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Nonlinearity in the Fed's Monetary Policy Rule

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Abstract

This paper investigates the nature of nonlinearities in the monetary policy rule of the US Fed using the flexible approach of Hamilton (2001a). We find that while there is significant evidence of nonlinearity for the period to 1979, there is little such evidence for the subsequent period. Possible asymmetries in the Fed's reactions to inflation deviations from target and the output gap in the 1960s and 70s may tell part of the story, but do not capture the entire nature of the nonlinearity. The inclusion of the interaction between inflation deviations and the output gap, as recently proposed, appears to characterize the nonlinear policy rule more adequately.

JEL *Classification*: E52, E58, C13

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1   Introduction

Since the early 1990s, research on monetary policy reaction functions has come with a rush from academic institutes, central bankers and private financial firms. In particular, the so-called Taylor rule (Taylor, 1993) has received considerable attention, in large part because this simple rule described the actual behaviour of the US Federal Funds rate rather surprisingly well. According to this rule, the Federal Reserve (or Fed, the US central bank) sets the Federal Funds interest rate using current values of real output and inflation in relation to their target values. In a similar context, Clarida, Gali and Gertler (1998, 2000) examine a forward-looking monetary policy reaction function in which the central bank proactively adjusts interest rates using expected future gaps in inflation and output compared with target values. The theoretical basis of linear reaction functions of this type rests on two key assumptions, namely that the central bank has a quadratic loss function and that the aggregate supply relation (Phillips curve) is linear.[2]

Recently, however, both of these assumptions have been challenged. In relation to the first, Nobay and Peel (1998), Cukierman (2000), Gerlach (2000), Ruge-Murcia (2001, 2002) and Bec, Salem and Collard (2002) consider asymmetric preference specifications for the central bank. Cukierman (2000) suggests that the political establishment views the costs of recessions as greater than the benefits of expansions. In a democratic society, an independent, but accountable, central bank cannot be totally insensitive to the wishes of the political establishment, so some of this asymmetry will appear in the loss function of the central bank. Gerlach (2000) finds that the Fed may have been more concerned by negative rather than by positive output gaps in the pre-1980 period, while Bec, Salem, and Collard (2002) extend his model

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[2]For more general study on monetary policy rule, see Clarida, Gali and Gertler (1999).

and conclude that the state of the business cycle (again measured by the output gap) is important for monetary policy in post-1982 U.S., French and German data. Instead of the output gap, Ruge-Murcia (2001, 2002) concentrates on asymmetric preferences with respect to unemployment, finding evidence for nonlinear behaviour of the central bank in OECD and G7 countries.

Turning to the second assumption, Schaling (1999) and Dolado, Maria-Dolores and Naveira (2002b) allow inflation to be a convex function of the output gap, implying a nonlinear aggregate supply (Phillips) curve. Combined with a quadratic loss function, the optimal policy rule is then also nonlinear, with the central bank increasing interest rates by a larger amount when inflation is above target than the amount it will reduce them when inflation is below target. Specifically, the reaction function derived by Dolado et al. (2002b) includes an interaction between expected inflation and the output gap. Empirical support for this interest rate rule is found in France, Germany and Spain over the 1980s and 1990s, but not in the US.

Dolado, Dolores and Ruge-Murcia (2002a) construct a general model, allowing the joint analysis of both types of departure from the linear-quadratic setup and in principle permitting the source of nonlinearities in the nominal interest rate rule to be traced back to central bank preferences, nonlinearities in the supply curve, or both. Their results imply that US monetary policy can be characterized by a nonlinear policy rule due to asymmetric inflation preferences of the Fed after 1983, but that the rule was linear prior to 1979.

Although this recent literature has provided evidence in favour of nonlinear monetary policy rules, all the empirical studies to date assume specific parametric models. In reality, however, we do not directly observe either the central bank's preferences or the aggregate Phillips curve in the economy, so that there exists an unbounded universe of possible alternative nonlinear specifications. Because rejection of linearity

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against a specific nonlinear alternative does not necessarily imply the validity of that nonlinear model, we believe that it is important to investigate the nature of any non-linearities in the central bank's reaction function while avoiding specific parametric assumptions. To this end, the present paper applies the methodology recently developed by Hamilton (2001a) to address this question. This approach provides a valid test of the null hypothesis of linearity against a broad range of alternative nonlinear models, consistent estimation of what the nonlinear relation looks like, and formal comparison of alternative nonlinear models. Hamilton (2001b), and Hamilton and Kim (2002) show that this methodology is very useful for characterizing the nonlinear relation between oil price changes and GDP growth and nonlinearity in the term structure, respectively.

Following Clarida et. al. (2000) and others, we consider the monetary policy reaction function for the postwar United States economy, both as a single sample and as subsamples before and after Volcker's appointment as the Fed Chairman in 1979. While we find no evidence of nonlinearity in post-1960 U.S. monetary policy using the whole period, the two subsamples show different results. More specifically, we find relatively strong evidence of nonlinearity for the pre-Volcker era, but little such evidence in the Volcker-Greenspan era. Our results are robust to whether the monetary policy rule is forward- or backward-looking and to different measures of both the output gap and inflation forecasts. We also explicitly test whether parametric representations previously suggested for the monetary policy rule capture all nonlinearity. Although we find some support for the hypothesis that the Fed reacted in an asymmetric way to the output gap in the 1960s and 70s, as suggested by Gerlach (2000), this type of asymmetric behaviour does not fully capture the nonlinearity. However, the specification in which the Fed reacts to the interaction between inflation and the output gap, as proposed in Dolado et al. (2002b), does

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adequately characterize the nonlinear policy rule during this period.

The plan of the paper is as follows. Section 2 reviews specifications for non-linear monetary policy rules suggested in the literature to date and proposes a flexible version of a nonlinear monetary policy rule. Section 3 reviews the Hamilton (2001a) methodology applied in this paper. Empirical results, including evaluation of specific nonlinear formulations, are in Section 4. Conclusions are offered in Section 5.

2 Nonlinear monetary policy rules

As noted in the Introduction, the linear monetary policy rules used by many authors are based on the assumption that the central bank has a quadratic loss function and the aggregate supply relation (Phillips curve) is linear[3]. These linear rules do quite well in describing monetary policy as implemented in many countries, including that of the Fed in the US. Nevertheless, it is important to appreciate the theoretical underpinnings of these rules and the grounds on which they have recently been challenged.

As usual, we assume that monetary policy is conducted by a central bank that chooses the sequence of short-term interest rates in order to minimize the present discounted value of its loss function which depends on both inflation and output in relation to their target values. Formally, the central bank faces the following problem:

$$\underset{\{i_{t+\tau}\}_{\tau=0}^{\infty}}{Min} E_t \sum_{\tau=0}^{\infty} \delta^{\tau} L(\widetilde{\pi}_{t+\tau}, \widetilde{y}_{t+\tau}), \tag{2.1}$$

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[3]Examples of the empirical application of these linear rules include Taylor (1993, 1999), Clarida et. al. (1998, 2000), Judd and Rudebusch (1998), Gerlach and Schnabel (2000).

such that

$$\pi_{t+1} \quad = \quad \pi_t + f(\widetilde{y}_t) + u_{t+1}, \tag{2.2}$$

$$\widetilde{y}_{t+1} \quad = \quad e\widetilde{y}_t + g(r_t) + \eta_{t+1}, \tag{2.3}$$

where δ is the discount factor, $L(.)$ is the unrestricted general loss function of the central banker, $f(.)$ and $g(.)$ are possibly nonlinear functions, π_t is the inflation rate at time t, i_t is the nominal interest rate, $\widetilde{\pi}_{t+\tau}$ is the expected inflation deviation from the inflation target (π^*) at time $t+\tau$, $\widetilde{y}_t$ is the output gap, $r_t = i_t - \pi_t$ is the real interest rate, and u_{t+1}, and η_{t+1} are shocks. Equations (2.2) and (2.3) describe the supply side (i.e., Phillips curve) and the aggregate demand of the economy respectively. As general AS and AD relations, we assume that $\frac{\partial f}{\partial \widetilde{y}_t} > 0, 0 \leq e < 1$, and $\frac{\partial g}{\partial r_t} < 0$. This is a generalization of the setup of Svensson (1997) that is the basis of many of the studies discussed below. The specific policy rule of a central bank depends on the functional forms of $L(.)$, $f(.)$, and $g(.)$, with the linear rule being a special case.

Cukierman (2000) specifies the loss function as $L(.) = 1/2 \left[\pi^2 + A\widetilde{y}^2 \right]$ if $\widetilde{y} < 0$ and $L(.) = \frac{1}{2}\pi^2$ otherwise, to capture the central bank's aversion to recessions. This loss function implies that the central banker dislikes inflation as well as negative output gaps, but given inflation the central banker is indifferent to positive output gaps. Bec et al. (2002) generalize this possible dependence on the state of the business cycle by using the loss function[4]:

$$
\begin{aligned}
L(\pi_t, \widetilde{y}_t) \quad = \quad & \frac{1}{2} \left[\widetilde{\pi}_t^2 + \omega_e \widetilde{y}_t^2 \right] \delta_{[\widetilde{y}_{t-p} > 0]} \\
& + \frac{1}{2} \left[\widetilde{\pi}_t^2 + \omega_r \widetilde{y}_t^2 \right] \delta_{[\widetilde{y}_{t-p} \leq 0]},
\end{aligned}
\tag{2.4}
$$

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[4]For the application of regime switching techniques to the measurement of monetary policy regimes, see Owyang and Ramey (2001).

where ω_e and ω_r are positive relative weights to output stabilization in expansion (e) and recession (r) respectively, p is the lag, and $\delta_{[.]}$ is the Heaviside or indicator function which is unity when the condition [.] holds, and zero otherwise. Based on (2.4) and linear dynamics in the AS/AD model, the central bank's optimal reaction function is state contingent and takes the following threshold type specification:

$$i_t = \left\{ \begin{array}{l} \alpha + \rho(L)i_{t-1} + \beta_e E_t \widetilde{\pi}_{t+k} + \gamma_e E_t \widetilde{y}_{t+q} + \varepsilon_t, \ \text{if } \widetilde{y}_{t-p} > 0 \\ \alpha + \rho(L)i_{t-1} + \beta_r E_t \widetilde{\pi}_{t+k} + \gamma_r E_t \widetilde{y}_{t+q} + \varepsilon_t, \ \text{if } \widetilde{y}_{t-p} \leq 0. \end{array} \right. \tag{2.5}$$

Rejection of the null hypotheses, $H_0 : \beta_e = \beta_r, \gamma_e = \gamma_r$ indicates asymmetric responses to the inflation and/or output gaps across the two states of the business cycle.

The studies Dolado et al. (2002a), (2002b) allow the Phillips curve (2.2) to be convex in the inflation-output gap through the use of the functional form $f(\widetilde{y}_t) = \frac{a\widetilde{y}_t}{1-a\phi\widetilde{y}_t}, f' > 0, f'' > 0, a > 0$ and $\phi \geq 0$.[5] Following Schaling (1999), Dolado et al. (2002b) also impose this form of nonlinearity in the output gap on the loss function through the specification of (2.1) as $L(.) = \frac{1}{2}(\widetilde{\pi}_{t+k})^2 + \frac{s}{2}f(\widetilde{y}_{t+q})^2$, where s measures the relative importance of stabilizing the output gap. The implication of $f(.)$ is that the central bank is more averse to positive output gaps than negative ones, so that interest rate increases are used aggressively to avoid the economy overheating. Minimising this loss function given the form of the Phillips curve leads to a nonlinear Euler equation, which (through a first-order Taylor series expansion) results in the policy rule:

$$i_t = c + \rho(L)i_{t-1} + \beta\widetilde{\pi}_{t+1} + \gamma\widetilde{y}_t + b(\widetilde{\pi}_{t+1}\widetilde{y}_t) + \varepsilon_t. \tag{2.6}$$

Note the inclusion of the multiplicative term in the inflation and output gaps, so that

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[5]Dolado et. al. (2002b) introduce $f(.)$ in the loss function, rather than the quadratic term $\widetilde{y}_t^2$, to enable them to derive a tractable closed-form solution for the optimal policy rule. See the Appendix of their paper.

rejection of the null $H_0 : b = 0$ provides evidence on this nonlinearity.

In Dolado et al. (2002a), the authors adopt the linex function in inflation deviations as the loss function, namely $L(\pi_t - \pi^*) = \frac{\exp(\theta(\pi_t - \pi^*)) - \theta(\pi_t - \pi^*) - 1}{\theta^2}$, where θ is a nonzero parameter[6]. As they discuss, this function permits different weights for positive and negative deviations of inflation from π^*, it implies that the size as well as the sign of a deviation are important, and it relaxes certainty equivalence. Hence, the central banker is allowed to exhibit a prudence motive and higher order moments of the inflation deviation (in addition to the mean) may play a role in the formulation of monetary policy. With these specific functional form assumptions for (2.1) and (2.2), combined with a linear form for (2.3), they derive the general form for the class of monetary policy rules they consider as:

$$i_t = \pi_t + f(\widetilde{y}_t) + \gamma\widetilde{y}_t + \frac{(1/a)(\pi_t - \pi^* + \theta\sigma_{\pi t}^2/2 + f(\widetilde{y}_t))}{1 - \phi(\pi_t - \pi^* + \theta\sigma_{\pi t}^2/2 + f(\widetilde{y}_t))}, \qquad (2.7)$$

where $\sigma_{\pi t}^2$ is the conditional variance of inflation. However, since linearity cannot be rejected empirically for the AS curve using US data, their monetary rule imposes the implied restriction $\phi = 0$ in the estimated form of

$$i_t = c + \rho(L)i_{t-1} + (1 - \rho)(\beta\pi_t + \gamma\widetilde{y}_t + d\sigma_{\pi t}^2) + \varepsilon_t. \qquad (2.8)$$

The time-varying conditional variance of inflation is parameterized using a GARCH(1,1) model and (2.8) is then estimated by GMM. The effect of inflation volatility is to introduce prudence in the loss function of the Fed, with values above target being weighted more heavily than those below.

Although each of the above specifications is plausible *a priori* and they may de-

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[6]For analytical tractability, Dolado et al. (2002a) assume that the central bank's loss function excludes output stabilization. They show in their Appendix that with the inclusion of an output gap term a closed-form solution cannot be obtained for the central bank's problem.

scribe certain properties of the nonlinear (asymmetric) relationship between interest rates and inflation/output gap deviations from their targets, these policy rules are driven by the specific assumptions made in each study about the central bank's preferences (the loss function) and/or the AS curve (Phillips curve). Perhaps surprisingly, no research appears to have yet allowed for possible nonlinearity in the AD relationship. While embedding nonlinearity in monetary policy rules in specific assumptions about the central bank's loss function or the AS/AD curves is logically attractive, it is also important to take a broader view.

In reality, we do not directly observe the preferences of the central bank, so that the assumption of a specific loss function might lead to incorrect inferences about the nature of nonlinearities in the monetary policy rule. Indeed, there is an unbounded universe of alternative nonlinear specifications of the policy rule depending on the functional forms of the loss function, the Phillips curve and the aggregate demand schedule. One logical way to avoid potential misspecification problems would be to leave the functions $L(.)$, $f(.)$, and $g(.)$ in equations (2.1) - (2.3) unrestricted and allow the data to tell us the form of the nonlinearity that is best supported by the data. This paper pursues that idea by using a flexible approach to nonlinear modeling recently suggested by Hamilton (2001a). Thus, allowing general nonlinearities in Fed's response to inflation and the output gap, we conjecture only the following flexible monetary policy rule:

$$i_t = \mu(E_t\widetilde{\pi}_{t+k}, E_t\widetilde{y}_{t+q}) + \gamma(L)i_{i-1} + \varepsilon_t, \tag{2.9}$$

where the function $\mu(.)$ is unrestricted, $1 - \gamma(L)$ is a stationary polynomial in the lag operator and ε_t is an error term. As discussed later, by applying Hamilton's (2001) methodology to infer the functional form of the monetary policy rule (2.9), we are

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able to evaluate each of the parametric rules discussed above. First, however, we describe the technique in the next section.

3   A flexible approach to nonlinear inference

Hamilton (2001a) proposes a new framework that combines the advantages of non-parametric and parametric methods. While the procedure does not assume any specific functional form for the conditional mean function, parameters are used to characterize this function and these parameters are estimated by maximum likelihood or Bayesian methods. Inference is based on classical econometric theory.

Consider the general nonlinear regression model

$$y_t = \mu(\mathbf{x}_t) + \boldsymbol{\gamma}' \mathbf{z}_t + \varepsilon_t, \tag{3.1}$$

where y_t is a scalar dependent variable, $\mathbf{x}_t$ and $\mathbf{z}_t$ are k- and p-dimensional vectors of explanatory variables, and ε_t is an error term with mean zero that is independent of $\mathbf{x}_t$ and $\mathbf{z}_t$ and of lagged values y_{t-j}, $\mathbf{x}_{t-j}$, $\mathbf{z}_{t-j}.(j = 1, 2, ...)$. In (3.1) we allow a subset of variables $\mathbf{z}_t$ for which the research is willing to assume linearity, thereby gaining efficiency by imposing this restriction. In our monetary policy application, $y_t = i_t$, $\mathbf{x}_t = (E_t \widetilde{\pi}_{t+1}, E_t \widetilde{y}_{t+1})'$, $\mathbf{z}_t = (i_{t-1}, i_{t-2})'$ for the forward-looking monetary policy rule and $\mathbf{x}_t = (\pi_t, \widetilde{y}_t)'$, $\mathbf{z}_t = (i_{t-1}, i_{t-2})'$ for the backward-looking rule. In contrast to previous analyses of nonlinear monetary policy rules, reviewed in the previous section, we treat the form of function $\mu(\cdot)$ as unknown. Following Hamilton (2001a), we view this function as the outcome of a random field. Specifically, the value of the function $\mu(\mathbf{x}_t)$ at $\mathbf{x}_t = \boldsymbol{\tau}$ is treated as being a Gaussian random variable with mean equal to the linear component $\alpha_0 + \boldsymbol{\alpha}' \boldsymbol{\tau}$ and variance λ^2, where $\alpha_0, \boldsymbol{\alpha}$, and λ are population parameters to be estimated. In the special case of $\lambda = 0$, then

$\mu(\mathbf{x}_t)$ is fixed and (3.1) becomes the usual linear regression model. In general, the parameter λ measures the overall extent of nonlinearity.

The basic idea of the method is that nonlinearity implies the values for $\mu(\mathbf{x}_t)$ and $\mu(\mathbf{x}_s)$ will be positively correlated for periods t and s whenever the vectors $\mathbf{x}_t$ and $\mathbf{x}_s$ are close to each other. The key is then parameterizing this correlation based on the distance measure $h_{st} = (1/2) \left[\sum_{i=1}^{k} g_i^2 (x_{is} - x_{it})^2 \right]^{1/2}$ where x_{it} denotes the ith element of the vector $\mathbf{x}_t$ and $g_1, g_2, ..., g_k$ are k additional parameters to be estimated. Hamilton proposes that $\mu(\mathbf{x}_s)$ should be uncorrelated with $\mu(\mathbf{x}_t)$ if $\mathbf{x}_s$ is sufficiently far away from $\mathbf{x}_t$. More precisely,

$$E\{[\mu(\mathbf{x}_s) - \alpha_0 - \boldsymbol{\alpha}' \mathbf{x}_s][\mu(\mathbf{x}_t) - \alpha_0 - \boldsymbol{\alpha}' \mathbf{x}_t]\} = 0 \quad \text{if } h_{st} > 1 \qquad (3.2a)$$

However, when $0 \le h_{st} \le 1$, this correlation should increase as h_{st} decreases, with the correlation going to unity as h_{st} goes to zero. In our context where the nonlinear part of the model includes $k = 2$ explanatory variables, then the correlation is assumed to be given by

$$Corr(\mu(\mathbf{x}_s), \mu(\mathbf{x}_t)) = H_2(h_{st}) \quad \text{if } 0 \le h_{st} \le 1 \qquad (3.2b)$$

where

$$H_2(h_{st}) = 1 - (2/\pi)[h_{st}(1 - h_{st}^2)^{1/2} + \sin^{-1}(h_{st})]. \qquad (3.3)$$

For the general specification and rationalization of this correlation, see Lemma 2.1 and Theorem 2.2 in Hamilton (2001a). It should be emphasized that $H_k(.)$ does not assume any parametric form for the functional relation $\mu(.)$ itself, but rather it parameterizes the correlation between pairs of random outcomes $\mu(\mathbf{x}_s)$ and $\mu(\mathbf{x}_t)$. The coefficient g_i determines the extent to which variation in the i-th element of $\mathbf{x}_t$ contributes to nonlinear variation in $\mu(\mathbf{x}_t)$. For g_i small, the value of $\mu(\mathbf{x}_t)$ changes

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little when the value of the corresponding explanatory changes, with $g_i = 0$ implying linearity of $\mu(\mathbf{x}_t)$ with respect to that variable.

Prior to estimation it is appropriate to determine whether nonlinearity exists by testing $H_0 : \lambda^2 = 0$. As is usual in nonlinear modelling, certain parameters are unidentified under the null of linearity. In the present context, this applies to $g_1, g_2, ..., g_k$. For the purpose of the nonlinearity test, Hamilton suggests that the lack of identification can be avoided by setting $g_i = 2 \left[k \left(T^{-1} \sum_{t=1}^{T} (x_{it} - \overline{x}_i)^2 \right) \right]^{-1/2}$, thereby scaling in terms of the individual sample standard deviations and the number of explanatory variables. Then, for T sample observations, the $(T \times T)$ matrix $\mathbf{H}$ of correlations can be formed, with the row s, column t element $H_k\{h_{st}\}$ given in (3.3) when $k = 2$ and $0 \leq h_{st} \leq 1$, or zero when $h_{st} > 1$. The Lagrange multiplier (LM) test of the null hypothesis can be obtained by using the residuals from an OLS linear regression of y_t on $(1, \mathbf{x}_t', \mathbf{z}_t')'$. Denoting the OLS residual vector by $\widehat{\varepsilon}$ and the OLS squared standard error as $\widetilde{\sigma}^2 = (T - k - p - 1)^{-1} \widehat{\varepsilon}' \widehat{\varepsilon}$, and the $(T \times T)$ projection matrix $\mathbf{M} = \mathbf{I}_T - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ where $\mathbf{X}$ is a $(T \times (1 + k + p))$ matrix whose tth row is given by $(1, \mathbf{x}_t', \mathbf{z}_t')$ and $\mathbf{I}_T$ is the $(T \times T)$ identity matrix, the test statistic is

$$\nu^2 = \frac{[\widehat{\varepsilon}'\mathbf{H}\widehat{\varepsilon} - \widetilde{\sigma}^2 tr(\mathbf{MHM})]^2}{\widetilde{\sigma}^4 (2tr\{[\mathbf{MHM} - (T - k - p - 1)^{-1}\mathbf{M}tr(\mathbf{MHM})]^2\})}. \tag{3.4}$$

Under the linearity null hypothesis, ν^2 has an asymptotic $\chi^2(1)$ distribution. Dahl's (2002) Monte Carlo investigations suggest that this test has good size and power properties against a variety of nonlinear alternatives.

In the presence of nonlinearity, Hamilton writes (3.1) as

$$\begin{aligned} y_t &= \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_t + \boldsymbol{\gamma}'\mathbf{z}_t + \lambda m(\mathbf{x}_t) + \varepsilon_t \tag{3.5} \\ &= \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_t + \boldsymbol{\gamma}'\mathbf{z}_t + u_t, \end{aligned}$$

where $m(.)$ is the realization of a scalar-valued Gaussian random field with mean zero, unit variance and covariance function given by (3.2a) and (3.2b). Assuming that the regression disturbance ε_t is i.i.d. $N(0, \sigma^2)$, the composite disturbance $u_t = \lambda m(\mathbf{x}_t) + \varepsilon_t$ is also Gaussian. With independence between $(\mathbf{x}_t', \mathbf{z}_t')'$ and ε_t, this specification implies a GLS regression model of the form

$$\mathbf{y}|\mathbf{X} \sim N(\mathbf{X}\boldsymbol{\beta}, \mathbf{P}_0 + \sigma^2 \mathbf{I}_T)$$

where $\mathbf{y} = (y_1, y_2, ..., y_T)'$, $\boldsymbol{\beta}$ is the $(1+k+p)$-dimensional vector $(\alpha_0, \boldsymbol{\alpha}', \boldsymbol{\gamma}')'$, and $\mathbf{P}_0$ is a $(T \times T)$ matrix whose row s, column t element is given by $\lambda^2 H_k(h_{st}) \delta_{[h_{st}<1]}$ with h_{st} is defined above, and the function $H_k(.)$ is specified in (3.3) for the case $k = 2$.

In addition to the linear regression parameters $(\alpha_0, \boldsymbol{\alpha}, \boldsymbol{\gamma})$ and σ^2, parameters to be estimated are the variance of the nonlinear regression error, λ^2, which governs the overall importance of the nonlinear component, and the parameters $(g_1, g_2, ..., g_k)$ determining the variability of the nonlinear component with respect to each explanatory variable in $\mathbf{x}_t$. As the above discussion implies, estimation and inference can be achieved by a GLS Gaussian regression. However, Hamilton (2001a) also describes the use of numerical Bayesian methods for the evaluation of the posterior distribution of any statistics of interest. The optimal inference of the value of the unobserved function $\mu(\mathbf{x}^*)$ at an arbitrary point $\mathbf{x}^*$ is given by

$$\widehat{\mu}(\mathbf{x}^*) = \alpha_0 + \alpha' \mathbf{x}^* + \mathbf{q}'(\mathbf{P}_0 + \sigma^2 \mathbf{I}_T)^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}), \tag{3.6}$$

where the $(T \times 1)$ vector $\mathbf{q}$ has tth element $\lambda^2 H_k(h_t^*) \delta_{[h_t^*<1]}$ for $h_t^* = (1/2) \left[\sum_{i=1}^{k} g_i^2 (x_{it} - x_i^*)^2 \right]^{1/2}$, in which x_{it} denotes the ith element of $\mathbf{x}_t$ and x_i^* denotes the ith element of $\mathbf{x}^*$. Hamilton shows that $\widehat{\mu}(\mathbf{x}^*)$ converges to the true value $\mu(\mathbf{x}^*)$ for any $\mu(.)$ from a broad class of continuous functions. This permits the calculation of confidence intervals,

using (3.6) along with its known standard error for each given parameter vector in conjunction with values of $\alpha_0, \boldsymbol{\alpha}, \boldsymbol{\gamma}, \sigma, \lambda$, and $\mathbf{g} = (g_1, g_2, ..., g_k)'$ generated from their posterior distributions, and examining the resulting distribution of inferences.

From a Monte Carlo investigation, Dahl (2002) shows that in many situations Hamilton's random field based estimator is substantially more accurate than the non-parametric spline smoother. He also finds that the procedure is useful in finite samples for characterizing a wide range of nonlinear time series models.

4 Empirical Results

Based on Hamilton's (2001a) methodology described in the previous section, the unrestricted monetary policy rule (2.9) can be rewritten as:

$$i_t \;\;=\;\; \mu(\mathbf{x}_t) + \alpha_2' \mathbf{z}_t + \varepsilon_t, \tag{4.1}$$

$$\mu(\mathbf{x}_t) \;\;=\;\; \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t + \lambda m(\mathbf{g} \odot \mathbf{x}_t), \tag{4.2}$$

where $\mathbf{x}_t \;=\; (E_t \widetilde{\pi}_{t+k}, E_t \widetilde{y}_{t+q})'$ is 2×1 vector and $\odot$ denotes element by element multiplication. The vector $\mathbf{z}_t$ contains lagged interest rates which capture interest rate smoothing by the Fed.[7] Following the theoretical discussion of previous models in Section 2, we assume that any nonlinearity in the Fed's reaction function relates only to the output gap and inflation, with lagged interest rates entering in a linear way.

In this section we report estimates of the central bank reaction function described by equations (4.1) and (4.2). We consider two different policy rules; (1) a flexible nonlinear forward-looking rule in line with Clarida et al. (1998, 2000), (2) a flexible nonlinear backward-looking rule of the type used by Taylor (1993). Following Clarida

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[7]For a discussion of the interest smoothing behaviour by the Fed, see Amato and Laubach (1999).

et. al. (2000) and Dolado et al. (2002a), we investigate two monetary regimes in U.S. economy, the pre-Volcker era (pre-1979) and the Volcker-Greenspan era (post-1979).

4.1 Data

Our data is quarterly from 1960:I to 2000:IV. Inflation is measured as the (annualized) rate of change of the GDP deflator (P_t) between two subsequent quarters: $\pi_t = 400 * (\ln(P_t) - \ln(P_{t-1}))$. The principal output gap measured we employ is the difference between real GDP and the estimate for potential real GDP constructed by the Congressional Budget Office (CBO). However, we also use output (real GDP) detrended by the HP filter of Hodrick and Prescott (1997). The interest rate is the average Federal Fund rate in the first-month of each quarter, expressed at annual rates. All these series were downloaded from the Federal Reserve Bank of St. Louis.

As a check on the robustness of the results of the forward-looking model to our constructed inflation forecast values, we also use corresponding actual inflation forecasts, specifically the median value from the *Survey of Professional Forecasters* and those of the Greenbook of the Federal Reserve Board[8]. These inflation forecast series were obtained from the Federal Reserve Bank of Philadelphia website.

We divide the sample into two main subperiods. The first (1960:I - 1979:II) encompasses the tenures of William M. Martin, Arthur Burns, and G. William Miller as Federal Reserve chairmen. The second (1979:III - 2000:IV) corresponds to the terms of Paul Volcker and Alan Greenspan. Previous analyses, including Clarida et al. (2000) and Dolado et al. (2002a), have indicated substantial differences in US

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[8]The *Survey of Professional Forecasters* is the oldest quarterly survey of macroeconomic forecasts in the US. The survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research (NBER), with the Federal Reserve Bank of Philadelphia taking it over in 1990. The Greenbook is produced before each meeting of the Federal Open Market Committee containing projections by the Research staff at the Board of Governors about how the economy will fare in the future. These projections are made available to the public after a lag of five years, and hence our Greenbook data ends in 1996:IV.

interest rate policy over these subperiods.

4.2 Forward-looking rule

As a generalization of the model of Clarida et al. (1998, 2000), the flexible nonlinear forward-looking rule can be written as

$$
\begin{aligned}
i_t \;=\;& c + \alpha E_t \widetilde{\pi}_{t+k} + \beta E_t \widetilde{y}_{t+q} + \gamma_1 i_{t-1} + \gamma_2 i_{t-2} \\
& + \sigma[\zeta m(g_1 E_t \widetilde{\pi}_{t+k}, g_2 E_t \widetilde{y}_{t+q}) + v_t],
\end{aligned}
\tag{4.3}
$$

where $c, \alpha, \beta, \gamma_1, \gamma_2, \sigma, \zeta, g_1$ and g_2 are parameters to be estimated, $v_t \sim N(0,1)$ and $m(.)$ denotes an unobserved realization from a Gaussian random field with mean zero, unit variance, and correlations given by (3.2a) and (3.2b). In comparison with (3.5), the innovation ε_t is written here as σ times v_t and the parameter λ is σ times ζ.

Clarida et al. (1998, 2000) use GMM to estimate the linear version of the equation (4.3) by replacing expected inflation and the output gap with their realized values. However, for our baseline estimation we generate $E_t\widetilde{\pi}_{t+k}$ and $E_t\widetilde{y}_{t+q}$ by estimating the processes for inflation and the output gap. Although this "generated regressors" approach could result in invalid inferences (Pagan, 1984), we guard against this to the extent that is practical by checking robustness to actual inflation forecasts[9]. To avoid overlapping forecast intervals and consequent problems with moving average errors, we assume that the target horizon is one-quarter for both inflation and the output gap (i.e., $k = q = 1$). Following Clarida et al. (2000), we also assume that two quarterly lags of i_t is sufficient to capture interest rate smoothing by the Fed and to account for serial correlation.

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[9]Ideally, we would also like to check robustness against actual output gap forecasts. Unfortunately, although Greenbook forecasts of output growth are published, the corresponding output gap forecasts are not. Orphanides (2001) reconstructs these output gap forecasts for the period 1987-1992, but this is not a sufficiently long time period for our purposes.

Our specifications for inflation and the output gap follow Gerlach and Smets (1999) and Aksoy et. al. (2002), so that for this purpose we assume that the functions $f(\widetilde{y}_t)$ and $g(r_t)$ in (2.2) and (2.3) respectively are linear. We also do not impose the unit root for inflation implicitly assumed in the former. More precisely, we assume that inflation is determined by the (CBO) output gap with a one period lag and past inflation rates, yielding the estimated forecasting equation:

$$\widehat{\pi}_t \;=\; \underset{(0.163)}{0.148} + \underset{(0.079)}{0.551}\pi_{t-1} + \underset{(0.090)}{0.059}\pi_{t-2} + \underset{(0.090)}{0.166}\pi_{t-3}$$
$$+\underset{(0.082)}{0.197}\pi_{t-4} + \underset{(0.035)}{0.145}\widetilde{y}_{t-1}. \tag{4.4}$$

The output gap is assumed to depend on previous output gaps and the average real interest rate over the year ending in the previous quarter. In the case of the CBO measure, this estimated equation yields the one-step ahead forecast for the outut gap as:

$$\widehat{\widetilde{y}}_t = \underset{(0.100)}{0.204} + \underset{(0.079)}{1.121}\widetilde{y}_{t-1} - \underset{(0.119)}{0.054}\widetilde{y}_{t-2} - \underset{(0.076)}{0.168}\widetilde{y}_{t-3} - \underset{(0.029)}{0.075}(\overline{r}_{t-1} - \overline{\pi}_{t-1}) \tag{4.5}$$

where $\overline{r}_t$ and $\overline{\pi}_t$ denote four-quarter (moving) averages of current and past interest and inflation rates. The lag lengths of inflation and the output gap in (4.4) and (4.5) respectively were chosen by AIC.[10] Since our sample is similar to Clarida et al. (2000), we use their estimates of the inflation target π^*, namely $\pi^* = 4.24$ in the pre-Volcker period (1960:I - 1979:II) and $\pi^* = 3.58$ in the Volcker-Greenspan period (1979:III - 2000:IV).[11] One other word is relevant about timing. As already noted,

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[10]The use of BIC did not qualitatively alter the results. When HP detrended output is used in (4.5), the lag length by AIC is unchanged at three.

[11]Dolado et al. (2002b) and Bec et al. (2002) assume that the inflation target is time-varying and use the index published in the reports of the Council of Economic Advisors as the inflation target measures for the U.S. However, as in Clarida et al. (2000), we assume that the inflation target is constant over the tenure of these FRB chairmen.

we use interest rate data for the first month of each quarter t, on the assumption that this captures the reaction of the Fed to the most recent information (relating to quarter $t-1$) about inflation and output. Therefore, following Clarida et al. (2000), although we refer to a one-quarter ahead forecast with $k = q = 1$ in (4.3), this forecast is for inflation and the output gap over the quarter t, since this is a whole quarter in advance of the information available when the relevant interest rate decision is made.

Columns 3 and 4 of Table 1 report the test statistic ν^2 of (3.4) for the null hypothesis of linearity in the forward-looking reaction function (4.3), computed separately for whole sample and the two subsamples. Over the whole sample, there is effectively no evidence against a linear policy rule. However, using the CBO output gap data, linearity is overwhelmingly rejected in the pre-Volcker era, but not in the Volcker-Greenspan period. The distinct results for the two subperiods is compatible with different monetary policies being conducted pre- and post-1979, a finding in line with Clarida et al. (2000) and other recent studies. Nevertheless, the lack of evidence of nonlinearity in the later period contrasts with the results of some other studies of this period, including Bec et al. (2002) and Dolado et al. (2002a), but agrees with Dolado et al. (2002b). Using HP detrended output, the evidence of nonlinearity in Table 1 for the pre-Volcker subsample is weaker than using the CBO measure, but the test statistic is significant at 10 percent. The other results using the HP filter confirm the findings obtained with CBO data.

Based on these results, we estimate a forward-looking nonlinear model for the first subperiod, but not for either the Volcker-Greenspan period or for the whole sample. Bayesian posterior estimates and their standard errors are reported in columns two and three of Table 2 for the parameters of (4.3) in the pre-Volcker subsample. When the CBO output gap measure is used, the coefficient on $E_t\widetilde{\pi}_{t+1}$ in the linear part is not statistically significant (indeed, this coefficient is negative and close to zero), but

the expected output gap and the interest rate of the previous quarter each linearly exert a positive effect on the interest rate. The nonlinear part $m(\mathbf{g} \odot \mathbf{x}_t)$ is significant collectively (as evidenced by the t-statistic for $\zeta = 0$), with inflation individually having a significant positive nonlinear impact through $\widehat{g}_1$. When HP detrended output is used, the nonlinear part overall is not statistically significant at 10 percent, but the general pattern of estimated coefficients is similar to the CBO case.

To assist interpretation, we fix values of one of inflation and the output gap at its sample mean and examine the consequences for the estimated reaction functions of changing the value of the other variable. This is achieved using the posterior distribution whose mean and standard deviation for each parameter are reported in the corresponding column of Table 2. Each value of the two variables gives a $\mathbf{x}^*$ of interest, with (3.6) used to compute the corresponding estimated conditional mean $\widehat{\mu}(\mathbf{x}^*)$. By generating a range of estimates of $\mu(\mathbf{x}^*)$, as explained in Section 3, 95 percent confidence intervals are also computed. However, these confidence regions are often relatively broad at extreme values for the variables, where little sample information is available, so that inferences on the shapes of these functions at the extremes must be treated with some caution.

Initially we fix $E_t\widetilde{y}_{t+1}$ at its sample mean while $E_t\widetilde{\pi}_{t+1}$ is allowed to vary, resulting in Figures 1a and 1b for the CBO and HP filtered output gap data respectively. In both cases, but particularly using the CBO measure, the figure indicates a more aggressive reaction by the Fed to expected inflation above than below the target. Indeed, for inflation beyond about 0.5 percent under target, the slope in Figure 1a is essentially flat, implying the same reaction by the Fed to any value of inflation below this threshold. When $E_t\widetilde{y}_{t+1}$ is varied, Figures 2a and 2b indicate a steeper slope in reaction to a negative output gap than for a positive one, implying that prior to 1979 the Fed may have reacted more strongly to output below than above potential. This

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is compatible with the notion that the Federal Reserve was more concerned about recession than expansion in the 1960s and 70s, as found by Gerlach (2000). Here the use of HP detrended output (Figure 2b) shows a distinct kink in the reaction for a negative output gap of around 0.8 percent, with a steeper slope to the left than to the right.

In sum, in a forward-looking context, our investigation finds little evidence against the hypothesis that the Fed operated a linear monetary policy rule when data over the whole sample or post-1979 are used. However, we find substantial support for a nonlinear policy rule in the pre-Volcker period, with graphical evidence suggesting that the Fed reacted more vigorously to expected inflation deviations above than below the inflation target and more strongly to output below than above potential. The latter is consistent with a recession aversion story in which policy makers care more about falls than increases in output.

The next two subsections consider the robustness of our results. Firstly, we use a backward-looking specification which avoids the need to forecast inflation and the output gap in Subsection 4.3, then we examine the robustness of our forward-looking model results to different inflation forecasts and to the dates used for the second subsample in Subsection 4.4.

4.3   Backward-looking rule

Our flexible nonlinear version of the backward-looking rule suggested by Taylor (1993) corresponds to specification (4.3) with both k and q set to 0. That is, the estimated model has the form

$$i_t = c + \alpha \pi_t + \beta \widetilde{y}_t + \gamma_1 i_{t-1} + \gamma_2 i_{t-2} + \sigma[\zeta m(g_1 \pi_t, g_2 \widetilde{y}_t) + v_t], \qquad (4.6)$$

where the subscript of t on inflation and the output gap correspond to the latest information on these variables available in the first month of quarter t. As in Taylor (1993), this equation is specified in terms of actual observed inflation and not the past deviation from target. The backward-looking model has the advantage over the forward-looking one that we do not need to specify the form of the forecasting equations used by the Fed for inflation and the output gap, since such equations are implicitly embedded in the nonlinear backward-looking model. Thus, nonlinear forms of the AS and/or AD equations (2.2)/(2.3) could be the source of any nonlinearity in (4.6).

The final two columns of Table 1 report the results of Hamilton's linearity test applied to (4.6) for the whole sample and two subsamples. These are qualitatively similar to those for the forward-looking rule. In particular, there is little evidence against linearity for the whole sample or the Volcker-Greenspan subsample, but significant evidence for the earlier subsample. This evidence is especially strong when using the CBO output gap measure, but now the HP detrended output data gives a significant test statistic at 5 percent and is close to significance at 1 percent.

Bayesian posterior estimates and their standard errors for the backward-looking flexible nonlinear policy rule (4.6) are reported in the final two columns of Table 2 for the pre-Volcker subsample. These are similar overall to those of the forward-looking rule. However, the linear component relating to neither inflation nor the output gap in the backward-looking case is significant. The coefficient on inflation is again very small and negative. The estimated linear response of the Fed to the actual output gap is less strong than that for the forecast value, with the coefficient $\widehat{\beta}$ in (4.6) not being statistically significant at even 10 percent. On the other hand, the importance of the overall nonlinear component in this backward-looking specification is underlined by the strong significance of $\widehat{\zeta}$ for both sets of estimates. The coefficients of both

inflation and the output gap individually are positive in the nonlinear component, although the only significant coefficient here is that of inflation in the model using the CBO output gap.

Figures 3 and 4 illustrate the estimated nonlinear functions $\widehat{\mu}(.)$ for this backward-looking case, using the same approach as in Figures 1 and 2. Once again, despite the negative sign and lack of significance of the linear inflation coefficient, Figure 3a (using CBO output gap) implies that the overall effect of inflation on the Fed's reaction function is positive. However, there is little response from the Fed to observed inflation (at an annual rate) of around 2 percent or less. Therafter, the response is effectively linear, until it flattens off at approximately 9 percent. Nevertheless, since the confidence interval gets wider as the rate of inflation increases, the estimated response at these higher rates is less reliable. The shape in Figure 3b, using HP filtered output, is similar, although the response flattens off at around 5 (rather than 9) percent.

The response of the Fed to the output gap shown in Figure 4a for the CBO output gap model is again positive overall. Although the shape is less smooth than the corresponding forward-looking graph (Figure 2a), within the central range of ± 2 percent it appears that the Fed reacts more aggressively to negative than positive output gaps. Using HP detrended output leads to a smoother estimated response to the output gap in Figure 4b. In particular, a kink can be seen at an output gap of a little over 1 pecent, with a stronger estimated response by the Fed below this value than above it.

In sum, there are no substantial differences between the estimates of the nonlinear forward-looking policy rule and that of the nonlinear backward-looking rule. The estimates in both cases suggest that the Fed reacted more strongly to output below than above potential output in the 1960s and 70s, and that it did not respond to

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changes in inflation provided that the rate remained low.

4.4 Robustness of the forward-looking estimates

In the estimates of nonlinear forward-looking policy rule discussed above, we used inflation forecasts generated by equation (4.4). Here, we consider alternative inflation forecasts, specifically actual one-quarter ahead forecasts made in real time by the *Survey of Professional Forecasters* and the Greenbook of the Federal Reserve Board of Governors. Table 3 reports the test statistic ν^2 of the null hypothesis of linearity for both inflation forecasts, when combined with our one-quarter ahead output gap forecasts for the CBO series through equation (4.5). However, results are not presented for the pre-Volcker subsample here because neither forecast series is available for the early part of this period. Nevertheless, we are able to investigate whether the apparent linearity found for the whole sample and the Volvker-Greenspan period is robust to the inflation forecast series.

The results using the inflation forecasts of the *Survey of Professional Forecasters* in Table 3 are in line with our results from Table 1, namely we do not reject linearity over either the whole sample or for the Volcker-Greenspan subsample. However, one potentially interesting finding is that the Greenbook forecasts yield a $p-value$ that is significant at 5 percent for this subsample. Although we do not pursue it here, this points to the potential value of further investigation into whether the use of these inflation forecasts constructed within the Fed may shed light on possible nonlinearity in the monetary policy rule over the Volcker-Greenspan period.

Since our second subsample includes the period when the Fed targeted nonborrowed reserves rather than Federal Funds rates, and interest rates were high and volatile, it is worth investigating whether this abnormal period has influenced the overall results for the Volcker-Greenspan era. To do this, we exclude 1979:III-1982:IV,

and repeat the linearity test. The results are included in Table 3. These confirm the lack of significance of the nonlinearity at the conventional level for either the generated inflation forecasts or those of the *Survey of Professional Forecasters* and for either output gap measure. However, as Greenbook data are available only to 1996IV, the test is not conducted using this inflation forecast series over this shorter period.

Thus, our overall conclusion is that the Fed has reacted in an essentially linear way to pin inflation down to its target in the Volcker-Greenspan period. This is in contrast to the conclusion of Dolado et al. (2002a) that the Fed's reaction since 1983 may have been asymmetric with respect to inflation. However, it agrees with Dolado et al. (2002b) who find that a nonlinear Phillips curve does not lead to asymmetries in US monetary policy in this period. Also, although Bec et al. (2002) model an asymmetric reaction to inflation for this period, their evidence is relatively weak and significant only at 3 percent.

4.5   Alternative nonlinear specifications

Our results provide strong evidence of a nonlinear policy rule in the pre-Volcker era. To examine whether the parametric models reviewed in Section 2 adequately capture this nonlinearity, we consider a formal statistical basis for comparing these alternative specifications with the nonlinearity revealed by the data through the flexible inference procedure. Note that the proposed alternative specifications in equations (2.5), (2.6) and (2.8), although nonlinear functions of the inflation deviation (or inflation) and the output gap, are linear functions of the parameters and thus they can be described as a linear regression model of the form

$$y_t = \alpha_0 + \boldsymbol{\alpha}'\mathbf{z}_t + \varepsilon_t \tag{4.7}$$

for suitable specifications $\mathbf{z}_t$. For example, with two lags of interest rates, equation (2.6) is a special case of (4.7) with $\mathbf{z}_t = (i_{t-1}, i_{t-2}, \widetilde{\pi}_{t+1}, \widetilde{y}_t, \widetilde{\pi}_{t+1}\widetilde{y}_t)'$. As such, we can test directly whether this particular specification for $\mathbf{z}_t$ adequately captures any nonlinearity in the data by comparing (4.7) with the more general model

$$y_t = \alpha_0 + \boldsymbol{\alpha}'\mathbf{z}_t + \lambda m(\mathbf{x}_t) + \varepsilon_t \tag{4.8}$$

for $\mathbf{x}_t = (\widetilde{\pi}_{t+1}, \widetilde{y}_t)'$ and $m(.)$ a realization of the random field whose correlations are characterized by (3.2a)/(3.2b). A test of $H_0 : \lambda = 0$ is now a test of whether the definition of $\mathbf{x}_t$ adequately captures the nonlinear dependence of y_t on $\widetilde{\pi}_{t+1}, \widetilde{y}_t$. This test is just adapted from testing the null hypothesis of linearity to testing the null hypothesis that the nonlinearity takes a particular known parametric form. In what follows, we consider four alternative nonlinear specifications for the forward-and backward-looking rules. Table 4 summarizes them.

Models 1F and 1B are threshold type models in which the Fed is allowed to react differently to positive and negative deviations of inflation from target and of the output gap. Models 2F and 2B are versions of the business cycle dependent model of Bec et al. (2002), and discussed as (2.5) above, where the Fed's reaction to the deviation of inflation from target and the output gap depends on the stage of the business cycle. Models 3F and 3B are Dolado et al.'s (2002a) specification in which asymmetric preferences lead to prudent behaviour whereby the Fed responds to the conditional variance of inflation; see (2.8). Following those authors, conditional volatility in inflation is parameterized through a GARCH(1,1) model, here applied to the residuals of our inflation forecasting equation (4.4). Finally, Models 4F and 4B are Dolado et al.'s (2002b) interaction model of inflation and the output gap, considered above as equation (2.6).

Table 5 reports the ν^2 test statistics for these alternative nonlinear specifications in the pre-Volcker era, using the CBO measure of the output gap. Given the very limited evidence we have found of nonlinearity in the monetary policy function over Volker-Greenspan period, we do not consider this later period. The nonlinearity evidenced by the test statistics for the pre-Volcker period in Table 1 is effectively undiminished in both versions of Model 3, implying that the introduction of inflation volatility does not account for this nonlinearity. We do not argue against the results of Dolado et al. (2002a) that such volatility may play a significant role in the Fed's monetary policy, but rather our conclusion is that it is not sufficient to characterize the nonlinearity during this period. The threshold-type Models 1 and 2 are generally more successful. Model 2F reduces the nonlinearity test statistic in the forward-looking model from 13.5 in Table 1 to 4.22 in Table 5, with the latter having a $p-value$ of 0.04. Thus, business cycle dependence in the model of Bec et al. (2002) may be part of the underlying source of the nonlinearity. Similarly, the backward-looking Model 1B, which also includes a business cycle dependence, is reasonably successful.

The most successful model overall, however, is the interaction model of inflation with the output gap (Models 4F, 4B). In this case, the ν^2 test statistic is not significant at any conventional level in the backward-looking specification, and is at the margin of significance at the 5 percent level for the forward-looking version. Therefore, this interaction apparently summarizes the nonlinearity in the monetary policy rule adequately, especially in the backward-looking specification.

This formal comparison suggests that the nature of nonlinearity in the monetary policy rule prior to 1979 might result partly from asymmetric reactions by the Fed to inflation deviations and (especially in the forward-looking case) the output gap, but more importantly from the Fed's reaction to the interaction between inflation

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and the output gap as in the model of Dolado et al. (2002b). As these authors point out, the intuition behind this interaction is plausible. If inflation is above target, the real interest rate will be below its equilibrium level, causing an increase in the output gap next period through the AD relation (2.3). Since in their model the Phillips curve (2.2) is convex, that anticipated future increase in the output gap will lead to greater anticipated inflationary pressure than in the linear case. To offset this latent inflationary pressure, the Fed should increase the interest rate at t by more than in the linear model. In line with Dolado et al. (2002b), our evidence above does not find nonlinearity of this type in the post-1979 period for the US. It is plausible that the Fed might have placed much more weight on the inflation target than the output gap during this recent period, and hence not reacted in a significant way to the interaction between these.

5   Concluding remarks

The linear monetary policy rule proposed by Taylor (1993) has since been widely used, including in the influential work of Clarida et al. (1998, 2000). However, this has recently been challenged on two grounds. Firstly, the central bank may have asymmetric responses to inflation deviations from target and/or to the output gap and, secondly, the underlying Phillips curve may be convex. Pursuing these two routes, contributors to the literature have specified sets of nonlinear relationships based on parametric models and provided some evidence in favor of nonlinearity in the policy rule. Our view is that because neither the preferences of the central bank nor the Phillips curve are directly observed, any inferences drawn from specific parametrizations should be confirmed against a flexible nonlinear specification. Detecting nonlinearity in a particular parametric form could otherwise lead to incorrect

conclusions about the validity of the particular model.

The contribution of this paper is to address this question using the framework of Hamilton (2001a) that explicitly parameterizes the set of nonlinear relations in a flexible way and takes into account uncertainty about the functional from in conducting hypothesis tests. We find that while there is quite strong evidence that the Fed operated a nonlinear monetary policy rule during the pre-Volcker period (1960-1979), the evidence is generally weak in Volcker-Greenspan era. Our results are relatively robust to whether the policy rule is forward- or backward looking, to output gap measures and to whether generated or actual inflation forecasts are used.

We also examine particular parametric specifications proposed in recent work in the context of the flexible, unrestricted framework. The notion that the Fed reacted more vigorously to inflation deviations above than below target and more strongly to output below than above potential in the 1960s and 70s characterizes the nature of the nonlinearity fairly successfully, but still leaves some unexplained nonlinearity. More promisingly, the interaction between inflation and the output gap, specified by Dolado, Maria-Dolores and Naveira (2002b) as arising through a convex Phillips curve, does rather well. Hence, we suggest that future structural models might build on this work to allow interactions of the dynamics of inflation and the output gap to influence the nonlinear monetary policy rule.

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Table 1. Tests of the linearity null hypothesis $\mu(\mathbf{x}_t) = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \boldsymbol{\alpha}_2'\mathbf{z}_t$

Sample	Output gap	Forward-looking rule		Backward looking rule	
(dates)	measure	Statistic	$p - value$	Statistic	$p - value$
Whole sample	CBO	0.603	0.438	0.036	0.849
(1960:I - 2000:IV)	HP	1.409	0.235	0.0002	0.988
Pre-Volcker	CBO	13.531	0.0002	12.25	0.0005
(1960:I - 1979:II)	HP	3.451	0.063	6.12	0.013
Volcker-Greenspan	CBO	0.499	0.480	0.507	0.476
(1979:III - 2000:IV)	HP	1.829	0.176	1.834	0.176

Note: CBO and HP denote the output gap estimate of the Congress Budget Office and Hodrick-Prescott detrended output respectively.

Table 2 Bayesian estimates of the flexible nonlinear policy rule in the pre-Volcker subsample

| | Forward-looking rule | | Backward-looking rule | |
| | Equation (4.3) | | Equation (4.6) | |
Coefficient	CBO	HP	CBO	HP
$\widehat{c}$	1.833** (0.588)	1.397** (0.500)	1.769** (0.824)	1.862** (0.814)
$\widehat{\alpha}$	-0.013 (0.213)	-0.060 (0.231)	-0.013 (0.165)	-0.072 (0.159)
$\widehat{\beta}$	0.176* (0.108)	0.314** (0.156)	0.104 (0.113)	0.178 (0.168)
$\widehat{\gamma}_1$	0.827*** (0.113)	0.851*** (0.117)	0.908*** (0.110)	0.853*** (0.111)
$\widehat{\gamma}_2$	-0.172 (0.116)	-0.084 (0.123)	-0.229** (0.109)	-0.120 (0.111)
$\widehat{\sigma}$	0.618*** (0.081)	0.623*** (0.093)	0.631*** (0.086)	0.594*** (0.083)
$\widehat{\zeta}$	1.814** (0.829)	1.526 (0.999)	1.774** (0.795)	1.910** (0.851)
$\widehat{g}_1$	0.372* (0.221)	0.361 (0.315)	0.260* (0.150)	0.252 (0.180)
$\widehat{g}_2$	0.162 (0.144)	0.514 (0.467)	0.209 (0.161)	0.336 (0.280)

Note: a) The values in parentheses are the standard errors of Bayesian posterior estimates with $N = 5000$ Monte Carlo simulations. b) ***, ** and * denote statistical significance at the 1%, 5% and 10% level, respectively, in a two-tailed $t-$test.

Table 3. Tests of the linearity null hypothesis $\mu(\mathbf{x}_t) = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \boldsymbol{\alpha}_2'\mathbf{z}_t$:

Robustness analysis of the forward-looking model

Sample	Inflation forecasts	Output gap measure	Statistic	$p-value$
Whole sample	*SPF* (68IV-00IV)	CBO	0.074	0.785
	Greenbook (65IV-96IV)	CBO	2.333	0.127
Volcker-Greenspan	*SPF* (79III-00IV)	CBO	0.472	0.492
	Greenbook (79III-96IV)	CBO	4.944	0.026
1983I - 2000IV	Own	CBO	0.255	0.613
	Own	HP	0.374	0.541
	SPF	CBO	1.109	0.292

Note: *SPF* indicates the median one-quarter ahead forecasts of the *Survey of Professional Forecasters.*

Table 4. Alternative nonlinear specifications

Model	Specification
1F	$i_t = c + \alpha_1 E_t \widetilde{\pi}_{t+1} + \alpha_2 E_t \widetilde{\pi}_{t+1} \delta_{[E_t \widetilde{\pi}_{t+1} > 0]} + \beta_1 E_t \widetilde{y}_{t+1} + \beta_2 E_t \widetilde{y}_{t+1} \delta_{[\widetilde{y}_{t+1} > 0]} + \gamma i_{t-1} + \varepsilon_t$
2F	$i_t = c + \alpha_1 E_t \widetilde{\pi}_{t+1} + \alpha_2 E_t \widetilde{\pi}_{t+1} \delta_{[\widetilde{y}_{t-1} > 0]} + \beta_1 E_t \widetilde{y}_{t+1} + \beta_2 E_t \widetilde{y}_{t+1} \delta_{[\widetilde{y}_{t-1} > 0]} + \gamma i_{t-1} + \varepsilon_t$
3F	$i_t = c + \alpha E_t \widetilde{\pi}_{t+1} + \beta E_t \widetilde{y}_{t+1} + \omega \sigma^2_{\pi_t} + \gamma i_{t-1} + \varepsilon_t$
4F	$i_t = c + \alpha E_t \widetilde{\pi}_{t+1} + \beta \widetilde{y}_t + \psi (E_t \widetilde{\pi}_{t+1} \widetilde{y}_t) + \gamma i_{t-1} + \varepsilon_t$
1B	$i_t = c + \alpha_1 \pi_t + \alpha_2 \pi_t \delta_{[\pi_t > \pi^*]} + \beta_1 \widetilde{y}_t + \beta_2 \widetilde{y}_t \delta_{[\widetilde{y}_t > 0]} + \gamma i_{t-1} + \varepsilon_t$
2B	$i_t = c + \alpha_1 \pi_t + \alpha_2 \pi_t \delta_{[\widetilde{y}_{t-1} > 0]} + \beta_1 \widetilde{y}_t + \beta_2 \widetilde{y}_t \delta_{[\widetilde{y}_{t-1} > 0]} + \gamma i_{t-1} + \varepsilon_t$
3B	$i_t = c + \alpha \pi_t + \beta \widetilde{y}_t + \omega \sigma^2_{\pi_t} + \gamma i_{t-1} + \varepsilon_t$
4B	$i_t = c + \alpha \pi_t + \beta \widetilde{y}_t + \psi (\pi_t \widetilde{y}_t) + \gamma i_{t-1} + \varepsilon_t$

Note: a) F and B denote forward- and backward-looking models, respectively. b) $\delta_{[.]}$ is unity if the statement [.] is true and 0 otherwise. c) $\pi^* = 4.24$, the assumed inflation target, as in Clarida et al. (2000).

Table 5. Tests of the linearity null hypothesis $\mu(\mathbf{x}_t) = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \boldsymbol{\alpha}_2'\mathbf{z}_t$ for

alternative nonlinear specifications in the pre-Volcker period

Rule	Model 1		Model 2		Model 3		Model 4	
	Stat.	$p-value$	Stat.	$p-value$	Stat.	$p-value$	Stat.	$p-value$
F	9.77	0.002	4.22	0.040	12.35	0.0004	3.85	0.05
B	4.88	0.027	10.63	0.001	19.46	1.0e-05	0.98	0.32

Note. F and B denote forward and backward-looking models, respectively.

Figure 1a. The effect of the inflation deviation
on the interest rate: FL, CBO
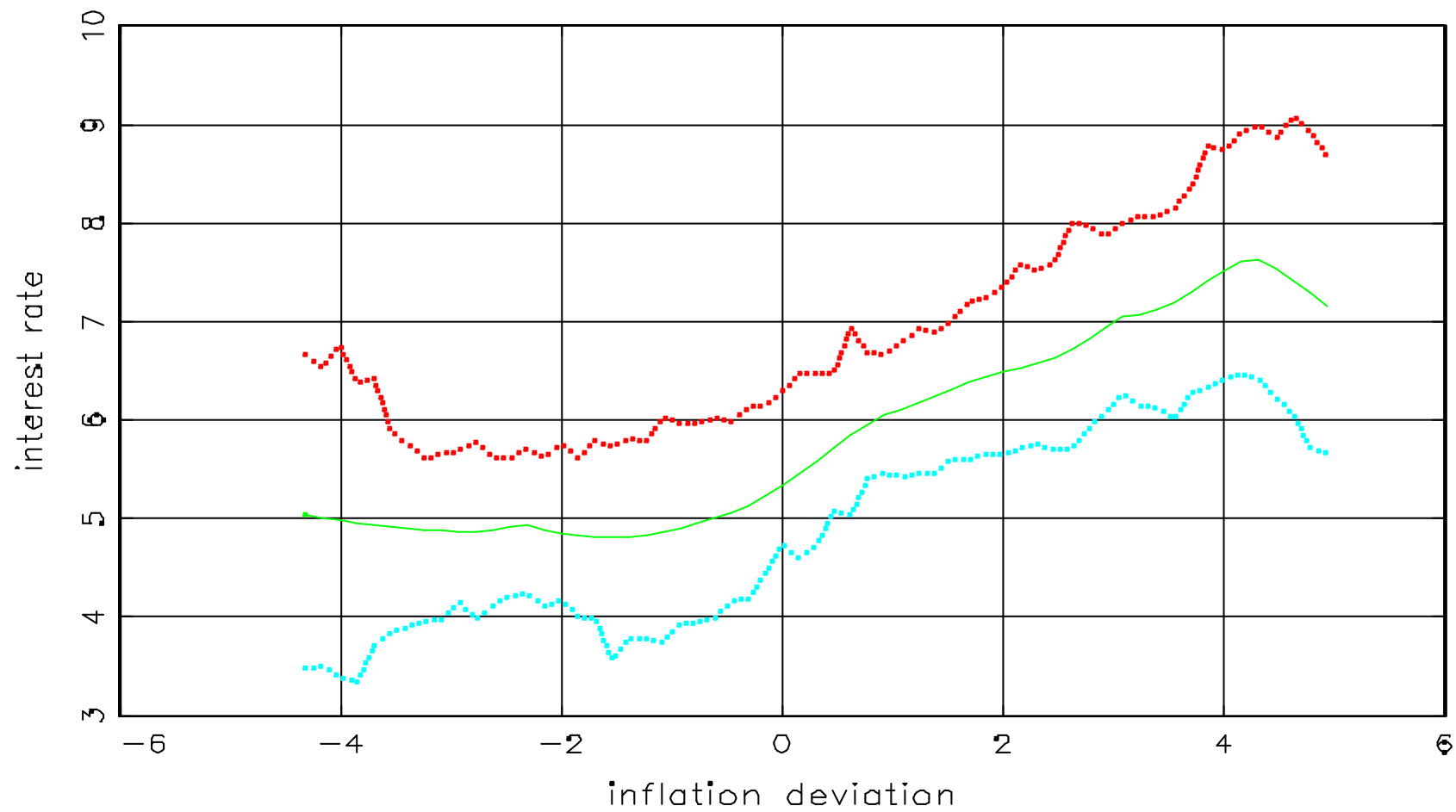
Figure 1b. The effect of the inflation deviation
on the interest rate: FL, HP
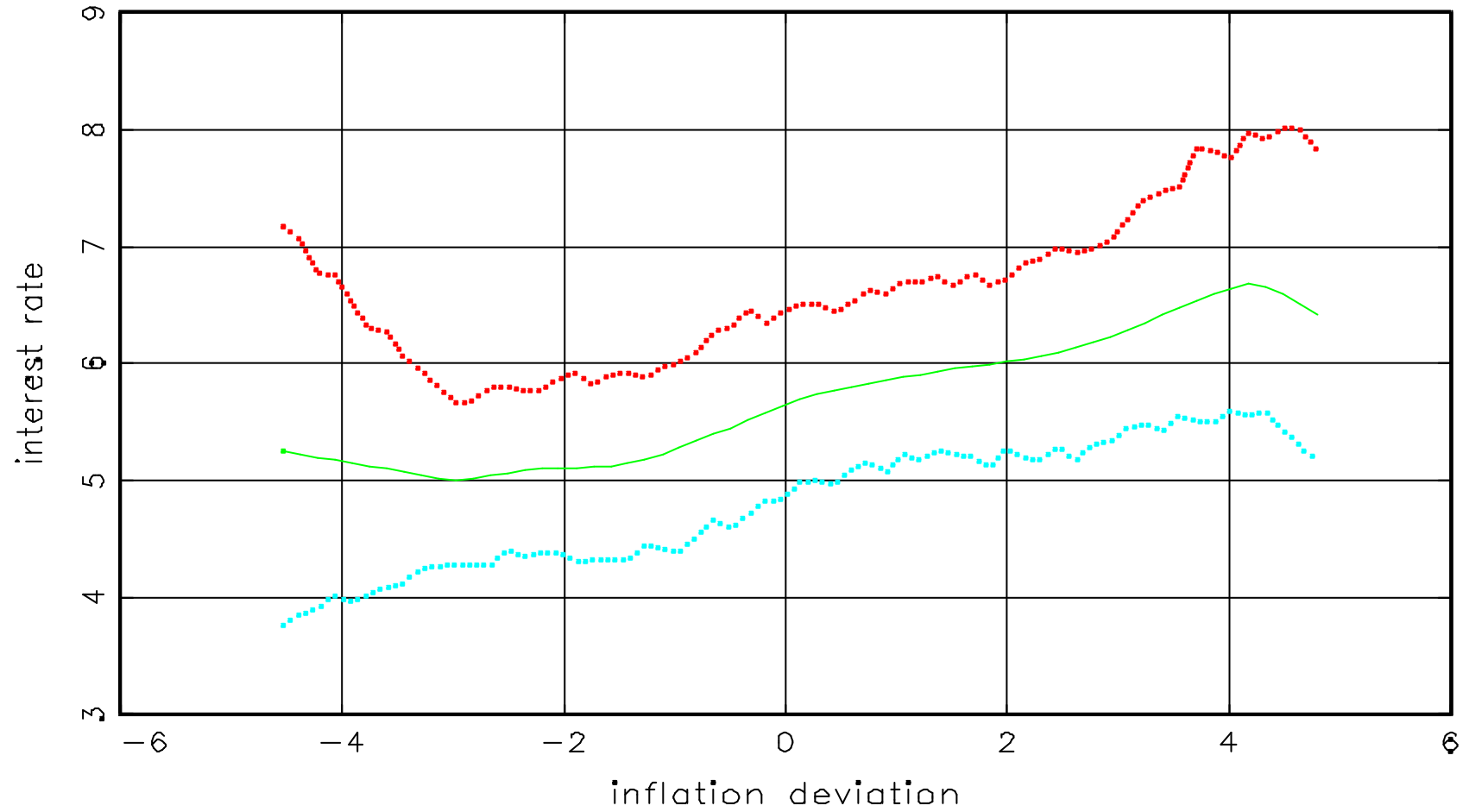
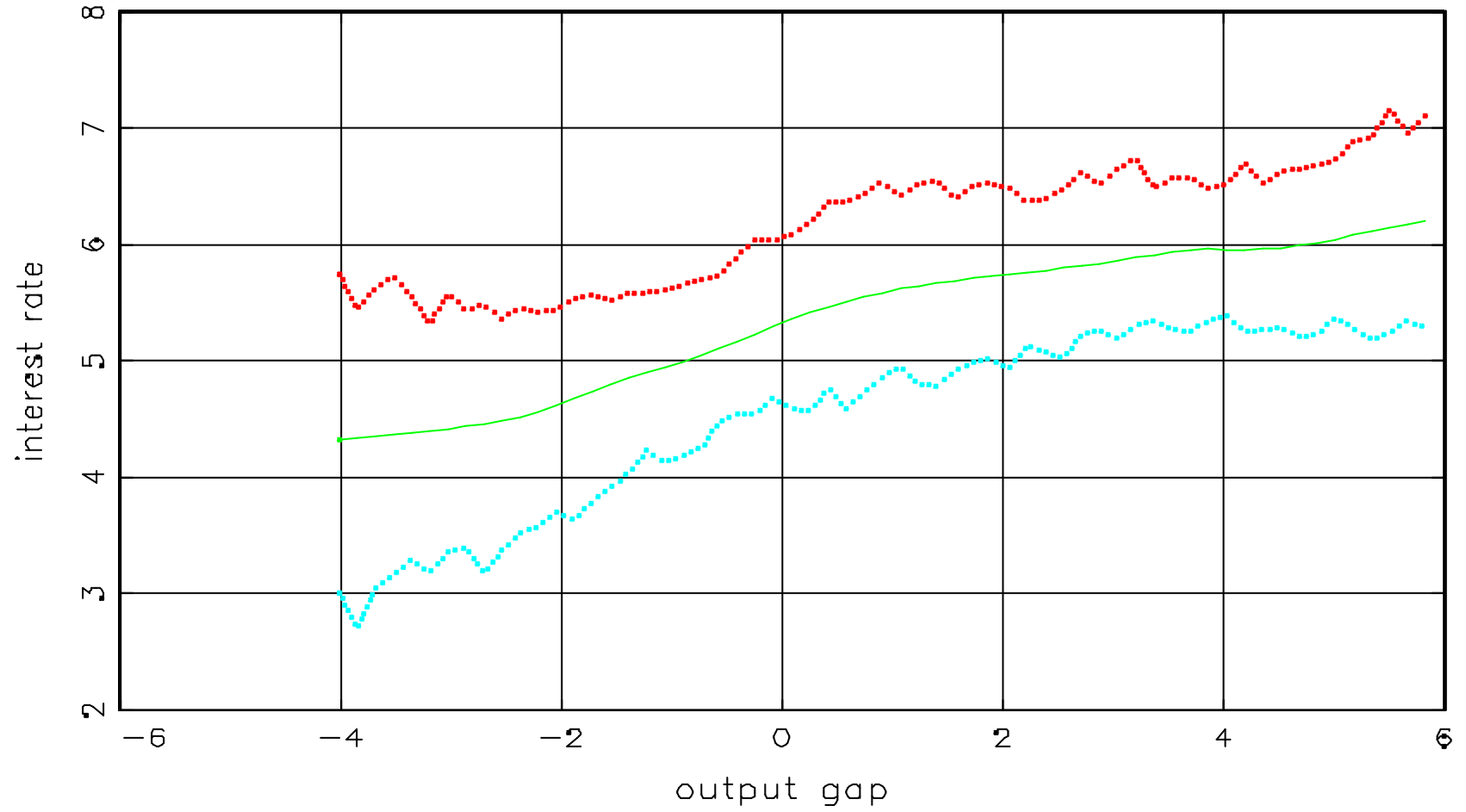Figure 2a. The effect of output gap
on the interest rate: FL, CBO

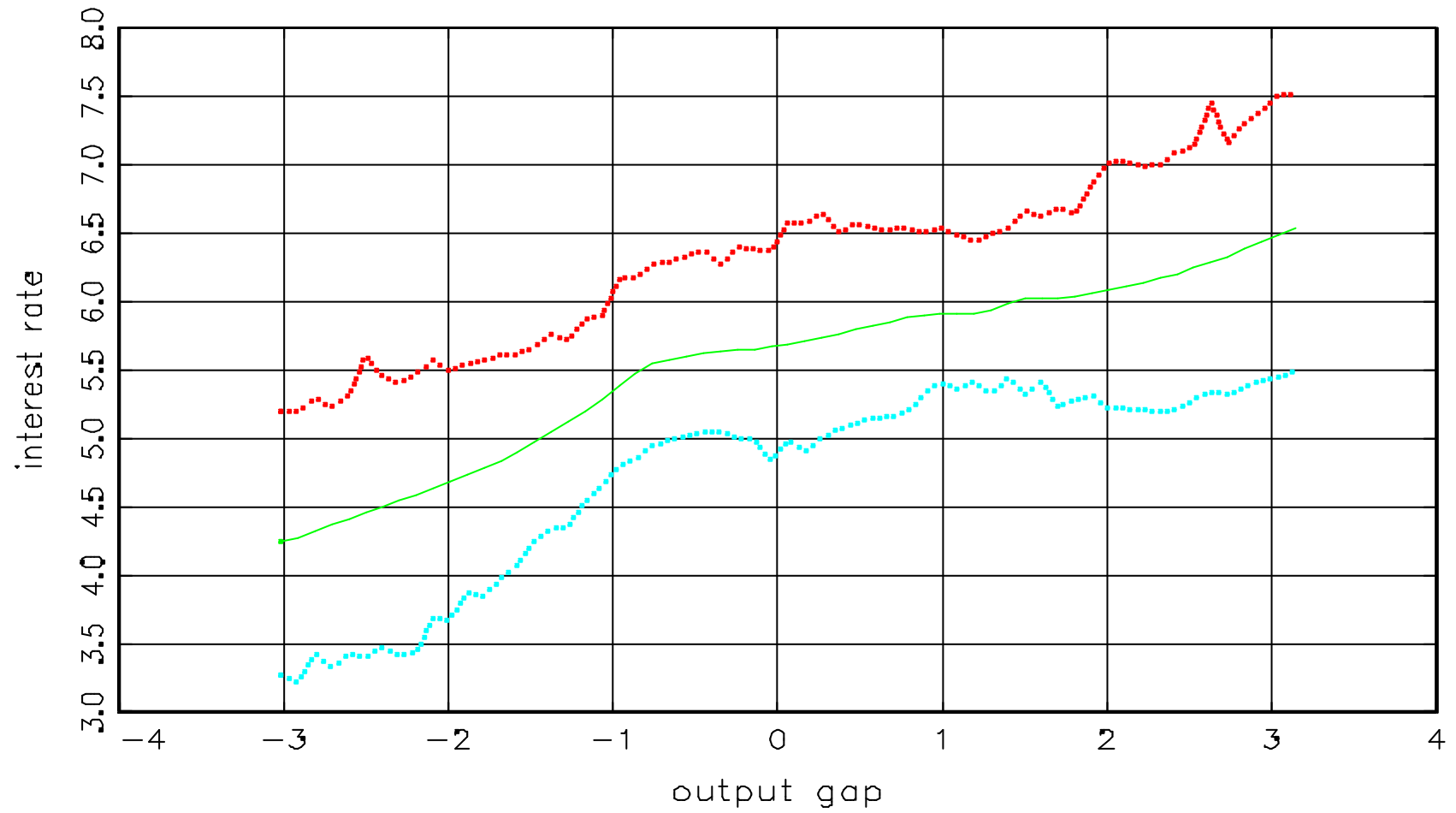Figure 2b. The effect of output gap
on the interest rate: FL, HP

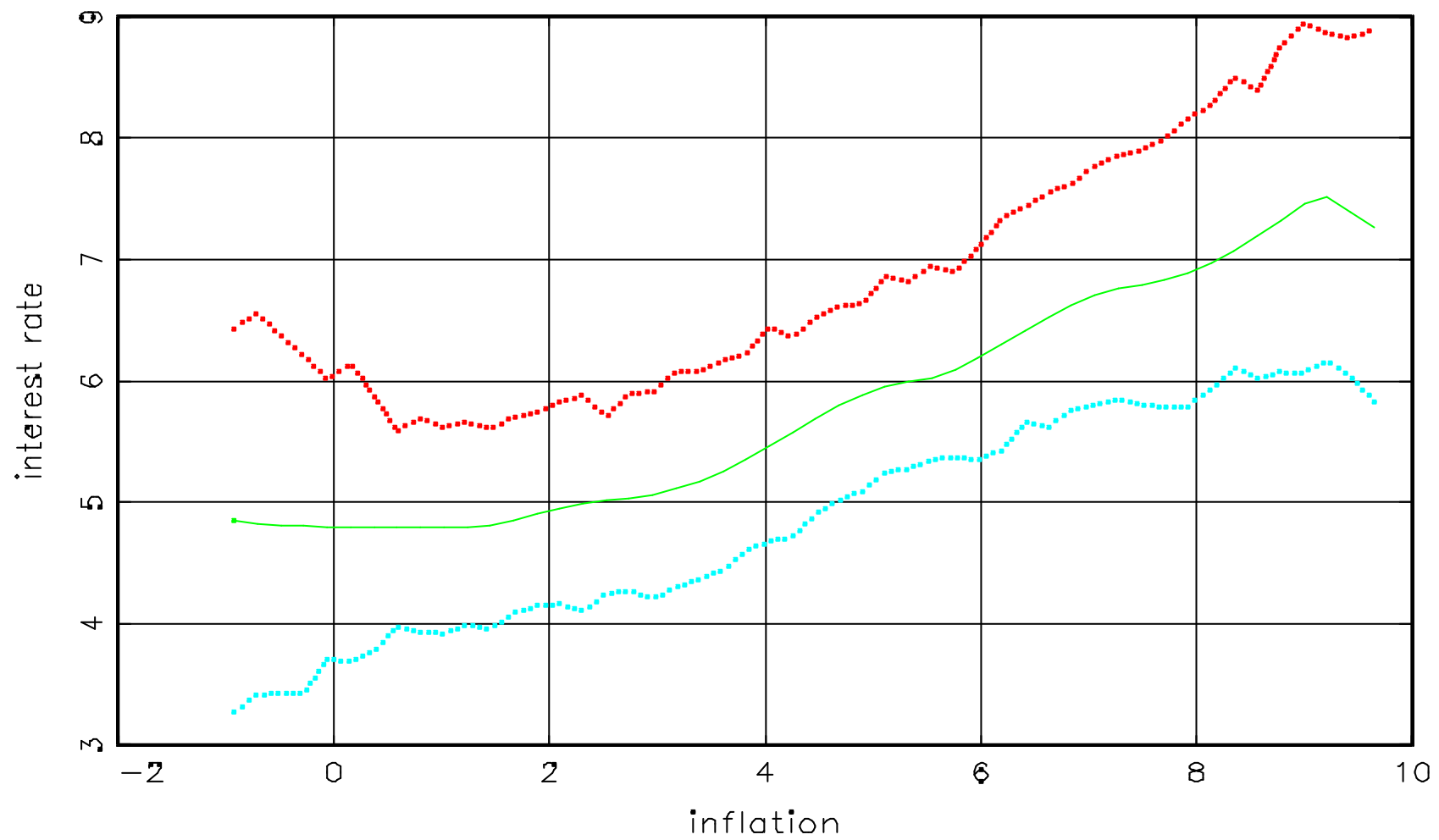Figure 3a. The effect of inflation on the interest rate: BL, CBO

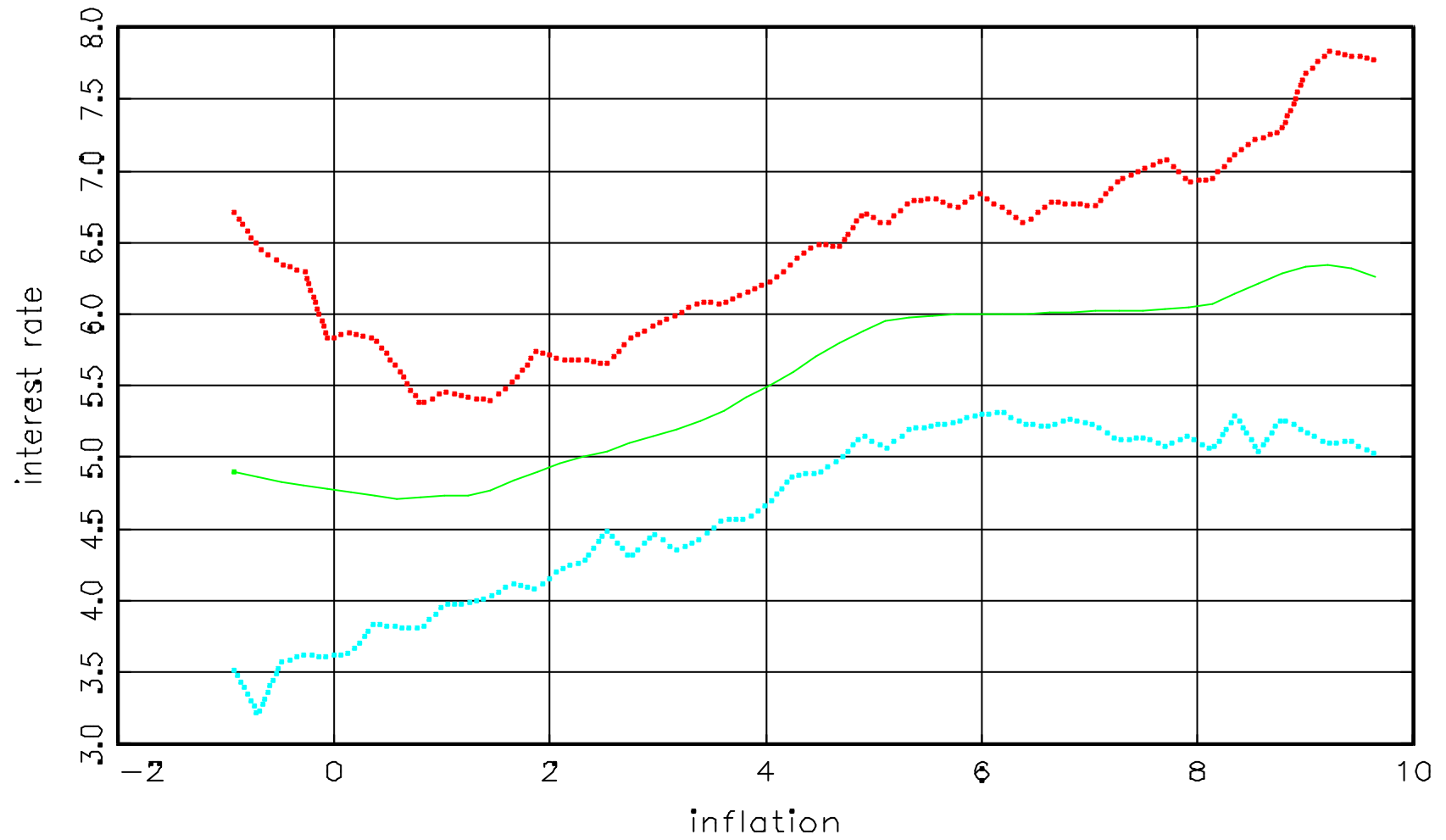Figure 3b The effect of inflation on the interest rate: BL, HP

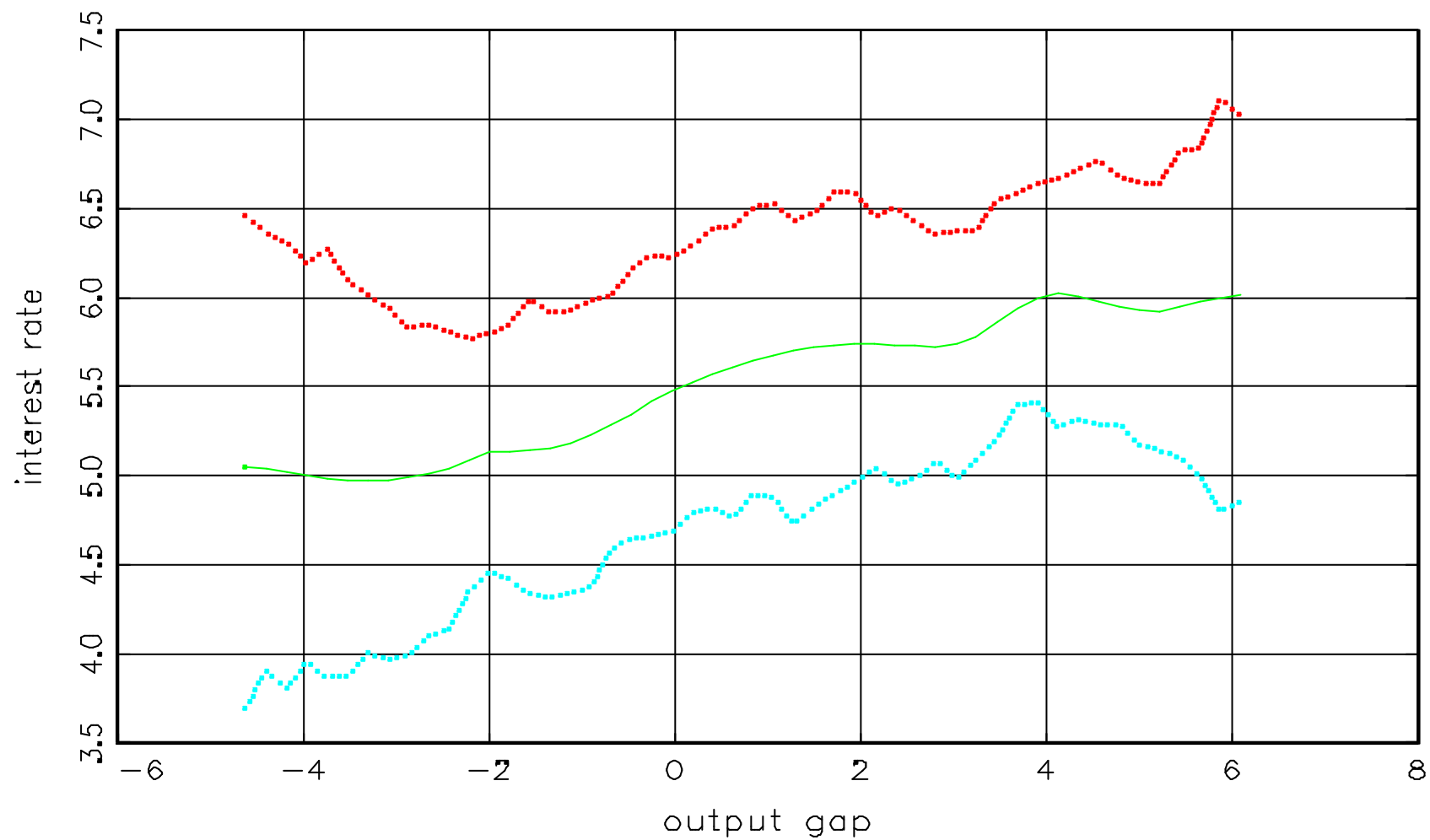Figure 4a The effect of output gap on the interest rate: BL, CBO

Figure 4b The effect of output gap on the interest rate: BL, HP