrium satisfies

$$m_{t+1} - g_{t+1} + \tau_{t+1} = \alpha m_t \quad \text{and} \quad Y_t < L \ . \tag{9}$$

If $S_t = B$, (6) implies a lower bound on the real interest rate:

$$\frac{I_{t+1} P_t}{P_{t+1}} \geq \delta \equiv \frac{1}{\omega^{-1}(1/(bz))} \ .$$

Note that $\delta < \alpha$ since ω is strictly increasing and since $z < \rho/(\rho - 1)$. (7) implies that a (Walrasian) full employment equilibrium satisfies

$$m_{t+1} - g_{t+1} + \tau_{t+1} \geq \delta m_t \quad \text{and} \quad Y_t = L \ . \tag{10}$$

An intertemporal equilibrium is a sequence $(m_t, Y_t)_{t \geq 0}$ satisfying the aggregate demand identity (8) together with either (9) (if $S_t = R$) or (10) (if $S_t = B$). Note that, for a given sequence of states S_t and fiscal parameters (g_t, τ_t), the set of intertemporal equilibria is independent (in real terms) of the initial money stock and of the sequence of nominal interest rates. Thus, money is neutral and superneutral, as is the case in competitive overlapping generations models (see Grandmont (1986)).[16]

[15] For a detailed treatment of all equilibrium cases, see the previous working paper version Kaas and Madden (2000b). See also footnote 17 below.

[16] It is worth mentioning that money need neither be neutral nor superneutral in models of

4 Endogenous business cycles

In this section we show how the model produces endogenous cycles and we suppose the "laissez–faire" case in which the government engages in no fiscal policy (LF):

(LF) For all $t \geq 0$, $g_t = \tau_t = 0$.

In this case, the bold curves in Figure 2 illustrate the equilibrium conditions (9) and (10). The line OAC indicates the curve $m_{t+1} = \delta m_t$, the line OBD is $m_{t+1} = \alpha m_t$. The curve OABE is the Walrasian equilibrium condition

$$Y_t = \frac{m_t}{1 - c\left(\frac{m_t}{m_{t+1}}\right)} = L \ , \tag{11}$$

which is upward–sloping if consumption in period t and $t+1$ are gross substitutes, i.e. $c' > 0$ (for instance, if u is CES with elasticity of substitution greater than 1). Condition (9) is described by the line OB and (10) is described by the curve ABE.[17]

It can be seen from Figure 2 that there are generically only steady states with full employment. When $\delta < 1$ and $\alpha \neq 1$, there exists a unique steady state $\bar{m}$ which is determinate (stable in the backward perfect foresight dynamics). When $\alpha < 1$, the only other intertemporal equilibria are inflationary equilibria converging to autarky ($m = 0$), zero production and zero employment. However, when $\alpha > 1$, there is scope for equilibrium cycles fluctuating between Walrasian equilibria (WE) in boom states and Keynesian unemployment equilibria (KU) in recession states. More specifically, whenever $\delta < 1$ and $\alpha > 1$, there may exist cycles between the curves OB and AB. Figure 2 illustrates a (deterministic) cycle of order 7 in which firms coordinate in five successive periods on state R and in two successive periods on state B. Below we will prove an existence theorem for general (k, l) cycles which are defined as deterministic cycles of order $k + l$ with k successive recession periods and

imperfect competition, even if there are no menu costs or other nominal rigidities, as has been stressed by Rankin (1992) and Rankin (1995). Rankin's results are due to the sensitivity of perfect foresight equilibria to the price forecast functions of consumers. These effects play no role in our model, however, since changes of the price in one sector will not affect the aggregate future price level under any reasonable assumption on the forecasting behaviour of consumers.

[17] A full employment equilibrium in state $S_t = R$ is on the segment BE, but it is not a Walrasian equilibrium since the trade unions set w_t^{CN} and firms make positive profits. An unemployment equilibrium in $S_t = B$ would also be on the line OB, but strictly below B.

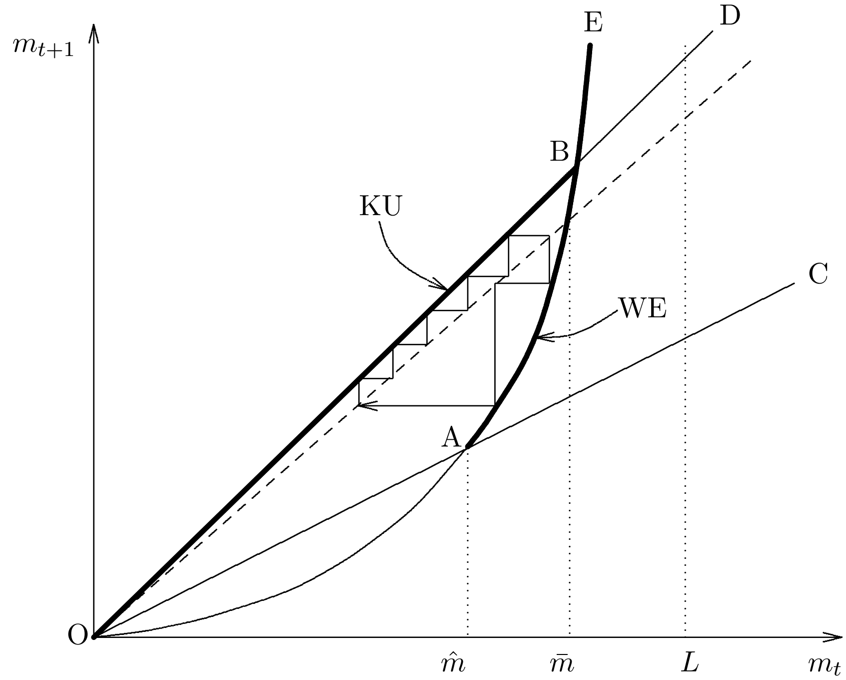

Figure 2: The equilibrium curves and a cycle of order 7.

l successive boom periods.[18] All (k, l) cycles turn out to be asymptotically stable in the local backward perfect foresight dynamics, so they are also stable in the dynamics under adaptive learning under some mild assumptions on the forecast function (see Grandmont and Laroque (1986)). Using the generic arguments of Azariadis and Guesnerie (1986), we also show the existence of non–degenerate stationary sunspot equilibria close to deterministic (k, l) cycles. Though our subsequent analysis is confined to the deterministic cycles, most arguments remain valid for the nearby stochastic cycles as well.

Theorem 1: Assume that the government follows the laissez–faire policy rule (LF) and let $\delta < 1 < \alpha$. Then there exists a $(k, 1)$ cycle if and only if $\alpha^k \delta \leq 1$. Whenever a $(k, 1)$ cycle exists, then there exists also a (k', l) cycle for any $1 \leq k' \leq k$ and any $l \geq 1$. Any (k, l) cycle is unique, and if a (k, l) cycle is interior, i.e. if the inequality in

[18] There are of course many more deterministic cycles fluctuating between recessions and booms more than once until they return back to their starting point or even irregular cycles, but they will not be considered here.

(10) is strict along the cycle, it is asymptotically stable in the local backward perfect forward dynamics and there exist non–degenerate stationary sunspot equilibria of cardinality $k + l$ close to this cycle.

Theorem 1 is related to Proposition 3 of Azariadis and Smith (1998) who consider a growth model with imperfect information in the credit market, multiple temporary equilibria, and an equilibrium dynamics fluctuating between a Walrasian regime and a regime of credit rationing. Their model has multiple steady states and a large variety of deterministic (and nearby stochastic) cycles spending an arbitrary number of periods in each of the two regimes. In our model, only the number of boom periods is indeterminate, whereas the number of recession periods is bounded above. This is because there is only one steady state in our model which is in the Walrasian regime.

The cycles of our model can exhibit some stylized features of business cycles. First, our cycles exhibit some types of business cycle asymmetries, as illustrated in Figure 3 showing the output time series of a (5,3) cycle. The first period of a recession is characterized by a large fall in output followed (when $k > 1$) by a gradual climb back to full employment. Moreover when $l > 1$, troughs are deeper than peaks are tall (i.e. there is negative skewness relative to the mean). Hence, the cycles exhibit steepness and deepness (see Sichel (1993)). They also exhibit sharpness (see McQueen and Thorley (1993)) since (when $l > 1$) growth rate changes at troughs are larger than at peaks. Second, booms are characterized by excess demand for labour, which can be interpreted as vacancies, unlike recessions: thus procyclical vacancies emerge. Third, since real wages are higher in booms than recessions we have procyclical real wages and countercyclical markups. Fourth, (7) implies that inflation is procyclical whenever the (exogenous) sequence of nominal interest rates is procyclical or acyclical.

We now examine briefly the empirical plausibility of our cycles condition. In the case of a Cobb–Douglas intertemporal utility function the conditions $\alpha\delta < 1$ and $\alpha > 1$ for which cycles exist are equivalent to

$$\frac{(N\rho - 1)(\rho - 1)}{N\rho^2} < b < \Big(\frac{(N\rho - 1)(\rho - 1)}{N\rho^2} z\Big)^{1/2} \quad . \tag{12}$$

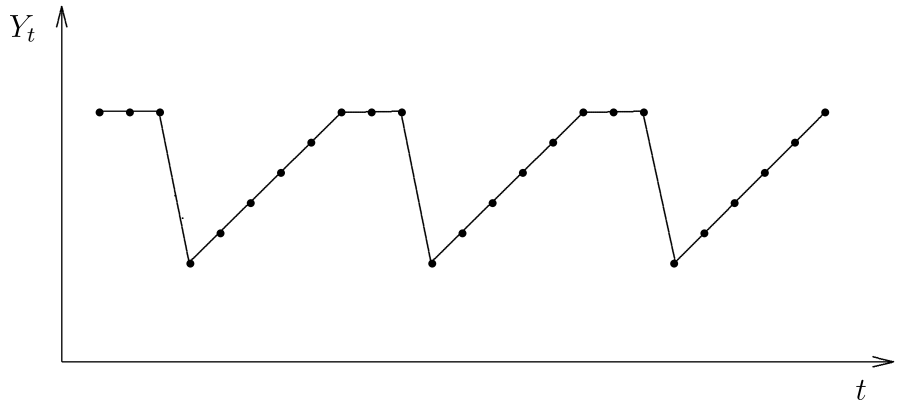

Figure 3: A steep, deep and sharp cycle ($k = 5$, $l = 3$).

Since the real reservation wage is $w_t^r/P_t = b(P_{t+1}/(P_t I_{t+1}))^{1-c}$ and since under (LF) the average real gross interest rate along each cycle is 1,[19] b is a reasonable measure of the ratio between the real reservation wage and labour productivity.[20] A labour share between 0.6 and 0.7 and a replacement ratio between 0.5 and 0.8 suggest that a rough interval of plausible values for b is $[0.3, 0.56]$. Furthermore, empirical studies report average demand elasticities just below 2 and markup factors between 1.2 and 1.4 (see Rotemberg and Woodford (1995, pp. 260–61)). Table 1 summarizes the upper and lower bounds for b of the cycles condition (12) for some values of ρ and N compatible with these studies. They are well in accordance with empirically plausible values for b.

	$N = 2$	$N = 3$	$N = 4$	$N = 5$
$\rho = 1.5$	[.222,.411]	[.259,.420]	[.278,.419]	[.289,.416]
$\rho = 2.0$	[.375,.558]	[.417,.569]	[.438,.570]	[.450,.568]

Table 1: Values of b leading to cycles.

[19]This follows from (7) and from $1 = (m_1/m_2)\cdots(m_{k+l-1}/m_{k+l})(m_{k+l}/m_1)$ along each (k, l) cycle.

[20]Labour productivity has been normalized to unity here, but if $y = A\ell$ is the production technology, b must be replaced by b/A in condition (12).

5 Fiscal policy

The purpose of this section is to show how appropriate fiscal policies can be effective in stabilizing cycles in a standard textbook Keynesian fashion. Our analysis concentrates on deterministic cycles, but we expect similar results for nearby stochastic sunspot cycles as well. We consider two different types of policies, a balanced budget policy in which the government raises spending and taxes during recessions, and an unbalanced budget fiscal policy in which the government runs deficit spending at the beginning of each recession and re–balances the budget at the end of the recession. Both types of policies raise output but do not affect equilibrium prices, the first with a fiscal multiplier of one, and the latter with a fiscal multiplier in excess of one. Later we point out some limitations of these results.

Theorem 1 describes endogenous cycles in the model without fiscal policy. To investigate the efficacy of fiscal policy, we assume the economy has been in a laissez–faire regime up to period $t = 0$, and has coordinated on a (k, l) cycle for some $k, l \geq 1$. At $t = 0$ the government announces its fiscal policy and we look at the impact of the (perfectly foreseen) policy on the laissez–faire (k, l) cycle. In particular, we assume that the selection of the cycle is invariant to the policy change. We further assume the government (like firms and trade unions) observes S_t and we look first at a balanced budget fiscal policy with $g_t = \tau_t$ for all t.

Suppose the laissez–faire (k, l) cycle has been $(m_1^*, \ldots, m_{k+l}^*)$ with outputs $(Y_1^*, \ldots, Y_{k+l}^*)$ where

$$m_{i+1}^* = \alpha m_i^* \quad \text{and} \quad Y_i^* = \frac{m_i^*}{1 - c(m_i^*/m_{i+1}^*)} < L, \ i = 1, \ldots, k, \tag{13}$$

$$m_{i+1}^* \geq \delta m_i^* \quad \text{and} \quad Y_i^* = \frac{m_i^*}{1 - c(m_i^*/m_{i+1}^*)} = L, \ i = k+1, \ldots, k+l, \tag{14}$$

(from now on identify $i = k + l + 1$ with $i = 1$). A natural Keynesian policy to cure the unemployment in the first k periods of the cycle is to introduce positive government spending during recessions:

(KBB) For all t, $g_t = \tau_t \geq 0$ if $S_t = R$ and $g_t = \tau_t = 0$ if $S_t = B$.

In particular, given the laissez–faire (k, l) cycle, suppose for $i = 1, \ldots, k$

$$g_t = g_i \quad \text{if} \quad S_t = \ldots = S_{t-i+1} = R \text{ and } S_{t-i} = B.$$

Thus g_i is the expenditure in the ith period of a recession. With this policy, a new (k, l) cycle $(m_1, \ldots, m_{k+l})$ with outputs $(Y_1, \ldots, Y_{k+l})$ has to satisfy

$$m_{i+1} = \alpha m_i \quad \text{and} \quad Y_i = \frac{m_i}{1 - c(m_i/m_{i+1}))} + g_i \leq L, \ i = 1, \ldots, k, \tag{15}$$

$$m_{i+1} \geq \delta m_i \quad \text{and} \quad Y_i = \frac{m_i}{1 - c(m_i/m_{i+1}))} = L, \ i = k+1, \ldots, k+l. \tag{16}$$

Suppose that $g_i \in [0, L - Y_i^*]$, $i = 1, \ldots, k$. Then it is immediate from (13), (14), (15) and (16), that there exists indeed a new cycle $(m_1, \ldots, m_{k+l})$, $(Y_1, \ldots, Y_{k+l})$ with the following features:

(i) For all $i = 1, \ldots, k+l$, $m_i = m_i^*$. Since the nominal money stock evolves under the balanced budget policy as under laissez–faire, prices also evolve as under laissez–faire. Hence, nominal prices and so wages are unaffected by the fiscal policy. The economy responds to the policy in a "fixprice" fashion.

(ii) For $i = k+1, \ldots, k+l$, $Y_i = Y_i^* = L$. Unsurprisingly the policy has no real effects during booms.

(iii) For $i = 1, \ldots, k$, $Y_i = Y_i^* + g_i$. During recessions the economy responds to the government expenditure with g_i producing a one–for–one increase in output. We have the classic textbook unit balanced budget multiplier operating. And if $g_i = L - Y_i^*$, $i = 1, \ldots, k$, output is stabilized at the full employment level throughout the cycle.

Theorem 2: Suppose the laissez–faire economy has coordinated on a (k, l) cycle up to period 0, with real balances $(m_1^*, \ldots, m_{k+l}^*)$ and outputs $(Y_1^*, \ldots, Y_{k+l}^*)$ given by (13) and (14). Suppose the government announces in period 0 a Keynesian balanced–budget fiscal policy (KBB) in which expenditures are $g_i \in [0, L - Y_i^*]$ in the ith period of a recession and zero throughout a boom. Then the policy produces a perfect foresight (k, l) cycle in which recession output levels increase with a balanced–budget multiplier of one and in which nominal prices and wages are as under laissez–faire. If $g_i = L - Y_i^*$, $i = 1, \ldots, k$, output is stabilized at the full employment level throughout the cycle.

In the textbook fixprice story, the balanced–budget multiplier of one gives way to a multiplier in excess of unity with unbalanced budgets where the government expenditure is not matched by current taxation. A similar outcome can emerge here. Start again from a laissez–faire (k, l) cycle given by (13) and (14) and consider the following policy (suppose $k \geq 2$):

(KUB) $g_t = g > 0$ if $S_t = R$ and $S_{t-1} = B$, and $g_t = 0$ otherwise.
$\tau_t = \tau = \alpha^{k-1} g > 0$ if $S_t = R$ and $S_{t-k} = B$, and $\tau_t = 0$ otherwise.

Here the government spends g in the first period of a recession and imposes the tax τ in the last recession period. The money supply thus expands at the beginning of the recession and contracts at the end. The restriction $\tau = \alpha^{k-1} g$ means that the contraction exactly matches the expansion since α is the real gross interest rate during recessions (see also the proof of Theorem 3). If g is small then the economy's response is again "fixprice", but now with multipliers (on g) in excess of unity for all periods of the recession.[21]

Theorem 3: Suppose the same laissez–faire starting point as Theorem 2. Now the government announces in period 0 a Keynesian unbalanced budget fiscal policy (KUB). If g is sufficiently small, then the policy produces a perfect foresight (k, l) cycle in which the output in each period of a recession increases by a multiplier (of g) in excess of unity, and in which nominal prices and wages evolve as under laissez–faire.

Theorems 2 and 3 show how Keynesian fiscal policies may produce fixprice responses with textbook multipliers in our model. We should point out that the unit multipliers of Theorem 2 result from the fact that labour supply is perfectly elastic at the reservation wage and inelastic at all other wages, which leads to a horizontal AS curve in the Keynesian regime. With a more general upward sloping labour supply schedule, fiscal policies may induce price changes. We would expect, however, that multipliers are close to the multipliers of Theorems 2 and 3 if the labour supply curve is close to ours.

Of course, policies outside the range of Theorems 2 and 3 may produce different prices from laissez–faire and so different multipliers. In the following we briefly report our results for some policy variations like a non–cyclical policy, excessive

[21] If the budget is re–balanced only in the first boom period instead of the last recession period, multipliers are still positive, but possibly less than unaty.

government spending, permanently unbalanced budgets or income taxation in the case of a Cobb–Douglas intertemporal utility function. Detailed derivations are contained in Appendix B.

When government spending is the same in all periods (booms as well as recessions), the balanced–budget multiplier is still positive but falls below unity. Government spending during booms produces inflation in all boom periods $k+1, \ldots, k+l$ which raises the reservation wage and thereby the union wage in period k. This leads again to higher inflation in period k and by the same argument to higher inflation and higher wages in all recession periods. The higher wages lead to lower employment and output levels than those obtained with a (KBB) policy in which government spending is raised only during recessions.

Similarly, excessive government spending or permanently unbalanced budgets lead to inflation in recession periods and thus to higher wage demands and higher unemployment in all previous recession periods. On the other hand, it can be shown that even a contractionary policy without government spending but with taxation at the end of a recession raises recession output levels with the same tax multipliers as a (KUB) policy with positive government spending. Thus, it is the deflationary effect of the contraction at the end of the recession which raises output and not the government spending at the beginning.

Finally, if the government levies a proportional tax on (wage and profit) income instead of a lump–sum tax, the reservation wage and thereby the union wage are directly adversely affected by taxation. A higher income tax rate has thus three effects on the recession output levels: a positive aggregate demand effect since a higher tax rate raises aggregate demand (similar to equation (3) in case of a lump–sum tax), a positive inflation effect if the increase of the tax rate is not matched by an increase of government spending (as in case of a lump–sum tax, a budget surplus decreases inflation which leads to lower wages in the previous period), but now also a negative labour supply (reservation wage) effect. Both in case of a (KBB) and in case of a (KUB) policy the negative labour supply effect offsets the two positive effects, and so Theorems 2 and 3 do not extend to the case of an income tax. Instead all multipliers become negative.[22]

[22]See also Molana and Moutos (1991) who show that the positive multipliers of Dixon (1987) and Mankiw (1988) become zero or negative if there is income taxation.

6 Conclusions

This paper contains several features which are new relative to the existing literature. First, the firms' coordination problem in our model provides a novel explanation for multiple equilibria under imperfect competition. Second, these multiple equilibria give rise to many endogenous cycles which exhibit several stylized business cycle features. And third, our policy analysis reproduces Keynesian textbook multipliers in a dynamic general equilibrium model without nominal rigidities.

Our multiplicity results use the assumption that sectoral labour markets are separated in the sense that workers can only work for firms in one sector, but nowhere else, and that these workers are represented by one sectoral trade union. It is worth pointing out, however, that the same multiplicity would emerge in an economy with one homogenous labour market and with a single trade union, in which workers can freely move between sectors but face a positive, arbitrarily small cost if they do not work in their home sector.[23] All wages in the intermediate range of Figure 1 produce again two equilibria: an unemployment equilibrium in which firms in all sectors coordinate on the Cournotian labour demand of Proposition 1; but there is also a full employment equilibrium in which all firms signal the excess labour demands of Proposition 2 since a lower labour demand of any one firm would not alter employment, output and price levels of all sectors when it is costly for the workers to move to another sector.

The firms' coordination problem can lead to a Walrasian temporary equilibrium even though there is imperfect competition on both the labour and the output markets. In a boom state, they create excess vacancies and behave as if they were price–takers whenever the wage is below the competitive wage. Trade unions anticipate this behaviour and set the competitive wage since it is the maximal wage consistent with full employment. In a recession state, the outcome is Keynesian unemployment, as would usually be expected in a model with imperfect competition on the output and the labour market. Fiscal policy in both regimes has opposite effects: in the Walrasian regime prices change and output stays constant, and vice versa in the Keynesian regime.

[23] A similar argument would apply if there were several separated labour markets, each comprising one trade union and many different sectors.

Appendix A: Proofs and derivations

Lemma A.1 If the rationing scheme is non–manipulable, symmetric and frictionless, and if firms $2, \ldots, N$ signal labour demands J, then firm 1 is rationed according to $\ell^1 = \min(J^1, c(J, \ldots, J))$ where the employment constraint is

$$c(J, \ldots, J) = \max\Big(\frac{L}{N}, L - (N-1)J\Big) \quad .$$

Proof: First consider $J > L/N$. If firm 1 signals $J^1 = J$, by symmetry all firms would be rationed to $\ell^j = L/N$, $j = 1, \ldots, N$. Thus, firm 1's employment constraint is $c(J, \ldots, J) = L/N > L-(N-1)J$. Second consider $J < L/N$. Then $c(J, \ldots, J) \geq L-(N-1)J$, since otherwise firm 1 would be rationed if it signalled $J^1 = L-(N-1)J$ which is impossible since the rationing scheme is assumed to be frictionless. Now suppose $c(J, \ldots, J) > L - (N-1)J$. If then firm 1 signals $J^1 = c(J, \ldots, J)$ it gets $\ell^1 = J^1$ and by symmetry all other firms are rationed to $\ell = (L-J^1)/(N-1) < J$. In particular, if firm 2 deviated to the higher labour demand $J^2 = J^1 > J$ it would still be rationed to $\tilde{\ell}^2 = \ell < J$ (because of non–manipulability), and because of symmetry firm 1 must then be rationed to $\tilde{\ell}^1 = \tilde{\ell}^2 = \ell$, too. All other firms signalling labour demands J get some $\tilde{\ell} \leq J$. But this contradicts the assumption that the rationing scheme is frictionless since total employment is $\tilde{\ell}^1 + \tilde{\ell}^2 + (N-2)\tilde{\ell} < NJ < L$ while total demand is $J^1 + J^2 + (N-2)J > NJ$. Therefore we must have $c(J, \ldots, J) = L - (N-1)J > L/N$. Finally, the case $J = L/N$ follows from continuity of c(.). $\square$

Proof of Proposition 3:

(a) follows easily from the fact that the unconstrained maximum of $w^{-\rho}(w - w^r)$ is $w = \rho w^r/(\rho - 1)$.

(b) If the union sets the wage $w = w^{WE}$, its payoff is $W_1 = (w^{WE} - w^r)L$. If $w^{WE} > \rho w^r/(\rho - 1)$, then the union's payoff

$$\Big(\frac{w^{CN}}{w}\Big)^\rho L(w - w^r)$$

is decreasing in w for $w > w^{WE}$ and less than W_1 at $w = w^{WE}$; so w^{WE} is the union optimum then. If $w^{WE} < \rho w^r/(\rho - 1)$, the best the union can get from $w > w^{WE}$ is at $w = \rho w^r/(\rho - 1)$ giving the payoff

$$W_2 = \Big(\frac{(\rho-1)w^{CN}}{\rho w^r}\Big)^\rho L \frac{1}{\rho - 1} w^r \ .$$

Therefore, $W_1 \geq W_2$ if and only if

$$\Phi(\bar{z}) \equiv \bar{z} - 1 - \Big(\frac{\rho N - 1}{\rho N}(1 - 1/\rho)\bar{z}\Big)^{\rho} \frac{1}{\rho - 1} \geq 0 \ , \tag{17}$$

where $\bar{z} \equiv w^{WE}/w^r$. Note that this inequality is not fulfilled at $\bar{z} = 1$, but that it is strictly fulfilled at $\bar{z} = \rho/(\rho - 1)$. Since Φ is strictly increasing on $[1, \rho/(\rho - 1)]$, there exists a unique $z \in (1, \rho/(\rho - 1))$ which fulfills (17) with equality, and any $\bar{z} \in (z, \rho/(\rho - 1))$ satisfies (17). Therefore, $w^{WE} \geq zw^r$ if and only if the union's optimal wage is w^{WE}. □

Derivation of the aggregate demand identity (3)

Consider a consumer supplying labour to some sector $\hat{s}$. The consumer maximizes $u(C_t, C_{t+1}) - b\ell_t$ subject to the budget constraints

$$\begin{aligned} \int_0^1 p_{st} c_{st} ds + \mu_t &\leq w_{\hat{s}t}\ell_t + \pi_t - \varphi_t \ , \\ \int_0^1 p_{s,t+1} c_{s,t+1} ds &\leq I_{t+1}\mu_t \ , \end{aligned}$$

where φ_t is a lump–sum tax, π_t is the consumer's profit income and μ_t is his money savings. Since all young consumers have identical homothetic preferences, the consumption demand for sector s output of all young consumers is

$$D^Y_{st} = \Big(\frac{p_{st}}{P_t}\Big)^{-\rho} c\Big(\frac{P_{t+1}}{P_t I_{t+1}}\Big) \frac{W^n_t}{P_t} \ ,$$

where W^n_t is the aggregate net (wage and profit) income of young consumers in period t, and $c : (0, \infty) \to (0, 1)$ is the propensity to consume, i.e. the fraction of income to be spent on first period consumption as a function of the real interest rate. Since M_{t-1} denotes the aggregate money savings of the old generation at the beginning of period t, and $g_t = (\int_0^1 g_{st}^{(\rho-1)/\rho} ds)^{\rho/(\rho-1)}$ is real government demand, the consumption demand of the old consumers and of the government are[24] are

$$D^O_{st} = \Big(\frac{p_{st}}{P_t}\Big)^{-\rho} \frac{M_{t-1}I_t}{P_t} \quad \text{and} \quad D^G_{st} = \Big(\frac{p_{st}}{P_t}\Big)^{-\rho} g_t.$$

Therefore, total demand for sector s output is

$$D_{st} = D^Y_{st} + D^O_{st} + D^G_{st} = \Big(\frac{p_{st}}{P_t}\Big)^{-\rho} Y_t$$

[24] The sector demand of the government can be obtained as the solution of the expenditure minimization problem subject to a given level of real government demand g_t.

where

$$Y_t = c\Big(\frac{P_{t+1}}{P_t I_{t+1}}\Big)\frac{W_t^n}{P_t} + \frac{M_{t-1}I_t}{P_t} + g_t$$

is aggregate demand. Since firms do not ration demand, sector output y_{st} equals demand D_{st}, and therefore the aggregate net income is $W_t^n = \int_0^1 p_{st}y_{st}ds - \varphi_t L = P_t(Y_t - \tau_t)$ where $\tau_t = (\varphi_t L)/P_t$ is the aggregate real lump–sum tax. Using the government's budget constraint $M_t = M_{t-1}I_t + P_t g_t - P_t\tau_t$, we obtain the aggregate demand identity (3) of the text.

Derivation of the reservation wage (4)

The utility of a young individual in sector $\hat{s}$ who consumes optimally can be written

$$\frac{w_{\hat{s}t}\ell_t + \pi_t - \varphi_t}{P_t\omega\Big(\frac{P_{t+1}}{P_t I_{t+1}}\Big)} - b\ell_t \ , \tag{18}$$

where $\omega(\theta) \equiv u(c(\theta), (1-c(\theta))/\theta)^{-1}$, π_t is profit income and φ_t is the lump–sum tax. Thus, all consumers in sector $\hat{s}$ want to supply labour if the sector wage exceeds the reservation wage of workers, $w_{\hat{s}t} > w_t^r \equiv bP_t\omega(P_{t+1}/(P_t I_{t+1}))$, so in this case labour supply is $L_{st} = L$. If $w_{\hat{s}t} < w_t^r$ labour supply is zero. It is also immediate from (18) that a utilitarian trade union aims to maximize $(w_{\hat{s}t} - w_t^r)\ell_{\hat{s}t}$ with $\ell_{\hat{s}t} = \int_0^L \ell_t^i di$ (where $\ell_t^i \in \{0, 1\}$ denotes employment of worker $i \in [0, L]$).

Proof of Theorem 1:

We assume first $c' > 0$ (we return to the case of a Cobb–Douglas utility function with $c' = 0$ below) and define $c(0) = \underline{c}$ and $c(\infty) = \bar{c}$. Then the Walrasian equilibrium condition (11) can be rewritten as $m_{t+1} = \beta(m_t)$ where $\beta(m) \equiv m/(c^{-1}(1 - m/L))$ is defined for $m \in ((1 - \bar{c})L, (1 - \underline{c})L)$ and is increasing. Let $\bar{m}$ be the fixed point of β, let $\hat{m}$ be the unique solution of $\beta(\hat{m}) = \delta\hat{m}$ (see Figure 2) and define $\hat{m}_l \equiv \beta^{-(l-1)}(\hat{m}) < \bar{m}$. Obviously, a sequence $m_1, \dots, m_k, m_{k+1}, \dots, m_{k+l}$ is a (k, l) cycle (along which the first k states are R and the remaining l states are B) if and only if m_{k+1} is a fixed point of the map $\psi(m) \equiv \alpha^k \cdot \beta^l(m)$ in the interval $[\hat{m}_l, \bar{m}]$ (m_{k+1} must not be smaller than $\hat{m}_l$ since then the $(l-1)$th iterate of β at m_{k+1} would be smaller than $\hat{m}$).

As has been noted above, a (k, l) cycle exists if and only if there is a fixed point of the map $\psi = \alpha^k \cdot \beta^l$ in the interval $[\hat{m}_l, \bar{m}]$. Note that

$$\beta'(m) \quad = \quad \frac{1}{c^{-1}(1 - m/L)} + \frac{m/L}{c^{-1}(1 - m/L)^2 c'(c^{-1}(1 - m/L))}$$

$$> \frac{1}{c^{-1}(1-m/L)} \; .$$

Hence, at any fixed point m^F of ψ we have

$$\begin{aligned}
\psi'(m^F) &= \alpha^k \cdot \beta'(m^F) \cdot \beta'(\beta(m^F)) \cdot \dots \cdot \beta'(\beta^{l-1}(m^F)) \\
&> \alpha^k \cdot \frac{1}{c^{-1}(1-m^F/L)} \cdot \dots \cdot \frac{1}{c^{-1}(1-\beta^{l-1}(m^F)/L)} \\
&= \alpha^k \cdot \frac{\beta(m^F)}{m^F} \cdot \frac{\beta^2(m^F)}{\beta(m^F)} \cdot \dots \cdot \frac{\beta^l(m^F)}{\beta^{l-1}(m^F)} = \frac{\alpha^k \cdot \beta^l(m^F)}{m^F} = 1 \; .
\end{aligned}$$

Since $\psi(\bar{m}) = \alpha^k \cdot \bar{m} > \bar{m}$, the condition $\psi(\hat{m}_l) \leq \hat{m}_l$ is necessary and sufficient for the existence of a fixed point m_{k+1} of ψ in $[\hat{m}_l, \bar{m}]$, and whenever a fixed point exists, it is unique. Since ψ describes also the local forward perfect foresight dynamics when the (k,l) cycle is interior, $\psi'(m_{k+1}) > 1$ implies stability of any interior (k,l) cycle in the local backward perfect foresight dynamics. If $l = 1$, $\hat{m} = \hat{m}_l$ and the condition $\psi(\hat{m}) \leq \hat{m}$ is equivalent to $\alpha^k \delta \leq 1$. Hence, a $(k,1)$ cycle exists iff $\alpha^k \delta \leq 1$. Suppose $\alpha^k \delta \leq 1$ and consider any $k' \leq k$ and $l \geq 1$. Since

$$\alpha^{k'} \beta(\hat{m}) = \alpha^{k'} \delta \hat{m} \leq \alpha^k \delta \hat{m} \leq \hat{m} \leq \hat{m}_l$$

we have $\psi(\hat{m}_l) = \alpha^{k'} \beta^l(\hat{m}_l) = \alpha^{k'} \beta(\hat{m}) \leq \hat{m}_l$ and thus the existence of a (k',l) cycle.

Finally, we have to show the existence of a non–degenerate stationary sunspot equilibrium with k unemployment states $S_1, \dots, S_k$ and l full employment states $S_{k+1}, \dots, S_{k+l}$ and positive transition probabilities π_i^j, $i, j = 1, \dots, k+l$. The utility maximization problem of the consumer under uncertainty has similar solutions as under certainty. The only difference is that the functions c and f are replaced by

$$c\Big(\frac{P_{t+1}^1}{P_t I_{t+1}}, \dots, \frac{P_{t+1}^{k+l}}{P_t I_{t+1}}, \Pi_i\Big) \quad \text{and} \quad f\Big(\frac{P_{t+1}^1}{P_t I_{t+1}}, \dots, \frac{P_{t+1}^{k+l}}{P_t I_{t+1}}, \Pi_i\Big) \; ,$$

where P_t is the price level in period t, $P_{t+1}^1, \dots, P_{t+1}^{k+l}$ are the expected price levels for the next period, and $\Pi_i = (\pi_i^1, \dots, \pi_i^{k+l})$ is the vector of transition probabilities assuming $S_t = S_i$. These functions are continuously differentiable in all arguments (since u is twice differentiable) and coincide with $c(P_{t+1}^j / P_t I_{t+1})$ and $f(P_{t+1}^j / P_t I_{t+1})$ if $\pi_i^j = 1$. The equilibrium conditions (9) and (10) change accordingly. Since a (k,l) cycle is a degenerate sunspot equilibrium with transition probabilities $\pi_{k+l}^1 = \pi_i^{i+1} = 1$, $i = 1, \dots, k+l-1$, and $\pi_i^j = 0$ else, the implicit function theorem yields a solution $(m_1, \dots, m_{k+l})$ of the equations in (9) and (10) when the transition matrix is close to the degenerate transition matrix (the inequalities in (9) and (10) are then also fulfilled since they hold strictly at the interior (k, l) cycle) and when the Jacobian of the equilibrium equations at the cycle is invertible. But this condition turns out to be equivalent to the condition $dm_{t+k+l}/dm_t|_{m_t=m_1} = \psi'(m_{k+1}) \neq 1$ which has been shown above.

Finally, in the case of a Cobb–Douglas utility function ($c' = 0$), c is a constant, and the curve OABE in Figure 2 is a vertical line (so that β and ψ are not defined). It is easy to see that a necessary and sufficient condition for the existence of a (k, l) cycle is simply $\alpha^k \delta \leq 1$ (in all boom periods, the cycle satisfies $m_t = \bar{m}$). $\square$

Proof of Theorem 3:

The conditions of a (k, l) cycle $(m_1, \ldots, m_{k+l})$, $(Y_1, \ldots, Y_{k+l})$ under a (KUB) policy are:

$$m_{i+1} = \alpha m_i \quad \text{and} \quad Y_i = \frac{m_i}{1 - c(m_i/m_{i+1})} < L, \; i = 1, \ldots, k-2, \tag{19}$$

$$m_k = \alpha m_{k-1} - \tau \quad \text{and} \quad Y_{k-1} = \frac{m_{k-1}}{1 - c(m_{k-1}/(m_k + \tau))} < L, \tag{20}$$

$$m_{k+1} = \alpha m_k \quad \text{and} \quad Y_k = \frac{m_k}{1 - c(m_k/m_{k+1})} + \tau < L, \tag{21}$$

$$m_{i+1} \geq \delta m_i \quad \text{and} \quad Y_i = \frac{m_i}{1 - c(m_i/m_{i+1})} = L, \; i = k+1, \ldots, k+l-1, \tag{22}$$

$$m_1 \geq \delta m_{k+l} + g \quad \text{and} \quad Y_{k+l} = \frac{m_{k+l}}{1 - c(m_{k+l}/(m_1 - g))} = L. \tag{23}$$

From (19), (20) and (21) we obtain $m_{k+1} = \alpha^k m_1 - \alpha\tau$ and using (22) and (23) we find

$$m_1 - g = \beta^l(m_{k+1}) = \beta^l(\alpha^k m_1 - \alpha\tau) = \beta^l(\alpha^k(m_1 - g)) \; .$$

Thus, $m_1 = m_1^* + g$, since m_1^* is the unique fixed point of $\beta^l(\alpha^k .)$. From (19) follows $m_i = m_i^* + \alpha^{i-1} g$ for $i = 1, \ldots, k-1$, (20) implies $m_k = \alpha^{k-1}(m_1^* + g) - \tau = \alpha^{k-1} m_1^* = m_k^*$, (21) implies $m_{k+1} = \alpha m_k^* = m_{k+1}^*$ and (22) yields $m_{k+i} = \beta^{i-1}(m_{k+1}^*) = m_{k+i}^*$ for all $i = 2, \ldots, l$. From this and (19), (20) and (21) we obtain the recession output levels:

$$Y_i = Y_i^* + \frac{\alpha^{i-1}}{1 - c(1/\alpha)} \cdot g \quad , \; i = 1, \ldots, k-1 \; ,$$
$$Y_k = Y_k^* + \alpha^{k-1} g \; .$$

If g is sufficiently small, we have $Y_i \leq L$ for all $i = 1, \ldots, k$. Moreover, all multipliers of g are greater than one. Furthermore, the inequalities in (22) are fulfilled since

$m_{i+1} = m^*_{i+1} \geq \delta m^*_i = \delta m_i$ for $i = k+1, \ldots, k+l-1$ and the inequality in (23) is fulfilled since $m_1 = m^*_1 + g \geq \delta m^*_{l+k} + g = \delta m_{l+k} + g$. Hence, $(m_1, \ldots, m_{k+l})$ is indeed a (k, l) cycle if g is sufficiently small.

Finally, we have to show that prices are the same as under laissez–faire. Consider some period t in which $S_t = B$ and $S_{t+1} = R$, and so $M_t = M^*_t$ (since the budget is balanced during a boom, the money stock at the end of the boom coincides with the laissez–faire money stock). Using $m_{t+1} = m^*_{t+1} + g$ (since $t+1$ is the first recession period), $M^*_{t+1} = I_{t+1} M^*_t$ and $M_{t+1} = I_{t+1} M_t + P_{t+1} g$, we obtain easily $P_{t+1} = P^*_{t+1}$. For $2 \leq i \leq k-1$ we know $M_{t+i} = I_{t+2} \cdots I_{t+i} M_{t+1}$ and $m_{t+i} = \alpha^{i-1} m_{t+1}$ from which we obtain

$$I_{t+2} \cdots I_{t+i} P_{t+1} = \alpha^{i-1} P_{t+i} \ , \tag{24}$$

for $i = 2, \ldots, k-1$. From $M_{t+k} = I_{t+2} \cdots I_{t+k} M_{t+1} - P_{t+k} \tau$ and $m_{t+k} = \alpha^{k-1} m_{t+1} - \tau$ we also obtain (24) for $i = k$. Since (24) also holds for (P^*_{t+1}, P^*_{t+i}) and since $P_{t+1} = P^*_{t+1}$, we have $P_{t+i} = P^*_{t+i}$ for all $i = 2, \ldots, k$. Equation (24) also implies that the budget is in fact re–balanced in period $t+k$, and since there is no fiscal policy between $t+k+1$ and $t+k+l$ we have $M_{t+i} = M^*_{t+i}$ and $P_{t+i} = P^*_{t+i}$ for all $i = k+1, \ldots, k+l$. □

Appendix B: Policy variations

We assume throughout this appendix a Cobb–Douglas utility function, so that the propensity to consume is a constant $c \in (0, 1)$ and the function f is $f(\theta) = \theta^{1-c}$.

Non–cyclical policy

Consider a balanced–budget policy in which government spending is the same in all periods (booms as well as recessions – e.g. the government cannot observe S_t), so $g_i = \tau_i = g$, $i = 1, \ldots, k+l$. With such a policy it turns out that a higher level of government activity g raises output during recessions, but with a multiplier below unity. In fact, if $\alpha^k \delta \leq 1$, it can be shown that for any $g \in [0, L)$, there exists a (k, l) cycle in which recession output levels are

$$Y_i = \alpha^{-(k+1-i)}(L - g) + g < L \ , \ i = 1, \ldots, k \ .$$

Thus the multiplier is $1 - \alpha^{-(k+1-i)} < 1$ and there is partial crowding out during recessions and full crowding out during booms.

Excessive spending

Starting from a (KBB) policy, suppose the government spends "excessively" during a recession period (but unlike the first case does not expend during booms). Specifically consider a laissez–faire (k,l) cycle where $k > 1$ and a (KBB) policy where $g_k = L - Y_k^*$ and $g_i = 0$ otherwise. From Theorem 2, the only policy impact is to raise Y_k from Y_k^* to L. Now suppose that the expenditure in k becomes slightly excessive: $g_k = L - Y_k^* + \varepsilon$ with some small $\varepsilon > 0$. Now the last recession period must involve full employment (in fact, (9) is replaced by $m_{k+1} - g_{k+1} + \tau_{k+1} = m_{k+1} \geq \alpha m_k$ and $Y_k = L$). Hence,

$$Y_k = \frac{m_k}{1-c} + L - Y_k^* + \varepsilon = L \quad \text{implies} \quad m_k = m_k^* - \varepsilon(1-c) \ .$$

Applying (9) for periods $i = 1, \ldots, k-1$ gives

$$m_i = m_i^* - \frac{1}{\alpha^{k-i}}\varepsilon(1-c) \ , \ Y_i = \frac{m_i}{1-c} = Y_i^* - \frac{\varepsilon}{\alpha^{k-i}} \ , \ i = 1, \ldots, k-1.$$

With $m_i = m_i^*$ and $Y_i = Y_i^*$, $i = k+1, \ldots, k+l$, we have a new (k,l) cycle satisfying (9) for $i = 1, \ldots, k-1$, $Y_k = L$ and $m_{k+1} \geq \alpha m_k$ for $i = k$ and (10) for $i = k+1, \ldots, k+l$ if ε is small enough. On this cycle recession prices (and so wages) are everywhere higher, and recession outputs are lower (the same in the last period k) than under the first (KBB) policy.

Permanently unbalanced budgets

Consider a policy of permanently unbalanced budgets in which the government never re–balances its budget by taxation. Suppose positive government spending during recessions $g_1, \ldots, g_k \geq 0$ and $g_i = 0$ otherwise, and no taxation $\tau_i = 0$ for all i. Then, if $g_1, \ldots, g_k$ are small enough, the equilibrium conditions (9) and (10) can be solved for a (k,l) cycle in which for $i = 1, \ldots, k-1$:

$$m_i = m_i^* - \alpha^{-(k-i)} g_k - \ldots - \alpha^{-1} g_{i+1} \ ,$$

and $m_i = m_i^*$ for $i = k, \ldots, k+l$. Recession outputs are $Y_k = Y_k^*$ and for $i = 1, \ldots, k-1$,

$$Y_i = Y_i^* - \frac{1}{1-c}\Big(\alpha^{-(k-i)} g_k - \ldots - \alpha^{-1} g_{i+1}\Big) \ ,$$

and so decreasing in $g_{i+1}, \ldots, g_k$. Similar to the excessive spending policy above, this policy produces inflation in periods $i = 2, \ldots, k$, and therefore raises wages and lowers output in periods $i = 1, \ldots, k-1$.

On the other hand, a modified (KUB) policy in which $g = 0$ but $\tau > 0$ leads to the same equilibrium conditions (19)–(23) as a (KUB) policy with positive government spending. Thus, tax multipliers on recession output levels are the same as for a (KUB) policy with positive government spending.[25]

Income taxation

Denote the income tax rate in period t again by $\tau_t \in [0, 1)$. The net income of young consumers is then $W_t^n = (1-\tau_t)P_t Y_t$ and the government's budget constraint $M_t = M_{t-1}I_t + P_t g_t - \tau_t P_t Y_t$. Using these two equations, the aggregate demand equation (8) is now replaced by

$$Y_t = \frac{m_t}{(1-c)(1-\tau_t)} \quad . \tag{25}$$

The reservation wage is now $w_t^r = bP_t f(P_{t+1}/(P_t I_{t+1}))/(1-\tau_t)$. These modifications imply that in the equilibrium conditions (5) and (6) the constant b has to be replaced by $b/(1-\tau_t)$. Using the same definitions of α and δ and the fact that f is also Cobb–Douglas, we find that there is an unemployment equilibrium in case of $S_t = R$ if

$$m_{t+1} - g_{t+1} + \tau_{t+1}Y_{t+1} = \alpha(1-\tau_t)^{1/(c-1)}m_t \quad \text{and} \quad Y_t < L \ , \tag{26}$$

and that there is a full employment equilibrium in case of $S_t = B$ if

$$m_{t+1} - g_{t+1} + \tau_{t+1}Y_{t+1} \geq \delta(1-\tau_t)^{1/(c-1)}m_t \quad \text{and} \quad Y_t = L \ , \tag{27}$$

where Y_t is given by (25).

Consider first a (KBB) policy: $\tau_t \geq 0$ if $S_t = R$, $\tau_t = 0$ otherwise, $g_t = \tau_t Y_t = \tau_t/(1-\tau_t) \cdot m_t/(1-c)$, and start again from a laissez–faire (k, l) cycle $(m_1^*, \ldots, m_{k+l}^*)$, $(Y_1^*, \ldots, Y_{k+l}^*)$. If $\tau_1, \ldots, \tau_k$ are sufficiently small, the equilibrium conditions (26) and (27) can be solved for a (k, l) cycle in which $m_i = m_i^*$, $i = k+1, \ldots, k+l$, and $m_i = \prod_{j=i}^k (1-\tau_j)^{1/(1-c)} m_i^*$, $i = 1, \ldots, k$. Recession outputs are

$$Y_i = (1-\tau_i)^{-1} \prod_{j=i}^{k} (1-\tau_j)^{1/(1-c)} \cdot Y_i^* \quad , \ i = 1, \ldots, k.$$

The first factor is the positive aggregate demand effect of the tax in period i, while the second factor is the negative labour supply effect of the income tax. Not only

[25] This result is only valid for a Cobb–Douglas utility function. In the general gross substitutes case, tax multipliers become even larger without government spending.

the higher reservation wage in period i lowers output in period i, but also the higher reservation wages in periods $i+1, \dots, k$ since they raise inflation in future periods which leads also to higher wage demands in period i. The net effect of the income tax rates on the recession output is unambiguously negative.

Consider next a (KUB) policy: $\tau_k = \tau \geq 0$, $\tau_i = 0$ otherwise and $g_1 = g = \alpha^{1-k}\tau Y_k = \alpha^{1-k}\tau/(1-\tau) \cdot m_k/(1-c)$, $g_i = 0$ else. Similar to the case of a lump–sum tax, the conditions for a (k,l) cycle are now:

$$\begin{aligned}
m_{i+1} &= \alpha m_i \quad \text{and} \quad Y_i = \frac{m_i}{1-c} < L,\ i = 1, \dots, k-2, \\
m_k &= \alpha m_{k-1} - \tau Y_k \quad \text{and} \quad Y_{k-1} = \frac{m_{k-1}}{1-c} < L, \\
m_{k+1} &= \alpha(1-\tau)^{1/(c-1)} m_k \quad \text{and} \quad Y_k = \frac{m_k}{(1-c)(1-\tau)} < L, \\
m_{i+1} &\geq \delta m_i \quad \text{and} \quad Y_i = \frac{m_i}{1-c} = L,\ i = k+1, \dots, k+l-1, \\
m_1 &\geq \delta m_{k+l} + g \quad \text{and} \quad Y_{k+l} = \frac{m_{k+l}}{1-c} = L.
\end{aligned}$$

If τ is small enough, these conditions can be solved for a (k,l) cycle as follows: $m_i = m_i^*$, $i = k+1, \dots, k+l$, and in period k, $m_k = (1-\tau)^{1/(1-c)} m_k^*$ and output $Y_k = (1-\tau)^{1/(1-c)-1} Y_k^*$ is falling in τ; the labour supply effect dominates the aggregate demand effect. In period $k-1$ we find $m_{k-1} = (1-\tau)^{1/(1-c)} m_{k-1}^* + \tau(1-\tau)^{1/(1-c)-1} Y_{k-1}^*$ and output is

$$Y_{k-1} = (1-\tau)^{1/(1-c)} \Big(1 + \frac{\tau}{(1-\tau)(1-c)}\Big) Y_{k-1}^* \ . \tag{28}$$

In periods $i = 1, \dots, k-2$, we have $m_i = \alpha^{-(k-i-1)} m_{k-1}$ and so outputs are

$$Y_i = \alpha^{-(k-i-1)} Y_{k-1} = (1-\tau)^{1/(1-c)} \Big(1 + \frac{\tau}{(1-\tau)(1-c)}\Big) Y_i^* \ . \tag{29}$$

In equations (28) and (29), the first factor is the negative labour supply effect, while the second factor contains both the positive inflation effect and the positive aggregate demand effect. It can be checked easily that the total effect is negative, i.e. increases in τ decrease output in periods $i = 1, \dots, k-1$.

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