Discussion Paper Series

Fearing the Worst: The Importance of Uncertainty for Inequality

By

Keith Blackburn and David Chivers

Centre for Growth and Business Cycle Research, Economic Studies,
University of Manchester, Manchester, M13 9PL, UK

March 2013
Number 182

Fearing the Worst: The Importance of Uncertainty for Inequality*

Keith Blackburn and David Chivers
Centre for Growth and Business Cycles Research
Department of Economics, University of Manchester

Abstract

We present an overlapping generations model in which aspirational agents face uncertainty about the returns to human capital. Investment in human capital requires external funding, implying a probability of bankruptcy that is greater the lower the human capital endowment of an agent. We show that agents with sufficiently low human capital endowments may experience such a strong influence of loss aversion that they abstain from human capital investment. We further show how this behaviour may be transmitted through successive generations to cause initial inequalities to persist. These results do not rely on any credit market imperfections.

JEL Classification: D31, D81, E24.

Keywords: Inequality, uncertainty, aspirations, loss aversion.

1 Introduction

Contemporary theories of income distribution are typically based on an appeal to some form of market imperfection which creates different incentives and opportunities for different individuals. Most prominent in this research are models that rely on the imperfect functioning of capital markets for one reason or another. The key implication of these models is that agents with

*Address for correspondence: Keith Blackburn, Department of Economics, University of Manchester, Manchester M13 9PL, England. **Tel:** 0161-275-3908. **Fax:** 0161-275-4928. **E-mail:** keith.blackburn@manchester.ac.uk.

insufficient wealth to serve as collateral for loans may be deterred or prevented from accessing profitable investment opportunities because of high costs of borrowing or rationed availability of credit. Moreover, any initial differences in individual wealth may turn out to be persistent (permanent) fixtures such that the limiting distribution of wealth is characterised by the same agent heterogeneity and income inequality as exists to begin with. From a macroeconomic perspective, the models also yield further insights by revealing how distributional outcomes can influence aggregate economic performance in terms of growth and development.

This paper does not seek to undermine the potential significance of (financial) market imperfections in determining the relative fortunes of individuals. Rather, its aim is to highlight another factor for consideration, one that is possibly of equal importance but that has hitherto been largely (and rather suprisingly) neglected by researchers. This is the role of risk in individual decision making when the outcomes of decisions are uncertain.

There are good reasons for thinking why aspects of risk and uncertainty may be important for issues of distribution. Not least of these is the precautionary motive for savings which suggests that agents' desire to insure themselves against uncertainty leads them to create a "buffer-stock" level of savings that increases with the degree of uncertainty. If one thinks of this motive as being stronger for less wealthy agents (as suggested by empirical observation), then one begins to realise why poorer members of the population may be less likely to undertake wealth-enhancing ventures when such ventures are relatively risky. In addition, by entertaining the notion of intergenerational linkages, one may also start to contemplate the possibility of history-dependent behaviour and, with this, the prospect of persistent inequality.

Our basic objective in this paper is to explore the idea that, in an uncertain environment, distributional outcomes may have as much to do with the structure of preferences as they have with the functioning of markets. We approach this by appealing to some recent advancements in decision theory. Specifically, we call upon the hypothesis of aspiration-induced loss aversion as a means of enlightening the type of behaviour that we envisage. This hypothesis is based on the notion that individuals, faced with some risky prospect, have concern over attaining (or not attaining) a certain level of wealth to which they aspire. Any outcome above (below) this aspiration level is regarded as a success (failure). The result of this is a value function that reflects individuals' weighted preferences over the overall probability of success and/or the overall probability of failure. These preferences can significantly influence the evaluation of a prospect and are absent from standard expected utility theory.

The concept of an aspiration level bears an obvious similarity to the concept of a reference point used in prospect theory. An important difference, however, is the way that outcomes are defined: models based on reference points take outcomes to be changes in wealth, whilst models based on aspiriation levels take outrcomes to be final states of wealth. As mentioned above, the preference for final state wealth to be above some aspiration level may have a significant influence on decision making. For example, individuals with relatively low levels of wealth to begin with will be relatively more wary about taking on risks as they are relatively less able to buffer themselves against bad outcomes. This provides the basic intuition underlying our analysis which shows how a sufficiently strong aversion to falling short of aspirations may induce the least wealthy agents to forego potentially profitable, but prohibitively risky, investment opportunities. Embedding these microfoundations in an overlapping generations framework, we also demonstrate how initial inequalities may persist as a long-run feature of distributional outcomes. The striking aspect of these results is that they are realised within the context of a financial environment in which borrowing and lending opportunties are unconstrained by any frictions. Rather than being the product of credit market imperfections, they are driven more fundamentally by the deep structure of preferences governing attitudes towards risk. We are unaware of any other analysis to offer a similar perspective and to establish similar insights.

The remainder of the paper is organised as follows. In Section 2 we provide a brief overview of the literature that forms the background to our investigation. In Section 3 we present the model that we use to conduct our analysis. In Section 4 we deduce the equilibrium behaviour of agents. In Section 5 we establish our main results. In Section 6 we comment on these results within the context of other research, as well as outlining some extensions of our analysis. In Section 7 we make a few concluding remarks.

2 Background Literature

The link between income distribution and the functioning of financial markets is formally articulated in a number of analyses that form a well-established and influential body of research (e.g., Aghion and Bolton 1997; Banerjee and Newman 1993; Blackburn and Bose 2001; Galor and Zeira 1993; Piketty 1997).[1] Broadly speaking, this research seeks to examine the extent to which

[1] This research developed alongside other work on income distribution that signalled a general revival of interest in the subject. Amongst this work are models of redistribution based on political motives (e.g., Alesina and Rodrik 1994; Persson and Tabellini 1994;

capital market imperfections of one form or another (such as asymmetric information or weak contract enforcement) can cause initial income inequalities to persist over time. The basic idea is illustrated by considering an environoment in which agents face a choice between two types of production, or investment, activity: the first is a low-cost, but low-yielding, venture (e.g., subsistence production), whilst the second is a high-cost, but high-yielding, endeavour (e.g., human capital investment). Agents obtain funding for the latter by using their own endowments of wealth and by acquiring loans from financial intermediaries if necessary. Because of capital market imperfections, the terms and conditions of loan contracts depend on agents' wealth status: poorer agents face higher costs of borrowing and/or higher requirements for collateral. The upshot is that there is a critical level of wealth below which agents are forced to undertake the low-yielding activity, whilst above which agents enjoy access to the high-yielding venture. In a dynamic setting this division of the population may endure through successive, interconnected generations of agents if other circumstances prevail, such as indivisibilities in investment. If so, then the limiting distribution of wealth is characterised by two steady states as initial inequalities persist to produce a polarisation between the rich and the poor. Evidently, since these cohorts engage in different activities with different productivities, the extent of inequality has implications for macroeconomic performance in terms of aggregate output and possibly growth.

To many observers, theories of income distribution based on capital market imperfections are compelling, not least because of the apparent pervasiveness of such frictions. Yet there is another feature of economies that is equally, if not more, pervasive and that may be worth just as much consideration: this is the existence of uncertainty, about which relatively little has been written in connection with income distribution. This is somewhat surprising, given the major role that risk can play in savings and investment decisions, the outcomes of which, being realised in the future, are typically fraught with uncertainty. For example, Krebs (2003) argues that investment in human capital is particularly susceptible to idiosyncratic, uninsurable labour risk (due to the unpredictability of employment opportunities and job-searching time), whilst Grossman (2008) lists a variety of other reasons why such investment is risky, not least of which is individuals' uncertainty about the distribution of post-educational earnings (because of changes in technology and demand conditions that may occur during the period of education). Another prime example of risky investment is the acquisition of equities, on

Perotti 1993) and models of inequality based on neighbourhood effects (e.g., Benabou 1992; Durlauf 1993; Fernandez and Rogerson 1996).

which some notable observations have been made, such as the tendency of individuals to hold a smaller proportion of risky assets in their portolios the greater is the degree of their income uncertainty and the lower is the level of their wealth (e.g., Guiso *et al.* 1996; Aiyagar 1994). From a distributional perspective, one's attention is particularly drawn to the last observation since it suggests a connection between wealth accumulation and the amount of risk that individuals are willing to bear. For this reason, it is important to understand how attitudes towards risk might influence decisions that govern the relative fortunes of individuals who do not share the same wealth status.

As discussed by Guiso and Paiella (2008), there is a prevailing consensus that individuals' aversion towards risk is decreasing in wealth. This property of decreasing absolute risk aversion stands in contrast to the more popular assumption of constant relative risk aversion. The latter is often used for practical purposes, and one may view its popularity more in terms of tractability rather than in terms of plausibility. The former might be less user-friendly, but it is conceptually more appealing. In particular, it accords with one's intuition that, in the words of Rabin (2000), "a dollar that helps us avoid poverty is more valuable than a dollar that helps us become very rich". Nevertheless, as pointed out by the same author, and emphasised further in Rabin and Thaler (2001), there remains a problem: under expected utility theory, risk averse behaviour for small stake gambles implies improbably high risk aversion for large stake gambles such that the marginal utility of wealth must decrease at an astronomical rate. Even if one ignores this, it is questionable whether decreasing absolute risk aversion is capable, by itself, of explaining why relatively poor agents choose not to pursue potentially wealth-enhancing opportunities.

One way of moving forward from the above is to think beyond the standard paradigm of expected utility by exploring other concepts in decision theory. Our attention is particularly drawn to the concept of loss aversion which entertains the idea that individuals have a stronger preference for avoiding losses than for acquiring gains. The concept was first introduced by Kahneman and Tversky (1979) in their pioneering work on prospect theory.[2] This theory makes two assertions about loss aversion: the first is that agents derive less utility from undertaking symmetric bets than they do from accepting the expected outcome with certainty; the second is that the extent of aversion to such bets increases with the size of the stake. The type of utility function that captures these features is one that exhibits reference-dependent

[2]This has become the dominant descriptive theory of decision making under uncertainty. It is part of the broader literature on non-expected utility, a comprehensive review of which can be found in Starmer (2000).

asymmetry: relative to some benchmark outcome, losses are weighted more heavily than gains (i.e., the utility function is steeper for bad states than for good states). Thus, what matters to an individual when faced with some gamble is not the total amount of income that he ends up with, but rather the amount of income relative to some reference level, deviations from which are evaluated differently depending on whether they are positive or negative. Diagramatically, the utility function is generally drawn S-shaped with a kink at the reference point, where it changes from being convex (in the domain of losses) to concave (in the domain of gains).[3]

The concept of loss aversion is particularly prominent in the field of behavioural economics and finance, where it has been subject to much investigation by decision theorists (e.g., Bleichrodt *et al.* 2009; Schmidt and Zank 2005) and applied by others to explain apparent anomalies and paradoxes, such as the endowment effect (e.g., Thaler 1980), the status quo bias (e.g., Samuelson and Zeckhauser 1988) and the equity premium puzzle (e.g., Benartzi and Thaler 1995). Its application in macroeconomics remains much more limited, and we are unaware of its use in any macro-type (dynamic general equilibrium) model of inequality and income distribution. Our aim in this paper is to develop such a model.[4]

Loss aversion draws attention to the importance of downside risk in shaping individuals' preferences. In models where this is motivated by reference points, outcomes are defined in terms of changes in wealth and the extent of loss aversion is reflected in the shape of the utility function independently of probabilities. As regards the latter aspect, it has been argued on the basis of experimental evidence that a major concern for individuals in evaluating risky prospects is the overall chances of success and failure. For example, Edwards (1954) shows that individuals prefer low probabilities of large losses to high probabilities of small losses, whilst Langer and Weber (2001) and Payne (2005) reveal that individuals pay special consideration to the probabilities of winning and losing as a whole.

To take account of the above, we turn to another, more recent, concept in decision theory - namely, aspiration levels. The basic idea of this is that individuals evaluate risky prospects according to their weighted preferences over the overall probabilities of success and failure, where success and failure are defined with respect to some aspirational outcome (e.g., Diecidue and Van de Ven, 2008). Individuals whose initial wealth is above (below) their aspiration level may be thought of as seeking to maintain (improve) their

[3] A selection of alternative functional forms can be found in Maggi (2004).

[4] Two recent macroeconomic applications of prospect theory are presented by Foellmi *et al.* (2011) and Rosenblatt-Wisch (2008). The focus of each these is on the aggregate implications of loss aversion in stochastic growth models.

status, implying the possibility of risk averse (risk loving) behaviour in the proximity of the threshold. The existence of aspiration levels has been detected in several empirical studies (e.g., Holthausen 1981; Mezias 1988) and it has been argued that individuals may use them as a means of simplifying complex decision problems (e.g., Langer and Weber 2001; Mezias *et al.* 2002). Like reference points, aspiration levels may be defined with respect to different target outcomes, such as maintaining initial wealth status, staying above the poverty line and avoiding situations of bankruptcy. Failure to meet such targets is assumed to result in a direct disutility (psychic) cost independently of any monetary cost. Whilst reference points and aspiration levels are closely related, there are important differences between them: the latter is a probabilistic (rather than purely behavioural) concept that takes outcomes to be final states of (rather than changes in) wealth, and that gives rise to a utility function which is discontinuous (rather than kinked) at the threshold point. The last of these features has the effect of reinforcing loss aversion.[5]

Part of the motivation for our analysis derives from ideas relating to precautionary savings behaviour, as discussed by Carroll (2001). The basic reason for such behaviour is that, when confronted by uninsurable risk, individuals save as a means of buffering themselves against future bad shocks. Suppose that agents have some target level of consumption. A negative income shock is of greater concern to poor agents than rich agents since the former stand a higher chance of failing to reach their consumption target. Less wealthy agents are therefore inclined to have a larger "buffer stock" of savings on which they can draw. To the extent that these savings are less productive than other, more risky, ways of disposing of income, initial inequalities may be reinforced. By way of further illustration, consider the following example which resonates more closely with our previous discussion. Imagine two agents who are faced with a gamble (a risky investment project, perhaps) that offers an equal chance of either winning or losing the same amount of money. Suppose that these agents are identical except in terms of their initial levels of wealth which lie above their common aspiration level (a poverty line, for example). In the worst case scenario, the poorer agent falls below the aspiration level, whilst the richer agent remains above it. Thus, although the rewards are comparatively higher for the former, the stake that is being risked is comparatively higher for him as well because of the higher probability of not meeting his aspirations. Fearing the worst, this may induce the poorer agent to abstain from the gamble (forego investment) altogether.

[5] For a broad comparison of aspiration level models and prospect theory, see Lopes and Oden (1999).

Similar to before, what this example illustrates is a tendency for less wealthy individuals to be less inclined to pursue risky, but potentially profitable, opportunities because the amount that they may lose is prohibitively costly for them (i.e., failing to achieve their aspirations counts for more than increasing their wealth). In this way, initial inequalities can cause diverse behaviour (gambling or not gambling, investing or not investing) which strengthens those inequalities and shapes distributional outcomes.

In the analysis that follows we seek to articulate the above ideas more rigorously in a simple theoretical model. The model describes an overlapping generations economy in which agents produce output using human capital, an initial distribution of which accounts for agent heterogeneity whilst lineage transfers of which account for intergenerational linkages. An agent accumulates human capital for himself by drawing on the human capital inherited from his parent and by undertaking his own self-improvements of knowledge and ability. The agent can maximise his human capital accumulation by making some physical investment of resources, for which he requires a loan from financial intermediaries. This is risky because the returns to human capital are uncertain and may not be high enough for the loan to be repaid, in which case the agent faces bankruptcy. Such an outcome is particularly unwelcome due to aspiration-induced loss aversion and is more likely to occur the less is the amount of inherited human capital. Against this background, we show how agents with sufficiently low endowments of human capital may have such a strong aversion to losses that they choose not to risk borrowing for human capital investment. We further show how this behaviour can be transmitted through successive generations to cause initial inequalities to persist. As indicated earlier, the striking aspect of these results is that they do not rely on any credit market imperfections, but rather are derived within the context of a financial environment that allows borrowing and lending to take place without impediment. In accordance with Carroll (2001), there are some individuals who do not borrow, not because they are unable to do so, but because they choose not to do so.

A recent related analysis to ours is that of Genicot and Ray (2012) who address the interesting matter of how aspirations are shaped and formed. In particular, the authors construct a model in which aspirations are co-determined endogenously with the distribution of income. The basic idea is that individuals form their aspirations with reference to the social environment (e.g., wealth distribution) which, in turn, evolves according to the development of aspirations. It is shown how persistent inequality may result from this. Whilst touching on similar issues, the focus of our own analysis is quite different. Like most of the literature, we do not seek to delve deeply into questions of what motivates and determines individual aspirations. Rather, taking aspirations as given, our concern is to examine how the uncertain prospect of attaining them might influence distributional outcomes through a differential impact of loss aversion on individuals' behaviour. We similarly show how persistent inequality may arise from this.

3 The Model

We consider a small open economy in which there is a constant population of mortal, reproductive agents measuring a size of unit mass. Each agent lives for two periods and belongs to a dynastic family of overlapping generations connected through tranfers of human capital. Each agent has one parent and one child, inheriting capabilities from the former and imparting capabilities to the latter. Each agent is a potential investor in human capital when young, and a producer and consumer of output when old. We proceed with our formal description of the economy with reference to the circumstances facing agents of generation t.

3.1 Preferences and Technologies

All agents have identical preferences defined over old-age consumption, or income, x_{t+1}, from which they derive a lifetime utility of $u_t = u(x_{t+1})$. Under standard expected utility theory, an agent's objective is to maximise $E(u_t)$. Our departure from this involves a non-expected utility approach based on aspiration level theory. In general, this theory posits an objective function that depends not only on the expected utility of a prospect, but also on the overall probability of success and/or the overall probability of failure in attaining some target outcome. Denoting these probabilities by P^s and P^f, respectively, one imagines individuals as maximising a value function of the form $V_t = E(u_t) + \mu P^s - \lambda P^f$, where μ and λ represent weighting parameters that measure the importance of success and failure.

As indicated earlier, the choice of target outcome, or aspiration level, is often somewhat arbitrary and open to different interpretations. In the context of the present analysis, however, there is a fairly natural candidate. As we shall see, agents' uncertainty about their incomes is essentially a matter of uncertainty about their ability to repay loans that are used to finance investments. If repayments can be made, then bankruptcy is avoided and an agent ends up with some positive level of income. If repayments cannot be made, then bankruptcy is incurred and an agent is left with no income. Given this, we are inclined to specify an agent's aspiration level as any positive amount of income, which is to say that agents aspire to avoid bankruptcy. Correspondingly, we associate bankruptcy ($x_{t+1} = 0$) as a measure of failure and non-bankruptcy ($x_{t+1} > 0$) as a measure of success. By way of simplification, we assume that agents care only about the former ($\mu = 0$), the overall probability of which is now denoted by $P^f = P(x_{t+1} = 0)$. For further convenience, we also assume that utility from consumption is linear, $u(x_{t+1}) = x_{t+1}$. These features are inconsequential for our main results, as we demonstrate later under various extensions of the model. As matters stand at present, the actual payoff to an agent is understood to be x_{t+1} if $x_{t+1} > 0$ and $-\lambda$ if $x_{t+1} = 0$, implying the sort of discontinuity which can account for loss aversion. The expected payoff to an agent is then given by the value function

$$V_t = E(x_{t+1}) - \lambda P(x_{t+1} = 0) \tag{1}$$

In the first period of life an agent makes a decision about his investment, i_t, in human capital.[6] In the spirit of other analyses (e.g., Banerjee and Newman 1993; Galor and Zeira 1993), we assume that there is a fixed cost of investment, $k > 0$, such that agents are faced with the binary choice of either $i_t = 0$ or $i_t = k$. Since all agents are endowed with zero resources to begin with, any of them who chooses the latter option must finance his investment by borrowing, a matter to which we return later. The concept of human capital in our model need not be restricted to including just knowledge and skills, but may be thought of more broadly as encompassing other personal attributes (most notably, health) that enhance productive efficiency. Whatever the interpretation, and whatever choice is made, an agent accumulates human capital, h_{t+1}, in a way that depends on the human capital inherited from her parent, h_t, augmented by some additional, but uncertain, component, γ_{t+1}. Specifically,

$$h_{t+1} = \begin{cases} \beta h_t + b(1 + \gamma_{t+1}) & \text{if } i_t = 0, \\ \beta h_t + B(1 + \gamma_{t+1}) & \text{if } i_t = k, \end{cases} \tag{2}$$

where $\beta \in (0, 1)$ and $B > b > 0$.

The term γ_{t+1} in (2) is a bounded random variable with known probability distribution, but unknown realised value at the time that agents make decisions. This variable may be thought of as capturing various personal characteristics (e.g., innate ability, health status and all-round functionality) that are randomly bestowed on agents and that agents become aware of during the course of human capital formation. For any given realisation of

[6]This is an investment of physical resources, rather than time or effort. The latter may be treated as being already subsumed into the behaviour of agents, who may be thought of as devoting a fixed amount of time or effort to human capital production when they are young.

γ_{t+1}, an agent ends up with more human capital if he invests resources in improving his capabilities than if he foregoes such investment. The basic role played by γ_{t+1} is to inject uncertainty into agents' future incomes by creating uncertainty about their future productive efficiency. An alternative approach would be to assume that output production is, itself, stochastic (due to technology shocks), in which case a similar analysis could be conducted to obtain essentially the same results. We choose the present modelling strategy arbitrarily and for no particular reason.[7] For simplicity, we assume that γ_{t+1} is uniformly distribution over the interval $(-c, c)$ with probability density function $f(\gamma_{t+1}) = \frac{1}{2c}$, where $c < 1$. The mean and variance of γ_{t+1} are therefore 0 and $\frac{c^2}{3}$, respectively. In view of the latter, a measure of uncertainty in our model is provided by c, an increase in which corresponds to a mean-preserving spread in the distribution of γ_{t+1}.

In the second period of life an agent produces output, y_{t+1}, using his human capital according to

$$y_{t+1} = Ah_{t+1}, \tag{3}$$

where $A > 0$. The agent's final consumption depends on what action he took when young. If the agent abstained from human capital investment ($i_t = 0$ in (2)), then he consumes all of his realised output, $A[\beta h_t + b(1 + \gamma_{t+1})]$. If the agent engaged in human capital investment ($i_t = k$ in (2)), then he consumes whatever output is left over after paying back lenders in return for his loan of k. Let R_{t+1} denote the rate of interest on loans so that $(1+R_{t+1})k$ is the size of loan repayment. Whether or not an agent is able to make this repayment depends on his realised production. The condition for repayment is $Ah_{t+1} - (1 + R_{t+1})k \geq 0$, or $A[\beta h_t + B(1 + \gamma_{t+1})] \geq (1 + R_{t+1})k$. If this condition is satisfied, then the agent is non-bankrupt and his consumption is $A[\beta h_t + B(1+\gamma_{t+1})] - (1+R_{t+1})k$. Conversely, if the condition is not satisfied, then the agent is bankrupt and his consumption is zero as lenders seize whatever output is produced. When holding with equality, the condition may be used to deduce a critical value of γ_{t+1} - $\widehat{\gamma}_{t+1}$, say - such that bankruptcy is avoided if $\gamma_{t+1} \geq \widehat{\gamma}_{t+1}$ and is unavoided if $\gamma_{t+1} < \widehat{\gamma}_{t+1}$. That is,

$$A[\beta h_t + B(1 + \widehat{\gamma}_{t+1})] = (1 + R_{t+1})k. \tag{4}$$

[7] Having said this, we note that our approach is well-motivated by our earlier discussion about the various risks associated with human capital investment. The formulation in (2) can be likened to the stochastic human capital production technologies used by Grossman (2008) and Krebs (2003), to whom we referred in our discussion. The main focus of these authors is on the role of human capital risk in influencing growth, though the former establishes this role with reference to initial wealth inequalities. Our own focus is centred primarily on distribution in an environment with initial human capital inequalities.

Naturally, $\widehat{\gamma}_{t+1}$ is increasing in R_{t+1} and decreasing in h_t: *ceteris paribus*, the higher is the interest rate on loans, or the lower is the inherited amount of human capital, the more productive must be a borrower if he is to be able to make his loan repayment. The probability that he is unable to do this - that is, the probability of bankruptcy - is given by $P(x_{t+1} = 0) = \int_{-c}^{\widehat{\gamma}_{t+1}} f(\gamma_{t+1})d\gamma_{t+1} = \frac{\widehat{\gamma}_{t+1}+c}{2c}$.

Given the above, we may summarise an agent's consumption profile under alternative scenarios as

$$x_{t+1} = \begin{cases} A[\beta h_t + b(1+\gamma_{t+1})] \text{ if } i_t = 0, \\ A[\beta h_t + B(1+\gamma_{t+1})] - (1+R_{t+1})k \text{ if } i_t = k,\ \gamma_{t+1} \geq \widehat{\gamma}_{t+1}, \\ 0 \text{ if } i_t = k,\ \gamma_{t+1} < \widehat{\gamma}_{t+1}. \end{cases} \tag{5}$$

In turn, we may compute from (1) the agent's expected payoff, $V_t|_{i_t}$, conditional on his decision about human capital investment when young. If no investment is made, we have

$$V_t|_{i_t=0} = A[\beta h_t + b]. \tag{6}$$

And if investment is made, we have

$$\begin{aligned} V_t|_{i_t=k} = &\int_{\widehat{\gamma}_{t+1}}^{c} \{A[\beta h_t + B(1+\gamma_{t+1})] - (1+R_{t+1})k\} f(\gamma_{t+1})d\gamma_{t+1} \\ &- \lambda \int_{-c}^{\widehat{\gamma}_{t+1}} f(\gamma_{t+1})d\gamma_{t+1}. \end{aligned} \tag{7}$$

Evidently, an agent will choose to invest if $V_t|_{i_t=k} \geq V_t|_{i_t=0}$. Our subsequent analysis of inequality is essentially concerned with identifying the circumstances under which this condition is satisfied (or not satisfied). Casual observation at this stage suggests a potentially important role for aspirations through the second term on the right-hand-side of (7). This term captures a borrower's expected utility loss associated with the possibility of ending up with zero income due to bankruptcy. Such a possibility does not arise if the agent foregoes human capital investment, an option that affords the agent less risk and that performs a similar role to precautionary savings.

3.2 Financial Markets

Any agent who chooses to spend resources on human capital accumulation must acquire external funding to finance the fixed investment cost of k. This funding is provided by competitive financial intermediaries that have access to a perfectly elastic supply of loanable funds at the world rate of interest,

$r > 0$. Intermediaries make loans to agents in the knowledge that bankruptcy may be declared. If so (i.e., if $\gamma_{t+1} < \widehat{\gamma}_{t+1}$), then agents' proclamations are verified and intermediaries appropriate whatever output is produced, $A[\beta h_t + B(1+\gamma_{t+1})]$. If not (i.e., if $\gamma_{t+1} \geq \widehat{\gamma}_{t+1}$), then intermediaries are paid back in full, earning a return of $(1+R_{t+1})k$. As indicated earlier, there are no credit market imperfections in our model. In particular, intermediaries do not face any problems of asymmetric information (e.g., observing the incomes of agents) or contract enforcement (e.g., preventing agents from absconding with loans). Issues that might otherwise arise from these - such as moral hazard, costly state verification and strategic defaulting - are therefore redundant and do not play any role in our analysis.[8]

Competition between intermediaries drives their expected profits to zero. Since the cost of borrowing is $(1+r)k$, this break-even condition is given by

$$\begin{aligned}(1+r)k = &\int_{\widehat{\gamma}_{t+1}}^{c} (1+R_{t+1})kf(\gamma_{t+1})d\gamma_{t+1} \\ &+ \int_{-c}^{\widehat{\gamma}_{t+1}} A[\beta h_t + B(1+\gamma_{t+1})]f(\gamma_{t+1})d\gamma_{t+1}. \qquad (8)\end{aligned}$$

For any given $\widehat{\gamma}_{t+1}$, this expression determines the contractual interest rate on loans, R_{t+1}. We may write the expression in a different way by combining it with (4) to obtain

$$\begin{aligned}(1+R_{t+1})k - (1+r)k = &\int_{-c}^{\widehat{\gamma}_{t+1}} A[\beta h_t + B(1+\widehat{\gamma}_{t+1})]f(\gamma_{t+1})d\gamma_{t+1} \\ &- \int_{-c}^{\widehat{\gamma}_{t+1}} A[\beta h_t + B(1+\gamma_{t+1})]f(\gamma_{t+1})d\gamma_{t+1}. \qquad (9)\end{aligned}$$

This shows the interest rate spread between lending and borrowing.[9] The size of spread depends on how much a lender expects to lose when a borrower claims that he is bankrupt and defaults on his loan (i.e., when $\gamma_{t+1} < \widehat{\gamma}_{t+1}$). To be sure, observe from (4) that the first integral term on the right-hand-side of (9) is equal to $\int_{-c}^{\widehat{\gamma}_{t+1}}(1+R_{t+1})kf(\gamma_{t+1})d\gamma_{t+1}$ which measures the expected amount of non-repayment when bankruptcy is declared. Conversely, the second integral term on the right-hand-side of (9) gives the expected amount

[8] We return to these issues later when we demonstrate how the results of our analysis are observationally equivalent to the results obtained from a model based on capital market imperfections.

[9] Results of this sort are fairly standard for the type of uncertain financial environment that we are considering (e.g., Agenor and Aizenman 1998a,b; Aizenman and Powell 2003; Azariadis and Chakraborty 1999).

of income that is seized from a defaulter. Accordingly, (9) implies that the contractual interest rate is set as a simple mark-up over intermediaries' cost of borrowing, where the size of mark-up is equal to the expected net income lost due to non-repayment of loans. This mark-up rule may be simplified to

$$(1+R_{t+1})k = (1+r)k + \frac{AB(\widehat{\gamma}_{t+1}+c)^2}{4c}. \tag{10}$$

As before, there is a positive relationship between R_{t+1} and $\widehat{\gamma}_{t+1}$: *ceteris paribus*, intermediaries set a higher contractual interest rate the more likely it is that bankruptcy will be declared.

4 Equilibrium Outcomes

There are two main considerations that dictate agents' behaviour towards human capital investment - the cost of borrowing to finance this investment and the possibility of not being able to meet this cost. The former is given by $(1 + R_{t+1})k$, which specifies the amount of loan repayment, whilst the latter is reflected in $\widehat{\gamma}_{t+1}$, which governs the probablity of bankruptcy. As indicated already, R_{t+1} and $\widehat{\gamma}_{t+1}$ are determined jointly in the sense that each one of them both influences and is influenced by the other. Solving for these variables is our starting point for characterising the equilibrium of the model. Having done this, we then proceed to deduce other equilibrium properties that form the basis of our subsequent analysis of inequality.

The expressions in (4) and (10) define a simultaneous equations system in R_{t+1} and $\widehat{\gamma}_{t+1}$. Under the parameter restriction $(1+r)k \leq A(\beta h_t + B) \leq (1+r)k + ABc$, there exists a unique feasible solution to this system, as given by[10]

$$\widehat{\gamma}_{t+1} = c - 2\sqrt{\frac{c[A(\beta h_t + B) - (1+r)k]}{AB}} \equiv \gamma(c, h_t), \tag{11}$$

$$R_{t+1} = r + \frac{AB[\gamma(c,h_t)+c]^2}{4ck} \equiv R(c, h_t). \tag{12}$$

There are two important factors on which $\widehat{\gamma}_{t+1}$ and R_{t+1} depend: the first is the extent of uncertainty about agents' incomes, as measured by c (the distributional support parameter on γ_{t+1}); the second is the amount of human capital that agents are initially endowed with, as given by h_t (the capabilities

[10] Details of the derivations can be found in an Appendix.

inherited from parents).[11] The result in (11) implies that $\gamma_c(c, h_t) > 0$ and $\gamma_h(c, h_t) < 0$, and the result in (12) reveals similarly that $R_c(c, h_t) > 0$ and $R_h(c, h_t) < 0$.[12] In words, the greater is the degree of uncertainty and/or the lower is the amount of inherited human capital, the higher is the probability of defaulting on loans and the higher is the contractual interest rate on loans. The effects of uncertainty are due to the fact that the loan repayment is a non-linear (specifically, concave) function of γ_{t+1}. To be sure, recall that the repayment is $A[\beta h_t + B(1 + \gamma_{t+1})]$ if $\gamma_{t+1} < \widehat{\gamma}_{t+1}$, but $(1 + R_{t+1})k$ if $\gamma_{t+1} \geq \widehat{\gamma}_{t+1}$. The expected repayment is therefore reduced by a mean-preserving spread in the distribution of γ_{t+1}. Intermediaries compensate for this by charging a higher interest rate on loans which increases the likelihood that defaulting will occur. The effects of inherited human capital operate in a similar way. A decrease in h_t reduces the expected loan repayment which raises the contractual interest rate and makes defaulting more likely.

Having established the above, we may now turn our attention to the equilibrium behaviour of agents. We do so by recalling the expression in (7) which gives an agent's expected payoff from investing in human capital. Using (8), we may re-write this expression as

$$\begin{aligned} V_t|_{i_t=k} &= \int_{-c}^{c} A[\beta h_t + B(1+\gamma_{t+1})] f(\gamma_{t+1}) d\gamma_{t+1} - (1+r)k \\ &\quad -\lambda \int_{-c}^{\widehat{\gamma}_{t+1}} f(\gamma_{t+1}) d\gamma_{t+1} \\ &= A(\beta h_t + B) - (1+r)k - \lambda \left(\frac{\widehat{\gamma}_{t+1} + c}{2c} \right). \end{aligned} \tag{13}$$

As stated earlier, an agent will choose to invest in human capital if doing so yields an expected payoff that is no less than the expected payoff from not investing - that is, if $V_t|_{i_t=k} \geq V_t|_{i_t=0}$. Using (6) and (13), together with (11), this condition can be written as $A(B - b) - (1 + r)k \geq \lambda \left[\frac{\gamma(h_t, c) + c}{2c} \right]$. We assume that the left-hand-side of this expression is strictly positive in order to avoid the degenerate case in which no agent ever invests. The right-hand-side is deduced to be an increasing function of c, a decreasing function of h_t and an increasing function of λ. Accordingly, the condition is less likely to be satisfied the greater is the degree of uncertainty, the lower is the amount of inherited human capital and the greater is the influence of

[11] Note that one does not need to assume that financial intermediaries are actually able to observe human capital directly. As shown in (11) and (12), $\widehat{\gamma}_{t+1}$ and R_{t+1} are functions of Ah_t, which is the output produced by parents. One needs only to assume that this is observable (in which case, of course, h_t can be trivially inferred anyway).

[12] Verification of these results is again contained in the Appendix.

aspirations. The crucial role played by aspirations is evident. As commented on previously, both a greater degree of uncertainty and a lower endowment of human capital makes bankruptcy more likely. Since agents aspire to avoid this, their aversion to it may incline them not to borrow to finance human capital investment when the probability of bankruptcy is high. Naturally, for any given probability of bankruptcy, the disincentive to borrow is stronger the greater is the influence of aspiration-induced loss aversion. In the absence of any such influence (i.e., when $\lambda = 0$) bankruptcy considerations play no role whatsoever.

5 Distribution and Inequality

The economy starts off with some initial distribution of human capital that accounts for agent heterogeneity and income inequality. Our principal concern is to study the role of aspiration-induced loss aversion in governing how the distribution changes over time and the extent to which initial inequalities may vanish or persist. We proceed to do this by determining the lineage dynamics for each dynasty that show the transition of human capital from one generation to the next. Then, for any given initial distribution of human capital, we may use this information to infer the dynamic processes operating at the aggregate level and thereby deduce possible long-run distribution outcomes.

From our previous analysis, an agent will choose to invest in human capital accumulation if $A(B-b) - (1+r)k \geq \lambda \left[\frac{\gamma(c,h_t)+c}{2c}\right]$. When holding with equality, this condition can be used to determine a critical inheritance of human capital - $\widehat{h}$, say - which is necessary for an agent to make such a choice. That is,

$$A(B-b) - (1+r)k = \lambda \left[\frac{\gamma(c,\widehat{h})+c}{2c}\right]. \tag{14}$$

Since $\gamma_h(c,h) < 0$, this expression implies that only those agents for whom $h_t \geq \widehat{h}$ will willingly borrow to finance human capital investment; all other agents for whom $h_t < \widehat{h}$ will choose not do this. Evidently, the precise value of the human capital threshold depends on certain key parameters. In particular, we may write $\widehat{h} = h(c,\lambda)$, where $h_c(c,\lambda) > 0$ and $h_\lambda(c,\lambda) > 0$.[13] Thus agents face a higher threshold the greater is the degree of uncertainty and/or the stronger is the influence of aspirations. The critical role played by the latter is again self-evident: in the absence of it ($\lambda = 0$), agents'

[13] As before, confirmation of these properties can be found in the Appendix.

inheritence of human capital would be irrelevant as all of them would invest in human capital accumulation since $A(B-b)-(1+r)k>0$.

Given the above, together with (2), we may conclude that the intergenerational evolution of human capital for an individual dynasty satisfies

$$h_{t+1}=\begin{cases}\beta h_t+b(1+\gamma_{t+1}) & \text{if } h_t<\widehat{h},\\ \beta h_t+B(1+\gamma_{t+1}) & \text{if } h_t\geq\widehat{h}.\end{cases} \tag{15}$$

These lineage transition equations are portrayed in Figure 1. Each of them corresponds to a stable stochastic difference equation which is bounded according to the bounds on γ_{t+1} (i.e., $\gamma_{t+1}\in(-c,c)$). The intersections with the 45^0 line are given by the stationary points associated with these bounds; that is,

$$h^*=\frac{b(1\pm c)}{1-\beta};\quad h^{**}=\frac{B(1\pm c)}{1-\beta}. \tag{16}$$

The transition equations are drawn under the parameter restriction $b(1+c)<(1-\beta)\widehat{h}<B(1-c)$ which we use for illustrative purposes and to make our analysis non-trivial.[14]

The long-run distribution of human capital in our economy is straightforward to characterise. The only investors in human capital accumulation are those agents who are well-endowed with human capital to begin with; these are agents for whom $h_0\geq\widehat{h}$, implying convergence to some high steady state equilibrium. All other agents who start off with relatively low human capital endowments remain forever as non-investors; these are agents for whom $h_0<\widehat{h}$, implying convergence to some low steady state equilibrium. In terms of income distribution, there is always the possibility that some generation of investors will find themselves bankrupt and worse off *ex post* than if they had not invested. Yet this does not affect the dynamics of human capital distribution and such agents are strictly better off *ex ante* in terms of their higher expected income from investing. The same remarks can be made about the actual and expected payoffs of agents.

As indicated already, the key factor in explaining our results is the existence of loss aversion caused by aspirations. The extent to which this impacts on an agent's behaviour depends on his likelihood of bankruptcy which, in turn, depends on his inherited human capital. The lower is this inheritance, the higher is the probability of defaulting and the stronger is the influence of loss aversion associated with this. For agents with $h_t<\widehat{h}$, the disutility

[14] For example, the restriction rules out the possibility that all agents automatically end up either investing or not investing in human capital, and also means that any lineage that chooses to invest at some point will never alter its choice subsequently.

suffered from bankruptcy is sufficiently high as to deter human capital investment. The number of agents for which this is true depends on both the extent of uncertainty and the strength of aspirational influence. This follows from the fact that, as noted above, $\widehat{h}$ is an increasing function of both c and λ. Thus, a higher value of either of these induces more agents to forego human capital investment.

6 Further Remarks

The foregoing analysis establishes our main results. In what follows we make a few additional observations about how these results relate to other relevant research and how they survive under some extensions of the model. As regards the former, two issues attract our attention - the role of capital market imperfections in explaining inequality and the effect of uncertainty on aggregate economic activity. As regards the latter, two modifications are considered - the generalisation of aspirational preferences and the introduction of initial wealth endowments.

6.1 Some Related Research

6.1.1 Inequality and Financial Markets

The persistence of inequality in our model is reflected in the existence of multiple, history-dependent long-run equilibria associated with threshold effects that explain how limiting outcomes depend on initial conditions. These are the key features of models of income distribution based on credit market imperfections (e.g., Banerjee and Newman 1993; Galor and Zeira 1993). The significant and novel aspect of our analysis is that it abstracts from any such imperfections and focuses purely on individuals' aspiration-based (loss averse) preferences as a means of accounting for inequality. With this in mind, it is interesting to note that our results are observationally equivalent to those obtained under a reformulation of our model in which the assumption of aspirations is replaced by an assumption of credit market imperfections. We demonstrate this as follows.

Suppose that aspirations are absent so that $\lambda = 0$ in (1). Instead, assume that there is an *ex post* informational asymmetry between borrowers and lenders such that only the former can directly observe the realisation of γ_{t+1}, whereas the latter must incur expenditures to make this observation. Such expenditures are incurred if a borrower claims bankruptcy since the claim needs to be verified in order to prevent a borrower from falsely declaring that he is unable to repay his loan. This is now a model of costly state verification, where the cost of verification, denoted v, provides a measure of credit market imperfections (e.g., Diamond 1984; Gale and Hellwig 1985; Townsend 1979).

The principal difference from our previous analysis is that the zero profit condition of intermediaries in (8) becomes

$$
\begin{aligned}
(1+r)k = & \int_{\widehat{\gamma}_{t+1}}^{c} (1+R_{t+1})kf(\gamma_{t+1})d\gamma_{t+1} \\
& + \int_{-c}^{\widehat{\gamma}_{t+1}} \{A[\beta h_t + B(1+\gamma_{t+1})] - v\} f(\gamma_{t+1})d\gamma_{t+1}. \qquad (17)
\end{aligned}
$$

In turn, the mark-up rule in (10) changes to

$$
(1+R_{t+1})k = (1+r)k + \frac{AB(\widehat{\gamma}_{t+1}+c)^2}{4c} + v\left(\frac{\widehat{\gamma}_{t+1}+c}{2c}\right). \qquad (18)
$$

The size of mark-up now reflects the expected verification cost, $v\left(\frac{\widehat{\gamma}_{t+1}+c}{2c}\right)$. As before, we may derive unique equilibrium values for $\widehat{\gamma}_{t+1}$ and R_{t+1} by solving the simultaneous equation system in (4) and (18). Doing this gives $\widehat{\gamma}_{t+1} = \gamma(c, h_t, v)$ and $R_{t+1} = R(c, h_t, v)$, where $\gamma_v(c, h_t, v) > 0$ and $R_v(c, h_t, v) > 0$. Thus, an increase in verification costs, v, has the same effect as an increase in uncertainty, c, or a decrease in inherited human capital, h_t: that is, there is an increase in the probability of bankruptcy and an increase in the interest rate on loans. Proceeding as before, we may also re-compute (13), the expected utility from human capital investment, as

$$
V_t|_{i_t=k} = A(\beta h_t + B) - (1+r)k - v\left(\frac{\widehat{\gamma}_{t+1}+c}{2c}\right). \qquad (19)
$$

And finally, we can establish the analogue of (14) for determining the critical level of inherited human capital, $\widehat{h}$:

$$
A(B-b) - (1+r)k = v\left[\frac{\gamma(\widehat{h}, c, v)+c}{2c}\right]. \qquad (20)
$$

It follows that $\widehat{h} = h(c, v)$, where $h_v(c, v) > 0$. The observational equivalence between these results and those established earlier is self-evident. Essentially, ν substitutes for λ in a way that makes the effects of credit market imperfections very similar to the effects of aspirations (or loss aversion). This result echoes the sentiments of Carroll (2001) who argues that, in many instances of uncertainty, the existence of a precautionary savings motive can generate behaviour that is virtually indistinguishable from the behaviour that emerges from the existence of liquidity constraints. The same is true with respect to loss aversion and credit market frictions. The outcomes may be similar in the sense that some individuals forego borrowing opportunities, but the reasons are fundamentally different: in the case of loss aversion, there is a self-imposed reluctance towards borrowing; in the case of credit constraints, there is an externally-imposed obstacle to borrowing.

6.2 Uncertainty and Macroeconomic Performance

The distributional effects of uncertainty in our model have implications for aggregate productive activity in the economy. This follows from the fact that the productivity of agents who invest in human capital is different from the productivity of agents who refrain from such investment. Since the number of investors and non-investors is determined by the degree of uncertainty, then so too is the total level of output. In this way, our analysis bears on other research that seeks to explore the link between uncertainty and macroeconomic performance.[15]

Let $g_t(h_t)$ be the probability density function of human capital at time t. Suppose that human capital is initially distributed over the interval $(\underline{h}, \overline{h})$ such that $\int_{\underline{h}}^{\overline{h}} g_t(h_t)dh_t = 1$ (corresponding to the unit mass of agents). In each period there is the same population of non-investors in human capital, $\int_{\underline{h}}^{\widehat{h}} g_t(h_t)dh_t$, and the same population of investors in human capital, $\int_{\widehat{h}}^{\overline{h}} g_t(h_t)dh_t$. From (2) and (15), the expected output of a non-investor is $A(\beta h_t + b)$, whilst the expected output of an investor is $A(\beta h_t + B)$. Accordingly, the expected total (or average) level of output in the economy is given by

$$\begin{aligned} Y_{t+1} &= \int_{\underline{h}}^{\widehat{h}} A(\beta h_t + b)g_t(h_t)dh_t + \int_{\widehat{h}}^{\overline{h}} A(\beta h_t + B)g_t(h_t)dh_t \\ &= A\beta \int_{\underline{h}}^{\overline{h}} h_t g_t(h_t)dh_t + A[b \int_{\underline{h}}^{\widehat{h}} g_t(h_t)dh_t + B \int_{\widehat{h}}^{\overline{h}} g_t(h_t)dh_t]. \end{aligned} \tag{21}$$

Recall that $\widehat{h} = h(c, \lambda)$, where $h_c(c, \lambda) > 0$ and $h_\lambda(c, \lambda) > 0$. Since $B > b$, it follows that, for any given $g_t(h_t)$, an increase in c, which increases $\widehat{h}$, implies

[15] For example, there is a large body of research that shows how uncertainty (or volatility) can influence long-term growth (either positively or negatively) through various factors (e.g., Aghion and Saint-Paul 1998; Blackburn and Varvarigos 2008; de Hek 1999; Jones *et al.* 2005; Martin and Rogers 2000). Whilst we do not consider long-term growth, our analysis identifies another factor that can create a link between uncertainty and macroeconomic performance - namely, the impact of uncertainty on distribution outcomes.

a decrease in the second right-hand-side term of (21), thus causing a decrease in Y_{t+1}. In words, a greater degree of uncertainty is associated with a lower average level of output as fewer agents choose to invest in human capital. It may also be noted that an increase in λ has the same effect, meaning that a stronger influence of aspirations (or loss aversion) reduces macroeconomic performance.

6.3 Some Extensions

6.3.1 Generalising Aspirations

Our treatment of aspirations in moulding agents' preferences led to the objective function in (1). We argued that, in the context of our analysis, a plausible assumption is that agents aspire to attain any positive level of income by avoiding bankruptcy. For convenience, we also assumed that, in terms of overall welfare, agents are affected only by the disappointment in falling short of their aspirations, not by the satisfaction from achieving them. In what follows we modify these features.

Let $x^* > 0$ denote some positive aspiration level of income. The direct (linear) utility of an agent is $u(x_{t+1} - x^*) = x_{t+1} - x^*$. The agent is also concerned about his overall prospects of both success and failure in attaining his aspiration level. Let $P(x_{t+1} \geq x^*)$ denote the probability of the former and $P(x_{t+1} < x^*)$ denote the probability of the latter. The agent's objective function (the analogue of (1)) is given by

$$V_t = E(x_{t+1} - x^*) + \mu P(x_{t+1} \geq x^*) - \lambda P(x_{t+1} < x^*). \tag{22}$$

Agents face the same technologies and opportunities as before. Investment in human capital implies the possibility of bankruptcy, with $\widehat{\gamma}_{t+1}$ in (4) defining a critical value of γ_{t+1} below (above) which such an event occurs (does not occur). The existence of a positive aspiration level leads us to define another critical value of γ_{t+1}, denoted $\widetilde{\gamma}_{t+1}$, above (below) which the aspiration level is attained (not attained). This is determined according to

$$A[\beta h_t + B(1 + \widetilde{\gamma}_{t+1})] = (1 + R_{t+1})k + x^*. \tag{23}$$

Evidently, $\widetilde{\gamma}_{t+1} = \widehat{\gamma}_{t+1} + \frac{x^*}{AB}$. Thus, an agent may find himself in one of three scenarios: $\gamma_{t+1} < \widehat{\gamma}_{t+1}$, in which case the agent is bankrupt and fails to achieve his aspirations; $\widehat{\gamma}_{t+1} \leq \gamma_{t+1} < \widetilde{\gamma}_{t+1}$, in which case the agent avoids bankruptcy, but still falls short of his aspirations; and $\gamma_{t+1} \geq \widetilde{\gamma}_{t+1}$, in which case the agent both avoids bankruptcy and attains his aspirations.

Given the above, we may deduce that $P(x_{t+1} \geq x^*) = \int_{\widetilde{\gamma}_{t+1}}^{c} f(\gamma_{t+1}) d\gamma_{t+1} = \frac{c - \widetilde{\gamma}_{t+1}}{2c}$ and $P(x_{t+1} < x^*) = \int_{-c}^{\widetilde{\gamma}_{t+1}} f(\gamma_{t+1}) d\gamma_{t+1} = \frac{\widetilde{\gamma}_{t+1} + c}{2c}$. An agent's expected payoff from human capital investment (the analogue of (7)) is therefore computed as

$$V_t|_{i_t=k} = \int_{\widetilde{\gamma}_{t+1}}^{c} \{A[\beta h_t + B(1+\gamma_{t+1})] - (1+R_{t+1})k\} f(\gamma_{t+1}) d\gamma_{t+1}$$
$$+ \mu \int_{\widetilde{\gamma}_{t+1}}^{c} f(\gamma_{t+1}) d\gamma_{t+1} - \lambda \int_{-c}^{\widetilde{\gamma}_{t+1}} f(\gamma_{t+1}) d\gamma_{t+1} - x^*. \qquad (24)$$

Since there is no change in the operation of financial markets, the expression in (8) continues to apply, as does the expression in (10). The equilibrium solutions for $\widetilde{\gamma}_{t+1}$ and R_{t+1} (obtained from (4) and (10)) are therefore still given by (11) and (12). As before, (8) may be used to convert (24) in to the following final version of an investor's expected payoff (the analogue of (13)):

$$V_t|_{i_t=k} = A(\beta h_t + B) - (1+r)k - x^* + \mu \left(\frac{c - \widetilde{\gamma}_{t+1}}{2c} \right) - \lambda \left(\frac{\widetilde{\gamma}_{t+1} + c}{2c} \right). \qquad (25)$$

For the sake of comparison with our main analysis, assume that non-investment in human capital acts as a safety option in the sense of always yielding an income that is above the aspiration level. Comparing the payoff from this, $V_t|_{i_t=0} = A[\beta h_t + b] - x^*$, with $V_t|_{i_t=k}$ in (25), the condition for an agent to invest in human capital is given by $A(B-b) - (1+r)k \geq \mu \left(\frac{\widetilde{\gamma}_{t+1} - c}{2c} \right) + \lambda \left(\frac{\widetilde{\gamma}_{t+1} + c}{2c} \right)$. Since $\widetilde{\gamma}_{t+1} = \gamma(h_t, c) + \frac{x^*}{AB} \equiv \Gamma(h_t, c)$, this condition may be used as before to deduce a critical value of h_t - that is, $\widehat{h}$ - above which an agent invests and below which an agent does not invest (the analogue of (14)):

$$A(B-b) - (1+r)k = \mu \left[\frac{\Gamma(c, \widehat{h}) - c}{2c} \right] + \lambda \left[\frac{\Gamma(c, \widehat{h}) + c}{2c} \right]. \qquad (26)$$

Clearly, the results of our main analysis (which can be recovered by setting $x^* = \mu = 0$) are not significantly altered by the generalisation of preferences reflected in (22).

6.3.2 Introducing Wealth

Intergenerational linkages in our model take place through the serendipitous intra-family transfers of human capital. In other models these linkages are the result of altruistic bequests of wealth which may influence opportunities for borrowing in the presence of capital market imperfections. We outline how our model may be extended to include a bequest motive.

Suppose that agents derive utility from their own consumption, c_{t+1}, and the bequests they leave to their offspring, q_{t+1}. *Ex post* (i.e., after incomes have been realised), their aim is to maximise $u(c_{t+1}, q_{t+1}) = \Phi c_{t+1}^{\phi} q_{t+1}^{1-\phi}$ ($\phi \in (0,1)$, $\Phi = [\phi^{\phi}(1-\phi)^{1-\phi}]^{-1}$) subject to $c_{t+1} + q_{t+1} = z_{t+1}$, where z_{t+1} is final income. They do this by choosing $c_{t+1} = \phi z_{t+1}$ and $q_{t+1} = (1-\phi)z_{t+1}$, implying an indirect utility of $U(z_{t+1}) = z_{t+1}$. An agent evaluates this with reference to some threshold outcome, z^*, that he aspires to exceed and that he fails to do so with probability $P(z_{t+1} \leq z^*)$. *Ex ante* (i.e., before any decisions have been made), the agents value function is

$$V_t = E(z_{t+1} - z^*) - \lambda P(z_{t+1} \leq z^*). \tag{27}$$

Suppose that parents invest bequests on behalf of their children (e.g., in a trust fund) who become entitled to their inheritance when old. Bequests can neither be used by agents to finance human capital investment when young nor appropriated by financial intermediaries should this investment result in bankruptcy. The expression for final income is therefore given by $z_{t+1} = x_{t+1} + (1+r)q_t$, where x_{t+1} is determined according to (5).

Assume that an agent's threshold income is simply his inheritance, $z^* = (1+r)q_t$. Thus agents aspire to be better off than they would be from relying solely on the altruism of their parents. This is equivalent to assuming that agents aspire to avoid bankruptcy ($x_{t+1} = 0$), meaning that (27) can be re-written as (1). All of our original results can be re-established, with straightforward implications for the dynamics of bequests, $q_{t+1} = (1-\phi)[x_{t+1} + (1+r)q_t]$.

7 Conclusions

This paper has sought to make a theoretical contribution to the literature on inequality and income distribution. Its approach has been to focus on the structure of preferences, rather than the functioning markets, as a way of explaining the diverse behaviour and diverse fortunes of individuals who face uncertainty. This offers a new perspective on why some individuals do not pursue potentially wealth-enhancing opportunities: the reason is not that they are prevented from doing so, but rather that it is not in their interests to do so. For such agents, the loss that may be incurred on a risky venture is simply too great to make the venture attractive, even if it offers the prospect of high rewards. For other agents, the same loss may be of much less concern so that the venture is taken on. These cohorts of individuals are the less wealthy and the more wealthy members of the population. The former may stand to gain relatively more if an investment goes well, but they also risk losing relatively more if the investment goes wrong. Under the influence of aspiration-induced loss aversion, it is the disliking of losses, rather than the liking of gains, that matters most to individuals, which is why the less wealthy of them may abstain from opportunities that could make them better off.

Our analysis raises important questions about the appropriateness and effectiveness of strategies aimed at redistribution. If inequality was the result of market imperfections, then the natural prescription for reducing inequality would be the attenuation of these imperfections. In the case of financial markets, for example, improvements in monitoring and enforcement would presumably make borrowing more accessible to greater numbers of individuals who might otherwise be denied loans. But if inequality was rooted in the deep structure of preferences, it is much less obvious what options are available and feasible. And if this source of inequality was mistaken for another (a possibility that we alluded to), then one may end up with well-meaning strategies that are ineffectual (and perhaps even worse if they are costly to implement). One possible approach suggested specifically by our analysis is the enhancement of human capital accumulation (e.g., through better quality public education and health programmes) that may push individuals over the human capital threshold by making their aspirations more attainable. In a broader context, to the extent that the poor wealth status of some individuals may make them unable (because of imperfections) or unwilling (because of preferences) to try to improve their status, lump-sum redistributions from the rich to the poor may offer the most straightforward means of reducing inequality. Further exploration of these issues is an avenue worth pursuing.

Appendix

The results in (11) and (12) are derived as follows. Combining (4) and (10) yields the quadratic equation

$$AB\widehat{\gamma}_{t+1}^{2} - 2ABc\widehat{\gamma}_{t+1} - \{4c[A(\beta h_t + B) - (1+r)k] - ABc^2\} = 0. \tag{A1}$$

Hence

$$\widehat{\gamma}_{t+1} = c \pm 2\sqrt{\frac{c[A(\beta h_t + B) - (1+r)k]}{AB}}. \tag{A2}$$

A necessary and sufficient condition for ruling out complex roots is $A(\beta h_t + B) \geq (1+r)k$. Given this, together with the fact that $\widehat{\gamma}_{t+1} \leq c$, the only possible solution to (A2) is when $2\sqrt{\cdot}$ enters negatively, as shown in (11). The restriction $A(\beta h_t + B) \leq (1+r)k + ABc$ ensures that $\widehat{\gamma}_{t+1} \geq -c$ as well. Given the result in (11), the result in (12) is obtained by appropriate substitution in (10).

The properties of the functions $\gamma(c, h_t)$ and $R(c, h_t)$ are established as follows. From (11) and (12), one finds that

$$\gamma_c(\cdot) = 1 - \sqrt{\frac{A(\beta h_t + B) - (1+r)k}{cAB}}, \tag{A3}$$

$$\gamma_h(\cdot) = -\frac{c\beta}{B}\sqrt{\frac{AB}{c[A(\beta h_t + B) - (1+r)k]}}, \tag{A4}$$

$$R_c(\cdot) = \frac{AB[\gamma(\cdot) + c]}{2kc^2}\left\{c\gamma_c(\cdot) + \sqrt{\frac{c[A(\beta h_t + B) - (1+r)k]}{AB}}\right\}, \tag{A5}$$

$$R_h(\cdot) = \frac{AB[\gamma(\cdot) + c]\gamma_c(\cdot)}{2kc}. \tag{A6}$$

Under the above parameter restrictions, it is deduced that $\gamma_c(\cdot) > 0$, $\gamma_h(\cdot) < 0$, $R_c(\cdot) > 0$ and $R_h(\cdot) < 0. A(B - b) - (1+r)k = \lambda\left[\frac{\gamma(c,\widehat{h})+c}{2c}\right]$

The properties of the function $\widehat{h} = h(c, \lambda)$ in (14) are found by totally differentiating this expression to obtain

$$0 = \left[\frac{\gamma(\cdot) + c}{2c}\right]d\lambda + \frac{\lambda[c\gamma_c(\cdot) - \gamma(\cdot)]}{2c^2}dc + \frac{\lambda\gamma_h(\cdot)}{2c}dh. \tag{28}$$

It follows that $h_c(\cdot) > 0$ and $h_\lambda(\cdot) > 0$.

References

Alesina, A. and D. Rodrik, 1994. Distributive politics and economic growth. *Quarterly Journal of Economics*, 109, 465-490.

Agenor, P.R. and J. Aizenman, 1998a. Contagion and volatility with imperfect credit markets. *IMF Staff Papers*, 45, 207-235.

Agenor, P.R. and J. Aizenman, 1998b. Volatility and the welfare costs of financial market integration. In P.R. Agenor, M. Miller, D. Vines and A. Weber (eds.), *The Asian Financial Crisis: Causes, Contagion and Consequences*, Cambridge University Press.

Aghion, P. and P. Bolton, 1997. A theory of trickle-down growth and development. *Review of Economic Studies*, 64, 151-172.

Aghion, P. and G. Saint-Paul, 1998a. On the virtue of bad times: an analysis of the interaction between economic fluctuations and productivity growth. *Macroeconomic Dynamcs*, 2, 322-344.

Aiyagar, S.R., 1994. Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics*, 109, 73-92.

Aizenman, J. and A. Powell, 2003. Volatility and financial intermediation. *Journal of International Money and Finance*, 22, 659-684.

Azariadis, C. and S. Chakraborty, 1999. Agency costs in dynamic economic models. *Economic Journal*, 109, 222-241.

Banerjee, A.V. and A.F. Newman, 1993. Occupational choice and the process of development. *Journal of Political Economy*, 101, 274-298.

Benabou, R., 1992. Heterogeneity, stratification and growth. Mimeo, MIT.

Benartzi, S. and R.H. Thaler, 1995. Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics*, 110, 73-92.

Blackburn, K. and N. Bose, 2003. A model of trickle-down through learning. *Journal of Economic Dynamics and Control*, 27, 445-466.

Blackburn, K. and D. Varvarigos, 2008. Human capital accumulation and output growth in a stochastic environment. Economic Theory, 36, 435-452.

Bleichrodt, H., U. Schmidt and H. Zank, 2009. Loss aversion in multi-attribute utility theory. *Management Science*, 55, 863-873.

Carroll, C.D., 2001. A theory of the consumption function, with and without liquidity constraints. *Journal of Economic Perspectives*, 15, 23-45.

de Hek, P.A., 1999. On endogenous growth under uncertainty. *International Economic Review*, 40, 727-744.

Diamond, D.W., 1984. Financial intermediation as delegated monitoring. *Review of Economic Studies*, 51, 393-414.

Diecidue, E. and J. Van de Ven, 2008. Aspiration level, probability of success and failure, and expected utility. *International Economic Review*, 49, 683-700.

Durlauf, S., 1993. Neighbourhood feedbacks, endogeous stratification and income inequality. Mimeo, University of Wisconsin.

Edwards, W., 1954. The Theory of Decision Making. *Psychological Bulletin*, 51, 380-417.

Fernandez, R. and R. Rogerson, 1996. Income distribution, communities and the quality of public education. *Quarterly Journal of Economics*, 111, 135-164.

Foellmi, R., R. Rosenblatt-Wisch and K.R. Schenke-Hoppe, 2011. Consumption paths under prospect utility in an optimal growth model. *Journal of Economic Dynamics and Control*, 35, 273-281.

Gale, D. and M. Hellwig, 1985. Incentive compatible debt contracts: the one period problem. *Review of Economic Studies*, 52, 647-663.

Galor, O. and J. Zeira, 1993. Income distribution and macroeconomics. *Review of Economic Studies*, 60, 35-52.

Genicot, G. and D. Ray, 2012. Aspirations and inequality. Mimeo, University of Georgetown.

Grossman, V., 2008. Risky human capital investment, income distribution and macroeconomic dynamics. *Journal of Macroeconomics*, 30, 19-42.

Guiso, L., T. Jappelli and D. Terlizzese, 1996. Income risk, borrowing constraints and portfolio choice. *American Economic Review*, 86, 158-172.

Guiso, L. and M. Paiella, 2008. Risk aversion, wealth and background risk. *Journal of the European Economic Association*, 6, 1109-1150.

Holthausen, D.M., 1981. A risk-return model with risk and return measured as deviations from a target return. *American Economic Review*, 71, 182-188.

Jones, L.E., R.E. Manuelli, H.E. Siu and E. Stacchetti, 2005. Fluctuations in convex models of endogenous growth I: growth effects. *Review of Economic Dynamics*, 8, 780-804.

Kahnemen, D. and A. Tversky, 1979. Prospect theory: an analysis of decision under risk. *Econometrica*, 47, 263-291.

Krebs, T., 2003. Human capital risk and economic growth. *Quarterly Journal of Economics*, 118, 709-744.

Langer, T. and M. Weber, 2001. Prospect theory, mental accounting and differences in aggregated and segregated evaluation of lottery portfolios. *Management Science*, 47, 716-733.

Lopes, L.L. and C. Oden, 1999. The role of aspiration level in risky choice: a comparison of cumulative prospect theory and SP/A theory. *Journal of Mathematical Psychology*, 43, 286-313.

Maggi, M.A., 2004. A characterisation of S-shaped utility finctions displaying loss aversion. Mimeo, Università degli Studi di Pavia.

Martin, P. and C.A. Rogers, 2000. Long-term growth and short-term economic instability. *European Economic Review*, 44, 359-381.

Mezias, S.J., 1988. Aspiration level effects: an empirical investigation. *Journal of Economic Behaviour and Organisation*, 10, 389-400.

Mezias, S.J., Y.R. Chen and P.R. Murphy, 2002. Aspiration level adaptation in an American Financial Services Organisation: a field study. *Management Science*, 48, 1285-1300.

Payne, J.W., 2005. It is whether you win or lose: the importance of the overall probabilities of winning or losing in risky choice. *Journal of Risk and Uncertainty*, 30, 5-19.

Perotti, R., 1993. Political equilibrium, income distribution and growth. *Review of Economic Studies*, 60, 755-776.

Persson, T. and G. Tabellini, 1994. Is inequality harmful for growth? *American Economic Review*, 84, 600-621.

Piketty, T., 1997. The dynamics of wealth distribution and the interest rate with credit rationing. *Review of Economic Studies*, 64, 173-189.

Rabin, M., 2000. Risk aversion and expected utility theory: a calibration theorem. *Econometrica*, 68, 1281-1292.

Rabin, M. and R.H. Thaler, 2001. Anomolies: risk aversion. *Journal of Economic Perspectives*, 15, 219-232.

Rosenblatt-Wisch, R., 2008. Loss aversion in aggregate macroeconomic time series. *European Economic Review*, 52, 1140-1159.

Samuelson, W.F. and R.J. Zeckhauser, 1988. Status quo bias in decision making. *Journal of Risk and Uncertainty*, 1, 7-59.

Schmidt, U. and H. Zank, 2005. What is loss aversion? *Journal of Risk and Uncertainty*, 30, 157-167.

Starmer, C., 2000. Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk. *Journal of Economic Literature*, 38. 332-382.

Thaler, R.H., 1980. Toward a positive theory of consumer choice. *Journal of Economic Behaviour and Organisation*, 1, 39-60.

Townsend, R., 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21, 265-293.

Figure 1
Human Capital Distribution