The University *of* Manchester

Discussion Paper Series

Competitive Wage Cycles with Imperfect Output Market Competition

By

Leo Kaas[*] and Paul Madden[†]

[*]Department of Economics, University of Vienna, Austria
[†]Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, UK

November 2002
Number 019

THE UNIVERSITY *of* MANCHESTER

Competitive wage cycles with imperfect output market competition*

Leo Kaas† Paul Madden‡

Abstract

We consider a model of a sector in which the same set of oligopolistic firms faces a common labour supply constraint. The wage is given in the short run, adjusting competitively in the longer run. When the costs of job creation are low relative to the degree of output market power, there exists no wage that clears the labour market in the short run, and at some wages there are two equilibria, one with involuntary unemployment and one with unfilled vacancies. The competitive wage dynamics produces a cycle with persistent labour market disequilibrium and recurrent periods of involuntary unemployment.

JEL classification: D43, D5, E24
Keywords: Involuntary unemployment, Imperfect competition, Multiple equilibria

---

*Acknowledgement.** We are grateful to Harald Uhlig for stimulating remarks. Financial support from the Economic and Social Research Council (UK) under grant L138251030 and from the Austrian Science Fund (FWF) is gratefully acknowledged.

†Department of Economics, University of Vienna, Hohenstaufengasse 9, 1010 Vienna, Austria, e-mail: leo.kaas@univie.ac.at

‡School of Economic Studies, Manchester University, Manchester M13 9PL, UK, e-mail: Paul.Madden@man.ac.uk

1 Introduction

The consequences for labour markets of imperfect competition in output markets have been extensively studied; relevant surveys are Dixon and Rankin (1994) and Silvestre (1993, 1995). In this paper we develop a model of labour market "disequilibrium" combined with imperfectly competitive output market equilibrium. The decisive feature of our model is that labour is sector–specific in the sense that the same set of oligopolistic firms faces a common labour supply constraint.[1] This sector–specificity of labour combined with imperfect output market competition gives rise to persistent labour market disequilibrium and competitive wage cycles.

In our model, the sector wage is given in the short run and adjusts in the longer run following competitive wage tâtonnement. At the given wage, firms create jobs (which involve small costs), and workers and jobs are matched according to a frictionless matching technology. When labour supply exceeds total jobs there is involuntary unemployment, and there are unfilled vacancies if the number of jobs exceeds labour supply. Employment of labour produces output which is sold, à la Cournot, at its market–clearing price. We show that there may be no wage at which the labour market clears in the short run and that at some wages there are two equilibria, one with involuntary unemployment and one with unfilled vacancies. If the sector wage adjusts in the longer run in a competitive fashion, falling if there is involuntary unemployment and rising when there are unfilled vacancies, then this tâtonnement process produces a persistent wage and employment cycle. Moreover, the cycles are compatible, albeit in a very stylized way, with various business cycle regularities (asymmetry, procyclical vacancies, countercyclical unemployment and markups).

A recurrent question in the existing literature is whether, with imperfect competition in the product market (e.g. monopolistic or Cournot), the resulting labour demand stays below labour supply at all positive wages. Such an outcome requires that marginal revenue becomes negative at a sufficiently low output level, or, equivalently, that output demand is sufficiently inelastic. Whilst the seminal paper of

[1]There is empirical evidence suggesting that industry–specificity contributes more to the wage profile than firm–specificity, thus providing insights about the importance of industry–specific human capital (see Neal (1995) and Parent (2000)).

Hart (1982) excludes this possibility by an assumption on the marginal revenue curve, Dehez (1985) and Silvestre (1990) show that such involuntary unemployment at all positive wages can occur in Hart's model with monopolistic (Dehez) or oligopolistic (Silvestre) firms. This literature has been refined and extended by Schultz (1992), d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1991), Kaas (1998) and Lasselle and Svizzero (2001). Thus there are a number of results which show how there may be no positive wage clearing the labour market, competitive wage adjustment forcing wages to zero.[2]

By contrast, we assume an elastic output demand function, implying that the Cournotian marginal revenue curve is positive, so that the conventional Cournotian labour demand tends to infinity as the wage tends to zero. Despite this assumption, there is no wage which clears the labour market and competitive wage adjustment produces a persistent disequilibrium cycle. The change is created by the assumption of sector–specific labour, and by allowing aggregate excess labour demand at given wages to be rationed, unlike the previous literature.[3] The symmetric stochastic job matching implies proportional rationing of such excess labour demands. If there are unfilled vacancies and a firm creates additional vacancies, it attracts labour from the other firms, but the cost of advertising vacancies stops the firms from unlimited overbidding behaviour which may otherwise accompany such manipulable rationing schemes. When the costs of advertising vacancies are not too high, it turns out that the incentives for firms to create an excess demand for labour are sufficiently strong to prevent the existence of a market–clearing wage. Suppose that, at a given wage w, Cournot competition leads to an unemployment outcome. If one firm (i say) attempted to increase its output supply a little by increasing its labour demand, it would be able to do so (the labour is available), aggregate output would increase and its price fall in the usual Cournotian fashion. But eventually the available labour supply constraint would bind, at which point the additional vacancies created by

[2]Strictly speaking, the labour market is in equilibrium at zero wage at which workers are indifferent between being employed or not.

[3]Hart (1982) and Silvestre (1990) clearly assume that firms are small in each sectoral labour market (hence labour is not sector–specific). However, Schultz (1992) and Lasselle and Svizzero (2001) have sector–specific labour, but simply assume that it is infeasible for firms to offer an excess demand for labour.

firm i would merely attract labour from the other firms, leaving aggregate employment, output and its price unchanged. This kinks upwards firm i's profit, leads to jumps in best responses, and ultimately to the non–existence of market–clearing wages and the competitive wage cycles.

The rest of the paper is organized as follows. Section 2 introduces the model, Section 3 derives the short–run equilibrium at a given wage, and Section 4 studies competitive wage cycles. Section 5 discusses robustness of the results, and Section 6 concludes. All proofs which are not included in the text are contained in the Appendix.

2 The model

We consider a sector of an economy in which $n \geq 2$ identical firms produce a homogenous output good from inputs of homogenous sector–specific labour under constant returns to scale.[4] Normalizing returns to unity, the technology of firm $i = 1, \ldots, n$ is $y_i = \ell_i$. The firms face a uniformly elastic output demand function whose inverse we denote by $P(y_1 + \ldots + y_n)$ which has constant elasticity $-\rho$ where $\rho \in (0, 1]$. On the labour market, firms face a common labour supply L, and the money wage is given in the short run. However, we allow the wage to adjust in the longer run according to a competitive wage tâtonnement process.

The usual argument for existence of a positive market–clearing wage with elastic output demand and sector–specific labour goes as follows: when there is Cournot competition between $n > 1$ firms, labour demand is a uniformly elastic function of the wage. Hence, as the wage tends to zero, labour demand tends to infinity, and vice versa, so the labour market must clear at some positive wage.[5] However, this argument is problematic since firms ignore the input supply constraint in the standard Cournot game. We assume instead that firms simultaneously decide about the number of jobs they create, anticipating that labour demand may be rationed

[4] We discuss below how our results generalize to decreasing or increasing returns to scale.

[5] In fact, in a Cournot oligopoly with $n > 1$ firms, involuntary unemployment at all positive wages can be excluded even for an inelastic demand when the demand elasticity is greater than $1/n$, whereas only a monopoly ($n = 1$) requires an elastic demand.

whenever the total number of jobs exceeds labour supply.

Specifically, firms create a number of jobs $J_i \geq 0$, $i = 1, \ldots, n$, and they pay a cost $c > 0$ for each job opening. As in the literature on labour market search (see e.g. Pissarides (2000)), these costs can be thought of as recruitment costs (capturing costs of advertising and interviewing) that are incurred with every job. After firms create jobs, workers and jobs are matched according to a frictionless and symmetric matching technology. Thus, the probability that a job is filled is the same for every firm, and it equals one whenever the number of workers exceeds the number of jobs. Hence, actual employment levels at the n firms are

$$\ell_i = J_i \min\Big(1, \frac{L}{J_1 + \ldots + J_n}\Big) \ , \ i = 1, \ldots, n \ . \tag{1}$$

With the workers matched to jobs, firms produce output to be sold at the market price $P(\ell_1 + \ldots + \ell_n)$. The firms therefore make profits

$$\pi^i(J_1, \ldots, J_n) \equiv (P(\ell_1 + \ldots + \ell_n) - w)\ell_i - cJ_i \quad , \ i = 1, \ldots, n,$$

where the employment levels ℓ_i follow from the matching technology (1). We are interested in Nash equilibria of the one–stage game in which firms create jobs simultaneously to maximize their profits at the given wage.[6] A *short–run equilibrium* is a symmetric pure–strategy Nash equilibrium $J_1 = \ldots = J_n = J$ of this game. By focusing on symmetric equilibria, it is sufficient to consider the best response problem of firm 1 only. Denote by $\pi(J_1, J) = \pi^1(J_1, J, \ldots, J)$ the profit of firm 1 provided that firms $2, \ldots, n$ offer the same number of jobs J. When the total number of jobs is less than the number of workers, there is unemployment, and firm 1's profit function is

$$\pi(J_1, J) = \pi^U(J_1, J) \equiv (P(J_1 + (n-1)J) - w - c)J_1 \ , \text{ if } J_1 + (n-1)J \leq L \ .$$

Hence, whenever there is unemployment, the firms face Cournotian profit functions with unit costs equal to $w + c$. On the other hand, when the total number of

[6] This game has an equivalent representation as a two–stage game in which firms set output prices simultaneously at the second stage, resulting in firms setting the market–clearing price in any second stage subgame. Unlike the result of Kreps and Scheinkman (1983), this result does not depend on the way consumers' demand is rationed at asymmetric prices (efficient, proportional etc.) and ensures that the Bertrand price setting produces an essentially Cournot outcome whenever demand is uniformly elastic (see Madden (1998)).

jobs exceeds the number of workers, there are excess vacancies, and firm 1's profit function is

$$\pi(J_1, J) = \pi^V(J_1, J) \equiv (P(L) - w)L\frac{J_1}{J_1 + (n-1)J} - cJ_1 \ , \ \text{if } J_1 + (n-1)J \geq L \ .$$

When there are excess vacancies, aggregate employment and output equal L, irrespective of the number of jobs at firm 1. Hence, firm 1 takes the output price $P(L)$ as given, but competes with the other firms for the number of workers which are allocated according to the proportional rationing scheme. The first derivatives of these two profit functions w.r.t. J_1 are

$$\pi_1^U(J_1, J) = P(J_1 + (n-1)J)\Big(1 - \rho\frac{J_1}{J_1 + (n-1)J}\Big) - w - c \ , \tag{2}$$

$$\pi_1^V(J_1, J) = (P(L) - w)L\frac{(n-1)J}{(J_1 + (n-1)J)^2} - c \ . \tag{3}$$

Obviously, π^V is strictly concave in J_1, and π^U is strictly concave in J_1 since demand is uniformly elastic. However, the composite profit function π needs not be concave, but has a kink at the transition from unemployment to excess vacancies. There are three types of potential short–run equilibria $J_1 = J$: a *market–clearing equilibrium* in which $J = L/n$, an *unemployment equilibrium* with $J < L/n$, and a *vacancy equilibrium* with $J > L/n$. We will now explore the conditions under which these equilibria exist.

3 The short–run equilibria

We are first interested in the existence of a market–clearing equilibrium. If w is a market–clearing wage, there must be an equilibrium $J_1 = J = L/n$. Since the profit function of firm 1 has a kink at $(J_1, J) = (L/n, L/n)$, $J_1 = L/n$ is a best response to $J = L/n$ if and only if $\pi_1^U(L/n, L/n) \geq 0 \geq \pi_1^V(L/n, L/n)$. Using (2) and (3), these conditions are equivalent to

$$P(L)\Big(1 - \frac{\rho}{n}\Big) - c \geq w \geq P(L) - \frac{n}{n-1}c \ .$$

Hence, these two conditions can be satisfied only if

$$c \geq P(L)\rho\frac{n-1}{n} \ . \tag{4}$$

If (4) is not satisfied, there exists no market–clearing equilibrium. When job creation costs are low relative to the degree of output market power (measured by the inverse of the output demand elasticity ρ), the profit function kinks upwards at the transition from unemployment to excess vacancies. Hence, when firms $i > 1$ each create $J = L/n$ jobs, firm 1 either finds profitable upward deviations to some $J_1 > L/n$ (when the wage is low) or to some $J_1 < L/n$ (when the wage is high), or to both (for intermediate wages). Consequently, no market–clearing equilibrium wage exists.[7]

Proposition 1:

(a) If (4) is satisfied (i.e. job creation costs are high), there exists a market–clearing equilibrium at any wage

$$w \in [P(L) - \frac{n}{n-1}c \ , \ P(L)(1 - \frac{\rho}{n}) - c] \ .$$

(b) If (4) is not satisfied (i.e. job creation costs are low), there exists no market–clearing equilibrium at any wage.

The non-existence result is due to the upward kink (or non–concavity) of profit functions. A further consequence of this non–concavity is a discontinuity of best response functions. Specifically, if (4) is satisfied, best response functions are continuous, and, as we will show in the following propositions, there exists a unique equilibrium at any given wage which depends continuously on the wage. On the other hand, if (4) is not satisfied, best response functions are discontinuous. As a result, there exist multiple equilibria for some wages, and the equilibrium correspondence is discontinuous.

We present next the results on the existence of unemployment equilibria. A formal proof is contained in the Appendix.

Proposition 2:

(a) If (4) is satisfied, there exists a unique unemployment equilibrium J for all

[7] It can also be shown that the existence of an asymmetric pure strategy market–clearing equilibrium requires a condition which is even stronger than (4).

$w > P(L)(1-\frac{\rho}{n})-c$ which is given by $P(nJ)(1-\frac{\rho}{n})=c+w$, and there exists no unemployment equilibrium for all other wages.

(b) If (4) is not satisfied, there exists

$$w^U \in \left(P(L)(1-\tfrac{\rho}{n})-c\ ,\ P(L)-c\frac{n}{n-1}\right)$$

such that there exists a unique unemployment equilibrium for all $w \geq w^U$, and there exists no unemployment equilibrium for all $w < w^U$.

Part (a) of this Proposition refers to the harmless case of continuous best response functions. In this case, unemployment tends to zero as the wage falls to the highest market–clearing wage $w = P(L)(1-\frac{\rho}{n})-c$. In part (b), best response functions are discontinuous. As the wage falls below the lowest unemployment wage w^U, it becomes profitable for firms to create a large number of jobs, so as to create an excess demand for labour and to attract workers from the other firms. When the firms do so, however, they suffer a decline in the output price (for output increasing to the full employment output) and they increase their total costs of job creation. But when c is low relative to the degree of market power, the gains from attracting workers are large relative to both the costs of creating excess vacancies and to the profit loss due to the fall in output price.

A similar picture emerges for the final case of vacancy equilibria. For high job creation costs, a vacancy equilibrium exists for all wages up to $P(L)-c\frac{n}{n-1}$ at which excess vacancies become zero. On the other hand, for low job creation costs (relative to the degree of market power), a vacancy equilibrium only exists up to a wage w^V above which firms prefer to create few jobs and unemployment, so as to gain from the increase in output price. The results are summarized in Proposition 3 which is again proven in the Appendix.

Proposition 3:

(a) If (4) is satisfied, there exists a unique vacancy equilibrium

$$J = \frac{(n-1)(P(L)-w)}{nc}\frac{L}{n}$$

for all $w < P(L)-c\frac{n}{n-1}$, and there exists no vacancy equilibrium for all other wages.

(b) If (4) is not satisfied, there exists

$$w^V \in \Big(P(L)(1-\tfrac{\rho}{n}) - c\ ,\ P(L) - c\frac{n}{n-1}\Big)$$

such that there exists a unique vacancy equilibrium for all $w \leq w^V$, and there exists no vacancy equilibrium for all $w > w^V$.

It is now clear from parts (a) of Propositions 1-3 that there is a unique short–run equilibrium at any wage when (4) is satisfied. If (4) is not satisfied, a short–run equilibrium would not exist for some wages if the lowest unemployment wage exceeded the highest excess vacancy wage, i.e. if $w^U > w^V$. However, existence of a short–run equilibrium can be guaranteed at all wages. As we show in the proof of Proposition 4, best response functions can have at most one discontinuity which must be an upward jump from an unemployment best response $J_1 < L-(n-1)J$ to an excess vacancy best response $J_1 > L-(n-1)J$ as J increases, and the best response is a continuous and concave function otherwise. This behaviour of best response functions leads to the existence of at least one symmetric pure strategy equilibrium at any wage (hence implying $w^U \leq w^V$). Moreover, a continuity argument implies that $w^U < w^V$.

Proposition 4: If (4) is not satisfied, $w^U < w^V$.

Propositions 1-4 are illustrated in Figure 1 and summarized in the following theorem.

Theorem:

(a) When job creation costs are high, there is a unique short–run equilibrium at any wage. The equilibrium exhibits unemployment for high wages, excess vacancies for low wages, and market clearing for intermediate wages.

(b) When job creation costs are low, there exists no market–clearing equilibrium wage. For high wages, there exists a unique unemployment equilibrium, for low wages there exists a unique equilibrium with excess vacancies, for intermediate wages there exists one equilibrium with unemployment and one with excess vacancies.

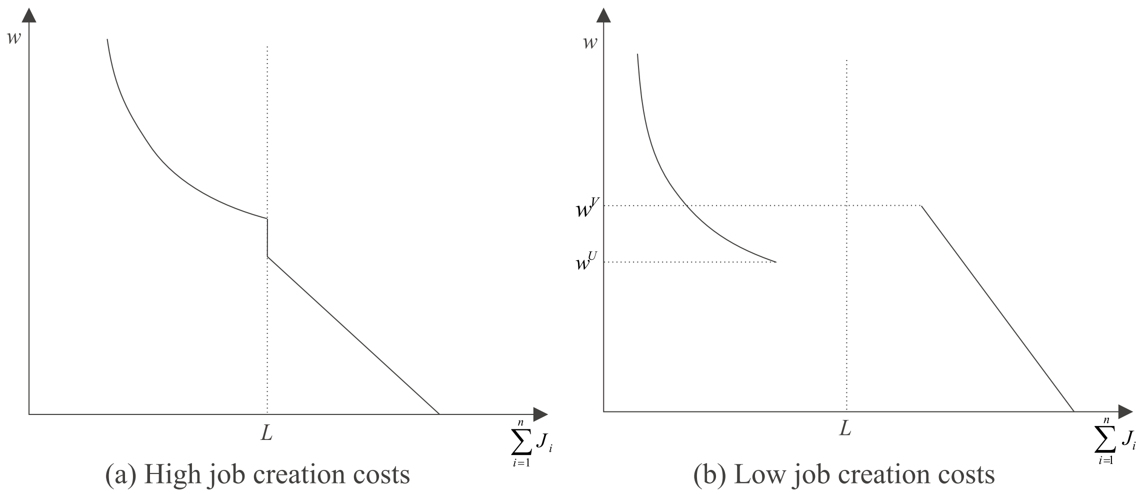

Figure 1: Short–run labour market equilibrium.

Note that, when job creation costs are zero, condition (4) is not satisfied, and Propositions 1 implies that there exists no market–clearing wage. Proposition 2 remains valid and implies that there are unemployment equilibria for all $w \geq w^U < P(L)$. However, short–run equilibria with excess vacancies fail to exist, as the proportional rationing scheme induces firms to drive job openings to infinity whenever the wage is less than $P(L)$ and whenever other firms signal an excess demand for labour. Such overbidding behaviour and failure of existence of short–run equilibria at given wages are well–known features of manipulable rationing mechanisms (see e.g. Bénassy (1982)). To guarantee existence, some non–manipulability of rationing is required. For instance, if we assume that firms have some capacity level $\bar{J} > L/n$ so that they cannot credibly attract workers beyond this level, vacancy equilibria at $J = \bar{J}$ exist for all $w \leq P(L)$ when $c = 0$.

4 Competitive wage cycles

We assumed so far that the wage is given for the period under consideration. When the labour market is in disequilibrium, however, firms and workers have strong incentives to adjust the wage. Instead of providing a rigorous microeconomic approach of wage determination (e.g. union or firm wage setting or worker–firm wage bargaining), we consider a standard competitive wage tâtonnement process: whenever there are excess vacancies, the wage goes up from one period to the next, and whenever there is unemployment, the wage goes down.[8]

Formally, if w_t is the wage in period t and if nJ_t denotes the number of jobs in period t, the wage tâtonnement process follows the dynamics

$$w_t = W(w_{t-1}, nJ_t - L) \quad , \;\; J_t \in \mathrm{SRE}(w_t) \; .$$

Here, $\mathrm{SRE}(w_t)$ denotes the set of short–run equilibria at wage w_t, and W is a wage adjustment function which is strictly increasing in both arguments and satisfies $W(w, 0) = w$ for all w. Obviously, only a market–clearing equilibrium wage can be a steady state of the wage dynamics. In order to avoid unstable overshooting, we assume that the wage tâtonnement function W does not react too sensitively to excess labour demand/supply. A sufficient condition is that $W_2(w, nJ(w) - L) < -1/(nJ'(w))$ where $J(w)$ denotes either an unemployment or a vacancy equilibrium. This condition implies that $dw_{t+1}/dw_t > -1$ so that overshooting wage cycles are excluded.

If (4) is satisfied (see Figure 1 (a)), $\mathrm{SRE}(w)$ is single–valued, and wages converge to some market–clearing equilibrium wage, provided that the above stability assumption is satisfied. If (4) is not satisfied (see Figure 1 (b)), the wage dynamics has no steady state and must follow some cyclical pattern around the interval $[w^U, w^V]$. Since there are multiple equilibria in this interval, firms are required to coordinate on one of them. It is reasonable to assume a "coordination rigidity" during wage tâtonnement in the sense that firms do not switch from an unemployment equilibrium to an excess vacancy equilibrium when the wage is above w^U. Vice versa, firms do not switch from a vacancy equilibrium to an unemployment equilibrium when the wage is below w^V. With such an assumption, the wage dynamics produces falling wages and increasing employment down the unemployment branch of Figure 1 (b), eventually flipping to the excess vacancy branch and a phase of increasing wages and falling vacancies, before flipping back to the unemployment branch, and so on.

[8] For related models with multiple labour market equilibria in which the wage is set by trade unions or by firms at a preceding stage, see Kaas and Madden (2000a) and Kaas and Madden (2000b).

Thus, in particular, involuntary unemployment is a persistent intermittent long–run feature of the competitive wage dynamics.

Though such a wage tâtonnement cycle describes labour market dynamics in a representative sector, its features are compatible with a few stylized business cycle regularities. Clearly, unemployment is countercyclical and vacancies are procyclical, so the cycles follow the shape of a Beveridge curve. Furthermore, markups are countercyclical. To see this, note that the markup factor (i.e. the ratio of output price over marginal costs) equals $\frac{n}{n-\rho} > 1$ when there is unemployment, but is one when there are excess vacancies since firms take then the output price as given.[9] Lastly, the cycle exhibits a business cycle asymmetry called steepness (see Sichel (1993)): the first period of unemployment (i.e. after the wage increases beyond w^V) is characterized by a large fall in output followed by a gradual increase in output when the wage falls. When the wage falls below w^U, output jumps back to the full employment level at which it remains during the following wage increase. Hence, a downswing is steeper than an upswing.[10]

Proposition 5:

(a) If job creation costs are high, the competitive wage tâtonnement process converges to a market–clearing equilibrium wage.

(b) If job creation costs are low, the competitive wage tâtonnement process produces a cyclical dynamics with alternating periods of involuntary unemployment and excess vacancies. The cycle exhibits procyclical vacancies, countercyclical unemployment, countercyclical markups, as well as steepness.

5 Robustness

Since condition (4) determines whether a market–clearing wage exists, it is interesting to know whether this condition is satisfied or not under a plausible param-

[9] Note, however, that firms make positive profits in spite of zero markups and a constant returns production function, since the "hiring technology" $cJ_1 = c(n-1)J\ell_1/(L-\ell_1)$ is strictly convex.

[10] When labour supply is upward–sloping (as assumed in the following section) the steepness property of the cycle is even more pronounced.

eterization of the model. Since we assumed rather specific functional forms (linear production function and inelastic labour supply), it is important to generalize condition (4) to more general fundamentals. In the following we extend Proposition 1 to a Cobb-Douglas production function with decreasing (or moderately increasing) returns to scale, and to a general upward–sloping labour supply function. We do not provide a generalization of the other propositions, though we conjecture strongly that they are robust as well.

Suppose now that each firm's production function is $y_i = \ell_i^\alpha$, $i = 1, \ldots, n$, for some $\alpha > 0$ which may not be too large to preserve concavity of the two relevant profit functions π^U and π^V. We assume a general upward–sloping labour supply function $L(w)$. If we denote by $k < 1$ the fraction of revenue spent on hiring, we can generalize Proposition 1 as follows:

Proposition 6: If a market–clearing wage exists, the fraction of revenue spent on hiring at the market–clearing wage must satisfy

$$k \geq \alpha\rho\frac{n-1}{n} \quad . \tag{5}$$

For the constant returns production function and a constant labour supply L, k equals $c/P(L)$ whenever $J = L/n$. Hence, (4) and (5) coincide. Note that (5) does not depend on the shape of the labour supply function which is due to the substitution of c by k. It is also possible to derive a generalized condition on the costs of job creation, but this requires some parametric specification of labour supply. In the proof of Proposition 6 we also provide a generalization of (4) when labour supply is iso–elastic: $L(w) = Aw^\gamma$.

We can now explore whether condition (5) is likely to be satisfied or not. Empirical studies report markup factors between 1.2 and 1.4 (see Rotemberg and Woodford (1995, pp. 260–61)). A labour share of about 0.7 suggests then that the employment elasticity of output α should be at least 0.7*1.2=0.84.[11] Rotemberg and Woodford also state that average demand elasticities are just below 2, so that 0.5 is a safe

[11] Typically α is set to 0.7 in calibrations, but with significant imperfect competition in the product market this number seems to be too low.

lower bound on the inverse demand elasticity ρ. Table 1 summarizes the lower bound on the hiring costs fraction of Proposition 6 for some values of n. Since it seems implausible that hiring costs are more than 20 percent of revenue, condition (5) is likely not to be satisfied.

	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$k \geq$	0.21	0.28	0.315	0.336

Table 1: Condition (5) for $\alpha = 0.84$ and $\rho = 0.5$.

6 Conclusions

With output market imperfect competition and given wages in the short run, the paper shows how there may be no wage which clears the labour market and a range of wages at which there are two short–run equilibria, one with involuntary unemployment and one with unfilled vacancies. If, in the longer run, wages adjust in a competitive fashion (falling when there is involuntary unemployment, rising with unfilled vacancies), a wage cycle emerges in which there are persistent and intermittent periods of involuntary unemployment, in particular. Thus competitive wage adjustment, based on a short–run "disequilibrium" model with imperfect output market competition, fails to eradicate involuntary unemployment both in the short run and the long run.

The novelties stem from the fact that in our single sector model the imperfectly competitive firms face a constraint on the labour available to them, and, in the short run, there will be firm employment rationing when there is excess demand for labour. With proportional rationing and small job creation costs, the results emerge since a firm's profit "kinks" when an increase of its available jobs creates aggregate excess demand for labour - the output price falls, à la Cournot, whilst there is still aggregate unemployment but remains constant thereafter, as aggregate employment and output remain constant. These kinks create the non–existence and multiplicity outcomes.

Appendix

Proof of Proposition 2:

From the first order condition $\pi_1^U(J,J) = 0$, a candidate unemployment equilibrium must satisfy $P(nJ)(1-\frac{\rho}{n}) = c + w$. Hence $J < L/n$ requires $w > P(L)(1-\frac{\rho}{n}) - c$. Suppose first that (4) is satisfied. This implies

$$P(L) - \frac{n}{n-1}c \leq (1-\frac{\rho}{n})P(L) - c < w \ ,$$

and thus

$$\pi_1^V(L-(n-1)J, J) = (P(L)-w)L\frac{(n-1)J}{L^2} - c < c\frac{n}{n-1}\frac{(n-1)J}{L} - c < 0 \ .$$

Hence, deviations to $J_1 \geq L-(n-1)J$ are not profitable, and J is an unemployment equilibrium.

Now suppose that (4) is not satisfied. Then at $w = (1-\frac{\rho}{n})P(L) - c$, $J = L/n$, $\pi_1^U(L/n, L/n) = 0$, and

$$\pi_1^V(L/n, L/n) = (P(L)-w)\frac{n-1}{n} - c = (\frac{\rho}{n}P(L)+c)\frac{n-1}{n} - c > 0 \ .$$

Therefore, at all wages close to $(1-\frac{\rho}{n})P(L) - c$, there is no unemployment equilibrium since there are profitable deviations to some $J_1 > L-(n-1)J$. On the other hand, consider $w = P(L) - \frac{n}{n-1}c > (1-\frac{\rho}{n})P(L) - c$ and the corresponding candidate unemployment equilibrium $J < L/n$. Then,

$$\pi_1^V(L-(n-1)J, J) = (P(L)-w)L\frac{(n-1)J}{L^2} - c = c\frac{nJ}{L} - c < 0 \ .$$

Hence there are no profitable deviations to some $J_1 \geq L-(n-1)J$, and J is an unemployment equilibrium. By continuity there exists a w^U (together with J_1, J) which satisfies

$$\pi^U(J,J) = \pi^V(J_1,J) \ , \ \pi_1^U(J,J) = \pi_1^V(J_1,J) = 0 \ .$$

To show that w^U is unique, denote by $B^U(J)$ (and $B^V(J)$) the best response functions under the hypothesis that the payoff function is π^U (π^V, resp.). Hence, these functions satisfy $\pi_1^U(B^U(J),J) = 0$ and $\pi_1^V(B^V(J),J) = 0$ for all J and for all w

(the wage is an implicit argument of the functions π^X and B^X, $X = U, V$). Define furthermore

$$F(J, w) \equiv \pi^U(B^U(J), J) - \pi^V(B^V(J), J) \ , \tag{6}$$

and let $J^U(w)$ be the candidate unemployment equilibrium at w (i.e. the unique J satisfying $B^U(J) = J$). Then, w^U is a solution of the equation $F(J^U(w), w) = 0$, and w^U is unique if we can show that

$$\frac{d}{dw} F(J^U(w), w) > 0 \quad \text{if} \quad F(J^U(w), w) = 0 \ . \tag{7}$$

If this condition holds, then $F(J^U(w), w) > 0$ for all $w > w^U$ and for all such w, $J^U(w)$ is an unemployment equilibrium, whereas for all $w < w^U$ there exists no unemployment equilibrium. Condition (7) is satisfied since $J^{U\prime}(w) < 0$ and since $F_1 < 0$ and $F_2 > 0$ for all relevant arguments as shown in the Lemma below. This completes the proof. $\square$

Lemma: Let $F(J, w)$ be defined as in (6), with w an implicit argument of π^X and B^X, $X = U, V$. Then, at any (J, w) such that $F(J, w) = 0$ and $B^U(J) < L - (n-1)J < B^V(J)$, we have

$$F_1(J, w) < 0 \quad \text{and} \quad F_2(J, w) > 0 \ .$$

Proof: Since the first order conditions $\pi_1^X = 0$, $X = U, V$, are satisfied, $F_1(J, w) < 0$ iff $\pi_2^U(B^U(J), J) < \pi_2^V(B^V(J), J)$. Using again the first order conditions $\pi_1^X = 0$, $X = U, V$, we have

$$\begin{aligned} \pi_2^U(B^U(J), J) &= (n-1)P'(B^U(J) + (n-1)J)B^U(J) \\ &= -(n-1)(P - w - c) = -(n-1)\frac{\pi^U(B^U(J), J)}{B^U(J)} \end{aligned}$$

and

$$\pi_2^V(B^V(J), J) = -(n-1)(P(L) - w)L\frac{B^V(J)}{(B^V(J) + (n-1)J)^2} = -(n-1)\frac{\pi^V(B^V(J), J)}{B^V(J)} \ .$$

Since we assume that $B^U(J) < L - (n-1)J < B^V(J)$ and $\pi^U(B^U(J), J) = \pi^V(B^V(J), J)$, we obtain $\pi_2^U(B^U(J), J) < \pi_2^V(B^V(J), J)$, and hence $F_1(J, w) < 0$.

Secondly, $F_2(J,w) > 0$ iff $\pi_w^U(B^U(J),J) > \pi_w^V(B^V(J),J)$ (again using $\pi_1^X = 0$, $X = U,V$). Since $\pi_w^U(B^U(J),J) = -B^U(J)$ and $\pi_w^V(B^V(J),J) = -B^V(J)L/(B^V(J) + (n-1)J)$, we obtain (after rearranging) the necessary and sufficient condition

$$B^V(J)(L - B^U(J)) > B^U(J)(n-1)J \ ,$$

which is satisfied because of our assumptions $B^U(J) < B^V(J)$ and $(n-1)J < L - B^U(J)$. □

Proof of Proposition 3:

A vacancy equilibrium must satisfy $\pi_1^V(J,J) = 0$. This yields $J = \frac{P(L)-w}{c}\frac{L}{n}\frac{n-1}{n}$, so that $J > L/n$ whenever $w < P(L) - c\frac{n}{n-1}$. Hence, if $w \geq P(L) - c\frac{n}{n-1}$, no vacancy equilibrium exists. Suppose now that $w < P(L) - c\frac{n}{n-1}$ and $J > L/n$. If (4) is satisfied, we have $w \leq P(L)(1-\rho/n) - c$ and

$$\begin{aligned} \pi_1^U(L-(n-1)J, J) &= P(L) + P'(L)(L-(n-1)J) - w - c \\ &> P(L) + P'(L)L/n - w - c \\ &= P(L)(1 - \tfrac{\rho}{n}) - w - c \geq 0 \ . \end{aligned}$$

Thus, J is a vacancy equilibrium at w since there are no profitable deviations to some $J_1 \leq L - (n-1)J$.

Now suppose that (4) is not satisfied. If $w = P(L) - c\frac{n}{n-1}$, $J = L/n$, and $\pi_1^V(L/n, L/n) = 0$, whereas

$$\pi_1^U(L/n, L/n) = P(L)(1-\tfrac{\rho}{n}) - w - c = \frac{c}{n-1} - P(L)\frac{\rho}{n} < 0 \ .$$

Hence, J is not an equilibrium since there are profitable deviations to some $J_1 < L - (n-1)J$. On the other hand, if $w \leq P(L)(1-\rho/n) - c$, there are no profitable deviations below $L-(n-1)J$. By continuity, there exists some w^V (together with J_1 and J) such that

$$\pi^U(J_1, J) = \pi^V(J,J) \ , \ \pi_1^U(J_1,J) = \pi_1^V(J,J) = 0 \ .$$

To show that w^V is unique, we employ the same argument as in the proof of Proposition 2. Denoting by $J^V(w)$ the candidate vacancy equilibrium at w (i.e. the solution of $B^V(J) = J$), w^V is a solution of the equation $F(J^V(w), w) = 0$ where F is defined as in (6). w^V is unique if $F(J^V(w), w)$ is upward–sloping in w. But this follows again from the Lemma and from the fact that J^V is downward sloping. Hence, for all $w \leq w^V$ there exists a vacancy equilibrium since $F(J^V(w), w) \leq 0$, and there is no vacancy equilibrium for any $w > w^V$. □

Proof of Proposition 4:

Suppose first that $w^U > w^V$. Then, according to Propositions 1, 2 and 3, there exists no symmetric pure strategy equilibrium for all wages $w \in (w^V, w^U)$. However, we will show in the following that there exists always a symmetric pure strategy equilibrium. This contradicts the assumption.

Let $J_1 = B(J)$ denote the best response function, and let $J_1 = B^U(J)$ and $J_1 = B^V(J)$ denote again the (unconstrained) best response functions under the hypothesis that the payoff function is π^U or π^V, respectively. Let $B^M(J) = L - (n-1)J$. Then, $B(J) = B^U(J)$ if $B(J) < L-(n-1)J$, $B(J) = B^V(J)$ if $B(J) > L-(n-1)J$, and $B(J) = B^M(J)$ if $B^U(J) \geq L-(n-1)J$ and $B^V(J) \leq L-(n-1)J$.

$B^X(J)$, $X = U, V, M$, are continuous, concave functions which cross the horizontal axis for sufficiently large J. Moreover, $B_1^V(0) = \infty$, $B^M(0) > 0$, and either $B^U(0) > 0$ (if $\rho < 1$) or $B_1^U(0) = \infty$ (if $\rho = 1$). Hence, each of these three functions crosses the diagonal at a unique positive J.

The best response function B is compound of the three functions B^U, B^V and B^M. If $B(J)$ is continuous, it must therefore also intersect the diagonal at some positive J. Hence, in this case there exists a symmetric pure strategy equilibrium. If $B(J)$ is discontinuous, any discontinuity must be a jump from B^V to B^U or from B^U to B^V (it is clear that there cannot be a discontinuous jump from or to B^M). We now show that there can only be a jump to above, i.e. from B^U to B^V as J increases. Therefore, there can be at most one discontinuity which must be an upward jump. A straightforward graphical argument shows that there must be (at least) one intersection of $B(J)$ with the diagonal. Hence, a symmetric pure strategy equilibrium exists.

To show that there can only be an upward jump, it is sufficient to show that the function $\pi^U(B^U(J),J) - \pi^V(B^V(J),J)$ is downward sloping at any J where $\pi^U(B^U(J),J) = \pi^V(B^V(J),J)$. But this has been shown in the Lemma. Therefore, we have shown that at least one symmetric pure strategy equilibrium exists which implies that $w^U \leq w^V$.

Now suppose that $w^U = w^V = w$. But this cannot be the case since $F(J^U(w),w) = 0$ and $F(J^V(w),w) = 0$, and since the Lemma implies that $F_1(J,w) < 0$ at any zero of F. Therefore, $w^U < w^V$. $\square$

Proof of Proposition 6:

With the production function $y_i = \ell_i^\alpha$ and a general upward–sloping labour supply function $L(w)$, we have

$$\begin{aligned}
\pi^U(J_1,J) &= P(J_1^\alpha + (n-1)J^\alpha)J_1^\alpha - (w+c)J_1 \\
\pi^V(J_1,J) &= P\Big(\frac{L(w)^\alpha(J_1^\alpha + (n-1)J^\alpha)}{(J_1+(n-1)J)^\alpha}\Big)\frac{J_1^\alpha L(w)^\alpha}{(J_1+(n-1)J)^\alpha} - w\frac{J_1 L(w)}{J_1+(n-1)J} - cJ_1
\end{aligned}$$

Lengthy but straightforward calculations show now that $\pi_1^U(L(w)/n, L(w)/n) \geq 0$ if and only if

$$\alpha\Big(\frac{L(w)}{n}\Big)^{\alpha-1} P\Big(n\Big(\frac{L(w)}{n}\Big)^\alpha\Big)\Big(1-\frac{\rho}{n}\Big) \geq w + c\ , \tag{8}$$

and that $\pi_1^V(L(w)/n, L(w)/n) \leq 0$ if and only if

$$\alpha\Big(\frac{L(w)}{n}\Big)^{\alpha-1} P\Big(n\Big(\frac{L(w)}{n}\Big)^\alpha\Big)\Big(1-\frac{1}{n}\Big) \leq w(1-\frac{1}{n}) + c\ . \tag{9}$$

With the substitution

$$k = \frac{cL(w)/n}{P(n(L(w)/n)^\alpha)(L(w)/n)^\alpha}\ ,$$

we find that condition (5) must be satisfied whenever (8) and (9) hold simultaneously. This proves Proposition 6.

For the labour supply function $L(w) = Aw^\gamma$ and the parameterization of the inverse demand by $P(Y) = BY^{-\rho}$, we can also derive from (8) and (9) a condition on c:

$$c \geq (H\rho)^{1/(1+\eta)}(1/\rho - 1)^{-\eta/(1+\eta)}(1-1/n)\ ,$$

where

$$H \equiv \alpha n^{(1-\alpha)(1-\rho)} A^{\alpha(1-\rho)-1} B \quad \text{and} \quad \eta \equiv \gamma(1-\alpha(1-\rho))\ .$$

One can easily check that this condition simplifies to (4) when $\alpha = 1$ and $\gamma = 0$. $\square$

References

Bénassy, J.-P. (1982): *The Economics of Market Disequilibrium.* Academic Press, New York.

d'Aspremont, C., R. Dos Santos Ferreira, and L. Gérard-Varet (1991): "Imperfect Competition, Rational Expectations, and Unemployment," in *Equilibrium Theory and Applications: Proceedings of the Sixth International Symposium in Economic Theory and Econometrics*, ed. by W. Barnett, B. Cornet, C. d'Aspremont, J. Gabszewicz, and A. Mas-Colell, pp. 353–381. Cambridge University Press, Cambridge.

Dehez, P. (1985): "Monopolistic Equilibrium and Involuntary Unemployment," *Journal of Economic Theory*, 36, 160–165.

Dixon, H., and N. Rankin (1994): "Imperfect Competition and Macroeconomics: A Survey," *Oxford Economic Papers*, 46, 171–199.

Hart, O. D. (1982): "A Model of Imperfect Competition with Keynesian Features," *Quarterly Journal of Economics*, 97, 109–138.

Kaas, L. (1998): "Multiplicity of Cournot Equilibria and Involuntary Unemployment," *Journal of Economic Theory*, 80, 332–349.

Kaas, L., and P. Madden (2000a): "Equilibrium Involuntary Unemployment under Oligempory," Mimeo, University of Vienna.

——— (2000b): "Imperfectly Competitive Cycles with Keynesian and Walrasian Features," Economics Series 83, Institute for Advanced Studies, Vienna.

Kreps, D., and J. Scheinkman (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 14, 326–338.

Lasselle, L., and S. Svizzero (2001): "The Impossibility of Underemployment with More than One Sector," *Oxford Economic Papers*, 53, 157–165.

Madden, P. (1998): "Elastic Demand, Sunk Costs and the Kreps–Scheinkman Extension of the Cournot Model," *Economic Theory*, 12, 199–212.

NEAL, D. (1995): "Industry–Specific Human Capital: Evidence from Displaced Workers," *Journal of Labor Economics*, 13, 653–677.

PARENT, D. (2000): "Industry–Specific Capital and the Wage Profile: Evidence from the National Longitudinal Survey of Youth and the Panel Study of Income Dynamics," *Journal of Labor Economics*, 18, 306–323.

PISSARIDES, C. (2000): *Equilibrium Unemployment Theory*. The MIT Press, Cambridge, MA, 2 edn.

ROTEMBERG, J., AND M. WOODFORD (1995): "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, Princeton, N.J. Princeton University Press.

SCHULTZ, C. (1992): "The Impossibility of Involuntary Unemployment in an Overlapping Generations Model with Rational Expectations," *Journal of Economic Theory*, 58, 61–76.

SICHEL, D. (1993): "Business Cycle Asymmetry: A Deeper Look," *Economic Inquiry*, 31, 224–236.

SILVESTRE, J. (1990): "There May be Unemployment When the Labour Market is Competitive and the Output Market is Not," *Economic Journal*, 100, 899–913.

——— (1993): "The Market–Power Foundations of Macroeconomic Policy," *Journal of Economic Literature*, 31, 105–141.

——— (1995): "Market Power in Macroeconomic Models: New Developments," *Annales d'Économie et de Statistique*, 37/38, 319–356.