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Credit Frictions, Collateral and the Cyclical Behaviour of the Finance Premium

By

P.R. Agénor, G.J. Bratsiotis, and D. Pfajfar

Centre for Growth and Business Cycle Research, Economic Studies,
University of Manchester, Manchester, M13 9PL, UK

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Credit Frictions, Collateral and the Cyclical Behavior of the Finance Premium

Pierre-Richard Agénor[*], George J. Bratsiotis[*], and Damjan Pfajfar[**]

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Abstract

This paper examines the behaviour of the finance premium following technology and monetary shocks in a Dynamic Stochastic General Equilibrium (DSGE) model where borrowers use a fraction of their production (output) as collateral. We show that this simple framework is capable of producing a countercyclical finance premium, while matching the macro dynamics of well-documented stylized facts. A key feature is the endogenous derivation of the default probability from break even conditions, that results in the loan rate being set as a countercyclical finance premium over the cost of borrowing from the central bank. The latter is shown to provide an accelerator effect through which shocks can amplify the loan spread and the dynamic response of macro variables.

JEL Classification Numbers: E31, E44, E52.

Keywords: finance premia; banks; collateral; asymmetric information; loan spreads.

[*]School of Social Sciences, University of Manchester, United Kingdom, and Centre for Growth and Business Cycle Research; [**]European Banking Center, CentER, Department of Economics, Faculty of Economics and Business Administration, University of Tilburg, P.O. Box 90153, NL-5000 LE Tilburg, Netherlands. We are grateful to Joshua Aizenman, Koray Alper, Chris Martin, and especially Roy Zilberman for excellent assistance. We also thank seminar participants for helpful discussions and comments on a previous draft. The views expressed in this paper are our own.

1 Introduction

Much of the recent research in Dynamic Stochastic General Equilibrium (DSGE) models with credit market imperfections focuses on the role of the finance premium and its dynamic behaviour in business fluctuations. A substantial amount of empirical evidence indicates to a *countercyclical* finance premium (see De Graeve 2008, Nolan and Thoenissen 2009, Gerali, et al 2010, Aliaga-Díazand Olivero 2011, etc.).[1] In general, in order to explain the behaviour of the finance premium the literature focuses on the borrowers' net worth. Yet, the assumptions made about the borrowers' net worth can result in opposing theoretical conclusions. In the Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999) models, the finance premium is countercyclical, whereas in the Carlstrom and Fuerst (1997) model the finance premium is procyclical. In the latter paper, the absence of entrepreneurial capital stock implies that shocks that increase the price of capital have no direct impact on borrowers' net worth and this results in a procyclical finance premium.[2] This result is also reached by others that build on the Carlstrom-Fuerst model, such as Gomes, Yaron and Zhang (2003) and De Fiore and Tristani (2009), who also produce a procyclical finance premium following a monetary shock. Faia and Monacelli (2007), also build on the Carlstrom-Fuerst model, but modify the behaviour of the probability of default in order to obtain a countercyclical finance premium. As the production of intermediate goods, in their model, is separated from the production of capital, with only the latter engaging into borrowing, the behaviour and productivity of the intermediate goods firms, do not affect the probability of default. A critical assumption therefore in Faia and Monacelli (2007) is to assume that the mean distribution of investment outcomes across capital producing entrepreneurs depends also positively on the state of aggregate productivity. This implies that a higher aggregate productivity and output, raise also, the income of borrowers, thus reducing the probability of default and producing a countercyclical risk premium.

This paper makes a simple point. Using a standard DSGE model we show that when a fraction of total production (or final output) can serve as the borrower's collateral, the model is capable of producing a countercyclical finance premium, following both technology and monetary policy shocks, while matching the macro dynamics of well-documented stylized facts, without further assumptions about the borrower's net worth, as those employed elsewhere the literature. Intuitively, most of the collateral that may

[1] As with most of the macro literature by *countercyclical* we refer to a falling premium as output rises, unlike some other literature that refers to the opposite effect.

[2] For a detailed comparison of these two models, see Walentin (2005).

serve as net worth (i.e. land, capital, or other fixed property) is reflected in the firm's production technology and hence, in equilibrium, in its final output. However, even in the case of a very simple labour-based technology, as employed for simplicity here, final (or future) output can also act as collateral, as often employed in farming, natural resources and other sectors. Building on Townsend (1979), Agénor and Aizenman (1998), and Aoki, Benigno and Kiyotaki (2010), we assume that firms use a fraction of their final (or future) output as collateral.[3] We employ and investigate this assumption within a DSGE model with sticky wages, where borrowing to cover labour costs is required by all goods producing firms. The risk in this model is born and assessed by the lender and it is reflected in the finance premium required over the risk-free refinance rate. We show that there is a probability of default that can be endogenously determined by the cut-off value of an idiosyncratic productivity shock. Using this and a break-even condition for banks, we derive a *countercyclical* finance premium over the risk-free refinance rate. The latter, in this model, is not the result of an assumption made about the mean distribution of investment outcomes. With all output producing firms engaging in borrowing, and with borrowing decisions made before shocks are realized, any innovation that reduces final output also reduces the real value of collateral that the lender receives in case of default. This means that falls in output endogenously increase the probability of default and raise the finance premium, thus generating a countercyclical finance premium. The finance premium is endogenously affected by the default probability, which in this model is itself a function of the loan rate and hence the finance premium. Through the latter effect, technology and monetary shocks amplify the spread between the loan and finance risk-free rates, thereby generating an accelerator effect.

The remainder of the paper is organized as follows. Section 2 presents the model, whereas section 3 discusses its steady state and equilibrium properties. Section 4 calibrates the model under technology and monetary shocks and section 5 concludes.

[3] Note that in Townsend's (1979) seminal work on the costly state verification framework, the realized value of bank-financed investment projects (which are all subject to an idiosyncratic shock) can be seized in case of default. In Agénor and Aizenman (1998) working capital must be paid for prior to production and output is subject to an idiosyncratic productivity shock. In that model, realized output is in a sense "produced" by loans, just as project outcomes depend on bank financing in Townsend's framework. Here we go a step further by interpreting the fraction of actual output that lenders can seize in case of default in the Agénor-Aizenman framework as ex post collateral that can be pledged by borrowers.

2 The Model

We consider an economy where a continuum of identical firms, $j \in (0, 1)$, use the labour services of all existing household types, $i \in (0, 1)$, to produce differentiated consumption goods. Households supply labour to firms, consume goods from all firms and at the end of each period receive profits from firms and banks. Firms need to cover labour costs entirely by borrowing but their production is subject to aggregate and idiosyncratic productivity shocks. The credit market is represented by a competitive commercial bank that receives deposits from households, borrows from the central bank and extend loans only to firms. Asymmetric information implies that firms must pledge a fraction of output as collateral. Deposit and loan rates are derived based on arbitrage conditions.

The timeline of the model is as follows. First, the bank receives deposits from households and liquidity borrowed from the central bank and makes decisions on its lending rate, subject to the idiosyncratic nature of its borrowers (firms) and the cost of borrowing from the central bank. Then shocks are realized, final goods become available and employment, loans and prices adjust by taking the going interest rate and loan rates as given. Only a fraction of firms can adjust their prices, as the rest are assumed to keep prices fixed in a Calvo-fashion. In the end of each period, loans must also be repaid and profits are distributed to households.

2.1 Households

The objective of household i is to maximize,

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s})^{1-1/\sigma}}{1-1/\sigma} - \eta_N \frac{(h_{i,t+s})^{1+\gamma}}{1+\gamma} \right), \tag{1}$$

where E_t is the expectations operator conditional on information available at t. C_t is aggregate consumption; $h_{i,t}$ is working time by household type i; $\beta \in (0, 1)$ a subjective discount factor, and σ, η_N, $\gamma > 0$. The household's budget constraint is,

$$P_t C_t + D_{t+1} = (1 + i_t^D) D_t + W_{i,t} h_{i,t} - T_t, \tag{2}$$

where i_t^D is nominal interest on deposits, D_t; $W_{i,t}$ is the nominal wage rate paid to household i; and T_t is a lump-sum tax.[4] The consumption index is, $C_t = (\int_0^1 C_{j,t}^{(\theta_p-1)/\theta_p} dj)^{\theta_p/(\theta_p-1)}$, where $C_{j,t}$ is the consumption of product j and $\theta_p > 1$. The demand for each differenti-

[4]Note that our analysis below assumes that commercial banks break even, hence for simplicity we assume zero profits accruing to households from the credit market.

ated good is, $C_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} C_t$, where the average price index, $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj\right)^{1/(1-\theta_p)}$. The first-order conditions of the above problem are,

$$C_t = E_t \left(\beta C_{t+1}^{-1/\sigma} (\frac{1+i_t^D}{1+\pi_{t+1}}) \right)^{-\sigma}, \tag{3}$$

$$\lim_{s\to\infty} E_{t+s}(1+i_{t+s}^D)^{-1} D_{t+s} = 0, \tag{4}$$

where $\pi_{t+1} = (P_{t+1} - P_t)/P_t$.

2.1.1 The Wage Setting

The wage setting follows a variant of Erceg, Henderson and Levin, (2000) and Smets and Wouters (2002), where each household i supplies a unique type of labour, $h_{i,t}$ but all types of labour are aggregated by a competitive labour contractor into one composite homogenous labour, $N_t = (\int_0^1 h_{i,t}^{\frac{\theta_w - 1}{\theta_w}} di)^{\frac{\theta_w}{\theta_w - 1}}$, where $\theta_w > 1$. The i^{th} household therefore faces the following demand curve for its labour, $h_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} N_t$, where W_t denotes the aggregate nominal wage paid for one unit of the composite labour, N_t, used in the production of each firm, $W_t = \left(\int_0^1 W_{i,t}^{1-\theta_w} di\right)^{\frac{1}{1-\theta_w}}$. In each period a constant fraction of $1 - \omega_w$ workers are able to re-optimize their wages while a fraction of ω_w index their wages according to last period's inflation rate, i.e. $W_{i,t} = \pi_{t-1} W_{i,t-1}$. From (1) and the above problem, the wage inflation is derived as[5],

$$\widehat{\pi}_t^W = \beta E_t \widehat{\pi}_{t+1}^W + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma_N \theta_w)} \left(\widehat{MRS}_{(C,N),t} - \widehat{W}_t^R \right), \tag{5}$$

where $MRS_{(C,N),t} = \eta_N N_t^{\gamma} C_t^{1/\sigma}$ and the real wage is defined as,

$$\widehat{W}_t^R = \widehat{W}_{t-1}^R + \widehat{\pi}_t^W - \widehat{\pi}_t, \tag{6}$$

where, $\widehat{\pi}_t$ is the log-linearized inflation rate as a deviation from its steady state.

2.2 Firms

The production of each firm relies on the labour services provided by the competitive labour contractor, but it is also subject to aggregate technology and idiosyncratic shocks,

$$Y_{j,t} = N_{j,t} Z_{j,t}, \qquad Z_{j,t} = A_t \varepsilon_{j,t}, \tag{7}$$

[5] The full derivation of the wage setting is provided in the Appendix.

where $N_{j,t}$ denotes the amount of labour services hired by firm j, at the nominal aggregate wage W_t; $Z_{j,t}$ is the total level of productivity of firm j; A_t is an aggregate technology shock and $\varepsilon_{j,t}$ is an idiosyncratic productivity uniform shock, distributed over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$. As employed elsewhere in the literature, the assumption of the uniform distribution is to pin down the cut-off point (see Faia and Monacelli 2007). The aggregate technology shock evolves in the conventional autoregressive process, $\log A_t = \left(1 - \rho^A\right) \log A + \rho^A \log A_{t-1} + \epsilon_t^A$, where, $\rho^A > 0$ and $A > 0$ is the steady state aggregate productivity level; ϵ_t^A is a normally distributed random shock with zero mean and a constant variance, σ_A.

Firms borrow from the bank to cover their expected wage costs, $W_t N_{j,t}$, at the gross nominal interest rate $1 + i_t^L$, and repay their loans at the end of each period. Let $L_{j,t}$ denote the nominal amount of borrowing by firm j at time t, the financing constraint in real terms is thus,[6]

$$L_{j,t}^R = W_t^R N_{j,t}, \tag{8}$$

where $L_{j,t}^R \equiv L_{j,t}/P_t$. The collateral that the bank can seize in case of default consists of a fraction, $\chi \in (0, 1)$, of the firm's production (i.e. final output), net of state verification and contract enforcement costs. Consequently, a firm will choose to default if

$$\chi Y_{j,t} < (1 + i_t^L) L_{j,t}^R, \tag{9}$$

where the left-hand side is firm j's actual repayment following a default, whereas the right-hand side is the contractual repayment, expressed in real terms. Let $\varepsilon_{j,t}^M$ be the cut-off point, below which default occurs; that is, the value of $\varepsilon_{j,t}$ for which (9) holds as an equality. Using (7) yields,

$$\chi(A_t \varepsilon_{j,t}^M) N_{j,t} = (1 + i_t^L) L_{j,t}^R, \tag{10}$$

Using (8), holding with equality, this expression can be rewritten as,

$$\varepsilon_{j,t}^M = \frac{(1 + i_t^L) W_t^R}{\chi A_t}. \tag{11}$$

[6] In a recent paper, Jermann and Quadrini (2012), assume that working capital includes not only payments to workers, but also payments to suppliers of investments, shareholders and bondholders.

2.3 Financial Intermediation

At the beginning of each period, the bank receives deposits from households and additional liquidity borrowed from the central bank, L_t^B, at the going refinance rate, i_t.[7] Assuming no required reserves, the (aggregate) balance sheet of the bank is, $L_t = D_t + L_t^B$,where $L_t = \int_0^1 L_{j,t}dj$. Perfect competition in the deposits market (and no required reserves) implies, $i_t^D = i_t$.

We next turn to the derivation of the lending rate, i_t^L. The bank faces the risk of default as firms' final output (i.e. at the end of period t), is subject to random shocks, hence contractual repayments are uncertain. A loan contract specifies a premium-inclusive lending rate that is set as a break-even condition. Specifically, this condition requires that in equilibrium the expected income from lending to firm j, is equal to the cost of borrowing these funds from the central bank at the given marginal cost, i_t. Let, $E_t S_t$ be the expected income from lending $L_{j,t}$, the break-even condition is,

$$E_t S_t = (1 + i_t) L_{j,t}, \tag{12}$$

To derive the finance premium, ϱ_t^L, that satisfies (12), we recall that in the event of default the bank seizes a fraction, χ, of the realized value of the firm's output, thus it receives a net repayment, $\chi Y_{j,t}$ (see eq. 9). We can thus write the expected income from loans in real terms as,

$$E_t S_t^R = \int_{\varepsilon_{j,t}^M}^{\bar{\varepsilon}} [(1 + i_t^L) L_{j,t}^R] f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [\chi Y_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}, \tag{13}$$

where $f(\varepsilon_{j,t})$ is the density function of $\varepsilon_{j,t}$. Equation (13) can be rewritten as,

$$E_t S_t^R = (1 + i_t^L) L_{j,t}^R - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [(1 + i_t^L) L_{j,t}^R - \chi Y_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}. \tag{14}$$

Substituting (11) for $(1 + i_t^L) L_{j,t}^R = \chi(A_t \varepsilon_{j,t}^M) N_{j,t}$, in the second term on the right-hand side of the above equation, we obtain,

$$E_t S_t^R = (1 + i_t^L) L_{j,t}^R - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [(\varepsilon_{j,t}^M - \varepsilon_{j,t}) \chi A_t N_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}. \tag{15}$$

[7] The additional liquidity borrowed from the central bank, here is equivalent to the assumption of an exogenous cash injection of $M_t - M_{t-1}$ in Ravenna and Walsh (2006). Here we assume that excess liquidity is covered by a nominal lump sum tax, i.e. $L_t^B = T_t$. This, together, with the bank's balance sheet, where at equilibrium $L_t = W_t N_t$, determines also the level of deposits, D_t.

Substituting the break-even condition (12) in real terms, $E_t S_t^R = (1 + i_t) L_{j,t}^R$, into (15), and dividing through by $L_{j,t}^R$ we obtain the loan rate as,[8]

$$i_t^L = i_t + \varrho_t^L, \tag{16}$$

where the *finance premium* is,

$$\varrho_t^L = \frac{\chi A_t N_t \int_{\underline{\varepsilon}}^{\varepsilon_t^M} (\varepsilon_t^M - \varepsilon_t) f(\varepsilon_t) d\varepsilon_t}{L_t^R}.$$

To obtain further insight, we use the properties of the idiosyncratic productivity shock. ε_t follows a uniform distribution over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$; its probability density therefore is $1/(\bar{\varepsilon} - \underline{\varepsilon})$ and its mean $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2$. Under these assumptions ϱ_t^L simplifies to,

$$\varrho_t^L = \left(\frac{\chi A_t}{W_t^R} \right) \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \Phi_t^2, \tag{17}$$

where $\Phi_t \in (0, 1)$ is the probability of default,

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \left(\frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right). \tag{18}$$

Thus, the loan rate is set as a premium over the going refinance rate. The finance premium is determined by the *ratio* of the size of real revenue lost in times of default (that is, for realizations of ε_t less than ε_t^M), to the real value of total loans (see also Bernanke Gertler and Gilchrist 1999). Specifically, the finance premium is shown to be a function of the expected size of loans, the size of χ and a quadratic function of the probability of default, Φ_t, that is itself determined (i.e. through ε_t^M) by the size of χ. As implied by (11), a higher fraction of collateral, χ, increases the cost of default, thereby reducing the frequency of defaults (hence ε_t^M); this reduces the default probability and the lending rate. If there is no default risk ($\Phi_t = 0$), the premium is zero, and $i_t^L = i_t$.

2.4 The New Keynesian Phillips Curve

Once the state of the economy is revealed, at the end of time t, each firm has a Calvo-type constant probability, ω_p, of keeping its price fixed at the previous period's price and a constant probability of $1 - \omega_p$ of adjusting to the new optimal price based on the new real marginal cost and treating the loan rate as given, (see eq. 16). Total real cost is,

[8] Because from (6) and (11) the size of $\varepsilon_{j,t/t-}^M$ depends only on χ, A_t, and W_t^R, and thus it is the same for all firms, in what follows we drop the subscript j.

$(1+i_t^L)W_t^R N_{j,t}$ and real marginal cost is, $mc_t^R = \frac{(1+i_t^L)W_t^R}{Z_{j,t}}$. Given this and constant returns to scale the firms' maximization problem can be expressed as,

$$E_t \sum_{s=0}^{\infty} \omega_p^s \Delta_{s,t+s} (\frac{P_{j,t}}{P_{t+s}} Y_{j,t+s} - mc_{t+s}^R Y_{j,t+s}), \tag{19}$$

where $\Delta_{s,t+s} = E_t \beta (C_{t+s}^{-1/\sigma} / C_t^{-1/\sigma})$, is the stochastic discount factor between time s and $t+s$. From (19), and taking the loan rate as given, the NKPC is

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \widehat{mc}_t^R, \tag{20}$$

where, $\widehat{x}$, is the log-linearized x, as a deviation from its steady state; $\lambda = (1-\omega_p)(1-\omega_p \beta)/\omega_p$, and $\widehat{mc}_t^R = mc(\widehat{i}_t^L, \widehat{\Phi}_t, \chi, shocks)$. If $\widehat{\Phi}_t = 0$, then $i_t^L = i_t$, and this corresponds to the standard cost channel of monetary policy, (see Ravenna and Walsh 2006). However, when $\widehat{\Phi}_t > 0$, technology and monetary shocks affect real marginal cost through multiple channels, as shown below.

2.5 Monetary Policy

Monetary policy is conducted through a conventional Taylor-type interest rate rule,

$$\widehat{i}_t = \rho^i \widehat{i}_{t-1} + (1-\rho^i)(\phi_y \widehat{Y}_t + \phi_\pi \widehat{\pi}_t) + \epsilon_t^i, \tag{21}$$

where, $\rho^i \in (0,1)$ captures the degree of interest rate smoothing; $\phi_y, \phi_\pi > 0$ are policy parameters and ϵ_t^i is a normally distributed random shock with zero mean and a constant variance, σ_i.

3 Equilibrium and Steady State

At equilibrium, markets for labour, goods, deposits, and credit must clear, thus at the macroeconomic equilibrium, aggregate output must be equal to aggregate consumption, $Y_t = C_t$. Note that the small fraction of output lost in times of default is already incorporated in these variables. This is because collateral in this model is already given as a fraction of output, and thus the level of output, and hence consumption, are endogenously affected by size of collateral and the probability of default.

At the steady state the deposit rate is, $i^D = i = \frac{1}{\beta} - 1$; the price and wage mark-ups are respectively, $\vartheta_p = \frac{\theta_p}{\theta_p - 1} = \frac{1}{mc} > 1$ and $\vartheta_w = \frac{\theta_w}{\theta_w - 1} > 1$; the marginal cost is, $mc = \frac{W}{P} \frac{(1+i^L)}{Z}$, where $\frac{W}{P} = \vartheta_w N^{\gamma} C^{\frac{1}{\sigma}}$ and $Z = A\mu_{\varepsilon}$; $\mu_{\varepsilon} = (\bar{\varepsilon} + \underline{\varepsilon})/2$, and the steady state loan rate is, $i^L = i^D + \frac{1}{2}\left(\frac{\chi\bar{\varepsilon}}{W/P}\right)\Phi^2$. From equations (11) and (18) for and taking into account that the steady state real price is unity, we obtain, $\Phi = \left(\frac{\varepsilon^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}\right)$ where $\varepsilon^M = \frac{\mu_{\varepsilon}}{\vartheta_p \chi}$.[9] Hence, the steady state probability of default is positive and depends endogenously on the mean productivity level of the firm, (i.e. μ_{ε}), the size of the firm's price markup, ϑ_p, and the degree of credit market imperfections, as measured by χ. Note, that unlike Faia and Monacelli (2007), the mean productivity of the idiosyncratic shock here is independent of A, and it is determined purely by the properties of the uniform distribution, i.e. $\mu_{\varepsilon} = (\bar{\varepsilon} + \underline{\varepsilon})/2$.

4 Parameterization and Simulations

To simulate the model we use the well-established parameterization proposed by Christiano Eichenbaum and Evans (2005), also used in Smets and Wouters (2003).

Table 1: Parameters of the Base Model

Parameters	Value	Description
β	0.99	Discount factor
γ	2.5	Inverse of the Frisch elasticity of labour supply
η_N	1.0	Labour Elasticity in utility function
σ	0.5	Consumption elasticity in utility function
θ_p	6.0	Price elasticity of product demand
θ_w	21.0	Wage elasticity of labour demand
ω_p	0.75	Price rigidity in Calvo price setting
ω_w	0.8	Wage rigidity in Calvo wage setting
A	1.0	Productivity Parameter
$\underline{\varepsilon}$	1.0	Idiosyncratic productivity shock lower range
$\bar{\varepsilon}$	1.35	Idiosyncratic productivity shock upper range
χ	0.97	Proportion of output seized in case of default
ρ^i	0.8	Degree of interest rate smoothing
ϕ_{π}	1.5	Interest rate response to inflation
ϕ_y	0.1	Interest rate response to output
ρ^A	0.8	Persistence in technology shock

As similar work and data on the cost of bankruptcy parameter, χ is limited, we assume that the fraction of actual output seized by the bank in case of default is $\chi = 97\%$;

[9] Note that here we have used the fact that at the symmetric price equilibrium, $mc = 1/\vartheta_p$.

that is we assume that an equivalent amount of 3% of output is spent in monitoring costs, verification costs, legal procedures etc.; however, we also consider a lower value of, $\chi = 0.85$. Note that for a typical value of a price markup, $\vartheta_p = 1.2$, and $\chi = 0.97$, and choosing, as in the rest of the literature, a value for the range of the idiosyncratic shock, $\underline{\varepsilon} = 1$, and $\bar{\varepsilon} = 1.35$, such that the model produces reasonable values, the steady-state value of the probability of default is $\Phi = 0.027$, or around 3%. This is very similar to the values assumed elsewhere in the literature. For example, Faia and Monacelli (2007) calibrate their model to generate an average bankruptcy rate of 3%, whereas Nolan and Thoenissen (2009) also assume that the probability of survival in business is approximately 97%.

4.1 Technology Shocks

Figure 1, shows the impulse responses to a 0.25% negative technology shock (ϵ_t^A), under the base model (Table 1), with $\chi = 0.97$ (solid line) and $\chi = 0.85$ (dotted line).

[Figure 1. Impulse Responses to a Technology Shock]

The fall in productivity reduces initially output and raises inflation and nominal interest rates, while as prices rise faster than nominal wages, real wages fall gradually. This response is consistent with much empirical evidence, (see Smets and Wouters 2003). In addition, the fall in productivity and output, in this model, reduces the size of collateral and this raises the probability of default. The latter effect is reflected on a higher lending rate, driven by a *countercyclical* finance premium over the risk-free refinance rate. The latter spread is gradually eliminated as output starts rising and the probability of default starts to fall. The *countercyclical* behaviour of the finance premium is also supported by a number of studies, (see above). Employment is shown to be procyclical, hence as output falls and the cost of borrowing increases (i.e. due to the higher default probability and loan spread), the demand for employment falls and so is the demand for loans; this is also in line with much empirical evidence. Note that with $\chi = 0.85$, the above effects are amplified. There is a higher probability of default, which is shown by a marked increase in the loan spread which is followed by a rise in inflation and a higher interest rate. The higher cost of borrowing increases the marginal cost, reduces output and employment and thus the demand for loans.

4.2 Monetary Shocks

Figure 2, shows the impulse responses to a 0.25% monetary shock (ϵ_t^i), using the base model with $\chi = 0.97$ and $\chi = 0.85$. Monetary shocks have a twofold effect in this model: (a) through the standard cost channel of monetary policy, that is the direct effect of i_t on i_t^L and (b) through the endogenous effect that a change in i_t has on i_t^L through the probability of default. Therefore monetary shocks here affect the probability of default and thus the borrowing constraint of the firm through various channels.[10] In particular, the fall in demand resulting from a higher refinance rate, results in a sharp fall in output, (see also Jermann and Quadrini 2012). The shock also reduces employment and wage inflation, but initially, and in line with the *price puzzle*, it raises the inflation rate as a result of both effects (a) and (b) above.[11] In line with much empirical evidence, prices respond much more sluggishly than output and the inflation rate remains positive for about 3-5 quarters before it becomes negative, (see Smets and Wouters 2003). As nominal wages catch up with prices, real wages start gradually to return to their steady state. The fall in output raises the probability of default and hence the lending rate, thus generating again a *countercyclical* finance premium.

[Figure 2. Impulse Responses to a Monetary Shock]

A lower χ raises the probability of default and, as with the technology shock, this raises the lending rate and the loan spread. In the case of a monetary shock this raises (through the effects (a) and (b) above), the real marginal cost which also raises the inflation rate. As expected, the higher cost of borrowing results in a lower demand for employment and loans.

5 Concluding Remarks

Using a simple DSGE framework, where a fraction of output is pledged by borrowing firms as collateral, we show how to derive, from break-even conditions, a loan rate that is driven by a countercyclical finance premium over the risk-free refinance rate. The finance premium is shown to be affected by the probability of default, both directly, but

[10] Note that in Jermann and Quadrini (2012), this is done through a stochastic financial shocks that affects directly the enforcement constraint.

[11] Note that in this model, the cost channel effect is strong and as a result the inflation rate is increasing substantially. This is because of the assumption that all production is based on borrowing and that all firms need to borrow all their working capital (i.e. all financing is external).

also through the way that the default probability is itself a function of the loan rate, the refinance rate and the firm's productivity. Thus the finance premium provides an accelerator effect through which both monetary and technology shocks can amplify the loan spread. The response of the macro variables to such shocks appears to be consistent with much of the established empirical literature, including the *price puzzle*. Similarly, the finance premium is countercyclical and causes the loan spread to peak within the first quarter following a shock and remain positive for about two years. Its dynamic behaviour therefore, is also consistent with much of the recent empirical literature on the cyclicality of loan rates, (i.e. Mojon and Peersman 2003, De Graeve 2008, Aliaga-Díaz and Olivero 2010, Gerali, Neri, Sessa and Signoretti, 2010).

Our simple framework can also be extended to account for investment, that may strengthen the role of output as collateral. As pointed out by Faia and Monacelli (2007), for example, credit frictions can have a large impact on the behaviour of investment and the price of capital.

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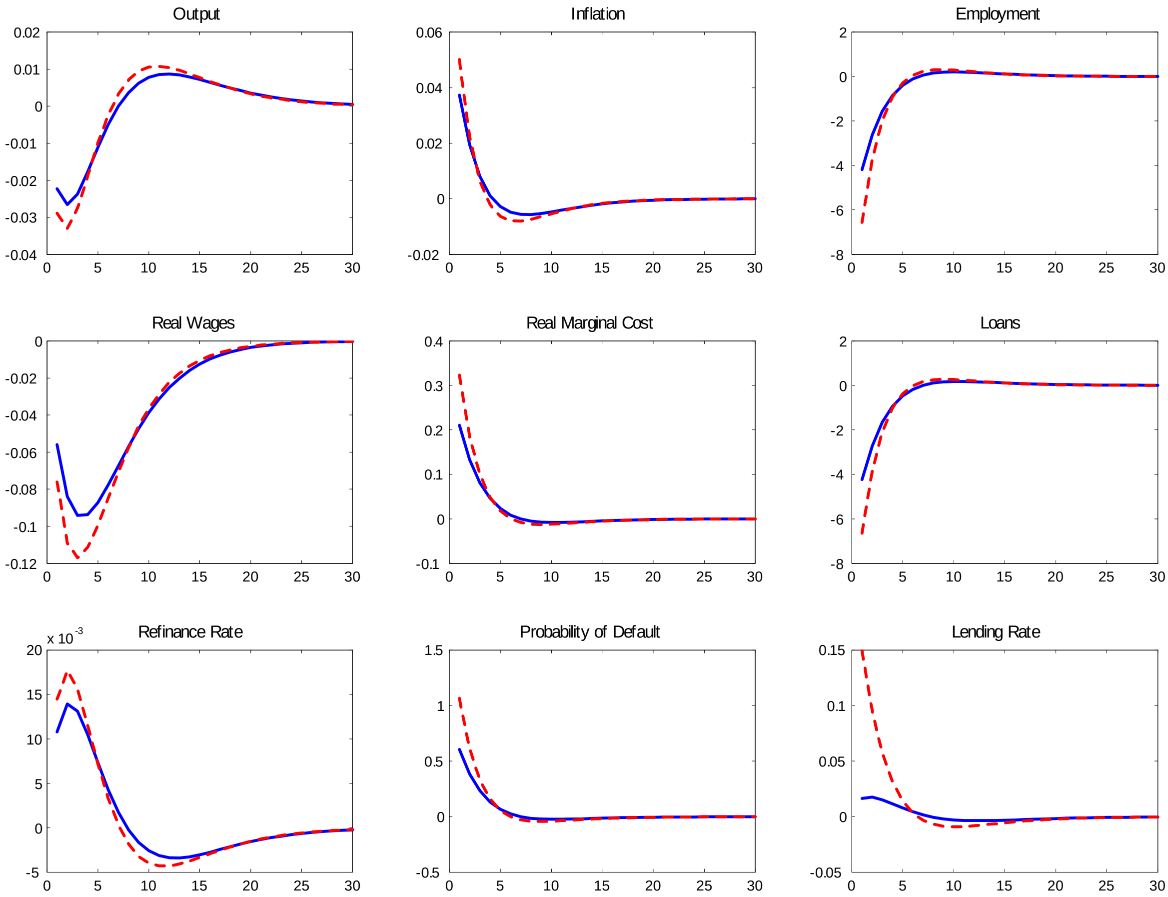

Figure 1. Impulse Responses to a Technology Shock, (— $\chi = 0.97$; - - - $\chi = 0.85$).

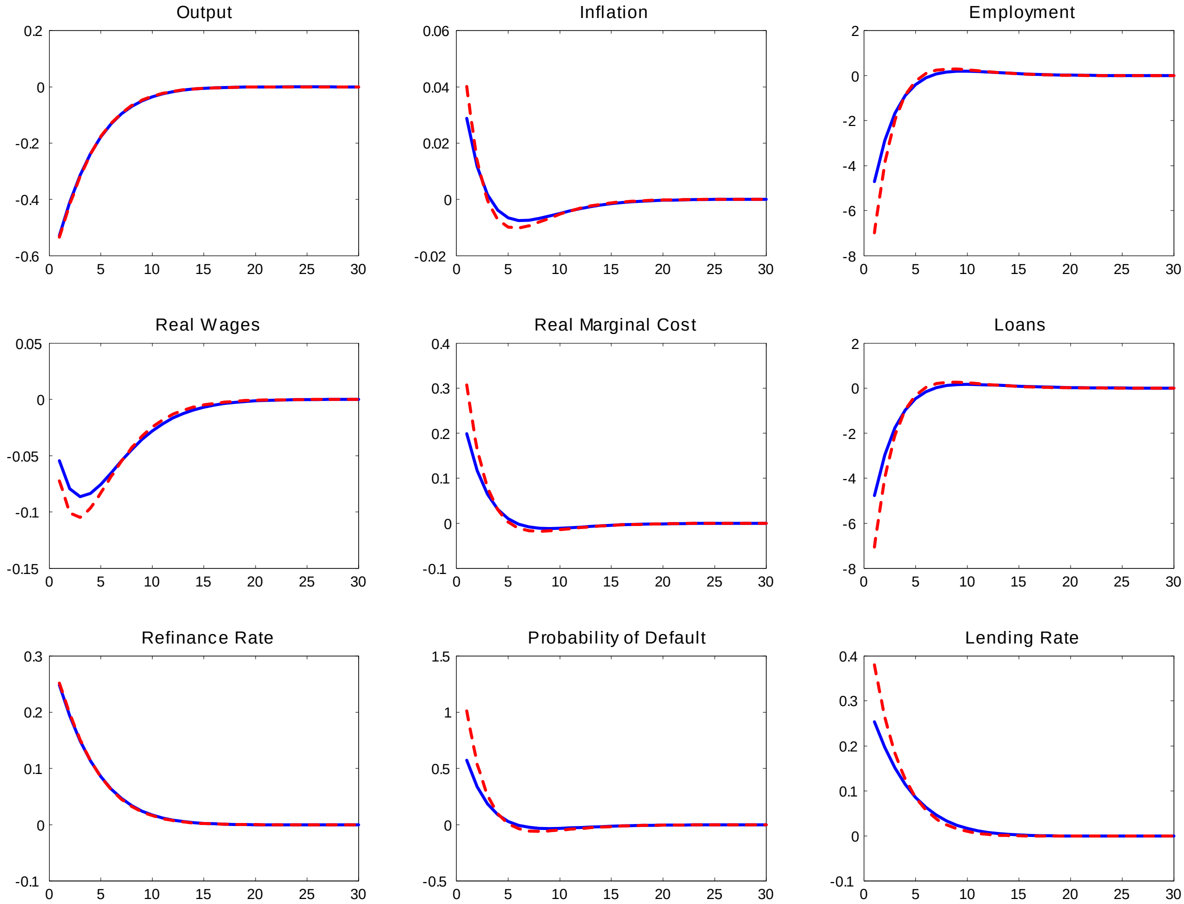

Figure 2. Impulse Responses to a Monetary Shock, (— $\chi = 0.97$; - - - $\chi = 0.85$).

Appendix (Not for Publication)

The Wage Setting

Following Erceg, Henderson and Levin, (2000) and Smets and Wouters (2002), the composite homogenous labour is,

$$N_t = \left(\int_0^1 h_{i,t}^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}, \tag{22}$$

with $\theta_w > 1$. The i^{th} household therefore faces the following demand curve for its labour,

$$h_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} N_t, \tag{23}$$

where W_t denotes the aggregate nominal wage paid for one unit of the composite labour, N_t, used in the production of each firm. Substituting (23) in (22), results in the economy wide wage equation,

$$W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} dj \right]^{\frac{1}{1-\theta_w}}. \tag{24}$$

We assume that each period a constant fraction of $1 - \omega_w$ workers are able to re-optimize their wages while a fraction of ω_w index their wages according to last period's inflation rate (π_{t-1}). These households therefore set their wages according to the following rule,

$$W_{i,t} = \pi_{t-1} W_{i,t-1}. \tag{25}$$

With indexed wages, if wages have not been set since period t, then at period $t + s$, the real wages for household j will be,

$$\frac{W_{i,t+s}}{W_{t+s}} = \frac{\Pi^s W_{i,t}}{W_{t+s}}, \tag{26}$$

where $\Pi^s = \pi_t \times \pi_{t+1} \times .... \times \pi_{t+s-1}$. Therefore, the demand for labour in period $t + s$ becomes,

$$h_{i,t+s} = \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\theta_w} N_{t+s}. \tag{27}$$

Households who set wages, maximize (1), subject to the budget constraint (2) and the demand for labour (27). From (1),

$$E_{t^+} \sum_{s=0}^{\infty} \beta^s \left[\mathbf{U}\left(C_{t+s}\right), \mathbf{V}(h_{i,t+s}) \right] = E_{t^+} \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(C_{t+s})^{1-1/\sigma}}{1 - 1/\sigma} - \eta_N \frac{(h_{i,t+s})^{1+\gamma}}{1+\gamma} \right\}$$

The first order condition with respect to $W_{i,t}$ results in,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\frac{\Pi^s W_{i,t}}{P_{t+s}} \mathbf{U}'_{C,t+s} - \frac{\theta_w}{1-\theta_w} \left(\mathbf{V}'_N \right)_{i,t+s} \right] h_{i,t+s} = 0. \tag{28}$$

If all households can re-optimize, (i.e. $\omega_w = 0$), equation (28) reduces to

$$\frac{W_t}{P_t} = -\frac{\theta_w}{\theta_w - 1}\frac{\mathbf{V}'_{N,t+s}}{\mathbf{U}'_{C,t+s}} = \frac{\theta_w}{\theta_w - 1} MRS_{(C,N),t}, \tag{29}$$

where $MRS_{(C,N),t} = \eta_N N_t^{\gamma} C_t^{1/\sigma}$. In equilibrium all re-optimizing households choose the same wage (W_t^*) and the optimal relative wage in a log-linearized form (denoted by hat) evolves according to,

$$\widehat{\left(\frac{W_t^*}{W_t}\right)} = \left(\frac{\omega_w}{1-\omega_w}\right)\widehat{\pi}_t^W, \tag{30}$$

where $\widehat{\pi}_t^W \equiv \widehat{W}_t - \widehat{W}_{t-1}$ is defined as the wage inflation. Finally, as in Erceg, Henderson and Levin, (2000) the wage inflation equation is shown to satisfy,

$$\widehat{\pi}_t^W = \beta E_t \widehat{\pi}_{t+1}^W + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma_N\theta_w)}(\widehat{MRS}_{(C,N),t} - \widehat{W}_t^R), \tag{31}$$

where the real wage is defined as,

$$\widehat{W}_t^R \equiv \widehat{\left(\frac{W_t}{P_t}\right)} = \widehat{W}_{t-1}^R + \widehat{\pi}_t^W - \widehat{\pi}_t, \tag{32}$$

where $\widehat{\pi}_t$ is the log-linearized rate of inflation.

The log-linearized system

Log-linearized variables are denoted by hat and represent log-deviations around their steady state values, or percentage point deviations in the case of interest rate and the inflation rate.[12] The log-linearized equations are as follows,

- Euler Equation,

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \sigma(\widehat{i_t^D} - E_t\widehat{\pi}_{t+1}), \tag{33}$$

where $E_t\widehat{\pi}_{t+1} = E_t\widehat{P}_{t+1} - \widehat{P}_t$ as the expected log deviation of inflation from its steady state value (assuming $\pi = 0$)

- Marginal Cost,

$$\widehat{mc}_t = \widehat{i_t^L} + \widehat{W}_t^R - \widehat{Z}_t.$$

- Total loans (in real terms)

$$\widehat{L}_t = \widehat{W_t^R} + \widehat{N}_t.$$

- Real Wages,

$$\widehat{W_t^R} = \widehat{W}_{t-1}^R + \widehat{\pi}_t^W - \widehat{\pi}_t.$$

[12] Log-linearized net interest rates are used as an approximation for log-linearized gross interest rates.

- Wage inflation, $\widehat{\pi}_t^W$, is

$$\widehat{\pi}_t^W = \beta E_t \widehat{\pi}_{t+1}^W + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma\theta_w)}(\widehat{MRS}_t - \widehat{W}_t^R),$$

where,

$$\widehat{MRS}_t = \frac{1}{\sigma}\widehat{C}_t + \gamma\widehat{Y}_t - \gamma\lambda_w\left(\widehat{W_t^R} + \widehat{i_t^L}\right) + \gamma(\lambda_w - 1)\widehat{Z}_t.$$

- Employment,

$$\widehat{N}_t = -\lambda_w\left(\widehat{W_t^R} + \widehat{i_t^L}\right) + \widehat{Y}_t + (\lambda_w - 1)\widehat{Z}_t.$$

- Productivity Shock,

$$\widehat{Z}_t = \widehat{\varepsilon}_t + \widehat{A}_t.$$

where, $\widehat{A_t} = \xi^A \widehat{A_{t-1}} + \alpha_t^A$.

- Probability of default (in percentage points),

$$\widehat{\Phi_t} = \Phi\left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}}\right)\left(\widehat{i_t^L} + \widehat{W_t^R} - \widehat{A_t}\right).$$

- Lending Rate,

$$\widehat{i_t^L} = \frac{1}{(1+i^L)}\left\{(1+i^D)\widehat{i_t^D} + \left(\frac{\chi A}{W^R}\right)\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\Phi^2\left(2\widehat{\Phi_t} - \widehat{W_t^R} + \widehat{A_t}\right)\right\}.$$

- NKPC:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \widehat{mc}_t^R,$$

where, $\lambda = (1-\omega_p)(1-\omega_p\beta)/\omega_p$.

- Interest Rate Policy Rule,$(\widehat{i_t} = \widehat{i_t^D})$,

$$\widehat{i_t} = \rho^i \widehat{i}_{t-1} + (1-\rho^i)(\phi_y \widehat{Y}_t + \phi_\pi \widehat{\pi}_t) + \epsilon_t^i.$$
