Excess Reserves, Monetary Policy and Financial Volatility

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Abstract

This paper examines the financial and real effects of excess reserves in a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with monopoly banking, credit market imperfections and a cost channel. The model explicitly accounts for the fact that banks hold excess reserves and they incur costs in holding these assets. Simulations of a shock to required reserves show that although raising reserve requirements is successful in sterilizing excess reserves, it creates a procyclical effect for real economic activity. This result implies that financial stability may come at a cost of macroeconomic stability. The findings also indicate that using an augmented Taylor rule in which the policy interest rate is adjusted in response to changes in excess reserves reduces volatility in output and inflation but increases fluctuations in financial variables. To the contrary, using a countercyclical reserve requirement rule helps to mitigate fluctuations in excess reserves, but increases volatility in real variables.

JEL Classification Numbers: E43, E52, E58

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# Introduction

Excess reserves have been a common feature of the banking system of many countries across the world.\(^1\) In developed countries, the phenomenon of excess reserves has become more apparent since the global financial crisis. For instance, in the case of the U.S., the sharp increase in excess reserves since 2008 occurred because risk averse banks stopped lending to each other and engaged in liquidity hoarding (see Ashcraft et al. (2009), Hilton and McAndrews (2010) and Jenkins (2010)). Others argued that the policy initiatives undertaken by the Federal Reserve in response to the crisis caused the quantity of bank reserves to surge (see Keister and McAndrews (2009) and Ennis and Wolman (2012)). Similarly, excess reserves have been growing rapidly in commercial banks in the Euro Area since the onset of the global economic crisis (see European Central Bank (2008) and Sol Murta and Garcia (2010)).

In developing economies, the problem of excess reserves is more prevalent. For several years, the banking system of some developing countries has recorded high persistent liquidity—for instance, inspection of the data over the last decade reveals that the ratio of excess reserves to total reserves has been greater than 15 percent for most of the period in Belize and above 25 percent in Jamaica. Given its importance to the monetary transmission process, several researchers have empirically examined the determinants of excess liquidity in developing economies. Contributions along these lines include Maynard and Moore (2005) for Barbados, Saxegaard (2006) for Sub-Saharan Africa, Khemraj (2007, 2009) for Guyana, Anderson-Reid (2011) for Jamaica, Pontes and Sol Murta (2012) for Cape Verde, Jordan et al. (2012) for The Bahamas and Primus et al. (2014) for Trinidad and Tobago. In most cases, these studies show that excess reserves appear to be a structural phenomenon.\(^2\)

As highlighted in Agénor and El Aynaoui (2010), the reasons for excess liquidity can be categorized into structural and cyclical factors. One structural factor is a low degree of financial development. Therefore, excess liquidity tends to be more persistent in countries with underdeveloped financial markets, such as inefficient payment systems, or an underdeveloped market for government securities (see Saxegaard (2006)). Another structural factor is a high degree of risk aversion. In an environment of increased uncertainty, risk averse banks charge a high risk premia to reduce demand for loans and to safeguard their loan portfolio. This leads to a voluntary build-up

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\(^1\) In most financial systems, excess liquidity refers to the maintenance by banks of a higher level of funds than is normally required to meet their statutory reserve requirements and settlement balances. Excess liquidity (or excess reserves) is measured as the difference between total bank reserves and required bank reserves. For this reason, the terms excess liquidity and excess reserves are used interchangeably in the literature.

\(^2\) Excess bank liquidity is also a problem for large developing countries, such as Brazil, Russia, India, China and Nigeria. Some of the other developing countries with the problem of excess reserves include Botswana (see Akinboade and Zachariah (1997)), Egypt (see Fielding and Shortland (2005)), Mexico (see Jallath-Coria et al. (2005)), Tanzania (see Aikeli (2006)), Turkey (see Tabak and Bankasi (2006)), Fiji (see Jayaraman and Choong (2012)), Indonesia (see Bathaluddin et al. (2012)), Costa Rica, El Salvador, Guatemala, Honduras, Nicaragua, Panama and the Dominican Republic (see Deléchat et al. (2012)), and Morocco (see El Hamma and Ejbari (2013)).
of excess liquid assets.\textsuperscript{3}

Regarding the cyclical factors, one of the main sources of excess reserves is foreign currency inflows (Ganley (2004) and Heenan (2005)). Current account inflows arise mainly through revenues received from commodity exports. Therefore, exporters of minerals, such as Botswana, and oil exporting economies, such as Nigeria and Trinidad and Tobago, observe huge surpluses in their current account when commodity prices are high on world markets. Particularly in oil-producing countries, a rise in oil prices results in an increase in government revenues. In many cases, these energy windfalls are used to finance an increase in government spending. As a result, the banking system of these economies holds large quantities of involuntary excess liquidity.\textsuperscript{4} Capital account inflows may arise from aid-related transfers, foreign direct investment and portfolio inflows. Another cyclical cause of excess liquidity is inflation. Because inflation leads to higher volatility in relative prices and an increase in riskiness of investment projects, it raises uncertainty about the value of collateral. This causes banks to demand a higher risk premium, which increases the lending rate. The higher loan rate can lead to a contraction in credit demand and increase excess reserves.

In an attempt to withdraw excess liquidity from the financial system, central banks have used open market operations, and issued central bank bills.\textsuperscript{5} Central banks have also issued long-term securities (bonds) and implemented special deposit facilities. In addition, reserve requirements have been used frequently to manage liquidity.\textsuperscript{6,7} For instance, between 2006 and 2009, the central banks of China, India and Trinidad and Tobago have all increased reserve requirements to mop up excess liquidity. Using reserve requirements can help the central bank or the government to reduce the quantity of excess reserves in the financial system without incurring interest costs, which are associated with the issuance of securities (Gray (2011)). Also, reserve requirements can be more effective because selling securities to sterilize excess liquidity can in fact be self-defeating as it can cause market interest rates to increase and stimulate capital inflows, thereby making the excess liquidity problem worse, if to begin with it resulted from large inflows (Lee (1997)). One disadvantage to note however is reserve requirements act as a tax on the financial sector (Central Bank of Trinidad and Tobago (2005) and Montoro and Moreno (2011)).

Excess liquidity impedes the transmission of monetary policy and undermines the ability of

\textsuperscript{3}Similarly, in a crisis environment where banks perceive an increase in the risk of default on loans, they may be unwilling to lend. For instance, Agénor et al. (2004) found that the contraction in bank credit and the associated increase in excess reserves in Thailand following the Asian financial crisis in the late 1990s resulted from supply related factors, which emanated from the banks.

\textsuperscript{4}According to Agénor and El Aynaoui (2010, p. 923), involuntary excess liquidity is "the involuntary accumulation of liquid reserves by commercial banks". Several researchers have examined the issue of involuntary excess liquidity. See for instance, Saxegaard (2006), Bathaluddin et al. (2012) and Primus et al. (2014).

\textsuperscript{5}See Nyawata (2012) for a discussion of the pros and cons of using Treasury bills and central bank bills to absorb liquidity.

\textsuperscript{6}Reserve requirement levels vary greatly across countries. It has been observed that these ratios are generally higher in developing countries, when compared to developed countries (see Table 1).

\textsuperscript{7}In recent years, a number of central banks have used reserve requirements as a liquidity management tool. See Montoro and Moreno (2011), Robitaille (2011) and Tovar et al. (2012) for further discussions.
monetary policy to stabilize the economy. It is important for excess bank reserves to be managed because tight liquidity conditions are necessary for the central bank to have leverage over the commercial banks to achieve its monetary objectives. Changes in the policy interest rate can be reflected in the interest rates in the banking sector if commercial banks do not hold substantial liquidity above the legal requirement—in this case, the central bank can effectively perform the function of monopoly supplier of domestic currency (Ganley (2004)). Furthermore, in the presence of excess liquidity, the effectiveness of monetary policy can be asymmetric. As empirically documented in Lebedinski (2007) for Morocco, when banks hold excess reserves they are more likely to respond in an asymmetric manner when adjusting deposit rates.

Moreover, excess reserves represent a challenge to monetary policy. When banks hold excess reserves, attempts by the monetary authorities to increase liquidity to stimulate demand will be ineffective. By contrast, if a central bank wants to pursue monetary tightening, an increase in the policy interest rate may not be transmitted to deposit and loan rates as banks holding excess reserves may not raise their interest rates. A lower deposit rate discourages savings and encourages consumption, whereas a lower loan rate stimulates demand for credit. These can therefore have inflationary consequences. Furthermore, when banks hold excess reserves, they are more inclined to lower credit standards and reduce loan rates to expand lending. This can lead to a rise in the risk of loan default and make the financial system more susceptible to crisis.

Despite the impact excess reserves can have on the transmission mechanism of monetary policy, there have been few attempts to examine the role of reserves in a New Keynesian general equilibrium framework. Two of the analytical contributions in this area are Christiano et al. (2010) and Samake (2010). Although both studies incorporated excess reserves into a general equilibrium model with banking, they did not model the fact that banks incur costs in holding these assets. It is important to have a well-defined demand for excess reserves to explicitly account for the rate of return on reserves and any opportunity cost of holding reserves.

This paper therefore explicitly models excess reserves in a Dynamic Stochastic General Equilibrium (DSGE) framework with monopoly banking, credit market imperfections and a cost channel. The model extends and modifies the framework in Agénor and Alper (2012), and integrates aspects of Glocker and Towbin (2012) and Agénor et al. (2013). In this framework, excess reserves are endogenous—as banks voluntarily demand this asset—and there are convex costs associated with holding these reserves as in Glocker and Towbin (2012). The model is calibrated based on Trinidad and Tobago data due to the fact that excess reserves have been growing very rapidly in that country. This research represents an essential contribution as it is the first attempt to apply a general equilibrium model with banking, to the Trinidad and Tobago economy, to examine the macroeconomic effects of excess reserves.

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8This is particularly important when banks hold excess reserves for precautionary purposes. In this case, an expansionary monetary policy will only raise the amount of excess reserves further, and will not help to expand credit.
The model is used to examine the financial and real effects of a productivity shock, an increase in the policy interest rate, a shock to the reserve requirement ratio and an exogenous increase in bank liquidity. Also, simultaneous shocks are applied to deposits and reserve requirements to examine whether increasing required reserves in response to a surge in liquidity is an effective measure. This experiment is another key contribution to this paper. Although it has been often observed in practice that raising reserve requirements can indeed offset an increase in liquidity, this is the first attempt to model this in a New Keynesian general equilibrium framework. In addition, the model is used to investigate the responses of the key variables following a liquidity shock when two alternative policy rules are used: an augmented Taylor rule in which the central bank adjusts its policy rate in response to changes in excess reserves, and a countercyclical rule in which the reserve requirement ratio reacts to deviations in excess reserves.

The results show that a negative supply shock and a contractionary monetary shock have the traditional effects. In the former case prices increase and output declines, while in the latter both prices and output fall. As the refinance rate rises following both types of shocks, the opportunity cost of holding excess reserves increases. The findings also indicate that although a positive shock to required reserves is successful in reducing excess reserves, the effect is expansionary for real economic activity. In the case of a liquidity shock, a simultaneous increase in reserve requirements can assist in reducing the quantity of excess reserves in the financial system. Furthermore, when an augmented Taylor rule which includes excess reserves is used, a liquidity shock has a less dampening effect on real variables but increases fluctuations in financial variables. To the contrary, using a countercyclical reserve requirement rule has the opposite effect for both real and financial variables.

The rest of this paper is organized as follows. Section 2 provides a review of the problem of excess reserves and its implications for monetary policy. Section 3 presents the model and Section 4 outlines the symmetric equilibrium. The key steady-state and log-linearized equations of the model are presented in Section 5. Section 6 provides a discussion of the calibration for Trinidad and Tobago. In Section 7, impulse response functions are used to discuss the findings from the policy experiments and other shocks to the model. Section 8 examines the dynamic effects of an increase in excess reserves under alternative policy rules, whereas Section 9 presents some sensitivity analysis to gauge the robustness of the results. The final section provides a summary of the main results.

2 Background

Managing excess liquidity in a small open economy with a fragmented market has been a challenge for the Central Bank of Trinidad and Tobago. The banking system of Trinidad and Tobago has been plagued with persistent high liquidity, which has risen to record levels in recent years. Figure 1 shows that commercial banks’ excess reserves as a percent of total reserves increased exponentially from an average of 7.0 percent in 2000 to 35.7 percent in 2013. The build-up of excess liquidity
in the banking system stems from the fact that Trinidad and Tobago is an energy producer, so high energy prices have significantly increased government revenues and expenditure.\textsuperscript{9} The limited capacity of the country to absorb the increased fiscal outlays has led to chronic excess liquidity in the banking system. Thus, the main source of liquidity is the government’s fiscal injections into the economy, which are financed by tax payments received from energy sector companies (Central Bank of Trinidad and Tobago (2005)). Inspection of the data reveals that over the period 2000 to 2006, the government’s domestic fiscal operations injected an equivalent of 5.5 percent of GDP in the economy, and 9.8 percent of GDP during 2007-2012. Figure 2 shows that these fiscal injections are positively correlated with the proportion of excess reserves to total reserves, and the ratio of government expenditure to GDP.

In Trinidad and Tobago, the repo rate is the official policy instrument used by the Central Bank to effect monetary policy. In light of the high levels of excess reserves, changes in the repo rate have little effect on liquidity and adjustments in the rate are not fully transmitted to money and credit market rates. Therefore, the monetary authority loses its ability to influence short-term interest rates. As a result of this the Central Bank had to increase reliance on the use of direct policy instruments—mainly the statutory reserve requirement—to effect monetary policy. Also, in 2006 the Bank re-introduced a secondary reserve requirement\textsuperscript{10} whereby commercial banks are required to hold an equivalent amount of 2 percent of their prescribed liabilities at the Central Bank. These secondary reserves balances are remunerated at 350 basis points below the repo rate. Another direct measure implemented since 2005 is a special deposit facility, whereby commercial banks are required to deposit specific amounts in interest bearing accounts.

Moreover, in order to withdraw some of the excess liquidity from the banking system, the Central Bank is consistently selling Treasury bills and notes. The Bank also issued special liquidity absorption bonds to complement the impact of regular open market operations. Because the government has to pay interest on these instruments, this adds pressure to the fiscal deficit. Furthermore, the surge in excess liquidity in recent years has intensified pressures in the foreign exchange market and has caused the Central Bank to increase the supply of foreign currency, to ensure the stability of the exchange rate. The increase in foreign exchange sales (mainly U.S. dollars) has helped to remove local currency from the financial system and in so doing, absorb liquidity. It has however been recognized that the high levels of liquidity in the financial system have outpaced the liquidity absorption measures undertaken by the Central Bank.

\textsuperscript{9}In 2012, the energy sector accounted for 44 percent of GDP, 54 percent of government revenue, and 81 percent of merchandise exports. Also, for the same period, the non-energy tradable sector and the nontradable sector accounted for 6 percent and 50 percent of GDP, respectively.

\textsuperscript{10}Previously, the secondary reserve requirement mandated financial institutions to keep a proportion of deposits in the form of Treasury bills. This helped to reduce bank loans to the government.
3 The Model

Consider an economy which contains seven classes of agents: identical infinitely-lived households indexed by \( h \in [0, 1] \), a final good-producing firm, a continuum of intermediate good-producing firms indexed by \( j \in [0, 1] \), a capital good producer, a commercial bank (a bank, for short), the central bank (whose responsibility is to regulate the commercial bank) and the government.\(^{11}\)

Households consume and supply labour to intermediate good-producing firms. Households also choose the real levels of cash, deposits and government bonds to hold at the beginning of the period. Intermediate good-producing firms use the labour provided by households and capital to produce a unique good that is sold on the monopolistically competitive market. The pricing mechanism of Rotemberg (1982) is used to account for the fact that intermediate good-producing firms incur a cost in adjusting prices. The final good-producing firm aggregates imperfectly substitutable intermediate goods into a single final good which is used for consumption, investment or government spending. The final good is sold at a perfectly competitive price. The capital good producer purchases the final good for investment and combines it with existing capital stock to produce new capital goods. In this model, wages are fully flexible and adjust to clear the market.

The commercial bank, which is owned by households, supplies credit in advance to intermediate good-producing firms to finance their short-term working capital needs. The bank also supplies credit to the capital good producer for investment financing. The bank’s supply of loans is perfectly elastic at the prevailing lending rate. These loans are extended prior to production or investment and are repaid at the end of the period. The bank pays interest on household deposits and central bank loans. In addition, the bank is required to hold minimum reserves against deposits at the central bank, and it has an explicit demand for excess reserves—the bank therefore holds excess reserves voluntarily. Because the bank does not hold involuntary reserves, the model does not account for any asymmetric effect on bank pricing behaviour. Total reserves at the central bank are remunerated at the reserve rate denoted by \( i^M \), which is constant. The bank determines the total reserve ratio, the deposit rate and the lending rate, and borrows from the central bank to finance any shortfall in funding. The central bank sets its policy interest rate using a Taylor-type rule and supplies all the credit demanded by the bank at the prevailing refinance rate. It is important to note that because there is a perfectly elastic supply of liquidity, the bank is not subject to (random) withdrawal risk which has been a key factor in reserve management models.\(^{12}\) Therefore, increased uncertainty about the size of cash withdrawals does not influence the quantity of excess bank reserves in this model.

The financial sector of the model incorporates features that are relevant to the structure of

\(^{11}\) In effect, the household and capital good producer can be thought of as one unit in this model, and this unit ignores the potential benefit of housing as collateral in capital good production while making housing choices. This point is further discussed later.

\(^{12}\) In reserve management models, the optimal level of reserves demanded by commercial banks is a function of deposit fluctuations (see Morrison (1966), Poole (1968) and Baltensperger (1980) for further discussions).
the Trinidad and Tobago economy. First, commercial banks in the economy have a voluntary
demand for excess reserves (see Primus et al. (2014)), which the model accounts for, given the
bank’s explicit demand for excess reserves. Second, commercial banks in Trinidad and Tobago are
required to hold primary and secondary reserves at the Central Bank—as highlighted in Section
2. Given that the secondary reserves are remunerated, the model extends this to consider the
case of interest being paid on the bank’s total reserve holding at the central bank—which is the
case in a few countries. Third, as noted in Section 2, the Central Bank conducts monetary policy
by using a short-term interest rate. A study by Primus (2012) found that a simple Taylor-type
rule can be used to describe the Central Bank’s interest rate setting behaviour. Therefore, in the
model, the central bank conducts monetary policy using a Taylor-type rule. Finally, the banking
system (as in most developing countries) has credit market imperfections and due to asymmetric
information problems, lending to firms is collateralized. The other sectors of the model are standard
for developing economies.

3.1 Households

Each household, \( h \), chooses consumption, labour supply to intermediate good-producing firms and
real monetary assets. The objective of a representative household is to maximize the following
utility function,

\[
U = E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{ht}}{1-\sigma} \right]^{1-\sigma} + \eta_N \ln(1 - N_{ht}) + \eta_X \ln X_{ht},
\]

where \( C_{ht} \) is household consumption, \( N_{ht} \) is the share of total time endowment (normalized to
unity) household \( h \) spent working, \( X_{ht} \) is a composite index of real monetary assets, \( \sigma > 0 \) gives
the intertemporal elasticity of substitution in consumption, \( \eta_N, \eta_X > 0 \) are preference parameters
with respect to leisure and money holdings respectively, \( \beta \in (0, 1) \) is the discount factor and \( E_t \) is
the expectation operator conditional on the information available at the beginning of period \( t \).

The composite monetary asset is a combination of real cash balances \( M_{ht}^H \) and real bank deposits
\( D_{ht} \), which can be represented by the following Cobb-Douglas function,

\[
X_{ht} = (M_{ht}^H)^{\nu} D_{ht}^{1-\nu},
\]

where \( \nu \in (0, 1) \).

Real wealth of household \( h \) at the end of period \( t \), \( A_{ht} \), is given by,

\[
A_{ht} = M_{ht}^H + D_{ht} + B_{ht}^H,
\]

where \( B_{ht}^H \) denotes holdings of one-period real government bonds.

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13 As mentioned in Section 1, high levels of excess reserves is a common feature of the banking system of several
developing countries. Therefore, although this research uses Trinidad and Tobago as a case study, this framework can
be applied to other high- and middle-income countries that have a similar financial structure.
At the beginning of period $t$, each household enters with $M_{ht-1}^H$ level of cash. Holding money balances yield no return, while deposits and government bonds yield gross returns of $(1 + i_t^D)$ and $(1 + i_t^B)$, respectively. Therefore, the total real returns from holding deposits and government bonds from period $t-1$, adjusted for the rate of inflation, are denoted respectively by $(1 + i_{t-1}^D)D_{ht-1}(P_{t-1} - P_t)$ and $(1 + i_{t-1}^B)B_{ht-1}(P_{t-1} - P_t)$, where $P_t$ represents the price of the final good.

In addition, households supply labour to intermediate good-producing firms, for which they receive a total real factor payment $N_{ht}$, where $\omega_t$ denotes the economy-wide real wage. Each household owns an intermediate good-producing firm so all the profits made by that firm, $J_{ht}^I$, are paid to the respective household. Also, each household receives a fixed fraction $\varphi_h \in (0, 1)$ of the bank’s profits, $J_{ht}^B$, and the capital good producer’s profits, $J_{ht}^K$, with $\int_0^1 \varphi_h dh = 1$. Each household is also required to pay a lump-sum tax, whose real value is $T_{ht}$.

The real budget constraint of household $h$ is,

$$M_{ht}^H + D_{ht} + B_{ht}^H \leq \omega_t N_{ht} - T_{ht} + M_{ht-1}^H\left(\frac{P_{t-1}}{P_t}\right) + (1 + i_{t-1}^D)D_{ht-1}\left(\frac{P_{t-1}}{P_t}\right)$$

$$+(1 + i_{t-1}^B)B_{ht-1}\left(\frac{P_{t-1}}{P_t}\right) + J_{ht}^I + \varphi_h J_{ht}^B + J_{ht}^K - C_{ht}.$$  \hspace{1cm} (4)

Each household maximizes lifetime utility with respect to $C_{ht}$, $N_{ht}$, $M_{ht}^H$, $D_{ht}$ and $B_{ht}^H$, taking $i_t^D$, $i_t^B$, $P_t$, and $T_{ht}$ as given. Maximizing (1) subject to (4) yields the following first order conditions:\footnote{Details of the derivations are shown in Appendix A.}

$$C_{ht}^{-1/\sigma} = \beta E_t \left[ (C_{ht+1})^{-1/\sigma} \left(\frac{1 + i_{t+1}^B}{1 + \pi_{t+1}}\right) \right],$$  \hspace{1cm} (5)

$$N_{ht} = 1 - \frac{\eta N(C_{ht})^{1/\sigma}}{\omega_t},$$  \hspace{1cm} (6)

$$M_{ht}^H = \eta_X \nu(C_{ht})^{1/\sigma}(1 + i_t^B),$$  \hspace{1cm} (7)

$$D_{ht} = \eta_X (1 - \nu)(C_{ht})^{1/\sigma}(1 + i_t^B),$$  \hspace{1cm} (8)

where $\pi_{t+1} = (P_{t+1} - P_t)/P_t$ is the inflation rate. The transversality condition is determined by the following equation,

$$\lim_{s \to \infty} E_{t+s} \Lambda_{t+s}\beta^\sigma(M_{t+s}^H) = 0.$$  \hspace{1cm} (9)

Equation (5) is the standard Euler equation which describes the optimal consumption path. Equation (6) represents the optimal labour supply which is positively related to the real wage and negatively related to consumption. Equation (7) shows that the demand for real cash balances depends positively on consumption and negatively on the opportunity cost of holding cash (measured by the rate of return on government bonds). Equation (8) denotes the real demand for deposits which is positively related to consumption and the deposit rate, and negatively related to the bond rate.
3.2 Final Good-Producing Firm

The final good producer assembles a continuum of imperfectly substitutable intermediate goods \( Y_{jt} \), indexed by \( j \in (0, 1) \), to produce the final good \( Y_t \), which is used for private consumption, government consumption and investment. The production technology for combining intermediate goods to produce the final good is given by the standard Dixit-Stiglitz (1977) technology,

\[
Y_t = \left\{ \int_0^1 [Y_{jt}]^{(\theta-1)/\theta} dj \right\}^{\theta/(\theta-1)},
\]

where \( \theta > 1 \) represents the elasticity of demand for each intermediate good.

Given the prices of intermediate goods, \( P_{jt} \), and the final good price, \( P_t \), the final good-producing firm chooses the quantities of intermediate goods to maximize its profits. The profit maximization problem of the final good producer is given by,

\[
\max_{Y_{jt}} P_t \left\{ \int_0^1 [Y_{jt}]^{(\theta-1)/\theta} dj \right\}^{\theta/(\theta-1)} - \int_0^1 P_{jt} Y_{jt} dj.
\]

The first-order condition with respect to \( Y_{jt} \) is,

\[
Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t.
\]

Equation (12) gives the demand for each intermediate good \( j \). Substituting (12) in (10) and imposing a zero-profit condition, the final good price is represented by,

\[
P_t = \left[ \int_0^1 (P_{jt})^{1-\theta} dj \right]^{1/(1-\theta)}.
\]

3.3 Intermediate Good-Producing Firms

Each intermediate good-producing firm, \( j \), produces a perishable good which is sold on a monopolistically competitive market. To produce these goods, each firm rents capital at the price \( r^K_t \) from the capital good producer and combines it with labour. The technology faced by each intermediate good-producing firm is given by the Cobb-Douglas production function,

\[
Y_{jt} = A_t K_{jt}^{\alpha} N_{jt}^{1-\alpha},
\]

where \( N_{jt} \) is household \( h = j \) labour hours, \( K_{jt} \) is the amount of capital rented by the firm, \( \alpha \in (0, 1) \) is the elasticity of output with respect to capital and \( A_t \) is a serially uncorrelated technology shock which follows a first-order autoregressive process, \( A_t = A_{t-1}^\rho \xi_t^A \), where \( \rho \in (0, 1) \) and \( \xi_t^A \sim N(0, \sigma_{\xi A}) \).

In order to pay wages in advance, firm \( j \) takes a loan from the bank at the beginning of the period. The amount borrowed is,

\[
L_{jt}^{FW} = \omega_t N_{jt},
\]
where $L_{jt}^{FW}$ represents the real value of loans demanded by intermediate good producers for all $t \geq 0$ and $\kappa^W \in (0, 1)$. Similar to Agénor et al. (2013), it is assumed that short-term loans for working capital do not carry any risk and are therefore contracted at a rate that reflects only the marginal cost of borrowing from the central bank, $i_t^R$, which is the refinance rate. The wage bill, inclusive of interest payments is $(1 + i_t^R)\kappa^W \omega_t N_{jt} + (1 - \kappa^W) \omega_t N_{jt}$. Rearranging this gives $(1 + \kappa^W i_t^R)\omega_t N_{jt}$, which shows the firm’s wage bill includes a constant share of financing of working capital needs. Thus, $\kappa^W$ indicates the strength of the cost channel; if $\kappa^W = 0$, no cost channel exists.

Intermediate good producers solve a two stage problem. In the first stage, given input prices, firms integrate capital and labour in a perfectly competitive market in order to minimize their total costs. The cost minimization problem for firm $j$ is,

$$\min_{N_{jt}, K_{jt}} [(1 + \kappa^W i_t^R)\omega_t N_{jt} + r^K K_{jt}].$$

Minimizing (16) subject to (14), the first-order conditions with respect to $N_{jt}$ and $K_{jt}$ equate the marginal products of capital and labour to their relative prices, from which the capital-labour ratio is obtained,

$$\frac{K_{jt}}{N_{jt}} = \left(\frac{\alpha}{1 - \alpha}\right) \frac{[(1 + \kappa^W i_t^R)\omega_t]}{r^K}.\hspace{1cm}(17)$$

The unit real marginal cost is,

$$mc_{jt} = \frac{[(1 + \kappa^W i_t^R)\omega_t]^{1-\alpha} (r^K)^\alpha}{\alpha^{\alpha}(1 - \alpha)^{1-\alpha} A_t}.$$\hspace{1cm}(18)

In the second stage, each firm chooses prices, $P_{jt}$, to maximize the discounted real value of current and future profits. Nominal price stickiness is introduced along the lines of Rotemberg (1982), by assuming that intermediate good-producing firms incur a cost in adjusting prices. These price adjustment costs, $PAC_{jt}$, which are measured in terms of aggregate output, $Y_t$, take the form,

$$PAC_{jt} = \frac{\phi_F}{2} \left(\frac{P_{jt}}{P_{jt-1}} - 1\right)^2 Y_t,$$\hspace{1cm}(19)

where $\phi_F \geq 0$ is the degree of price stickiness.

Thus, the profit maximization problem for the intermediate good producer is,

$$\max_{P_{jt}} E_t \sum_{t=0}^{\infty} \beta^t A_t J^I_{jt},$$\hspace{1cm}(20)

where $\beta^t A_t$ is the firm’s discount factor for period $t$, with $A_t$ representing the marginal utility gained from consuming an additional unit of profit. Real profits, $J^I_{jt}$, are defined as,

$$J^I_{jt} = Y_{jt} - mc_{jt} Y_{jt} - PAC_{jt}.$$\hspace{1cm}(21)

Substituting (12) and (19) in (21) and taking $mc_{jt}$, $P_t$ and $Y_t$ as given, the first-order condition with respect to $P_{jt}$ is,
(1 - \theta)\Lambda_t \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t \frac{Y_t}{P_t} + \theta \Lambda_t mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\theta-1} Y_t \frac{Y_t}{P_t} - \Lambda_t \phi_F \left\{ \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) Y_t \frac{Y_t}{P_{jt-1}} \right\} + \beta \phi_F E_t \left\{ \Lambda_{t+1} \left( \frac{P_{jt+1}}{P_{jt}} - 1 \right) \left( \frac{P_{jt+1}}{P_{jt}^2} \right) Y_{t+1} \right\} = 0. \tag{22}

Equation (22) gives the adjustment process of the nominal price \( P_{jt} \). When there is no price adjustment cost (\( \phi_F = 0 \)), the price equals a mark-up over the real marginal cost,

\[ P_{jt} = \left( \frac{\theta}{\theta - 1} \right) mc_{jt} P_t. \tag{23} \]

In a symmetric equilibrium \( P_{jt} = P_t \) for all \( j \); hence the real marginal cost equals the reciprocal of the mark-up, \( mc_t = (\theta - 1)/\theta \).

### 3.4 Capital Good Producer

In the economy, all the capital is owned by the capital good producer who employs a linear production function to produce capital goods. As in Agénor et al. (2013), at the beginning of each period, the capital good producer purchases \( I_t \) of the final good from the final good producer. Because payments for these final goods must be made in advance, the capital good producer borrows from the bank,

\[ L^{I,F}_t = I_t, \tag{24} \]

where \( L^{I,F}_t \) denotes real loans made to the capital good producer for investment purposes. The total costs faced by the capital good producer at the end of period \( t \) for buying an amount \( I_t \) of the final good is \( (1 + i^L_t)L_t \), where \( i^L_t \) is the lending rate.

The capital good producer combines undepreciated capital from the previous period, with investment to produce new capital goods. New capital goods, denoted as \( K_{t+1} \), are given by,

\[ K_{t+1} = I_t + (1 - \delta)K_t - \frac{\Theta_K}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t, \tag{25} \]

where \( K_t = \int_0^1 K_{jt}d_j \), \( \delta \in (0, 1) \) gives the constant rate of depreciation and \( \Theta_K > 0 \) measures the magnitude of adjustment costs. The capital good producer rents the new capital stock to intermediate good-producing firms at the rate \( r^K_t \).

The capital good producer chooses the amount of capital stock in order to maximize the value of the discounted stream of dividend payments to the household. The optimization problem of the
capital good producer is given by,\(^\text{15}\)

\[
\max_{K_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \Lambda_t J_t^K,
\]

where real profits, \(J_t^K\), can be denoted as,

\[
J_t^K = r_t^K K_t - (1 + i_t^B) I_t.
\]

Maximizing (26) subject to (25), the first-order condition is,

\[
E_t r^K_{t+1} = (1 + i_t^B) E_t \left\{ \left[ 1 + \Theta_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] \left( \frac{1 + i_t^B}{1 + \pi_{t+1}} \right) \right\} - E_t \left\{ (1 + i_t^L) \left[ (1 - \delta) + \Theta_K \frac{K_{t+2}}{K_{t+1}} \right] \right\}.
\]

Equation (28) shows the expected rental rate of capital is a function of the current and expected loan rates, the cost of adjusting capital across periods, the bond rate, the depreciation rate and the inflation rate. The opportunity cost of investing in physical capital is measured by the real rate of return on government bonds. If the capital good producer does not borrow at the beginning of the period, and there are no adjustment costs \((\Theta_K = 0)\),

\[
E_t r^K_{t+1} = E_t \left( \frac{1 + i_t^B}{1 + \pi_{t+1}} \right) - 1 + \delta.
\]

Equation (29) is the standard arbitrage condition which implies that capital is produced up to the point where the (expected) rental rate of capital is equal to the (expected) real interest rate on government bonds, plus depreciation.

### 3.5 Commercial Bank

The bank receives deposits \(D_t\) from households at the start of each period. These deposits are used to finance loans to intermediate good-producing firms to cover wage payments and to the capital good producer for investment. Therefore, combining (15) and (24), total lending, \(L_t^F\), in real terms is,

\[
L_t^F = \kappa^W \omega_t N_t + I_t.
\]

Given households’ deposits and total loans to firms, to finance any shortfall in funding, the bank borrows from the central bank, \(L_t^P\), for which it pays a net interest rate \(i_t^R\).

\(^{15}\)As noted earlier, the household and capital good producer in this model can be thought of as a single unit with respect to housing choices. Agénor et al. (2013) assume further that in case of default the capital seized by the bank is returned immediately and in its entirety to the household, who turns it back instantly to the capital good-producing firm. As a result, and as implicitly assumed in (27), the capital good producer does not internalize the risk of default, that is, the possibility that it could lose the fraction of the housing stock that it used to secure bank loans. See Agénor et al. (2014) for an alternative treatment.
Assets of the commercial bank at the beginning of period $t$ consist of real loans to firms and real total reserve holdings, $TR_t$, whereas its liabilities comprise of real loans from the central bank and real deposits. The bank’s balance sheet is thus,

$$L_t^F + TR_t = L_t^R + D_t. \tag{31}$$

Total reserves comprise of excess reserves, $ER_t$, and required reserves, $RR_t$, which are the compulsory minimum amount of reserves the bank must hold at the central bank. Thus,

$$TR_t = ER_t + RR_t, \tag{32}$$

where total reserves are a portion $\mu^R_t$ of deposits and required reserves are a percent $\mu_t$ of deposits. Therefore, $TR_t = \mu_t^R D_t$ and $RR_t = \mu_t D_t$; where $\mu_t^R, \mu_t \in (0, 1)$. Using these in (32), excess reserves are therefore determined residually,$^{16}$

$$ER_t = (\mu_t^R - \mu_t)D_t. \tag{33}$$

Total reserves held at the central bank are remunerated at the rate $i^M$,\textsuperscript{17} where $i^M < i^R_t$. The bank therefore chooses the total reserve ratio, the deposit rate and the lending rate to maximize its present discounted value of real profits. Hence, the bank’s profit maximization problem is,

$$\max_{\{\mu_t^R, 1+i_t^D, 1+i_t^L\}} E_t^\infty \sum_{t=0}^\infty \beta^t \Lambda_t J_t^B, \tag{34}$$

where $E_t^-$ is the expectations operator based on information available at the beginning of period $t$ and $J_t^B$ represents real bank profits at the end of period $t$.

Therefore, expected real bank profits can be defined as,

$$E_t^-(J_t^B) = (1 + \kappa^W_i R_t) L_t^{F,W} + Q_t^F (1 + i_t^L) L_t^{F,I} + (1 - Q_t^F) \kappa^C K_t$$

$$+ (1 + i^M) TR_t - (1 + i_t^D) D_t - (1 + i_t^R) L_t^B - \Phi (\mu_t^R - \mu_t) D_t,$$ \tag{35}

where $\kappa^C \in (0, 1)$ and $Q_t^F \in (0, 1)$ is the repayment probability.

From equation (35), the first term on the right-hand side, $(1 + \kappa^W_i R_t) L_t^{F,W}$, shows repayment on loans to intermediate good-producing firms. The second term, $Q_t^F (1 + i_t^L) L_t^{F,I}$, represents expected repayment on loans to the capital good producer, providing that there is no default. The third term, $(1 - Q_t^F) \kappa^C K_t$, denotes the bank’s earnings in case of default, where $1 - Q_t^F$ represents the probability of default. This term therefore shows real effective collateral, given by a fraction $(\kappa^C)$ of the real capital stock. The expression $(1 + i^M) TR_t$ denotes the principal plus interest gained

\textsuperscript{16}In principle, the bank should determine directly excess reserves; however, it is more convenient to solve for total reserves first, and use this solution to determine excess reserves.

\textsuperscript{17}A few studies have discussed how interest on reserves can be used as a policy instrument (see Goodfriend (2002), Ennis and Weinberg (2007), Keister et al. (2008), Keister and McAndrews (2009), and Kashyap and Stein (2012)).
from total reserves, whereas \((1 + i^D_t)D_t\) represents the principal and interest paid on real deposits, and \((1 + i^R_t)L^B_t\) reflects the gross repayments to the central bank. Similar to Glocke and Towbin (2012), the final term, \(\Phi(\mu^{TR}_t - \mu_t)D_t\), is included to represent the convex costs of holding reserves, which are proportional to the amount of real deposits. Thus,

\[
\Phi(\mu^{TR}_t - \mu_t) = -\Phi C_1 (\mu^{TR}_t - \mu_t) + \frac{\Phi C_2}{2} (\mu^{TR}_t - \mu_t)^2 + \varepsilon^R_t. \tag{36}
\]

From equation (36), \(\Phi C_1\) and \(\Phi C_2\) are cost function parameters. The linear term, \(\Phi C_1 (\mu^{TR}_t - \mu_t)\), determines steady-state deviations from the required reserve ratio. A positive deviation from the ratio may generate small benefits because holding excess reserves reduces the costs of liquidity management. Intuitively, if the bank fails to meet the reserve requirement it has to face the penalty rate for funds borrowed from the central bank. The quadratic term, \(\frac{\Phi C_2}{2} (\mu^{TR}_t - \mu_t)^2\), indicates that negative deviations from the required ratio may generate large costs. For instance, the central bank may impose a higher penalty rate in cases where there are large negative deviations from its target, and at the same time, cease remuneration of excess reserves.\(^\dagger\) The last term, \(\varepsilon^R_t\), represents a cost shock.

From the balance sheet constraint (31), and given that \(L^F_t\) and \(D_t\) are determined by the private agents’ behaviour, borrowing from the central bank can be solved for residually. Therefore, using \(TR_t = \mu^{TR}_t D_t\) in (31) yields,

\[
L^B_t = L^F_t - (1 - \mu^{TR}_t)D_t. \tag{37}
\]

Using \(TR_t = \mu^{TR}_t D_t\) and substituting (36) and (37) in (35) gives the bank’s static optimization problem,

\[
\max_{\{\mu^{TR}_t, 1 + i^D_t, 1 + i^R_t\}} \left\{ (1 + \kappa^W) \lambda^{TR}_L L^F W + Q^F_t (1 + i^L_t) L^{F,I} + (1 - Q^F_t) \kappa^C K_t + (1 + i^M_t) \mu^{TR}_t D_t - (1 + i^D_t) D_t - (1 + i^R_t) [L^F_t - (1 - \mu^{TR}_t) D_t] - \left[ -\Phi C_1 (\mu^{TR}_t - \mu_t) + \frac{\Phi C_2}{2} (\mu^{TR}_t - \mu_t)^2 + \varepsilon^R_t \right] D_t \right\}. \tag{38}
\]

The first-order condition with respect to \(\mu^{TR}_t\) is,

\[
\mu^{TR}_t = \mu_t + \frac{(1 + i^M) + \Phi C_1 - (1 + i^R)}{\Phi C_2}. \tag{39}
\]

The difference between the total reserve ratio and the required reserve ratio \(\mu^{TR}_t - \mu_t\), represents the excess reserve ratio, \(\mu^{ER}_t\), which is given by,

\[
\mu^{ER}_t = \frac{(1 + i^M) + \Phi C_1 - (1 + i^R)}{\Phi C_2}. \tag{40}
\]

\(^\dagger\) These responses, however, are not explicitly accounted for in the model.
The first-order condition with respect to $1 + i^D_t$ is,

$$
(1 + i^M_t)\mu_t^{TR}\left(\frac{\partial D_t}{\partial (1 + i^D_t)}\right) - (1 + i_t^D)(\frac{\partial D_t}{\partial (1 + i^D_t)}) - D_t
$$

$$
+ (1 + i^R_t)(1 - \mu_t^{TR})(\frac{\partial D_t}{\partial (1 + i^D_t)}) - \Phi(\frac{\partial D_t}{\partial (1 + i^D_t)}) = 0,
$$

using $\eta_D = (\frac{\partial D_t}{\partial (1 + i^D_t)})(\frac{1 + i^D_t}{D_t})$ to represent the constant interest elasticity of the supply of deposits by the household results in,

$$
(1 + i^D_t) = (1 + \frac{1}{\eta_D})^{-1} [(1 + i^M_t)\mu_t^{TR} + (1 - \mu_t^{TR})(1 + i^R_t) - \Phi(\mu_t^{TR} - \mu_t)],
$$
or,

$$
1 + i_t^D = (1 + \frac{1}{\eta_D})^{-1}[1 + (1 + i^R_t) - \mu_t^{TR}(i^R_t - i^M_t)] + \Phi(\mu_t^{TR} - \mu_t) - \frac{\Phi\mu_t^{TR} - \mu_t}{2}.
$$

The first-order condition with respect to $1 + i^L_t$ is,

$$
Q_t^F L_t^{F,I} + Q_t^F (1 + i_t^L) \frac{\partial L_t^{F,I}}{\partial (1 + i_t^L)} - (1 + i_t^R) \frac{\partial L_t^{F,I}}{\partial (1 + i_t^L)} = 0,
$$

using $\eta_L = \frac{\partial L_t^{F,I}}{\partial (1 + i_t^L)} (1 + i_t^L)$ to denote the interest elasticity of demand for loans for investment yields,

$$
Q_t^F + Q_t^F (1 + i_t^L) \frac{\eta_L}{(1 + i_t^L)} - (1 + i_t^R) \frac{\eta_L}{(1 + i_t^L)} = 0,
$$
or,

$$
1 + i_t^L = \frac{1 + i_t^R}{Q_t^F[\eta_L^{-1} + 1]}.
$$

Equation (40) represents the bank’s excess reserve ratio. This shows that $\mu_t^{ER}$ increases with $i^M$ but falls with $i_t^R$. Therefore, the excess reserve ratio is decreasing in the spread between the interest rate on reserves and the refinance rate. By holding an additional unit of excess reserves at the central bank, the bank benefits by gaining $1 + i^M$; at the same time, it saves because it does not have to borrow from the central bank to meet reserve requirements. By contrast, the bank also incurs costs for holding the extra unit of excess reserves. Therefore, equation (40) balances the costs and the benefits of holding excess reserves. From equation (41), the (gross) interest rate on deposits depends on the marginal cost of borrowing from the central bank, which is lowered in the presence of remunerated reserves by the difference between the refinance rate and the interest rate on reserves. The deposit rate also depends on the costs associated with holding excess reserves. Equation (42) shows that the (gross) lending rate depends positively on the marginal cost of borrowing from the central bank and negatively on the repayment probability, $Q_t^F$. 

17
As in Agénor and Alper (2012), the repayment probability is taken to depend positively on "micro" and "macro" factors, namely, the real effective collateral-loan ratio and economic activity. Therefore, $Q_F^t$ increases with the collateral provided by firms and falls with the amount borrowed. Hence,

$$Q_F^t = \phi_0 \left( \frac{K_t}{C_L F^T L_t} \right)^{\phi_1} \left( \frac{Y_t}{Y} \right)^{\phi_2},$$

where $\phi_0, \phi_1, \phi_2 > 0$ and $(Y_t/Y)$ represents the output gap, with $Y$ denoting the steady-state value of output\(^{19}\) under fully flexible prices.

### 3.6 Central Bank

The central bank’s assets consist of government bonds, $B_t^C$, and loans to the commercial bank, $L_t^B$, whereas its liabilities consist of total reserves, $TR_t$, and currency supplied to households and firms, $M_t^s$. Therefore, the central bank’s balance sheet is given by,

$$B_t^C + L_t^B = TR_t + M_t^s.$$  \hfill (44)

Using $TR_t = \mu_t^{TR} D_t$ and rearranging, equation (44) becomes,

$$M_t^s = B_t^C + L_t^B - \mu_t^{TR} D_t.$$  \hfill (45)

Equation (45) shows the supply of currency is matched by government bonds and central bank loans extended to the commercial bank, less the fraction of deposits held at the central bank.

In this economy, the central bank sets the policy interest rate using a Taylor-type rule (see Taylor (1993)), and supplies all the liquidity the bank needs through a standing facility. The policy rule is of the form,

$$i_t^R = \chi i_{t-1}^R + (1 - \chi) [r + \pi_t + \varepsilon_1 (\pi - \pi^T) + \varepsilon_2 \ln (\frac{Y_t}{Y})] + \epsilon_t,$$

where $\chi \in (0,1)$ measures the degree of interest rate smoothing, $r$ is the steady-state value of the real interest rate on bonds, $\pi_t$ represents current inflation, $\pi^T \geq 0$ is the central bank’s inflation target, $\varepsilon_1, \varepsilon_2 > 0$ measure the relative weights on inflation deviations from its target and the output gap, respectively and $\epsilon_t$ is a serially correlated shock with constant variance, which follows a first order autoregressive process of the form,

$$\epsilon_t = \rho \epsilon_{t-1} \exp(\xi_t),$$

where $\rho \in (0,1)$ and $\xi_t \sim N(0, \sigma_{\xi}^2)$ is a serially correlated random shock with zero mean. The standard specification (46) will be extended later in the text to include a measure of excess reserves, in order to examine the dynamic effects of an increase in bank reserves under an interest rate rule which reacts directly to changes in liquidity.

\(^{19}\)Similar to other studies (see, for instance, Meh and Moran (2010)), the output gap is measured in terms of deviations from its steady-state value.
3.7 Government

The government purchases the final good, collects taxes, and issues one-period risk-free bonds, $B_t$, which are held by the central bank, $B^C_t$, and households, $B^H_t$. Total bonds can be denoted by, $B_t = B^C_t + B^H_t$. The government’s real budget constraint is given by,

$$B_t + T_t + i_t^{R} L_{t-1}^B P_{t-1}^B P_t^{-1} + i_t^{-1} B^C_{t-1} P_{t-1}^C P_t^{-1} - i^M T R_{t-1} P_{t-1}^T P_t = (48)$$

$$G_t + (1 + i_{t-1}) B_{t-1} P_{t-1}^t,$$

where $G_t$ denotes real government spending and $T_t$ represents real lump-sum tax revenues. The sum of the terms $i_t^{R} L_{t-1}^B P_{t-1}^B P_t^{-1} + i_t^{-1} B^C_{t-1} P_{t-1}^C P_t^{-1}$ and $i^M T R_{t-1} P_{t-1}^T P_t$ (adjusted for the rate of inflation) comes from the assumption that the net income earned by the central bank from lending to the commercial bank, holding government bonds and holding reserves from the commercial bank, respectively, is transferred to the government at the end of each period.

Government purchases represent a constant fraction, $\psi \in (0,1)$, of output of the final good,

$$G_t = \psi Y_t.$$  

(49)

4 Symmetric Equilibrium

In a symmetric equilibrium, all firms producing intermediate goods are identical so they produce the same output, and prices are the same across firms. Also, all households supply the same number of labour hours. Therefore, $K_{jt} = K_t$, $N_{jt} = N_t$, $Y_{jt} = Y_t$, $P_{jt} = P_t$, for all $j \in (0,1)$.

It is necessary for equilibrium conditions in the credit, deposit, goods and cash markets to be satisfied.\(^{20}\) The supply of loans by the commercial bank and supply of deposits by households are perfectly elastic at the prevailing interest rates; as a result, the markets for loans and deposits always clear. To satisfy equilibrium in the goods markets, production must be equal to aggregate demand. Thus, using (19),

$$Y_t = C_t + G_t + I_t + \frac{\phi_F}{2} (\frac{1 + \pi_t}{1 + \pi} - 1)^2 Y_t.$$  

(50)

The equilibrium condition of the market for cash is,

$$M_t^s = M_t^H + M_t^F,$$

(51)

where $M_t^F = \int_0^1 M_{j,t}^F dj$ represents the total cash holdings of intermediate good-producing firms and the capital good producer. It is assumed that bank loans to all firms are extended in the form of cash such that, $L_t^F = M_t^F$. Substituting this in (51), $M_t^s = M_t^H + L_t^F$. Replacing $M_t^s$ from (45) gives,

$$B^C_t + L^B_t - \mu_t^{TR} D_t = M_t^H + L_t^F.$$  

(52)

\(^{20}\)The equilibrium condition of the market for government bonds is eliminated by Walras’ Law.
Using $L_t^B$ from (37) into (52) gives,

$$
\bar{B}^C = M_t^H + D_t.
$$

(53)

Given that the total stock of bonds held by the central bank is constant, equation (53) implies that real cash balances are inversely related to real bank deposits. This equation also represents the money market equilibrium condition, from which the equilibrium bond rate is obtained.

5 Steady State and Log-Linearization

This section presents some of the key steady-state and log-linearized equations of the model. Further details on all the steady-state equations are shown in Appendix B, whereas the log-linearized equations are outlined in Appendix C.

Given that the steady state is characterized by zero inflation, from equation (5), the steady-state value of the bond rate (which is equal to the refinance rate) is given by,

$$
1 + i^B = 1 + i^R = 1 + r = \frac{1}{\beta}.
$$

The equality between $i^B$ and $i^R$ implies that the commercial bank has no incentive to borrow from the central bank in order to buy government bonds.

The steady-state deposit and lending rates are given by,

$$
1 + i^D = (1 + \frac{1}{\eta_D})^{-1}(1 + i^R) - \mu^T R (i^R - i^M)
$$

$$
+ \Phi_{C1} (\mu^T R - \mu) - \frac{\Phi_{C2}}{2} (\mu^T R - \mu)^2,
$$

$$
1 + i^L = \frac{1 + i^R}{Q^F [\eta_L^1 + 1]}.
$$

In the steady state, the repayment probability is inversely related to the firm’s assets over its liabilities,

$$
Q^F = \phi_0 \left( \frac{\kappa C K}{L F T} \right) \phi_1.
$$

The steady-state value of the excess reserve ratio is given by,

$$
\mu_{ER} = \frac{(1 + i^M) + \Phi_{C1} - (1 + i^R)}{\Phi_{C2}}.
$$

In order to solve the model, each variable is log-linearized around a non-stochastic, zero-inflation steady state. The log-linearized deposit rate is denoted by,$^{21}$

---

$^{21}$The reserve requirement ratio is exogenous in this model. Therefore, $\mu_t = 0$ except for the case when there is a shock to the variable.
\[ i_t^D = \frac{1}{(1 + i_D^D)(1 + \frac{1}{\eta_D})^{-1}} \left\{ (1 - \mu^{TR}) (1 + i_t^R) - \mu^{TR} \mu_t^{TR} (i_t^R - i^M) \right\} + \left[ \Phi_{C1} - \Phi_{C2} (\mu^{TR} - \mu) \right] (\mu^{TR} \mu_t^{TR} - \mu_t^t) \].

Log-linearizing the lending rate yields,
\[ i_t^L = i_t^R - \hat{Q}_t^F, \]
where, a linear approximation of the repayment probability gives,
\[ \hat{Q}_t^F = \phi_2 \hat{Y}_t + \phi_1 \left( \hat{K}_t - \hat{F}_t^{FI} \right). \]

From (40), the log-linearized excess reserve ratio is,
\[ \hat{\mu}_t^{ER} = -\left(1 + i_t^R \right) \frac{i_t^R}{\Phi_{C2} \mu_t^{ER}}. \]

Log-linearizing (22) gives the New Keynesian Phillips Curve (see Galí (2008) and Walsh (2010)), which states that current inflation depends on firms’ marginal costs and expected inflation,
\[ \hat{\pi}_t = \frac{(\theta - 1)}{\phi_F} \hat{m}_c_t + \beta E_t \hat{\pi}_{t+1}. \]

Log-linearizing equation (18), marginal costs are given by,
\[ \hat{m}_c_t = (1 - \alpha) (\hat{\kappa}_t^W i_t^R + \hat{\omega}_t) + \alpha \left( \frac{1 + r^K}{r^K} \right) \hat{r}_t^K - \hat{A}_t. \]

This equation shows that marginal costs depend positively on the real wage and the rental rate of capital, and negatively on aggregate supply shocks. Also, because \( \kappa^W > 0 \) based on the calibration, marginal costs are directly affected by changes in \( i_t^R \) (which represents the interest rate on short-term loans for working capital).

### 6 Calibration

The model is calibrated for Trinidad and Tobago due to the fact that the banking system in that country has recorded high persistent liquidity. The main data sources are The Central Bank of Trinidad and Tobago and The Ministry of Finance of Trinidad and Tobago. Because Trinidad and Tobago is a high-income developing economy (see The World Bank (2014)), it can be difficult to get estimates for some of the parameters. Hence, in cases where country-specific parameters are not readily available, estimates based on other studies for high- and middle-income countries are used. The calibration can therefore be applied to other developing countries that have the problem of excess bank reserves.

A summary of the parameter values is provided in Table 2. Regarding the parameters for the household, the steady-state value of beta (quarterly) for Trinidad and Tobago is calculated using,
\( \beta = 1/1 + r. \) This gives a value of beta equal to 0.985. The intertemporal elasticity of substitution, \( \sigma \), is taken to be 0.5, which is in line with estimates for middle-income countries (see Agénor and Montiel (2015)). Similar to Agénor and Alper (2012), the preference parameter for leisure, \( \eta_N \), is calibrated at 1.8. The preference parameter for composite monetary assets, \( \eta_X \), is set at 0.02, which is consistent with the values used in existing studies for other developing countries. Furthermore, the relative share of cash in narrow money, \( \nu \), is calibrated to be 0.2, consistent with the available data for Trinidad and Tobago for the period 2007-2011.

For the production side, the elasticity of demand for intermediate goods, \( \theta \), is 10.0, which corresponds to a steady-state mark-up rate of 11.1 percent. The share of capital in output of intermediate goods, \( \alpha \), is 0.3 and is consistent with estimates for developing countries. The cost channel parameter, \( \kappa^W \), is set at 0.45. Using the method proposed in Keen and Wang (2007), the value of the adjustment cost parameter for prices, \( \phi_F \), is calculated as 65. As is standard in the literature, the depreciation rate for capital is set equal to 0.034. Also, the adjustment cost parameter for investment, \( \Theta_K \), is set at 18.

In considering the parameters characterizing bank behaviour, the effective collateral-loan ratio, \( \kappa^C \), is set at a value of 0.05 which is consistent with the evidence in Trinidad and Tobago. There is little information on the values for the cost function parameters, \( \Phi_{C1} \) and \( \Phi_{C2} \). Hence, these coefficients are calibrated such that the differential between the steady-state total reserve ratio and the required reserve ratio is 4.5 percent, which is close to the actual spread observed in the recent data for Trinidad and Tobago. Using this approach gives a value of 0.35 for \( \Phi_{C1} \) and 7.5 for \( \Phi_{C2} \). The elasticity of the repayment probability with respect to collateral, \( \phi_1 \), is set at a relatively low value, 0.02; whereas the elasticity of the repayment probability with respect to cyclical output, \( \phi_2 \), is 0.2, as in Agénor et al. (2012).

On the central bank side, the required reserve ratio, \( \mu \), is set at 0.17, as imposed by the Central Bank of Trinidad and Tobago according to legislation. Similar to Agénor and Alper (2012), the lagged value of the policy rate in the interest rate rule, \( \chi \), is set to 0. The calibration therefore implies that there is direct interest rate smoothing from the central bank’s policy response. The parameters for the response of the refinancing rate to inflation deviations from its target and to output growth, \( \varepsilon_1 \) and \( \varepsilon_2 \), are set to 1.5 and 0.1, respectively, which are standard values estimated for Taylor-type rules in middle-income countries. The degree of persistence in the supply shock, \( \rho_{\Delta} \), and the shock to the refinancing rate, \( \rho_e \), are both set to 0.4. Finally, the share of government spending in output is estimated as 0.15, which is in line with the actual value observed for the period 2007-2011 in Trinidad and Tobago.

The steady-state deposit rate is calibrated at 1.05 percent, which is the actual value on average for the period 2007-2011.\(^{23}\) It is important to note that the low deposit rate is due to the high

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\(^{22}\)The average real bond rate, \( r \), for the period 2007-2011 is 1.52 percent. Using this value, \( \beta \) is equal to 0.985.

\(^{23}\)This represents the deposit rate announced by commercial banks for ordinary savings.
liquidity environment, which depresses the short-term interest rate.\textsuperscript{24} Further, the data show that the prime lending rate for Trinidad and Tobago is 10.7 percent on average during 2007-2011. This value is therefore used for the steady-state loan rate. Banks in Trinidad and Tobago earn no interest on primary reserve requirements. However, the Central Bank pays interest on secondary reserve requirements, as discussed in Section 2. The interest rate paid on reserves is set at a low value of 0.25 percent, which satisfies the condition that the interest rate on total reserves is less than the refinance rate. The ratio of excess reserves to total reserves in the steady state is 20.9 percent, which is close to the value observed in the recent data for Trinidad and Tobago. Further, in the steady state, the proportion of deposits held as total reserves is 21.5 percent. The steady-state value of the repayment probability is 97 percent; this implies the default probability is around 3 percent. The steady-state ratio of consumption to output is 68.1 percent, which is close to the value observed for the period 2007-2011. The cash plus deposit to output ratio in the steady state is set at 45.2 percent, which is in line with the actual value for the period 2007-2011. Also, in the steady state the ratio of investment to output is set at 16.8 percent, while the collateral-to-loan ratio is 1.47.

7 Policy Analysis

This section uses impulse response functions to study the dynamic effects of four shocks to the model. All the figures show the percent deviation of the variables from their steady-state values, with the exception of the total reserve ratio, the ratio of excess reserves to total reserves, inflation and the interest rate variables which are expressed in percentage points. The first case examines the impact of a negative supply shock; then the transmission of monetary policy following an increase in the central bank’s refinance rate is analyzed. The next experiment investigates the impact of a shock to reserve requirements. Following this, I examine the response of the model to a liquidity shock, taking the form of an increase in bank deposits. Finally, simultaneous shocks are administered to deposits and the required reserve ratio.

7.1 Negative Supply Shock

Figure 3 shows the impulse response functions of some of the main variables of the model following a one percent negative productivity shock. The direct effect of the shock is an immediate decline in output, and a rise in the marginal production costs, which in turn exerts an upward pressure on prices. As the rise in inflation dominates the fall in output, the policy rate, which is determined by the Taylor rule, rises as a result. The higher policy rate leads to a direct increase in the deposit rate, which in turn raises the demand for bank deposits, and reduces borrowing from the central bank. From the central bank’s balance sheet, a fall in loans to the commercial bank reduces the supply of currency, and therefore to restore equilibrium in the money market, the demand for cash must

\textsuperscript{24}Similarly, inspection of the data shows that the rise in excess reserves in the U.S. and Euro Area in recent years was associated with a sharp fall in the short-term interest rate.
fall. Because the central bank’s real bond holdings, which determine the total monetary assets, are fixed, the bond rate adjusts to clear the money market. Therefore, to reduce the demand for cash, the bond rate increases, which, through intertemporal substitution, leads to a fall in the level of current consumption. Overall, the higher rate of return on deposits and bonds increases households’ demand for these financial assets, and lowers their consumption.

A key point to note is that based on the calibration, the cost channel exists ($\kappa^W > 0$). Thus, owing to the fact that marginal costs depend directly on the policy interest rate, an increase in this rate tends to further raise firms’ marginal costs as it increases the labour costs of production. Furthermore, the increase in the refinance rate also translates to an immediate rise in the loan rate, which lowers the demand for investment and the level of physical capital over time. The collateral-to-loan ratio increases on impact as loans for investment fall by more than the value of collateral. Because the collateral effect is dominated by the cyclical output effect, the repayment probability falls, causing the loan rate to increase further, which in turn exerts an upward pressure on the rental rate of capital. Based on the calibration, the higher rental rate of capital offsets the fall in the level of physical capital, the rise in both labour supply and the refinance rate, causing real wages to increase upon the impact of the shock.\textsuperscript{25} The rise in real wages, in turn, creates further upward pressure on firms’ marginal costs.

In this model, excess reserves are positively related to their rate of return, but depend negatively on the refinance rate. Therefore, because the interest rate paid on reserves is fixed by the central bank, an increase in the marginal cost of borrowing from the central bank lowers the level of excess reserves. As the refinance rate and the other interest rates in the banking sector increase, the costs of holding excess reserves are higher. Put differently, there is a higher opportunity cost of holding excess reserves when the marginal cost of borrowing from the central bank increases. Thus, provided that the interest rate on reserves remains unchanged, the rise in other short-term interest rates indicates that banks can earn a higher return from investing in other assets, so they reduce demand for excess reserves. Given that the excess reserve ratio decreases, and that the required reserve ratio remains constant, the total reserve ratio also falls.

### 7.2 Monetary Policy Shocks

This section examines the transmission of two monetary policy shocks: an increase in the refinance rate and an increase in the reserve requirement ratio.

Figure 4 illustrates the general equilibrium effects of a one percent increase in the refinance rate. A rise in the policy interest rate raises the deposit and loan rates immediately. As in the previous case, the rise in the deposit rate increases households’ demand for bank deposits, and reduces their

\[\text{The value of capital, labour supply, the refinance rate and the rental rate of capital were calculated. The results showed that the increase in the rental rate of capital offsets the total (negative) value of all the other variables, bringing about an increase in real wages.}\]
incentive to hold cash. The higher level of bank deposits lowers both central bank borrowing and the money supply. Consequently, the government bond rate increases to reduce demand for cash and to restore equilibrium in the money market. In response to the rise in the bond rate, current consumption and output fall. At the same time, a higher loan rate leads to a reduction in loans for investment, which in turn causes the collateral-to-loan ratio to increase. However, as the drop in output dominates the rise in the collateral-to-loan ratio, the repayment probability falls. In this case, real wages fall by more than the value of physical capital, placing downward pressure on the rental rate of capital. The decline in marginal costs, which results from the drop in the rental rate of capital and real wages, creates a downward pressure on inflation.

Similar to the case of the negative productivity shock, the higher refinance rate increases the opportunity cost of holding excess reserves. As a consequence, the bank demands less excess reserves. The reduction in the quantity of excess reserves leads to an immediate fall in the level of total bank reserves.

Figure 5 shows the effects of a one percent increase in the minimum reserve requirement ratio, \( \mu_t \). Given that the required reserve rate is exogenous in this model, to assess the impact of an increase, it is assumed that \( \mu_t \) is stochastic and follows a first-order autoregressive process of the form:

\[
\mu_t = \mu_{t-1} \exp(\xi_t). 
\]

The impulse response functions show that an increase in reserve requirements does indeed lead to a reduction in the excess reserve ratio.\(^{26}\) In general, because the required reserve ratio goes up and the excess reserve ratio falls, the net effect on the total reserve ratio is a priori ambiguous; given our calibration, the net effect is positive, as shown in the simulations. Because the deposit rate is set as a mark-down on the total reserve ratio, a rise in total reserves leads to a fall in the interest rate on deposits. Consequently, household deposits fall, while borrowing from the central bank and the money supply increase. Equilibrium in the money market requires an increase in the demand for cash, which is brought about through a reduction in the bond rate, which in turn leads to a higher level of current consumption and output. The policy rate increases in response to the rise in output.

Furthermore, an increase in the marginal cost of borrowing from the central bank leads to a rise in the loan rate, which results in a higher rental rate of capital, a decline in investment and a lower capital stock over time. Primarily owing to the higher rental rate of capital, real wages increase. As observed previously, a higher lending rate leads to a reduction in investment loans, and a rise in the collateral-to-loan ratio. The higher output, along with the increase in the collateral-to-loan ratio, causes the repayment probability to rise. Nevertheless, the increase in the refinance rate dominates the response of the repayment probability, such that the loan rate rises although mitigated due to the fall in the perception of risk. Marginal costs increase because of three simultaneous effects:

\(^{26}\)Agénor and El Aynaoui (2010), using a simple (static) model with credit market imperfections, also found that raising reserve requirements can help to sterilize excess liquidity. Therefore, central banks use the required reserve ratio as a liquidity management tool (see Montoro and Moreno (2011), Robitaille (2011) and Tovar et al. (2012) for further discussions).
the increase in the refinance rate, higher real wages and the rise in the rental rate of capital. The increase in firms’ production costs creates an upward pressure on prices, which ultimately leads to an amplified rise in the refinance rate.

In a related study, Glocker and Towbin (2012) used an open economy model with competitive banking to examine whether reserve requirements are effective as a monetary policy tool. Although the focus of the Glocker-Towbin study was principally on reserve requirements and some of the features of the model were different to the one used in this study, the simulations of an increase in reserve requirements reveal similar findings. In both studies the results show that under an interest rate rule an increase in reserve requirements widens the spread between lending and deposit rates.\textsuperscript{27} The higher lending rate reduces investment and can lead to a fall in output. Owing to the fact that the bank holds excess reserves, it is fully responsive to the increase in reserve requirements and therefore cuts the deposit rate to induce households to reduce their demand for deposits. The lower level of deposits, in turn, leads to a lower ratio of excess reserves, while the fall in the deposit rate stimulates consumption. The total net effect of the shock on output depends on the relative changes in consumption and investment. Based on the calibration, the effect of the increase in consumption dominates the fall in investment, causing output to rise. This implies that changes in reserve requirements are procyclical, with respect to economic activity.\textsuperscript{28}

It is important to note that the effects of changes in the reserve requirement ratio on economic activity depend—to a great extent—on the use of other monetary policy instruments. In the case where mainly direct policy tools are used to effect monetary policy, a higher required reserve ratio can lead to an increase in demand for deposits by banks, which will cause the deposit rate to increase, and inflation and output to fall. However, under an interest rate rule, the effect of a higher level of reserve requirements acting as a tax on banks dominates, so the central bank accommodates the increase in required reserves, and the spread between the lending and deposit rates increases.

In the case of a decrease in reserve requirements, the deposit rate rises and the loan rate falls. The fall in the loan rate increases investment, while the rise in the deposit rate reduces consumption. Usually, the overall effect brings about a fall in output because the drop in consumption dominates the increase in investment. This is contrary to the conventional view of monetary policy whereby a fall in required reserves brings about an expansionary effect. In keeping with the conventional view, the findings from a study by Areosa and Coelho (2013) showed that a decrease in reserve requirements caused output to increase. It must however be noted that in their study, the condition specified for the loan rate, which was not derived optimally, ensured that a lower reserve requirement ratio generated an overall countercyclical effect.

\textsuperscript{27} Montoro and Moreno (2011) and Tovar et al. (2012) also pointed out that an increase in required reserves acts as a tax on banks, so the spread between deposit and lending rates widens.

\textsuperscript{28} See Baltensperger (1982) and Horrigan (1988) for further discussions on the impact of changes in reserve requirements on economic stability.
7.3 Liquidity Shock: Increase in Bank Deposits

As illustrated in Figure 6, the direct effect of an exogenous one percent increase in bank deposits is an immediate rise in the ratio of excess reserves to total reserves. Given that required reserves remain constant, the increase in excess reserves leads to a rise in total reserves. The higher level of bank deposits also reduces borrowing from the central bank, thereby lowering money supply. Similar to the cases of the supply shock and the monetary shock, to restore equilibrium in the money market, the bond rate rises, which, through intertemporal substitution, results in a reduction in current consumption and a fall in output. In response to the drop in output, the policy interest rate falls, leading to a downward pressure on the loan rate, which in turn reduces the rental rate of capital. A lower rental rate of capital increases the demand for physical capital and investment. As observed previously, the fall in the rental rate of capital reduces real wages; in turn, the fall in both variables results in a drop in the firms’ marginal costs and thus inflation. The decrease in prices leads to an amplified drop in the policy rate. Also, the lower loan rate stimulates investment, leading to a fall in the collateral-to-loan ratio, which combined with the contraction in output, causes the repayment probability to fall. The lower refinance rate attenuates the response of the repayment probability to the shock, leading to an amplified decline in the lending rate.

7.4 Simultaneous Shocks to Deposits and Reserve Requirements

The results from the previous sections show that a shock to deposits increases excess reserves, but a shock to the reserve requirement ratio reduces the amount of excess liquidity in the banking system. Therefore, this section investigates the impact of simultaneous one percent shocks to deposits and required reserves. The premise for this experiment is to examine if there is an increase in excess liquidity, and at the same time the central bank responds in a non-systematic manner by raising reserve requirements, whether fluctuations in excess reserves can be reduced. Figure 7 shows the impulse response functions of the shock to deposits alone, and the joint shocks. The results show indeed that when there is a positive shock to liquidity, and there is a simultaneous increase in reserve requirements, the volatility in excess reserves is reduced. Therefore, this finding provides formal evidence (in line with the practical evidence on central bank policymaking), that raising reserve requirements, when there is an exogenous increase in liquidity, helps to sterilize excess reserves. It should also be noted that under the combined shock, there is lower volatility for the interest rate variables. Notably, fluctuations in inflation and output are also reduced.

8 Policy Rules for Managing Excess Reserves

This section examines the macroeconomic effects of a one percent liquidity shock when two alternative policy rules are used—an augmented Taylor rule and a countercyclical rule for the required reserve ratio.
8.1 An Augmented Taylor Rule

First, consider the case where the central bank adjusts its policy rate directly in response to changes in excess reserves. The rationale for this is to examine how effective a policy rule which responds to fluctuations in excess reserves may be in mitigating the volatility in the main variables of the model, under a shock to deposits. The augmented interest rate rule in its log-linear form is given by,

\[
\hat{i}_t^R = \chi \hat{i}^R_{t-1} + (1 - \chi)[\varepsilon_1(\hat{\pi}_t) + \varepsilon_2(\hat{Y}_t) + \varepsilon_3(\hat{ER}_{TR_t})] + \epsilon_t,
\]

where \( \hat{ER}_{TR_t} \) denotes the ratio of excess reserves to total reserves. Primus (2012) estimated an augmented Taylor rule for Trinidad and Tobago which included a measure of excess reserves. The results from her study showed the relative weight corresponding to deviations in excess reserves from steady state, \( \varepsilon_3 \), is \(-0.03\). The negative coefficient indicates that in response to an increase in excess reserves, the central bank lowers the policy rate to reduce incentive for banks to hold excess liquidity. Intuitively, if the penalty rate for not meeting the reserve requirement is low, commercial banks would tend to reduce their holdings of excess reserves. By contrast, if the penalty rate is high, banks would voluntarily hold excess reverses as a measure of precaution to avoid any shortfall in liquidity. A reduction in the policy rate is also expected to lower the deposit rate, which in turn discourages household deposits.

The estimated value of \(-0.03\) was quite low and had only a marginal effect on the policy rule and the model by extension. As a result of this, alternative values of \(-0.05\) and \(-0.2\) are considered for \( \varepsilon_3 \). To assess the impact of an augmented policy rule on volatility, I compare the asymptotic standard deviations and the relative standard deviations of the main variables in the model under a liquidity shock with the augmented Taylor rule and the standard Taylor rule (see Table 3). Figure 8 compares the simulations of a shock to deposits under both rules when \( \varepsilon_3 \) is set at \(-0.2\). The results from Figure 8 and Table 3 indicate that the augmented Taylor rule is effective in reducing the volatility of key macroeconomic variables such as inflation, consumption and output (among other variables), following an exogenous increase in deposits. In addition, the refinance rate, the loan rate, the deposit rate, the total reserve ratio and the ratio of excess reserves to total reserves are slightly more volatile. Further, under the augmented rule, the more volatile negative reaction of the loan rate stimulates investment; as a result, fluctuations in investment rise.

8.2 A Countercyclical Reserve Requirement Rule

The second case investigates the macroeconomic effects of a policy rule in which the required reserve ratio is determined by its previous value, a fraction of its steady-state value, and deviations in excess reserves. Therefore, in this approach, the required reserve ratio is endogenous and serves a countercyclical role for managing changes in excess reserves. The central bank is assumed to set
the reserve requirement rule according to the following,

\[ \hat{\mu}_t = (1 - \tau)\mu + \zeta (ER_{TR} - TR_t) + \tau \hat{\mu}_{t-1}, \]

where \( \tau \) denotes the degree of persistence in the policy rule and \( \zeta \), which measures deviations in the ratio of excess reserves to total reserves from its steady state, is an indicator of cyclical conditions. There is little information on the values for \( \tau \) and \( \zeta \); hence, for illustrative purposes a value of 0.12 is used for \( \tau \), and \( \zeta \) is set at 0.85. Thus, there is a relatively low degree of persistence in changes of the required reserve ratio. Also, the positive value for \( \zeta \) means that the central bank increases required reserves when there is a rise in excess bank liquidity.

To assess whether the countercyclical rule can help to reduce volatility, I compare the asymptotic standard deviations and the relative standard deviations of the main variables when the reserve requirement ratio is endogenous and exogenous to the model. The results from Table 4 and the impulse response functions in Figure 9 show that the countercyclical reserve requirement rule is successful in reducing fluctuations in excess reserves and total reserves, but at the expense of inflation and output being more volatile. The results also indicate that the reserve requirement rule has no effect on the other macroeconomic variables in the model.

9 Sensitivity Analysis

To test the robustness of the results, this section examines the responses of the key variables in the model under alternative parameter configurations. Specifically, I examine the case where there is a stronger cost channel effect, an increase in the reserve requirement ratio and a higher level of excess reserves.

9.1 Higher Cost Channel Effect

This experiment examines the case where firms finance all their working capital needs from bank loans, that is, from (15) an increase in \( \kappa^W \) from 0.45 to 1, under a one percent shock to the policy interest rate. In general, the results (which are not reported here to save space) are similar to the benchmark case (Figure 4).\textsuperscript{29} The main difference is that there is a slight increase in volatility of the financial variables—that is, the excess to total reserve ratio, the total reserve ratio, the deposit rate, the loan rate and the bond rate. Intuitively, if firms use bank loans to finance all their working capital needs, there is a stronger (direct) cost channel effect; so the higher refinance rate is reflected in the financial variables.

\textsuperscript{29}Given that the results are not significantly different, further empirical testing can be conducted to determine whether the cost channel parameter—which is positive—is also significant for Trinidad and Tobago (see, for instance, Malikane (2012)).
9.2 Increase in the Reserve Requirement Ratio

This experiment evaluates the impact of a more aggressive use of reserve requirements under a one percent increase in the policy rate. Thus, the reserve requirement ratio was raised by 3 percent, which implies that $\mu$ increases from 17 percent to 20 percent. The results, which are presented in Figure 10, show that on impact of the shock, the excess reserve ratio falls by more when the reserve requirement ratio is higher. Therefore, a contractionary monetary policy is more successful in reducing excess reserves, if reserve requirements are used more aggressively. In addition, the financial variables are slightly more volatile when the reserve requirement ratio is higher.

9.3 Increase in the Excess Reserve Ratio

This experiment examines the impact of a higher excess reserve ratio, under a monetary policy shock and a shock to the reserve requirement ratio. To quantify these effects, the cost function parameters, $\Phi_{C1}$ and $\Phi_{C2}$, are changed from 0.35 and 7.5 to 0.2 and 2.2 respectively, so the excess reserve ratio, $\mu^{ER}$, is increased by 4 percent.

Figure 11 shows the results of a higher excess reserve ratio under a one percent increase in the policy interest rate. The findings reveal that when there is a higher level of excess reserves (that is the case where $\Phi_{C1} = 0.2$ and $\Phi_{C2} = 2.2$), the interest rate variables are slightly more volatile but the real variables remain unchanged. Also, given that the bank holds more excess reserves, on impact of the shock volatility in the excess reserve ratio and the total reserve ratio is amplified, and both variables fall by substantially more compared to the case where the proportion of excess reserves is lower.

Furthermore, the case of an increase in the excess reserve ratio under a one percent positive shock to the reserve requirement ratio was considered. The results, which are not reported here to save space, show that the ratio of excess reserves to total reserves is highly more volatile on impact of the shock—as compared to the benchmark case (Figure 5). However, fluctuations in the total reserve ratio are mitigated because of the higher reserve requirement ratio.

10 Concluding Remarks

The purpose of this paper was to examine the dynamic effects of excess reserves in a New Keynesian general equilibrium model with banking. For this purpose, I extend and modify the framework presented in Agénor and Alper (2012) and Agénor et al. (2013), by allowing the monopoly bank to hold excess reserves. As in Glocker and Towbin (2012), the model explicitly accounts for the fact that banks incur convex costs in holding excess reserves, which are proportional to their deposit holdings. Similar to the practice in a few countries, the bank receives interest payments on reserves from the central bank. Other notable features of the model are that it accounts for credit market imperfections and it incorporates a cost channel because intermediate good-producing firms must
borrow in advance to finance their working capital needs. Also, the supply of bank loans to firms is perfectly elastic at the prevailing lending rate, and the central bank’s supply of liquidity is perfectly elastic at the policy interest rate.

The model, which was calibrated for Trinidad and Tobago, was used to explain the main macroeconomic variables responses to a negative supply shock, a shock to the refinance rate, a shock to reserve requirements and a liquidity shock. I also examined the case where there were simultaneous shocks to reserve requirements and liquidity. The simulations show that under both a negative supply shock and a contractionary monetary shock, the refinance rate increases, leading to a rise in demand for financial assets such as deposits and bonds. The higher policy rate also increases the opportunity cost of holding excess reserves, leading the bank to reduce demand for these assets; as a result, excess reserves fall. The simulations of the positive shock to the required reserve ratio show that an increase in reserve requirements leads to an immediate fall in excess reserves. However, although a higher level of required reserves mitigates the volatility in excess reserves, there is an expansionary effect on inflation and output. This result is similar to a study by Glockler and Tobin (2012) that found an increase in reserve requirements reduces the deposit rate, which in turn stimulates economic activity. Thus, although raising reserve requirements helps to mitigate fluctuations in reserves, it creates a procyclical effect. This implies that financial stability may come at a cost of macroeconomic stability. This therefore raises the question of the optimal combination of instruments which will ensure that the objectives of macroeconomic stability and financial stability are achieved.

Furthermore, the results from the joint shocks indicate that the impact of an exogenous rise in bank deposits can be less dampening on macroeconomic variables if the required reserve ratio increases simultaneously. This experiment therefore supports the decision—which has indeed been practiced by many central banks—to raise reserve requirements in order to sterilize excess reserves. This finding is also in line with many other contributions in the literature, which suggest that central banks should raise reserve requirements to reduce excess reserves (see for instance Agénor and El Aynaoui (2010), Gray (2011), Montoro and Moreno (2011), Robitaille (2011) and Tovar et al. (2012)).

In addition, I examine two policy rules aimed at reducing macroeconomic fluctuations under a liquidity shock: an augmented interest rate rule which includes a measure of excess liquidity and an endogenous countercyclical reserve requirement rule. Given that banks have a voluntary demand for excess reserves, the rationale for the first rule is to investigate whether the volatility of key macroeconomic and financial variables can be reduced when the policy interest rate responds to changes in excess reserves. Therefore, in the case where there is a positive deviation in excess reserves, the central bank is likely to respond by reducing the policy interest rate. A lower refinance rate makes borrowing from the central bank less costly and discourages banks from holding excess reserves. In the second policy experiment I examine whether a countercyclical reserve requirement rule can
reduce fluctuations in excess reserves. The findings show that although the augmented Taylor rule is successful in reducing volatility in real variables, the financial variables of the model become more volatile. The results from the countercyclical rule indicate that if the reserve requirement ratio is used in a countercyclical fashion, it can help to stabilize fluctuations in excess reserves, but at the expense of inflation and output being slightly more volatile.

The Central Bank of Trinidad and Tobago has been using reserve requirements to reduce excess reserves as mentioned in Section 2. This research suggests that the Central Bank can use a countercyclical reserve requirement rule to adjust the required reserve ratio. One advantage of using this rule is that it can help to reduce the adverse effects associated with sharp increases (or decreases) in reserve requirements. Also, the Central Bank should continue to use Treasury bills and notes as these can help to reduce excess reserves and influence market interest rates. Thus, the use of short-term instruments can help to reduce the effects of an increase in reserve requirements on the deposit rate. Furthermore, owing to the fact that changes in the policy rate are weakly transmitted to real variables because of high financial system liquidity, using an augmented interest rate rule which is adjusted in response to deviations in excess reserves can help to mitigate the effects of an increase in liquidity on real variables.
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Table 1. Reserve Requirement Levels in Various Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Required Reserve Ratios (%) (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>None</td>
</tr>
<tr>
<td>Belize</td>
<td>8.5</td>
</tr>
<tr>
<td>Brazil</td>
<td>44*</td>
</tr>
<tr>
<td>Canada</td>
<td>None</td>
</tr>
<tr>
<td>Cape Verde</td>
<td>18</td>
</tr>
<tr>
<td>China</td>
<td>20**</td>
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<tr>
<td>Croatia</td>
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<td>Euro Area</td>
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<td>Guyana</td>
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<tr>
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<td>Malawi</td>
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<td>The Bahamas</td>
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<td>Trinidad and Tobago</td>
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<td>United Kingdom</td>
<td>None</td>
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<td>United States</td>
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</tbody>
</table>

Notes: The reserve requirement ratio refers to holdings of domestic currency liabilities; *Reserve requirements on demand deposits; **For large financial institutions; ***Primary reserve requirements.

Source: Author’s survey.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.985</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.5</td>
<td>Elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\eta_N$</td>
<td>1.8</td>
<td>Relative preference for leisure</td>
</tr>
<tr>
<td>$\eta_X$</td>
<td>0.02</td>
<td>Relative preference for money holdings</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
<td>Share parameter in index of money holdings</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>10.0</td>
<td>Elasticity of demand, intermediate goods</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
<td>Share of capital in output, intermediate good</td>
</tr>
<tr>
<td>$\phi_F$</td>
<td>65</td>
<td>Adjustment cost parameter, prices</td>
</tr>
<tr>
<td>$\Theta_K$</td>
<td>18</td>
<td>Adjustment cost parameter, investment</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.034</td>
<td>Depreciation rate of capital</td>
</tr>
<tr>
<td><strong>Bank</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa^C$</td>
<td>0.05</td>
<td>Effective collateral-loan ratio</td>
</tr>
<tr>
<td>$\Phi_{C1}$</td>
<td>0.35</td>
<td>Linear cost function parameter</td>
</tr>
<tr>
<td>$\Phi_{C2}$</td>
<td>7.5</td>
<td>Quadratic cost function parameter</td>
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<tr>
<td>$\phi_1$</td>
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<td>Elasticity of risk premium with respect to collateral</td>
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<tr>
<td>$\phi_2$</td>
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<td>Elasticity of risk premium with respect to cyclical output</td>
</tr>
<tr>
<td>$\kappa^W$</td>
<td>0.45</td>
<td>Share of financing working capital</td>
</tr>
<tr>
<td><strong>Central Bank</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.17</td>
<td>Reserve requirement ratio</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0</td>
<td>Degree of persistence in interest rate rule</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>1.5</td>
<td>Response of refinance rate to inflation deviations</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>0.1</td>
<td>Response of refinance rate to output growth</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.15</td>
<td>Share of government spending in output</td>
</tr>
<tr>
<td><strong>Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_A (\rho_L)$</td>
<td>0.4</td>
<td>Degree of persistence, supply shock (shock to refinance rate)</td>
</tr>
</tbody>
</table>
Table 3. Standard Deviations under Standard Taylor Rule and Augmented Taylor Rule with Increase in Bank Deposits

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\varepsilon_3 = 0$</th>
<th>$\varepsilon_3 = -0.03$</th>
<th>$\varepsilon_3 = -0.05$</th>
<th>$\varepsilon_3 = -0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refinance rate</td>
<td>0.0049</td>
<td>0.0049</td>
<td>1.0000</td>
<td>0.0050</td>
</tr>
<tr>
<td>Loan rate</td>
<td>0.0046</td>
<td>0.0046</td>
<td>1.0000</td>
<td>0.0047</td>
</tr>
<tr>
<td>Bond rate</td>
<td>0.0012</td>
<td>0.0012</td>
<td>1.0000</td>
<td>0.0012</td>
</tr>
<tr>
<td>Deposit rate</td>
<td>0.0038</td>
<td>0.0038</td>
<td>1.0000</td>
<td>0.0039</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0031</td>
<td>0.0030</td>
<td>0.9677</td>
<td>0.0028</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0021</td>
<td>0.0020</td>
<td>0.9524</td>
<td>0.0019</td>
</tr>
<tr>
<td>Total reserve ratio</td>
<td>0.0031</td>
<td>0.0031</td>
<td>1.0000</td>
<td>0.0031</td>
</tr>
<tr>
<td>Excess-total reserves</td>
<td>0.0116</td>
<td>0.0117</td>
<td>1.0086</td>
<td>0.0118</td>
</tr>
<tr>
<td>Output</td>
<td>0.0016</td>
<td>0.0015</td>
<td>0.9375</td>
<td>0.0014</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0001</td>
<td>0.0005</td>
<td>5.0000</td>
<td>0.0008</td>
</tr>
<tr>
<td>Rental rate capital</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.8750</td>
<td>0.0007</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.0109</td>
<td>0.0100</td>
<td>0.9174</td>
<td>0.0095</td>
</tr>
<tr>
<td>Marginal cost</td>
<td>0.0138</td>
<td>0.0129</td>
<td>0.9348</td>
<td>0.0123</td>
</tr>
<tr>
<td>Repayment prob.</td>
<td>0.0003</td>
<td>0.0003</td>
<td>1.0000</td>
<td>0.0003</td>
</tr>
<tr>
<td>Collateral-loan ratio</td>
<td>0.0001</td>
<td>0.0005</td>
<td>5.0000</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Notes: *Sd. Dev. is the standard deviation; ** Rel. S.D. denotes the relative standard deviation.
Table 4. Standard Deviations when the Reserve Requirement Ratio is Exogenous and Endogenous

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\mu_t = 0$ Sd. Dev.</th>
<th>$\hat{\mu}<em>t = (1 - \tau)\mu + \zeta(ER</em>{TR_t}) + \tau\hat{\mu}_{t-1}$ Sd. Dev.*</th>
<th>Rel. S.D.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refinance rate</td>
<td>0.0049</td>
<td>0.0049</td>
<td>1.0000</td>
</tr>
<tr>
<td>Loan rate</td>
<td>0.0046</td>
<td>0.0046</td>
<td>1.0000</td>
</tr>
<tr>
<td>Bond rate</td>
<td>0.0012</td>
<td>0.0012</td>
<td>1.0000</td>
</tr>
<tr>
<td>Deposit rate</td>
<td>0.0038</td>
<td>0.0038</td>
<td>1.0000</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0031</td>
<td>0.0032</td>
<td>1.0323</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0021</td>
<td>0.0021</td>
<td>1.0000</td>
</tr>
<tr>
<td>Total reserve ratio</td>
<td>0.0031</td>
<td>0.0024</td>
<td>0.7742</td>
</tr>
<tr>
<td>Excess-total reserves</td>
<td>0.0116</td>
<td>0.0078</td>
<td>0.6724</td>
</tr>
<tr>
<td>Output</td>
<td>0.0016</td>
<td>0.0017</td>
<td>1.0625</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0001</td>
<td>0.0001</td>
<td>1.0000</td>
</tr>
<tr>
<td>Rental rate capital</td>
<td>0.0008</td>
<td>0.0008</td>
<td>1.0000</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.0109</td>
<td>0.0109</td>
<td>1.0000</td>
</tr>
<tr>
<td>Marginal cost</td>
<td>0.0138</td>
<td>0.0138</td>
<td>1.0000</td>
</tr>
<tr>
<td>Repayment prob.</td>
<td>0.0003</td>
<td>0.0003</td>
<td>1.0000</td>
</tr>
<tr>
<td>Collateral-loan ratio</td>
<td>0.0001</td>
<td>0.0001</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Notes: *Sd. Dev. is the standard deviation; ** Rel. S.D. denotes the relative standard deviation.
Figure 1. The Percent of Excess Reserves to Total Reserves in Trinidad and Tobago

Source: Central Bank of Trinidad and Tobago Database.
Figure 2. Net Domestic Fiscal Injections (NDFI) as a percent of GDP, Excess Reserves to Total Reserves (ER_TR) and Government Expenditure (GOV'T EXP) to GDP

Source: Central Bank of Trinidad and Tobago Database.
Figure 3. Negative Supply Shock

Absolute deviations from baseline, unless otherwise indicated.
Absolute deviations from baseline, unless otherwise indicated.
Figure 5. Increase in Reserve Requirement Ratio

Absolute deviations from baseline, unless otherwise indicated.
Figure 6. Increase in Bank Deposits

Absolute deviations from baseline, unless otherwise indicated.
Figure 7. Increase in Bank Deposits and Simultaneous Shocks to Deposits and Reserve Requirements

Absolute deviations from baseline, unless otherwise indicated.
Figure 8. Increase in Bank Deposits under Standard Taylor Rule and Augmented Taylor Rule

Absolute deviations from baseline, unless otherwise indicated.
Figure 9. Increase in Bank Deposits when the Reserve Requirement Ratio is Exogenous and Endogenous

Absolute deviations from baseline, unless otherwise indicated.
Figure 10. Increase in the Required Reserve Ratio under a Contractionary Monetary Policy Shock

Absolute deviations from baseline, unless otherwise indicated.
Figure 11. Increase in the Excess Reserve Ratio under a Contractionary Monetary Policy Shock

Absolute deviations from baseline, unless otherwise indicated.
Appendix A
Solution to Optimization Problems

Solutions to the optimization problems of the household:

Substituting (2) in (1) subject to (4), the Lagrangian function for the household problem can be written as,

\[ L = E_t \sum_{t=0}^{\infty} \beta^t \frac{[C_{ht}]^{1-\sigma}}{1-\sigma} + \eta_N \ln(1 - N_{ht}) + \eta_X \ln[(M_{ht}^{H})^{\nu} D_{ht}^{1-\nu}] \]

\[ + \Lambda_t \{ \omega_t N_{ht} - T_{ht} + M_{ht-1}^{H} \frac{P_{t-1}}{P_t} + (1 + i_t^D) D_{ht-1} - \frac{P_{t-1}}{P_t} \}
\]

\[ + (1 + i_{t-1}^B) B_{ht-1}^{H} + J_t^I + \varphi_t J_t^B + J_t^K - C_{ht} - M_{ht}^{H} - D_{ht} - B_{ht}^{H} \]

where \( \Lambda_t \) denotes the Lagrange multiplier. Let \( \pi_{t+1} = (P_{t+1} - P_t) / P_t \) denote the inflation rate; maximizing (A1) with respect to \( C_{ht}, N_{ht}, M_{ht}^{H}, D_{ht} \) and \( B_{ht}^{H} \), taking \( i_t^D, i_t^B, P_t \), and \( T_{ht} \) as given yields the following first order conditions,

\[ C_{ht}^{-1/\sigma} = \Lambda_t, \]  
\[ \eta_N \frac{1}{1 - N_{ht}} - \Lambda_t \omega_t = 0, \]
\[ \frac{\eta_X \nu}{M_{ht}^{H}} - \Lambda_t + \beta E_t \left( \frac{\Lambda_{t+1}}{1 + \pi_{t+1}} \right) = 0, \]
\[ \frac{\eta_X (1 - \nu)}{D_{ht}} - \Lambda_t + \beta E_t \left\{ \Lambda_{t+1} \left( \frac{1 + i_t^D}{1 + \pi_{t+1}} \right) \right\} = 0, \]
\[ \beta E_t \left( \frac{\Lambda_{t+1}}{1 + \pi_{t+1}} \right) = \frac{\Lambda_t}{1 + i_t^B}. \]

The transversality condition is given by,

\[ \lim_{s \to \infty} E_{t+s} \Lambda_{t+s} \beta^s (M_{t+s}^{H}) = 0. \]

Combining (A2) and (A6), the Euler equation is,

\[ C_{ht}^{-1/\sigma} = \beta E_t \left[ (C_{ht+1})^{-1/\sigma} \left( \frac{1 + i_t^B}{1 + \pi_{t+1}} \right) \right]. \]

Using (A2) in (A3) and rearranging yields the supply of labour equation,

\[ N_{ht} = 1 - \frac{\eta_N (C_{ht})^{1/\sigma}}{\omega_t}. \]

Substituting (A6) in (A4) and using (A2) gives the demand for real cash balances,

\[ M_{ht}^{H} = \frac{\eta_X \nu (C_{ht})^{1/\sigma} (1 + i_t^B)}{i_t^B}. \]
Combining (A2), (A5) and (A6), the real demand for bank deposits is,

\[ D_{ht} = \frac{\eta X(1-\nu)(C_{ht})^{1/\sigma}(1+i^{B})}{i^{B} - i^{D}}. \]  

(A11)

Solutions to the optimization problems of the final good-producing firm:
The profit maximization problem of the final good producer is given by,

\[ \max_{Y_{jt}} P_{t} \left\{ \int_{0}^{1} Y_{jt}^{(\theta-1)/\theta} \, dj \right\}^{\theta/(\theta-1)} - \int_{0}^{1} P_{jt} Y_{jt} \, dj. \]  

(A12)

The first-order condition with respect to \( Y_{jt} \) gives,

\[ P_{t} \left( \int_{0}^{1} Y_{jt}^{\theta-1/\theta} \, dj \right)^{1/\theta-1} Y_{jt}^{-1/\theta} - P_{jt} = 0. \]

Given that \( Y_{jt}^{1/\theta} = \left\{ \int_{0}^{1} Y_{jt}^{(\theta-1)/\theta} \, dj \right\}^{1/(\theta-1)} \), this can be written as,

\[ Y_{jt} = \left( \frac{P_{jt}}{P_{t}} \right)^{-\theta} Y_{t}. \]  

(A13)

Equation (A13) gives the demand for each intermediate good \( j \).

Using (A13) in the final good production function (10), then making \( P_{t} \) the subject, the final good price can be denoted as,

\[ P_{t} = \left[ \int_{0}^{1} (P_{jt})^{1-\theta} \, dj \right]^{1/(1-\theta)}. \]  

(A14)

Solutions to the optimization problems of intermediate good-producing firms:
The first-stage minimization problem for firm \( j \) is,

\[ \min_{N_{jt},K_{jt}} \left[ (1 + \kappa^{W} i_{t}^{R}) \omega_{t} N_{jt} + r_{t}^{K} K_{jt} \right]. \]  

(A15)

Minimizing (A15) subject to (14), the Lagrangian function for this problem is,

\[ \mathcal{L}^{IG} = (1 + \kappa^{W} i_{t}^{R}) \omega_{t} N_{jt} + r_{t}^{K} K_{jt} + \lambda_{t} \left[ Y_{jt} - A_{t} K_{jt}^{\alpha} N_{jt}^{1-\alpha} \right], \]  

(A16)

where \( \lambda_{t} \) denotes the Lagrange multiplier. The first-order condition with respect to \( N_{jt} \) yields,

\[ (1 + \kappa^{W} i_{t}^{R}) \omega_{t} = \lambda_{t} (1-\alpha) \frac{Y_{jt}}{N_{jt}}. \]  

(A17)

The first-order condition with respect to \( K_{jt} \) gives,

\[ r_{t}^{K} = \alpha \frac{Y_{jt}}{K_{jt}} \lambda_{t}. \]  

(A18)

Combining (A17) and (A18) gives,

\[ N_{jt} = \frac{(1-\alpha)}{\alpha} \frac{r_{t}^{K}}{(1 + \kappa^{W} i_{t}^{R}) \omega_{t}} K_{jt}. \]  

(A19)
With $Y_{jt} = 1$, the constraint (14) can be rewritten as $1 - A_t K_{jt}^\alpha N_{jt}^{1-\alpha} = 0$. Using (A19) in this gives,

$$K_{jt} = A_t^{-1} \left[ \frac{\alpha}{(1-\alpha)} (1 + \kappa W^R i_t) \omega_t \right]^{1-\alpha}. \quad (A20)$$

From (A19) and (A20), $N_{jt}$ is,

$$N_{jt} = A_t^{-1} \left[ \frac{\alpha}{(1-\alpha)} (1 + \kappa W^R i_t) \omega_t \right]^{-\alpha}. \quad (A21)$$

Combining (A20) and (A21) the capital-labour ratio is,

$$\frac{K_{jt}}{N_{jt}} = \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{1 + \kappa W^R i_t \omega_t}{r_t^K} \right). \quad (A22)$$

Combining (A17) and (A18) yields,

$$[(1-\alpha) + \alpha] \lambda_t = \lambda_t = \left[ (1 + \kappa W^R i_t \omega_t) N_{jt} + r_t^K K_{jt} \right]. \quad (A23)$$

This implies that $\lambda_t$ is also equal to the unit real marginal cost, $mc_{jt}$. Using (14) and (A22) in this gives the unit real marginal cost as,

$$mc_{jt} = \left[ (1 + \kappa W^R i_t \omega_t) \right]^{1-\alpha} \left( \frac{r_t^K}{\alpha \alpha (1-\alpha) A_t^{1-\alpha}} \right). \quad (A24)$$

The second-stage optimization problem for the intermediate good producer is,

$$\max_{P_{jt}} P_t \sum_{t=0}^\infty \beta^t \Lambda_t \left\{ \left[ \frac{P_{jt}}{P_t} \right]^{1-\theta} Y_t - mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t - \left( \frac{\phi_F}{2} \left( \frac{P_{jt}}{P_{jt-1}} \right) - 1 \right)^{2} Y_t \right\}. \quad (A25)$$

Taking $mc_{jt}$, $P_t$ and $Y_t$ as given, the first-order condition with respect to $P_{jt}$ is,

$$(1-\theta)\Lambda_t \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t + \theta \Lambda_t mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\theta-1} Y_t - \Lambda_t \phi_F \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) \frac{Y_t}{P_{jt-1}} = 0. \quad (A25)$$

Solutions to optimization problems of the capital good producer:

The optimization problem of the capital good producer is given by,

$$\max_{K_{t+1}} \sum_{t=0}^\infty \beta^t \Lambda_t \left\{ r_t^K K_t - (1 + i_t^L) \left[ K_{t+1} - (1-\delta)K_t + \frac{\Theta_K}{2} \left( \frac{K_{t+1}}{K_t} \right)^2 \right] \right\}. \quad (A26)$$
The first-order condition with respect to $K_{t+1}$ is,

$$
\beta \Lambda_{t+1} r_{t+1}^K - \Lambda_t (1 + i_t^L) + \beta \Lambda_{t+1} (1 + i_{t+1}^L) (1 - \delta)
$$

$$
-(1 + i_t^L) \Lambda_t \Theta_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{K_t}{K_t} - (1 + i_{t+1}^L) \beta \Lambda_{t+1} \frac{\Theta_K}{2} \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right)^2
$$

$$
+(1 + i_{t+1}^L) \beta \Lambda_{t+1} \Theta_K \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} = 0.
$$

Using some algebraic manipulations and substituting equation (A6) in this gives,

$$
E_t r_{t+1}^K = (1 + i_t^L) r_t \left\{ \left[ 1 + \Theta_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \left( \frac{1 + i_t^B}{1 + \pi_{t+1}} \right) \right] \right\}
$$

$$
- E_t \left\{ (1 + i_{t+1}^L) \left[ (1 - \delta) + \frac{\Theta_K}{2} \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right] \right\}.
$$
Appendix B

Steady-State Equations

This section presents the steady-state values of the variables in the model. These values are computed by dropping the time subscripts from the variables.

From (46), the policy rate in the steady state is,

\[ i^R = r + \pi + \varepsilon_1(\pi - \pi^T). \] (B1)

In the steady state, inflation is equal to its target value. Thus,

\[ \pi = \pi^T. \] (B2)

Using this result in (B1) gives the steady-state value of the refinance rate,

\[ i^R = r + \pi. \] (B3)

From equation (5), the steady-state value of the bond rate (which is equal to the real interest rate) is,

\[ \frac{1 + i^B}{1 + \pi} = 1 + r = \frac{1}{\beta}. \]

In the case where the inflation target is equal to zero, there is zero inflation in the steady state, \( \pi = \pi^T = 0 \). Thus,

\[ 1 + i^B = 1 + i^R = 1 + r = \frac{1}{\beta}. \] (B4)

In the steady state, capital adjustment costs are zero,

\[ \frac{\Theta_K}{2} \left( \frac{K}{K} - 1 \right)^2 K = 0. \] (B5)

Using (B5) in (25), total investment in the steady state is,

\[ I = \delta K. \] (B6)

From equation (50), the steady-state equilibrium condition of the goods market yields \( Y = C + G + I \).

Using \( G = \psi Y \) from (49) and (B6) in this, the steady-state value of consumption is given by,

\[ C = (1 - \psi) Y - \delta K. \] (B7)

Using (B5) in (28) gives the steady-state value of the rental rate of capital,

\[ r^K = [1 + i^B] - (1 - \delta)[1 + i^L] > r. \] (B8)

In the steady state, the total reserve ratio and excess reserve ratio from (39) and (40) respectively are,

\[ \mu^{TR} = \mu + \left[ \frac{(1 + i^M) + \Phi_{C1} - (1 + i^R)}{\Phi_{C2}} \right], \] (B9)
\[
\mu_{ER} = \frac{(1 + i^M) + \Phi C_1 - (1 + i^R)}{\Phi C_2}.
\] (B10)

The ratio of excess reserves to total reserves is,
\[
ER_{TR} = \frac{\mu_{ER}}{\mu_{TR}}.
\] (B11)

Total reserves in the steady state are,
\[
TR = (\mu + \mu_{ER}) D.
\] (B12)

From (41), the steady-state value of the deposit rate is,
\[
1 + i^D = (1 + \frac{1}{\eta_D})^{-1}(1 + i^R) - \mu_{TR}(i^R - i^M)
\] + \Phi C_1(\mu_{TR} - \mu) - \frac{\Phi C_2}{2}(\mu_{TR} - \mu)^2].
\] (B13)

Using (42), the steady-state value of the lending rate is given by,
\[
1 + i^L = \frac{1 + i^R}{Q^F \left[\eta_L^{-1} + 1\right]}.
\] (B14)

The steady-state value of the repayment probability from (43) is,
\[
Q^F = \phi_0(\frac{\kappa C K}{L F_T})^{\phi_1}.
\] (B15)

In the steady state, the collateral-to-loan ratio, CL, is,
\[
CL = \frac{\kappa C K}{L F_T}.
\] (B16)

Using (B4) in (7), the steady-state value for real cash balances is,
\[
M^H = \frac{\eta_X p C^{1/\sigma}}{1 - \beta}.
\] (B17)

From (8), the steady-state value of real bank deposits is,
\[
D = \frac{\eta_X (1 - \nu) C^{1/\sigma} (1 + i^B)}{i^B - i^D}.
\] (B18)

The steady-state value of labour supply from equation (6) is given by,
\[
N = 1 - \frac{\eta_N C^{1/\sigma}}{\omega}.
\] (B19)

From (14), output of intermediate goods is,
\[
Y = AK^\alpha N^{1-\alpha}.
\] (B20)

The capital-labour ratio in the steady state from (17) is,
\[
\frac{K}{N} = (\frac{\alpha}{1 - \alpha})\left[\frac{(1 + \kappa W i^R)\omega}{r^R}\right].
\] (B21)
Using (B4) and (B8) in (B21), the steady-state real wage is,

\[ \omega = \left( \frac{1 - \alpha}{\alpha} \right) \frac{K \left[ \beta^{-1} - (1 - \delta) \right] (1 + i^L)}{N (1 + \kappa^W \theta^R)}. \]  \hspace{1cm} (B22)

In the steady state, the price adjustment equation (A25) is,

\[ (1 - \theta) + \theta mc = 0, \]

whereas, the steady-state value of the marginal cost is,

\[ mc = \frac{\theta - 1}{\theta}. \]  \hspace{1cm} (B23)

From (15), the steady-state level of loans demanded by intermediate good producers is,

\[ L^{F,W} = \kappa^W \omega N, \]  \hspace{1cm} (B24)

whereas, using (24), loans demanded by the capital good producer is,

\[ L^{F,I} = I. \]  \hspace{1cm} (B25)

Combining (B24) and (B25) total loans are,

\[ L^F = \kappa^W \omega N + I. \]  \hspace{1cm} (B26)

From (37), the steady-state level of borrowing from the central bank is,

\[ L^B = L^F - (1 - \mu^R^R)D. \]  \hspace{1cm} (B27)

The money market equilibrium condition in the steady state is,

\[ B^C = M^H + D. \]  \hspace{1cm} (B28)

From (48), taxes in the steady state are,

\[ T = G + i^M^T^R + i^B^B^H - i^R^L^B. \]  \hspace{1cm} (B29)
Appendix C

Log-Linearized Equations

The log-linearized equations of the model are presented in this section. Variables with a hat represent percentage point deviations for interest rate variables, inflation, the total reserve ratio and the excess reserve ratio from the steady state, and log-deviations around a non-stochastic steady state for the other variables.

Log-linearizing private consumption (equation (5)) gives,

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \sigma (\hat{i}_t^B - E_t \hat{\pi}_{t+1}) , \]

where \( \hat{\pi}_{t+1} \) is defined as,

\[ E_t \hat{\pi}_{t+1} = E_t \hat{P}_{t+1} - \hat{P}_t. \]

Log-linearizing the demand for cash (equation (7)) gives,

\[ \hat{M}_t^H = \frac{1}{\sigma} \hat{C}_t - \left[ \frac{\beta}{1 - \beta} \right] \hat{i}_t^B . \]

The demand for deposits from (8) is,

\[ \hat{D}_t = \frac{1}{\sigma} \hat{C}_t + \frac{1 + \hat{i}_t^D}{\hat{i}_t^D - \hat{i}_t^B} \left[ \hat{i}_t^D - \hat{i}_t^B \right] . \]

From (6), labour supply in its log-linear form is,

\[ \hat{N}_t = \frac{\eta_N(C)^{1/\sigma}}{\omega - \eta_N(C)^{1/\sigma}} \left\{ \hat{\omega}_t - \frac{\hat{C}_t}{\sigma} \right\} . \]

From (17), labour demand can be derived as,

\[ \hat{N}_t = \hat{K}_t - \kappa^W \hat{i}_t^R - \hat{\omega}_t + \left( \frac{1 + \hat{r}_K^K}{\hat{r}_K^K} \right) \hat{r}_t^K . \]

Log-linearizing the rental rate of capital from (28) gives,

\[ \hat{r}_t^K = \frac{(1 + \hat{i}_t^L)(1 + \hat{i}_t^B)}{1 + \hat{r}_K^K} \left\{ \hat{i}_t^B + \hat{i}_t^L - \hat{\pi}_{t+1} + \Theta_K \left( E_t \hat{K}_{t+1} - \hat{K}_t \right) \right\} \]

\[ - \frac{(1 + \hat{i}_t^L)}{1 + \hat{r}_K^K} \left\{ (1 - \delta) \hat{i}_t^L + \Theta_K E_t \left( \hat{K}_{t+2} - \hat{K}_{t+1} \right) \right\} . \]

Log-linearizing equation (A25) gives the New Keynesian Phillips Curve,

\[ \hat{\pi}_t = \frac{(\theta - 1)}{\phi_F} \hat{m}c_t + \beta E_t \hat{\pi}_{t+1} . \]

A log-linear approximation of the marginal cost, equation (18), yields,

\[ \hat{m}c_t = (1 - \alpha)(\kappa^W \hat{i}_t^R + \hat{\omega}_t) + \alpha \left( \frac{1 + \hat{r}_K^K}{\hat{r}_K^K} \right) \hat{r}_t^K - \hat{A}_t . \]
From (14), output of intermediate goods in its log-linear form is,
\[ Y_t = \hat{A}_t + (1 - \alpha) \hat{N}_t + \alpha \hat{K}_t. \] (C10)

From (39) and (40) the log-linearized total reserve ratio and excess reserve ratio are given by,
\[ \hat{\mu}_t^{TR} = \frac{\mu_t^{TR}}{\mu^{TR}_t} - \frac{(1 + i^R)(\bar{\mu}_t^R)}{\Phi_{C2} \mu^{TR}_t}, \] (C11)
\[ \hat{\mu}_t^{ER} = \frac{-(1 + i^R)\bar{\mu}_t^R}{\Phi_{C2} \mu^{ER}_t}. \] (C12)

The ratio of excess reserves to total reserves in its log-linear form is,
\[ \hat{E}R_{-TR_t} = \frac{\hat{\mu}_t^{TR} - \hat{\mu}_t^{ER}}{\mu^{TR}_t - \mu^{ER}_t}. \] (C13)

Total reserves are,
\[ \hat{TR}_t = \frac{1}{\mu^{TR}_t} \left\{ (\hat{\mu}_t + \hat{D}_t) \mu D + \left( \frac{\hat{\mu}_t^{ER} + \hat{D}_t}{\mu^{ER}_t} \right) \mu^{ER} D \right\}. \] (C14)

From (41), the deposit rate is given by,
\[ i_t^D = \frac{1}{(1 + i^D)(1 + \frac{1}{\eta_D})^{-1}} \left\{ (1 - \mu^{TR}_t) (1 + i^R)\bar{\mu}_t^R - \mu^{TR}_t \bar{\mu}_t^{TR} (i^R - i^M) \right\} \] (C15)
\[ + \left[ \Phi_{C1} - \Phi_{C2} (\mu^{TR}_t - \mu) \right] \left( \mu^{TR}_t \bar{\mu}_t^{TR} - \mu \bar{\mu}_t \right). \]

Log-linearizing the lending rate, equation (42), gives,
\[ \hat{i}_t^L = i_t^R - \hat{Q}_t^F. \] (C16)

From (43), the repayment probability in its log-linear form is,
\[ \hat{Q}_t^F = \phi_2 \hat{Y}_t + \phi_1 \left( \hat{K}_t - \hat{L}_t^{F,I} \right). \] (C17)

The linearized equation for the collateral-to-loan ratio is given by,
\[ \hat{C}L_t = \hat{K}_t - \hat{L}_t^{F,I}. \] (C18)

From (46), the central bank policy rate is determined by,
\[ i_t^R = \chi i_{t-1}^R + (1 - \chi) [\varepsilon_1(\hat{\pi}_t) + \varepsilon_2(\hat{Y}_t)] + \varepsilon_t. \] (C19)

Using (15), total loans to intermediate good producers in log linear form can be written as,
\[ \hat{L}_t^{L,W} = \hat{N}_t + \hat{\omega}_t, \] (C20)
whereas, the capital good producer’s demand for credit from (24) is,
\[ \hat{L}_t^{F,I} = \hat{I}_t. \] (C21)
Log-linearizing equation (30), total loans to firms are,

\[
\hat{L}_t^F = \frac{\kappa^W \omega N}{L^F} \left( \hat{N}_t + \hat{\omega}_t \right) + \frac{\hat{I}_t}{L^F}. \tag{C22}
\]

From (37), borrowing from the central bank is,

\[
\hat{L}_t^B = \frac{L^F \hat{L}_t^F}{L^B} - \frac{D}{L^B} \left[ \hat{D}_t - \mu^{TR} \hat{D}_t - \mu^{TR} \hat{\mu}_t^{TR} \right]. \tag{C23}
\]

Log-linearizing (53) gives the money market equilibrium, from which \( \hat{i}_t^B \) is obtained,

\[
M^H \hat{M}_t^H + D \hat{D}_t = 0. \tag{C24}
\]

New capital goods from equation (25) is given by,

\[
\hat{K}_{t+1} = \frac{I}{K} \hat{I}_t + (1 - \delta) \hat{K}_t. \tag{C25}
\]

From (50), the log-linearized equation for the equilibrium condition of the goods market which is used to determine investment is,

\[
Y \hat{Y}_t (1 - \psi) = C \hat{C}_t + \delta K \hat{I}_t. \tag{C26}
\]

Taxes from (48) are,

\[
T \hat{T}_t - G \hat{G}_t = i^M TR (\hat{R}_{t-1} - \hat{\pi}_t) + \left[ i^B \hat{B}^C - (1 + i^B) \hat{B} \right] \hat{\pi}_t + (1 + i^B) (\hat{B} - \hat{B}^C) \hat{i}_t^{B+1} - L^B (1 + i^R) \hat{i}_t^R - i^R L^B (\hat{L}_t^B - \hat{\pi}_t). \tag{C27}
\]