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Child Labor, Intra-Household Bargaining and Economic Growth

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Abstract

This paper develops a three-period, gender-based overlapping generations model of economic growth with heterogeneity in parental preferences, endogenous intra-household bargaining, and child labor in home production by girls. Improved access to infrastructure reduces the amount of time parents find optimal for their daughters to spend on household chores, thereby allowing them to allocate more time to studying at home. The model is calibrated for a low-income country and various quantitative experiments are conducted, including an increase in the share of public spending on infrastructure, an increase in time allocated by mothers to their daughters, and a decrease in fathers' preference for their daughters' education. Our analysis shows that poor access by families to infrastructure may provide an endogenous explanation, complementary to those focusing solely on social norms and cultural values, for the persistence of child labor at home and gender inequality in low-income countries.

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1 Introduction

The role of women in promoting growth and development continues to occupy center stage in policy debates and academic circles alike. Much evidence suggests that gender inequality in terms of access to education, health, formal sector employment, and income continues to be a significant constraint on human development and growth in many developing countries.[1] For instance, although in many countries gender parity has been achieved in primary and secondary school enrollment, in many others—especially in Sub-Saharan Africa—girls go to school much less frequently than boys. In developing countries, nearly 1 of every 5 girls who enrol in primary school do not complete their primary education and only 43 percent of girls attend secondary school (UNICEF (2007, 2012*a*)). In low-income countries, only 5 to 10 percent of students are female. According to the International Labour Office (2012, p. 16), the gender gap in the labour force participation rate decreased globally in the 1990s from 27.9 to 26.1 percentage points. However, between 2002 and 2012, it remained largely constant. In 2012, the labor force participation rate for women was only 31.8 percent in South Asia, compared with 81.3 percent for men; for Latin America and the Caribbean, these rates were 49.6 and 79.5, and for the Middle East, 18.7 and 74.3, respectively. When they do work, women often face less favorable employment opportunities and often end up in "bad jobs," with poor prospects of escaping precarity and vulnerability.

The causes of gender inequality (both at home and in the workplace) are complex and include a wide range of economic and noneconomic factors, such as social norms, cultural values, religious beliefs, and inadequate social institutions. A few contributions on this issue have focused on women's bargaining power—or lack thereof—in the family as a possible structural cause of inequality between husbands and wives.[2] Basu (2006), for instance, developed a collective household model in which spouses have different utility functions and the power balance in the family is endogenously related

[1] See Blackden and Bhanu (1999), Blackden et al. (2006), Herz and Sperling (2004), Morrison et al. (2007), Momsen (2009), Jütting et al. (2010), and World Bank (2011).

[2] Another strand of the literature has focused on endogenizing social institutions themselves, at the local or national levels. Strulik (2011) for instance studied how community attitudes affect school attendance and child labor, and how aggregate behavior of the community feeds back onto the formation of schooling attitudes. His analysis has obvious implications for gender inequality as well.

to decisions made by the family itself, via consumption and labor supply. His analysis shows that in some cases the equilibrium outcome may be characterized by persistence in gender inequality.

This paper follows the same perspective, but focuses on an alternative mechanism through which intra-household (cooperative) bargaining may matter for gender equality and economic growth: the impact of women's bargaining power on girls' human capital accumulation. The premise of our analysis is that bargaining between spouses may bias the allocation of family resources toward girls and may have major effects on their ability to accumulate human capital when young—thereby affecting their productivity and capacity to generate income in adulthood. In addition, we also take a *macro* perspective on women's bargaining power, by emphasizing the role of access to infrastructure. If such access is poor, girls may be forced by their parents to engage in household chores—an important form of child labor, as discussed by Fors (2012)—thereby limiting their ability to generate human capital in childhood and restraining their bargaining power in adulthood. The weaker women's intra-household bargaining is, to begin with, the greater the adverse effect on girls' time allocated to home schooling and the weaker their human capital later in life. Thus, poor access to infrastructure may explain persistence in gender inequality.

To conduct our analysis we develop a three-period, gender-based overlapping generations (OLG) model of endogenous growth in which parental preferences are heterogeneous and only girls are involved in child labor (in the form of work at home, that is, time allocated to household chores, rather than work outside the home).[3] This is consistent with the evidence for a wide range of developing countries. Webbink et al. (2012), for instance, in an extensive study of 16 African and Asian countries, found that about 30 percent of African children and 11 percent of Asian children work over 15 hours a week in what they call *hidden* child labor—family and business work. Girls

[3]Gender-based OLG models include a seminal paper by Galor and Weil (1996), and subsequent contributions by Greenwood et al. (2005), de la Croix and Vander Donckt (2010), Agénor (2017), and Agénor and Canuto (2015). Other important recent contributions on the economics of gender include Fernández (2013), and Doepke and Tertilt (2014), which is further discussed later. None of these papers, however, considers jointly the issues of child labor and gender inequality, as we do here. Greenwood et al. (2005), in particular, focus on the liberation of adult women and emphasize the effect of technology on the price of durable goods.

are more involved in housework whereas boys tend to work in the family business. In the same vein, in a study for Bolivia, Zapata et al. (2011) found that girls are 51 percent more likely than boys to be out of school and working, mostly in domestic activities; this probability is even higher for indigenous girls.[4] In Guatemala, more than 90 percent of child domestic workers are girls (UNICEF (2007, p. 48)). Also related to our purpose, Reggio (2011) found that in Mexico an increase in a mother's bargaining power—measured in terms of ownership of family assets and the decision-making process related to those assets—is associated with fewer hours of work, including housework, for her daughters but not for her sons.

In the model, intra-household bargaining is endogenous and depends on the relative level of human capital of men and women. Girls' time is combined with access to infrastructure to produce home goods and parents choose how much time their daughters must allocate to home production. The key mechanism that we highlight is that improved access to infrastructure reduces the amount of time that parents find optimal for their daughters to spend on household chores, which allows them therefore to allocate more time to studying at home—thereby enhancing the human capital that they build in childhood and use in adulthood. In turn, this increase in human capital, to the extent that it occurs at a relatively faster rate than boys' human capital, may improve women's bargaining power. If mothers value relatively more the education of their daughters, this shift in bargaining power may further reduce the amount of time that the family finds optimal for girls to spend in home production. The benefits of improved access to infrastructure over time and across generations are therefore magnified.[5] Thus, our analysis shows that poor access by families to infrastructure may provide an endogenous, "macro" explanation, complementary to studies emphasizing social norms and attitudes, and religious or cultural factors, for the persistence of child labor at home and gender inequality in low-income countries.

The remainder of the paper is organized as follows. Section 2 describes the model, whereas Section 3 characterizes the balanced growth equilibrium and illustrates ana-

[4] See also the references in Edmonds (2008) and Webbink et al. (2012).

[5] The shift in bargaining power may also tilt the allocation of the family's resources toward children in general and girls in particular—a mechanism for which there is much empirical evidence, even though we do not dwell much on it in the present paper.

lytically the transitional and steady-state effects of an increase in public investment in infrastructure, taking into account endogenous intra-household bargaining. Section 4 presents a benchmark calibration for low-income countries. The approach that we propose here is to calibrate the steady-state solution of the model and focus therefore on the long-run effects of policy and exogenous shocks because of the fact that many of these shocks are structural in nature and unlikely to produce tangible economic results in the short-run. In Section 5, several experiments designed to illustrate the properties of the model are discussed, including (again) an increase in investment in infrastructure, a reallocation of mothers' time toward girls, a reduction in the sensitivity of women's bargaining power, and a reduction in fathers' preference for their daughters' education. Section 6 offers some concluding remarks and discusses possible extensions of the analysis.

2 The Model

We now present a three-period, gender-based overlapping generations (OLG) model of economic growth with public capital that incorporates intra-household bargaining.

Formally, we consider an OLG economy where two goods are produced, a marketed commodity and a home good. Individuals live for three periods, denoted $t-1$, t, $t+1$: childhood, adulthood (or middle age) and retirement. The marketed commodity can be either consumed in the period it is produced or stored to yield capital at the beginning of the following period. Each individual is either male or female, and is endowed with one unit of time in childhood and adulthood, and zero units in old age. Children are born with the same innate abilities and depend on their parents for consumption and any spending associated with schooling. Girls and boys are endowed with one unit of time. But whereas boys allocate their time between school and homework only, girls allocate their time between school, homework, and household chores.[6] The latter activities are viewed here as a form of child labor, an expression often used to refer to work outside the home. Mothers' time allocated to child rearing and market work is

[6] Note that we do not consider child "domestic workers," that is, children (girls, for the most part) who work in other people's households, doing domestic chores, caring for children, etc.; see UNICEF (1999, 2012*b*) and International Labour Office (2013).

considered exogenous.[7]

All individuals, both males and females, work in middle age. The only source of income is therefore wages in the second period of life, which serve to finance family consumption in adulthood and old age. At the beginning of adulthood, individuals of each gender also meet randomly with someone of the opposite sex to form a family. All income is pooled, and couples therefore become joint decision makers. For simplicity, once married, individuals do not divorce; couples retire together and die together.[8] Each couple produces a constant number $n \geq 2$ of children. It is also assumed that parents' preferences over boys and girls are the same, and that they have control over the gender composition of their family, so that half of their children are daughters and half of them sons. Rearing children involves only parental time; for simplicity, we abstract from spending on marketed commodities to feed them and send them to school. Male spouses allocate inelastically all their time to market work. Due to exogenous factors (such as social or cultural norms), mothers incur the whole time cost involved in rearing children.[9]

In addition to individuals, the economy is populated by firms and an infinitely-lived government. Firms produce marketed commodities using public capital in infrastructure as an input, in addition to male and female labor and private capital. Home production (which affects positively the family's utility) combines girls' time and infrastructure services. Only girls are engaged in home production. The government invests in infrastructure and spends on education, as well as some unproductive items. It taxes the wage income of adults (males and females), but not the interest income of retirees. It cannot borrow and therefore must run a balanced budget in each period. Finally, all markets clear.

[7] Of course we could assume that mothers also allocate some time to home production—as documented by a significant body of evidence, reviewed in Agénor (2012). But as long as such time is supplied inelastically, this would not alter our analysis. Endogenizing women's time allocation could be pursued along the lines discussed for instance in Agénor (2017) and Agénor et al. (2014). However, our focus here is solely on girls' child labor, and we therefore abstract from that issue. We return to it in our concluding remarks.

[8] The assumption that spouses die together, or soon after the passing of the other, is consistent with the evidence on the so-called *broken heart syndrome*—clinically known as stress cardiomyopathy—according to which sudden emotional stress related to the loss of a close family member can trigger acute heart failure.

[9] Thus, our analysis does not address that source of gender bias; see Zhang et al. (1999) for instance.

2.1 Home Production

Home production (which includes cooking dinner, doing laundry, cleaning the house, etc.) involves combining girls' time, in proportion $\varepsilon_t^{g,P}$, and infrastructure services.[10] Production, Q_t, takes place under decreasing returns to scale:

$$Q_t = [0.5n\varepsilon_t^{g,P} + \zeta^P(\frac{K_t^I}{K_t^P})]^{\pi^Q}, \tag{1}$$

where the superscript g is used to identify girls, $0.5n$ the number of daughters in the family, K_t^I is the stock of public capital in infrastructure, K_t^P the aggregate stock of private capital, $\pi^Q \in (0,1)$, and $\zeta^P \in (0,1)$ a coefficient that parameterizes the degree of efficiency of infrastructure services relative to girls' time. A low (high) degree of efficiency between girls' time and is thus captured by a value of ζ^P close to zero (unity). Thus, greater access to roads or electricity allows girls to devote less time to home production. With better access to roads, for instance, girls do not need to walk long hours to fetch water and collect wood, especially in rural areas (see Food and Agriculture Organization (2010)). Access to infrastructure is not excludable but subject to congestion (and thus partially rival), as discussed next.[11]

2.2 Market Activity

Firms are identical and their number is normalized to unity. They produce a single nonstorable commodity, using male effective labor, $E_t^m N_t^{m,i}$ (where E_t^m is average male human capital and $N_t^{m,i}$ male raw labor), female effective labor, defined as $\varepsilon^{f,W} E_t^f N_t^{f,i}$ (where E_t^f is average female human capital, $N_t^{f,i}$ female raw labor, and $\varepsilon^{f,W}$ time allocated by mothers to market work), private capital, $K_t^{P,i}$, and public infrastructure. Although public capital is nonexcludable, it is partially rival because of congestion effects; for simplicity, congestion is taken to be proportional to the aggregate private capital stock, $K_t^P = \int_0^1 K_t^{P,i} di$. Thus, the more firms use public infrastructure services

[10] The model could be extended to account for the use of marketed goods as inputs in the production of home good, as for instance in Siegel (2012. However, this would complicate significantly the analysis without adding much insight, given the issue at stake.

[11] The assumption of nonexcludability (no agent, individual or firm, can prevent other agents from using it concomitantly) is important here to justify the introduction of the *aggregate* stock of public capital in the production functions for the home and market goods.

in the production process (as measured by their private capital stock), the smaller the stock of those assets available for use by firms and (from (1)) households.

The production function of individual firm $i \in (0,1)$ takes the form

$$Y_t^i = (\frac{K_t^I}{K_t^P})^{\alpha}(E_t^m N_t^{m,i})^{\beta}(\varepsilon^{f,W} E_t^f N_t^{f,i})^{\beta}(K_t^{P,i})^{1-2\beta}, \tag{2}$$

where $\beta \in (0,1)$. The elasticity of output with respect to male and female labor is assumed to be the same.[12]

With the price of the marketed good normalized to unity, profits of firm i in the final sector, $\Pi_{i,t}$, are given by

$$\Pi_{i,t} = Y_t^i - (w_t^m E_t^m N_t^{m,i} + w_t^f E_t^f \varepsilon^{f,W} N_t^{f,i}) - r_t K_t^{P,i},$$

where r_t is the rental rate of private capital (which is also the rate of return on savings), w_t^m the effective male wage, and w_t^f the effective female wage.

Given our emphasis on intra-household bargaining (or gender inequality in the family), we abstract from gender discrimination in the workplace.[13] Thus, profit maximization with respect to private inputs, taking factor prices as given, yields

$$w_t^m = \frac{\beta Y_t^i}{E_t^m N_t^{m,i}}, \quad w_t^f = \frac{\beta Y_t^i}{\varepsilon^{f,W} E_t^f N_t^{f,i}}, \quad r_t = \frac{(1-2\beta)Y_t^i}{K_t^{P,i}}. \tag{3}$$

In equilibrium, the superscript i can be dropped. And given that men and women are in equal numbers in the adult population ($N_t^m = N_t^f$),[14]

$$w_t^m = (\frac{\varepsilon^{f,W} E_t^f}{E_t^m})w_t^f. \tag{4}$$

[12]In practice, 2β is in the range 0.6-0.7, consistent with the observed share of labor income in output.

[13]The two issues may not be unrelated; discrimination against women in the labour market could justify the assumption that only girls are used as child labor and only mothers spend time on child rearing. Indeed, Chichilnisky (2008) studied a game with incomplete information about women's work at home and in the marketplace. Expectations about women's lower wages lead to women bearing the brunt of household chores, and this, in turn, hampers their productivity and lowers their wages in the marketplace. Inequality at home fosters inequality in the marketplace and vice versa, and both combine to generate persistence in the gender gap.

[14]Because $\varepsilon^{f,W} \in (0,1)$, if $E_t^f < E_t^m$, assuming that women are paid less than their marginal product as a result of gender discrimination (see Agénor (2017)) would be necessary to ensure that (4) delivers the condition $w_t^m > w_t^f$.

Given that all firms are identical, and that their number is normalized to 1, $K_t^P = K_t^{P,i}$ $\forall i$, and aggregate output Y_t is, from (2),

$$Y_t = \int_0^1 Y_t^i di = (k_t^I)^\alpha (E_t^m N_t^m)^\beta (E_t^f \varepsilon^{f,W} N_t^f)^\beta (K_t^P)^{1-2\beta},$$

where $k_t^I = K_t^I / K_t^P$ is the public-private capital ratio. Equivalently, this expression can be rewritten as

$$Y_t = (k_t^I)^\alpha (\frac{E_t^m N_t^m}{K_t^P})^\beta (\frac{E_t^f N_t^f}{K_t^P})^\beta (\varepsilon^{f,W})^\beta K_t^P. \quad (5)$$

2.3 Time Allocation and Utility

To raise their children mothers must spend $\varepsilon^{f,R} \in (0,1)$ units of time on each of them but (as noted earlier) no direct cost in terms of marketed commodities.

In addition to raising children, mothers allocate time to market activity (in proportion $\varepsilon^{f,W}$). The time that females can devote to market activity is thus[15]

$$\varepsilon^{f,W} = 1 - n\varepsilon^{f,R}. \quad (6)$$

Let ε^S denote the fixed and indivisible amount of time that boys and girls must both allocate to formal (out of home) schooling. The time allocated by boys (identified with the superscript b) and girls to home schooling is thus

$$\varepsilon^{b,L} = 1 - \varepsilon^S, \quad (7)$$

$$\varepsilon_t^{g,L} = 1 - \varepsilon^S - \varepsilon_t^{g,P}, \quad (8)$$

where, as noted earlier, $\varepsilon_t^{g,P}$ denotes (endogenous) the amount of time that girls allocate to home production.[16] For simplicity, we do not explicitly account for the fact that (older) girls may also allocate time to rearing their (younger) siblings, given that we consider only one period in childhood.

[15] As noted earlier, because we assume that the fertility rate is exogenous and constant at n, and $\varepsilon^{f,R}$ is exogenous, $\varepsilon^{f,W}$ is exogenous as well. We introduce them explicitly, however, because these are important to provide a realistic numerical calibration of the model, as discussed later on.

[16] Note that only girls' time is endogenously related to public infrastructure, whereas the amount of time $\varepsilon^{b,L}$ that boys spend in home schooling is exogenous.

The family's (collective) utility takes the composite form

$$U_t = \varkappa_t U_t^f + (1 - \varkappa_t) U_t^m, \tag{9}$$

where U^j is partner j's utility function and $\varkappa_t \in (0, 1)$ is a weight that measures the wife's bargaining power in the household decision process. Perfect equality corresponds therefore to $\varkappa_t = 0.5$. As shown by Doepke and Tertilt (2014), maximizing (9) subject to appropriate constraints and for $\varkappa_t$ given yields an outcome that is similar to the solution of a Nash bargaining problem in which the couple maximizes the weighted product of the two partners' marital surpluses and the outside option is given by the utility achieved upon divorce.[17]

Families consume both the marketed commodity and the good produced at home. Assuming that the consumption of children is subsumed in the family's consumption, the sub-utility functions are given by, with $j = m, f$,

$$U_t^j = \eta_C^j \ln c_t^{t-1} + \eta_Q \ln Q_t + \eta_E^j \ln e_{t+1}^f + \frac{1}{1+\rho} \ln c_{t+1}^{t-1}, \tag{10}$$

where c_t^{t-1} and c_{t+1}^{t-1} are the family's total consumption in adulthood and old age, respectively, e_{t+1}^f a unit of human capital of a female, and $\rho > 0$ a common discount rate. Coefficients η_C^j measure the relative preference for today's consumption, η_Q the family's common relative preference for the home produced good, and η_E^j the relative preference for girls' education. The restrictions $\eta_C^f < \eta_C^m$ and $\eta_E^f > \eta_E^m$ are also imposed. Thus parents benefit equally from consumption of the home good; η_Q does not depend on j. But women are less concerned than men about current consumption ($\eta_C^f < \eta_C^m$) and care more about the human capital of their daughters ($\eta_E^f > \eta_E^m$). Thus, there is intergenerational altruism, but it matters more for mothers. These facts are well documented in a number of studies, including UNICEF (2007), World Bank (2011), and Doepke and Tertilt (2014). Note that only the marketed commodity is consumed in old age.

A male (female) adult in period t is endowed with e_t^m (e_t^f) units of human capital. Each unit of human capital earns an *effective* market wage, w_t^m for men and w_t^f for women, per unit of time worked.

[17] Doepke and Tertilt also developed a noncooperative model of household bargaining that has similar implications to the type of cooperative bargaining framework used here.

The family's budget constraints for periods t and $t+1$ are given by

$$c_t^{t-1} + s_t = (1-\tau)w_t^T, \tag{11}$$

$$c_{t+1}^{t-1} = (1+r_{t+1})s_t, \tag{12}$$

where $\tau \in (0,1)$ a constant tax rate, s_t savings, and w_t^T *gross* wage income of the family, defined as

$$w_t^T = e_t^m w_t^m + \varepsilon^{f,W} e_t^f w_t^f. \tag{13}$$

From equations (11) and (12), the family's consolidated budget constraint is

$$c_t^{t-1} + \frac{c_{t+1}^{t-1}}{1+r_{t+1}} = (1-\tau)w_t^T. \tag{14}$$

Assuming cooperative bargaining, families maximize (9), taking $\varkappa_t$, factor prices and the tax rate as given, subject to (10) and (14), with respect to c_t^{t-1}, c_{t+1}^{t-1}, and $\varepsilon_t^{g,P}$, which as shown below affects girls' human capital in adulthood.[18]

2.4 Human Capital Accumulation

Boys and girls have access to the same "out of home" learning technology. However, each group's education outcomes depend also on the amount of time that parents devote to tutoring them at home.

Let e_{t+1}^j, $j = m, f$ be the human capital of males and females born in period t and used in period $t+1$. The production of either type of human capital requires several inputs. First, it depends on the time mothers allocate to tutoring their children. Specifically, mothers subdivide their total amount of time allocated to child rearing, $\varepsilon^{f,R}$, which is set exogenously, into fixed fractions $\chi^R \in (0,1)$ allocated to sons and $1-\chi^R$ allocated to daughters.[19] A bias in parental preferences toward boys can therefore be captured by assuming that $\chi^R > 0.5$.

[18]Note that it could also be assumed that the amount of time that girls spent in household chores, $\varepsilon_t^{g,P}$, generates a positive externality in terms of the total time that mothers devote to child rearing, $\varepsilon^{f,R}$, thereby affecting their participation in market activities. However, due to indivisibilities in labor supply this effect is likely to be relatively small in practice and is abstracted from for simplicity.

[19]The analysis could be extended to account for boys' education in the family's utility function and solve optimally for the time that mothers allocate to them, χ^R. In the present setting, we take it as being determined by social norms.

Second, knowledge accumulation depends on average government spending on education per child, $G_t^E/n0.5N_t$, where N_t is the number of adults alive in period t, itself given by

$$N_t = \; n0.5N_{t-1}, \tag{15}$$

that is, the number of children born in period $t-1$, n, times the number of families formed in $t-1$, $0.5N_{t-1}$.[20]

Third, human capital accumulation depends on a mother's human capital. Because individuals are identical within a generation, a mother's human capital at t is equal to the average human capital of the previous generation, E_t^f. Finally, although time spent in school affects equally the human capital of boys and girls at $t+1$, girls' human capital depends also on the amount of time that they allocate to school-related activities at home.[21]

Thus, abstracting from gender-based discrimination in the public education system itself, and assuming no depreciation for simplicity, the human capital that men and women have in the second period of life is[22,23]

$$e_{t+1}^m = (\frac{G_t^E}{n0.5N_t})^{\nu_1}(E_t^f)^{1-\nu_1}(\chi^R\varepsilon^{f,R})^{\nu_2}(\varepsilon^S)^{\nu_3}, \tag{16}$$

$$e_{t+1}^f = (\frac{G_t^E}{n0.5N_t})^{\nu_1}(E_t^f)^{1-\nu_1}[(1-\chi^R)\varepsilon^{f,R}]^{\nu_2}(\varepsilon^S+\varepsilon_t^{g,L})^{\nu_3}, \tag{17}$$

where $\nu_1 \in (0,1)$, ν_2, $\nu_3 > 0$.

[20] We therefore abstract from the possibility that government spending in education may itself be subject to gender bias; see, for instance, Masterson (2012).

[21] Our analysis would remain conceptually the same as long as time allocated to home schooling affects boys and girls differently, with a higher marginal effect for girls. The key point is that the time that girls can allocate to school-related activities at home (if any) is determined residually, given the time constraint, time spent in school, and time spent in household chores, which is determined by the family's utility maximization problem.

[22] For tractability, the human capital technology is taken to exhibit constant returns to scale in government spending and the average human capital of mothers. Note also that we abstract from the impact of infrastructure on human capital; see for instance Yamauchi et al. (2011) for some country evidence and Agénor (2012) for an overview. Accounting for this externality would strengthen the main policy conclusion of this paper, if it is stronger for girls; otherwise it would not affect the relative human capital ratio and therefore would not affect women's bargaining power.

[23] In principle the term $\varepsilon^S + \varepsilon^{b,L}$, rather than ε^S alone, should enter in equation (16), to ensure symmetry with (17). However, $\varepsilon^{b,L}$ is constant in the analysis and this would not have any qualitative effect on the behavior of bargaining power and human capital accumulation.

Combining equations (16) and (17) yields

$$\frac{e^f_{t+1}}{e^m_{t+1}} = (\frac{1-\chi^R}{\chi^R})^{\nu_2}(\frac{\varepsilon^S + \varepsilon^{g,L}_t}{\varepsilon^S})^{\nu_3}, \tag{18}$$

which shows, all else equal, that the more girls are able to allocate time to studying at home, the higher their human capital will be relative to boys. A reduction in χ^R (that is, an increase in rearing time allocated to daughters) also raises the relative level of girls' human capital.

2.5 Government

As noted earlier, the government taxes only the wage income of adults. It spends G^I_t on infrastructure investment, G^E_t on education, and G^U_t on other (not directly productive) items. All its services are provided free of charge. It cannot issue bonds and must therefore run a balanced budget:

$$G_t = \Sigma G^j_t = \tau(w^m_t E^m_t N^m_t + w^f_t \varepsilon^{f,W} E^f_t N^f_t). \tag{19}$$

Shares of spending are all assumed to be constant fractions of government revenues:

$$G^j_t = \upsilon_j \tau(w^m_t E^m_t N^m_t + w^f_t \varepsilon^{f,W} E^f_t N^f_t), \quad j = E, I, U \tag{20}$$

where $\upsilon_j \in (0, 1)$ for all j. Combining these equations therefore yields

$$\sum \upsilon_j = 1. \tag{21}$$

Assuming full depreciation for simplicity, public capital in infrastructure evolves according to[24]

$$K^I_{t+1} = G^I_t. \tag{22}$$

2.6 Bargaining Power

We now examine what determines women's bargaining power, $\varkappa_t$. In the literature, women's bargaining power has been related to, or measured by, a variety of measures:

[24] Although here we focus on the case where only the flow of public investment determines the accumulation of public capital in infrastructure, in the Appendix we consider the more general case where existing public capital is an essential input in the production of public capital in infrastructure.

the male-female ratio of earned incomes, the share of assets that they hold within the household or patterns of decision-making within the household (as revealed by surveys), and women's access to financial services.[25] However, it has been found that several of these measures are highly correlated with relative educational outcomes (see, for instance, Frankenberg and Thomas (2003)). Accordingly, here the relative bargaining power of women is assumed to evolve as a function of an autonomous component $\bar{\varkappa} \in (0,1)$ and of the relative levels of average human capital of husband and wife:

$$\varkappa_t = \bar{\varkappa}^{1-\gamma_B}[(\frac{E_t^f}{E_t^m})^{\mu_B}]^{\gamma_B}, \tag{23}$$

where $\gamma_B \in (0,1)$ measures the relative importance of the endogenous component of bargaining power; $1-\gamma_B$ measures therefore the importance of extraneous factors, such as social norms and cultural values. The parameter $\mu_B \geq 0$ measures the sensitivity of the endogenous component of bargaining power to relative stocks of human capital.

2.7 Market-Clearing Condition

The asset-market clearing condition requires equality between savings and investment, or equivalently, that tomorrow's private capital stock be equal to today's savings by adult workers. Given that s_t is savings per family, that the number of families is $(N_t^m + N_t^f)/2$, and that $N_t^m = N_t^f$,

$$K_{t+1}^P = 0.5(N_t^m + N_t^f)s_t = N_t^f s_t, \tag{24}$$

where again for simplicity full depreciation is assumed.

3 Balanced Growth Path

A *competitive equilibrium* in this model is a sequence of prices $\{w_t^m, w_t^f, r_{t+1}\}_{t=0}^{\infty}$, allocations $\left\{c_t^{t-1}, c_{t+1}^{t-1}, s_t, \varepsilon_t^{g,P}\right\}_{t=0}^{\infty}$, physical capital stocks $\left\{K_{t+1}^P, K_{t+1}^I\right\}_{t=0}^{\infty}$, human capital stocks $\{E_{t+1}^m, E_{t+1}^f\}_{t=0}^{\infty}$, a constant tax rate, and constant spending shares such that, given initial stocks $K_0^P, K_0^I > 0$ and $E_0^m, E_0^f > 0$, individuals maximize utility, firms

[25] See for instance Doss (1996, 2013), Frankenberg and Thomas (2003), Anderson and Eswaran (2009), Angel-Urdinola and Wodon (2010), and Quisumbing (2010).

maximize profits, markets clear, and the government budget is balanced. In equilibrium, it must also be that $e_t^j = E_t^j$ for $j = m, f$. A *balanced growth equilibrium* is a competitive equilibrium in which c_t^{t-1}, c_{t+1}^{t-1}, K_{t+1}^P, K_{t+1}^I, E_{t+1}^m, E_{t+1}^f grow at the constant, endogenous rate $1+\gamma$, and the rate of return on private capital, girls' time allocation, and bargaining power are all constant.

As shown in the Appendix, the solution of the model yields

$$k_t^I = J = \frac{\upsilon_I \tau}{\sigma(1-\tau)}, \quad \forall t \tag{25}$$

where σ is the family's propensity to save, defined as

$$\sigma = \frac{1}{1+(1+\rho)\eta_C} < 1. \tag{26}$$

Equation (25) implies that the public-private capital ratio is constant over time.

As also shown in the Appendix, solving the family's optimization problem leads to the following solution for girls' time allocated to home production, allowing for the fact that they may *not* go to school at all—in which case all their time is devoted to household chores:

$$\varepsilon^{g,P} = \min[\frac{\Lambda_1 - \zeta^P J}{\Lambda_2}, 1], \tag{27}$$

where

$$\Lambda_1 = 0.5n\eta_Q \pi^Q \eta_E^{-1} \nu_3^{-1},$$

$$\Lambda_2 = 0.5n + \Lambda_1,$$

and, for $h = C, E$,

$$\eta_h = \varkappa \eta_h^f + (1-\varkappa)\eta_h^m = \eta_h^m + (\eta_h^f - \eta_h^m)\varkappa. \tag{28}$$

Given the restrictions imposed earlier, $\eta_C^f < \eta_C^m$ and $\eta_E^f > \eta_E^m$, equation (28) implies that

$$\frac{d\eta_C}{d\varkappa} < 0, \quad \frac{d\eta_E}{d\varkappa} > 0.$$

Combining (8) and (27) yields girls' time allocated to school-related activities at home, assuming that they *do* attend school:

$$\varepsilon^{g,L} = \max\left\{1 - \varepsilon^S - \frac{\Lambda_1 - \zeta^P J}{\Lambda_2}, 0\right\}. \tag{29}$$

This result shows that improved family access to infrastructure reduces the amount of time parents find optimal for their daughters to spend on household chores and therefore allows daughters to allocate more time to studying at home. Moreover, equations (27)-(29) show that a higher family preference for girls' education (a higher η_E) reduces the optimal amount of time that girls must allocate to household chores. Because η_E depends positively on women's bargaining power, $\varkappa$, it follows that an increase in $\varkappa$ contributes to improving women's human capital, independently of any other effect.

Equations (27) and (29) also show the possibility of a *stagnating equilibrium*, as in Bell and Gersbach (2009) for instance: indeed, if access to public capital is too low, it is possible for $(\Lambda_1 - \zeta^P J)/\Lambda_2 > 1 - \varepsilon^S$, even while $(\Lambda_1 - \zeta^P J)/\Lambda_2 \leq 1$, in which case $\varepsilon^{g,L} = 0$. In those conditions, parents will choose *not* to send their daughters to school, implying therefore no school-related activities at home.[26] The critical value of the public-private capital ratio above which schooling takes place for girls is thus $(\Lambda_1 - \zeta^P J_C)/\Lambda_2 = 1 - \varepsilon^S$, so that

$$J_C = \frac{\Lambda_1 - \Lambda_2(1 - \varepsilon^S)}{\zeta^P}. \tag{30}$$

Figure 1 illustrates the behavior of $\varepsilon^{g,L}$ and $\varepsilon^{g,P}$ as a function of J. For $J = 0$, equation (27) implies that $\varepsilon^{g,P} = \Lambda_1/\Lambda_2 < 1$. However, for parents to actually send their daughters to school, $\varepsilon^{g,P}$ must actually be less than $1 - \varepsilon^S$, given that the amount of time that they must allocate to that activity is indivisible. For $0 < J < J_C$, $\varepsilon^{g,P}$ remains above $1 - \varepsilon^S$, so girls do not attend school at all. As a result, $\varepsilon^{g,P} = 1$ and $\varepsilon^{g,L} = 0$. As J increases above J_C, $\varepsilon^{g,P}$ jumps down from 1 to either $1 - \varepsilon^S$ or some value below that (from point A to point B, for instance) and continues to fall afterward. At the same time, $\varepsilon^{g,L}$ starts increasing from its initial value of 0, reaching a maximum at $1 - \varepsilon^S$, which is obtained when $\varepsilon^{g,P} = 0$, that is, from (27), when $J \geq J_H = \Lambda_1/\zeta^P$.[27]

As shown in the Appendix, the model can be condensed into a single first-order

[26] In that case, of course, the model generates a corner solution in which no capital is accumulated and output is zero.

[27] For simplicity, we assume that there is no minimum amount of time that girls must allocate to home production.

difference equation in $x_t^f = K_t^P/e_t^f N_t^f$, the private capital-effective female labor ratio:

$$x_{t+1}^f = \Gamma_5 (x_t^f)^{(1-2\beta)(1-\nu_1)}, \tag{31}$$

where

$$\Gamma_5 = \Gamma_4 J^{\alpha(1-\nu_1)} (\varepsilon^S + \varepsilon^{g,L})^{-\nu_3[1+\beta(1-\nu_1)]},$$

$$\Gamma_4 = \Gamma_3 \Gamma_1^{1-\nu_1},$$

$$\Gamma_1 = \left\{ (\frac{\chi^R}{1-\chi^R})^{\nu_2} \varepsilon^{f,W} (\varepsilon^S)^{\nu_3} \right\}^{\beta},$$

$$\Gamma_3 = \left\{ \frac{2\beta(1-\tau)\sigma}{[(1-\chi^R)\varepsilon^{f,R}]^{\nu_2} n^{1-\nu_1} 0.5} \right\} (\upsilon_E \tau 2\beta)^{-\nu_1},$$

with the growth rate of output given by

$$1 + \gamma_{t+1} = \frac{Y_{t+1}}{Y_t} = \Gamma_1 (\frac{1}{\varepsilon^S + \varepsilon^{g,L}})^{\beta\nu_3} J^{\alpha} (\frac{1}{x_{t+1}^f})^{2\beta} 2\beta(1-\tau)\sigma. \tag{32}$$

Stability of the adjustment process described by (31) requires $|(1-2\beta)(1-\nu_1)| < 1$, which always holds. The steady-state solution of (31) is

$$\tilde{x}^f = \Gamma_5^{1/\Pi_1}, \tag{33}$$

where $\Pi_1 = 1 - (1-2\beta)(1-\nu_1) > 0$. Substituting this solution in (32) gives the steady-state growth rate of output:

$$1 + \gamma = \Gamma_1 (\varepsilon^S + \varepsilon^{g,L})^{-\beta\nu_3} J^{\alpha} (\tilde{x}^f)^{-2\beta} 2\beta(1-\tau)\sigma. \tag{34}$$

The adjustment process corresponding to (31) is illustrated by the concave curve XX in the right-hand side panel of Figure 2. The left-hand side panel in the figure displays the convex curve GG, which corresponds to (32) and shows the relationship between the growth rate of output $1+\gamma_{t+1}$ and the private capital-effective female labor ratio x_{t+1}^f. The initial equilibrium obtains at points A and B.

Note also that using (18), (23), and (29), with $J \geq J_C$,

$$\varkappa = \bar{\varkappa}^{1-\gamma_B} [(\frac{1-\chi^R}{\chi^R})^{\nu_2} (\frac{\varepsilon^S + \varepsilon^{g,L}}{\varepsilon^S})^{\nu_3}]^{\mu_B \gamma_B} = \varkappa(J), \tag{35}$$

with $\varkappa' > 0$. Thus, women's bargaining power is also positively related to access to infrastructure. Note also that in the particular case where $J = J_C$, with J_C defined in (30), $\varepsilon^{g,L} = 0$ and women's bargaining power is independent of the public-private capital ratio—even though girls are actually allowed to attend school.

To illustrate analytically the long-run effects of public capital, consider the impact of a budget-neutral increase in the share of government spending on infrastructure, financed by a cut in unproductive spending, that is, $d\upsilon_I + d\upsilon_U = 0$.[28] As shown in the Appendix, an increase in υ_I raises the public-private capital ratio, time allocated by girls to home schooling, and women's bargaining power, but it has an ambiguous effect on the private capital-effective female labor ratio and the steady-state growth rate. The reason for the latter is as follows. The increase in the public-private capital ratio has a direct, positive effect on growth, which reflects its impact on overall productivity of private inputs. In turn, the increase in productivity tends to increase the demand for (male and female) labor. At the same time, the increase in girls' time allocated to home schooling raises directly their human capital and the effective supply of female labor. There is also an indirect effect on that variable because the initial relative increase in women's human capital raises their bargaining power (to an extent that depends on the parameter ν_3), which increases the family's preference for girls' education, η_E, and induces parents (as discussed earlier) to further reduce their daughters' time allocated to household chores.[29] However, because both the private capital stock and the effective supply of female labor increase, the change in the ratio of these variables is ambiguous and so is its impact on growth.

This ambiguity is illustrated in Figure 2 as well. As can be inferred from (29), (33), and (34), and given that both J and $\varepsilon^{g,L}$ increase, curves XX and GG may shift either upward or downward following an increase in υ_I. The figure illustrates the case where GG shifts upward (which implies, for a given value of the private capital-effective female labor ratio, that the direct effect of the public-private capital ratio dominates its indirect effect of girls' time allocated to home schooling, $\varepsilon^{g,L}$), whereas XX shifts either up or down. In the first case, the new equilibrium is at A' and B', characterized

[28]Assuming instead that the increase in infrastructure investment is financed by a cut in education spending (as discussed later in the numerical experiments) would not alter the fundamental ambiguities discussed here.

[29]An increase in $\varkappa$, as noted earlier, also tends to reduce η_C which, from (26), tends to increase the savings rate, and thus the private capital stock. This tends to mitigate the increase in the public-private capital ratio, but not to reverse it.

by a higher private capital-effective female labor ratio and a lower growth rate. In the second case, the new equilibrium is at A'' and B'', characterized now by both a lower private capital-effective female labor ratio and a higher growth rate. However, if curve GG shifts *downward*, the equilibrium outcome could be a higher private capital-effective female labor ratio and a *lower* steady-state growth rate.

The foregoing discussion suggests therefore that, even accounting for a positive effect of improved access to infrastructure on women's bargaining power (and thus girls' time allocated to human capital accumulation), the net effect on growth may not be positive. To explore this issue further, we now turn to a numerical analysis.

4 Calibration

To further examine the conditions under which improved access to infrastructure may have an adverse effect on growth, the model is calibrated using average data for low-income countries for the period 2000-09 (unless otherwise indicated) and simulated under different parameter configurations. We use data provided by the *World Development Indicators* (WDI) database of the World Bank, UNESCO and UNICEF surveys, supplemented as needed with information from specific papers.

For *households*, the annual discount rate is set at 0.04, a fairly conventional choice. This implies that the discount factor is equal to 0.96 on a yearly basis. Interpreting a period as 20 years in this OLG framework yields the intergenerational discount factor $[1/(1+0.04)]^{20} = 0.456$.

To calibrate $\varkappa$, as defined in (35), requires setting eight parameters: γ_B, $\bar{\varkappa}$, χ^R, ν_2, ν_3, μ_B, ε^S, and $\varepsilon^{g,L}$ (or, equivalently from (8), $1-\varepsilon^{g,P}$). The coefficients ν_2 and ν_3 are equal to 0.3 and 0.4, respectively, as discussed below. In the absence of survey-based data, the parameters μ_B and γ_B are set at "neutral" values of 1 and 0.5, respectively. Thus, in the initial equilibrium, women's bargaining power depends equally on factors (social norms and values) that are outside the scope of the model and on relative human capital stocks. Sensitivity analysis with respect to μ_B is reported later on.

Even though there is much informal evidence in favor of bias in mothers' rearing time allocation toward boys, survey data provide little information on its magnitude.

We therefore assume that such bias exists in the initial equilibrium but is quite moderate; we therefore set $\chi^R = 0.6$ and treat it as a shift parameter later on.

To calibrate girls' schooling time, we use a combination of data from UNESCO surveys and UNICEF's Multiple Indicator Cluster Surveys (MICS), round 4, for low-income Sub-Saharan African countries.[30] According to UNESCO data, entrance age (for boys and girls) in primary school is on average 6, and exit age from secondary school is 18. A period is 20 years, so schooling in childhood is 12 years.[31] According to the same source, the number of school days per year in developing countries typically varies between 180 and 209 days in secondary schools, with the number of teaching hours varying between 30 and 34 hours per week. Using the lower estimate, 49.3 percent of each year is spent in school; multiplied by 12, this means that the effective number of years in mandatory schooling is about 5.9 years. Again, with a period representing 20 years in the model, the proportion of girls' time spent in school could thus be measured as 0.29. However, this number is too high for many low-income developing countries, where many girls do not attend school at all. Data suggest also a greater number of school days lost due to illness and other factors. Accordingly, we choose a slighlty lower value and set $\varepsilon^S = 0.2$.

The private savings rate, σ, is set at 12 percent, which corresponds to the average value for low-income countries reported in Agénor (2017). Using the definition of σ given in (26) implies $1/[1 + (1 + \rho)\eta_C] = 0.12$, an expression that can be solved for η_C:

$$\eta_C = (\frac{1}{1+\rho})[(\frac{1}{0.12}) - 1]. \tag{36}$$

With the intergenerational discount factor equal to 0.456, this expression yields $\eta_C = 3.34$. We assume that families value consumption of the marketed good and the home good equally, so that the parameter η_Q is set at the same value as η_C.

In the *home production sector*, the parameter ζ^P is set to unity to capture a high degree of efficiency of infrastructure services, and the curvature of nonmarket production function is set initially at $\pi^Q = 0.8$ to capture rapidly decreasing marginal returns in terms of these two inputs. This seems to be a more reasonable assumption, in a low-income context, than the relatively low values used in the literature (see for instance

[30] See http://mics.unicef.org/surveys.

[31] See http://stats.uis.unesco.org/unesco/ReportFolders/ReportFolders.aspx

Kimura and Yasui (2010)). For sensitivity analysis, lower values of $\zeta^P = 0.8$ and $\pi^Q = 0.55$ (which implies weaker marginal returns to inputs in home production) will also be used.

Time allocated by girls to home production can also be estimated from a sample of recent MICS results for low-income countries. These surveys provide information on children's time allocated to child labor, both in the home and outside the home (including as domestic workers). In general, they indicate that the majority of children aged 5-14 years who are attending school are also involved in child labour activities. Results for Ghana for instance (based on a 2011 survey) indicate that 60.2 percent of girls aged 5-11, and 85.3 percent of girls aged 12-14, are engaged in household chores for less than 28 hours per week (MICS 2011). Of the 97 percent of the children aged 5-14 years attending school, 35 percent are also involved in child labour activities. In Sierra Leone (MICS 2010), 62.7 percent of girls aged 5-11, and 87.1 percent of girls aged 12-14, are engaged in household chores for less than 28 hours per week. Time allocated solely to household chores varies between 4 and 6 hours a day. Similar results are obtained for Nigeria (MICS 2011) and Gambia (MICS 2012).[32] We assume that girls require up to 12 hours of sleeping and "idle" (or leisure) time per day, so that available time for other activities is also 12 hours. We also assume that they allocate autonomously about 2 hours to personal care per day, or equivalently $2/12 = 0.17$ units of time. Using the upper estimate of 6 hours of household chores a day, the (normalized) time that girls allocate to home production, $\varepsilon^{g,P}$, is thus set to 0.5. Accounting for formal schooling and personal care, this implies therefore that $\varepsilon^{g,L}$, which is determined residually from (8), is equal to 0.13 units of time. Thus, initially, girls allocate 13 percent of their time to school-related activities.

The initial bargaining power of women is set at $\varkappa = 0.3$. This ratio measures women's relative human capital stock, which corresponds (as hypothesized in the analytical model) to the main determinant of bargaining power. In one of the few empirical studies available on the topic, Reggio (2011) found an average estimate of women's bargaining power in the family of the order of 0.46, with a standard deviation of 13 percent.

[32] In the same vein, Togunde and Carter (2006) found that in Nigeria children spend on average 4 hours a day of work (some of it outside the home), while 20 percent work 5 to 6 hours a day.

Our benchmark value is thus well within a two-standard error deviation confidence interval.[33] Using the estimate of $\varepsilon^{g,L}$, expression (35) can be solved for the parameter $\bar{\varkappa}$:

$$\bar{\varkappa} = [0.3^2 \cdot (\frac{0.6}{1-0.6})^{0.3} \cdot (\frac{0.2}{0.2+0.13})^{0.4}] = 0.08.$$

Using the estimate of $\varepsilon^{g,P}$, η_E is calibrated as follows. Given the value of the fertility rate, $n = 4.7$, considered by Baldacci et al. (2004*a*, Table 1), and from the values given above and the definitions of Λ_1 and Λ_2, $\Lambda_1 = \eta_E^{-1}15.7$ and $\Lambda_2 = \eta_E^{-1}15.7 + 2.35$. Substituting these results, with $J = 0.148$ as shown below, in (27) yields

$$\varepsilon^{g,P} = 0.5 = \frac{\eta_E^{-1}15.7 - 1 \cdot 0.148}{\eta_E^{-1}15.7 + 2.35},$$

which can be solved for the relative preference for education:

$$\eta_E = 5.94. \tag{37}$$

In the absence of observable data on the gender-specific preference parameters η_C^f, η_C^m and η_E^f, η_E^m, we calibrate them (for consistency) on the basis of the family-wide values of η_C and η_E derived earlier, and by using estimates of gender wage gaps in low-income countries. The relative male-female earning gap for Sub-Saharan African countries reported in Nopo et al. (2012, Table 4a, first column) is 31 percent, which corresponds to $(w^m - w^f)/w^f$ in the model. From this result, the (unweighted) average wage rate is thus $w = 0.5(w^m + w^f) = 0.5[1 + (1 + 0.31)^{-1}]w^m = 0.882w^m$, so that the gross premium for male workers is $w^m/w = 1.134$. In practice, gender wage gaps reflect a number of observable and nonobservable factors, including gender bias at home and in the market place, as well as individual preferences and attitudes toward work. Without much additional information, however, we assume that the male-specific preference parameters in the model, relative to the family-wide values, fully reflect the male wage premium. We therefore set $\eta_C^m = 1.134\eta_C = 1.134 \cdot 3.34 = 3.79$ and $\eta_E^m = \eta_E/1.134 = 5.93/1.134 = 5.23$. Given that $\varkappa = 0.3$, the female-specific parameters can be determined residually using (28), (36), and (37):

$$\eta_C^f = \frac{\eta_C - \eta_C^m(1-\varkappa)}{\varkappa} = \frac{3.34 - 3.79(1-0.3)}{0.3} = 2.31,$$

[33] Given our focus on low-income countries, where women's relative human capital is weaker than in upper-income developing countries, a lower value than Reggio's estimate for Mexico is warranted. Using an alternative value of 0.4 would in any case make little difference to the results.

$$\eta_E^f = \frac{\eta_E - \eta_E^m(1-\varkappa)}{\varkappa} = \frac{5.94 - 5.23(1-0.3)}{0.3} = 7.61,$$

which imply indeed that $\eta_C^f < \eta_C^m$ and $\eta_E^f > \eta_E^m$.[34]

To derive an estimate of the time allocated by mothers to market work, we first calculate total time that they allocate to child rearing by multiplying the fertility rate, $n = 4.7$, by the time allocated to each child as estimated by Agénor et al. (2014) for low-income countries, $\varepsilon^{f,R} = 0.053$ percent. We also account for two autonomous components of mothers' time that were excluded from the time constraint (6) for simplicity: personal care and home production. In standard fashion, suppose that available time for an adult in a normal working day is 16 hours (assuming therefore 8 hours of sleep in a 24-hour day). Common estimates from household surveys suggest that women allocate about 2 hours to personal care and about 2.5 hours to home production (see Agénor et al. (2014)). Thus total time allocated to market work is thus $(1 - 0.053 \cdot 4.7)16 - 2 - 2.5 = 7.5$ hours, which therefore implies that $\varepsilon^{f,W} = 7.5/16 = 0.47$.

In the *marketed good production sector*, the elasticities of production of final goods with respect to public capital and each type of labor, α and β, are set equal to 0.15 and 0.35, respectively. Both values are taken from Agénor (2011) and are consistent with the empirical evidence. The first parameter, for instance, is close to the average estimated by Bom and Ligthart (2014) from a large number of studies. This yields a value of the elasticity of output with respect to private capital equal to $1 - 2\beta = 0.3$, again in line with the empirical evidence.

In the *human capital sector*, the elasticity with respect to government spending on education, ν_1, is set equal to 0.4. The elasticity with respect to time allocated by mothers to child rearing, ν_2, is set equal to a relatively low value, 0.3. Both values are consistent with those reported in Agénor (2017). The elasticity with respect to time allocated by girls to home schooling, ν_3, is set equal to 0.4. Sensitivity analysis with respect to ν_1 and ν_3 is also reported later on.[35]

[34] Initial gender-specific values of η_C^m, η_E^m and η_C^f, η_E^f do not actually have much effect on the results reported later because it is the *average* values η_C, η_E that matter, and these values change relatively little across experiments. In turn, this is because bargaining power $\varkappa$ itself does not change by large amounts, given the size of the shocks that we consider.

[35] Because mother's time allocated to child rearing is constant, the value of ν_2 matters only for the

The effective tax rate on wages, τ, is calculated by multiplying the average ratio of tax revenues to GDP for low-income countries, equal to 15.05 percent for the period 2001-08, estimated by Baldacci et al. (2004*b*, Table 1), divided (to match the model's definition) by the average share of labor income for developing countries estimated by Guerriero (2012), 0.701.[36] Thus, $\tau = 21.5$ percent. To estimate the initial share of government investment on infrastructure, υ_I, we use as a starting point the ratio of total public investment to GDP in low-income countries calculated by Gupta et al. (2011, Table 1) for the period 2000-09. Because public investment includes non-infrastructure related outlays, we assume, based on the evidence reported in Foster and Briceño-Garmendia (2010), that about 40 percent of that amount (or 1.4 percent) really consists of infrastructure investment. The share υ_I can therefore be estimated by 0.014/0.215, that is, $\upsilon_I = 6.5$ percent. The initial share of government spending on education, υ_E, is based on the average estimated from WDI for the years 2004, 2006, and 2007 and is set at 0.171. These numbers imply from the budget constraint that the share of spending on other items is $\upsilon_U = 0.764$.

From the model's solution (25), and the above values for σ, υ_I and τ, the equilibrium value of the public-private capital ratio is

$$J = \frac{0.065 \cdot 0.215}{0.12(1 - 0.215)} = 0.148,$$

which implies therefore that public capital is a relatively scarce factor in the economy, consistent with the evidence for low-income countries (see for instance Foster and Briceño-Garmendia (2010)).

The benchmark parameter values are summarized in Table 1. Based on these values, the model is solved for the steady-state value of private capital-effective labor ratio, $\tilde{x}^f$, using (31) and $1 + \gamma$, together with the solutions for J, $\varepsilon^{g,L}$, and $\tilde{x}^f$, to determine the growth rate of output. A multiplicative constant is also introduced, in order to yield an annual growth rate of marketed output per worker equal to 3.3 percent, the

experiment involving changes in χ^R. But given the magnitude of the shock to χ^R that we consider, its impact is muted.

[36] The estimate used is the corrected measure LS5 proposed by Guerriero, which (importantly for developing countries) accounts for self-employed workers, while considering the possibility for them to generate some capital income.

average growth rate of low-income countries during the period 1975-2000, considered by Baldacci et al. (2004*b*).

5 Quantitative Experiments

To illustrate the role of policy and gender-related exogenous shocks (namely, autonomous changes in social norms regarding gender) in the model, we consider several experiments: an increase in investment in infrastructure (aimed at promoting access to rural roads, power grids, and so on), a reallocation of mothers' time toward girls (which eventually improves their bargaining power in adulthood), a reduction in the sensitivity of the endogenous component of bargaining power to relative stocks of human capital, and a reduction in fathers' preference parameter for their daughters' education.[37]

In all cases we focus on steady-state effects and assume that the initial public-private capital ratio is sufficiently high to ensure that it remains above the critical value J_C defined in (30) yet below the upper value J_H, above which $\varepsilon^{g,P} = 0$ and $\varepsilon^{g,L} = 1 - \varepsilon^S$. Thus, we consider an initial equilibrium in which the economy experiences positive, albeit low, economic growth.

To summarize the simulation results, we focus on the following variables: girls' time allocation, women's bargaining power, the public-private capital ratio, and the growth rate of marketed output.

5.1 Investment in Infrastructure

We consider first the effects of a budget-neutral increase in the share of public expenditure on infrastructure investment, υ_I, from an initial value of 0.065 to 0.105, under two alternative financing assumptions: first, financing by a cut in unproductive spending, as in the analytical experiment reported earlier ($d\upsilon_I + d\upsilon_U = 0$) and second, financing by a cut in spending on education ($d\upsilon_I + d\upsilon_E = 0$).[38] The first experiment helps to

[37] A number of other experiments could be conducted with the model. However, those that have been selected illustrate well a broad range of gender-based policies.

[38] The type of offsetting cuts in education spending that we have in mind here do *not* involve cuts in pay or outlays on school supplies, which could affect the productivity of teachers and children—thereby mitigating the benefits of spending reallocation emphasized here. Rather, one can think of these cuts as involving reductions in spending on a bloated and possibly corrupt bureaucracy.

highlight changes in girls' time allocation, whereas the second helps to emphasize the policy trade-offs that policymakers may face in allocating their resources.

5.1.1 Cut in Unproductive Spending

The results of an increase in infrastructure investment financed by a cut in unproductive spending are displayed in Table 2, for different values of some key structural parameters.

Consider first the impact under the benchmark case. The direct effect of the shock is of course an increase in the public-private capital ratio J (which rises overall from an initial value of 0.148 to 0.239, or 0.091 percentage points) thereby promoting growth. In addition, an increase in the share of government spending on infrastructure lowers girls' time allocated to home production. This, in turn, raises time allocated to home schooling and girls' human capital accumulation, and thus eventually women's bargaining power in the family.

With the benchmark parameter values, the results (shown in bold in Table 2) indicate that the net effect of the increase in the share of investment spending has a net positive effect on growth, of the order of 0.17 percentage points. At the same time, time allocated by girls to home production falls (by about 1.9 percentage points), whereas both time allocated by girls to home schooling and the relative bargaining power of women in family increase (by about 1.9 percentage points again and 0.3 percentage points, respectively). The table reports results for a lower $\pi^Q = 0.55$ as well; in that case, the policy weakens the reduction in girls' time allocated to home production, and mitigates the increase in time allocated to home schooling and the relative bargaining power of women, but with no discernible effect on output growth.

The table also indicates results for two alternative values of ν_3, the elasticity of human capital with respect to girls' time allocated to home schooling, equal to 0.2 and 0.6, for comparison with the benchmark case of 0.4. An increase, say, in ν_3 has both direct and indirect effects on girls' time allocation. On the one hand, the sensitivity of time allocated by girls to home production, $\varepsilon^{g,P}$ to the public-private capital ratio, J, becomes stronger, because parents internalize the higher marginal benefit of additional schooling. Consequently, an increase in J triggered by a rise in the share of public spending on infrastructure has now a larger marginal impact on $\varepsilon^{g,P}$. On the other, for a given ratio of human capital, a higher ν_3 strengthens the effect of a reduction (increase) in time allocated by girls to home production (home schooling) on girls' human capital and women's relative bargaining power. This in turn tends to increase the family's preference for female education, η_E, which also helps to magnify the effect of an increase in the public-private capital ratio on $\varepsilon^{g,P}$. In addition, the increase in women's bargaining power lowers the preference for current consumption, η_C, which also lowers the preference for home production, η_Q. This also contributes to reducing the time that parents find optimal to require their daughters to allocate to household chores. Thus, both the direct effect and the indirect effects (through η_E and η_Q) operate in the same direction. Finally, because the lower preference for current consumption increases the household savings rate, an increase in ν_3 has a positive effect on growth.[39] As shown in the table, in the case where $\nu_3 = 0.6$, instead of 0.4, time allocated by girls to household chores (home schooling) does indeed fall (increase) by more than in the benchmark case; and because women's bargaining power also increases by more (given their relatively higher human capital stock), the net effect on growth is also stronger—the steady-state growth rate increases now by 0.19 percentage points, instead of 0.17. Opposite effects hold for a lower value of $\nu_3 = 0.2$.

5.1.2 Cut in Education Spending

The results of an increase in infrastructure investment financed *fully* by a cut in education spending are displayed in Table 3, again for a range of values of some key structural parameters. To illustrate potential trade-offs, we focus on two key parameters: ν_1 (the elasticity of human capital with respect to government spending on education) and ν_3 (the elasticity of human capital with respect to girls' time allocated to home schooling).[40]

[39] From (27), the marginal effect of J on $\varepsilon^{g,P}$ is measured by $-\zeta^P/\Lambda_2$. From the definition of Λ_1 and Λ_2, a higher ν_3 lowers directly Λ_1, and thus Λ_2, for η_E and η_Q given; this strengthens the marginal effect of an increase in J on $\varepsilon^{g,P}$. At the same time, an increase in ν_3, through a higher $\varepsilon^{g,L}$, strengthens women's bargaining power, $\varkappa$. This raises indirectly η_E and lowers η_Q (given that, in our calibration, $\eta_Q = \eta_C$), which both combine to lower Λ_1 and thus Λ_2 further—thereby also contributing to a higher marginal effect of J on $\varepsilon^{g,P}$.

[40] Values of the remaining parameters are the same as those used in the benchmark case described in Table 1.

The intuition about the role of ν_1 and ν_3 is clear; the lower the elasticity of human capital with respect to government spending on education, or the higher the elasticity of human capital with respect to girls' time allocated to home schooling, the more productive investment in infrastructure is compared to spending on education, and the more likely it is that the net impact on the growth rate is positive. The channels through which these effects operate, however, are different. This is captured by (35), where a change in ν_1 has no effect on women's relative human capital stock (in contrast to ν_3), so the only channel through which ν_1 can *potentially* affect the relative bargaining power of women is an indirect one, operating through a change in time allocated by girls to home production.

Table 3 illustrates three sets of outcomes: ν_1 varying between 0.1 and 0.6 for ν_3 fixed at its benchmark value of 0.4, and ν_3 varying between 0.4 and 0.9 for ν_1 fixed at its benchmark value of 0.4 and a lower value of 0.1, to capture weak marginal effects of government spending on education.[41] The benchmark results are shown in bold in the table. When ν_3 is fixed, the effect of a change in ν_1 on girls' time allocation and bargaining power is much the same, because ν_1 has no quantitatively significant effect on these variables. However, the important point here is that in the case where $\nu_1 = 0.4$, as in the benchmark case, a comparison of the results in Tables 2 and 3 shows that when the increase in spending on infrastructure is financed by a cut in spending on education (which adversely affects the rate of human capital economy, for boys and girls alike, as noted earlier), the net effect on growth is either negative or negligible—despite the fact that girls are able to reallocate their time from household chores to studying. This result illustrates well the trade-offs that arise when budget-neutral changes in government expenditure involve a reallocation across productive outlays.

The table also shows that for ν_1 fixed at 0.4, increases in ν_3 (as noted earlier) have both direct and indirect effects (through changes in women's relative bargaining power) on girls' time allocation, which all combine to magnify the positive effect of improved access to infrastructure on the time that girls allocate to studying. At the same time, greater women's bargaining power contributes to higher growth through its

[41] A low value of ν_1 is quite often used in simulation studies focusing on developing countries; see for instance Agénor (2011) and the references therein.

impact on savings. Nevertheless, a result similar to the one reported earlier obtains: the financing of higher spending on infrastructure by a concomitant reduction in spending on education translates into a *negative* effect on growth, despite the benefit associated with women's relatively higher human capital stock and increased power on allocating household resources. As ν_3 increases (falls) this adverse effect is mitigated (magnified), but the trade-off persists.

However, when the marginal effect of government spending of education is weaker, as captured by $\nu_1 = 0.1$, the results in the table indicated that the net effect on growth turns slighlty *positive*, even in the benchmark case of $\nu_3 = 0.4$. It is also increasing in the value of ν_3; indeed, for $\nu_3 > 0.7$, the effect on the steady-state growth rate of output is of the order of 0.1 percentage points. Put differently, because girls are able to reallocate a larger fraction of their time toward human capital accumulation and to improve in so doing their bargaining power later in life, a policy that entails higher spending on infrastructure may still promote growth—even if it involves a fully offsetting cut in spending on education. The practical implication, of course, is not that governments should stop providing books in order to build roads, but rather that they should exercise careful judgment when making spending decisions in determining what are the main constraints to girls' access to education.

The foregoing discussion has focused on the case of a high degree of efficiency of infrastructure services, that is, $\zeta^P = 1$. As a result, the benefits of an increase in infrastructure investment on girls' time allocation and human capital accumulation, and therefore on economic growth, are magnified. By implication, a lower degree of efficiency ($\zeta^P < 1$) would mitigate these benefits, implying that (in contrast to the case illustrated in Table 3) increases in public investment that are fully offset by cuts in education spending may *not*, even with low values of ν_1 and high values of ν_3, generate a positive effect on growth. However, a smaller share of financing of a higher υ_I by a cut in education spending would restore this result. For instance, with $\zeta^P = 0.8$, an increase in υ_I of the same magnitude as before, but combined now with only a 20 percent financing by a cut in education spending, would again generate long-run growth of the order of 0.1 percentage points.

5.2 Allocation of Mothers' Time toward Girls

Consider a reduction in time allocated to sons χ^R, and thus a concomitant increase in time allocated to daughters, from an initial value of 0.6 to 0.5 (see Table 2). This may capture changes in social norms and attitudes toward women, unrelated to direct policy changes. By definition, this policy has no impact on mothers' total time allocated to child rearing, which remains at $n\varepsilon^{f,R} = 0.25$. In the present setting (where rearing time affects schooling outcomes in childhood), if mothers allocate relatively less time to their sons, their human capital and productivity later in life will also be relatively lower when compared to their daughters. By implication, effective male labor supply will tend to fall relative to women's effective labor supply. In turn, the relative increase in women's human capital stock promotes growth and raises their bargaining power, which translates into a reduction in the family's preference parameter for current consumption. The family's propensity to save and the level of savings therefore increase, and so does the stock of private capital. This positive effect on growth is mitigated by the congestion effect associated with the higher propensity to save (which entails a fall in the public-private capital ratio), but overall the net impact on the growth rate remains positive.

5.3 Reduction in Sensitivity of Women's Bargaining Power

Consider a reduction in μ_B, which measures the sensitivity of women's bargaining power to changes in their relative stock of human capital, from an initial value of unity to 0.2 (see Table 2). At the initial levels of human capital, the fall in μ_B reduces women's bargaining power; in turn, this lowers the family's preference for girls' education and raises its preference for current consumption. The first effect translates into more time in household chores for girls, which eventually weakens their bargaining position in adulthood—thereby magnifying the initial change in time allocation. The lower rate of human capital accumulation by girls is also detrimental to growth. The second effect translates into a lower family savings rate and a lower stock of private capital, which has an adverse effect on growth. However, this effect is mitigated by the fact that a lower private capital stock weakens the magnitude of congestion effects. Overall, the decrease in μ_B reduces the relative bargaining power of women by about 0.9 percentage points and exerts a negligible effect on steady-state growth. The key point is that the endogenous mechanism that relates women's bargaining power, girls' time allocation, and human capital accumulation tends to magnify the initial shift in the bargaining function.

5.4 Decrease in Fathers' Preference for Daughters' Education

Consider a decrease in fathers' preference for girls' education, η_E^m, from an initial value of 5.23 to a value of 3.23 (for illustrative purposes), with η_E^f remaining constant at 7.61. As shown in Table 2, this shift (which leads to an immediate reduction in the family-wide preference for girls' education, η_E) translates into an increase in the optimal amount of time that girls must allocate to household chores, or equivalently (assuming that the new optimal value for $\varepsilon^{g,P}$ remains less than $1-\varepsilon^S$, to avoid a corner solution), a decrease in time that they allocate to school-related activities at home. In turn, this translates into a lower relative capital stock for females, and therefore a weakening in their bargaining power later in life. The initial reduction in the family-wide preference parameter η_E is thus magnified (see (28)). Overall, the parameter η_E drops from an initial value of 5.94, as given in (37), to 4.47.

The impact of this experiment on steady-state growth is illustrated in Table 2. The fact that women accumulate less human capital is, by itself, detrimental to growth. In addition, because women's preference for current consumption is lower than that of men ($\eta_C^f < \eta_C^m$), the reduction in their bargaining power increases the average family preference parameter for today's consumption, η_C (see again (28)), from the initial value of 3.34 to 3.37. Thus, the family's savings rate, defined in (26), decreases slightly. At the aggregate level, the decrease in savings translates into a lower private capital stock in the steady state, which adversely affects growth; at the same time, however, a lower private capital stock weakens the magnitude of congestion effects, which enhances the impact of public capital on growth. The net effect on the growth rate is, nevertheless, negative, of the order of 0.2 percentage points. As also shown in Table 2, symmetric results are obtained for an increase in η_E^f, which for illustrative purposes is shown as a rise from the initial value 7.61 to 15.61.

Finally, note that, given the parsimonious nature of the model, it is likely that the growth effects of changes in women's bargaining power are underestimated. Indeed, in the foregoing analysis we abstracted from the fact that families spend a fraction of their resources on children, and that such spending may improve the quality of their education or their health (through improved nutrition and cognitive skills). Suppose that the share of family spending on each child, θ^R, is a weighted average of the preferred shares of spending by fathers and mothers, η_R^f and η_R^m, and that mothers have a higher preference for spending on children ($\eta_R^f > \eta_R^m$), out of concern for their well-being. This is well documented in the literature (see Schultz (2002), Smith et al. (2003), Roushdy (2004), Ahmed (2006), UNICEF (2007), World Bank (2011), and Doepke and Tertilt (2014)). Suppose also that not only education, as is the case here, but also health (as for instance in Agénor et al. (2014)), display persistence over time. In such conditions an increase in women's bargaining power may raise children's chances of survival through infancy, their performance in school, and their productivity in adulthood—thereby promoting growth. If these effects are strong enough to compensate for the impact of lower family savings on physical capital accumulation—a likely outcome if initial levels of health and human capital are relatively low—the growth effect of policies that are conducive to women exerting greater control over family resources would be magnified.

6 Concluding Remarks

The purpose of this paper was to study the growth effects of externalities associated with intra-household bargaining and the role of access to infrastructure (or lack thereof) on girls' time allocation. To that end we presented a three-period, gender-based overlapping generations (OLG) model that accounts for heterogeneity in parental preferences, human capital accumulation, infrastructure, and growth. In contrast to boys, only girls' time allocated to household chores was assumed to be endogenously related to access to infrastructure. Mothers care more than fathers about the human capital of their daughters (they are more intergenerationally altruistic towards girls) and men care more about current consumption than women. Fundamentally, in the paper gender inequality is an equilibrium outcome that is linked not only to social norms and cultural values but also to the way household members endowed with individual preferences interact with each other and make decisions about girls' time allocation.

The long-run properties of the model were characterized and its properties were illustrated by considering the impact of an increase in spending on infrastructure. The model was then calibrated using data for low-income countries and then used to analyze numerically the effects of not only an increase in spending on infrastructure, but also a reduction in fathers' preference for their daughters' education and a reallocation of mothers' time toward girls. These experiments were conducted by considering alternative values of the parameters that were deemed essential to understanding their effects. The results show that policies aimed at promoting an increase in family access to infrastructure may have significant benefits for girls (in terms of education outcomes), as well as in terms of economic growth. This policy may lead to a reduction in girls' time devoted to household chores, which may in turn allow them to build more human capital—with persistent effects on productivity and wages in their adult life, as well as improved bargaining power in terms of resource allocation within the family. If mothers have a relatively higher preference than fathers for their daughters' education, this increase in women's bargaining power may further reduce the amount of time that the family finds optimal for girls to spend on household chores. The benefits of improved access to infrastructure are therefore magnified. Importantly, the analysis shows that these effects may occur even when an increase in government expenditure on infrastructure is financed by a reduction in spending on education. Intuitively, even if spending less on education may mitigate the ability of both males and females to accumulate human capital, thereby reducing growth, higher spending on infrastructure promotes girls' human capital accumulation and their bargaining power in the family, thereby offsetting any direct, adverse effect on growth. The practical policy implications of these results cannot be overemphasized: in poor countries where access to infrastructure is, to begin with, limited, promoting girls' education and reducing gender inequality may well require *at the margin* to allocate more public resources to infrastructure investment, rather than education (as advocated by Schultz (2002) for instance). This is especially important if offseting changes in education expenditure come from spending reductions on an inefficient or corrupt schooling bureaucracy—a common feature of education systems in low-income countries (see for instance UNESCO (2009, Chapter 3)).

Our analysis could be extended in several directions. A first and relatively straightforward set of extensions would be to endogenize fertility, account for family spending on children, and to relate it to parental preferences. As noted earlier, if mothers have a relatively higher preference for children's education and health (a well-documented fact), the growth effects of policies that contribute to increasing women's bargaining power would likely be magnified. By how much growth increases is an empirical matter that would be worth exploring quantitatively. In addition, with endogenous fertility, accounting for the fact that family resources are partly allocated to children would also help to examine how changes in intra-household bargaining affect the demographic transition, through the well-known trade-off between the quantity and the quality of offspring.

A second extension would be to consider alternative intra-household bargaining schemes. As noted in the text, several of the alternative measures of women's bargaining power used in practice (such as relative wages or the relative share of assets that women hold within the household) are likely to be highly correlated with relative educational outcomes—the measure used in this paper. However, one possibly important measure that we do not capture is greater access by women to financial services. Although the addition of a financial sector would add some significant degree of complexity to the model, it would be a fruitful way to examine the impact of access to microfinance, for instance, on women's control of family resources and their implications for children's health, girls' education, gender equality, and economic growth.

A third extension would be to endogenize mothers' time allocation as well, along the lines for instance of Agénor and Agénor (2014), Agénor (2017), and Agénor and Canuto (2015), and assume that home production requires mothers' and daughters' time—both of which are determined optimally to maximize the family's utility. This would also be consistent with the large body of evidence (including most recently Cubas (2016)) which shows that improved household access to basic infrastructure is associated with higher female labor force participation rates. If endogenous, mother's time allocation would be another margin through which the household can respond to changes in the environment that induce the household to increase/decrease daughters' hours of housework. Mothers' ability to alter their housework hours could also prevent daughters' educational attainment to be negatively influenced by bargaining power differences between their parents as well as changes in the environment that increase the opportunity cost of their time. A key issue then would be how much the family values mothers' time (given its higher opportunity cost, in terms of the market wage) relative to daughters' time. In addition, the degree of intergenerational altruism, which in this paper operates from mothers to daughters, could operate in the opposite direction, with important consequences on mothers' time allocation today. Indeed, if mothers expect their daughters to provide substantial support to their parents in their old age, they may be more willing to engage in home production today and "liberate" their daughters' time, thereby allowing them to engage more in human capital accumulation.[42] However, in practice, it is often boys who are groomed to provide old age support, so it is not clear that this "reverse altruism effect" would prove to be particularly strong.

A fourth extension would be to introduce child labor for both gender types, with parents using girls to perform household chores (as in the present setting) and boys to smooth family income by engaging in market-related activities outside the home, such as farming or family business. This would be consistent with the evidence on the division of labor often imposed on children, as discussed earlier. And because education outcomes and access to infrastructure would likely affect (directly or indirectly) the market wage that boys earn, this would allow a richer analysis of wage gaps and gender inequality in poor countries, as well as the type of public policies that may affect their evolution. However, as long as improved access to infrastructure has a sizable effect on girls' time allocated to education, their ultimate effect on labor market returns for women may continue to dominate the effect for men; as a result, women's bargaining

[42] If mothers expect a more educated daughter to be able to marry a more educated man, with therefore a higher income potential and a greater capacity to provide financial support in their old age, they may also be more willing to invest more time today in household chores.

power may again improve relatively more and the main conclusions of the present paper would not be qualitatively altered.

Finally, the model provides a number of general, qualitative implications that can be assessed with formal econometric techniques. First, persistence in gender inequality should be lower in countries where households have higher access to public infrastructure. Second, the intergenerational correlation between the educational attainment of mothers and daughters should be lower for countries where families have greater access to public infrastructure. Third, if the mechanism that relates child labor and education in adulthood applies only to girls (as hypothesized in the model), these relationships should only be significant for mothers and daughters but not for fathers and sons. Fourth, time allocated to housework by girls should be negatively correlated with the education of women in future generations. In the introduction, a number of studies that have looked at some of these patterns (in Bolivia, Mexico, and Sub-Saharan Africa) were identified and used as motivation for focusing our analysis on girls' time allocation. Other studies focusing on the relationship between child labor and educational attainment are also consistent with the predictions of the model; as documented by UNICEF (2007, p. 27) for instance, in developing countries children with uneducated mothers are on average at least twice as likely to be out of school than children whose mothers attended primary school. Another study of children aged 7 to 14 years in Sub-Saharan Africa found that 73 percent of children with educated mothers were in school, compared with only 51 percent of children whose mothers lacked schooling.[43]

However, as far as we know there are no formal, quantitative studies focusing squarely on the relationship between public infrastructure, gender inequality, and child labor. To conduct such analysis a possible avenue would be to start with the country data from UNICEF's Multiple Indicator Cluster Surveys, mentioned earlier. There are two potentially difficult issues thatneed to be addressed in this context. First, there are significant differences across countries in social norms, religious beliefs, and cultural values with respect to the role of women; and although such variables may be difficult to measure and to standardize across countries, they need to be controlled for. Second,

[43] In a study of Brazil, Emerson and Souza (2007) document the fact that a mother's education has a greater positive impact than a father's education on daughters' school attendance.

the UNICEF surveys would need to be matched with comparable surveys that provide information on access to infrastructure at the household level; to our knowledge, such information is fairly limited at the moment.

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Table 1
Calibration for Low-Income Countries: Benchmark Case

Parameter	Value	Description
Households		
ρ	0.04	Annual discount rate
$\varkappa$	0.3	Bargaining power parameter
μ_B	1.0	Sensitivity of bargaining power to ratio of human capital stocks
γ_B	0.5	Weight of endogenous component of bargaining power
χ^R	0.6	Proportion of mothers' rearing time allocated to boys
σ	0.12	Family's savings rate
n	4.7	Gross fertility rate
$\eta_C = \eta_Q$	3.34	Pref. parameters, consumption in adulthood and home production
η_C^m, η_C^f	$3.79, 2.31$	Pref. parameters, male and female, consumption in adulthood
η_E^m, η_E^f	$5.23, 7.61$	Pref. parameters, male and female, children's education
Time allocation		
ε^S	0.2	Time allocated by boys and girls to schooling
$\varepsilon^{g,P}$	0.5	Time allocated by girls to household chores
$\varepsilon^{g,L}$	0.13	Time allocated by girls to homework
$\varepsilon^{f,R}$	0.053	Time allocated by mothers to child rearing (unit time)
$\varepsilon^{f,W}$	0.47	Time allocated by mothers to market work
Home production		
ζ^P	1.0	Efficiency parameter
π^Q	0.8	Curvature of home production function
Market production		
α	0.15	Elasticity with respect to public-private capital ratio
β	0.35	Elasticity with respect to male labor and female labor
Human capital		
ν_1	0.4	Elasticity with respect to government spending on education
ν_2	0.3	Elasticity with respect to mothers' time allocated to child rearing
ν_3	0.4	Elasticity with respect to girls' time allocated to home schooling
Government		
τ	0.215	Effective tax rate on wages
υ_I	0.065	Share of government spending on infrastructure
υ_E	0.171	Share of government spending on education

Table 2
Quantitative Experiments

Increase in infrastructure investment 1/		Absolute deviations from baseline			
	Baseline	Benchmark	$\pi^Q = 0.55$	$v_3 = 0.2$	$v_3 = 0.6$
Time allocated by girls to					
Household chores	0.5	**-0.0190**	-0.0178	-0.0122	-0.0232
Home schooling	0.13	**0.0190**	0.0178	0.0122	0.0232
Relative bargaining power of women	0.3	**0.0034**	0.0032	0.0021	0.0053
Public-private capital stock ratio	0.148	**0.0910**	0.0910	0.0924	0.0892
Output growth rate	0.033	**0.0017**	0.0017	0.0015	0.0019

Other shocks 2/		Absolute deviations from baseline			
	Baseline	$\mu_B = 0.2$	$\chi^R = 0.5$	$\eta_E^m = 3.23$	$\eta_E^f = 15.61$
Time allocated by girls to					
Household chores	0.5	0.0020	-0.0041	0.0755	-0.0966
Home schooling	0.13	-0.0020	0.0041	-0.0755	0.0966
Relative bargaining power of women	0.3	-0.0094	0.0196	-0.0152	0.0158
Public-private capital stock ratio	0.148	0.0005	-0.0011	0.0009	-0.0009
Output growth rate	0.033	-0.0001	0.0014	-0.0023	0.0024

Notes: π^Q is the curvature of home production function and set equal to 0.8; v_3 is the elasticity of human capital with respect to girls' time allocated to home schooling and set equal to 0.4; μ_B is the sensitivity of bargaining power to human capital stocks and set equal to 1.0; χ^R is the proportion of mothers' rearing time allocated to boys and set equal to 0.6; η_E^m and η_E^f are preference parameters of males and females for children's education, respectively. They are equal to 5.23 and 7.61, respectively in the benchmark case.

1/ Increase in υ_I from 0.065 to 0.105, financed by a cut in υ_U.

2/ Decrease in μ_B from 1 to 0.2, decrease in χ^R from 0.6 to 0.5, decrease in η_E^m from 5.23 to 3.23 and increase in η_E^f from 7.61 to 15.61.

Source: Authors' calculations.

Table 3
Increase in Infrastructure Investment, Financed by a Cut in Education Spending 1/

v_3 is fixed at 0.4	Baseline	Absolute deviations from baseline					
		$v_1 = 0.1$	$v_1 = 0.2$	$v_1 = 0.3$	$v_1 = 0.4$	$v_1 = 0.5$	$v_1 = 0.6$
Time allocated by girls to household chores	0.5	-0.0190	-0.0190	-0.0190	**-0.0190**	-0.0190	-0.0190
Time allocated by girls to home schooling	0.13	0.0190	0.0190	0.0190	**0.0190**	0.0190	0.0190
Relative bargaining power of women	0.3	0.0034	0.0034	0.0034	**0.0034**	0.0034	0.0034
Public-private capital stock ratio	0.148	0.0910	0.0910	0.0910	**0.0910**	0.0910	0.0910
Output growth rate	0.033	0.0002	-0.0004	-0.0009	**-0.0013**	-0.0017	-0.0021
v_1 is fixed at 0.4	Baseline	Absolute deviations from baseline					
		$v_3 = 0.4$	$v_3 = 0.5$	$v_3 = 0.6$	$v_3 = 0.7$	$v_3 = 0.8$	$v_3 = 0.9$
Time allocated by girls to household chores	0.5	**-0.0190**	-0.0213	-0.0232	-0.0248	-0.0260	-0.0270
Time allocated by girls to home schooling	0.13	**0.0190**	0.0213	0.0232	0.0248	0.0260	0.0270
Relative bargaining power of women	0.3	**0.0034**	0.0043	0.0053	0.0065	0.0078	0.0092
Public-private capital stock ratio	0.148	**0.0910**	0.0901	0.0892	0.0882	0.0870	0.0858
Output growth rate	0.033	**-0.0013**	-0.0012	-0.0011	-0.0010	-0.0010	-0.0009
v_1 is fixed at 0.1	Baseline	Absolute deviations from baseline					
		$v_3 = 0.4$	$v_3 = 0.5$	$v_3 = 0.6$	$v_3 = 0.7$	$v_3 = 0.8$	$v_3 = 0.9$
Time allocated by girls to household chores	0.5	-0.0190	-0.0213	-0.0232	-0.0248	-0.0260	-0.0270
Time allocated by girls to home schooling	0.13	0.0190	0.0213	0.0232	0.0248	0.0260	0.0270
Relative bargaining power of women	0.3	0.0034	0.0043	0.0053	0.0065	0.0078	0.0092
Public-private capital stock ratio	0.148	0.0910	0.0901	0.0892	0.0882	0.0870	0.0858
Output growth rate	0.033	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008

Notes: v_1 is the elasticity of human capital with respect to government spending on education and v_3 is the elasticity of human capital with respect to girls' time allocated to home schooling. Both are set equal to 0.4 in the benchmark case.

1/ Increase in υ_I from 0.065 to 0.105, financed by a cut in υ_E.

Source: Authors' calculations.

Figure 1
Access to Infrastructure and Girls' Time Allocation

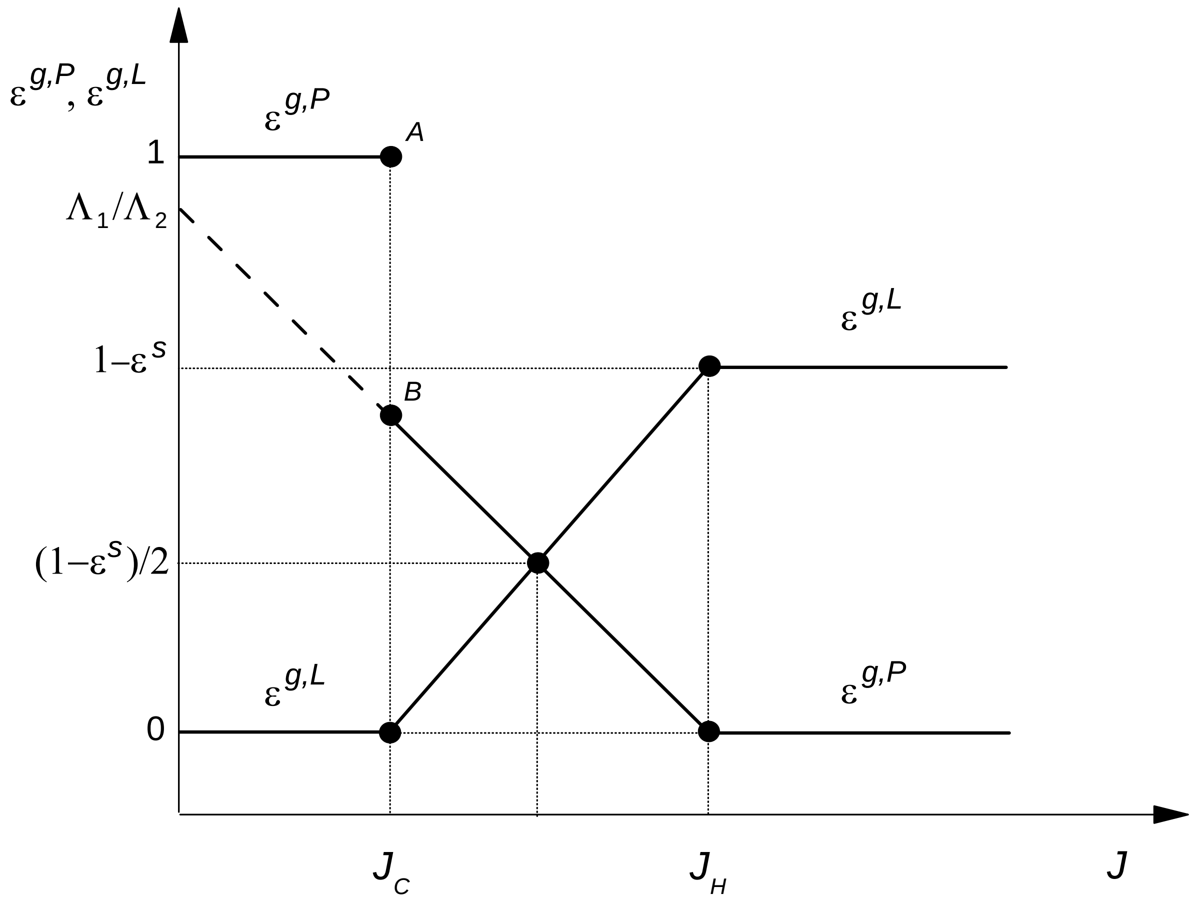
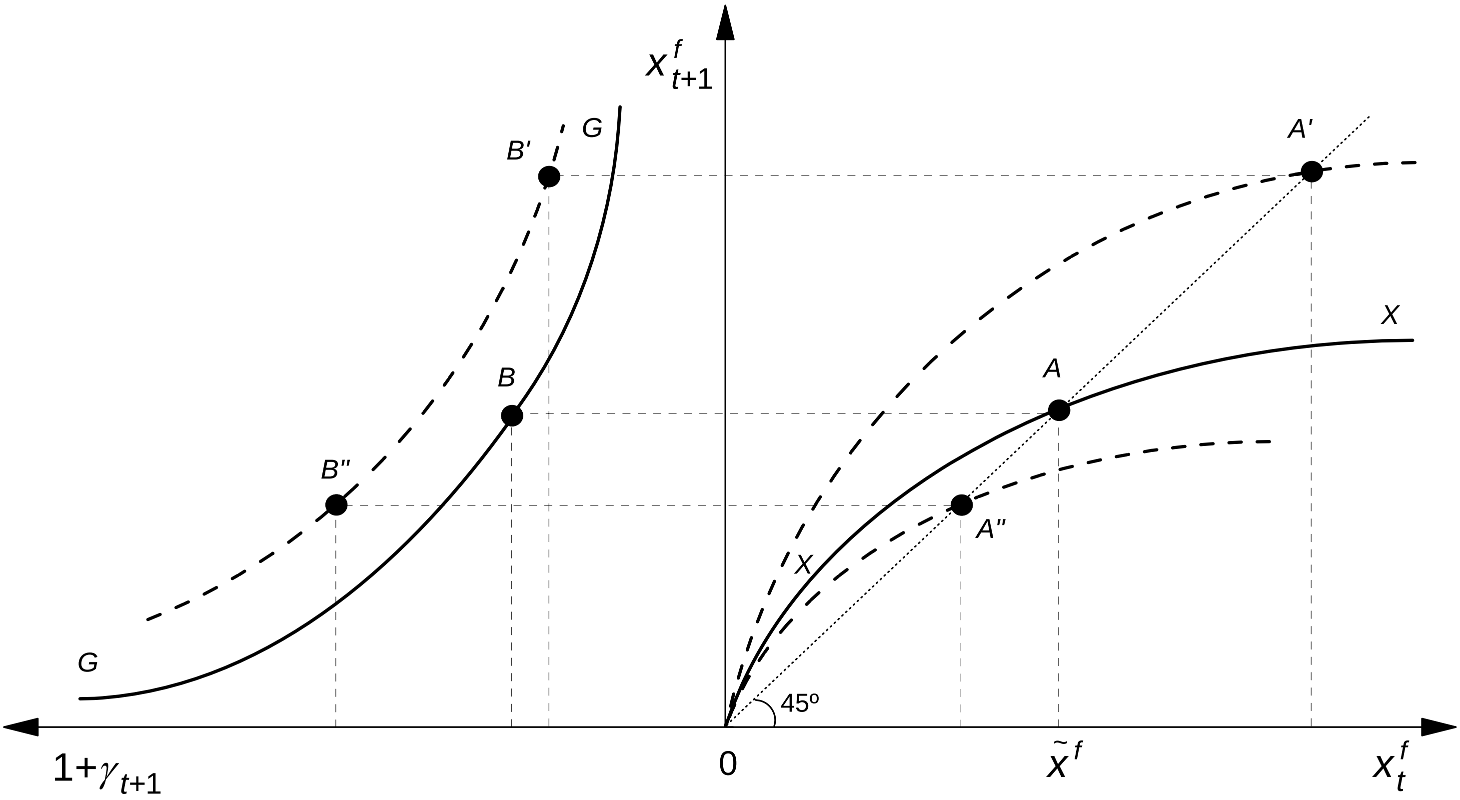


Figure 2
Equilibrium and Increase in Spending on Infrastructure



Appendix

Consider first the family's optimization problem. Substituting (1) in (10), and the result in (9) yields

$$U_t = \left[\varkappa\eta_C^f + (1-\varkappa)\eta_C^m\right]\ln c_t^{t-1} + \eta_Q\pi^Q \ln(0.5n\varepsilon_t^{g,P} + \zeta^P k_t^I) \tag{A1}$$

$$+\left[\varkappa\eta_E^f + (1-\varkappa)\eta_E^m\right]\ln e_{t+1}^f + \frac{1}{1+\rho}\ln c_{t+1}^{t-1},$$

Define, for $h = C, E$,

$$\eta_h = \varkappa\eta_h^f + (1-\varkappa)\eta_h^m = \eta_h^m + (\eta_h^f - \eta_h^m)\varkappa.$$

Given the restrictions discussed in the text, $\eta_C^f < \eta_C^m$, and $\eta_E^f > \eta_E^m$. Thus,

$$\frac{d\eta_C}{d\varkappa} < 0, \quad \frac{d\eta_E}{d\varkappa} > 0.$$

If women's bargaining power increases, the family will value consumption today less and therefore spend less today (saving more in the process), and it will value the education of children more.

Using the above definitions, the collective utility function (A1) takes the form

$$U_t = \eta_C \ln c_t^{t-1} + \eta_Q\pi^Q \ln(0.5n\varepsilon_t^{g,P} + \zeta^P k_t^I) + \eta_E \ln e_{t+1}^f + \frac{1}{1+\rho}\ln c_{t+1}^{t-1}, \tag{A2}$$

where $k_t^I = K_t^I/K_t^P$. From equations (3), dropping the index i and given that $N_t^m = N_t^f$,

$$w_t^m = (\frac{\varepsilon^{f,W} E_t^f}{E_t^m})w_t^f,$$

which can be substituted in (13) to give, with $E_t^j = e_t^j$, for $j = m, f$,

$$w_t^T = e_t^m w_t^m + e_t^f \varepsilon^{f,W} w_t^f = 2e_t^f \varepsilon^{f,W} w_t^f. \tag{A3}$$

In turn, this expression can be substituted in the budget constraint (14) to give

$$2(1-\tau)e_t^f \varepsilon^{f,W} w_t^f - c_t^{t-1} - \frac{c_{t+1}^{t-1}}{1+r_{t+1}} = 0. \tag{A4}$$

From (17), and noting from (8) that $\varepsilon^S + \varepsilon_t^{g,L} = 1 - \varepsilon_t^{g,P}$, the human capital of females in $t+1$ is

$$e_{t+1}^f = (\frac{G_t^E}{n0.5N_t})^{\nu_1}(E_t^f)^{1-\nu_1}[(1-\chi^R)\varepsilon^{f,R}]^{\nu_2}(1-\varepsilon_t^{g,P})^{\nu_3}. \tag{A5}$$

Families maximize (A2) subject to (A4) and (A5), with respect to c_t^{t-1}, c_{t+1}^{t-1}, $\varepsilon_t^{g,P}$and with $\varepsilon_t^{g,L}$ solved residually from (8). First-order conditions yield the familiar Euler equation

$$\eta_C \frac{c_{t+1}^{t-1}}{c_t^{t-1}} = \frac{1+r_{t+1}}{1+\rho}, \tag{A6}$$

together with

$$\frac{\eta_Q \pi^Q 0.5n}{0.5n\varepsilon_t^{g,P} + \zeta^P k_t^I} = \frac{\eta_E \nu_3}{1-\varepsilon_t^{g,P}},$$

or equivalently

$$0.5n\varepsilon_t^{g,P} + \zeta^P k_t^I = \Lambda_1(1-\varepsilon_t^{g,P}), \tag{A7}$$

where

$$\Lambda_1 = 0.5n\eta_Q \pi^Q \eta_E^{-1} \nu_3^{-1}.$$

Substituting (A6) in the intertemporal budget constraint (A4) yields

$$c_t^{t-1} = [\frac{(1+\rho)\eta_C}{1+(1+\rho)\eta_C}]2(1-\tau)e_t^f \varepsilon^{f,W} w_t^f. \tag{A8}$$

Thus, from (11), (A3), and (A8), family savings, s_t, is equal to

$$s_t = 2(1-\tau)\sigma e_t^f \varepsilon^{f,W} w_t^f, \tag{A9}$$

where σ is the marginal propensity to save, defined as

$$\sigma = \frac{1}{1+(1+\rho)\eta_C} < 1. \tag{A10}$$

From equation (A7), we have

$$\min \varepsilon_t^{g,P} = [\frac{\Lambda_1 - \zeta^P k_t^I}{\Lambda_2}, 1], \tag{A11}$$

where

$$\Lambda_2 = 0.5n + \Lambda_1. \tag{A12}$$

This equation can be substituted in (8), together with (A12), to give

$$\varepsilon_t^{g,L} = \max\left\{1 - \varepsilon^S - \frac{\Lambda_1 - \zeta^P k_t^I}{\Lambda_2}, 0\right\}. \tag{A13}$$

To study the dynamics in the economy, substitute (A9) in (24) to give

$$K_{t+1}^P = N_t^f s_t = N_t^f 2(1-\tau)\sigma e_t^f \varepsilon^{f,W} w_t^f, \tag{A14}$$

that is, substituting for w_t^f from (3) and dividing by K_t^P,

$$\frac{K_{t+1}^P}{K_t^P} = 2\beta(1-\tau)\sigma(\frac{Y_t}{K_t^P}). \tag{A15}$$

Equations (20) can be rewritten as given that $N_t^m = N_t^f$,

$$G_t^j = \upsilon_j \tau (w_t^m E_t^m + w_t^f \varepsilon^{f,W} E_t^f) N_t^f, \quad j = I, E, U$$

that is, using (A3),

$$G_t^j = \upsilon_j \tau 2 e_t^f \varepsilon^{f,W} w_t^f N_t^f.$$

Substituting for w_t^f from (3) gives

$$G_t^j = \upsilon_j \tau 2 \beta Y_t. \tag{A16}$$

To study the dynamics, in this Appendix, we start from a more general formulation of (22), that is,

$$K_{t+1}^I = (G_t^I)^{\mu_I} (K_t^I)^{1-\mu_I}, \tag{A17}$$

where $\mu_I \in (0,1)$. As in Agénor (2012*b*, Chapter 1), we assume that the production of new public capital requires combining the flow of investment and the existing capital stock.

Substituting (A16) for $j = I$ in (A17) gives

$$K_{t+1}^I = (\frac{G_t^I}{K_t^I})^{\mu_I} K_t^I = (\frac{\upsilon_I \tau 2 \beta Y_t}{K_t^I})^{\mu_I} K_t^I = (\upsilon_I \tau 2 \beta)^{\mu_I} (\frac{Y_t}{K_t^I})^{\mu_I} K_t^I,$$

or equivalently

$$\frac{K_{t+1}^I}{K_t^I} = (\upsilon_I \tau 2 \beta)^{\mu_I} \left(k_t^I\right)^{-\mu_I} (\frac{Y_t}{K_t^P})^{\mu_I}, \tag{A18}$$

where $k_t^I = K_t^I / K_t^P$ is the public-private capital ratio.

Combining (A15) and (A18) yields

$$k_{t+1}^I = \frac{(\upsilon_I \tau 2 \beta)^{\mu_I}}{2\beta(1-\tau)\sigma} \left(k_t^I\right)^{1-\mu_I} (\frac{Y_t}{K_t^P})^{-(1-\mu_I)}. \tag{A19}$$

To fully specify the dynamics of k_{t+1}^I, the expression Y_t/K_t^P must therefore be solved for. First, rewrite equation (5) here for convenience:

$$\frac{Y_t}{K_t^P} = (k_t^I)^{\alpha} (\frac{E_t^m N_t^m}{K_t^P})^{\beta} (\frac{E_t^f N_t^f}{K_t^P})^{\beta} (\varepsilon^{f,W})^{\beta}.$$

This equation can be rewritten as

$$\frac{Y_t}{K_t^P} = (k_t^I)^{\alpha} (\varepsilon^{f,W})^{\beta} (\frac{1}{x_t^m})^{\beta} (\frac{1}{x_t^f})^{\beta}, \tag{A20}$$

where $x_t^j = K_t^P / e_t^j N_t^j$ is the private capital-effective labor j ratio.

Because $N_t^m = N_t^f$, and given that from (18) to eliminate e_t^m,

$$x_t^m = \frac{K_t^P}{e_t^m N_t^m} = \frac{K_t^P}{e_t^m N_t^f} = \frac{K_t^P}{e_t^f N_t^f} (\frac{e_t^f}{e_t^m}) = x_t^f (\frac{1-\chi^R}{\chi^R})^{\nu_2} (\frac{\varepsilon^S + \varepsilon_t^{g,L}}{\varepsilon^S})^{\nu_3}.$$

Substituting this result in (A20), together with (8), yields

$$\frac{Y_t}{K_t^P} = \Gamma_1(\frac{1}{\varepsilon^S + \varepsilon_t^{g,L}})^{\beta\nu_3}(k_t^I)^\alpha(\frac{1}{x_t^f})^{2\beta}, \tag{A21}$$

where

$$\Gamma_1 = \left\{(\frac{\chi^R}{1-\chi^R})^{\nu_2}(\varepsilon^{f,W})(\varepsilon^S)^{\nu_3}\right\}^\beta.$$

Substituting (A21) into (A19) yields

$$k_{t+1}^I = \Gamma_2(\frac{1}{\varepsilon^S + \varepsilon_t^{g,L}})^{-\beta\nu_3(1-\mu_I)}\left(k_t^I\right)^{(1-\mu_I)(1-\alpha)}(\frac{1}{x_t^f})^{-2\beta(1-\mu_I)}, \tag{A22}$$

where

$$\Gamma_2 = \frac{(\upsilon_I\tau 2\beta)^{\mu_I}}{2\beta(1-\tau)\sigma}\Gamma_1^{-(1-\mu_I)}.$$

From (A22), it is clear that as long as $\mu_I = 1$, k_t^I is constant $\forall t$ at

$$J = \frac{\upsilon_I\tau 2\beta}{2\beta(1-\tau)\sigma} = \frac{\upsilon_I\tau}{(1-\tau)\sigma},$$

given the definition of Γ_2.

The dynamic equation for x_{t+1}^f is now derived. From (A16), with $j = E$,

$$\frac{G_t^E}{N_t} = \upsilon_E\tau 2\beta(\frac{Y_t}{N_t}).$$

Substituting this result into (17) yields

$$e_{t+1}^f = (\frac{\upsilon_E\tau 2\beta}{n})^{\nu_1}(\frac{Y_t}{0.5N_t})^{\nu_1}(E_t^f)^{1-\nu_1}[(1-\chi^R)\varepsilon^{f,R}]^{\nu_2}(\varepsilon^S + \varepsilon_t^{g,L})^{\nu_3}. \tag{A23}$$

From (15) for $t+1$, (A15), (A23) and given that $N_{t+1}^f = 0.5N_{t+1}$,

$$x_{t+1}^f = \frac{K_{t+1}^P}{e_{t+1}^f N_{t+1}^f} = \Gamma_3(\frac{Y_t}{0.5e_t^f N_t})^{1-\nu_1}(\varepsilon^S + \varepsilon_t^{g,L})^{-\nu_3}, \tag{A24}$$

where

$$\Gamma_3 = \left\{\frac{2\beta(1-\tau)\sigma}{[(1-\chi^R)\varepsilon^{f,R}]^{\nu_2}n^{1-\nu_1}0.5}\right\}(\upsilon_E\tau 2\beta)^{-\nu_1}.$$

By definiton $Y_t/0.5e_t^f N_t = (Y_t/K_t^P)x_t^f$. Using (A21) to substitute for Y_t/K_t^P yields therefore

$$\frac{Y_t}{0.5e_t^f N_t} = \Gamma_1(\frac{1}{\varepsilon^S + \varepsilon_t^{g,L}})^{\beta\nu_3}(k_t^I)^\alpha(x_t^f)^{1-2\beta}.$$

Substituting this result in (A24), together with (8), yields

$$x_{t+1}^{f} = \Gamma_4(k_t^I)^{\alpha(1-\nu_1)}(x_t^f)^{(1-2\beta)(1-\nu_1)}(\varepsilon^S+\varepsilon_t^{g,L})^{-\nu_3[1+\beta(1-\nu_1)]}, \quad \text{(A25)}$$

where

$$\Gamma_4 = \Gamma_3\Gamma_1^{1-\nu_1}.$$

To determine the growth rate of output per worker, it is convenient to note first that $Y_{t+1}/N_{t+1} = (Y_{t+1}/K_{t+1}^P)(K_{t+1}^P/N_{t+1})$. Now, using (15), (A15), and (A21) for $t+1$ yields

$$\frac{Y_{t+1}}{N_{t+1}} = \Gamma_1(\frac{1}{\varepsilon^S+\varepsilon_{t+1}^{g,L}})^{\beta\nu_3}(k_{t+1}^I)^{\alpha}(x_{t+1}^f)^{-2\beta}2\beta(1-\tau)\sigma(\frac{Y_t}{n0.5N_t}). \quad \text{(A26)}$$

The balanced growth rate of output per worker is thus

$$1+\gamma_{Y/N} = \Gamma_1(\frac{1}{\varepsilon^S+\tilde{\varepsilon}^{g,L}})^{\beta\nu_3}\frac{(\tilde{k}^I)^{\alpha}}{0.5n}(\tilde{x}^f)^{-2\beta}2\beta(1-\tau)\sigma,$$

where, from the equation (A13),

$$\tilde{\varepsilon}^{g,L} = \max\left\{1-\varepsilon^S-\frac{\Lambda_1-\zeta^P\tilde{k}^I}{\Lambda_2}, 0\right\}, \quad \text{(A27)}$$

and $\tilde{k}^I$ and $\tilde{x}^f$ are the steady-state solutions obtained by setting $\Delta k_{t+1}^I = \Delta x_{t+1}^f = 0$ in (A22) and (A25):

$$\tilde{k}^I = \left\{\Gamma_2(\frac{1}{\varepsilon^S+\tilde{\varepsilon}^{g,L}})^{-\beta\nu_3(1-\mu_I)}(\frac{1}{\tilde{x}^f})^{-2\beta(1-\mu_I)}\right\}^{1/\Pi_1}, \quad \text{(A28)}$$

$$\tilde{x}^f = \{\Gamma_4(\tilde{k}^I)^{\alpha(1-\nu_1)}(\varepsilon^S+\tilde{\varepsilon}^{g,L})^{-\nu_3[1+\beta(1-\nu_1)]}\}^{1/\Pi_2}, \quad \text{(A29)}$$

where

$$\Pi_1 = 1-(1-\mu_I)(1-\alpha),$$
$$\Pi_2 = 1-(1-2\beta)(1-\nu_1) > 0.$$

To determine the growth rate of output proceeds in the same way. From (A21) for $t+1$,

$$Y_{t+1} = \Gamma_1(\frac{1}{\varepsilon^S+\varepsilon_{t+1}^{g,L}})^{\beta\nu_3}(k_{t+1}^I)^{\alpha}(\frac{1}{x_{t+1}^f})^{2\beta}K_{t+1}^P,$$

that is, using (A15),

$$1+g_{t+1} = \frac{Y_{t+1}}{Y_t} = \Gamma_1(\frac{1}{\varepsilon^S+\varepsilon_{t+1}^{g,L}})^{\beta\nu_3}(k_{t+1}^I)^{\alpha}(\frac{1}{x_{t+1}^f})^{2\beta}2\beta(1-\tau)\sigma,$$

which yields the steady-state growth rate:

$$1+\boldsymbol{\gamma} = \Gamma_1(\frac{1}{\varepsilon^S+\tilde{\varepsilon}^{g,L}})^{\beta\nu_3}(\tilde{k}^I)^{\alpha}(\tilde{x}^f)^{-2\beta}2\beta(1-\tau)\sigma. \quad \text{(A30)}$$

Let $\mu_I = 1$, as in the text. This implies, as can be inferred from (A28), that $\tilde{k}^I$ is constant at J (as shown in (25)) and that from (A29) $\tilde{x}^f$ is equal to (33). Using then (25) and (33), as well as (A27) and (A30), it can be verified that the log derivatives of J, $\tilde{x}^f$, $\varepsilon^{g,L}$, and $1+\gamma$ with respect to υ_I are, with $\varkappa$ given,

$$\left.\frac{d\ln J}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} = \frac{1}{\upsilon_I} > 0, \tag{A31}$$

$$\left.\frac{d\ln\tilde{x}^f}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} = \left\{\alpha(1-\nu_1) - \frac{\nu_3[1+\beta(1-\nu_1)]\zeta^P J}{\Lambda_2(\varepsilon^S+\varepsilon^{g,L})}\right\}\frac{1}{\upsilon_I\left[1-(1-2\beta)(1-\nu_1)\right]} \lesseqgtr 0, \tag{A32}$$

$$\left.\frac{d\ln(\varepsilon^S+\varepsilon^{g,L})}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} = \frac{\zeta^P J}{\upsilon_I\Lambda_2(\varepsilon^S+\varepsilon^{g,L})} > 0, \tag{A33}$$

$$\left.\frac{d\ln(1+\gamma)}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} = \alpha\left.\frac{d\ln J}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} \tag{A34}$$

$$-\beta\nu_3\left.\frac{d\ln(\varepsilon^S+\varepsilon^{g,L})}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} - 2\beta\left.\frac{d\ln\tilde{x}^f}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} \lesseqgtr 0.$$

Substituting (A31)-(A33) in (A34) gives

$$\left.\frac{d\ln(1+\gamma)}{d\upsilon_I}\right|_{d\upsilon_I+d\upsilon_U=0} = \frac{\alpha}{\upsilon_I} - \frac{\beta\nu_3\zeta^P J}{\upsilon_I\Lambda_2(\varepsilon^S+\varepsilon^{g,L})}$$

$$-2\beta\left\{\alpha(1-\nu_1) - \frac{\nu_3[1+\beta(1-\nu_1)]\zeta^P J}{\Lambda_2(\varepsilon^S+\varepsilon^{g,L})}\right\}\frac{1}{\upsilon_I\left[1-(1-2\beta)(1-\nu_1)\right]} \lesseqgtr 0.$$

This result is discussed in the text. With $\varkappa$ endogenously related to J, as implied by (35), both η_C and η_E become also endogenous, and the above expressions become even more complex and ambiguous.