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Optimal Fiscal Management of Commodity Price Shocks

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Abstract

This paper analyses how low-income countries should optimally respond, through fiscal policy, to commodity price shocks. The model accounts for imperfect access to world capital markets and a variety of externalities associated with public infrastructure, including utility benefits, a direct complementarity effect with private investment, and reduced distribution costs. However, public capital is also subject to congestion and absorption constraints, with the latter affecting the efficiency of infrastructure investment. The model is parameterized and used to examine the transmission process of a temporary resource price shock under a benchmark case (cash transfers) and alternative fiscal rules, involving either higher public spending or accumulation in a sovereign fund. The optimal allocation rule between spending today and asset accumulation is determined so as to minimize a social loss function defined in terms of the volatility, relative to the benchmark case, of private consumption and either the nonresource primary fiscal balance or a more general index of macroeconomic stability, which accounts for the volatility of the real exchange rate. Sensitivity analysis is conducted with respect to various structural parameters and model specification.

JEL Classification Numbers: F41, H41, H54

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1 Introduction

The design of fiscal policy in developing countries with large endowments of non-renewable natural resources continues to generate much debate among economists and policymakers. While the economic rents created by high commodity prices, or the discovery of new reserves, provide an opportunity to promote economic growth and human development in these countries, managing those resources effectively has proven to be a daunting challenge. Time and again the abundance of natural resources has led to a so-called *natural resource curse*, which has taken the form of increased rent-seeking, a weakening of institutions, and heightened risks of conflict and civil wars.[1] Global commodity price volatility has often translated into domestic macroeconomic instability, especially in resource-rich developing countries where exports and government revenues are highly concentrated. This volatility has often been related to the procyclical response of fiscal policy to commodity price movements (Céspedes and Velasco (2014)). At the same time, large and sustained inflows of foreign exchange associated with resource windfalls have led to Dutch disease effects, that is, a contraction of domestic production of nonresource traded goods resulting from increased demand for nontraded goods and a real exchange rate appreciation.

Much of the debate on natural resource management has been dominated by the permanent income hypothesis (PIH) approach. The standard PIH approach implies that, for a country where the only source of government revenues is resource income, the intertemporal budget constraint is satisfied when the nonresource primary deficit is limited to the perpetuity value of resource wealth, that is, the present value of all future resource revenue. From that perspective, the PIH provides a benchmark for the nonresource primary fiscal balance that can be financed indefinitely.[2]

However, some recent research has questioned the relevance of the PIH for resource-rich low-income countries. In particular, it has been argued that the PIH is not appropriate

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[1] For a review of the literature on the resource curse, see Frankel (2010) and van der Ploeg (2011).

[2] With projections for nonresource revenue, the nonresource primary balance benchmark also provides an estimate of the "sustainable" level of expenditure (Baunsgaard et al. (2012), Lundgren et al. (2013)). Note that, in this analysis, there is no distinction between government consumption and total public spending (which includes investment), even though strictly speaking the PIH is a model of (optimal) consumption behavior. See Attanasio and Weber (2010) for a review of the PIH.

for these countries, because it ignores the fact that they suffer from significant weaknesses in terms of access to core physical infrastructure.[3] Indeed, the narrow interpretation of the PIH—that resource windfalls should be saved in their entirety in the form of financial assets held abroad—ignores the fact that lack of access to infrastructure imposes severe constraints on economic activity in poor countries, and that investments in infrastructure, given their lumpy nature, may need to be raised significantly over the short to medium run to create a Big Push (see Agénor (2010; 2012, Chapter 6)). As documented by Geiregat and Yang (2013), infrastructure indicators in resource-rich developing countries continue to be either worse (for instance, paved roads) or no better (access to improved water, for instance), than in nonresource-rich developing countries, regardless of their income per capita status.[4]

Moreover, in the presence of a strong complementarity effect between public capital and private investment, and large externalities in terms of education and health—as documented in the recent literature—the rate of return on public investment in infrastructure can potentially be very high. Thus, a fiscal rule that requires investing all income from a resource windfall in safe financial assets held in a sovereign wealth fund may have a high opportunity cost in the medium run, in terms of foregone growth in physical assets (both private and public) and output.

The key issue therefore has been to devise more flexible fiscal management rules that allow governments, in response to resources windfalls, to allocate sufficient resources to meet short-term needs in infrastructure investment and possibly other components of productive spending (in education and health, most notably), while at the same time maintaining fiscal and macroeconomic stability, achieving long-term fiscal sustainability, and ensuring adequate savings for future generations. At the same time, it is important to account for absorption capacity constraints, which could hamper the quality and effectiveness of government outlays. Indeed, infrastructure spending itself may be inefficient due

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[3] See for instance Foster and Briceño-Garmendia (2010) and Andersen and Dalgaard (2013). The (in)appropriateness of the conventional PIH prescription has been discussed extensively in the literature; see Collier et al. (2010), Gelb and Grassmann (2010), Gelb (2011), van der Ploeg (2011), Baunsgaard et al. (2012), International Monetary Fund (2012), Lundgren et al. (2013), and van den Bremer and van der Ploeg (2013), considering the large development needs of low-income developing countries.

[4] See Foster and Briceño-Garmendia (2010) and Freund and Rocha (2010) for the role of transport costs—the highest of any region—in Sub-Saharan Africa.

to a lack of administrative or managerial talent within government. As a result, increased investment may not translate into increased production capacity and higher levels of productivity in future periods. This aspect of the debate on the consequences of resource abundance is particularly important for the low-income countries (such as Côte d'Ivoire, Ghana, Kenya, Senegal, and Uganda) that have recently discovered new resources but at the same time continue to suffer from institutional weaknesses in their ability to select, manage, and evaluate large and complex investment projects.

A number of analytical contributions, including Takizawa et al. (2004), Matsen and Torvik (2005), Venables (2010), van der Ploeg (2011, 2012), van der Ploeg and Venables (2011, 2013), Dagher et al. (2012), Berg et al. (2013), Richmond et al. (2013), van den Bremer and van der Ploeg (2013), and Araujo et al. (2016), have attempted to address these issues. Several of these contributions bring together elements of the literature on the optimal management of resource windfalls and the literature on the Dutch disease, which considers more broadly the macroeconomic consequences of large inflows of capital (in the form of aid, remittances, or private capital flows, rather than government-related flows only). While spending patterns determine the extent to which these windfalls generate Dutch disease effects, relative price changes associated with the Dutch disease may also have an impact on optimal spending allocation. Some papers also provide theoretical support to the view that public investment in infrastructure may dominate saving in foreign assets as an optimal strategy to manage resource revenue in economies where, to begin with, borrowing opportunities on world capital markets and access to infrastructure are limited.

In an important contribution, van der Ploeg and Venables (2013) developed a deterministic, perfect foresight model in which the lack of home-grown capital creates supply rigidities and relative prices are endogenous. They argued that an optimal revenue management policy typically consists of three elements. First, investment and capital in the nontradable sector should be built up rapidly; second, consumption should increase gradually to its new long-run value; and third, foreign assets should be managed to ensure that domestic spending (consumption and investment) is on an efficient path. In a central case this involves keeping resource revenues offshore until absorption constraints have been relaxed. But compared to the PIH prescription (holding the entire windfall in foreign assets) it may be optimal to place less revenue in offshore funds in the long run, because of the need to finance higher public investment, but more in the short run, because of absorption constraints. Dagher et al. (2012), Berg et al. (2013), and Richmond et al. (2013) develop closely related, three-sector dynamic stochastic general equilibrium (DSGE) models with a financial side to discuss both fiscal and monetary policy responses to natural resource price shocks. In these models, to capture absorptive capacity constraints, the efficiency of public investment is negatively related to the level of investment itself. In the same vein, Araujo et al. (2016) developed a two-sector model with private and public investment, as well as several frictions, including absorptive capacity constraints, inefficiencies in investment, and borrowing constraints. The model is used to study how these frictions, and the behavior of investment, affect current account dynamics associated with commodity price shocks. However, none of these contributions addresses directly the issue of the optimal allocation of resource windfalls in a stochastic environment.

This paper contributes to the ongoing debate by developing a DSGE model to study the optimal fiscal response to resource price shocks in a low-income country where access to infrastructure is limited. The focus of the analysis is on newly resource-rich economies that face the problem of managing revenues associated with a positive commodity (oil, for short) price shock. Specifically, the model is used to compare macroeconomic outcomes under alternative fiscal approaches to investing oil revenue, and to determine the optimal allocation between saving and investment. It explicitly accounts for imperfect access to world capital markets, government spending on infrastructure, the inefficiency of such spending, and absorptive capacity constraints. In addition, the traded sector benefits from a learning-by-doing externality. Hence, nonresource export sectors temporarily hit by worsening competitiveness are unable to fully recover when resources run out (see van Wijnbergen (1984)).

In modeling the externalities associated with public capital, the model goes beyond the existing literature by accounting for direct utility benefits, a direct complementarity effect with private investment, and reduced distribution costs. Accounting for these externalities is important to understand the nature of the trade-offs that governments face between spending today or tomorrow. At the same time, public capital is also subject to congestion, and absorption constraints, which depend on the relative scale of investment itself, adversely affect the efficiency of infrastructure spending. The combination of these features makes the model of interest in its own right. Importantly as well, the paper argues that the optimal allocation of resources must be determined on the basis of a social loss function that combines not only a measure of household welfare (namely, consumption volatility) but also an indicator of fiscal or macroeconomic volatility. This matters because in practice policymakers in poor countries have shown time and again concern with the instability that commodity price shocks (as noted earlier) often impart to their economies. In such conditions, an optimal rule based solely on maximizing household welfare may not provide an adequate foundation for policy advice. Put differently, my analysis is based on the *revealed preferences* of policymakers, rather than considering a benevolent agent that maximizes the utility of the representative household.

The model is parameterized and used first to examine the transmission process of a temporary resource price shock under a benchmark case (cash transfers, that is, the windfall is transferred entirely to citizens) and alternative fiscal rules, involving either higher public spending or accumulation in a sovereign fund. under the first two rules, the model is able to account for the key stylized facts associated with a resource windfall, namely, the Dutch disease effects alluded to earlier. On a more normative basis, the optimal allocation rule between spending today and asset accumulation is then determined so as to minimize a social loss function defined in terms of the volatility, relative to the benchmark case, of private consumption and either the nonresource primary balance or a more general index of macroeconomic stability, which accounts for the volatility of the real exchange rate. In either case, the key result is that the choice between spending now and spending later involves a *dynamic volatility trade-off.* Initially, full spending creates a lot of volatility; increasing the share of a resource windfall that is saved (or equivalently, reducing the share spent today) tends at first to *reduce* that volatility. However, as the proportion of resource revenues that is saved continues to rise, the interest income from the assets held in the sovereign fund become larger, and this tends to raise spending over time, which tends to *increase* volatility once again. In addition, depending on the structure of the model and its calibration, there may also be a trade-off *between* the volatility measures that are accounted for in the social loss function. Sensitivity analysis is conducted with respect to various structural parameters and model specification. In general, neither full spending, nor full saving, represents an optimal fiscal response to resource windfalls. In all cases, the results show that the optimal policy is always better, in terms of its impact on macroeconomic volatility, than an unconditional cash transfer policy.

The paper is organized as follows. Section 2 describes the model, whereas section 3 discusses the steady-state solution and log-linearization. Section 4 presents a parameterization for a "typical" low-income country, whereas Sections 5 and 6 examine the transmission process of a temporary resource price shock under a benchmark case (cash transfers) and alternative fiscal rules, involving full allocation of the resource windfall to either public spending or accumulation in a sovereign fund. Instead of defining these rules in terms of investment in infrastructure only, a broader concept of government spending is used, to reflect the fact that poor countries face limited access not only to infrastructure services but also to education and health services. In Section 7 the optimal allocation rule between spending today and asset accumulation is determined so as to minimize a fundamental social loss function, defined in terms of the volatility of private consumption and the nonresource primary balance relative to the benchmark case. A more general function is also defined, in terms of consumption volatility and macroeconomic volatility, which involves not only fiscal volatility but also real exchange rate volatility. Section 8 presents some sensitivity analysis, mainly to illustrate how the optimal allocation rule changes in response to various structural parameters and model specification. The final section offers some concluding remarks.

2 The Model

In line with much of the literature, consider an open economy with three sectors producing a non-renewable resource (referred to as crude oil, or simply oil from now on, and identified with superscript O), a nonoil tradable good (identified with superscript T), and a nontradable good (identified with superscript N). Oil output, Y_t^O, is a flow endowment owned by the government; its extraction requires no use of factor resources. It is not consumed domestically and only provides an additional source of tax revenue from exportables.[5] Tradable output, Y_t^T, and nontradable output, Y_t^N, are produced competitively. The nontradable good is a perishable, pure consumption good, whereas the nonoil tradable good is a mixed good, which can be either consumed or invested. Private investment (equipment) consists of tradables only, whereas public investment consists of both tradables and nontradables.[6] Capital and labor are both perfect mobile between the tradable and nontradable sectors.

The price of oil is exogenously determined outside the home country and denominated in foreign currency. The world price of a unit of the nonoil tradable good is unity and purchasing power parity (PPP) holds at the wholesale level for these goods. Thus, prices measured in foreign currency are equivalent to relative prices expressed in units of the tradable good at the wholesale level. However, PPP does not hold at the retail level for nonoil tradable goods due to distribution costs—including retail services, marketing and advertising, and (most importantly for the issue at hand) local transportation services. Production inputs are perfectly mobile, so factor returns are equalized across sectors.

Both households and the government spend on tradables and nontradables and can borrow on world capital markets.[7] However, the cost of borrowing incorporates a risk premium, which depends on the country's external debt (scaled by tradable output) and the composition of output, which measures the capacity to repay.

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[5] One could think of a country producing crude oil, which is not refined at home, and importing transformed products such as liquefied petroleum gas, gasoline, kerosene, and so on.

[6] Private investment is viewed here as consisting of imported equipment, whereas public consumption, defined later, is viewed as consisting to a large extent of wages and salaries. These assumptions are fairly reasonable for a low-income country. Changing them—by assuming that private investment also involves some spending on nontradables as well, or that government spending consists also in part of tradables—would affect the magnitude of the response of the real exchange rate, rather than its direction.

[7] Many low-income countries in Sub-Saharan Africa are now able to borrow on international financial markets; in several cases, these countries have been able to sell Eurobonds (denominated in euros or US dollars) at lower premia than some troubled European economies. According to market estimates reported by the Financial Times, African countries raised more than \$11bn in foreign-currency sovereign bonds in 2013, up from \$6bn in 2012 and \$1bn in 2000. A few corporate entities in Sub-Saharan Africa have also recently issued Eurobonds. See Sy (2013) for a brief discussion of the causes, both cyclical and structural, of these trends and their relation with the behavior of commodity prices.

2.1 Oil Production and Prices

Oil production follows an exogenous deterministic process, such that

$$\frac{Y_t^O}{\tilde{Y}^O} = (\frac{Y_{t-1}^O}{\tilde{Y}^O})^{\rho_{Y^O}}, \tag{1}$$

where $\tilde{Y}^O$ is the steady-state value of Y_t^O and $\rho_{Y^O} \in (0,1)$ is an autoregressive coefficient, which (as discussed later) is related to the rate of depletion of oil resources.

The country's oil production is assumed to be small relative to world supply. The price of oil on world markets (relative to the foreign-currency price of nonoil tradables), P_t^O, follows therefore an exogenous process, which is assumed to be stochastic:

$$\frac{P_t^O}{\tilde{P}^O} = (\frac{P_{t-1}^O}{\tilde{P}^O})^{\rho_{P^O}} \exp(\epsilon_t^{P^O}), \tag{2}$$

where $\tilde{P}^O$ is the steady-state value of P_t^O, $\rho_{P^O} \in (0,1)$ is an autoregressive coefficient, and $\epsilon_t^{P^O}$ a normally distributed random shock with zero mean and a constant variance. Thus, any oil windfall due to a shock to prices is only temporary.

2.2 Nonoil Tradable Production

Nonoil tradable goods are produced competitively using labor, in quantity L_t^T, private capital, K_t^T, and public capital, K_t^I. The production function of these goods is given by

$$Y_t^T = Q_{t-1}^T (L_t^T)^{\beta} (K_t^T)^{1-\beta} (\frac{K_t^I}{K_t^P})^{\omega_T}, \tag{3}$$

where $\beta \in (0,1)$, $\omega_T > 0$, K_t^P is the aggregate private capital stock (defined later), and Q_t^T a productivity factor, which operates with a lag to capture gradual diffusion effects. Although public capital is nonexcludable, it is partially rival and subject to congestion. For simplicity, congestion is measured in terms of the aggregate stock of private capital.[8]

Given equation (3), firms maximize profits, defined as $\Pi_t^T = Y_t^T - w_t L_t^T - r_t^K K_t^T$,where w_t is the economy-wide wage rate (measured in terms of foreign currency) and r_t^K the economy-wide rental rate of capital. Factor prices and the aggregate private capital stock

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[8] More generally, congestion could be measured in the form $(K_t^P)^{\zeta}$ where $\zeta \geq 0$. See Eicher and Turnovsky (2000) for a discussion of congestion in models of growth with public capital.

are taken as given in solving this optimization problem. First-order conditions take the standard form

$$w_t = \beta(\frac{Y_t^T}{L_t^T}), \tag{4}$$

$$r_t^K = (1-\beta)(\frac{Y_t^T}{K_t^T}). \tag{5}$$

Productivity in the tradable sector is endogenous and evolves according to a learning-by-doing mechanism (external to firms), along the lines discussed by Torvik (2001). Specifically, productivity increases with the share of the population employed in the tradable sector:

$$Q_t^T = Q^{T,0}(\frac{L_t^T}{L_t})^{\nu_T}, \tag{6}$$

where $Q^{T,0} > 0$ and $\nu_T \in (0,1)$. Thus, the production process is such that workers learn from each other, so that their average productivity in a given period depends on the fraction of the labor force employed in that sector in the past. By implication, an appreciation of the real exchange rate that leads to a relative decline in the size of the tradable sector (as measured by the share of that sector in total employment) would be magnified by a reduction over time in productivity.[9]

2.3 Nontradable Production

Nontradable goods are produced competitively using also labor, L_t^N, private capital, K_t^N, and public capital. The production function is given by

$$Y_t^N = Q^N(L_t^N)^{\eta}(K_t^N)^{1-\eta}(\frac{K_t^I}{K_t^P})^{\omega_N}, \tag{7}$$

where $\eta \in (0,1)$, $\omega_N > 0$, and Q^N is a time-invariant productivity parameter. Thus, there are no learning spillovers between production sectors.

Given (7), firms maximize profits, defined as $\Pi_t^N = P_t^N Y_t^N - w_t L_t^N - r_t^K K_t^N$,where P_t^N is the retail price of nontradable goods, taking w_t, r_t and K_t^P as given. The first-order conditions are

$$z_t w_t = \eta(\frac{Y_t^N}{L_t^N}), \tag{8}$$

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[9] This is consistent with analyses of the Dutch disease effect by various other authors emphasizing productivity losses. See, for instance, Adam and Bevan (2006) in the context of aid flows.

$$z_t r_t^K = (1-\eta)(\frac{Y_t^N}{K_t^N}), \tag{9}$$

where $z_t = 1/P_t^N$ is defined as the *unadjusted* real exchange rate.

2.4 Distribution Sector

The sale of physical units of nonoil tradable goods to consumers requires using γ_t units of the nontradable good, which are spent in distribution services. This implies that the law of one price does not hold: the cost of distributing nonoil traded goods introduces a wedge between the producer and the consumer price of these goods.[10] As in Burstein et al. (2003), the distribution sector is perfectly competitive so the retail price of tradable goods is equal to the marginal cost. This implies that

$$P_t^T = 1 + \gamma_t P_t^N, \tag{10}$$

where P_t^T is the retail price of tradable goods.[11]

A significant fraction of distribution costs are assumed to consist of transportation costs, which are related negatively to access to (congested) infrastructure:

$$\gamma_t = \gamma^0 (\frac{K_t^I}{K_t^P})^{-\varrho}, \tag{11}$$

where $\gamma^0 > 0$ and $\varrho \in (0,1)$.

2.5 Households

Consumption decisions follow a two-step process: households first determine the optimal path of total consumption over time, and then allocate that amount at each moment in time between spending on nonoil tradables and nontradables. Specifically, in the first stage the representative household chooses sequences of consumption, C_{t+s}, labor, L_{t+s}, capital, K_{t+s+1}^P, and debt, D_{t+s+1}^P, for $s = 0, 1, ...\infty$, and given K_t^P and D_t^P, in order to maximize lifetime utility:

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda^s \left\{ \frac{[C_{t+s}(K_{t+s}^I)^{\eta_I}]^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} - \frac{\eta_L}{1+\psi} L_{t+s}^{1+\psi} \right\}, \tag{12}$$

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[10] In similar fashion, in Christiano et al. (2010) imported goods must be combined with domestic inputs before being passed onto final domestic users.

[11] Because there are no distribution costs in the nontradable sector, there is no difference between the wholesale and retail prices of nontradable goods.

where $\mathbb{E}_t$ is the expectations operator, conditional on information available up to period t, $\Lambda \in (0,1)$ is a discount factor, $\varsigma > 0$ the intertemporal elasticity of substitution in consumption, ψ the inverse of Frisch elasticity of labor supply, and $\eta_I, \eta_L > 0$ preference parameters. As in some recent contributions focusing on the dual nature of public goods, public capital interacts with consumption to generate direct utility benefits.[12]

At the beginning of period t, firms receive from households the stock of capital (which can be freely reallocated across production sectors) that they acquired at date $t-1$. The stock of private capital evolves therefore according to

$$K_t^P = (I_{t-1}^P)^{\varphi_K}(\frac{K_{t-1}^I}{K_{t-1}^P})^{1-\varphi_K} + (1-\delta^P)K_{t-1}^P - \Gamma(K_t^P, K_{t-1}^P), \tag{13}$$

where I_t^P is private investment, which consists only of spending on tradables, $\delta^P \in (0,1)$ a constant rate of depreciation, $\varphi_K \in (0,1)$, and $\Gamma()$ an adjustment cost function. In a novel fashion, equation (13) captures a *direct* complementarity effect between private investment and public capital, by assuming that gross private investment must be combined with (congested) public capital to generate *effective* investment. Thus, public capital is an essential input to private capital formation; it affects private investment not only through the rate of return to private capital and intertemporal consumption-saving decisions (as conventionally assumed in this type of models) but also directly through the ability to build private capital. The standard case corresponds to $\varphi_K = 1$.

The capital adjustment cost function is specified as

$$\Gamma(K_t^P, K_{t-1}^P) = 0.5\kappa(\frac{K_t^P}{K_{t-1}^P} - 1)^2 K_{t-1}^P, \tag{14}$$

where $\kappa > 0$ is an adjustment cost parameter.[13]

Households own both types of firms. Their net income consists of after-tax nonoil income, which is used to service their foreign debt, consume, invest, and pay lump-sum taxes, T_t^L; shortfalls in resources are offset by increases in foreign debt. Let D_t^P denote

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[12] See Agénor (2008), Chatterjee and Ghosh (2011), and Economides et al. (2011). In the first contribution, the interaction between public capital and consumption operates through the public provision of health services. Alternatively, as in Chatterjee and Turnovsky (2012) for instance, public capital could be made to interact with leisure to generate utility benefits.

[13] As in Turnovsky (1996), it could be assumed that public capital lowers adjustment costs associated with private investment. However, because the model is log-linearized for its solution, this extension would not alter the dynamics.

household foreign-currency debt; the representative household's end-of-period budget constraint is thus given by

$$D_{t+1}^P = (1 + r_t^W)D_t^P - (1 - \tau)(Y_t^T + z_t^{-1}Y_t^N) + C_t + I_t^P + T_t^L, \tag{15}$$

where $\tau \in (0, 1)$ and r_t^W is the interest rate that domestic agents face on world capital markets, which is defined later.

In the first stage of the optimization problem, households maximize (12) subject to (13)-(15). As shown in Appendix A, the first-order conditions are[14]

$$\mathbb{E}_t C_{t+1}^{-\varsigma^{-1}} = \frac{C_t^{-\varsigma^{-1}}}{\Lambda(1 + r_t^W)} \mathbb{E}_t[(\frac{K_t^I}{K_{t+1}^I})^{\eta_I(1-\varsigma^{-1})}], \tag{16}$$

$$L_t = [\frac{(1 - \tau)w_t C_t^{-\varsigma^{-1}}}{\eta_L (K_t^I)^{-\eta_I(1-\varsigma^{-1})}}]^{1/\psi}, \tag{17}$$

$$\mathbb{E}_t \left\{ \left[\kappa(\frac{K_{t+1}^P}{K_t^P} - 1) + 1 \right]^{-1} \left[(1 - \tau)r_{t+1}^K + 1 - \delta^P + \frac{\kappa}{2}(\frac{\Delta(K_{t+2}^P)^2}{(K_{t+1}^P)^2}) \right] \right\} = 1 + r_t^W, \tag{18}$$

together with the appropriate transversality conditions on K_t^P and D_t^P. Equation (16) boils down to the standard Euler equation when $\eta_I = 0$. Equation (17) defines labor supply, and (18), where $\Delta(K_{t+2}^P)^2 = (K_{t+2}^P)^2 - (K_{t+1}^P)^2$, equates the expected return on capital (net of marginal adjustment costs) with the world interest rate.[15]

Consumption is a bundle of nonoil tradables and nontradables, C_t^T and C_t^N:

$$C_t = (C_t^N)^\theta (C_t^T)^{1-\theta}, \tag{19}$$

where $\theta \in (0, 1)$.

Nominal consumption spending is $P_t^T C_t^T + P_t^N C_t^N$, that is, using (10), $(1 + \gamma_t P_t^N)C_t^T + P_t^N C_t^N$. In the second stage of the optimization problem, the representative household therefore maximizes (19) subject to the static budget constraint

$$C_t = (1 + \gamma_t z_t^{-1})C_t^T + z_t^{-1}C_t^N, \tag{20}$$

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[14] In solving this optimization problem I assume that the household does not internalize the complementarity effect. This is equivalent to setting $\varphi_K = 1$ in (13).

[15] With $\kappa_t = 0\ \forall t$, this expression takes the simpler arbitrage form $(1 - \tau)\mathbb{E}_t r_{t+1}^K = r_t^W + \delta^P$, which equates the (expected) marginal product of capital to its user cost, as measured by the rate at which households can borrow and lend, augmented by the capital depreciation rate.

where real consumption is measured (consistent with (15)) in terms of the world price of nonoil tradables. The solution is given by

$$C_t^N = \theta z_t C_t, \tag{21}$$

$$C_t^T = (\frac{1-\theta}{1+\gamma_t z_t^{-1}})C_t = (1-\theta)(\frac{z_t}{x_t})C_t, \tag{22}$$

where $x_t = P_t^T/P_t^N$ is the "adjusted" (for distribution costs) real exchange rate.[16] From (10),

$$x_t = \frac{1+\gamma_t P_t^N}{P_t^N} = z_t + \gamma_t, \tag{23}$$

which implies, given (11), that the adjusted real exchange rate depends directly on the public-private capital ratio.[17]

2.6 Government

The government receives revenues from oil production, T_t^O, taxes on nonoil income, T_t^{NO}, as well as lump-sum taxes, T_t^L. It also receives interest income on the stock of foreign-currency assets, F_t, held in a resource fund (if any), at the net interest rate r_t^F. Total revenue, measured in foreign-currency terms, is thus given by

$$T_t = T_t^O + T_t^{NO} + T_t^L + r_t^F F_t, \tag{24}$$

or equivalently

$$T_t = P_t^O Y_t^O + \tau(Y_t^T + z_t^{-1} Y_t^N) + T_t^L + r_t^F F_t. \tag{25}$$

The government buys nonoil tradable and nontradable goods at producer prices; its purchases, measured in foreign-currency terms, are given by G_t, whose determination depends (as discussed later) on the fiscal rule in place. Government spending is allocated in fixed fractions to investment I_t^G (measured in foreign-currency-terms) and consumption, C_t^G, which consists only of nontraded goods:

$$I_t^G = \upsilon^G G_t, \tag{26}$$

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[16] The Lagrangian for this problem is $L = C_t = (C_t^N)^\theta (C_t^T)^{1-\theta} + \mu[C_t - (1+\gamma_t z_t^{-1})C_t^T - z_t^{-1} C_t^N]$, where μ is a Lagrange multiplier. The first-order conditions are $\theta(C_t^T/C_t^N)^{1-\theta} = \mu z_t^{-1}$, and $(1-\theta)(C_t^T/C_t^N)^{-\theta} = \mu(1+\gamma_t z_t^{-1})$, that is, $C_t^N/C_t^T = [\theta/(1-\theta)][z_t/(1+\gamma_t z_t^{-1})]$. Substituting this result for C_t^T in the budget constraint (20) yields (21). Similar manipulations yield (22).

[17] Without distribution costs, the ratio z_t/x_t in (22) is of course unity.

$$C_t^G = (1 - \upsilon^G) z_t G_t, \tag{27}$$

where $\upsilon^G \in (0,1)$. Thus,

$$G_t = I_t^G + z_t^{-1} C_t^G. \tag{28}$$

In turn, public investment is allocated in fixed proportions between spending on nontraded goods, $I_t^{G,N}$, and spending on nonoil traded goods (such as imported machines), $I_t^{G,T}$:

$$I_t^{G,N} = \upsilon^{G,N} z_t I_t^G, \quad I_t^{G,T} = (1 - \upsilon^{G,N}) I_t^G, \tag{29}$$

where $\upsilon^{G,N} \in (0,1)$. By implication,

$$I_t^G = I_t^{G,T} + z_t^{-1} I_t^{G,N}. \tag{30}$$

The stock of public capital evolves according to

$$K_t^I = (1 - \delta^G) K_{t-1}^I + \varphi_{t-1} I_{t-1}^G, \tag{31}$$

where $\varphi_t \in (0,1)$ is an indicator of efficiency of spending on infrastructure and $\delta^G \in (0,1)$ is the depreciation rate.[18] In line with some contributions to the literature on public capital and growth (see Agénor (2009) and Rioja (2013)), the depreciation rate δ^G could be endogenized by relating it to the ratio of maintenance spending to the existing stock of capital. In the same vein, the rate of depreciation of the *private* capital stock, δ^P, could be related to the same ratio, to capture the fact that poorly-maintained public capital tends to accelerate the physical decay of private sector assets. However, these effects are unlikely to be significant in a short-term context and are therefore abstracted from for simplicity.[19]

To capture absorption capacity constraints, the efficiency parameter is assumed to be negatively related with the ratio of public investment to public capital:

$$\varphi_t = \varphi_0 (\frac{I_t^G}{K_t^I})^{-\varphi_1}, \tag{32}$$

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[18] This specification was first proposed by Pritchett (2000), in a discussion of growth accounting, and first used in a formal macro/development setting by Agénor (2010). Araujo et al. (2016) adopt the same assumption.

[19] Note also that if maintenance spending is proportional to the existing capital stock, as one would expect if wear and tear of fixed assets is fully provisioned for, the ratio of these two variables would be constant and so would be the depreciation rate.

where $\varphi_0, \varphi_1 > 0$. Thus, as investment (in proportion of the capital stock) increases, absorptive constraints tend to develop, possibly at an increasing rate ($\varphi_1 > 1$); this, in turn, tends to slow the rate of public capital accumulation and mitigate the benefits of higher public investment.[20]

The government issues foreign-currency denominated debt, D_t^G, at the world interest rate r_t^W, to finance its deficit. The government's flow budget constraint is thus given by[21]

$$D_{t+1}^G = (1 + r_t^W) D_t^G + G_t - T_t. \tag{33}$$

In principle, the transversality condition, $\lim_{T \to \infty} D_{t+T+1}^G / [\Pi_{j=0}^T (1 + r_{t+j}^W)] \leq 0$ would need to be imposed to rule out explosive paths of public debt. However, in the simulations reported later it will be assumed that the government does not issue additional debt to finance its deficit; sustainability issues therefore do not arise.[22]

The nonresource primary balance, B_t^{NO}, is defined as

$$B_t^{NO} = T_t^{NO} + T_t^L - G_t. \tag{34}$$

2.7 Market-Clearing Conditions

Total output, Y_t, measured in foreign-currency terms, can be defined as

$$Y_t = Y_t^T + z_t^{-1} Y_t^N + P_t^O Y_t^O. \tag{35}$$

The market-clearing condition of the nontradable sector equates supply of nontradables to demand, consisting of purchases by households and the government, public investment, and distribution costs:

$$Y_t^N = C_t^N + C_t^G + I_t^{G,N} + \gamma_t C_t^T. \tag{36}$$

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[20] By contrast, in Berg et al. (2013) and related contributions, the efficiency parameter is subject to a threshold effect related to the *level* of public investment. An alternative approach, as in van der Ploeg (2012) and van den Bremer and van der Ploeg (2013), would be to assume that public investment is subject to adjustment costs, which fall with the amount invested.

[21] If the government can draw freely on the asssets held by the sovereign fund, the overall fiscal balance, equation (33), would take the form $\Delta D_{t+1}^G - \Delta F_{t+1} = r_t^W D_t^G + G_t - T_t$.

[22] See van der Ploeg and Venables (2013) and Araujo et al. (2016) for a discussion of public external borrowing in a related context.

Total labor is allocated between the two production sectors. Thus, the labor market equilibrium condition is given by[23]

$$L_t = L_t^N + L_t^T. \tag{37}$$

The capital market equilibrium condition indicates that the stock of capital acquired last period is allocated today across the two production sectors:[24]

$$K_{t-1}^P = K_t^T + K_t^N. \tag{38}$$

In the benchmark case, and some subsequent experiments, no resources are accumulated in the sovereign wealth fund; thus

$$F_{t+1} = F_t = 0. \tag{39}$$

Combining the household and government budget constraints, together with the equilibrium condition (36) and the accumulation rule (39), yields the debt accumulation equation (or equivalently, the negative of the current account balance):[25]

$$\begin{aligned} D_{t+1} - F_{t+1} = (1 + r_t^W)D_t - (1 + r_t^F)F_t + C_t^T \\ +I_t^{G,T} + I_t^P - Y_t^T - P_t^O Y_t^O, \end{aligned} \tag{40}$$

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[23] Partial labor mobility across sectors could be introduced along the lines proposed by Horvath (2000) for instance or in Harris-Todaro fashion, as in Agénor and Aizenman (1996). However, much of the evidence suggests a high degree of cross-sectoral labor mobility in low-income countries; see Agénor (2006).

[24] The assumption that capital is perfectly mobile across sectors captures the idea that redeploying production from tradables to nontradables can be done quickly, within the same production units, at no cost.

[25] Adding the private sector and government budget constraints (15) and (33) yields $D_{t+1}^P + D_{t+1}^G = (1+r_t^W)(D_t^P+D_t^G)+C_t+I_t^P+G_t-(Y_t^T+z_t^{-1}Y_t^N+P_t^OY_t^O)-[T-\tau(Y_t^T+z_t^{-1}Y_t^N)-P_t^OY_t^O-T_t^L]$, that is, defining $D_{t+j} = D_{t+j}^P+D_{t+j}^G$, $j=0,1$, and using (25) to substitute for T_t, $D_{t+1} = (1+r_t^W)D_t+C_t+I_t^P+G_t-(Y_t^T+z_t^{-1}Y_t^N+P_t^OY_t^O)-r_t^FF_t$. Using (20) to substitute for $C_t = (1+\gamma_tz_t^{-1})C_t^T+z_t^{-1}C_t^N$ and (28) and (30) to substitute for $G_t = I_t^{G,T}+z_t^{-1}I_t^{G,N}+z_t^{-1}C_t^G$, yields in turn $D_{t+1} = (1+r_t^W)D_t+(1+\gamma_tz_t^{-1})C_t^T+z_t^{-1}C_t^N+I_t^P+I_t^{G,T}+z_t^{-1}I_t^{G,N}+z_t^{-1}C_t^G-(Y_t^T+z_t^{-1}Y_t^N+P_t^OY_t^O)-r_t^FF_t$, which can be rearranged as $D_{t+1} = (1+r_t^W)D_t+C_t^T+I_t^P+I_t^{G,T}-Y_t^T-P_t^OY_t^O-[z_t^{-1}Y_t^N-\gamma_tz_t^{-1}C_t^T-z_t^{-1}C_t^N-z_t^{-1}I_t^{G,N}-z_t^{-1}C_t^G]-r_t^FF_t$. The expression in brackets on the right-hand side corresponds to the equilibrium condition of the market for nontraded goods, equation (36). Substituting that equation in the previous expression and subtracting (39) from the resulting expression gives (40). With partial resource accumulation, (39) is replaced by $F_{t+1} = (1-\phi^F)F_t+\chi T_t^O$, and $P_t^OY_t^O$ in (25) is replaced by $(1-\chi)P_t^OY_t^O$, with $\phi^F, \chi \in (0,1)$. The only change in (40) is that $1+r_t^F$ is replaced by $1+r_t^F-\phi^F$ if management fees (defined as ϕ^FF_t) are paid to nonresidents.

where $D_t = D_t^P + D_t^G$ denotes total foreign liabilities, measured in foreign-currency terms. This condition states that changes in the economy's net foreign liabilities, $D_t - F_t$, are given by the difference between the country's tradable spending (given by the sum of debt service, private consumption of, and government investment spending on, nonoil tradables, and private investment), and tradable income (which consists of nonoil tradable output, oil income, and the return on resources invested in the sovereign fund).

2.8 World Interest Rate

The interest rate earned by the country's sovereign fund, r_t^F, is set equal to the constant risk-free world interest rate, $r^{W,R}$:[26]

$$r_t^F = r^{W,R}. \tag{41}$$

By contrast, the market cost of foreign borrowing depends on the world risk-free rate and a risk premium, PR_t:

$$r_t^W = (1 + r^{W,R})(1 + PR_t) - 1. \tag{42}$$

The premium itself is defined as being positively related to the country's *government* debt-tradable output ratio, and negatively to the country's tradable over nontradable output, which represents an indicator of the capacity to raise foreign exchange and service external debt:[27]

$$PR_t = (\frac{D_t^G}{Y_t^T + P_t^O Y_t^O})^{pr_1} (\frac{Y_t^T + P_t^O Y_t^O}{z_t^{-1} Y_t^N})^{-pr_2}, \tag{43}$$

where pr_1, $pr_2 > 0$. Thus, all else equal, an increase in oil output lowers the risk premium through both a reduction in the debt-tradable output ratio and an increase in the tradable-nontradable output ratio. In line with the evidence on the determinants of foreign borrowing spreads, it is the balance sheet of the *government* that affects the premium, rather than the net liabilities of domestic agents.

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[26] In practice, the rate of return on assets held by sovereign funds tends to be significantly higher than the world risk-free interest rate. However, as long as the differential is assumed constant, this would have no effect on the analysis.

[27] For the first effect, see for instance Bhandari et al. (1990), Murphy (1991), and Turnovsky (1997), as well as other contributions in the sovereign debt literature. The second effect was introduced in Agénor and Aizenman (1999). Note also that the premium could be endogenized by relating it to *individual* debt, as in Agénor (1997), which implies that households internalize the impact of their decisions on the cost of borrowing.

A competitive equilibrium in this framework consists of sequences of allocations $\{C_t^N, C_t^T, I_t^P, D_t, F_t, L_t^N, L_t^T, K_t^P, K_t^N, K_t^T, G_t\}_{t=0}^{\infty}$, final good and factor input prices, $\{P_t^N, P_t^T, w_t, r_t^K\}_{t=0}^{\infty}$, such that, taking as given K_{-1}^P, K_{-1}^I, D_{-1}, F_{-1}, the exogenous processes $\{P_t^O, Y_t^O\}_{t=0}^{\infty}$, constant policy parameters τ, υ^G, and $\upsilon^{G,N}$, and constant public debt,

a) $\{C_t, C_t^N, C_t^T, L_t, I_t^P, D_t^P, K_t^P\}_{t=0}^{\infty}$ solve households' optimization problem;

b) $\{L_t^N, K_t^N\}$ solve the nontradable good firm's optimization problem;

c) $\{L_t^T, K_t^T\}$ solve the nonoil tradable good firm's optimization problem;

d) the government sets a sequence of spending $\{G_t\}_{t=0}^{\infty}$, its components $\{C_t^G, I_t^G\}_{t=0}^{\infty}$, and a sequence of lump-sum taxes $\{T_t^L\}_{t=0}^{\infty}$, so that its flow and lifetime budget constraints are satisfied; and

e) market-clearing conditions for nontradable goods, labor, private capital, and nonoil tradable goods (equations (36), (37), (38), and (40), respectively) are satisfied.

3 Steady State and Log-Linearization

The steady-state equilibrium is characterized in Appendix B. Most of the equilibrium conditions are standard; in particular, from (18) the rental rate of capital is equal to $(1-\tau)\tilde{r}^K = \tilde{r}^W + \delta^P$, whereas from (13) and (14) private investment is equal to $\tilde{I}^P = \delta^P \tilde{K}^P$ when $\varphi_K = 1$.[28] And from the Euler equation (16), because the equilibrium stock of public capital is constant, the steady-state world interest rate is given by the standard expression $r^W = \Lambda^{-1} - 1$. At the same time, as can be inferred from from (26) and (31), the steady-state public capital stock is $\tilde{K}^I = \varphi \upsilon^G \tilde{G}/\delta^G$; from (3), (7), and (11), this value affects the steady-state values of nonoil tradables and nontradables output, as well as distribution costs.

The model is log-linearized in Appendix C. Most of the equations are fairly standard; for convenience, and because they matter for specifying the fiscal rules that are studied later on, some equations pertaining to fiscal variables are reported here. Specifically, the log-linearized equations for total revenues (24) and the government budget constraint (33)

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[28] In what follows steady-state values are denoted by tildes without time script, whereas percentage point deviations for interest rates and log-deviations for the other variables are denoted with a caret.

with constant debt are given by

$$\tilde{T}\hat{T}_t = \tilde{T}^O\hat{T}_t^O + \tilde{T}^{NO}\hat{T}_t^{NO} + \tilde{T}^L\hat{T}_t^L + (1+\tilde{r}^F)\tilde{F}(\hat{r}_t^F + \hat{F}_t) - \tilde{F}\hat{F}_t, \tag{44}$$

$$\tilde{T}\hat{T}_t = \left(1+\tilde{r}^W\right)\tilde{D}^G\hat{r}_t^W + \tilde{G}\hat{G}_t. \tag{45}$$

These equations can be combined and solved for either lump-sum taxes, $\hat{T}_t^L$ or government spending, $\hat{G}_t$, depending on the fiscal rule in place (which also determines changes in assets held in the sovereign fund, $\hat{F}_t$).

For later convenience, the log-linearized equation for the nonoil primary balance is

$$\tilde{B}^{NO}\hat{B}_t^{NO} = \tilde{T}^{NO}\hat{T}_t^{NO} + \tilde{T}^L\hat{T}_t^L - \tilde{G}\hat{G}_t. \tag{46}$$

4 Parameterization

To understand the likely impact of an oil windfall on low-income countries in general the model is parameterized for a "typical" poor country that has recently begun to produce oil, whose reserves will take only a few decades for depletion, and has not yet accumulated substantial financial assets in a sovereign fund. Data sources include the World Bank's *African Development Indicators* database, the IMF's *World Economic Outlook* (WEO) database, as well as parameter estimates from various published papers.

For households, the intertemporal discount factor is set at 0.898, based on the estimates of the real interest rate and the depreciation rate of private capital provided below. The intertemporal elasticity of substitution, ς, is set at 0.2, in line with the evidence for low-income countries reported in Agénor and Montiel (2015, Chapter 11). The Frisch elasticity of labor supply is set at 0.125 (so that $\psi = 8$) to capture a fairly inelastic supply of labor. This is a fairly reasonable assumption for low-income countries.[29] The preference parameter η_L is set at a fairly low value, 0.14. The share of nontradables in total consumption, θ, is set at 0.55, in line for instance with Pieschacón (2012) and Rabanal and Tuesta (2013).[30] The elasticity η_I is set initially at a relatively low value, 0.08, and sensitivity analysis is reported later on.

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[29] Berg et al. (2013) for instance use a value of 10.

[30] Dagher et al. (2012) use a value of 0.4, while at the same time using a share of nontradable output in total output of 0.61, using national accounts data for Ghana.

The rate of depreciation of private capital, δ^P, is set at 0.045, which is consistent with some of the values reported by Bu (2006) for Sub-Saharan African countries. The parameter that measures the strength of adjustment costs to private capital is set at a high level, $\kappa = 25$, to match a slow response of private investment to shocks. This is the same value used in Berg et al. (2013) and follow-up studies. The direct complementarity effect of public capital on private investment is assumed to be initially strong; accordingly, $\varphi_K = 0.5$. Sensitivity analysis with a weaker effect (higher φ_K) is also reported later on.

For the oil sector, the degree of persistence in production, ρ_{YO}, is computed such that at half-life 50 percent of the oil reserves are still left in the ground. If proven oil reserves are expected to last 30 years, the formula yields $(\rho_{YO})^{15} = 0.5$, or equivalently $\rho_{YO} = 0.955$. For oil prices, the degree of persistence is set at $\rho_{PO} = 0.93$ and the standard deviation of the nonsystematic shock ϵ_t^{PO} at 0.25, as in Maliszewski (2009), based on econometric estimates using real spot prices in the world oil market.

For the nonoil sector, elasticities of production with respect to labor, β and η, are set equal to 0.6 and 0.7, respectively, to capture the fact that production in the nontradable sector is relatively more labor intensive ($\eta > \beta$). These values are consistent with the range of estimates of the share of labor income for developing countries obtained by Guerriero (2012) for instance. The evidence on the strength of learning-by-doing effects in tradable activities is somewhat ambiguous (see Syverson (2011)); consequently, the elasticity of the productivity factor in the tradable sector with respect to the share of the labor force engaged in that sector, ν_T, is kept at a fairly low value, 0.03. The elasticities with respect to public capital, ω_N and ω_T, are set initially at the same value in both sectors, 0.17, which corresponds to the long-run value estimated through meta-regression analysis by Bom and Ligthart (2014, Table 4) for core public capital. This value may underestimate the true elasticity of output with respect to public infrastructure, either because it does not account for the human capital or other externalities associated with public capital (Agénor and Neanidis (2015)) or because it is based on studies that did not use efficiency-adjusted measures of the public capital stock. Sensitivity analysis is thus reported later on.

In the distribution sector, the sensitivity of distribution costs with respect to the public-private capital ratio, ϱ, is set at 4, to capture an initial situation where the lack of access to infrastructure has significant marginal effects on these costs. The initial value of the distribution cost parameter, γ, is set at 0.5, as in Burnside et al. (2006) and Oviedo and Singh (2013) for instance.

Regarding the government, the nonoil tax rate, τ, is set equal to the average tax revenue-to-GDP ratio calculated by Baldacci et al. (2004, Table 1) for a group of 39 low-income countries, 15.1 percent. The initial ratio of noninterest spending in GDP, G/Y, is set equal to 17.1 percent to capture an initial position of fiscal imbalance. Based on an estimate of debt service (itself calculated from values of D^G and r^W reported next), the steady-state solution of the government overall fiscal balance (33), which gives $\tilde{T} = \tilde{r}^W \tilde{D}^G + \tilde{G}$, can be used to calibrate the initial value of lump-sum taxes, $\tilde{T}^L$. The initial share of infrastructure investment in government spending, υ^G, is set at 0.114, or equivalently 1.9 percent of GDP. Thus, the case considered here is consistent with the range of estimates of investment shares reported in Foster and Briceño-Garmendia (2010) for Sub-Saharan Africa. The parameter that captures the allocation of investment in infrastructure to nontraded goods, $\upsilon^{G,N}$, is set at 0.62, within the range of estimates of the share of nontradables in total investment for Côte d'Ivoire, Gabon, Ghana, and Uganda reported by Bems (2008, Table 8). Using the average value for the 30 Sub-Saharan African countries (excluding South Africa) in the sample compiled by Dabla-Norris et al. (2012, Table 1), the efficiency parameter for public investment, φ, is set at 0.37.[31] The parameter φ_1 is set at the low value of 0.05 initially, which implies that φ_0, which is solved for residually, is equal to 0.33. The rate of depreciation of public capital, δ^G, is set at 0.035, in line with Agénor et al. (2008) and Dagher et al. (2012) for instance.

Government debt as a share of GDP, D^G/Y, is set equal to 23.7 percent, which is equal to the ratio of external debt to GDP for Sub-Saharan Africa in 2010 reported in the WEO database. Thus, all public debt is assumed to result from foreign borrowing.[32] From the estimates compiled by Boyce and Ndikumana (2012, Table 1) for 33 countries in Sub-Saharan Africa, the average stock of private capital flight in the same year represented 78.7

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[31] Dabla-Norris et al. (2012) define their metric on a range of 1 to 4 with an average value of 1.47 for the 30 countries; this value was simply divided by 4 to obtain an indicator between zero and unity.

[32] Although domestic public debt has increased in recent years in low-income countries, it remains largely concentrated in the hands of the central bank and commercial banks; see Bua et al. (2014). Note also that in all the simulations reported next, the stock of public debt is assumed constant. However, because the country-specific interest rate changes, debt service changes as well.

percent of GDP. Thus, the economy's net stock of external debt, as a share of GDP, can be calibrated at $23.7-78.7 = -55.0$ percent. By this metric, in the initial equilibrium the country is a net creditor to the rest of the world. However, due to market imperfections only the *public* debt matters in the determination of the risk premium.

The world risk-free interest rate (in foreign-currency terms), $r^{W,R}$, is set initially at 0.017, which corresponds to the difference between recent averages on nominal yields on U.S. treasury 30-year bonds and an average rate of U.S. inflation of 2.0 percent.[33] To estimate the country risk premium, the spread on sovereign bonds issued by Kenya on international financial markets is used. Recent averages on nominal yields on 30-year sovereign bonds issued by that country is 13.3 percent; the risk premium (in foreign-currency terms) can thus be calculated as $[(1 + 0.113)/(1 + 0.017)] - 1 = 0.094$.[34] This also implies that the household discount factor is equal to $1/(1+0.113) = 0.898$, as noted earlier.[35] The elasticities of the risk premium with respect to the debt-tradable output ratio and the tradable-nontradable output ratio, pr_1 and pr_2, respectively, are set at 0.8 and 0.0 initially, and sensitivity analysis is reported later on. Assets of the resource fund in proportion of GDP, F/Y, are set initially at a fairly low value, 0.01; the rate of return on these assets, as noted earlier, is set equal to the world risk-free rate.

Table 1 summarizes the benchmark parameter values whereas Table 2 presents initial steady-state values. The share of nontradable output in total output is equal to 59.5 percent, the share of oil production is 7.8 percent (and 39.6 percent of total government revenues), and the share of nonoil tradable output is 32.5 percent. These numbers are in line with those found in some other studies for developing countries (see for instance Dagher et al. (2012), Pieschacón (2012, Table 2)). The rate of return on private capital is 17.8 percent and the private capital-output ratio is about 2, consistent with the evidence for developing countries (see World Bank (2013, Chapter 1)). Private consumption represents 75.7 percent of output and private investment is 11.7 percent, the latter in line with the low values observed in poor countries. Government consumption accounts for 20

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[33] See http://www.investing.com/.

[34] By way of comparison, van der Ploeg and Venables (2011) estimate from cross-country regressions a mean value for the interest rate spread of the order of 6.3 percent.

[35] This expression is derived from the steady-state relationship between the world interest rate and the discount factor, $\tilde{r}^W = \Lambda^{-1} - 1$. See Appendix B.

percent of output and public investment (as noted earlier) is 1.9 percent. The nonresource primary balance is in surplus initially, at 6 percent of output. The initial public-private capital ratio is 9.2 percent, consistent with the assumption of an infrastructure-constrained economy.

5 Benchmark Experiment: Cash Transfers

Suppose that the country considered cannot hedge against commodity price risk and experiences at period t an unanticipated and temporary positive shock to the real price of oil, P_t^O.[36] The resource windfall corresponds therefore to the log-difference between actual oil revenues and their steady-state value, as defined in the log-linearized version of the model, weighted by their steady-state value, $\tilde{T}^O\hat{T}_t^O$.

In the benchmark experiment, this windfall is assumed to be transferred entirely and directly to households. Cash transfers, especially *conditional* transfers, have become an increasingly popular tool of development policy; although these programs started primarily in middle-income countries (see for instance Fiszbein and Schady (2009) and Baird et al. (2013)), they are gaining rapid ground in low-income countries such as Ghana, Kenya, and Uganda.

A number of arguments can be put forward to justify this choice. The first is that because citizens, in effect, own the natural resources, the most appropriate approach is to transfer any windfall revenues back to them; even if these transfers are not conditional on specific actions by the recipients (in terms of spending on children's health and education for instance), they may help to alleviate poverty directly.[37] The second is that the distribution of resource rents to citizens can mitigate the governance risks associated with natural resource revenues (see Gelb and Grasmann (2010)). In that perspective, the resource curse is a symptom of societies characterized by high levels of corruption and a weak system of checks and balances on political decision-making; by taking resources away from politicians and putting them directly in the hands of the citizenry, waste can

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[36] See Borensztein et al. (2013) for a discussion of the reasons why countries do not, or cannot, hedge against these risks. This assumption is particularly appropriate here given that we consider a low-income country with limited access to world capital markets.

[37] See Fuentes (2011) for a discussion of political pressures to distribute resource revenues across groups in the case of Chile.

be avoided. The model considered in this paper does not account for overlapping generations and the role of health and education spending on productivity; it is therefore not adequate to discuss *conditional* cash transfers.[38] However, in the model, cash transfers are not purely unproductive; households can allocate their resources not only to consumption but also to capital accumulation, thereby increasing future income and consumption.

Thus, with the *cash transfer rule*, there is no asset accumulation ($\hat{F}_t = 0$) and lump-sum taxes are reduced by the amount of the windfall in oil revenues:

$$\tilde{T}^L \hat{T}_t^L = -\tilde{T}^O \hat{T}_t^O, \tag{47}$$

with government spending $\hat{G}_t$ solved residually from the government budget constraint (45). Substituting (47) in equation (44) and setting $\hat{F}_t = \hat{r}_t^F = 0$ yields $\tilde{T}\hat{T}_t = \tilde{T}^{NO}\hat{T}_t^{NO}$. In turn, substituting this result in (45) yields

$$\tilde{G}\hat{G}_t = \tilde{T}^{NO}\hat{T}_t^{NO} - \left(1 + \tilde{r}^W\right) \tilde{D}^G \hat{r}_t^W. \tag{48}$$

The results of this experiment for a 10 percent temporary shock in P_t^O are shown in Figure 1. On impact, the transfer of resources to households creates a temporary wealth effect which raises total consumption and consumption of both traded and (at the initial exchange rate) nontraded goods. The increase in the demand for nontraded goods leads to a real appreciation; to maintain equilibrium in the labor market, the product wage (measured in terms of the price of nonoil traded goods) must increase. This increase, however, is less than proportional compared to the movement in the real exchange rate, implying that the product wage in the nontradable sector falls. There is therefore a shift on the supply side toward the production of nontradables. At the same time, the increase in consumption raises the demand for leisure and lowers labor supply; total employment falls while at the same time workers are reallocated from the production of nonoil tradables to nontradables. At first, the drop in the former exceeds the expansion of the latter, implying that total output falls. This tends to lower nonoil tax revenues. And because the relative size of the oil sector is small, total output of tradable goods also falls, thereby

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[38] In poor countries limited administrative capacity and institutional weaknesses in general may also make it difficult to implement a conditional cash transfer program. Accounting for these weaknesses would also be important to assess the macroeconomic effects of these programs.

(from (43)) raising the risk premium on world capital markets through a higher debt-tradable output ratio. The increase in the cost of borrowing tends to reduce (through the arbitrage condition with the rental rate of capital) private investment and the rate of accumulation of private capital, which mitigates the expansion in the nontradable sector.[39] Because nonoil revenues fall and the world interest rate increases, equation (48) implies a contraction in total government spending—and in public investment as well. The impact of lower investment on the stock of public capital is partly mitigated by an increase in investment efficiency, due to a relaxation of absorption constraints. However, because the private capital stock falls proportionally more—not only because of the higher cost of borrowing, alluded to earlier, but also as a result of a reverse complementary effect—the public-private capital ratio increases, thereby increasing productivity of private inputs and promoting activity in both production sectors. The fall in government spending also mitigates the positive demand-side effect stemming from higher private consumption on the real exchange rate. The nonresource primary balance to output ratio weakens substantially on impact, despite the fall in government spending.

Over time, the initial drop in nonoil output is reversed; nonoil tax revenues begin to increase, and public spending starts to recover, thereby inducing a gradual recovery in public investment. Although the efficiency of investment falls somewhat due to absorption constraints, the stock of public capital also begins to recover. Because the increase in the risk premium is fairly persistent, private investment remains depressed for a long while. Despite the recovery in labor supply (which mirrors the behavior of consumption) the persistent drop in the private capital stock leads to a contraction in output of nontradables, whereas the gradual reversal in the behavior of the real exchange rate (which generates expenditure-switching effects), the sustained increase in the public-private capital ratio, benefits production of traded goods. Productivity in the traded goods sector rises also, through learning-by-doing externalities. At the same time, the increase in the public-private capital ratio raises the marginal product of labor, thereby also contributing to the recovery in employment, and reduces distribution costs, which mitigate the effect of the shock on the (adjusted) real exchange rate.

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[39] The increase in the world interest rate also tends to reduce private consumption today through the intertemporal effect, thereby mitigating the initial increase associated with the wealth effect.

Thus, a cash transfer policy does generate an expansion in aggregate demand and a real exchange rate appreciation. However, because government spending (which is determined residually in this experiment) falls, so does public investment and the public capital stock. There is therefore an adverse supply-side effect which magnifies the drop in output of nonoil traded goods and mitigates the expansion of output in the nontraded goods sector induced by the increase in profitability. The Dutch disease effect is mitigated over time, as the increase in public capital benefits the supply side. The latter result is consistent with other studies—such as Agénor et al. (2008) in the context of aid, and Berg et al. (2013) in the context of resource price shocks—that have emphasized the productivity effects of infrastructure. In the present model, however, the transmission process is more complex, given that there are a number of additional channels through which public capital affects the economy.

Figures 2 to 6 study the sensitivity of these results with respect to changes in five parameters: a higher elasticity of utility with respect to the public-private capital ratio, η_I, from 0.08 to 0.2; a higher elasticity of output in both production sectors to the public-private capital ratio, ω_N and ω_T, from 0.17 to 0.25, a value consistent with the upper end of the estimates reported by Bom and Lightart (2014) and the general equilibrium estimates of Agénor and Neanidis (2015); a smaller degree of complementarity between public and private investment, as measured by φ_K, from 0.5 to 0.6; a lower elasticity of the risk premium with respect to the debt-tradable output ratio, pr_1, from 0.8 to 0.6; and a higher elasticity of the risk premium with respect to the tradable-nontradable output ratio, pr_2, from 0.0 to 0.2.[40] In each case, the benchmark results obtained in Figure 1 are also displayed.

Figure 2 presents the results associated with a higher η_I. The future drop in the public-private capital stock is brought forward through a lower rate of increase in private consumption. The appreciation of the real exchange rate is therefore mitigated. On impact, in fact, there is a small depreciation. At the same time, the demand for leisure falls and labor supply increases, which magnifies the expansion in output in the nontradable sector (which is more labor intensive, to begin with) and amplifies the contraction in the nonoil tradable sector. As a result, the fall in output in the nonoil tradable sector

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[40] The case of a higher φ_1, which measures the magnitude of absorption constraints, is discussed later.

is amplified, and so is the drop in total output. Because nonoil tax revenues are now falling by more initially, government spending also falls by more, the rate of accumulation of public capital is slower, and the increase in the public-private capital ratio is less pronounced.

Figure 3 displays the results associated with higher ω_N and ω_T. On impact, the higher degree of complementarity between public capital and labor raises employment in the nontraded goods sector by more; this is met in part by a smaller drop in labor supply and a reallocation of workers away from the nonoil tradable sector, which implies a larger fall in output from that sector. As a result, total output falls by more than in the benchmark case of $\omega_N = \omega_T = 0.17$, and so do nonoil revenues. The drop in government spending and infrastructure investment is thus more significant, implying that the public capital rises by less than before. The future drop in the public-private capital stock is brought forward through a smaller increase in private consumption. The drop in output implies also a more significant increase in the risk premium, which lowers consumption. Even though the cost of borrowing is now higher, and the public-private capital ratio (which determines the magnitude of the complementarity effect) lower, investment now falls by less than before because of the smaller increase in consumption. The combination of a lower level of aggregate demand and higher output of nontraded goods implies that the real exchange rate now depreciates on impact.

Figure 4 shows the results associated with a higher φ_K. Compared to those obtained with the benchmark set of parameters, the drop in the public-private capital ratio has now a smaller impact on private investment. As a result, private investment falls by less than before (it actually expands), implying that the fall in the public-private capital ratio is now even more significant. As a result, current consumption increases by less (it actually falls). The output and employment effects are the same as in the case of higher ω_N and ω_T and once again the real exchange rate depreciates, as a result of higher output of, and lower spending on, nontraded goods.

Figure 5 presents the results associated with a smaller pr_1. The cyclical movement in almost all the variables is magnified, because the response of the risk premium, and the cost of borrowing abroad, in response to the initial fall in nonoil tradable output is now more pronounced. By and large, however, the results remain in the same direction, except for the real exchange rate, which depreciates on impact.

Finally, Figure 6 shows the results associated with a higher pr_2. On impact, the ratio of (oil and nonoil) tradable output to nontradable output, adjusted for changes in relative prices, increases; as a result, the risk premium, and the cost of borrowing abroad, fall by more than before. This lowers saving and raises consumption today by more, implying a larger drop in investment. The higher demand for leisure reduces labor supply, which in turn tends to raise wages. However, because the larger increase in private consumption translates into a real appreciation, the product wage in the nontradable sector increases by more than in the tradable sector. As a result, employment falls by more in the nontradable sector and is partly reallocated to the tradable sector, where the drop in output is mitigated. Consequently, the drops in nonoil revenues, government spending, and infrastructure investment are also all less pronounced than in the initial case, implying a higher public-private capital ratio.

6 Alternative Fiscal Rules

As alternatives to the benchmark experiment of cash transfers, two fiscal policy rules are now considered. As discussed in the literature, both rules have been much debated in the literature. Temporary resource windfalls may create (through fiscal expansion) sizable aggregate demand pressures and macroeconomic volatility in the short run. Even though supply-side effects may mitigate these effects in the longer run, fiscal rules, in the form of asset accumulation in a sovereign fund, can play an important role in smoothing fluctuations and stabilizing the economy. Under both rules, it is assumed again that public debt is constant; the government budget is balanced through lump-sum taxes.

With the *full spending rule*, $\hat{F}_t = 0$ again (no asset accumulation) and oil revenues are spent entirely on both consumption and investment:

$$\tilde{G}\hat{G}_t = \tilde{T}^O\hat{T}_t^O, \tag{49}$$

with lump-sum taxes $\hat{T}_t^L$ solved residually from the government budget constraint (45).

After substituting for $\hat{T}_t$ from (44), using (49), equation (45) yields

$$\tilde{T}^L\hat{T}_t^L = -\tilde{T}^{NO}\hat{T}_t^{NO} - (1+\tilde{r}^F)\tilde{F}(\hat{r}_t^F + \hat{F}_t) + \tilde{F}\hat{F}_t + \left(1+\tilde{r}^W\right)\tilde{D}^G\hat{r}_t^W,$$

or, with $\hat{F}_t = \hat{r}_t^F = 0$:

$$\tilde{T}^L \hat{T}_t^L = -\tilde{T}^{NO} \hat{T}_t^{NO} + \left(1 + \tilde{r}^W\right) \tilde{D}^G \hat{r}_t^W. \tag{50}$$

With the *full saving rule* (or full asset accumulation), the log-linearized law of motion for the stock of assets is given by

$$\tilde{F} \hat{F}_{t+1} = (1 - \phi^F) \tilde{F} \hat{F}_t + \tilde{T}^O \hat{T}_t^O, \tag{51}$$

where $\phi^F \in (0, 1)$ is a coefficient that measures a (small) management fee, levied on the stock of assets held in the sovereign fund.[41] Deviations in government spending from its steady-state value are now limited to changes in the flow of interest income generated from the fund:[42]

$$\tilde{G} \hat{G}_t = (1 + \tilde{r}^F) \tilde{F} (\hat{r}_t^F + \hat{F}_t) - \tilde{F} \hat{F}_t. \tag{52}$$

Because oil revenues are no longer a direct resource for the government, $\tilde{T}^O \hat{T}_t^O$ must be taken out of the definition of $\hat{T}_t$, implying therefore that equation (44) becomes

$$\tilde{T} \hat{T}_t = \tilde{T}^{NO} \hat{T}_t^{NO} + \tilde{T}^L \hat{T}_t^L + (1 + \tilde{r}^F) \tilde{F} (\hat{r}_t^F + \hat{F}_t) - \tilde{F} \hat{F}_t.$$

Substituting this result in the government budget constraint (45) and using (52) implies that lump-sum taxes are once again determined by (50).

The first experiment is consistent with the view, advocated in a number of recent contributions, that low-income countries should use resource windfalls to address their development needs. In contrast to some of these studies, however, the view here is that these development needs are not limited to infrastructure investment; poor countries also face pressing needs in terms of education and health services. Even though the supply of these services are not explicitly modeled, this is captured by assuming that resource windfalls are allocated to total government spending, as stated in (49).[43] The

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[41] With $\phi^F = 0$, a temporary increase in external savings due to an oil windfall would lead to a permanent increase in the stock of external assets. A positive value of ϕ^F (however small) is necessary to eliminate the unit root in F_t and to calculate the asymptotic variances that are needed later on.

[42] In level terms, equations (51) and (52) correspond to $F_{t+1} = (1 - \phi_F) F_t + T_t^O$ and $G_t = r_t^F F_t$. Note that because assets of different maturities are not explicitly introduced, there is actually no distinction between a sovereign *wealth* fund and a sovereign *liquidity* fund, which has a much shorter-term focus, to smooth expenditures. As a result the term sovereign fund is used throughout.

[43] Another reason to consider total spending rather than investment only is that sustained increases in investment, I_t^G, must be accompanied by higher spending on maintenance (and thus, C_t^G) as well, to avoid rapid depreciation of the capital stock.

second experiment corresponds to the so-called bird-in-hand policy: the government limits spending from a windfall to the stream of returns from accumulated financial assets (see Collier et al. (2010)).[44] As reported in Table 1, the management fee, ϕ^F, is set at a low value, 0.25 percent of the stock of assets, a number in line with current practices of international asset management companies.[45]

The results of a 10 percent temporary shock in P_t^O under the two rules, and for two values of the parameter that measures the strength of absorption constraints, φ_1 (the benchmark value of 0.05 and a higher value of 0.5), are shown in Figures 7 and 8. With the full spending rule Dutch disease effects are magnified, compared to the benchmark case (cash transfers) represented in Figure 1. The main reason is that now government spending rises (rather than falls), implying higher investment and a higher public capital stock. The appreciation of the real exchange rate is thus more significant, despite the reduction in distribution costs. As a result, the reallocation of resources from the tradable sector to the nontradable sector, and the expansion in production in the latter, are magnified. The fall in total output of tradable goods and the associated increase in the risk premium lead again to a contraction in private investment, and to an increase in consumption, this time through the intertemporal effect. With a greater incidence of absorption constraints, the efficiency of public investment drops markedly, and the public capital stock accumulates at a slower rate. In turn, this mitigates the expansion in the nontradable sector over time but overall the direction of the effects are the same. Lump-sum taxes now increase, thereby moderating the increase in private consumption.

With the full saving rule, assets held in the sovereign fund increase rapidly (as a share of output) and stabilize at about 300 percent of GDP.[46] The interest income from the fund is used to finance both consumption and investment, in proportion of initial spending allocations. The increase in the cost of borrowing abroad leads to lower consumption

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[44] This policy ensures that projected future spending from oil resources remains relatively stable, even if oil revenues unexpectedly dry up. Under such a rule, per capita consumption from the oil wealth may increase with time as financial assets are gradually accumulated, public capital is built up, nonoil tax revenues increase, implying transfers from current to future generations.

[45] See the Sovereign Wealth Fund Institute website, http://www.swfinstitute.org/.

[46] The ratio of sovereign fund assets to GDP grows rapidly because of the size of the shock, its degree of persistence, and the fact that aggregate output falls initially.

today, thereby raising private savings and investment. The real exchange rate now depreciates on impact, and this induces downward pressure on the product wage (measured in terms of the price of nonoil traded goods) to maintain labor market equilibrium. This effect is magnified by the increase in labor supply, implying that the product wage in the nontradable sector increases by more than the product wage in the tradable sector. This would normally induce a reallocation of labor from the nontradable to the tradable sector. At the same time, however, the increase in private investment and in the private capital stock benefit mostly the nontradable sector, where it is a relatively scarce factor to begin with.[47] This raises the marginal product of labor there—sufficiently so for labor to flow to that sector and for output to expand. Thus, even though the policy entails a depreciation (in contrast to the full spending case), the net effect is still a shift in production toward nontradables. Over time, the contraction in household consumption is mitigated and so is the expansion of investment. Because public investment does increase, the public capital stock also rises (despite the drop in efficiency). Moreover, this increase is initially less substantial than the increase in the private capital stock, implying that the public-private capital ratio falls at first, before recovering.

The thrust of the foregoing analysis is that, under either a spending rule or a cash transfer rule, the model is capable of reproducing some of the key stylized facts associated with a resource windfall in a commodity-dependent economy. The key difference with more standard models, of course, is the presence of public capital which affects the supply- and demand-sides of the economy through a variety of channels. In particular, although increases in public investment may help to fuel aggregate demand and magnify a real appreciation in the short run, over time the positive supply-side effects of public capital work in the opposite direction. Because of its ability to reproduce these basic facts, the model provides a natural benchmark to study the main issue of interest in this paper—the optimal allocation of resource windfalls, possibly "in between" the pure spending, cash transfer, and saving rules discussed earlier.

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[47] In the full spending experiment, the reallocation of private capital from the tradable to the nontradable sector is much less significant.

7 Optimal Allocation of Resource Windfalls

The foregoing analysis has focused on "pure" fiscal rules compared to a benchmark case involving cash transfers. However, a key policy issue is the following: given dire infrastructure needs and significant constraints on absorption capacity, but also concerns about household welfare and macroeconomic volatility, what should be the optimal allocation of a resource windfall between spending today and spending tomorrow, through accumulation in a resource fund? Put differently, how should precautionary buffers be determined? As noted earlier, this issue is of great practical concern to a number of low-income countries, particularly those that are highly vulnerable to volatility and uncertainty of resource revenue as a result of a high degree of concentration of their exports. My analysis therefore needs to account for the *revealed preferences* of policymakers, in addition to pure welfare considerations.

Conceptually, this can be addressed by assuming that the government's goal is to allocate a fraction $\chi \in (0,1)$ of the oil windfall to a sovereign fund and a fraction $1-\chi$ to current spending, which includes infrastructure investment. Put differently, the hybrid rule to be considered here combines the two pure rules considered earlier; the partial increase in investment makes it possible to build a stabilization buffer against future oil revenue shocks. With $\chi < 1$, the government raises not only spending today but also all future spending by using some of the current windfall to increase its assets held in the sovereign fund.

Formally, the resource accumulation rule is now, instead of (51),

$$\tilde{F}\hat{F}_{t+1} = (1-\phi^F)\tilde{F}\hat{F}_t + \chi\tilde{T}^O\hat{T}_t^O, \tag{53}$$

whereas the spending rule generalizes (52) to give

$$\tilde{G}\hat{G}_t = (1-\chi)\tilde{T}^O\hat{T}_t^O + (1+\tilde{r}^F)\tilde{F}(\hat{r}_t^F + \hat{F}_t) - \tilde{F}\hat{F}_t. \tag{54}$$

The term $\tilde{T}^O\hat{T}_t^O$ must also now be replaced by $(1-\chi)\tilde{T}^O\hat{T}_t^O$ in defining total taxes in (44). Substituting the resulting expression for $\hat{T}_t$ in the government budget constraint (45) and using (54), it can be established that lump-sum taxes are once again determined by (50).

Most existing contributions have focused on utility-based measures of social welfare, abstracting from other considerations that may be important for policymakers.[48] The stability criterion that I propose in this paper is to determine χ so as to minimize a *fundamental* social loss function defined as a weighted geometric average of the volatility of private consumption—which is detrimental to welfare if households are risk averse—and the volatility of the nonresource primary balance, σ_C^χ and $\sigma_{B^{NO}}^\chi$, normalized with respect to the volatility indicators corresponding to the benchmark experiment (cash transfers), σ_C^B and $\sigma_{B^{NO}}^B$:

$$\mathcal{L}_t^F(\chi) = (\frac{\sigma_C^\chi}{\sigma_C^B})^\mu(\frac{\sigma_{B^{NO}}^\chi}{\sigma_{B^{NO}}^B})^{1-\mu}, \tag{55}$$

where $\mu \in (0,1)$. Thus, if the government sets policy solely on the basis of household welfare (respectively fiscal stability) considerations, $\mu = 1$ (respectively $\mu = 0$); in the general case, the lower μ is, the larger the concern with fiscal stability.

An alternative stability criterion is to determine χ so as to minimize a *generalized* social loss function that involves a weighted average of the volatility of private consumption (as before) and a broader measure of macroeconomic volatility, defined in terms of a weighted average of the volatility of the nonoil primary balance and the volatility of the real exchange rate, with equal weights as a point of reference:[49]

$$\mathcal{L}_t^G(\chi) = (\frac{\sigma_C^\chi}{\sigma_C^B})^\mu[(\frac{\sigma_{B^{NO}}^\chi}{\sigma_{B^{NO}}^B})^{0.5}(\frac{\sigma_z^\chi}{\sigma_z^B})^{0.5}]^{1-\mu}, \tag{56}$$

where again $\mu \in (0,1)$.

The positive (rather than purely normative) approach proposed here to measuring social welfare is aimed at addressing two distinct considerations. The first is household welfare, which in general is adversely affected the volatility of private consumption. The second is either fiscal volatility or more generally macroeconomic volatility. The volatility of the nonoil primary balance (defined in (46)) aims to capture movements in fiscal variables that are not linked to fluctuations in oil prices, whereas movements in the real

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[48] Maliszewski (2009) for instance discusses derivations of optimizing rules under various social welfare functions and *ad hoc* rules, but his model has no productive public goods. The same issue arises in the analysis of Engel et al. (2013). van der Ploeg and Venables (2013) and van den Bremer and van der Ploeg (2013) account for public capital but continue to focus on utility-based measures of social welfare.

[49] Alternatively, the volatility of the composition of nonoil output, $z_t^{-1}Y_t^N/Y_t^T$ could also be considered. However, in this model, this measure is highly correlated with the volatility of the real exchange rate.

exchange rate are taken to capture changes in a key relative price, fluctuations in which are often viewed as a key symptom of macroeconomic instability (see Agénor and Montiel (2015)).[50] Thus, the benefits of the self-insurance (or precautionary buffers) provided by a sovereign fund against large commodity price shocks may extend beyond fiscal stability to overall macroeconomic stability, as captured by fluctuations in the real exchange rate.

Table 3 shows the value of the fundamental loss function (55), calculated on the basis of (unconditional) asymptotic variances, for μ and χ both varying between 0 and 1 with a grid of 0.1.[51] The use of asymptotic variances means therefore that the policymaker is concerned not only about volatility "today" (say, the next few quarters) but also about volatility "tomorrow" (in fact, into the indefinite future). The results show that while the function is decreasing in μ for χ given, it has a convex shape in χ for μ given. The reason is that, as shown in Figure 9, volatility of both consumption and the nonresource primary balance are convex in χ. Intuitively, spending all the revenues associated with a windfall creates a lot of volatility in the economy. As χ increases, more of the windfall is saved; the reduction in today's spending tends at first to reduce that volatility. However, as χ continues to rise, the interest income from the assets held in the sovereign fund becomes larger, and this tends to raise spending over time—thereby increasing volatility once again. This effect is not symmetric, because the shock is temporary and the increase in spending associated with interest income (whose level depends also on the risk-free interest rate and the management fee paid on asset held in the sovereign fund) is more gradual than the reduction in spending initially associated with a higher χ. In the present case this means less volatility in consumption but more volatility in the nonresource primary balance for high values of χ, while the opposite holds with low values of χ. Nevertheless, there is a fundamental *dynamic volatility trade-off* between spending now and spending later with respect to *each* volatility measure in the loss function.

In addition, there may also be a (static) trade-off *between* these volatility measures. If the proportion of the windfall that is spent today is large (or χ is small), volatility

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[50] See Baunsgaard et al. (2012) for a more general discussion of the nonresource primary balance as an indicator of fiscal sustainability. As suggested by Lundgren et al. (2013, p. 34), nonoil output was also used as a scaling variable; but this had no significant effect on any of the results.

[51] From the level form associated with (53), $\tilde{F} = \chi\tilde{T}^O/\phi^F$, which therefore depends on χ. However, when performing the grid search over alternative values of χ, and given the focus on nonsystematic shocks, the initial steady-state level of foreign assets is kept constant.

in private consumption (and the real exchange rate) may be large, but volatility in the nonresource primary balance may be small, depending on the behavior of aggregate output and the debt-output ratio. If aggregate output falls (because the drop in nonoil traded output is large), the debt-output ratio will increase, and so will the risk premium and the world interest rate. Because lump-sum taxes increase with debt service payments, this may dampen movements in the nonresource primary balance.[52] However, as the proportion of the windfall that is spent today falls (that is, as χ increases), consumption volatility may be mitigated, and so will the appreciation of the real exchange rate and the fall in output of nonoil traded goods; as a result, volatility in the nonresource primary balance may be magnified. Which trade-off dominates depends in general on the structure of the model, its calibration, and the degree of persistence of the price shock.

In the present case, the convexity of the loss function means that there is a value of χ for which the loss function is minimized.[53] That value is shown in bold, red numbers in the table. For $\mu = 0.5$ for instance, which implies that the government attaches equal weight to consumption volatility and fiscal volatility, the optimal value of χ is also 0.5. In general, the higher the weight attached to consumption volatility (the higher μ is, or conversely, the lower the weight $1 - \mu$ attached to fiscal volatility), the higher the share of the resource windfall that the government should save. These results are also illustrated in the three-dimensional diagram of Figure 10. In addition, the results show that the optimal policy is always better, in terms of overall volatility, than a pure cash transfer policy; the value of the loss function is always less than unity for the optimal χ. The key reason for this is of course the fact that the policy has no direct effect on access to public services.

The grid of 0.1 used in Table 3 is sufficient to illustrate the main point of the analysis. Of course, a more refined grid would give a more precise value of χ. Table 3 illustrates this case when χ varies between 0.4 and 0.5, by intervals of 0.01. The results show again that, as μ increases, the optimal value of χ also tends to increase.

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[52] From (46) and (50), $\tilde{B}^{NO}\hat{B}_t^{NO} = \left(1+\tilde{r}^W\right)\tilde{D}^G\hat{r}_t^W - \tilde{G}\hat{G}_t$. The first term corresponds to the net increase in lump-sum taxes.

[53] In fact, given that consumption volatility itself follows a U-shape pattern, a loss function that accounts solely for that variable (out of concern only for household welfare) would yield a well-defined solution for the optimal χ.

Table 5 shows the determination of the optimal value of χ when the generalized loss function (56), is used, again with a grid of 0.1. The results show that at low values of μ the optimal allocation parameter is now higher, compared to Table 3, because of concerns with the volatility of the real exchange rate. Again, a finer grid would provide more precise values, especially for the cases where the choice of χ appears to be the same as with the fundamental loss function (for instance, for $\mu = 0.5$ to 1.0). At this stage, however, the intuition is fairly clear. In the same vein, changing the relative weights attached to fiscal volatility and real exchange rate volatility is a rather straightforward exercise, which would not change the fundamental intuition of the methodology proposed here.[54]

It is worth noting that, in the case of a negative shock, the intuition is of course symmetric, with χ representing now the proportion of the resources that are *taken out* of the sovereign fund—assuming, of course, that the fund is sufficiently capitalized to begin with. With small withdrawals (χ low) the adverse shock creates a lot of volatility, in particular through a concomitant contraction in government spending. As χ increases (more and more resources previously saved are withdrawn from the fund), the adverse effect of the initial shock on spending is mitigated and volatility decreases at first. But as χ continues to rise and public outlays increase, volatility starts increasing again—at a slower rate now, given that (all else equal) the interest income generated by the lower level of assets held in the sovereign fund becomes smaller. Thus, the volatility curve takes again the same convex shape as shown in Figure 9.

8 Sensitivity Analysis

To assess the robustness of the results established in the previous sections, I now consider changes in some key parameters and model specification. Instead of focusing on the impact of these changes on the transmission mechanism of commodity price shocks under alternative fiscal rules (as was done earlier for some alternative parameter values in the benchmark case), the emphasis here is on their implications for the optimal allo-

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[54] Experiments (which are not reported here to save space) suggest that a higher weight on real exchange rate volatility in the loss function (56) implies an optimal value of χ that is higher.

cation between spending today and spending tomorrow, that is, the optimal value of χ. Specifically, the following changes are considered: tighter absorption constraints; a higher elasticity of output to public infrastructure; an alternative specification of the risk premium on world capital markets; an investment-biased spending rule; imperfect mobility of physical capital across sectors; and public-private sharing of resource revenues.

8.1 Absorption Constraints

Consider as before a higher incidence of absorption constraints, as captured by a higher φ_1. Intuitively, a weaker ability to design and manage investment projects implies that when public investment increases at higher rates efficiency falls, thereby mitigating the increase in the public capital stock and making output, private consumption, and other macroeconomic variables *less* volatile. Thus, all else equal, a higher φ_1 should imply a *lower* optimal χ, that is, less saving accumulation in the sovereign fund, compared to the benchmark case.

Table 6 shows the results for the fundamental loss function, with a value of φ_1 increasing from 0.05 in the benchmark case to 0.5. The results suggest that, for some values of μ, the optimal value of χ is smaller than in Table 3. For $\mu = 0.6$ for instance, the optimal χ is 0.5, rather than 0.6. For other values of μ, however, a grid of 0.1 is too wide to pass judgment. Accordingly, Table 7 shows the results for χ varying between 0.40 to 0.50, with a grid of 0.01, as in Table 4. A comparison of Tables 5 and 8 shows for instance that for $\mu = 0.2$, the optimal χ is 0.43 in the benchmark case, whereas it is 0.42 with a higher incidence of absorption constraints. A similar result holds for $\mu = 0.3$ and $\mu = 0.4$. For $\mu = 0.5$ to 1.0, the grid would need to be refined to three decimal points to establish clear-cut results, but again this is fairly intuitive.

8.2 Elasticity of Output to Public Capital

Consider as before a higher elasticity of output in both production sectors with respect to public capital, ω_T and ω_N. Intuitively, this should lead to a higher optimal value of spending (lower χ), compared to the benchmark case. Table 8 shows the results for the fundamental loss function, with values of ω_T and ω_N increasing from 0.17 in the benchmark case to 0.25. A comparison of the results in Tables 4 and 9 shows that the optimal value of χ is indeed lower. For $\mu = 0.3$ for instance, the optimal value is 0.5 in the benchmark case, whereas it is 0.4 for higher values of ω_T and ω_N; for $\mu = 0.6$ or 0.7, the optimal values are 0.6 and 0.4, respectively. A finer grid confirms these results. Although they are not reported here to save space, they indicate that for χ varying between 0.40 and 0.50 for instance, for $\mu = 0.3$ the optimal value is 0.47 in the benchmark case (as shown in Table 4) but only 0.40 for higher values of ω_T and ω_N.

8.3 Alternative Specifications of the Risk Premium

In the risk premium function defined earlier (see equation (43)), public external debt was scaled by total tradable output and in the composition of output total tradable output also appeared. In that sense, oil resources may improve directly international creditworthiness.[55] However, as noted by van der Ploeg (2012, p. 520), in practice credit rating agencies often view natural resources as a *weakness* (attributed to increased political risk, delayed fiscal reform, and lack of production diversification), instead of a strength. Indeed, as noted by van den Bremer and van der Ploeg (2013, p. 8), there is no conclusive empirical support for oil windfalls alleviating the debt premium paid on international capital markets. If so then, it is D^G/Y_t^T that should appear in (43). Moreover, markets may internalize the fact that government assets are being accumulated in the sovereign fund; if so then yet another alternative specification of the risk premium may take the form[56]

$$PR_t = (\frac{D^G - F_t}{Y_t^T})^{pr1}(\frac{Y_t^T}{z_t^{-1}Y_t^N})^{-pr2}.$$

However, experiments with the latter specification did not prove stable for values of χ above a relatively low threshold. Intuitively, the rapid increase in assets in the sovereign fund lowers net foreign liabilities of the government, thereby lowering the risk premium. Because households face the same (premium-inclusive) interest rate on world markets as the government, the drop in the cost of foreign borrowing translates in turn into a rapid, and unsustainable, buildup of debt by households.[57]

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[55] *Future* oil resource revenues could also be securitized as a way to gain improved access to international capital markets; however, for low-income countries the resources generated in that way remain paltry.

[56] As in Agénor and Yilmaz (2013), it could be assumed also that a fraction of public capital is used directly as collateral.

[57] This outcome would be different, of course, if as in Agénor (1997), households face a specific risk

8.4 Investment-Biased Spending Rule

In the foregoing analysis, it was assumed that resource windfalls or interest income from assets held in the sovereign fund are used to finance government spending in general (consumption and investment). An alternative policy, much discussed in the literature, involves spending only on infrastructure. In the full spending, full saving, and hybrid cases, the term $\tilde{G}\hat{G}_t$ on the left-hand side in equations (49), (52), and (54) is therefore replaced by $\tilde{I}^G\hat{I}_t^G$. Substituting either one of these expressions in (45), together with (44), and noting that $\tilde{G}\hat{G}_t = \tilde{I}^G\hat{I}_t^G + \tilde{z}^{-1}\tilde{C}^G(\hat{C}_t^G - \hat{z}_t)$, gives

$$\tilde{T}^L\hat{T}_t^L = -\tilde{T}^{NO}\hat{T}_t^{NO} + \tilde{z}^{-1}\tilde{C}^G(\hat{C}_t^G - \hat{z}_t) + \left(1+\tilde{r}^W\right)\tilde{D}^G\hat{r}_t^W, \tag{57}$$

which replaces (50).

Rather than studying an “investment-only” spending rule, an “investment-biased” rule is examined instead. This rule takes the form of a simultaneous increase in the *share* of government spending allocated to infrastructure investment, υ^G. This policy is more realistic (and also more feasible politically) given that, as noted earlier, low-income countries face not only severe infrastructure needs but are also confronted with high demands on other components of spending, especially related to human development.

Table 9 shows the results for the fundamental loss function, with a value of υ^G increasing from 0.114 in the benchmark case to 0.24. A comparison of Tables 4 and 10 shows that, as expected, the optimal value of χ is in general lower now, given that a higher public capital stock mitigates the volatility of relative prices and consumption

8.5 Imperfect Intersectoral Capital Mobility

In the model presented earlier it was assumed that private capital is perfectly mobile across sectors. While this assumption has some appeal (especially if shifting resources from producing one type of goods to another occurs can be done quickly and within the same production units) it is worth considering the case where mobility is imperfect, as in some contributions. In Morshed and Turnovsky (2004) and Chatterjee and Mursagulov (2012) for instance, there are convex costs of transferring private capital across sectors.

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premium that depends on their own level of borrowing.

Accordingly, suppose now that once capital is installed, it is difficult and expensive to unbolt and reallocate it quickly to another sector. Thus, there are constraints not only in changing the aggregate stock of capital, but also its allocation between sectors in the short run. However, in the longer run, there are no costs to moving physical assets. Thus, as in Morshed and Turnovsky (2004), there is (capital) factor specificity for each sector in the short run and perfect factor mobility across sectors in the long run.

Formally, equation (38) is now replaced by a CES function in terms of K_t^N and K_t^T:

$$K_{t-1}^P = [\zeta_K (K_t^T)^{(\eta_K - 1)/\eta_K} + (1-\zeta_K)(K_t^N)^{(\eta_K-1)/\eta_K}]^{\eta_K/(\eta_K-1)}, \tag{58}$$

where $\zeta_K \in (0,1)$ is the share of capital in the traded sector in the initial steady state, and $\eta_K > 0$ is the elasticity of substitution between K_t^N and K_t^T. In the short sun, $r_t^{K,T}$ and $r_t^{K,N}$ are different and the aggregate rental rate of capital is given by

$$r_t^K = [\zeta_K^{\eta_K} (r_t^{K,T})^{1-\eta_K} + (1-\zeta_K)^{\eta_K} (r_t^{K,N})^{1-\eta_K}]^{1/(1-\eta_K)}, \tag{59}$$

with, in the steady state, $\tilde{r}^{K,T} = \tilde{r}^{K,N} = \tilde{r}^K$.[58]

To perform the simulations, the traded goods sector is taken to be more capital intensive, $\zeta_K = 0.6$, and the elasticity of substitution to be $\eta_K = 0.5$. The results, which are not reported here to save space, are fairly intuitive. Imperfect capital mobility does dampen the impact of a commodity price shock on the volatility of output (and thus of nonoil tax revenues) and consumption, compared to the perfect mobility case. The volatility of the nonoil resource balance is smaller as well, and so is the volatility of the real exchange rate. As a result, and regardless of whether the fundamental or the extended social loss function is used, the optimal value of χ is uniformly smaller than before.

8.6 Public-Private Sharing of Resources Revenues

In the foregoing analysis it was assumed that oil revenues accrue entirely to the state. In practice, the exploitation of natural resources typically involves a public-private partnership, with residents and nonresidents, and revenue sharing. To account for this, suppose

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[58] In solving for the steady state with imperfect capital mobility, a multiplicative constant is added to equation (58) to ensure that the solution of K^P matches the value obtained under perfect capital mobility. The steady-state value of r^K remains tied, as before, to the sum of the world interest rate and the depreciation rate.

that—even though oil production does not require the direct use of private inputs—the government receives only a fraction $\tau^O \in (0,1)$ of an oil windfall, and that out of the fraction $1 - \tau^O$ that accrues to the private sector a fraction $\psi^O \in (0,1)$ is distributed to domestic households. After changing the household budget constraint (15) and the revenue equation (25), the main change to the model is that the coefficient of the term $P_t^O Y_t^O$ in the savings-investment balance (40) is now $-[\psi^O + (1-\psi^O)\tau^O]$ instead of -1. Clearly, if nonresidents do not receive any fraction of the windfall that does not accrue to the government ($\psi^O = 1$), nothing changes compared to the previous analysis; less spending by the government is simply matched by higher spending by the private sector. However, when $\psi^O < 1$, $\psi^O + (1-\psi^O)\tau^O$ is also less than unity. This implies that a resource windfall has a weaker effect on aggregate demand, and therefore the initial real appreciation is mitigated. How much so depends, in general, on the values of τ^O and ψ^O. With $\tau^0 = 0.5$ and $\psi^O = 0.8$ for instance, the behavior of the real exchange rate would not be significantly affected. In terms of the optimal value of χ, this change would also have no discernible impact with the fundamental value of the loss function, whereas with the generalized function the optimal χ would tend to be (slightly) lower as a result of reduced macroeconomic volatility.

By and large, the sensitivity analysis confirms the key result highlighted earlier: the optimal choice of χ internalizes the dynamic trade-off associated with saving today, and spending tomorrow, the resources associated with a resource windfall. While changes in structural parameters and model specification do affect (as can be expected) the slope of that trade-off, especially those that alter the supply-side effects of public capital, they do not affect the fundamental result—the existence of a U-shape relationship between volatility and the share of the windfall saved.[59]

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[59] Another set of experiments would involve switching off one by one each of the four channels through which public capital operates (utility benefits, interaction with private investment in the formation of private capital, endogenous distribution costs, and endogenous efficiency of public investment). While the results, which are not reported here to save space, indicate that these channels do affect the slope of the dynamic trade-off associated with commodity price shocks, they also show that they do not affect either the fundamental insights of the analysis.

9 Concluding Remarks

The purpose of this paper has been to develop a dynamic stochastic model to study the optimal fiscal response to commodity price shocks in a small open low-income country. The model accounts for imperfect access to world capital markets and a variety of externalities associated with public infrastructure, including direct utility benefits, a direct complementarity effect with private investment, and reduced distribution costs. The latter assumption implies that delivering traded goods to consumers requires distribution services, which utilize nontraded goods as its sole input; the intensity of distribution costs varies inversely with access to infrastructure. At the same time, however, public capital is subject to congestion and absorption constraints, with the latter affecting the efficiency of infrastructure investment. It was argued that, to serve as foundation for practical policy advice in countries where commodity price shocks are a key source of instability, the optimal allocation of resource windfalls should be determined by minimizing a social loss function that combines not only a measure of household welfare but also an indicator of fiscal or macroeconomic volatility.

The model was parameterized and used to examine the transmission process of a temporary resource price shock under a benchmark case (cash transfers, that is, the windfall is transferred entirely to citizens) and alternative fiscal rules, involving either higher public spending or accumulation in a sovereign fund.[60] The optimal allocation rule between spending today and asset accumulation was determined so as to minimize a social loss function defined in terms of the volatility, relative to the benchmark case, of private consumption and either the nonresource primary balance or a more general index of macroeconomic stability, which accounts for the volatility of the real exchange rate. In either case, it was argued that the choice between spending now and spending later involves a *dynamic volatility trade-off* with respect to each volatility measure in the loss function. Initially, full spending creates a lot of volatility; increasing the share of a resource windfall that is saved (or equivalently, reducing the share spent today) tends at first to *reduce* that volatility. However, as the proportion of resource revenues that is saved

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[60] Note also that there is an ongoing debate regarding whether sovereign wealth funds should finance domestic projects (especially in infrastructure) directly, just like development banks. This raises, however, a host of issues; see Gelb et al. (2014) for a discussion.

continues to rise, the interest income from the assets held in the sovereign fund become larger, and this tends to raise spending over time, which tends to *increase* volatility once again. In addition, depending on the structure of the model and its calibration, there may also be a trade-off *between* the volatility measures that are accounted for in the social loss function. Sensitivity analysis was conducted with respect to various structural parameters and model specification. In all cases, the results show that the optimal policy is always better, in terms of overall volatility, than an unconditional cash transfer policy.

The paper uses a DSGE framework to study the effects of temporary commodity price shocks. Even though these shocks can be persistent, entailing therefore prolonged transitional dynamics, they do not have long-run effects. As a result, only fluctuations around the steady-state equilibrium are analyzed. Yet, the fact that public capital exerts supply-side effects means that even temporary shocks could have permanent level effects, implying therefore that the steady state could change following these shocks.

Even though in the present setting changes across steady states are not accounted for, it is important to take into account the supply-side effects of productive public expenditures. The reason is that these effects matters from the perspective of mitigating volatility as well: they tend to offset the demand-side effects associated with the impact of the windfall on consumption and investment, and may therefore have a substantial impact on the volatility of the real exchange rate—and by implication the distribution of output and expenditure. Put differently, accounting for the transitional effects of productive public investment is important to discuss the *short-run* stabilization issue considered in the paper, even though the longer-run implications of this investment on the steady-state equilibrium are not explicitly accounted for.

Another issue relates to the fact that the paper uses a first-order log-linearization to solve the model. It is now well established that first-order approximations are insufficient in some cases, especially for conducting utility-based welfare analysis or when studying the dynamics of asset prices. In such cases, second- and higher-order approximations may be more accurate. However, there are good reasons to believe that a first-order approximation may be a reasonable approach in the present case. First, only small shocks are considered, implying limited deviations from the initial steady state. Second, there are no asset demand equations and no asset prices in the model; the fact that the riskiness of an asset may be related to the variance of the underlying shocks is not a concern. Third, the paper focuses directly on asymptotic variances, not expected welfare, in defining the policy criterion; the degree of curvature of the utility function (which militates in favor of higher-order approximations) does not play a direct role in that regard. Fourth, higher-order approximation techniques remain difficult to implement with models (like this one) that are relatively large—the so-called "curse of dimensionality" problem. Nevertheless, even though it still remains to be seen if these techniques work for larger models, future research on managing resource windfalls should address both issues of switches in steady states and nonlinearities.

Future research could also consider various extensions of the model. First, nominal rigidities in prices and wages could be introduced, as in Lartey (2008), Shi and Xu (2010), and Heer and Schubert (2012) for instance. However, given the focus of the paper on fiscal policy (rather than monetary policy), accounting for these rigidities would not alter much the qualitative features of the results. Intuitively, nominal price rigidities would mostly mitigate the magnitude of real exchange rate movements on impact, compared to the case of flexible nontraded good prices considered here. Second, domestic use of commodities (by households as final goods, and by firms as intermediate goods in the production process) could also be introduced; again, as can be inferred from the discussion in Pieschacón (2012), the implications are fairly intuitive in terms of the transmission process and the optimal allocation rule. Third, although it was made clear that my analysis is based on the revealed (policy) preferences of policymakers, a full political economy framework would help to provide stronger microfoundations for the loss function used—not only in terms of the variables used but also the relative weight μ of each objective in that function. For instance, fiscal and macroeconomic volatility could create a potential (electoral) cost for the incumbent; but voters could also dislike consumption volatility and this could be internalized by the incumbent.

Fourth, it would be worth extending the analysis to consider yet more rules for putting resources into, and pulling resources out, of the sovereign fund than those considered here. For instance, instead of assuming (as was done here) that interest income only is transferred to the government budget, it could be assumed, as done in Norway for instance, that all commodity resources are allocated to the sovereign fund, and that a fraction of total fund assets (interest *and* principal) is withdrawn each period. Formally, the *Norway rule* (in levels) would take the form $F_{t+1} = (1 - \phi^F)F_t + T_t^O$, with $G_t = \lambda(1 + r_t^F)F_t$, and $\lambda > 0$. In Norway's case $\lambda = 0.04$ but in the present context the issue would be to solve for the *optimal* value of λ. Some recent research has shown that a rule along those lines could perform quite well for a low-income country (Hassler et al. (2013)), but the model used is too simple to be reliable.

Finally, other types of productive public goods, such as education and health services, could be explicitly accounted for and their implications for the optimal allocation of resource windfalls examined.

However, while using alternative solution techniques and extending the analytical framework along the lines described above could be valuable in their own right, it is not clear that they would affect the basic intuition and the fundamental methodological contribution of this paper. Indeed, the analysis was able to answer a crucial, practical question for policymakers in low-income countries confronted with highly volatile movements in resource prices: how much of a windfall should be spent today and how much should be set aside in a sovereign fund, given pressing needs to provide productive goods and promote development, while at the same time accounting for absorption constraints? Rather than considering only utility-based measures of social welfare, as in some previous contributions, the recommendation in this paper is that in addressing this issue it is important to focus on second moments—the volatility not only of private consumption (which is highly correlated with household welfare) but also macroeconomic volatility, in the form either of a narrow indicator of fiscal volatility, or a broader measure that also involves fluctuations in the real exchange rate. If the objective is to minimize a social loss function that explicitly accounts for these measures, dynamic trade-offs imply that in general neither full spending, nor full saving, would represent an optimal fiscal response to resource windfalls.

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Table 1

Parameterization: Benchmark Case

Parameter	Value	Description
Households		
Λ	0.898	Discount factor
ς	0.2	Intertemporal elasticity of substitution
η_L	0.14	Preference parameter, labor in utility function
η_I	0.08	Preference parameter, public capital
ψ	8	Inverse of Frisch elasticity of labor supply
θ	0.55	Share of nontradables in private consumption
κ	25	Adjustment cost parameter, private investment
φ_K	0.5	Strength of complementarity effect, private investment
δ^P	0.045	Depreciation rate, private capital
Oil production		
ρ_{Y^O}	0.955	Persistence parameter, oil output
ρ_{P^O}	0.93	Persistence parameter, world oil price
Nonoil production		
β, η	$0.6, 0.7$	Labor shares, tradable and nontradable sectors
ν_T	0.03	Strength of learning-by-doing effect, tradable sector
ω_N, ω_T	0.17	Elasticity of output wrt public capital
Distribution cost		
ϱ	4.0	Sensitivity to public-private capital ratio
Government		
τ	0.151	Nonoil revenue ratio
ψ^G	0.171	Share of total government spending in output
υ^G	0.114	Share of spending on infrastructure investment
$\upsilon^{G,N}$	0.62	Share of infrastructure investment on nontraded goods
φ	0.37	Efficiency parameter, public investment
φ_1	0.05	Absorption constraint parameter, public investment
δ^G	0.035	Depreciation rate, public capital
ϕ_F	0.0025	Management fee on sovereign assets
χ	$0, 1$	Fraction of resource windfalls allocated to a sovereign fund
μ	$0, 1$	Importance of consumption volatility in government loss function
Risk premium		
pr_1	0.8	Elasticity wrt tradable-nontradable output ratio

Table 2
Initial Steady-State Values
(In percent of total output, unless otherwise indicated)

Variable	Value	Description
Y^O	0.078	Oil output
$z^{-1}Y^N$	0.595	Nontradable output
Y^T	0.325	Nonoil tradable output
L^N/L	0.743	Share of employment in nontradable sector
C	0.757	Private consumption
$z^{-1}C^N$	0.820	Consumption of nontradables
C^T	0.325	Consumption of tradables
I^P	0.117	Private investment
K^P	2.009	Private capital stock
I^G	0.019	Public investment
G	0.171	Total government spending
T^{NO}	0.139	Nonoil revenues
T^O	0.08	Oil revenues
T	0.198	Total revenues
B^{NO}	0.06	Nonoil primary balance
D^G	0.237	Government foreign liabilities
D	-0.55	Total foreign liabilities
K^I/K^P	0.092	Public-private capital ratio (absolute value)
$r^{W,R}$	0.017	World risk-free interest rate (in percent)
PR	0.094	Risk premium (in percent)
F	0.01	Assets in sovereign wealth fund

Table 3

Optimal Allocation Rule under the Fundamental loss Function, Benchmark Parameters

		μ										
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
χ = 0.0												
C	0.72522	1.0383	1.0017	0.9664	0.9323	0.8995	0.8678	0.8372	0.8077	0.7792	0.7517	0.7252
B^{NO}	1.03830											
χ = 0.1												
C	0.63931	0.9495	0.9127	0.8773	0.8433	0.8106	0.7791	0.7489	0.7199	0.6919	0.6651	0.6393
B^{NO}	0.94955											
χ = 0.2												
C	0.55863	0.8849	0.8451	0.8071	0.7708	0.7362	0.7031	0.6715	0.6413	0.6125	0.5849	0.5586
B^{NO}	0.88486											
χ = 0.3												
C	0.48581	0.8498	0.8036	0.7599	0.7185	0.6795	0.6425	0.6076	0.5745	0.5433	0.5137	0.4858
B^{NO}	0.84977											
χ = 0.4												
C	0.42487	**0.8480**	**0.7913**	**0.7385**	0.6892	0.6432	0.6002	0.5602	0.5227	0.4878	0.4553	0.4249
B^{NO}	0.84797											
χ = 0.5												
C	0.38159	0.8796	0.8092	0.7443	**0.6847**	**0.6298**	**0.5794**	0.5329	0.4902	0.4510	0.4148	0.3816
B^{NO}	0.87964											
χ = 0.6												
C	0.36233	0.9414	0.8557	0.7778	0.7069	0.6426	0.5840	**0.5309**	**0.4825**	**0.4386**	**0.3986**	**0.3623**
B^{NO}	0.94143											
χ = 0.7												
C	0.37087	1.0279	0.9283	0.8383	0.7571	0.6837	0.6174	0.5576	0.5035	0.4547	0.4107	0.3709
B^{NO}	1.02791											
χ = 0.8												
C	0.40542	1.1334	1.0227	0.9228	0.8326	0.7513	0.6779	0.6116	0.5519	0.4980	0.4493	0.4054
B^{NO}	1.13344											
χ = 0.9												
C	0.46019	1.2532	1.1338	1.0257	0.9279	0.8394	0.7594	0.6870	0.6215	0.5623	0.5087	0.4602
B^{NO}	1.25322											
χ = 1.0												
C	0.52894	1.3836	1.2567	1.1415	1.0369	0.9418	0.8555	0.7770	0.7058	0.6411	0.5823	0.5289
B^{NO}	1.38357											

Source: Author's calculations.

Table 4

Optimal Allocation Rule under the Fundamental loss Function, Benchmark Parameters, χ varying between 0.40 and 0.50

		μ										
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
χ = 0.40												
C	0.42487	**0.8480**	**0.7913**	0.7385	0.6892	0.6432	0.6002	0.5602	0.5227	0.4878	0.4553	0.4249
B^{NO}	0.84797											
χ = 0.41												
C	0.41965	0.8497	0.7918	0.7379	0.6876	0.6408	0.5971	0.5565	0.5186	0.4832	0.4503	0.4196
B^{NO}	0.84966											
χ = 0.42												
C	0.41460	0.8517	0.7925	0.7375	0.6863	0.6386	0.5942	0.5530	0.5146	0.4788	0.4456	0.4146
B^{NO}	0.85169											
χ = 0.43												
C	0.40975	0.8541	0.7936	**0.7374**	0.6852	0.6366	0.5916	0.5497	0.5108	0.4746	0.4410	0.4098
B^{NO}	0.85405											
χ = 0.44												
C	0.40507	0.8567	0.7949	0.7375	0.6843	0.6349	0.5891	0.5466	0.5071	0.4705	0.4366	0.4051
B^{NO}	0.85674											
χ = 0.45												
C	0.40063	0.8598	0.7966	0.7380	0.6837	0.6335	0.5869	0.5437	0.5038	0.4667	0.4324	0.4006
B^{NO}	0.85976											
χ = 0.46												
C	0.39636	0.8631	0.7985	0.7387	0.6834	0.6322	0.5849	0.5411	0.5006	0.4631	0.4284	0.3964
B^{NO}	0.86311											
χ = 0.47												
C	0.39233	0.8668	0.8007	0.7397	**0.6833**	0.6313	0.5831	0.5387	0.4977	0.4597	0.4247	0.3923
B^{NO}	0.86677											
χ = 0.48												
C	0.38853	0.8707	0.8032	0.7410	0.6835	0.6305	0.5816	0.5366	0.4950	0.4566	0.4212	0.3885
B^{NO}	0.87074											
χ = 0.49												
C	0.38493	0.8750	0.8061	0.7425	0.6840	0.6300	0.5804	0.5346	0.4925	0.4536	0.4179	0.3849
B^{NO}	0.87504											
χ = 0.50												
C	0.38159	0.8796	0.8092	0.7443	0.6847	**0.6298**	**0.5794**	**0.5329**	**0.4902**	**0.4510**	**0.4148**	**0.3816**
B^{NO}	0.87964											

Source: Author's calculations.

Table 5

Optimal Allocation Rule under the Generalized Loss Fundction, Benchmark Parameters

		μ										
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
χ = 0.0												
C	0.7252	0.8220	0.8118	0.8017	0.7917	0.7819	0.7721	0.7625	0.7530	0.7436	0.7344	0.7252
B^{NO}	1.0383											
z	0.6508											
χ = 0.1												
C	0.6393	0.7328	0.7229	0.7131	0.7034	0.6939	0.6845	0.6752	0.6660	0.6570	0.6481	0.6393
B^{NO}	0.9495											
z	0.5656											
χ = 0.2												
C	0.5586	0.6564	0.6459	0.6356	0.6254	0.6154	0.6056	0.5959	0.5863	0.5770	0.5677	0.5586
B^{NO}	0.8849											
z	0.4870											
χ = 0.3												
C	0.4858	0.5965	0.5843	0.5725	0.5608	0.5495	0.5383	0.5274	0.5167	0.5062	0.4959	0.4858
B^{NO}	0.8498											
z	0.4187											
χ = 0.4												
C	0.4249	0.5575	0.5425	0.5280	0.5138	0.5001	0.4867	0.4736	0.4609	0.4486	0.4366	0.4249
B^{NO}	0.8480											
z	0.3665											
χ = 0.5												
C	0.3816	**0.5453**	**0.5261**	**0.5077**	**0.4899**	**0.4727**	0.4561	0.4401	0.4247	0.4098	0.3955	0.3816
B^{NO}	0.8796											
z	0.3380											
χ = 0.6												
C	0.3623	0.5651	0.5405	0.5170	0.4946	0.4731	**0.4525**	**0.4328**	**0.4140**	**0.3960**	**0.3788**	**0.3623**
B^{NO}	0.9414											
z	0.3392											
χ = 0.7												
C	0.3709	0.6166	0.5860	0.5570	0.5294	0.5031	0.4782	0.4545	0.4320	0.4106	0.3902	0.3709
B^{NO}	1.0279											
z	0.3699											
χ = 0.8												
C	0.4054	0.6929	0.6568	0.6225	0.5900	0.5592	0.5300	0.5024	0.4761	0.4513	0.4277	0.4054
B^{NO}	1.1334											
z	0.4236											
χ = 0.9												
C	0.4602	0.7860	0.7450	0.7062	0.6694	0.6345	0.6014	0.5701	0.5404	0.5122	0.4855	0.4602
B^{NO}	1.2532											
z	0.4929											
χ = 1.0												
C	0.5289	0.8898	0.8447	0.8019	0.7612	0.7227	0.6860	0.6513	0.6183	0.5869	0.5572	0.5289
B^{NO}	1.3836											
z	0.5722											

Source: Author's calculations.

Table 6

Optimal Allocation Rule under the Fundamental loss Function, $\varphi_1 = 0.5$

						μ					
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\chi = 0.0$												
C	0.7022	1.0463	1.0054	0.9661	0.9283	0.8920	0.8571	0.8236	0.7914	0.7605	0.7307	0.7022
B^{NO}	1.0463											
$\chi = 0.1$												
C	0.6187	0.9563	0.9155	0.8765	0.8392	0.8034	0.7692	0.7364	0.7050	0.6750	0.6462	0.6187
B^{NO}	0.9563											
$\chi = 0.2$												
C	0.5412	0.8900	0.8468	0.8057	0.7666	0.7294	0.6940	0.6603	0.6283	0.5978	0.5688	0.5412
B^{NO}	0.8900											
$\chi = 0.3$												
C	0.4726	0.8531	0.8041	0.7580	0.7146	0.6736	0.6349	0.5985	0.5642	0.5319	0.5014	0.4726
B^{NO}	0.8531											
$\chi = 0.4$												
C	0.4175	**0.8493**	**0.7910**	**0.7368**	0.6863	0.6393	0.5954	0.5546	0.5166	0.4812	0.4482	0.4175
B^{NO}	0.8493											
$\chi = 0.5$												
C	0.3816	0.8791	0.8087	0.7439	**0.6844**	**0.6296**	**0.5792**	**0.5328**	0.4901	0.4509	0.4148	0.3816
B^{NO}	0.8791											
$\chi = 0.6$												
C	0.3706	0.9393	0.8558	0.7798	0.7106	0.6475	0.5900	0.5376	**0.4898**	**0.4463**	**0.4067**	**0.3706**
B^{NO}	0.9393											
$\chi = 0.7$												
C	0.3866	1.0245	0.9294	0.8431	0.7648	0.6938	0.6293	0.5709	0.5179	0.4698	0.4262	0.3866
B^{NO}	1.0245											
$\chi = 0.8$												
C	0.4266	1.1291	1.0244	0.9294	0.8432	0.7650	0.6941	0.6297	0.5713	0.5183	0.4702	0.4266
B^{NO}	1.1291											
$\chi = 0.9$												
C	0.4847	1.2483	1.1356	1.0331	0.9399	0.8550	0.7778	0.7076	0.6438	0.5857	0.5328	0.4847
B^{NO}	1.2483											
$\chi = 1.0$												
C	0.5552	1.3782	1.2584	1.1491	1.0492	0.9580	0.8748	0.7988	0.7293	0.6660	0.6081	0.5552
B^{NO}	1.3782											

Source: Author's calculations.

Table 7

Optimal Allocation Rule under the Fundamental loss Function, φ_1 = 0.5, and χ varying between 0.40 and 0.50

							μ					
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
χ = 0.40												
C	0.4175	**0.8493**	**0.7910**	0.7368	0.6863	0.6393	0.5954	0.5546	0.5166	0.4812	0.4482	0.4175
B^{NO}	0.8493											
χ = 0.41												
C	0.4129	0.8508	0.7914	0.7362	0.6849	0.6371	0.5927	0.5514	0.5129	0.4771	0.4439	0.4129
B^{NO}	0.8508											
χ = 0.42												
C	0.4085	0.8526	0.7921	**0.7359**	0.6837	0.6352	0.5902	0.5483	0.5094	0.4733	0.4397	0.4085
B^{NO}	0.8526											
χ = 0.43												
C	0.4044	0.8548	0.7931	0.7359	0.6829	0.6336	0.5879	0.5455	0.5062	0.4697	0.4358	0.4044
B^{NO}	0.8548											
χ = 0.44												
C	0.4005	0.8573	0.7944	0.7362	0.6823	0.6323	0.5859	0.5430	0.5032	0.4663	0.4321	0.4005
B^{NO}	0.8573											
χ = 0.45												
C	0.3967	0.8601	0.7961	0.7368	0.6819	0.6311	0.5841	0.5406	0.5004	0.4631	0.4286	0.3967
B^{NO}	0.8601											
χ = 0.46												
C	0.3932	0.8633	0.7980	0.7376	**0.6818**	0.6303	0.5826	0.5386	0.4978	0.4602	0.4254	0.3932
B^{NO}	0.8633											
χ = 0.47												
C	0.3900	0.8667	0.8002	0.7388	0.6821	0.6297	0.5814	0.5367	0.4955	0.4575	0.4224	0.3900
B^{NO}	0.8667											
χ = 0.48												
C	0.3869	0.8705	0.8027	0.7402	0.6825	0.6294	0.5804	0.5352	0.4935	0.4550	0.4196	0.3869
B^{NO}	0.8705											
χ = 0.49												
C	0.3841	0.8746	0.8056	0.7419	0.6833	**0.6293**	0.5796	0.5338	0.4917	0.4528	0.4171	0.3841
B^{NO}	0.8746											
χ = 0.50												
C	0.3816	0.8791	0.8087	0.7439	0.6844	0.6296	**0.5792**	**0.5328**	**0.4901**	**0.4509**	**0.4148**	**0.3816**
B^{NO}	0.8791											

Source: Author's calculations.

Table 8

Optimal Allocation Rule under the Fundamental loss Function, $\omega_N = \omega_T = 0.25$

							μ					
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\chi = 0.0$												
C	0.5660	1.0966	1.0264	0.9607	0.8992	0.8417	0.7878	0.7374	0.6902	0.6460	0.6047	0.5660
B^{NO}	1.0966											
$\chi = 0.1$												
C	0.5014	1.0073	0.9394	0.8761	0.8171	0.7620	0.7106	0.6628	0.6181	0.5764	0.5376	0.5014
B^{NO}	1.0073											
$\chi = 0.2$												
C	0.4457	0.9464	0.8777	0.8141	0.7550	0.7003	0.6495	0.6024	0.5587	0.5182	0.4806	0.4457
B^{NO}	0.9464											
$\chi = 0.3$												
C	0.4029	**0.9195**	**0.8467**	0.7796	0.7178	0.6610	0.6086	0.5604	0.5160	0.4752	0.4375	0.4029
B^{NO}	0.9195											
$\chi = 0.4$												
C	0.3771	0.9297	0.8495	**0.7762**	**0.7092**	**0.6480**	**0.5921**	**0.5410**	**0.4944**	0.4517	0.4127	0.3771
B^{NO}	0.9297											
$\chi = 0.5$												
C	0.3721	0.9757	0.8860	0.8046	0.7307	0.6635	0.6025	0.5472	0.4969	**0.4512**	**0.4098**	**0.3721**
B^{NO}	0.9757											
$\chi = 0.6$												
C	0.3886	1.0528	0.9530	0.8626	0.7807	0.7067	0.6396	0.5789	0.5240	0.4743	0.4293	0.3886
B^{NO}	1.0528											
$\chi = 0.7$												
C	0.4241	1.1550	1.0449	0.9452	0.8551	0.7736	0.6998	0.6331	0.5728	0.5181	0.4687	0.4241
B^{NO}	1.1550											
$\chi = 0.8$												
C	0.4743	1.2761	1.1558	1.0469	0.9483	0.8589	0.7780	0.7047	0.6383	0.5781	0.5236	0.4743
B^{NO}	1.2761											
$\chi = 0.9$												
C	0.5352	1.4112	1.2808	1.1625	1.0550	0.9575	0.8690	0.7887	0.7158	0.6497	0.5897	0.5352
B^{NO}	1.4112											
$\chi = 1.0$												
C	0.6034	1.5568	1.4160	1.2880	1.1715	1.0656	0.9693	0.8816	0.8019	0.7294	0.6634	0.6034
B^{NO}	1.5568											

Source: Author's calculations.

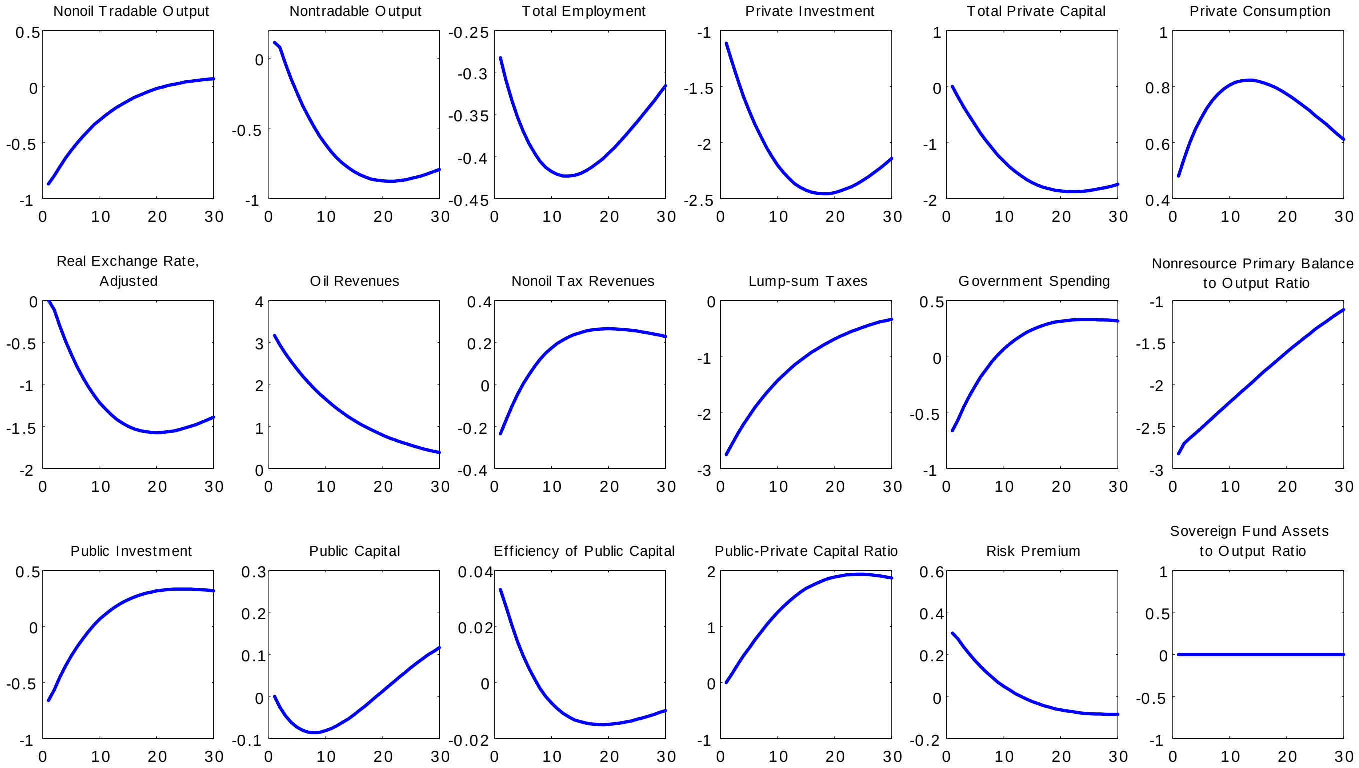
Table 9

Optimal Allocation Rule under the Fundamental loss Function, $\upsilon^{G} = 0.24$

		μ										
VARIABLE	Rel SD	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\chi = 0.0$												
C	0.7937	0.9950	0.9727	0.9510	0.9297	0.9089	0.8886	0.8688	0.8493	0.8304	0.8118	0.7937
B^{NO}	0.9950											
$\chi = 0.1$												
C	0.7070	0.9178	0.8941	0.8711	0.8487	0.8268	0.8055	0.7848	0.7646	0.7449	0.7257	0.7070
B^{NO}	0.9178											
$\chi = 0.2$												
C	0.6308	0.8696	0.8422	0.8156	0.7898	0.7648	0.7407	0.7173	0.6946	0.6727	0.6514	0.6308
B^{NO}	0.8696											
$\chi = 0.3$												
C	0.5692	**0.8555**	**0.8214**	**0.7886**	**0.7571**	0.7268	0.6978	0.6700	0.6432	0.6175	0.5929	0.5692
B^{NO}	0.8555											
$\chi = 0.4$												
C	0.5274	0.8770	0.8335	0.7922	0.7529	**0.7156**	**0.6801**	**0.6464**	**0.6143**	0.5838	0.5549	0.5274
B^{NO}	0.8770											
$\chi = 0.5$												
C	0.5102	0.9317	0.8772	0.8260	0.7777	0.7322	0.6894	0.6491	0.6112	**0.5755**	**0.5418**	**0.5102**
B^{NO}	0.9317											
$\chi = 0.6$												
C	0.5200	1.0142	0.9486	0.8873	0.8300	0.7764	0.7262	0.6793	0.6354	0.5944	0.5560	0.5200
B^{NO}	1.0142											
$\chi = 0.7$												
C	0.5556	1.1183	1.0428	0.9723	0.9066	0.8453	0.7882	0.7350	0.6853	0.6390	0.5958	0.5556
B^{NO}	1.1183											
$\chi = 0.8$												
C	0.6123	1.2387	1.1544	1.0759	1.0027	0.9345	0.8709	0.8117	0.7564	0.7050	0.6570	0.6123
B^{NO}	1.2387											
$\chi = 0.9$												
C	0.6850	1.3710	1.2791	1.1934	1.1134	1.0387	0.9691	0.9041	0.8435	0.7869	0.7342	0.6850
B^{NO}	1.3710											
$\chi = 1.0$												
C	0.7691	1.5122	1.4133	1.3210	1.2346	1.1539	1.0784	1.0079	0.9421	0.8805	0.8229	0.7691
B^{NO}	1.5122											

Source: Author's calculations.

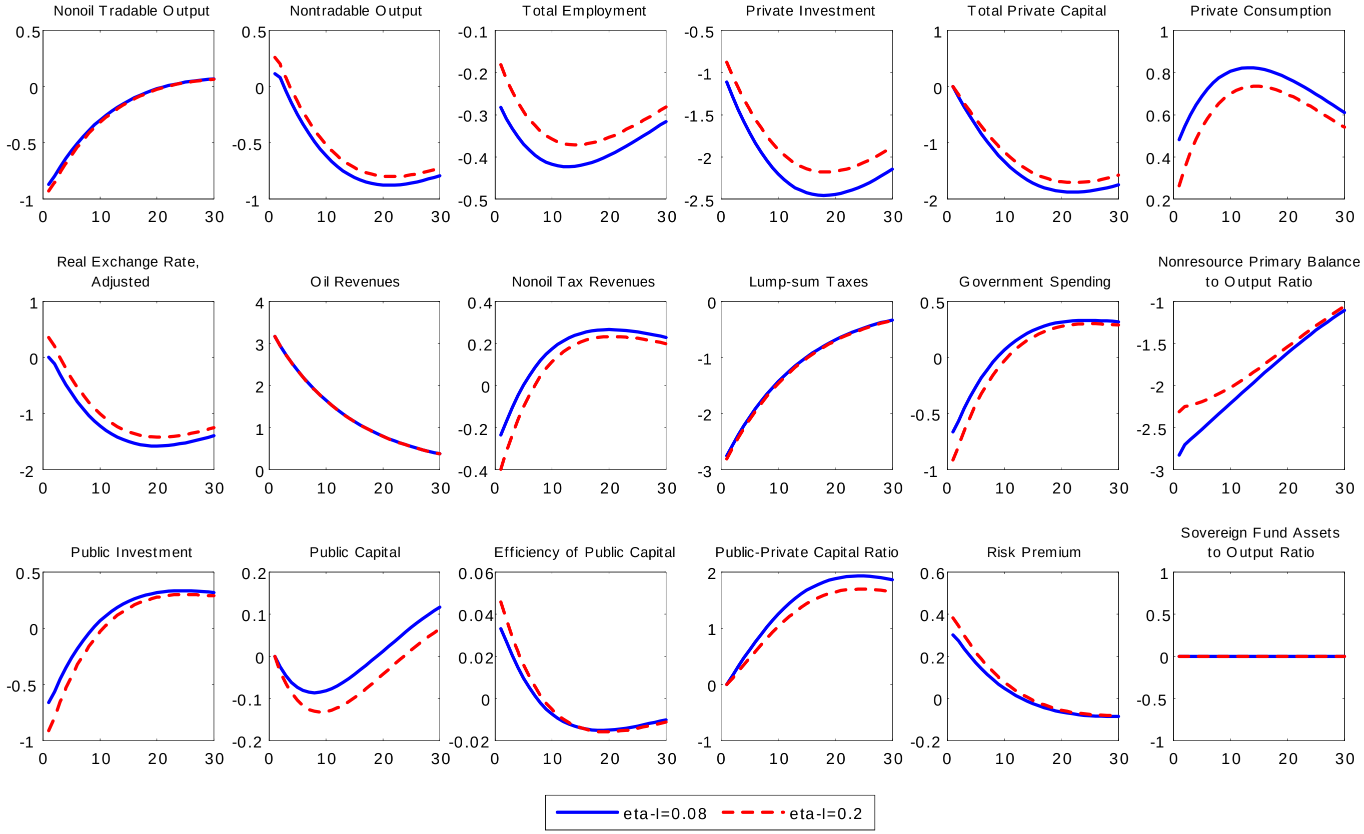
Figure 1
Benchmark Experiment: Cash Transfers to Households



Absolute deviations from baseline, unless otherwise indicated.

Figure 2

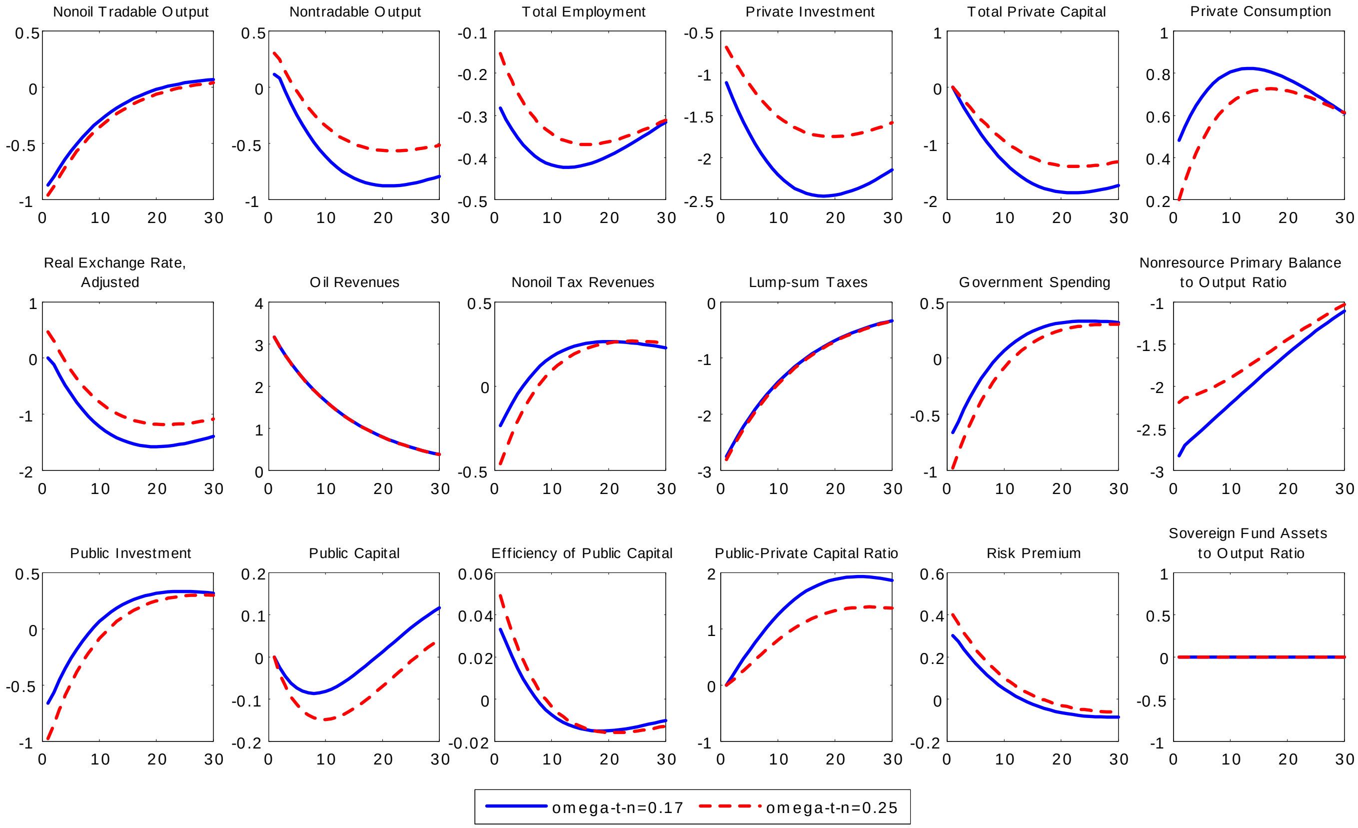
Benchmark Experiment: Higher η_I from 0.08 to 0.2



Absolute deviations from baseline, unless otherwise indicated.

Figure 3

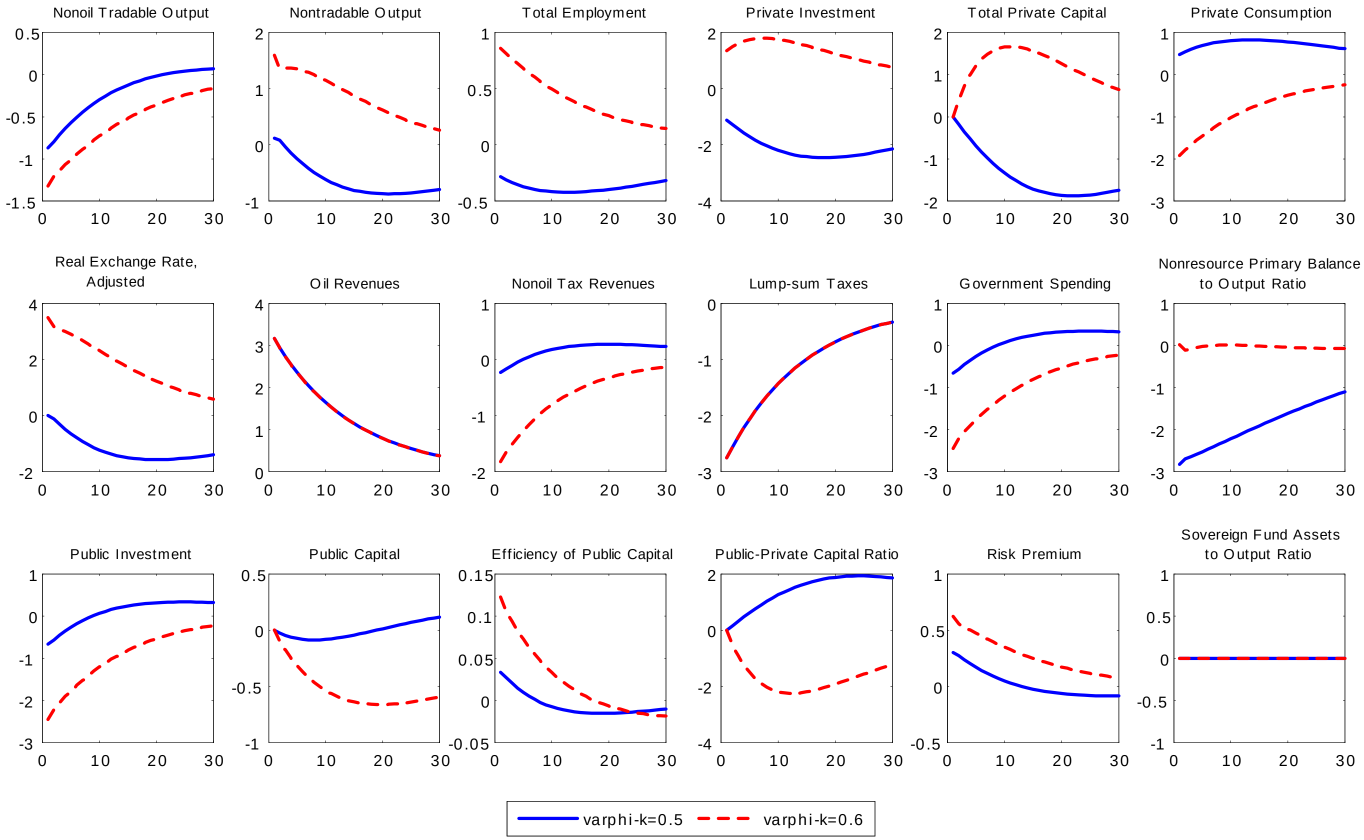
Benchmark Experiment: Higher ω_T, ω_N from 0.17 to 0.25



Absolute deviations from baseline, unless otherwise indicated.

Figure 4

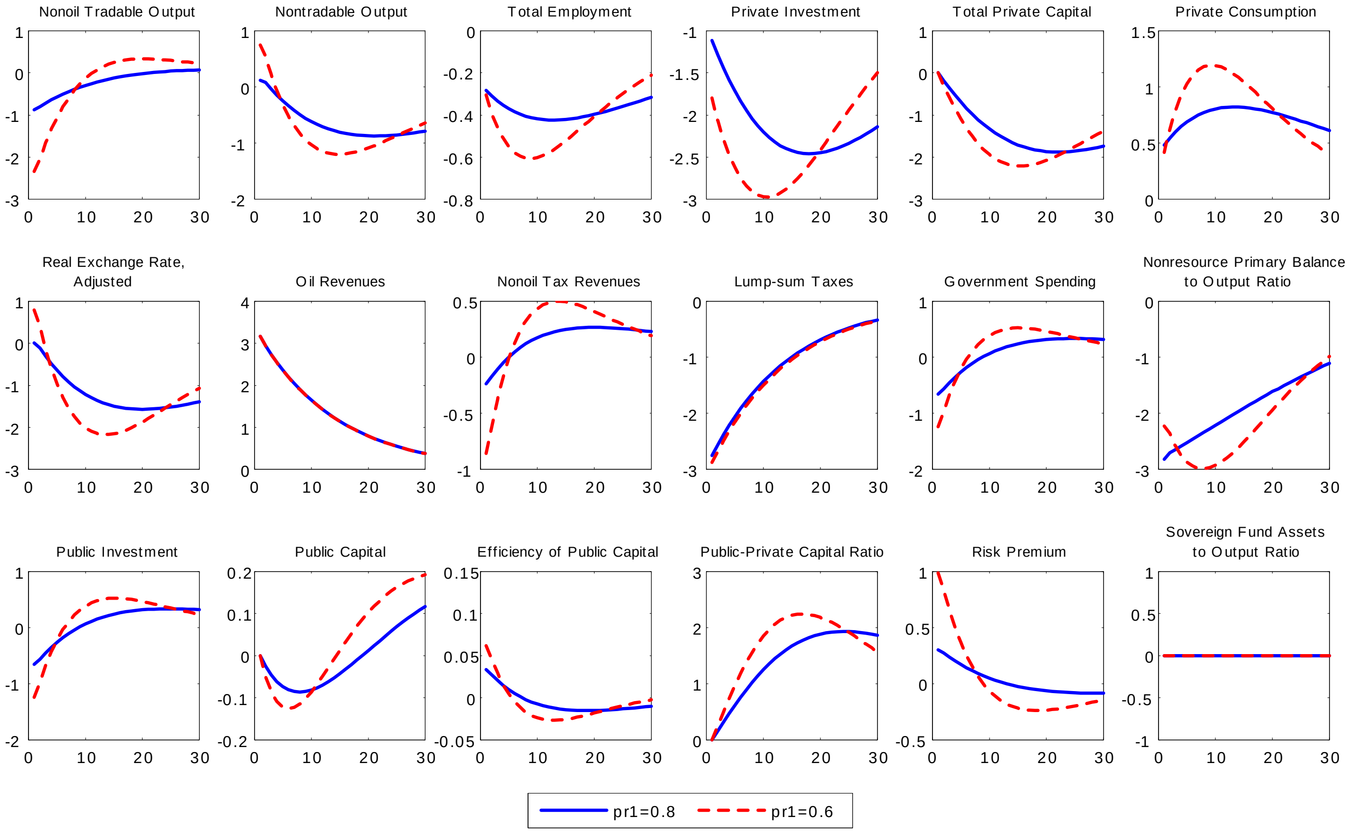
Benchmark Experiment: Higher φ_K from 0.5 to 0.6



Absolute deviations from baseline, unless otherwise indicated.

Figure 5

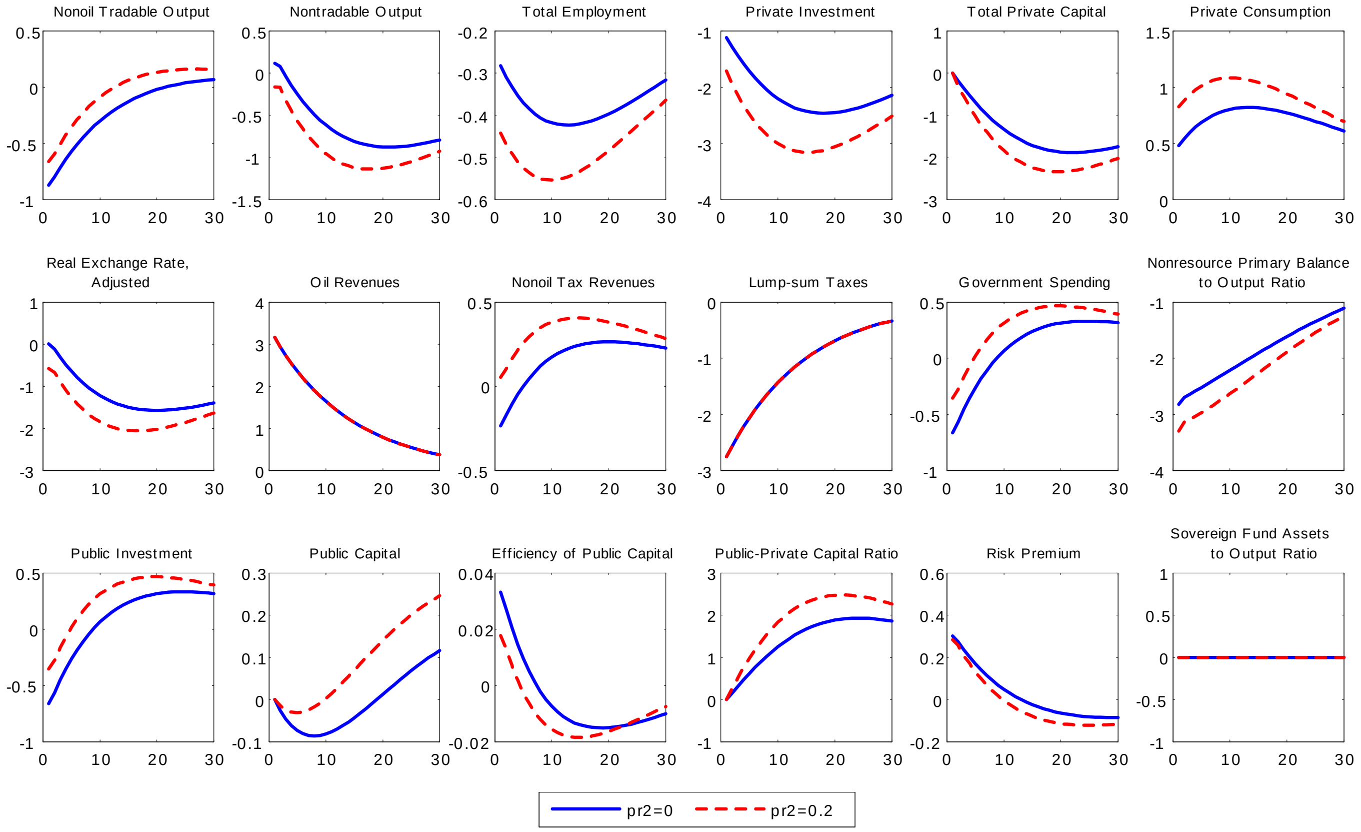
Benchmark Experiment: Lower pr_1 from 0.8 to 0.6



Absolute deviations from baseline, unless otherwise indicated.

Figure 6

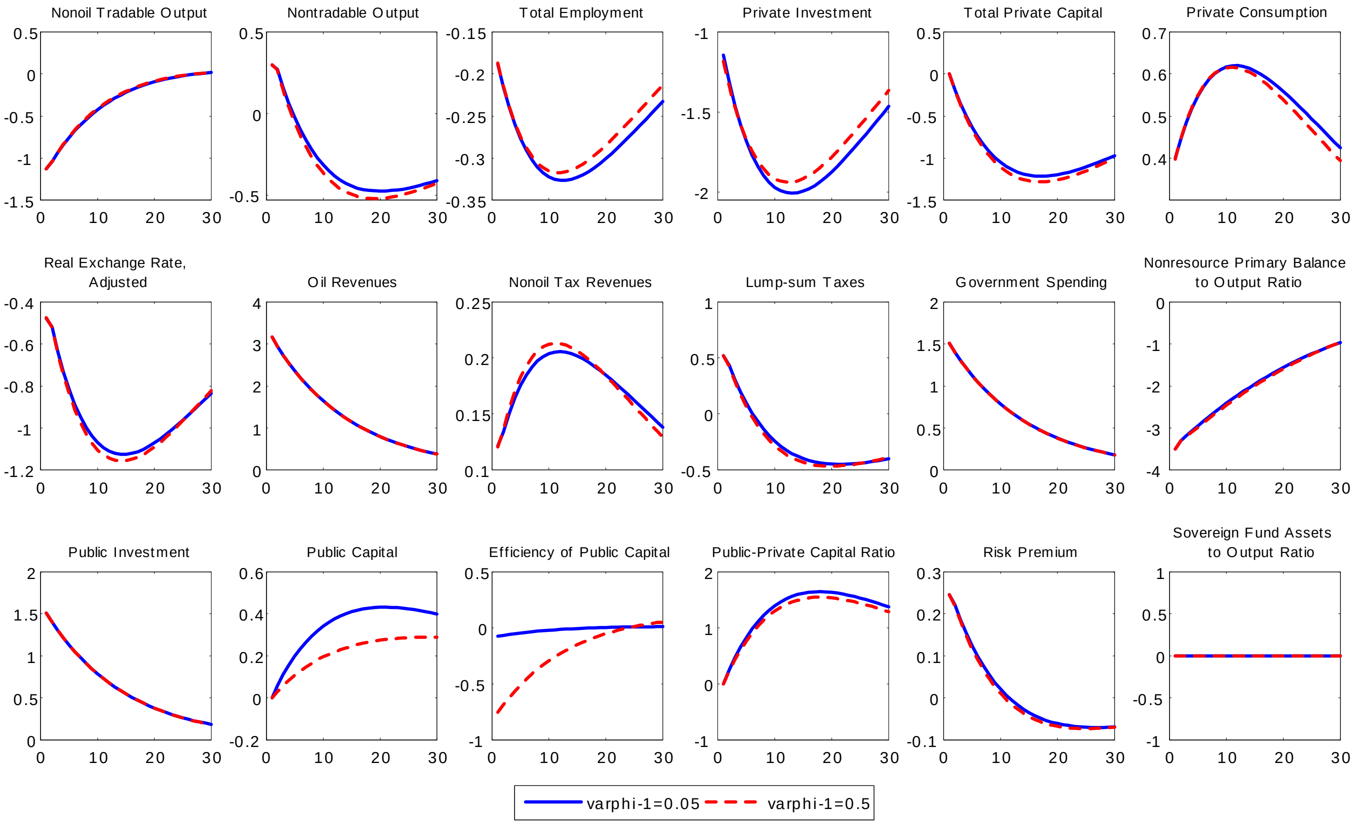
Benchmark Experiment: Higher pr_2 from 0 to 0.2



Absolute deviations from baseline, unless otherwise indicated.

Figure 7

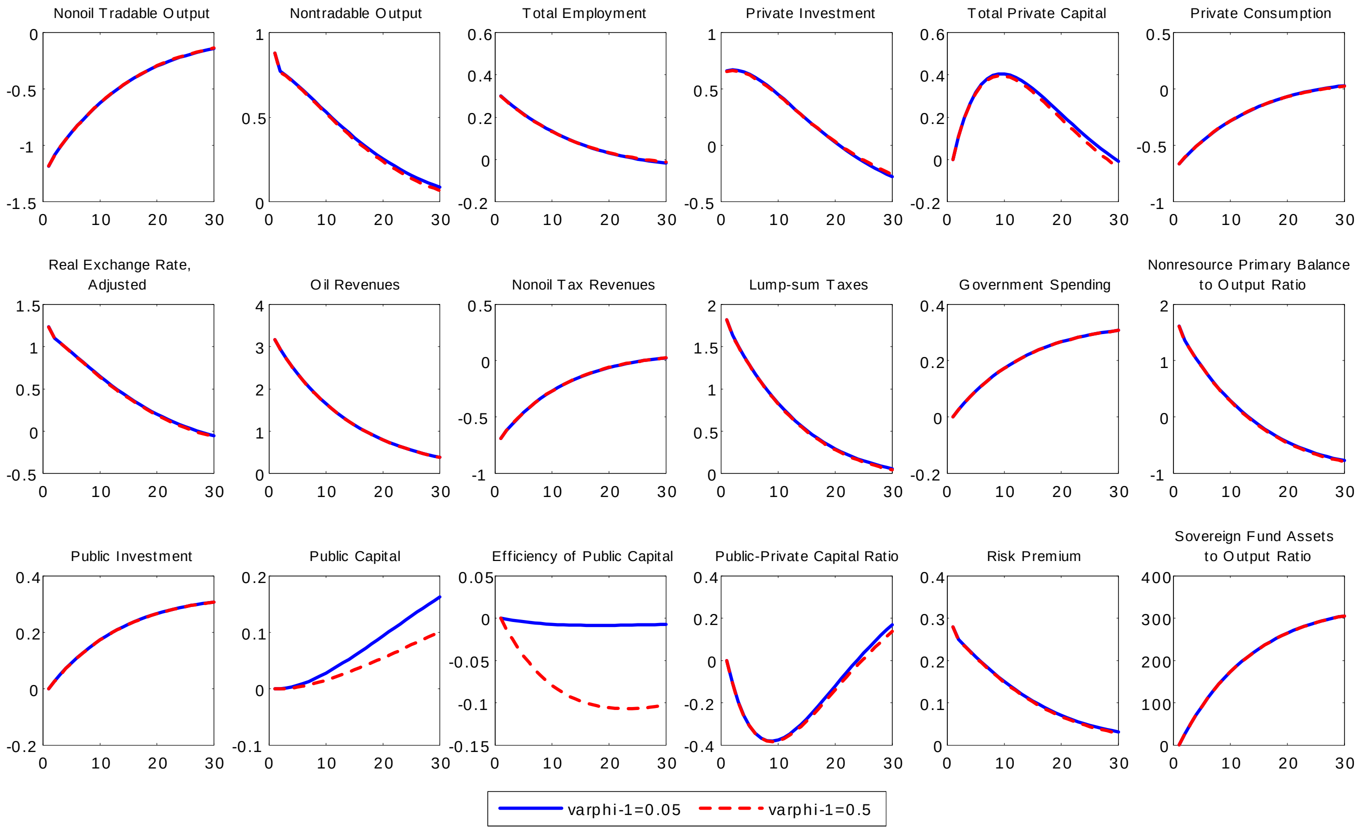
Full Spending Experiment: Higher φ_1 from 0.05 to 0.5



Absolute deviations from baseline, unless otherwise indicated.

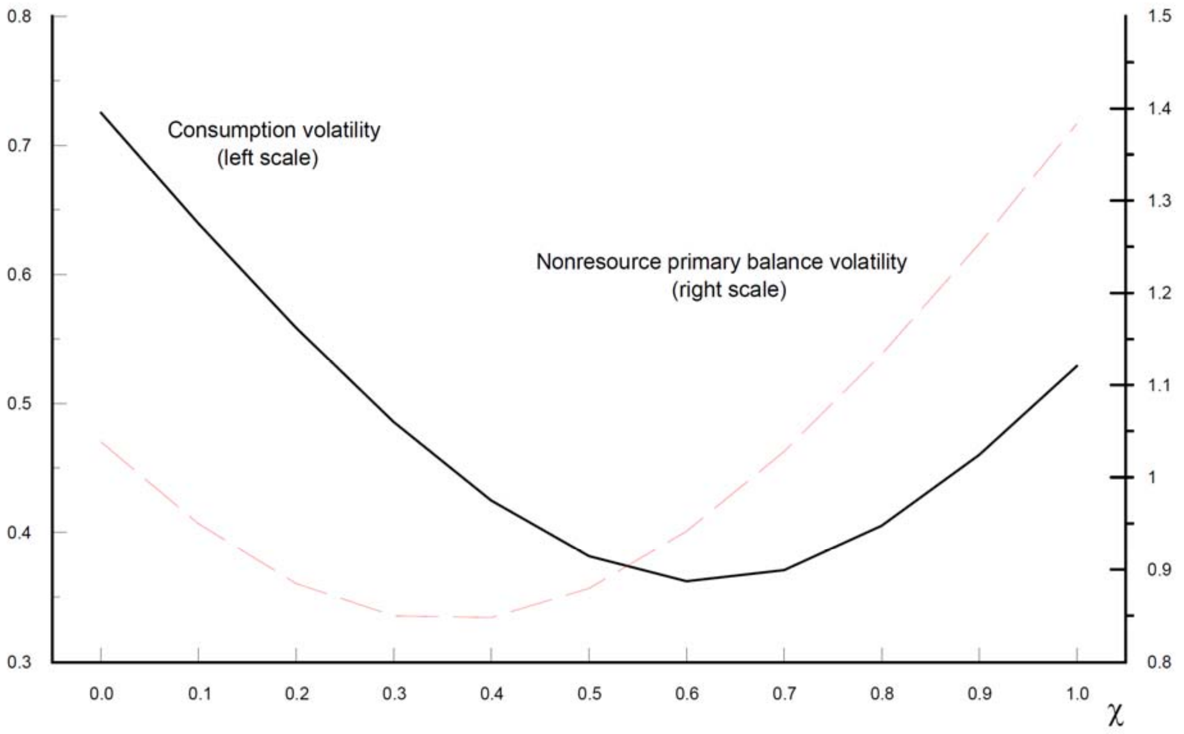
Figure 8

Full Saving Experiment: Higher φ_1 from 0.05 to 0.5



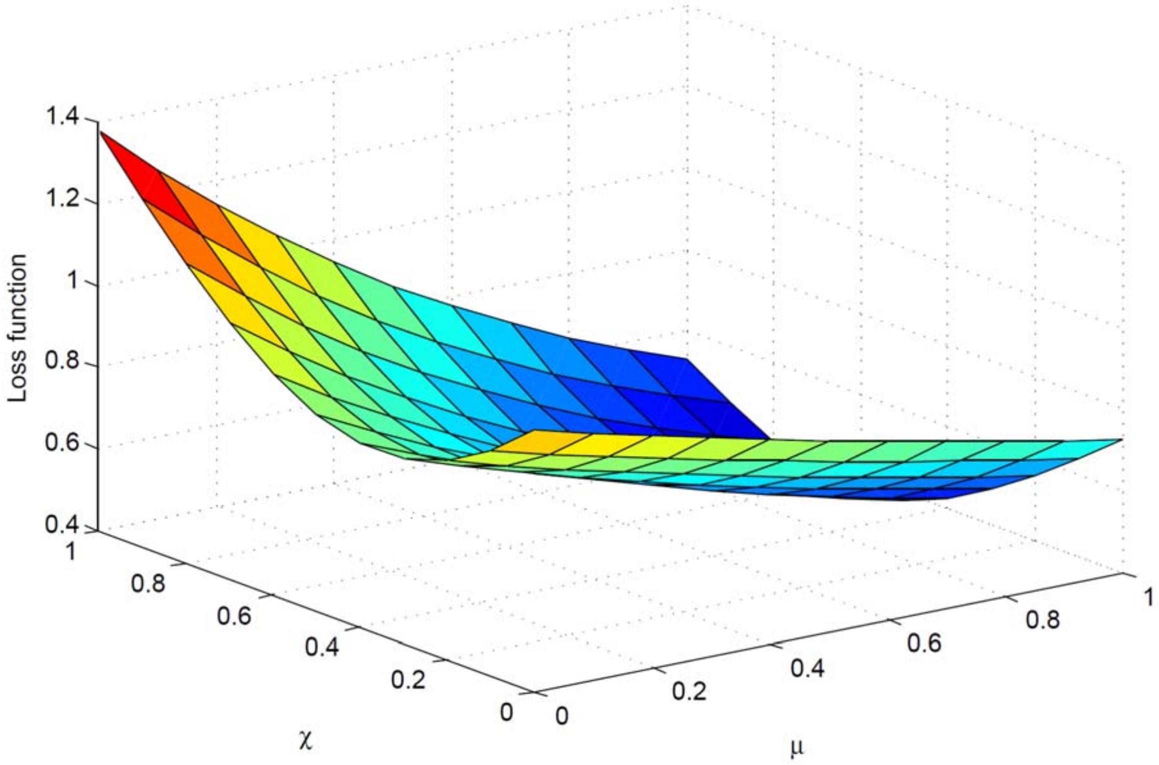
Absolute deviations from baseline, unless otherwise indicated.

Figure 9
Volatility of Consumption and the Nonresource Primary Balance as a Function of the share of the Resource Windfall Saved



Note: χ is the share of the resource windfall allocated to accumulation in a sovereign fund. Volatility measures are the relative (normalized) standard deviations reported in the first column of Table 4.

Figure 10
Optimal Saving-Spending Allocation Parameter with Fundamental Loss Function



Note: χ is the share of the resource windfall allocated accumulation in a sovereign fund, whereas μ is the relative weight on (normalized) private consumption volatility in the loss function.