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Entry and Exit with Financial Frictions

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Abstract This paper considers a model of firm dynamics to study how well aggregate shocks account for fluctuations in the entry and exit of establishments. To do this, I construct measures of aggregate financial and technology shocks. Under reasonable parameters, the model indicates that financial shocks (and not technology shocks) have contributed to the majority of cyclical fluctuations in entry and exit rates. In particular, the reduction in entry and the increase in exit during the 2007-09 recession have contributed to the slow recovery of output and hours that followed.

Keywords: business cycles, firm dynamics, entry, exit, financial frictions

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1 Introduction

Fluctuations in real GDP and employment are a natural starting point for any study of business cycles. However, fluctuations in establishment[1] entry and exit rates are often overlooked. Entry and exit rates were notably affected during the Great Recession, with entry rates falling by 17% and exit rates rising by 21%. Nevertheless, models of the business cycle typically have abstracted from entry and exit. This omission is not inconsequential, as entry and exit have been found to amplify the response of output and hours to aggregate shocks (e.g., see Clementi and Palazzo (2014)). For this reason, I construct a model of firm dynamics to address how much measured aggregate technology and financial shocks account for cyclical movements in entry and exit rates. Under reasonable parameters, the model predicts that financial shocks (and not technology shocks) account for most of the observed cyclical changes in entry and exit rates.

Building on Hopenhayn (1992), I assume that establishments face idiosyncratic productivity shocks and must pay fixed costs both to enter and to operate. However, in addition to an aggregate technology shock, I add an aggregate financial shock as well. While both Lee and Mukoyama (2013) and Clementi and Palazzo (2014) also augment Hopenhayn (1992) with aggregate technology shocks, this paper goes further by adding financial shocks as well. Financial shocks are assumed to be a direct source of aggregate fluctuations, as in Benk et al. (2005), Gilchrist and Zakrajšek (2011) and Jermann and Quadrini (2012). In this model, they are assumed to affect the marginal cost of hiring an additional worker. This is similar to how financial frictions have been modeled in the literature recently, in that they introduce a wedge between the marginal product of labor and the wage (e.g., see Jermann and Quadrini (2012) and Arellano et al. (2012)).

The financial shock is motivated by several financial frictions modeled in the literature. On the one hand, it reflects binding collateral constraints such as those in Kiyotaki and Moore (1997). However, since it operates through the labor wedge, it is most similar to the collateral constraints seen in Jermann and Quadrini (2012). On the other hand, it is motivated by the "credit spread puzzle," the observation in the corporate finance literature that default risk accounts for a small fraction of corporate bond spreads (both levels and changes).[2] This is similar in principle to the excess bond premium in Gilchrist and Zakrajšek (2012). It is also motivated by the portion of the financial frictions literature[3] which stresses

[1]An establishment (or plant) is one location of a firm. A firm may consist of many establishments.

[2]For example, see Collin-Dufresne et al. (2001), Elton et al. (2001), Huang and Huang (2003), Houweling et al. (2005) and Driessen (2005). See also Gourio (2013).

[3]For example, Bernanke and Gertler (1989) build on the "costly state verification" model of Townsend (1979) and Gale and Hellwig (1985) to generate endogenously an external finance premium. Carlstrom and Fuerst (1997) and Bernanke et al. (1999) build on this model by embedding the agency problem in a general

the role of the external finance premium. Consequently, the financial shock (and not the technology shock) is correlated with several quantitative indicators of financial distress, such as lending standards and credit spreads.

To construct empirical measures of both the technology and financial shocks, I utilize the procedure of Chari et al. (2007). That is, the sequence of shocks is set to guarantee that cyclical fluctuations of output and hours in the model match the data. This method parallels approaches used in the literature, such as measuring productivity shocks as the Solow residual or using an enforcement constraint to measure a financial shock as in Jermann and Quadrini (2012). Nevertheless, this procedure is more general, as it requires fewer assumptions to implement. With these measured shocks, it is then possible to evaluate the contribution of each shock to cyclical fluctuations in entry and exit rates, as well as output and hours. To do this, each shock is then fed back into the model, both in combination and one at a time.

Under reasonable parameters, the model indicates that financial shocks account for the majority of fluctuations in entry and exit rates. This has been true since at least the early 1990s, when the data on entry and exit begin. Specifically, the financial shock alone explains 69.2% of cyclical fluctuations in entry rates, and 51.6% of exit rates. Meanwhile, the technology shock alone accounts for practically none of the observed cyclical fluctuations in entry and exit rates. This result has significant implications for the aggregate economy, as entry and exit have amplified and propagated the effects of these financial shocks. In particular, the reduction in entry and the increase in exit during the 2007-09 recession have contributed to the slow recovery in output and hours that followed. Clementi and Palazzo (2014) obtained a similar result in a model with only technology shocks. Therefore, this paper highlights how entry and exit have further propagated the effects of financial shocks.

Moreover, financial shocks are also a much better explanation for both output and hours between 1964-I and 2014-I. Specifically, financial shocks account for 87.9% of cyclical fluctuations in output and 111.4% of hours. Meanwhile, technology shocks only explain 10.7% of output and -11.5% of hours over the same time period. Therefore, for hours, the technology shock actually tends to dampen fluctuations in hours caused by the financial shock. Furthermore, this paper shows that the importance of financial shocks has grown considerably since the 1980s. Between 1964-I and 1983-IV, financial shocks account for 20.4% of output and 78.3% of hours. Meanwhile, technology shocks account for 70.3% of output and 18.0% of hours. Therefore, before 1984, technology shocks are a relatively better explanation for output while financial shocks do better explaining hours. However, after 1984, financial shocks have since contributed to the vast majority of fluctuations in both output and hours. Specifically, between 1984-I and 2014-I, financial shocks account for essentially all of the

equilibrium model.

fluctuations in output and 117.4% of hours. Meanwhile, technology shocks account for none of fluctuations in output and -16.9% of hours.

The result that financial shocks have grown in importance since the 1980s is a finding that, up to now, has only been hinted at in the literature. On the one hand, Chari et al. (2007) found that the efficiency wedge (i.e., the technology shock) was a better explanation for output during the early 1980s recession, while the labor wedge was a better explanation for hours. Since the financial shock manifests itself in the labor wedge in this paper, this paper is consistent with those results. On the other hand, Gilchrist and Zakrajšek (2011) and Jermann and Quadrini (2012) also found a strong role for financial shocks in explaining cyclical fluctuations in output and hours. However, Gilchrist and Zakrajšek (2011) focus only on the 2007-09 recession, and the analysis of Jermann and Quadrini (2012) begins in 1984. Yet, Jermann and Quadrini (2012) chose to begin their analysis in 1984 because that was also a time in which financial flows (e.g., payouts to debt and equity) became more volatile. Moreover, the credit shock in Benk et al. (2005) does appear to become more volatile in the 1980s, but their analysis (naturally) excludes the 2007-09 recession. Nevertheless, by considering a long time period going back to 1964, this paper is able to explicitly identify the growing importance of direct financial shocks.

This paper is related to many studies which model the dynamics of heterogeneous firms. In particular, Cooley and Quadrini (2001) consider a model of firm dynamics and financial frictions, but they do not consider aggregate shocks and exit is exogenous in their model. Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006) both study firm dynamics and financial frictions with endogenous exit, but abstract from aggregate uncertainty. Both Gomes and Schmid (2010) and Arellano et al. (2012), however, do model entry and exit endogenously. However, the model of exit in these two papers is a bit stylized, as exit is only assumed to occur when firms default. Nevertheless, they do not focus on the effect of aggregate shocks on entry and exit. Furthermore, Arellano et al. (2012) model financial frictions, which introduces a wedge between the expected marginal product of labor and the wage, as in this paper. However, the key aggregate shock in their paper is an uncertainty shock.

Others, however, have studied the effect of aggregate shocks on entry and exit. Samaniego (2008) considers endogenous entry and exit, and characterizes the transition path between two steady states. In contrast to this paper, he finds that a reasonably-sized aggregate productivity shock has little effect on entry and exit rates. Lee and Mukoyama (2013) find the same for exit rates, but not for entry rates.[4] Meanwhile, Clementi and Palazzo (2014)

[4]Lee and Mukoyama (2013) also document that the exit rate of manufacturing firms is less cyclical than the entry rate. However, this same pattern is not observed in the data considered in this paper.

find significant effects on both entry and exit. Therefore, relative to these studies, the contribution is two-fold. First, it includes a financial shock in addition to a productivity shock, making it possible to evaluate the relative significance of the two shocks for entry and exit. Second, this paper highlights the theoretical determinants of the sensitivity of entry and exit to aggregate shocks. Of particular importance is the hazard rate, which measures, at the margin, how many more producers would enter (or exit) relative to the number of producers which actually enter (or exit), given a marginal change in the aggregate state. The strong response of entry and exit to aggregate shocks in this paper is due to a calibration in which the hazard rate is high in the steady state.

This paper is organized as follows. Section 2 discusses the features of the data on entry and exit this paper seeks to explain. Section 3 presents an overview of the model. Section 4 characterizes the model. Section 5 discusses the construction of the aggregate shocks and Section 6 reviews the model's calibration. Section 7 examines the main results and Section 8 concludes.

2 Observations on Entry and Exit

To motivate the study of entry and exit, I first focus on some observations from the data. Specifically, I utilize establishment birth and death rates from the BLS's Business Employment Dynamics (BED) survey. In the BED survey, an establishment death is defined to occur when an establishment reports zero employment in the third month of a quarter and does not report positive employment in the third month of the next four quarters. The establishment death rate (i.e., exit rate) is then defined as the number of deaths divided by the average of the total number of establishments who operated today and the total number who operated in the previous quarter. The establishment birth rate (i.e., entry rate) is defined analogously. This strict definition of entry and exit eliminates most temporary or seasonal entry and exit. The data for entry begins in 1993-II, while the data for exit begins in 1992-III. This time period covers two recessions: the 2001 recession and the 2007-09 recession.

[Figure 1 about here.]

The top panel of Figure 1 plots historical establishment entry and exit rates in the United States. This shows historical entry rates to be procyclical and exit rates to be countercyclical, facts which have been documented for manufacturing establishments by Lee and Mukoyama (2013). More precisely, the contemporaneous correlation of HP-filtered entry rates with HP-filtered real GDP is 0.58. Meanwhile, the analogous correlation for exit rates -0.22.

However, both entry and exit rates tend to lead real GDP. The correlation of entry rates with real GDP 1-quarter ahead is 0.64. Meanwhile, the correlation of exit rates with real GDP 3-quarters ahead is -0.55.

At a quarterly frequency, entry and exit rates have typically averaged about 3%. This rate of turnover is higher than the rates reported by Lee and Mukoyama (2013) for manufacturing establishments between 1972 and 1997 (roughly 6% at an annual frequency). In the BED survey, these entering and exiting establishments account for roughly 20% of the jobs created and destroyed each quarter. Therefore, they are a significant contributor to job creation and destruction. During the 2001 recession, exit rates increased but there was little effect on entry rates. In contrast, during the 2007-09 recession, both entry and exit rates were affected. Furthermore, while Lee and Mukoyama (2013) also document that exit rates are less cyclical than entry rates, this pattern is not apparent in the BED survey.

Cyclical fluctuations in entry and exit also have important implications for the cyclicality of the number of operating establishments. Decreases in entry rates and increases in exit rates both tend to decrease the growth of the number of establishments. Therefore, the observation that entry is procyclical and exit is countercyclical implies that the growth rate of the number of establishments will tend to slow down during recessions. This fact can be seen in the bottom panel of Figure 1, which plots the total number of establishments, expressed as the percent deviation from a log-linear trend. Therefore, this suggests that entry and exit help propagate the effects of aggregate shocks to the economy. A negative technology shock, for example, may have a larger and more persistent effect on aggregate output and employment through its effect on entry, exit and the number of operating establishments.

3 Model

In this model, time is discrete and the unit of observation can be thought to be an establishment or a producer. Throughout this paper, the words "producer," "establishment" or "entrepreneur" may be used interchangeably when discussing the theory. This model is based on Hopenhayn (1992) in that producers face idiosyncratic productivity shocks and must pay fixed costs both to enter and to operate. However, to account for cyclical fluctuations in output, hours, entry and exit, I add aggregate financial and technology shocks.

In the following subsections, the components of the model are described in detail.

3.1 Producers

The economy is populated by a continuum of producers who are perfectly competitive. Labor is the only input in the producer's production function, $f(z, s, n) = zsn^{\gamma}$, where z is aggregate productivity, s is idiosyncratic productivity and n is the labor input. It is assumed that $\gamma \in (0, 1)$, implying that there are decreasing returns to scale at the producer level. One way to interpret diminishing returns to scale at the producer-level is to think of the "span of control" models of Rosen (1982) and Lucas (1978). Here a "producer" can be interpreted as consisting of an entrepreneur and n units of labor. The idiosyncratic productivity, s, can reflect heterogeneity in the skill of managers and diminishing returns to scale is a consequence of the diminishing returns of an entrepreneur in managing larger operations. However, management is not being modeled directly and entrepreneurs earn positive profits because of diminishing returns to scale. Although there are decreasing returns to scale at the producer level, there still are constant returns to scale in the aggregate because the producer can be replicated. With perfect competition, producer-level diminishing returns allow for heterogeneity to exist in equilibrium and prevents the most productive producers from taking over the market completely.

Idiosyncratic productivity is assumed to follow an AR(1) process. This process is given by

$$\ln s' = \rho_s \ln s + \varepsilon_s \tag{1}$$

where s' is the next-period shock and ε_s is an independent innovation drawn from $N(0, \sigma^2_{\varepsilon s})$. In addition, producers face two aggregate shocks: (1) an aggregate productivity shock, and (2) an aggregate financial shock. The current aggregate technology shock is denoted by z and the current financial shock is denoted by θ. I will discuss precisely how θ enters the model in Section 3.1.1. I assume that these aggregate shocks follow a VAR(1) process given by

$$\begin{bmatrix} \ln z' \\ \ln \theta' \end{bmatrix} = \begin{bmatrix} a_z \\ a_\theta \end{bmatrix} + \begin{bmatrix} \rho_{z,z'} & \rho_{\theta,z'} \\ \rho_{z,\theta'} & \rho_{\theta,\theta'} \end{bmatrix} \begin{bmatrix} \ln z \\ \ln \theta \end{bmatrix} + \begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \end{bmatrix} \tag{2}$$

where (z', θ') are the next-period shocks and $(\varepsilon_z, \varepsilon_\theta)$ are normally distributed innovations with mean zero and covariance matrix $\mathbf{\Sigma}$. Assuming $\sigma^2_{\varepsilon z}$ is the variance of ε_z, $\sigma^2_{\varepsilon\theta}$ is the variance of ε_θ, and $\rho_{\varepsilon_z,\varepsilon_\theta}$ is the correlation of ε_z and ε_θ, $\mathbf{\Sigma}$ is parameterized as follows

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma^2_{\varepsilon z} & \rho_{\varepsilon_z,\varepsilon_\theta}\sigma_{\varepsilon z}\sigma_{\varepsilon\theta} \\ \rho_{\varepsilon_z,\varepsilon_\theta}\sigma_{\varepsilon z}\sigma_{\varepsilon\theta} & \sigma^2_{\varepsilon\theta} \end{bmatrix}.$$

I now discuss how financial shocks enter the model.

3.1.1 Financial Frictions

To model financial frictions, I take a reduced-form approach. Specifically, given the wage w, I assume producers choose today's labor n to maximize profits:

$$\pi(z, \theta, s; w) = \max_n \left\{ f(z, s, n) - \frac{wn}{\theta} \right\} \tag{3}$$

The first order condition implies that the optimal choice of n must satisfy $w/\theta = f_n(z, s, n)$. Therefore, the financial shock θ drives a wedge between the wage and the marginal product of labor. This is consistent with how financial frictions have been modeled in several recent papers, such as Jermann and Quadrini (2012) and Arellano et al. (2012). Since it is assumed that $f(z, s, n) = zsn^\gamma$, the optimal labor demand of an individual producer is given by:

$$n^d(z, \theta, s; w) = \left[\frac{\gamma \theta z s}{w} \right]^{1/(1-\gamma)}. \tag{4}$$

By driving a wedge between the wage and the marginal product of labor, the financial shock directly affects labor demand. Consequently, a "negative" financial shock refers to a scenario in which θ is lower and thus labor demand is lower.

This setup can be rationalized by an intra-period working capital constraint. This assumption can be motivated by the observation that inputs such as labor and materials have to be purchased in advance. Specifically, at the beginning of the period, the producer chooses today's employment n and pays wages wn. To partially finance the wage bill, producers issue a non-defaultable intra-temporal bond at the beginning of the period. With the bond, the producer promises to re-pay b at the end of the period. In return, the producer receives qb at the beginning of the period, where q is the price of the intra-temporal bond. Assuming that producers only finance a fraction κ of wages with debt, this will imply that producers choose b under the constraint that $qb = \kappa wn$.

Financial frictions then consist of two components. The first is that the price of the intra-temporal bond is $q = 1 - \phi$. With no frictions, the price of this bond should be 1. When $\phi > 0$, this indicates that producers are paying a premium for debt. The second friction is that producers face a collateral constraint. Namely, it is assumed that the promised debt payment cannot exceed some fraction of output:

$$b \leq \xi f(z, s, n).$$

When ξ is lower, the collateral constraint is tighter. When the collateral constraint binds, the producer's labor demand is given by:

$$n^d = \left[\frac{\xi(1-\phi)zs}{\kappa w}\right]^{1/(1-\gamma)}.$$

This is equivalent to the reduced-form labor demand in Equation 4 when $\theta = \xi(1-\phi)/(\kappa\gamma)$. Therefore, low values of θ can arise from three sources: (1) a tighter collateral constraint (i.e., lower ξ), (2) a higher premium on debt (i.e., higher ϕ), and (3) a situation in which producers are more dependent on external finance (i.e., higher values of κ).

Therefore, this model specification simultaneously incorporates multiple views of financial frictions seen in the literature. On the one hand, it includes a collateral constraint. Similar to Jermann and Quadrini (2012), the collateral constraint in this model drives a wedge between the wage and the marginal product of labor. On the other hand, it allows for a external finance premium, which is stressed in the financial frictions literature. This premium is motivated by the "credit spread puzzle," the observation in the corporate finance literature that default risk accounts for a small fraction of corporate bond spreads (both levels and changes). Related to this is the excess bond premium in Gilchrist and Zakrajšek (2012).

Nevertheless, in Section 5, only θ will be measured. Consequently, it is not necessary to make specific assumptions about the value of κ. While it will not be possible to separate out the effects of the individual financial frictions, this approach is general enough to capture multiple sources of frictions. Moreover, as will be seen in Section 5, the measured financial shock (and not the technology shock) turns out to be highly correlated with quantitative measures of financial distress, such as lending standards and credit spreads.

3.1.2 Exit Decision

Each period the producer must pay a fixed operating cost, $c_f > 0$, denominated in terms of the output good. If the producer chooses to operate today, it pays the fixed cost, earns profits today and continues to tomorrow. Otherwise, it exits. When making the exit decision, an entrepreneur compares the value of operating with the value of exiting, which is zero. Denote by $V(\mathbf{x}, s)$ a producer's value of operating in the current period, where $\mathbf{x}$ is the aggregate state today and s is the producer's idiosyncratic productivity. This value function is defined later in Equation 8. The aggregate state $\mathbf{x}$ is a vector of state variables which includes z and θ, the current-period technology and financial shocks, respectively. Therefore, the producer will exit today if and only if $V(\mathbf{x}, s) < 0$. This implies that there exists an exit cutoff productivity, $\underline{s}(\mathbf{x})$, such that the producer will exit if and only if $s < \underline{s}(\mathbf{x})$. This cutoff is defined as the value $\underline{s}$ such that

$$V(\mathbf{x}, \underline{s}) = 0. \tag{5}$$

Because $\underline{s}(\mathbf{x})$ depends on the aggregate state, aggregate shocks will influence exit rates by shifting this productivity threshold. In particular, during good times, $\underline{s}(\mathbf{x})$ will be lower, reducing the fraction of incumbent producers who will exit.

3.1.3 Entry Decision

As in Clementi and Palazzo (2014), I assume that there is a finite mass $\bar{M}_e$ of prospective entrants every period. Each potential entrant receives a "signal" s_e about its productivity, where s_e is drawn from a log normal distribution with mean μ_g and standard deviation σ_g. If a potential entrant with signal s_e chooses to operate today, it would immediately begin operation and its idiosyncratic productivity would be s_e. After a potential entrepreneur makes the decision to enter, it pays a fixed entry cost, $c_e \geq 0$. In contrast to Hopenhayn (1992), potential entrants know s_e when they pay the fixed entry cost.[5] The advantage of this assumption is that it introduces endogenous selection into entry.[6] As a consequence, only most productive producers will be able to enter during bad times.

In making its entry decision, a potential entrant compares the value of operating it would receive if it enters against the total cost of entry. Thus, a potential entrant with signal s_e will enter if and only if $V(\mathbf{x}, s_e) \geq c_e$. This implies that there is an entry cutoff for potential entrants, $\bar{s}_e(\mathbf{x})$, where $\bar{s}_e$ is defined as the value of s_e such that

$$V(\mathbf{x}, \bar{s}_e) = c_e. \tag{6}$$

A potential entrant will enter if and only if $s_e \geq \bar{s}_e(\mathbf{x})$. Because $\bar{s}_e(\mathbf{x})$ depends on the aggregate state, aggregate shocks will influence entry rates by shifting this threshold. In particular, during good times, $\bar{s}_e(\mathbf{x})$ will be lower, increasing the fraction of potential entrants who will enter. In addition, since $c_e \geq 0$, it follows that $\underline{s}(\mathbf{x}) \leq \bar{s}_e(\mathbf{x})$. In other words, all entrants will choose to operate.

3.1.4 Transition Rule for Distribution of Producers

Given the description of the entry and exit conditions, it is now possible to define the law of motion for the distribution of producers. First, define $g(\cdot)$ to be a function over the current period's signal, s_e. This function represents the distribution of potential entrants. $\bar{M}_e = \int_0^\infty g(s_e) ds_e$ is then the total mass of potential entrants. In this model, $g(s_e) = \bar{M}_e \varphi(s_e)$,

[5] The model of entry in this paper is also consistent with a modified version of Hopenhayn (1992), in which the entry cost c_e varies with $\bar{M}_e$. In particular, suppose c_e varies with $\bar{M}_e$ as follows: $c_e = \bar{c}_e \bar{M}_e^{\eta}$. Recall that $\bar{M}_e$ is endogenous in Hopenhayn (1992). However, if $\eta \to \infty$, the equilibrium value of $\bar{M}_e$ is fixed at 1. Therefore, when η is very large, that model of entry is the same as assuming $c_e = 0$ in this paper.

[6] This is also the reason Lee and Mukoyama (2013) modeled entry with a "two-step" process.

where $\varphi(\cdot)$ is the probability density function for a log normal random variable with mean μ_g and standard deviation σ_g. Similarly, define $\mu(\cdot)$ to be a function over the current period's idiosyncratic shock, s. This function represents the distribution of incumbent producers at the beginning of the period before the entry and exit decisions are made. $M = \int_0^\infty \mu(s)ds$ is then the total mass of incumbent producers. In contrast to $g(\cdot)$, the distribution $\mu(\cdot)$ is endogenously determined in this model.

Let $\mathbf{x} = (z, \theta, \mu)$ be the vector of aggregate state variables. Given today's aggregate state $\mathbf{x}$, tomorrow's incumbent distribution μ' will be given by

$$\mu'(s') = \int_{\underline{s}(\mathbf{x})}^{\infty} h(s'|s)\mu(s)ds + \int_{\bar{s}_e(\mathbf{x})}^{\infty} h(s'|s_e)g(s_e)ds_e. \tag{7}$$

The function $h(s'|s)$ is the conditional probability density function for s', as determined by the process assumed in Equation 1. The first term represents the mass of incumbents who do not exit today and transition to s' tomorrow. The second term represents the mass of potential entrants who enter today and transition to s' tomorrow.

3.1.5 Producer's Optimization Problem

The producer's problem can now be formulated recursively. Let $\mathbf{x} \equiv (z, \theta, \mu)$ be the vector of aggregate state variables. Define $V(\mathbf{x}, s)$ as the value of continuing for an entrepreneur with aggregate state $\mathbf{x}$ and idiosyncratic productivity s in the current period, after any dividends from the operations of the current period have been issued. Then, $V(\mathbf{x}, s)$ is defined as the function which solves the following Bellman equation:

$$V(\mathbf{x}, s) = \pi(z, \theta, s; w(\mathbf{x})) - c_f + \frac{1}{1+r} E\left[\max\left\{V(\mathbf{x}', s'), 0\right\} | z, \theta, s\right] \tag{8}$$

subject to

$$\mu' = T_\mu(\mathbf{x}).$$

$T_\mu(\mathbf{x})$ is the transition rule defined in Equation 7 and $w(\mathbf{x})$ is the equilibrium wage. If the producer operates, it earns profits $\pi(z, \theta, s; w(\mathbf{x}))$ and pays the fixed cost c_f. Next period, the incumbent producer receives $V(\mathbf{x}', s')$ if it operates and zero otherwise.

3.2 Labor Supply

The supply of labor is assumed to be given by the function

$$N^s(w) = (w/\psi)^\nu \tag{9}$$

where ν is the Frisch elasticity of labor supply.[7] The parameter ψ will just be used to normalize the wage to 1 in the steady state. By assuming an elastic labor supply, the equilibrium wage will change in response to aggregate technology and financial shocks.

3.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium can then be defined as follows. A recursive competitive equilibrium consists of (i) the value function $V(\mathbf{x}, s)$ (ii) policy function $n(\mathbf{x}, s)$ (iii) cutoff rules $\underline{s}(\mathbf{x})$, $\bar{s}_e(\mathbf{x})$ (iv) wage function $w(\mathbf{x})$ and (v) law of motion $T_\mu(\mathbf{x})$ such that

1. $V(\mathbf{x}, s)$ solves the Bellman equation given by Equation 8.

2. The policy rule $n(\mathbf{x}, s) = n(z, \theta, s; w(\mathbf{x}))$, where $w(\mathbf{x})$ is the equilibrium wage and $n(z, \theta, s; w)$ is the solution to Equation 3.

3. The cutoff rules, $\underline{s}(\mathbf{x})$ and $\bar{s}_e(\mathbf{x})$, are given by Equations 5 and 6, respectively.

4. The wage $w(\mathbf{x})$ clears the labor market for aggregate state $\mathbf{x}$. First, define aggregate labor demand, given wage $\mathbf{x}$ and w, as follows:

   $$N^d(\mathbf{x}, w) = \int_{\underline{s}(\mathbf{x})}^{\infty} n(z, \theta, s; w)\mu(s)ds + \int_{\bar{s}_e(\mathbf{x})}^{\infty} n(z, \theta, s_e; w)g(s_e)ds_e.$$

   The function $n(z, \theta, s; w)$ is the solution to Equation 3. Thus, the wage w clears the labor market when $N^d(\mathbf{x}; w) = N^s(w)$, where $N^s(\cdot)$ is the labor supply function defined in Equation 9.

5. The actual transition rule for the distribution of producers, $T_\mu(\mathbf{x})$, is given by Equation 7, implying that it is consistent with the transition rule assumed by producers.

[7] This setup is observationally equivalent to a general equilibrium model in which labor is supplied by households with preferences linear in consumption and separable in consumption and leisure.

4 Characterization of Model

With the model fully specified, I now characterize theoretically how financial shocks influence entry and exit. I then describe how fluctuations in entry and exit rates propagate the effects of aggregate financial and technology shocks on aggregate output and hours.

4.1 Entry and Exit Rate Elasticities

To understand the impact aggregate shocks have on entry and exit rates, I construct aggregate-shock elasticities for entry and exit rates. In what follows, it is useful to define $\hat{\mu}(x) = \mu(e^x)e^x$ and $\hat{g}(x) \equiv g(e^x)e^x$. Note that $\hat{\mu}(\cdot)$ and $\hat{g}(\cdot)$ are equivalent to $\mu(\cdot)$ and $g(\cdot)$, but defined over log productivity. Furthermore, the entry rate is defined to be $m_e \equiv M_e/M$, where $M_e \equiv \int_{\bar{s}_e}^{\infty} g(s_e)ds_e = \int_{\ln \bar{s}_e}^{\infty} \hat{g}(x)dx$ is the mass of entrants and $M \equiv \int_0^{\infty} \mu(s)ds = \int_{-\infty}^{\infty} \hat{\mu}(x)dx$ is the mass of incumbent producers. Similarly, the exit rate is defined to be $m_x \equiv M_x/M$, where $M_x \equiv \int_0^{\underline{s}} \mu(s)ds = \int_{-\infty}^{\ln \underline{s}} \hat{\mu}(x)dx$ is the mass of exiting producers. Note that the definitions of entry and exit rates in the model differ slightly from their definition in the data (see Section 2). Nevertheless, since entry and exit rates are small, these two measures are approximately equal. Moreover, let $G_c(a) \equiv \int_a^{\infty} g(s_e)ds_e = \int_{\ln a}^{\infty} \hat{g}(x)dx$ be the mass of potential entrants with a signal higher than a and let $M(a) \equiv \int_0^a \mu(s)ds = \int_{-\infty}^{\ln a} \hat{\mu}(x)dx$ be the mass of incumbent producers at the beginning of the period with productivity less than a. Furthermore, define $h_e(s) \equiv g(s)s/G_c(s)$ and $h_x(s) \equiv \mu(s)s/M(s)$ to be the hazard rates for entry and exit, respectively.

Then, elasticities can be derived for entry and exit rates as follows:

$$
\begin{aligned}
\frac{d \ln m_e}{d \ln \theta} &= -h_e(\bar{s}_e)\frac{d \ln \bar{s}_e}{d \ln \theta} & \qquad \frac{d \ln m_e}{d \ln z} &= -h_e(\bar{s}_e)\frac{d \ln \bar{s}_e}{d \ln z} \\
\frac{d \ln m_x}{d \ln \theta} &= h_x(\underline{s})\frac{d \ln \underline{s}}{d \ln \theta} & \qquad \frac{d \ln m_x}{d \ln z} &= h_x(\underline{s})\frac{d \ln \underline{s}}{d \ln z}
\end{aligned}
$$

For each elasticity, there are two components which determine the quantitative impact of an aggregate technology or financial shock. The first is the hazard rate, which reflects how many entrepreneurs are near the productivity cutoff threshold relative to all entrepreneurs who enter or exit, respectively. The second is the slope of the corresponding productivity cutoff, which reflects how much the productivity cutoff responds to the aggregate shock. These elasticities do not readily admit a functional form, which requires the model to be solved numerically to quantify the effect that aggregate shocks will have on entry and exit rates.

Nevertheless, it can be seen qualitatively what determines these elasticities. In particular, consider the the exit hazard rate. This hazard rate can be re-written as follows:

$$h_x(\underline{s}) \equiv \frac{\mu(\underline{s})\underline{s}}{M(\underline{s})} = \frac{\hat{\mu}(\ln \underline{s})/M}{M(\underline{s})/M} = \frac{\hat{\mu}(\ln \underline{s})/M}{m_x} \tag{10}$$

where M is the total mass of incumbents at the beginning of the period. The function $\hat{\mu}(\ln s)/M$ is the probability density function (over $\ln s$) representing the distribution of incumbent producers. Thus, the numerator of Equation 10 is the probability density at the exit threshold, $\ln \underline{s}$. Meanwhile, the denominator is just m_x, the fraction of incumbent producers who exit (i.e., the exit rate).

Computationally, it turns out that there are two determinants of the hazard rate $h_x(\underline{s})$. The first is the exit rate m_x. It turns out that the higher the exit rate, m_x, the lower the hazard rate $h_x(\underline{s})$. Given the distribution of incumbent entrepreneurs, varying m_x can be interpreted as varying the default threshold $\ln \underline{s}$. Then, suppose that m_x increases and thus $\ln \underline{s}$ increases. Increasing m_x will tend to increase the denominator in Equation 10, which tends to decrease $h_x(\underline{s})$. However, the numerator may increase as well. If $\ln \underline{s}$ is sufficiently large, increasing $\ln \underline{s}$ will tend to cause $\hat{\mu}(\ln \underline{s})/M$ to fall. In that case $h_x(\underline{s})$ unambiguously falls. But for low $\ln \underline{s}$, $\hat{\mu}(\ln \underline{s})/M$ increases as well. From the computations it turns out that the increase in the numerator $\hat{\mu}(\ln \underline{s})/M$ is smaller than the increase in the denominator m_x. Thus, in general, the exit hazard rate tends to decrease with m_x.

[Figure 2 about here.]

The second determinant of the exit hazard rate is the shape of the incumbent producer distribution. To see how the distribution matters, consider Figure 2. Figure 2 considers a scenario in which the incumbent distribution exhibits less dispersion in idiosyncratic productivity, but the exit rate m_x is the same as before. In this case, to match the same exit rate m_x, $\hat{\mu}(\ln \underline{s})/M$ will have to be higher. Therefore, the lower the dispersion in productivity, the higher the hazard rate will be at the exit threshold. The dispersion in productivity is largely determined by the assumed process for idiosyncratic productivity in Equation 1. When $\sigma_{\varepsilon s}/\sqrt{1-\rho_s^2}$ is higher, the incumbent distribution will exhibit more dispersion in productivity, and thus the hazard rate will be lower for a given exit rate, m_x.

The determinants of the entry hazard rate are analogous to those for exit. As with exit, re-write the entry hazard rate as follows:

$$h_e(\bar{s}_e) = \frac{g(\bar{s}_e)\bar{s}_e}{G_c(\bar{s}_e)} = \frac{\hat{g}(\ln \bar{s}_e)/\bar{M}_e}{G_c(\bar{s}_e)/\bar{M}_e} = \frac{\hat{g}(\ln \bar{s}_e)/\bar{M}_e}{\bar{m}_e} \tag{11}$$

where $\bar{M}_e = G_c(0) = \int_0^\infty g(s_e)ds_e$ is the total mass of potential entrants and $\bar{m}_e \equiv G_c(\bar{s}_e)/\bar{M}_e$ is the fraction of potential entrants who enter in a period. The function $\hat{g}(\ln s_e)/\bar{M}_e$ is the probability density function (over $\ln s_e$) representing the distribution of potential entrants. Thus, the numerator of Equation 11 is this probability density at the entry threshold. Meanwhile, the denominator of Equation 11 is the entry probability $\bar{m}_e$ and not the entry rate m_e.

Recall that $g(s_e)/\bar{M}_e$ is exogenously given. In the calibration in Section 6, $g(s_e)/\bar{M}_e$ is assumed to be the log normal probability density function with mean μ_g and standard deviation σ_g. In this case, it trivially follows that (1) the entry hazard rate will decrease when $\bar{m}_e$ increases, and (2) given $\bar{m}_e$, the hazard rate will be higher when σ_g is lower.

4.2 Aggregate Output and Hours

Given that aggregate shocks will influence entry and exit, it can now be seen how this affects the aggregate economy. Given aggregate state $\mathbf{x} = (z, \theta, \mu)$, let $Y(\mathbf{x})$ and $N(\mathbf{x})$ denote aggregate output and hours in equilibrium. Then, define $\Gamma(\mathbf{x})$ as follows:

$$\Gamma(\mathbf{x}) \equiv \int_{\underline{s}(\mathbf{x})}^{\infty} s^{1/(1-\gamma)} \mu(s) ds + \int_{\bar{s}_e(\mathbf{x})}^{\infty} s_e^{1/(1-\gamma)} g(s_e) ds_e. \tag{12}$$

Intuitively, $\Gamma(\mathbf{x})$ is the productivity-weighted mass of incumbents and entrants who operate today. $\Gamma(\mathbf{x})$ will tend to be higher when there are a lot of producers operating. Then, using Y_t, N_t, and Γ_t as short-hand notation for $Y(\mathbf{x}_t)$, $N(\mathbf{x}_t)$ and $\Gamma(\mathbf{x}_t)$, aggregate output must satisfy the following relationship:

$$Y_t = z_t N_t^{\gamma} \Gamma_t^{1-\gamma}. \tag{13}$$

With exogenous entry and exit, Γ_t is constant. However, with endogenous entry and exit, Γ_t is another channel through which aggregate shocks will be propagated. During boom periods, not only will N_t increase, but so will Γ_t, further amplifying the effect of aggregate shocks.

Furthermore, cyclical fluctuations in Y_t and N_t can be related to cyclical fluctuations in z_t, θ_t and Γ_t. To express each variable as the percentage deviation from its steady state value, I indicate this using a circumflex (e.g., $\hat{Y}_t$ is the percent deviation of Y_t from its steady state value). Then, $\hat{Y}_t$ and $\hat{N}_t$ can be related to $\hat{z}_t$ and $\hat{\theta}_t$ and $\hat{\Gamma}_t$ as follows:

$$\hat{Y}_t = \frac{1}{1+\nu(1-\gamma)} \left[(1+\nu)\hat{z}_t + \gamma\nu\hat{\theta}_t + (1+\nu)(1-\gamma)\hat{\Gamma}_t\right] \tag{14}$$

$$\hat{N}_t = \frac{\nu}{1+\nu(1-\gamma)} \left[\hat{z}_t + \hat{\theta}_t + (1-\gamma)\hat{\Gamma}_t\right]. \tag{15}$$

Now suppose that there is an unexpected increase in z_t or θ_t. On impact, the contribution of entry and exit to the dynamics will be small as the effect on Γ_t will be small. However, over time, the effect on Γ_t will be larger, and this will have an additional effect on both output and hours.

5 Measurement of Shocks

To measure the sequence of technology and financial shocks, I apply the procedure of Chari et al. (2007). Specifically, I set the sequence of $\{z_t, \theta_t\}$ so that model-predicted fluctuations in output and hours match those seen in the data. To be more precise, let $\hat{Y}_t^m$ and $\hat{N}_t^m$ denote the cyclical component of model-predicted output and hours, respectively, at date t. Output and hours are de-trended with log-linear trend. Let $\hat{Y}_t^d$ and $\hat{N}_t^d$ be the corresponding series from the data. Then, I constructed the sequence of $\{z_t, \theta_t\}$ so that $\hat{Y}_t^m = \hat{Y}_t^d$ and $\hat{N}_t^m = \hat{N}_t^d$ for all t. Using this procedure, it is not possible to identify the level of z_t and θ_t. Nevertheless, the precise level of these shocks is irrelevant, as only changes in z_t and θ_t are relevant for the results. Consequently, I normalized the level of $\ln z_t$ and $\ln \theta_t$ to zero by assuming $a_z = a_\theta = 0$. Then, using this procedure, I measured the technology and financial shocks between 1964-I and 2014-I. For Y_t^d, I used quarterly seasonally adjusted real GDP from the BEA's National Income and Product Accounts. For N_t^d, I used the quarterly average of aggregate weekly hours of production and non-supervisory employees from the BLS's Current Employment Statistics (CES) survey.

This procedure parallels approaches used in the literature where the model is used to identify an exogenous sequence of shocks. Examples include using the Solow residual to measure a technology shock, or using an enforcement constraint to measure a financial shock (as in Jermann and Quadrini (2012)). However, it is a more general approach since it requires less stringent assumptions to identify the shocks. For example, the Solow residual only identifies the technology shock in models where there are constant returns-to-scale. In contrast, this procedure does not require the assumption of constant returns-to-scale to identify the technology shock from the data.

[Figure 3 about here.]

The top panel of Figure 3 plots the resulting technology shocks against output. The contemporaneous correlation of the technology shock with real GDP is 0.41 (see Table 1). However, the technology shock is essentially uncorrelated with hours, as the contemporaneous correlation with hours is -0.01. Also shown in Figure 3 is the Solow residual. Since there is no capital in the model, the Solow residual is calculated assuming that quarterly changes in the capital stock are zero. Since the model matches both output and hours, the model-generated Solow residual matches the Solow residual in the data, given both aggregate shocks. From Figure 3, it can be seen that the Solow residual is highly correlated with the technology shock. In fact, the contemporaneous correlation is 0.86. Moreover, from Equation 13, it can be seen why the Solow residual does not exactly identify the technology shock. Namely, it ignores fluctuations in Γ_t, which also contribute to fluctuations in output.

[Table 1 about here.]

Meanwhile, the bottom panel of Figure 3 plots the measured financial shock against hours. Compared to the technology shock, the financial shock is more volatile. Whereas the standard deviation of $\ln z_t$ is 1.3%, the standard deviation of $\ln \theta_t$ is 4.4%. Moreover, the financial shock is also highly correlated with output and hours. The contemporaneous correlation of the financial shock with hours is 0.92, while the correlation with output is 0.64.

Furthermore, from Figure 3 it can be seen that financial shocks have become more volatile over time. To see this precisely, consider the observations before and after 1984. I focus on this particular date for two reasons. First, it roughly corresponds to the beginning of the Great Moderation (e.g., see Stock and Watson (2002)). Second, as documented by Jermann and Quadrini (2009) and Jermann and Quadrini (2012), the volatility of financial flows (e.g., payouts to equity and debt) increased after 1984. As reported in Table 1, the standard deviation of $\ln \theta_t$ between 1964-I and 1983-IV was 2.7%. However, between 1984-I and 2014-I, this standard deviation almost doubled to become 5.1%. In contrast, the volatility of technology shocks remained essentially the same over these two time periods. Moreover, the correlation of the financial shock with output and hours both increased after 1984. At the same time, the correlation of the technology shock with output and hours both decreased after 1984. In fact, after 1984, the technology shock has exhibited a small negative correlation with hours.

Moreover, the financial shock is correlated with other quantitative indicators of financial distress. For example, the left-panel of Figure 4 plots a measure of lending standards reported by the Federal Reserve's Senior Loan Officer Opinion Survey. This particular measure is the net percentage of domestic banks tightening lending standards for commercial and industrial (C&I) loans to large and middle-market firms.[8] Since this is a measure of the changes in lending standards, a tightening of lending standards should correlate with decreases in θ_t.

[8]Large and middle-market firms are defined as those with annual sales of $50 million or more. Small firms are the rest. The term "net percentage" means the fraction of banks that have tightened standards minus the fraction of banks that have eased standards.

Therefore, the left-panel of Figure 4 compares the Federal Reserve's measure of lending standards to $-\varepsilon_{\theta,t}$, which is the negative of innovations to financial shocks. This shows that $-\varepsilon_{\theta,t}$ tracks reasonably well with the Federal Reserve's measure of lending standards. In fact, the contemporaneous correlation between the two measures is 0.59. Meanwhile, the contemporaneous correlation between $\varepsilon_{z,t}$ and the Federal Reserve's measure of lending standards is only 0.08.

[Figure 4 about here.]

Furthermore, the financial shock is also correlated with credit spreads. The right-panel of Figure 4 plots $-\varepsilon_{\theta,t}$ against the Baa-Aaa credit spread. The Baa-Aaa credit spread is the difference between the Baa and Aaa interest rates reported in the Federal Reserve's H.15 release. It turns out that the innovations $\varepsilon_{\theta,t}$ are negatively correlated with credit spreads. The contemporaneous correlation between $\varepsilon_{\theta,t}$ and the Baa-Aaa credit spread is -0.42. Meanwhile, the contemporaneous correlation between $\varepsilon_{z,t}$ and the credit spread was only -0.03. Therefore, this seems to confirm that the estimated financial shock (and not the technology shock) reflects financial market conditions. The shock θ_t tends to decrease both when lending standards tighten and when credit spreads increase.

6 Calibration

The model can now be calibrated. Table 2 lists the calibrated parameters.

[Table 2 about here.]

6.1 Productivity Process Parameters

After using the procedure outline in Section 5, the sequence of aggregate shocks $\{z_t, \theta_t\}$ was fit to the VAR(1) process in Equation 2. This yielded estimates $\rho_{z,z'} = 0.883$, $\rho_{\theta,z'} = -0.022$, $\rho_{z,\theta'} = 0.223$ and $\rho_{\theta,\theta'} = 1.003$. Moreover, $\sigma_{\varepsilon z} = 0.0051$ and $\sigma_{\varepsilon\theta} = 0.0092$, while $\rho_{\varepsilon_z,\varepsilon_\theta} = -0.492$.

Furthermore, the persistence of the idiosyncratic productivity shock was set to $\rho_s = 0.850$ and the standard deviation of the innovation was set to $\sigma_{\varepsilon s} = 0.103$. These parameters were chosen to be consistent with values assumed in the literature. At an annual frequency, Khan and Thomas (2013) assume a persistence of 0.659 and an innovation standard deviation of 0.118. Meanwhile, Clementi and Palazzo (2014) assume a persistence of 0.55 and an innovation standard deviation of 0.22, at an annual frequency. Meanwhile, the values used in this paper are consistent with $\rho_s = 0.653$ and $\sigma_{\varepsilon s} = 0.135$ at an annual frequency.[9] Therefore, the values of $(\rho_s, \sigma_{\varepsilon s})$ are similar to the values assumed by Khan and Thomas (2013).

6.2 Parameters Directly Related to Entry and Exit

I now consider the parameters which directly influence the behavior of entry and exit. First, consider the mass of potential entrants born each period, $\bar{M}_e$, which matters for the number of producers which operate in the steady state. If $\bar{M}_e$ is doubled, the equilibrium number of producers which operate in the steady state is doubled as well. Given that $\bar{M}_e$ merely changes the scale of the economy, $\bar{M}_e$ normalized to 1. Next, consider the fixed operating cost, c_f. This is set to target an average entry or exit rate of 3.2%. This is the average quarterly establishment entry rate between 1993-II and 2007-III from the BLS's Business Employment Dynamics (BED) survey. As seen in Section 4.1, the target exit rate is critical for the sensitivity of exit rates to aggregate shocks.

While the level of m_x is critical for the sensitivity of exit rates to aggregate shocks, the level of the entry probability $\bar{m}_e$ is critical for the sensitivity of entry rates. This was seen in Section 4.1. However, either c_e or μ_g could be used to target the entry probability $\bar{m}_e$. Increasing c_e would tend to increase the entry threshold $\bar{s}_e(\mathbf{x})$. Given the distribution of potential entrants, $g(s_e)$, a higher entry threshold would tend to reduce $\bar{m}_e$. The alternative to this is to reduce μ_g below zero. This would shift the potential entrant distribution $g(s_e)$ to the left. Then, holding the entry threshold constant, this would tend to cause $\bar{m}_e$ to fall. The latter approach does not require $\bar{s}_e$ to be above $\underline{s}$ to calibrate to a low $\bar{m}_e$. In contrast, calibrating c_e, by pushing $\bar{s}_e$ above $\underline{s}$, would tend to push up the relative size of entrants. For low $\bar{m}_e$, entrants would need to be larger than incumbent producers, which is not true in the data. Therefore, in the benchmark calibration, I set $c_e = 0$, and calibrated μ_g to target $\bar{m}_e$. As for $\bar{m}_e$, there is little guidance in the data for what $\bar{m}_e$ should be. Consequently, I set $\bar{m}_e$ to guarantee that the volatility of entry rates in the model was roughly consistent with the data. This required that I set $\bar{m}_e$ to a low number. Therefore, in the benchmark economy, I set $\bar{m}_e = 10\%$.

[9] Some caution is required when converting between annual and quarterly frequencies. Given the parameters for a productivity process at a quarterly frequency, (ρ_q, σ_q), the corresponding annual parameters (ρ_a, σ_a) were set so that one would obtain (ρ_a, σ_a) if the quarterly AR(1) process were estimated at an annual frequency.

6.3 Remaining Parameters

The Frisch elasticity of labor supply, ν, was set to 1.5. According to Keane and Rogerson (2012), values commonly assumed in the literature range between 1 and 2. As for returns to scale in the model, I assumed that $\gamma = 0.6$. This implies that the labor share will be 60%. Given the estimation procedure for the aggregate technology and financial shocks, these parameters are not particularly important. The effect of different values for ν or γ will be offset by different estimates for the aggregate shocks. And finally, since the model period is assumed to be a quarter, the risk-free rate r was chosen to be 1%. The labor disutility parameter ψ was set to normalize the wage to 1 in the steady state.

7 Results

The model was solved using dynamic programming techniques. Because the distribution of incumbents in the aggregate state $\mathbf{x}$ is an infinite-dimensional object, I applied the algorithm of Krusell and Smith (1998). When applying this algorithm, one potential problem is that forecast errors may accumulate over time. Applying the suggested accuracy check in Den Haan (2010), I find that this does not occur. Appendix A goes into more specific detail on the numerical methods used and reports the results of several accuracy tests.

Starting with the steady state distribution of producers, I feed back into the model the whole history of shocks measured in Section 5. To evaluate the quantitative importance of each shock, I then consider a *technology shock only* economy and a *financial shock only* economy. In both economies, I still feed both shocks into the model. However, in the *technology shock only* economy, I solve the model assuming an individual producer's profits are $\pi(z, \bar{\theta}, s; w)$, where $\pi(z, \theta, s; w)$ is defined in Equation 3 and $\bar{\theta} = 1$ is the mean value of θ. Meanwhile, in the *financial shock only* economy, I solve the model assuming profits are $\pi(\bar{z}, \theta, s; w)$, where $\bar{z} = 1$ is the mean value of z. Therefore, in each of these experiments, the direct effect of one shock is eliminated, but the forecasting effect is retained. This is the accounting procedure outlined in Chari et al. (2007).

The model's predictions for output, hours, entry and exit rates are then compared to those observed in the data. The technology and financial shocks are measured between 1964-I and 2014-I. Meanwhile, the data for entry begin in 1993-II and the data for exit begin in 1992-III. This means that the model was simulated for 114 quarters (or 28.5 years) until it could generate results on entry and exit that could be compared to the data.

7.1 Entry and Exit Rates

Figure 5 plots the model's predictions for entry and exit rates against the data. For the data, I used establishment birth and death rates from the BED survey. Both the model-generated rates and the data are de-trended with a linear trend. The left three panels plot entry rates and the right three panels plot exit rates. The top row of Figure 5 plots the model's prediction for entry and exit rates in the *technology shock only* economy. The middle row shows the results for the *financial shock only* economy. And lastly, the bottom row shows the results of the benchmark economy in which both shocks are fed into the model.

[Figure 5 about here.]

With both shocks, the model does well accounting for magnitude and cyclicality of both entry and exit rates. During the 2007-09 recession, the two shocks together can account for the decrease in entry rates and the increase in exit rates that occurred. Moreover, during the 2001 recession, the model with both shocks can partially account for the increase in exit rates that occurred. However, Figure 5 indicates that financial shocks are a stronger explanation for the behavior of both entry and exit rates than technology shocks. In particular, during the 2007-09 recession, the *financial shock only* economy predicts a decrease in entry and an increase in exit, consistent with the data. During the same time period, however, the *technology shock only* economy counterfactually predicts an increase in entry and a decrease in exit. Similarly, around the 2001 recession, the *financial shock only* economy accurately predicts an increase exit rates while the *technology shock only* economy does not. And finally, while the *technology shock only* economy predicts an increase in entry, the *financial shock only* economy predicts a fall in entry during the 2001 recession. In the data, however, there was at best a small decrease in entry rates.

To quantify the contribution of the each shock to cyclical fluctuations in entry and exit rates, I project the model's predictions onto the data. Specifically, let $\hat{x}_{d,t}$ denote the percent deviation of x from trend in the data, where x is the variable of interest (e.g., entry or exit rates). Analogously, suppose $\hat{x}_{m,t}$ is the predicted percent deviation from trend in the model, where $m \in \{z\theta, z, \theta\}$. In other words, $\hat{x}_{z\theta,t}$ is the model's prediction in the benchmark economy with both shocks, while $\hat{x}_{z,t}$ and $\hat{x}_{\theta,t}$ correspond to the predictions of the *technology shock only* and *financial shock only* economies, respectively. Then, I perform the following simple linear regression:

$$\hat{x}_{m,t} = \beta_{m,c} + \beta_{m,d}\hat{x}_{d,t} + u_{m,t}. \tag{16}$$

Note that $\beta_{m,d} = \rho_{m,d}\sigma_m/\sigma_d$, where $\rho_{m,d}$ is the correlation of $\hat{x}_{m,t}$ and $\hat{x}_{d,t}$, σ_m is the standard deviation of $\hat{x}_{m,t}$ and σ_d is the standard deviation of $\hat{x}_{d,t}$. Intuitively, $\beta_{m,d}$ captures the relative variation of $\hat{x}_{m,t}$ that is correlated with the data. Even if σ_m/σ_d is high, $\beta_{m,d}$ will still be low if $\hat{x}_{m,t}$ is uncorrelated with $\hat{x}_{d,t}$. Similarly, even if $\hat{x}_{m,t}$ is highly correlated with $\hat{x}_{d,t}$, $\beta_{m,d}$ will be small if σ_m/σ_d is small. Consequently, $\beta_{m,d}$ can be interpreted as the fraction of $\hat{x}_{d,t}$ explained by the model.

I then estimate $\beta_{m,d}$ for entry and exit rates. The resulting numbers, which are reported in Table 3, indicate that financial shocks account for the majority of fluctuations in both entry and exit rates. Specifically, the *financial shocks only* economy accounts for 69.2% of fluctuations in entry rates. In contrast, the technology shock accounts for -4.0%, a number which is statistically insignificant. Overall, both shocks together account for 68.4% of fluctuations in entry rates. Meanwhile, financial shocks account for 51.6% of fluctuations in exit rates, while technology shocks account for -9.8%. However, the estimate of -9.8% is statistically insignificant. Together, both shocks account for 44.6% of fluctuations in exit rates. Either way, financial shocks alone account for most of the fluctuations in entry and exit predicted by the benchmark economy with both shocks.

[Table 3 about here.]

These results indicate that aggregate shocks have a large effect on entry and exit, in contrast to Samaniego (2008). As discussed in Section 4.1, the corresponding hazard rates directly influence the sensitivity of entry and exit to aggregate shocks. Therefore, the results of this paper are partly driven by a calibration in which these hazard rates are large in the steady state. Nevertheless, calibrating to a smaller hazard rate would reduce the sensitivity of entry and exit to both financial and technology shocks, leaving the relative contribution of financial shocks unchanged.

7.2 Effect of Entry and Exit on Output and Hours

Since financial shocks are a strong explanation for the cyclical behavior of entry and exit rates, they are also a strong explanation for the number of operating establishments observed in Figure 1. The important question is whether this has significant consequences for the aggregate economy. The effect of $\hat{\Gamma}_t$ on output and hours suggests that it does (e.g., see Equations 14 and 15). To see this explicitly, consider Figure 6. The left panel of Figure 6 compares the benchmark economy's prediction for aggregate output (with both shocks) to an economy in which entry and exit are exogenous. The right panel of Figure 6 does the same for aggregate hours. In the case of exogenous entry and exit, Γ_t is constant, and thus $\hat{\Gamma}_t = 0$ in Equations 14 and 15. Since the benchmark economy with both shocks matches the data for output and hours, this illustrates the effect that endogenous entry and exit has on output and hours (through Γ_t).

[Figure 6 about here.]

With exogenous entry and exit, the standard deviation of $\hat{Y}_t$ would fall from 3.8% to 2.5% and the standard deviation of $\hat{N}_t$ would fall from 4.6% to 3.8%. Moreover, the quarterly autocorrelation of $\hat{Y}_t$ would fall from 0.984 to 0.950, and the quarterly autocorrelation of $\hat{N}_t$ would fall from 0.984 to 0.978. Therefore, by directly influencing entry and exit rates, the effect of financial shocks on aggregate output and hours is amplified. Moreover, entry and exit have also made aggregate output and hours more persistent. Consider for example the response of the economy after the 2007-09 recession. With exogenous entry and exit, the recovery in both output and hours would have been much faster. This indicates that the reduction in entry and the increase in exit rates during the 2007-09 recession have contributed to the slow recovery of output and hours that followed.

The result that entry and exit amplify and propagate the effects of aggregate shocks has been seen in Clementi and Palazzo (2014). Devereux et al. (1996), Bilbiie et al. (2012), Jaimovich and Floetotto (2008) and Chatterjee and Cooper (2014) have found similar results in models with monopolistic competition. However, in Clementi and Palazzo (2014), technology shocks were the only source of aggregate uncertainty. Yet, this paper shows that financial shocks (and not technology shocks) have been the main driver of observed cyclical fluctuations in entry and exit rates since the 1990s. This result is not inconsequential, as entry and exit have further propagated the effects of financial shocks on output and hours.

7.3 Output and Hours

While financial shocks are a strong explanation for entry and exit rates, they have also directly contributed to fluctuations in output and hours. To see this, consider Figure 7, which plots the model's predictions for output and hours against the data. The top row of Figure 7 plots output and hours under the *technology shock only* economy. The bottom row shows output and hours under the *financial shock only* economy. By construction, output and hours coincide with the data in the benchmark economy with both shocks. For output, I used quarterly seasonally adjusted real GDP from the BEA's National Income and Product Accounts. For hours, I used the quarterly average of aggregate weekly hours of production and non-supervisory employees from the BLS's Current Employment Statistics (CES) survey. Output and hours in the model and in the data are both de-trended with a log-linear trend.

[Figure 7 about here.]

From Figure 7, it can be seen that the *financial shock only* economy accounts for a significant fraction of observed fluctuations in output and hours. However, the role of both financial and technology shocks has not been stable over time. In fact, before 1984, output and hours under the *technology shock only* economy have been positively correlated with the data. However, after 1984, this correlation has become zero or negative. Meanwhile, output and hours under *financial shock only* economy have been positively correlated with the data throughout the entire sample. Nevertheless, as financial shocks became more volatile after 1984, their importance for output and hours as grown as well.

To quantify precisely the overall contribution of the two shocks to cyclical fluctuations in output and hours, I estimate $\beta_{m,d}$ for output and hours (see Equation 16). Table 4 reports the resulting estimates of $\beta_{m,d}$. Over the entire sample, financial shocks account for most of the fluctuations in output and hours. Specifically, the *financial shock only* economy explains 87.9% of fluctuations in output, while the *technology shock only* economy explains 10.7%. Moreover, the *financial shock only* economy explains 111.4% of fluctuations in hours, while the technology shock explains -11.5%. Consequently, these results indicate that the technology shock actually tends to dampen fluctuations in hours caused by the financial shock.

[Table 4 about here.]

As noted earlier, however, the relative contribution of financial shocks to fluctuations in output and hours has not been stable over time. Considering only the observations between 1964-I and 1983-IV, technology shocks account for 70.3% of fluctuations in output, while financial shocks only account for 20.4%. Over the same time period, technology shocks account for 18.0% of fluctuations in hours while financial shocks account for 78.3%. In other words, before 1984, technology shocks do better explaining output while financial shocks do better explaining hours. In contrast, between 1984-I and 2014-I, financial shocks account for 104.2% of fluctuations in output, while the technology shock accounts for -3.9%. In fact, the estimate of -3.9% is statistically insignificant. Moreover, over the same time period, financial shocks account for 117.4% of fluctuations in hours, while the technology shock explains -16.9%. Therefore, after 1984, financial shocks are a better explanation for both output and hours. Moreover, where the technology shock amplified fluctuations in hours before 1984, it has dampened them after 1984.

Consequently, the result that financial shocks have been growing in importance since the 1980s is one which has only been hinted at in the literature. On the one hand, Chari et al. (2007) find that the efficiency wedge (i.e., the technology shock) does a better job than the labor wedge in accounting for the behavior of output during the Great Depression and the early 1980s recession. In contrast, the labor wedge does relatively better accounting for the behavior of hours. Consistent with these results, this paper finds that the labor wedge (i.e., financial shocks in this paper) does better than technology shocks in explaining fluctuations in hours before 1984, while technology shocks do relatively better explaining output. On the other hand, Jermann and Quadrini (2012) measure a direct financial shock from an enforcement constraint and a technology shock as the Solow residual. Consistent with this paper, they find that financial shocks (which also operate though the labor wedge in their paper) account for most of the observed fluctuations in *both* output and hours between 1984-I and 2010-II. Yet, Jermann and Quadrini (2012) chose to begin their analysis in 1984 because that was also a period during which financial flows (e.g., payouts to equity and debt) became more volatile. Nevertheless, by considering a long time period going back to 1964, this paper is able to explicitly identify the growing importance of direct financial shocks for output and hours.

8 Conclusion

This paper develops a model of firm dynamics to study how well aggregate shocks account for cyclical fluctuations in the entry and exit of establishments, as well as output and hours. To do this, I construct measures of aggregate technology and financial shocks and feed them into the model. The result is that financial shocks have been growing in importance since the 1980s. Before 1984, technology shocks are a relatively better explanation for output, while financial shocks better account for hours. However, after 1984, financial shocks have become significantly more volatile. Consequently, they have since become the better explanation for both output and hours. In fact, after 1984, the technology shock accounts for practically none of output, and has actually dampened fluctuations in hours caused by the financial shock.

However, the model indicates that financial shocks are also a much stronger explanation for the behavior of entry and exit rates than technology shocks. This has been true for at least since the early 1990s, when the data on entry and exit begin. This has further influenced the aggregate economy, as the model shows that entry and exit have propagated and amplified the effects of financial shocks. Not only have entry and exit made output and hours more volatile, they have made them more persistent as well. In fact, if it were not for the effect of financial shocks on entry and exit, the recovery of output and hours after the 2007-09 recession would have been much faster.

References

Albuquerque, Rui, and Hugo A. Hopenhayn (2004) 'Optimal lending contracts and firm dynamics.' *Review of Economic Studies* 71(2), 285–315

Arellano, Cristina, Yan Bai, and Patrick Kehoe (2012) 'Financial markets and fluctuations in volatility.' Unpublished

Benk, Szilárd, Max Gillman, and Michal Kejak (2005) 'Credit shocks in the financial deregulatory era: Not the usual suspects.' *Review of Economic Dynamics* 8(3), 668–687

Bernanke, Ben, and Mark Gertler (1989) 'Agency costs, net worth, and business fluctuations.' *American Economic Review* 79(1), 14–31

Bernanke, Ben, Mark Gertler, and Simon Gilchrist (1999) 'The financial accelerator in a quantitative business cycle framework.' In *Handbook of Macroeconomics,* ed. John B. Taylor and Michael Woodford, vol. 1, Part C (Elsevier) chapter 21, pp. 1341–1393

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz (2012) 'Endogenous entry, product variety, and business cycles.' *Journal of Political Economy* 120(2), 304–345

Carlstrom, Charles T., and Timothy S. Fuerst (1997) 'Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.' *American Economic Review* 87(5), 893–910

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan (2007) 'Business cycle accounting.' *Econometrica* 75(3), 781–836

Chatterjee, Satyajit, and Russell Cooper (2014) 'Entry and exit, product variety, and the business cycle.' *Economic Inquiry.* Forthcoming

Clementi, Gian Luca, and Berardino Palazzo (2014) 'Entry, exit, firm dynamics, and aggregate fluctuations.' Unpublished

Clementi, Gian Luca, and Hugo A. Hopenhayn (2006) 'A theory of financing constraints and firm dynamics.' *Quarterly Journal of Economics* 121(1), 229–265

Collin-Dufresne, Pierre, Robert S. Goldstein, and J. Spencer Martin (2001) 'The determinants of credit spread changes.' *Journal of Finance* 56(6), 2177–2207

Cooley, Thomas F., and Vincenzo Quadrini (2001) 'Financial markets and firm dynamics.' *American Economic Review* 91(5), 1286–1310

Den Haan, Wouter J. (2010) 'Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents.' *Journal of Economic Dynamics and Control* 34(1), 79–99

Devereux, Michael B., Allen C. Head, and Beverly J. Lapham (1996) 'Aggregate fluctuations with increasing returns to specialization and scale.' *Journal of Economic Dynamics and Control* 20(4), 627–656

Driessen, Joost (2005) 'Is default event risk priced in corporate bonds?' *Review of Financial Studies* 18(1), 165–195

Elton, Edwin J., Martin J. Gruber, Deepak Agrawal, and Christopher Mann (2001) 'Explaining the rate spread on corporate bonds.' *Journal of Finance* 56(1), 247–277

Gale, Douglas, and Martin Hellwig (1985) 'Incentive-compatible debt contracts: The one-period problem.' *Review of Economic Studies* 52(4), 647–663

Gilchrist, Simon, and Egon Zakrajšek (2011) 'Monetary policy and credit supply shocks.' *IMF Economic Review* 59(2), 195–232

_ (2012) 'Credit spreads and business cycle fluctuations.' *American Economic Review* 102(4), 1692–1720

Gomes, Joao F., and Lukas Schmid (2010) 'Equilibrium credit spreads and the macroeconomy.' Unpublished

Gourio, François (2013) 'Credit risk and disaster risk.' *American Economic Journal: Macroeconomics* 5(3), 1–34

Hopenhayn, Hugo A. (1992) 'Entry, exit and firm dynamics in long run equilibrium.' *Econometrica* 60(5), 1127–1150

Houweling, Patrick, Albert Mentink, and Ton Vorst (2005) 'Comparing possible proxies of corporate bond liquidity.' *Journal of Banking and Finance* 29(6), 1331–1358

Huang, Jing-Zhi, and Ming Huang (2003) 'How much of corporate-treasury yield spread is due to credit risk?: A new calibration approach'

Jaimovich, Nir, and Max Floetotto (2008) 'Firm dynamics, markup variations, and the business cycle.' *Journal of Monetary Economics* 55(7), 1238–1252

Jermann, Urban, and Vincenzo Quadrini (2009) 'Financial innovations and macroeconomic volatility'

— (2012) 'Macroeconomic effects of financial shocks.' *American Economic Review* 102(1), 238–271

Keane, Michael, and Richard Rogerson (2012) 'Micro and macro labor supply elasticities: A reassessment of conventional wisdom.' *Journal of Economic Literature* 50(2), 464–476

Khan, Aubhik, and Julia K. Thomas (2013) 'Credit shocks and aggregate fluctuations in an economy with production heterogeneity.' *Journal of Political Economy* 121(6), 1055–1107

Kiyotaki, Nobuhiro, and John Moore (1997) 'Credit cycles.' *Journal of Political Economy* 105(2), 211–248

Krusell, Per, and Anthony A. Smith, Jr. (1998) 'Income and wealth heterogeneity in the macroeconomy.' *Journal of Political Economy* 106(5), 867–896

Lee, Yoonsoo, and Toshihiko Mukoyama (2013) 'Entry, exit, and plant-level dynamics over the business cycle.' Unpublished

Lucas, Jr., Robert E. (1978) 'On the size distribution of business firms.' *The Bell Journal of Economics* 9(2), 508–523

Rosen, Sherwin (1982) 'Authority, control, and the distribution of earnings.' *The Bell Journal of Economics* 13(2), 311–323

Samaniego, Roberto M. (2008) 'Entry, exit and business cycles in a general equilibrium model.' *Review of Economic Dynamics* 11(3), 529–541

Stock, James H., and Mark W. Watson (2002) 'Has the business cycle changed and why?' *NBER Macroeconomics Annual* 17, 159–218

Tauchen, George (1986) 'Finite state markov-chain approximations to univariate and vector autoregressions.' *Economics Letters* 20(2), 177–181

Townsend, Robert M. (1979) 'Optimal contracts and competitive markets with costly state verification.' *Journal of Economic Theory* 21(2), 265–293

A Computational Method

The producer's value functions are approximated by value function iteration. The aggregate state $\mathbf{x}$ includes μ, which is an infinite-dimensional object. Therefore, I applied the algorithm of Krusell and Smith (1998). In particular, I assumed that agents only perceived the aggregate state as $(z, \theta, \bar{\Gamma})$, where $\bar{\Gamma}$ is defined as follows:

$$\bar{\Gamma} \equiv \int_0^\infty s^{1/(1-\gamma)}\mu(s)ds + \int_0^\infty s_e^{1/(1-\gamma)}g(s_e)ds_e.$$

In other words, $\bar{\Gamma}$ is the productivity-weighted mass of incumbents and potential entrants. This slightly differs from $\Gamma(\mathbf{x})$, defined in Equation 12. When $f(z, s, n) = zsn^\gamma$, the optimal labor demand $n(z, \theta, s; w)$ is given by Equation 4. Aggregating across all producers and combining with the labor supply function, this implies that the equilibrium wage should satisfy

$$\ln w = c + \frac{1}{1 + \nu(1-\gamma)} \left[\ln z + \ln \theta + (1-\gamma)\ln \Gamma(\mathbf{x})\right] \tag{A.1}$$

where c is a constant which depends only on model parameters. Since $\Gamma(\mathbf{x})$ depends on the entry and exit cutoffs, in the numerical calculations, I approximated Equation A.1 with the following:

$$\ln w \approx \frac{1}{1 + \nu(1-\gamma)} \left[\ln z + \ln \theta + (1-\gamma)\left(\ln \bar{\Gamma} - a_\Gamma\right)\right] \tag{A.2}$$

where a_Γ is the steady state level of $\ln \bar{\Gamma}$. First, I used $\bar{\Gamma}$ instead of $\Gamma(\mathbf{x})$. Second, I calibrated ψ in the labor supply function so that the wage is normalized to 1 in the steady state. After simulating the model with an artificial sequence of shocks, $\{z_t, \theta_t\}_{t=1}^T$, for $T = 20,500$ and dropping the first 500 observations, the maximum error (in absolute value) from this approximation was 0.16%.

Furthermore, since agents need to know tomorrow's $\bar{\Gamma}$ to determine wages in the future, I suppose that $\bar{\Gamma}$ obeys the following law of motion:

$$\ln \bar{\Gamma}' = \beta_0 + \beta_1 \ln \bar{\Gamma} + \beta_2 \ln z + \beta_3 \ln \theta \tag{A.3}$$

Then, to solve the model, the following algorithm is used.

1. Construct an artificial sequence of technology shocks, $\{z_t\} = (z_1, z_2, \ldots, z_T)$ and an artificial sequence of financial shocks, $\{\theta_t\} = (\theta_1, \theta_2, \ldots, \theta_T)$, for $T = 10,500$.

2. Start with an initial guess for $(\beta_0, \beta_1, \beta_2, \beta_3)$.

3. Approximate the value function $V(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$ using the algorithm from Appendix A.1.

4. Approximate the steady state distribution of incumbent producers, $\mu(\ln s)$, using the algorithm in Appendix A.2.

5. Starting with the steady state distribution of producers, simulate the economy under $\{z_t\}$ and $\{\theta_t\}$ using the algorithm in Appendix A.3.

6. Using the simulation results, estimate $(\beta_0', \beta_1', \beta_2', \beta_3')$ via the following OLS regression:

$$\ln \bar{\Gamma}_{t+1} = \beta_0' + \beta_1' \ln \bar{\Gamma}_t + \beta_2' \ln z_t + \beta_3' \ln \theta_t + u_t.$$

   Drop the first 500 observations. If $(\beta_0', \beta_1', \beta_2', \beta_3')$ is sufficiently close to the initial guess, $(\beta_0, \beta_1, \beta_2, \beta_3)$, stop. Otherwise, return to step 2. To ensure convergence, use as the new guess a convex combination of $(\beta_0', \beta_1', \beta_2', \beta_3')$ and $(\beta_0, \beta_1, \beta_2, \beta_3)$.

7. With the converged values for $(\beta_0, \beta_1, \beta_2, \beta_3)$, simulate the economy with the sequence of measured aggregate productivity and financial shocks.

A.1 Approximation of Value Function

The producer's value function is approximated by value function iteration. In this case, the producer's value function has four state variables, current-period productivity $\ln s$ and the aggregate shocks $(\ln z,\ \ln \theta,\ \ln \bar{\Gamma})$. This value of operating is then jointly solved by iterating on the producer's Bellman equation. The details of the algorithm can then be described as follows.

1. Define the grid points for the state variables $(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$.

2. Starting with the initial guess for the value function, $V_n(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$, calculate the set of liquidation cutoffs, $\underline{s}(\ln z, \ln \theta, \ln \bar{\Gamma})$, and entry cutoffs, $\bar{s}_e(\ln z, \ln \theta, \ln \bar{\Gamma})$. To interpolate the value function in between grid points, the log of V_n is interpolated with quadrilinear interpolation.

3. For each $(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$ on the grid, given the optimal choice of n, approximate the producer's value, $V_{n+1}(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$. The key difficulty is the need to calculate the continuation values in the value of operating and the value of waiting. For accuracy reasons, the processes for z, θ and s are *not* approximated with a discrete Markov process as in Tauchen (1986). Thus, this integral is numerically approximated using an adaptive integration routine from the GNU Scientific Library.

4. Keep iterating until $V_n(\ln z, \ln \theta, \ln \bar{\Gamma}, \ln s)$ converges.

A.2 Approximation of Steady State Distribution

Let $\mu(\ln s)$ be the density representing the distribution of incumbent producers over productivity at the beginning of the period. Given the steady state entry cutoff, $\bar{s}_e$, and the steady state exit cutoff, $\underline{s}$, the steady state μ is calculated by iterating on the transition rule for the distribution of incumbent producers. The details of the algorithm can then be described as follows.

1. Define a grid for the idiosyncratic productivity $\ln s$ and the signal $\ln s_e$.

2. Given the value function $V(\ln z, \ln\theta, \ln\bar{\Gamma}, \ln s)$, calculate the entry cutoff $\bar{s}_e$ and exit cutoff $\underline{s}$ for $\ln z = 0$, $\ln\theta = 0$ and $\ln\bar{\Gamma} = a_\Gamma$. Given the law of motion for $\bar{\Gamma}$ in Equation A.3, a_Γ is computed as $a_\Gamma = \beta_0/(1-\beta_1)$. This yields the steady state entry and exit productivity cutoffs.

3. Start with some initial guess for the distribution of incumbent producers, $\mu_0(\ln s)$.

4. Given $\mu_n(\ln s)$, compute $\mu_{n+1}(\ln s')$ by iterating on transition rule defined in Equation 7. These integrals are numerically approximated using an adaptive quadrature routine from the GSL Scientific Library. When calculating this integral, the prior distribution, $\mu_n(\ln s)$, is interpolated using linear interpolation.

5. Keep iterating until $\mu_n(\ln s)$ converges.

A.3 Simulation

To simulate the model by feeding in a history of aggregate shocks, $\{z_t, \theta_t\}$, the following approach was used.

1. Take as a given the producer's value function $V(\ln z, \ln\theta, \ln\bar{\Gamma}, \ln s)$, which implicitly depends on $(\beta_0, \beta_1, \beta_2, \beta_3)$.

2. Initialize the distribution $\mu_0(\ln s)$, to its steady state value.

3. For each t, given the initial distribution $\mu_t(\ln s)$ at the beginning of the period, do the following:

   (a) Solve for $\underline{s}(\ln z_t, \ln\theta_t, \ln\bar{\Gamma}_t)$ and $\bar{s}_e(\ln z_t, \ln\theta_t, \ln\bar{\Gamma}_t)$.

   (b) Calculate the mass of entrants ($M_{e,t}$), the mass of exiting producers ($M_{x,t}$), the mass of incumbent producers (M_t), entry rates ($m_{e,t}$) and exit rates ($m_{x,t}$), as follows:

$$
\begin{aligned}
M_{e,t} &= \bar{M}_e \left[1 - \Phi\left(\frac{\ln \bar{s}_e(\ln z_t, \ln \theta_t, \ln \bar{\Gamma}_t) - \mu_g}{\sigma_g}\right)\right] \\
M_{x,t} &= \int_{-\infty}^{\ln \underline{s}(\ln z_t, \ln \theta_t, \ln \bar{\Gamma}_t)} \mu_t(\ln s) d \ln s \\
M_t &= \int_{-\infty}^{\infty} \mu_t(\ln s) d \ln s \\
m_{e,t} &= M_{e,t}/M_t \\
m_{x,t} &= M_{x,t}/M_t.
\end{aligned}
$$

Note that $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

(c) Using Equation 7, calculate the distribution, $\mu_{t+1}(\ln s)$, for next period.

A.4 Accuracy of Forecasting Rule

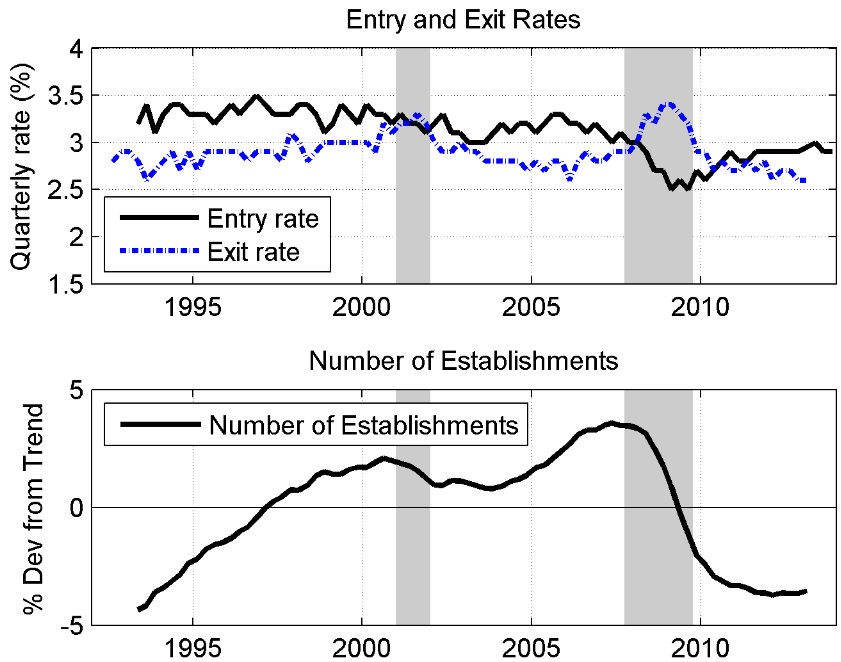
As noted before, the algorithm of Krusell and Smith (1998) to solve the model. In the benchmark economy with both shocks, the parameters of the forecasting rule in Equation A.3 were estimated to be $(\beta_0, \beta_1, \beta_2, \beta_3) = (0.12532, 0.90925, 0.24896, 0.07828)$. Two standard measures used to assess the accuracy of the forecasting rule are the R^2 and root mean square error of the regression equation. The R^2 turned out to be 0.99992 and the root mean square error was 2.4176×10^{-4}.

However, as demonstrated by Den Haan (2010), these tests are flawed. Therefore, following Den Haan (2010), I assess the accuracy of the forecasting rule by calculating the maximum error between the actual $\bar{\Gamma}$ and the forecasted $\bar{\Gamma}$ generated by the rule without updating. More specifically, I first construct an artificial sequence of technology and financial shocks, $\{z_t, \theta_t\}_{t=1}^T$ for $T = 20,500$. Next, I simulate the model under $\{z_t, \theta_t\}_{t=1}^T$. After doing this, I obtain a sequence of realized moments $\{\bar{\Gamma}_t\}_{t=1}^T$. Next, I drop the first 500 observations. Using only the forecasting rule and the initial realized value, $\bar{\Gamma}_{501}$, I generate a forecast for $\bar{\Gamma}_t$ for all $t > 501$. I denote this forecast by $\hat{\bar{\Gamma}}_t$. I then compute the simulated percentage error as $\hat{e}_t = \ln \hat{\bar{\Gamma}}_t - \ln \bar{\Gamma}_t$ for all $t > 501$.

[Figure 8 about here.]

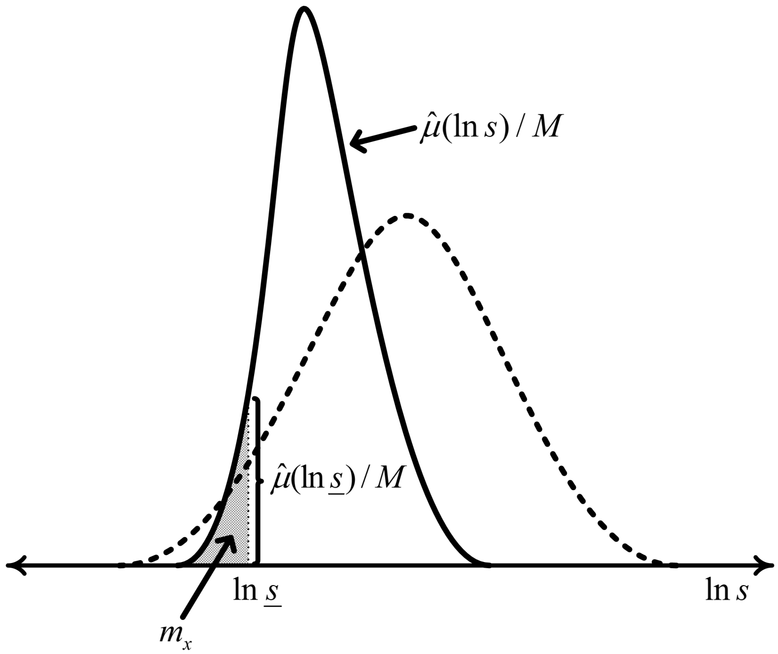
The top panel of Figure 8 plots a histogram of the simulated forecast error, $\hat{e}_t$. The mean error was -0.0089%, indicating that the forecasting rule was essentially unbiased. The maximum error (in absolute value) was 0.51% and only 0.015% of observations had a forecast error above 0.5% in absolute value. Moreover, the bottom panel of Figure 8 plots $\hat{e}_t$ against t, showing that the forecast errors do not accumulate.

Figure 1: Historical Establishment Entry and Exit Rates



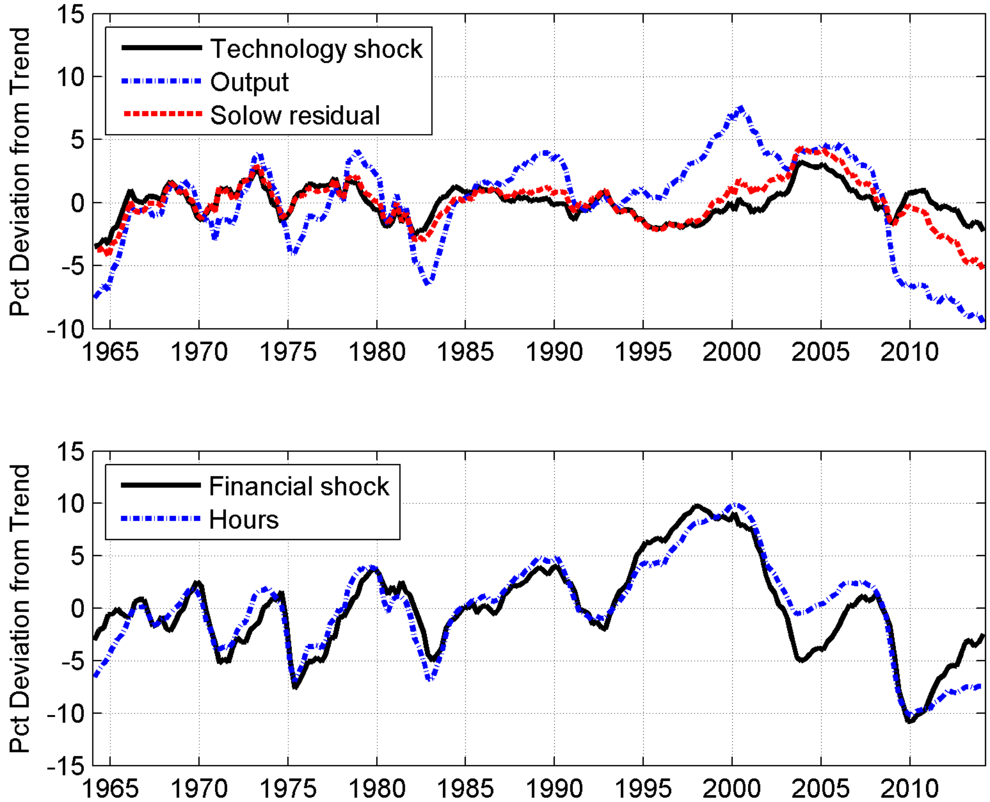
Note: The top panel plots establishment entry and exit rates. The establishment entry rate measures the number of new entrants as a percentage of the average of the previous and current total number of establishments. Exit rates are defined analogously. Both series are seasonally adjusted. The bottom panel plots the number of establishments over time, de-trended by a log-linear trend. NBER recession dates are highlighted.
Source: Business Employment Dynamics from the Bureau of Labor Statistics

Figure 2: Exit Hazard Rate with Less Productivity Dispersion



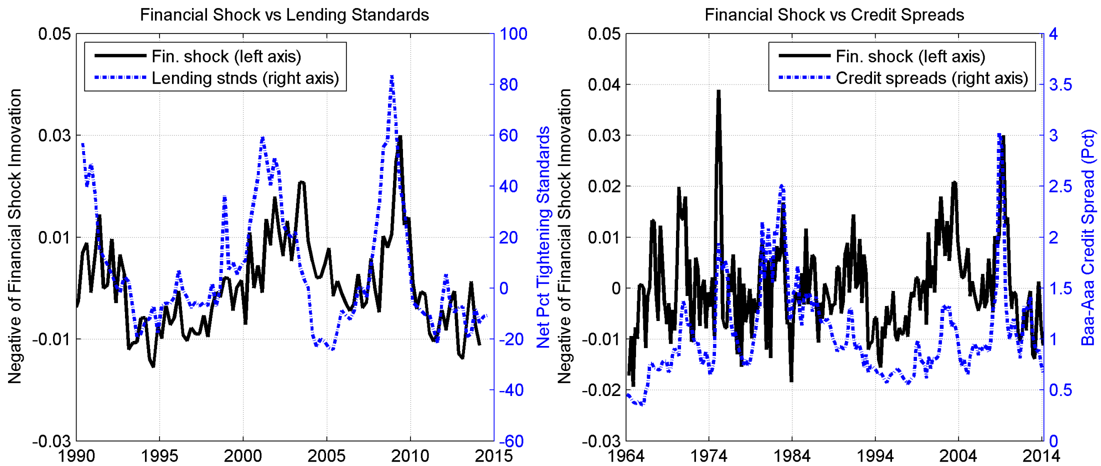
Note: This depicts how the exit hazard rate changes when the productivity dispersion of incumbent entrepreneurs decreases. The exit rate, m_x, does not change because it is calibrated to target a specific value. Thus, the height of the probability density function at the exit cutoff, $\ln \underline{s}$, must increase and the hazard rate will be higher.

Figure 3: Measured Aggregate Shocks



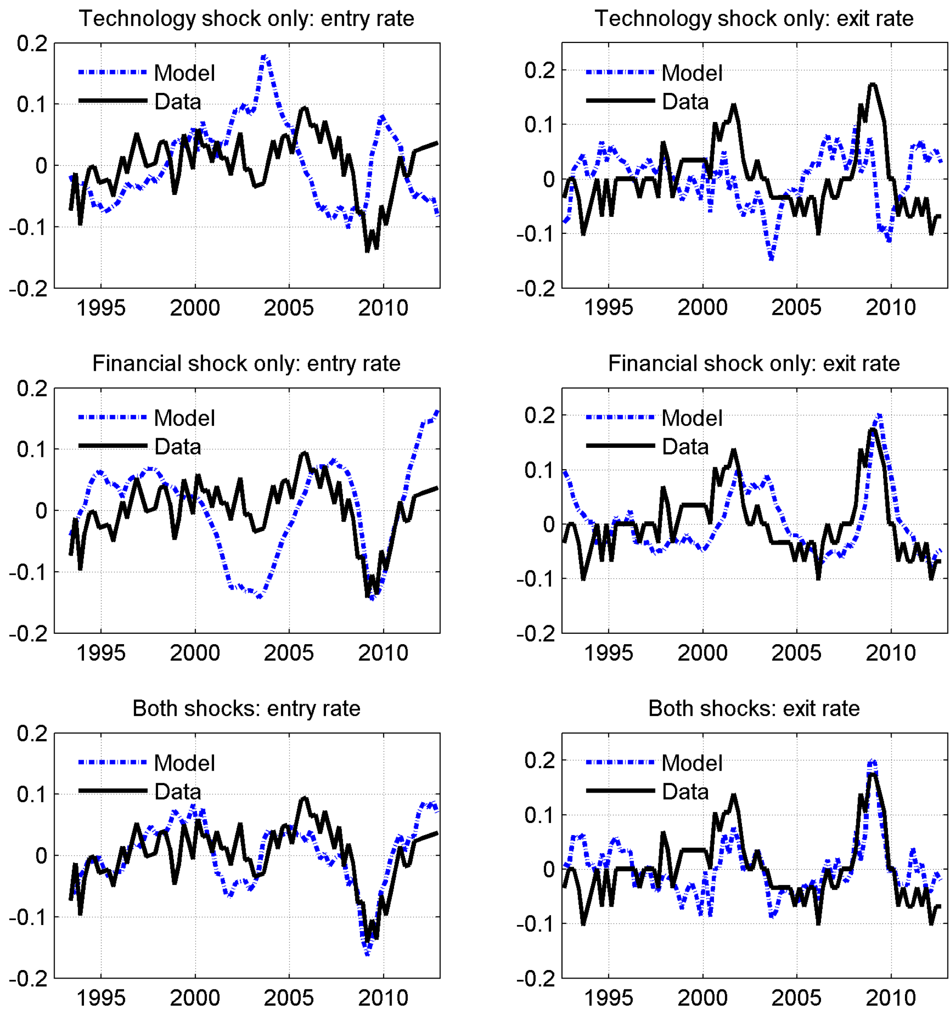
Note: The top panel plots the measured technology shock, output and the Solow residual. The bottom panel plots the measured financial shock against hours. Output (Y) is quarterly seasonally adjusted real GDP from the BEA's National Income and Product Accounts. Hours (N) is obtained as the quarterly average of aggregate weekly hours of production and non-supervisory employees from the BLS's Current Employment Statistics survey. The Solow residual is calculated as $SR = Y/N^{\gamma}$, where $\gamma = 0.6$ is the labor share, Y is output and N is hours. Each series is de-trended with a log-linear trend.

Figure 4: Financial Shocks vs Lending Standards and Credit Spreads



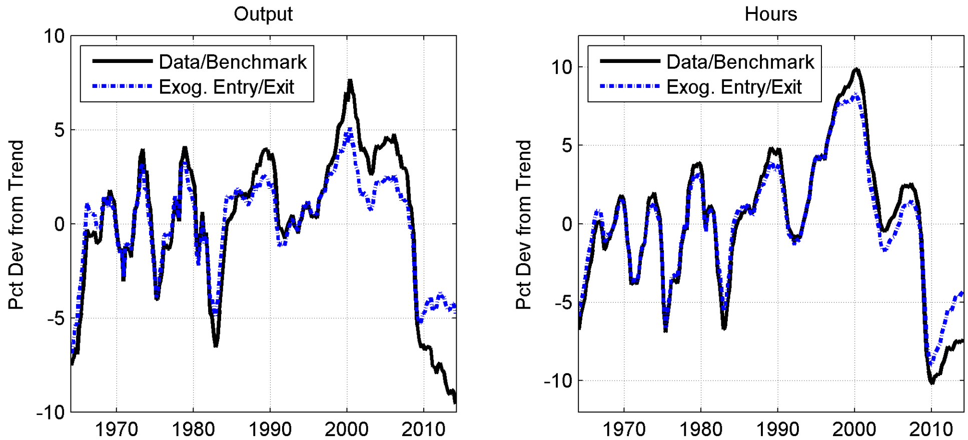
Note: The left panel compares the negative of innovations to financial shocks ($-\varepsilon_\theta$) to lending standards from the Federal Reserve's Senior Loan Office Opinion survey. The measure of lending standards is the net percentage of domestic banks tightening lending standards for commercial and industrial (C&I) loans to large and middle-market firms. The right panel compares $-\varepsilon_\theta$ to Baa-Aaa credit spreads. The Baa-Aaa credit spread is the difference between interest rates on Baa and Aaa corporate bonds, as reported in the Federal Reserve's H.15 release.

Figure 5: Benchmark Results for Entry and Exit Rates



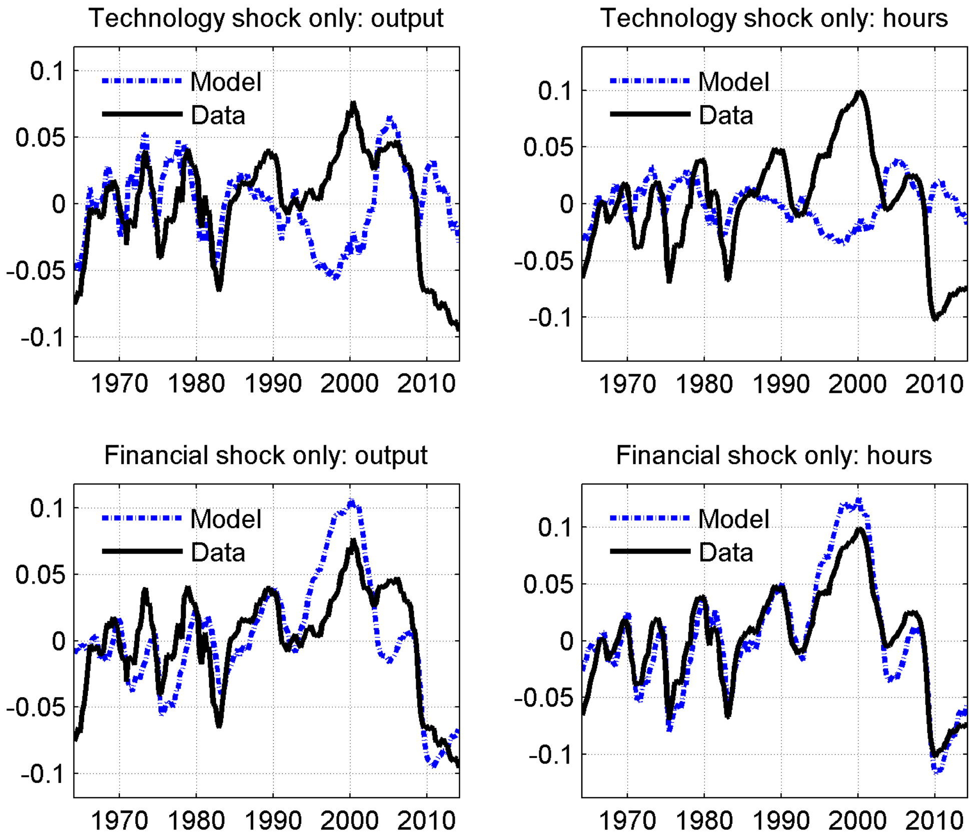
Note: Both the model-generated series and the data are de-trended with a linear trend. Data for entry and exit rates are obtained from the BLS's Business Employment Dynamics survey.

Figure 6: Effect of Entry and Exit on Output and Hours



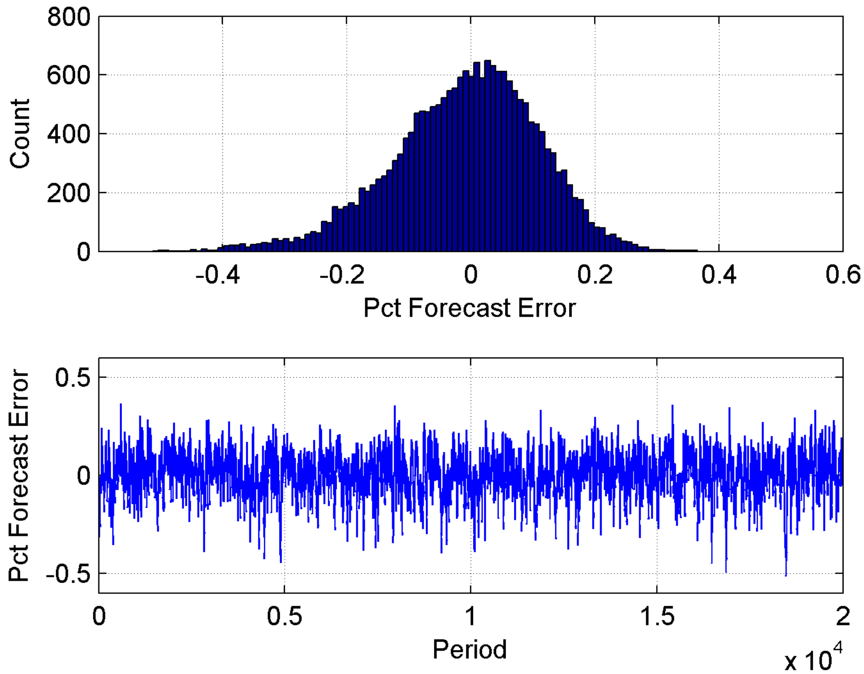
Note: The left panel plots the model's prediction for output in the benchmark economy with both shocks. Also plotted is the model's prediction when entry and exit are exogenous. The right panel does the same for hours. Output and hours with exogenous entry and exit are obtained from Equations 14 and 15 by setting $\hat{\Gamma}_t = 0$.

Figure 7: Benchmark Results for Output and Hours



Note: Output and hours are both de-trended with a log-linear trend. Note that in the benchmark economy with both shocks, both output and hours coincide with the data by construction. The data on output is obtained as quarterly seasonally adjusted real GDP from the BEA's National Income and Product Accounts. Hours is obtained as the quarterly average of aggregate weekly hours of production and non-supervisory employees from the BLS's Current Employment Statistics survey.

Figure 8: Accuracy of Forecasting Rule



Note: This figure plots the results of the accuracy check suggested by Den Haan (2010). Forecasts for the future aggregate state are generated by the rule without updating. The top panel plots the histogram of the resulting forecast errors, and the bottom panel plots these forecast errors over time.

Table 1: Summary Statistics for Measured Shocks

	Technology Shock	Financial Shock
1964-I to 2014-I		
Standard Deviation	0.013	0.044
Correlation with Output	0.410	0.644
Correlation with Hours	-0.012	0.917
1964-I to 1983-IV		
Standard Deviation	0.014	0.027
Correlation with Output	0.706	0.438
Correlation with Hours	0.288	0.825
1984-I to 2014-I		
Standard Deviation	0.013	0.051
Correlation with Output	0.299	0.671
Correlation with Hours	-0.134	0.929

Note: For $x_t = z_t$ and $x_t = \theta_t$, three statistics are reported: (1) the standard deviation of $\ln x_t$, (2) the correlation of $\ln x_t$ with $\hat{Y}_t^d$ and (3) the correlation of $\ln x_t$ with $\hat{N}_t^d$. $\hat{Y}_t^d$ and $\hat{N}_t^d$ are the cyclical components of output and hours, respectively, in the data. Output and hours are both de-trended with a log-linear trend.

Table 2: Parameter Values

Parameter	Value	Notes
		Process for Aggregate Shocks
a_z	0.000	Normalize to zero
a_θ	0.000	Normalize to zero
$\rho_{z,z'}$	0.883	Match output and hours
$\rho_{\theta,z'}$	-0.022	Match output and hours
$\rho_{z,\theta'}$	0.223	Match output and hours
$\rho_{\theta,\theta'}$	1.003	Match output and hours
$\sigma_{\varepsilon z}$	0.0051	Match output and hours
$\sigma_{\varepsilon\theta}$	0.0092	Match output and hours
$\rho_{\varepsilon_z,\varepsilon_\theta}$	-0.492	Match output and hours
		Process for Idiosyncratic Productivity Shock
ρ_s	0.850	Consistent with literature
$\sigma_{\varepsilon s}$	0.103	Consistent with literature
		Signal Distribution for Potential Entrants
μ_g	-0.534	Target probability potential entrant enters = 10%
σ_g	0.196	Set to $\sigma_{\varepsilon s}/\sqrt{1-\rho_s^2}$
		Production Function Parameter
γ	0.600	Set labor share to 60%
		Labor Supply Parameters
ψ	0.980	Normalize wage to 1 in steady state
ν	1.500	Consistent with literature
		Risk-Free Rate
r	0.010	Set risk-free rate to 4% annually
		Entry and Exit Parameters
$\bar{M}_e$	1.000	Normalize to 1
c_f	0.203	Target entry/exit rate = 3.2%
c_e	0.000	Set to zero

Table 3: Benchmark Results for Entry and Exit Rates

	$\rho_{m,d}$	σ_m/σ_d	$\beta_{m,d}$	p-val	R^2
Entry Rate					
Technology shock only	-0.029	1.378	-0.040	0.797	0.001
Financial shock only	0.422	1.640	0.692	0.000	0.178
Both shocks	0.630	1.087	0.684	0.000	0.397
Exit Rate					
Technology shock only	-0.119	0.820	-0.098	0.288	0.014
Financial shock only	0.538	0.959	0.516	0.000	0.290
Both shocks	0.539	0.828	0.446	0.000	0.291

Note: Let $\hat{x}_{m,t}$ denote cyclical fluctuations in the model-generated series (either entry or exit rates) and let $\hat{x}_{d,t}$ denote the corresponding cyclical fluctuations in the data. $\rho_{m,d}$ is the correlation of $\hat{x}_{m,t}$ with $\hat{x}_{d,t}$. σ_m is the standard deviation of the $\hat{x}_{m,t}$, while σ_d is the standard deviation of $\hat{x}_{d,t}$. $\beta_{m,d} = \rho_{m,d}\sigma_m/\sigma_d$ is the coefficient obtained by regressing $\hat{x}_{m,t}$ against $\hat{x}_{d,t}$. Also reported is the R^2 of this regression and the p-value for the null hypothesis that $\beta_{m,d} = 0$. See Equation 16. All time series are de-trended with a linear trend.

Table 4: Benchmark Results for Output and Hours

	$\rho_{m,d}$	σ_m/σ_d	$\beta_{m,d}$	p-val	R^2
Output, 1964-I to 2014-I					
Technology shock only	0.142	0.752	0.107	0.044	0.020
Financial shock only	0.749	1.174	0.879	0.000	0.561
Both shocks	1.000	1.001	1.001	0.000	1.000
Output, 1964-I to 1983-IV					
Technology shock only	0.733	0.959	0.703	0.000	0.538
Financial shock only	0.283	0.722	0.204	0.011	0.080
Both shocks	1.000	1.002	1.002	0.000	1.000
Output, 1984-I to 2014-I					
Technology shock only	-0.055	0.703	-0.039	0.548	0.003
Financial shock only	0.825	1.263	1.042	0.000	0.681
Both shocks	1.000	1.001	1.001	0.000	1.000
Hours, 1964-I to 2014-I					
Technology shock only	-0.291	0.393	-0.115	0.000	0.085
Financial shock only	0.942	1.182	1.114	0.000	0.888
Both shocks	1.000	0.999	0.999	0.000	1.000
Hours, 1964-I to 1983-IV					
Technology shock only	0.304	0.592	0.180	0.006	0.092
Financial shock only	0.791	0.990	0.783	0.000	0.625
Both shocks	1.000	0.998	0.998	0.000	1.000
Hours, 1984-I to 2014-I					
Technology shock only	-0.473	0.358	-0.169	0.000	0.224
Financial shock only	0.964	1.217	1.174	0.000	0.930
Both shocks	1.000	1.000	1.000	0.000	1.000

Note: Let $\hat{x}_{m,t}$ denote cyclical fluctuations in the model-generated series (either output or hours) and let $\hat{x}_{d,t}$ denote the corresponding cyclical fluctuations in the data. $\rho_{m,d}$ is the correlation of $\hat{x}_{m,t}$ with $\hat{x}_{d,t}$. σ_m is the standard deviation of the $\hat{x}_{m,t}$, while σ_d is the standard deviation of $\hat{x}_{d,t}$. $\beta_{m,d} = \rho_{m,d}\sigma_m/\sigma_d$ is the coefficient obtained by regressing $\hat{x}_{m,t}$ against $\hat{x}_{d,t}$. Also reported is the R^2 of this regression and the p-value for the null hypothesis that $\beta_{m,d} = 0$. See Equation 16. All time series are de-trended with a log-linear trend.