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On the stability properties of optimal interest rules under learning

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Abstract

In recent literature on monetary policy, it has been argued that a sensible policy rule should be able to induce learnability of the fundamental equilibrium: if private agents update their beliefs over time using adaptive learning technques, they should be able to converge towards rationality. Evans and Honkapohja (2003) showed that in a New Keynesian model an expectations based rule has such a desirable property, while a fundamentals based one does not. In order to implement an expectations based rule, though, the policymaker needs to observe private sector expectations. We show that there exists an alternative rule, based only on fundamentals, that can achieve the same positive results in terms of stability of private sector's learning dynamics. Moreover, such a rule is learnable by the policymaker, and the combined learning dynamics of the private sector and the central bank make the economy converge to the fundamental equilibrium.

Key words: Monetary policy, expectations, learning, E-stability.

JEL classification: E52, E58, D84.

Introduction

The analysis of monetary policy is often conducted under the assumption of rational expectations (RE), where agents are supposed to have all the relevant information to make their economic decisions, and to use that information efficiently. Works have shown, though, that policy rules that perform well under rationality might actually fail to stabilize the economy when agents form their expectations as adaptive learners (see e.g., Bullard and Mitra (2002), Evans and Honkapohja (2003) and Evans and Honkapohja (2006)), as they do not allow agents to learn the fundamental equilibrium over time.

In particular, in a New Keynesian forward-looking model, an optimal discretionary policy rule derived under the hypothesis of rational expectations and that responds only to fundamental shocks leads to a system that is characterized by equilibrium indeterminacy and E-instability[1] of the minimum state variable (MSV)[2] rational expectations equilibrium (REE): in other words, the economy has multiple stable RE equilibria, and agents are not able to learn the fundamental one. E-instability depends on the fact that, in deriving the optimal policy rule, perfect rationality from the part of the agents is assumed: when this assumption doesn't hold, expectations represent an important factor for determining the system dynamics, and should be explicitly considered when designing the policy rule. Evans and Honkapohja (2003) show indeed that in this case the outcome can be improved by responding to private expectations: not only the system in this case has a unique non explosive REE, but the equilibrium is also E-stable, which means that it can be learned in real time by agents.[3] The implementation of such an expectations based rule, though, requires the policymaker to be able to observe timely the expectations of the public.

The aim of this paper is to investigate whether it is possible for a policymaker, by responding only to fundamental shocks, to achieve the same positive results in terms of E-stability of equilibrium that can be achieved using an expectations based rule. We will show that this is indeed possible, but policy parameters must be updated over time to keep track of changes in private sector's expectations.

Intuitively, responses to shocks can substitute the necessary response to expectations only if shocks and expectations are correlated. We know that when expectations are rational, they are indeed a function of the shocks, and so there is the necessary correlation. But when expectations are not rational, can they still be considered functions (only) of fundamental shocks? The answer is yes, under our hypothesis of learning, which assumes that people recurrently estimate a correctly specified model, one in which only fundamental shocks matter. We leave for future work to consider the case in which agents learn by estimating a misspecified model, or one that possibly includes also sunspot components.

The rational expectations hypothesis allows a direct map from shocks to current endogenous variables. If expectations are not rational, the map should be from shocks and expectations to endogenous variables. But expectations, on their part, are also function of these shocks. Thus a direct map from shocks to endogenous variables is still possible, even though it will change over time as expectational parameters are updated through learning. Lucas (1976) in his famous critique showed that under RE policy parameters affect expectations and therefore the stochastic process for the endogenous variables is not invariant to policy decisions.[4] We show here instead

[1] For the relevance of the E-stability concept, and for an exaustive guide to its analysis, see Evans and Honkapohja (2001).

[2] The concept of MSV was first introduced by McCallum (1983). For a discussion of the role of this solution concept in rational expectations models, see McCallum (1999).

[3] This link relies on the E-stability principle. Again, see Evans and Honkapohja (2001).

[4] Evans and Ramey (2006) present an extension of the critique to a framework where agents have adaptive, rather than rational, expectations.

how expectations affect (optimal) policy parameters under learning and argue that this is an important aspect for policymakers to take into consideration.

We must note here that in this paper we focus on the problem of E-stability, and largely ignore that of determinacy. In this respect, we follow McCallum (2009) and argue that determinacy is neither a necessary nor a sufficient condition for monetary policy analysis: once we take into account information feasibility of an equilibrium, the essential point is that a solution be learnable on the basis of the information generated within the economy. Moreover, we note that discussing determinacy under the assumption of adaptive learning from the part of agents makes little sense: the problem of multiple stable equilibria to which the term indeterminacy refers to, arises only under the assumption of rational expectations. Once we assume that agents gain the necessary information to make their decisions using recursive learning algorithms based on specific perceived laws of motion, the problem of indeterminacy loses its original strength. In particular in our model, since there is only one REE consistent with the forecasting model used by agents, the only question that remains is whether beliefs of agents will or will not converge, over time, to that equilibrium.[5]

In addition to showing the existence of a policy rule that is based only on fundamentals but is equivalent to an expectations based rule under learning (and for this reason we label it expectations equivalent fundamentals based (EEFB) rule), we will also show that such a rule is learnable by the central bank (CB). In other words, the CB can gain all the information needed to implement such rule from past experience, without the need to obtain additional information. In this respect, our rule is superior to an expectations based one: while in fact Evans and Honkapohja (2003) show that with an expectations based rule the CB could learn the structural parameters needed to implement such a policy, still the policymaker would need to observe private sector's expectations in order to implement it. With an EEFB rule, instead, the CB can rely solely on its adaptive learning activity to implement this policy. Borrowing the terminology of Taylor (1993) and McCallum and Nelson (1999), we can say that an EEFB rule is operational, since it responds only to exogenous variables and it requires only information available at the time the interest rate is set.

We will also analyze combined learning dynamics for the private sector and the policymaker, and show that under such dynamics the economy converges to the fundamental REE.

An additional result of this paper is that the EEFB rule is robust to policy errors: we will in fact show that even if policy parameters are not exactly as prescribed by the EEFB rule, still E-stability prevails and agents can learn the relevant equilibrium.

We will also use our EEFB rule to investigate what changes in policy responses are called for by the learning activity of agents. Orphanides and Williams (2004), in a different setting, show that when the private sector doesn't have RE and is instead adaptively learning the structure of the economy from data, the central bank should implement a more aggressive policy in terms of its response to inflation. We therefore use our EEFB rule to try and shed some light on how the response to fundamental shocks should change when agents are learning. It turns out that the answer is not univocal, and it depends on the relative position of agents' beliefs over the learning activity compared to their rational expectations values.

The structure of the paper is as follows: in Section 1 we lay down the model and show both the fundamentals based rule and the expectations based rule that have been proposed in the literature; in Section 2 we derive the EEFB rule, present the E-stability analysis under such rule and discuss the changes in the response to shocks called for by the learning activity of agents; Section 3 shows that the EEFB rule is learnable by the policymaker; Section 4 analyses E-stability with combined learning for the private sector and the CB; Section 5 considers the effect of policy errors on equilibrium stability; Section 6 presents simulations of real time dynamics under different policy specifications; and Section 7 concludes.

[5] In models with lag endogenous variables, there could be more than one REE consistent with a correctly specified forecasting model, and the question of whether one or more of those equilibria are non explosive under RE would arise. In this case learnability could be used as a selection criterion among them.

1 The model

The framework we use is a standard forward-looking New Keynesian model, as presented in Clarida, Gali and Gertler (1999). The equations describing the economic system are:

$$x_t = -\varphi\left(i_t - \hat{E}_t\pi_{t+1}\right) + \hat{E}_t x_{t+1} + g_t \tag{1}$$

$$\pi_t = \lambda x_t + \beta\hat{E}_t\pi_{t+1} + u_t. \tag{2}$$

Equation (1) is a forward-looking IS equation, obtained by log-linearizing the consumption Euler equation that arises from the household's optimal saving decision: x_t is the output gap, the deviation of the output from its potential level, i_t is the interest rate, which is here taken to be the policy instrument, and π_t is the inflation rate. $\hat{E}$ indicates expectations, not necessarily rational.[6]

Equation (2) is a forward-looking Phillips curve derived under the assumption of staggered nominal price setting by optimizing monopolistically competitive firms. Individual pricing decisions are aggregated and the ensuing relation is log-linearized about the steady state. All parameters in (1)-(2) are positive, and $\beta < 1$.

The shocks follow AR(1) processes:

$$g_t = \mu g_{t-1} + \tilde{g}_t \tag{3}$$

$$u_t = \rho u_{t-1} + \tilde{u}_t \tag{4}$$

with μ, $\rho \in (0, 1)$. The stochastic process g_t represents a demand shock coming from potential output or government expenses, while u_t summarizes any cost push shock to marginal costs. Innovations $\tilde{g}_t$ and $\tilde{u}_t$ are i.i.d. processes with zero mean and variances $\sigma^2_{\tilde{g}}$ and $\sigma^2_{\tilde{u}}$ respectively. As it is common in the literature, we will assume that at time t agents can observe time-t shocks, but only endogenous variables up to time $t-1$.

1.1 Fundamentals based policy rule

The policy problem is to minimize expected deviations of output gap and inflation from their target levels. Here for simplicity we assume that both those targets are zero, which means that the policymaker aims to reach the potential output while driving inflation to zero. The policy objective function is then

$$\min E_t \sum_{i=0}^{\infty} \beta^i \left(\alpha x^2_{t+i} + \pi^2_{t+i}\right). \tag{5}$$

Parameter α weighs the relative importance of the two target variables in the loss function: with $\alpha = 0$, we have pure inflation targeting. β is the discount factor for the policymaker, here taken to be the same as that of the private sector in equation (2).

We will consider here only the case of discretionary policy. In this case the first order condition for optimal policy is

$$\lambda\pi_t + \alpha x_t = 0. \tag{6}$$

The system (1), (2) and (6) under the RE assumption gives the reaction function for the interest rate:

$$i_t = \gamma^R_c + \gamma^R_u u_t + \gamma^R_g g_t \tag{7}$$

[6] Preston (2005) points out that when expectations are not rational, equations (1) and (2) do not actually represent optimality conditions for the houlehold and firm's problems. In this case, he claims, forecasts for the whole future paths of inflation and output gap should be considered. Honkapohja, Mitra and Evans (2002) defend instead this formulation as a sensible way to represent behavioral rules under adaptive learning.

with[7]

$$\gamma_c^R = 0 \tag{8}$$

$$\gamma_u^R = \frac{\lambda(1-\rho)+\varphi\alpha\rho}{\varphi(\lambda^2+\alpha(1-\beta\rho))} \tag{9}$$

$$\gamma_g^R = \frac{1}{\varphi}. \tag{10}$$

As shown in different works (see e.g. Svensson and Woodford, 2004 and Evans and Honkapohja, 2003) the New Keynesian model when closed with this policy rule not only is indeterminate, i.e. there are multiple stable REE, but also the MSV solution, based only on fundamentals, is E-unstable and thus can not be adaptively learned in real time by agents.

1.2 Expectations based policy rule

We now follow Evans and Honkapohja (2003) and derive the optimal policy for this economy without imposing rationality on the part of agents. The system is still described by equations (1), (2) and (6). From (2) and (6) we get

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\hat{E}_t\pi_{t+1} + \frac{\alpha}{\alpha+\lambda^2}u_t \tag{11}$$

and combining (1) with (6) results

$$\pi_t = \frac{\alpha\varphi}{\lambda}i_t - \frac{\alpha\varphi}{\lambda}\hat{E}_t\pi_{t+1} - \frac{\alpha}{\lambda}\hat{E}_t x_{t+1} - \frac{\alpha}{\lambda}g_t. \tag{12}$$

Equations (11) and (12) give the expectations based policy rule

$$i_t = \gamma_c^E + \gamma_\pi^E\hat{E}_t\pi_{t+1} + \gamma_x^E\hat{E}_t x_{t+1} + \gamma_u^E u_t + \gamma_g^E g_t, \tag{13}$$

with

$$\gamma_c^E = 0 \tag{14}$$

$$\gamma_\pi^E = 1 + \frac{\lambda\beta}{\varphi(\alpha+\lambda^2)}, \quad \gamma_x^E = \frac{1}{\varphi} \tag{15}$$

$$\gamma_u^E = \frac{\lambda}{\varphi(\alpha+\lambda^2)}, \quad \gamma_g^E = \frac{1}{\varphi}. \tag{16}$$

Equations (1), (2) and (13) represent the new system describing the evolution of the economy, given private sector's expectations, when the central bank implements policy rule (13). In Evans and Honkapohja (2003) it is shown that this system is determinate under RE and that the (unique) equilibrium is E-stable. This means that there is only one stable RE equilibrium and this equilibrium is learnable by agents using adaptive learning techniques such as recursive least squares (RLS).

[7] Note that here $\gamma_c^R = 0$ because we have assumed in (5) that the target levels for inflation and output are zero. Assuming target levels different from zero would not change any of the results of the paper.

2 Policy rule and private sector learning

By responding to private expectations, the policymaker can obtain E-stability of equilibrium, so that if agents hold beliefs that depart from rationality, they can learn the REE and conform to it over time. But in order to implement such rule, the central bank must be able to observe the expectations of the public.

Under learning, private agents form expectations using a forecasting model that is updated over time: we assume that this model is correctly specified, i.e., it includes all and only the variables that are relevant for the economy.[8] Under this conditions, we want here to investigate whether it is possible for the policymaker to implement a policy that responds only to fundamental shocks and still be able to induce learnability of the fundamental equilibrium.

Given the structural model (1)-(2), the only fundamental variables that drive this economy are the exogenous shocks, and therefore perceived laws of motion (PLMs) for inflation and output consistent with the MSV solution take the form

$$\pi_t = a_0 + a_1 u_t + a_2 g_t \tag{17}$$
$$x_t = b_0 + b_1 u_t + b_2 g_t. \tag{18}$$

These PLMs represent the model that agents recurrently estimate over time and use to make their forecasts. We assume, as it is common in recent literature on monetary policy, that private agents learn from data using a stochastic recursive algorithm, and in particular one that implements RLS.

Agents could use a constant gain or a decreasing gain algorithm for their learning activity: the choice affects convergence of beliefs to the RE values. With a decreasing gain, agents have infinite memory and don't discard past data as time goes on: when the value of $1/t$ is chosen for the gain, and appropriate initial conditions are specified, this procedure corresponds to standard least squares estimation. In this case, beliefs can converge towards the RE values. Constant gain is instead equivalent to rolling windows regression, and is more appropriate when the economy is perceived to be subject to structural changes. In this case parameter values remain noisy at the limit: they can not converge point-wise towards their REE values, but can converge towards a stationary distribution around those values. We will consider in this paper only the decreasing gain case, but we conjecture that all the main arguments presented in this work would carry over to the constant gain case as well.

The learning algorithms used by agents therefore are of the form

$$\theta_t = \theta_{t-1} + \zeta_t R_t^{-1} z_{t-1}(y_{t-1} - \theta'_{t-1} z_{t-1}) \tag{19}$$
$$R_t = R_{t-1} + \zeta_t (z_{t-1} z'_{t-1} - R_{t-1}), \tag{20}$$

where θ_t is the vector of parameters to be estimated and z_{t-1} is the vector of exogenous variables determining the endogenous variable y_{t-1}. ζ_t is the gain parameter, which we set equal to $1/t$ in order to implement RLS. In our case, θ corresponds either to the vectors of a_i or b_i, z_{t-1} is the vector $[1\ u_{t-1}\ g_{t-1}]'$ and y_{t-1} is either π_{t-1} (when estimating a_i) or x_{t-1} (when θ corresponds to b_i).[9]

In order to find the optimal policy rule based only on fundamentals, the policymaker solves the maximization problem (5) subject to (1), (2), a policy rule of the form

$$i = \gamma_c^{EE} + \gamma_u^{EE} u_t + \gamma_g^{EE} g_t \tag{21}$$

[8] In other words, we leave aside in this work the problem of misspecified models and sunspot equilibria.

[9] It is common practice to use data only through time $t-1$ when estimating parameters at time t used to form expectations for time $t+1$ endogenous variables. This avoids a simultaneity problem between parameter estimates and current endogenous variables.

and expectations formation equations

$$\hat{E}_t \pi_{t+1} = a_0 + a_1 \rho u_t + a_2 \mu g_t \tag{22}$$

$$\hat{E}_t x_{t+1} = b_0 + b_1 \rho u_t + b_2 \mu g_t, \tag{23}$$

which follow from the PLMs (17) and (18) and from the AR form for the shocks (3) and (4). We assume that ρ and μ are known: otherwise they could be consistently estimated, given that these shocks are exogenous and observable.

Solving the policymaker's problem, we find that policy rule (21) has coefficients

$$\gamma_{c,t}^{EE} = \left(1 + \frac{\lambda\beta}{\varphi\left(\alpha + \lambda^2\right)}\right) a_{0,t} + \frac{1}{\varphi} b_{0,t} \tag{24}$$

$$\gamma_{u,t}^{EE} = \frac{\lambda}{\varphi\left(\alpha + \lambda^2\right)} + \left(\rho + \frac{\lambda\beta\rho}{\varphi\left(\alpha + \lambda^2\right)}\right) a_{1,t} + \frac{\rho}{\varphi} b_{1,t} \tag{25}$$

$$\gamma_{g,t}^{EE} = \frac{1}{\varphi} + \left(\mu + \frac{\lambda\beta\mu}{\varphi\left(\alpha + \lambda^2\right)}\right) a_{2,t} + \frac{\mu}{\varphi} b_{2,t}, \tag{26}$$

which give the optimal fundamentals based policy rule when people are not rational but form instead their expectations by estimating the parameters in correctly specified PLMs.[10] As already anticipated before, we label this policy expectations equivalent fundamentals based (EEFB) rule. Note that the optimal policy parameters vary over time, as people update their estimates for $a's$ and $b's$ in their PLMs.[11]

This is the crucial difference with the fundamentals based policy rule derived under the assumption of rationality. With time-invariant policy parameters, the drift in the estimated parameters of agents due to their learning activity is transmitted directly onto the reduced form of the model (or actual low of motion). The policy rule must thus change over time in order to adapt to expectations-induced changes in the transmission channel from shocks to macroeconomic outcomes.

Noting that private expectations under learning are given by (22) and (23), we can rearrange terms in (21) and rewrite it exactly as policy rule (13). This is possible because expectations are a specific and well defined function of fundamentals.

Note moreover that, under RE, the expectational parameters a_i and b_i are

$$a_0^{RE} = 0, \quad a_1^{RE} = \frac{\alpha}{\lambda^2 + \alpha(1 - \beta\rho)}, \quad a_2^{RE} = 0 \tag{27}$$

$$b_0^{RE} = 0, \quad b_1^{RE} = -\frac{\lambda}{\lambda^2 + \alpha(1 - \beta\rho)}, \quad b_2^{RE} = 0 \tag{28}$$

[10] This policy is optimal in a restricted sense, since the policymaker does not try to exploit actively the learning mechanism of agents.

[11] This time-dependence has been made explicit in (24)-(26) by adding a subscript t to the coefficients $a's$ and $b's$. In the rest of the paper, we follow a common practice in the learning literature and for simplicity of notation we avoid the subscript t on the time varying expectatonal and policy parameters.

and policy parameters in (21) coincide with their RE values in (7):

$$\gamma_c^{EE} = \left(1 + \frac{\lambda\beta}{\varphi\left(\alpha + \lambda^2\right)}\right) a_0 + \frac{1}{\varphi} b_0 = 0 = \gamma_c^R \tag{29}$$

$$\gamma_u^{EE} = \frac{\lambda}{\varphi\left(\alpha + \lambda^2\right)} + \left(\rho + \frac{\lambda\beta\rho}{\varphi\left(\alpha + \lambda^2\right)}\right) a_1 + \frac{\rho}{\varphi} b_1 = \frac{\lambda(1-\rho) + \varphi\alpha\rho}{\varphi(\lambda^2 + \alpha(1-\beta\rho))} = \gamma_u^R \tag{30}$$

$$\gamma_g^{EE} = \frac{1}{\varphi} + \left(\mu + \frac{\lambda\beta\mu}{\varphi\left(\alpha + \lambda^2\right)}\right) a_2 + \frac{\mu}{\varphi} b_2 = \frac{1}{\varphi} = \gamma_g^R. \tag{31}$$

This shows that as agents approach rationality, the policy rule approaches the fundamentals based rule and the economy converges to the MSV REE. In other words, once agents beliefs have converged to their RE values and the economy is in the MSV equilibrium, the fundamentals based rule and the EEFB rule prescribe the same response to shocks.

Policy parameters (24)-(26) allow us also to investigate what modifications does the learning activity of agents introduce into the optimal response of the policymaker to shocks.

By looking at the policy response to g_t, we can see that, out of the RE equilibrium, the policy coefficient γ_g^{EE} in (21) depends on the expectational parameters a_2 and b_2, which, if not at their RE value of zero, could be either positive or negative: if they are positive, so that a positive shock to demand is perceived by private agents to increase output and inflation, then the policy response to the same shock prescribed under learning (γ_g^{EE} in (26)) is stronger than the optimal response under RE (γ_g^R in (10)). On the other hand, if a_2 and b_2 are negative, so that a positive shock to demand is perceived by private agents to decrease output and inflation, then the policy response to the same shock prescribed under learning is milder than the optimal response under RE. This means that when agents are learning, the CB has to modify its behavior in response to shocks to account for the effect of private expectations on the economy: if agents perceive the demand shock to push up inflation and output, the CB need to be more aggressive in order to dampen such beliefs, while if agents perceive the shock to decrease output and inflation, the CB needs to be more soft in its response, to account for the depressing effect on output and inflation already introduced by agents' beliefs.

If we instead look at the policy response to u_t, comparison is more difficult. Nevertheless, we can see that if a_1 or b_1 (note that this second one is negative in equilibrium) are higher than their REE values, this implies that $\gamma_u^{EE} > \gamma_g^R$. This means that if a negative supply shock (i.e., a positive u_t, that increases production costs) is perceived by agents to have a stronger positive impact (in the sense of increasing) on inflation (i.e., $a_1 > a_1^{RE}$), the CB should respond more strongly to it, while if the same shock is perceived to have a stronger negative effect on output (i.e., $b_1 < b_1^{RE}$), the response of the CB to the shock should be milder.

These results indicate that the CB, in its response to shocks, needs to take into account what is the perceived effect of those shocks on inflation and output by the public, and modify its behavior accordingly. The CB, in other words, needs to modify its behavior in order to compensate for the imperfect beliefs of agents regarding the effect that shocks have on the economy, otherwise beliefs of agents would become self fulfilling and diverge away from rationality. Suppose, for example, that agents believe a supply shock has a stronger impact on inflation that what would be under RE: they will expect higher inflation, and this will lead to higher actual current inflation, unless the CB modifies its response to the shock to dampen its effect on inflation (by responding more strongly to it) and thus prove agents' beliefs wrong and guide them back towards rationality.

2.1 E-stability analysis

We will show now that in the New Keynesian model here considered, the EEFB rule (21) is able to induce learnability of the fundamental equilibrium. We will start by showing that policy rule (7) is not able to induce E-stability in the model, and then show how the analysis change when the EEFB rule is used.

Using structural model (1)-(2), policy rule (7) and expectation formation schemes (22)-(23), we obtain the ALMs for inflation and output

$$x_t = [\varphi a_0 + b_0] + \left[\rho\left(\varphi a_1 + b_1\right) - \varphi\gamma_u^R\right] u_t + \left[\mu\left(a_2\varphi + b_2\right) - \varphi\gamma_g^R + 1\right] g_t \tag{32}$$

$$\begin{aligned}\pi_t = {} & [(\varphi a_0 + b_0)\lambda + \beta a_0] + \left[\left(\rho\left(\varphi a_1 + b_1\right) - \varphi\gamma_u^R\right)\lambda + \beta\rho a_1 + 1\right] u_t + \\ & + \left[\left(\mu\left(a_2\varphi + b_2\right) - \varphi\gamma_g^R + 1\right)\lambda + \beta\mu a_2\right] g_t.\end{aligned} \tag{33}$$

Mapping PLMs (17)-(18) into the above two ALMs, we obtain the T-maps

$$a_0 \Longrightarrow (\beta + \varphi\lambda)\, a_0 + \lambda b_0 \tag{34}$$

$$a_1 \Longrightarrow (\beta\rho + \lambda\rho\varphi)\, a_1 + \rho\lambda b_1 - \varphi\lambda\gamma_u^R + 1 \tag{35}$$

$$a_2 \Longrightarrow (\varphi\mu\lambda + \beta\mu)\, a_2 + \mu\lambda b_2 - \varphi\lambda\gamma_g^R + \lambda \tag{36}$$

$$b_0 \Longrightarrow \varphi a_0 + b_0 \tag{37}$$

$$b_1 \Longrightarrow \rho b_1 + \rho\varphi a_1 - \varphi\gamma_u^R \tag{38}$$

$$b_2 \Longrightarrow \mu b_2 + \mu\varphi a_2 - \varphi\gamma_g^R + 1. \tag{39}$$

The MSV REE is a fixed point of these maps, and E-stability is governed by the local asymptotic behavior of the system of ODEs defined as follows[12]

$$\dot{a}_0 = (\beta + \varphi\lambda - 1)\, a_0 + \lambda b_0 \tag{40}$$

$$\dot{a}_1 = (\beta\rho + \lambda\rho\varphi - 1)\, a_1 + \rho\lambda b_1 - \varphi\lambda\gamma_u^R + 1 \tag{41}$$

$$\dot{a}_2 = (\varphi\mu\lambda + \beta\mu - 1)\, a_2 + \mu\lambda b_2 - \varphi\lambda\gamma_g^R + \lambda \tag{42}$$

$$\dot{b}_0 = \varphi a_0 \tag{43}$$

$$\dot{b}_1 = (\rho - 1)\, b_1 + \rho\varphi a_1 - \varphi\gamma_u^R \tag{44}$$

$$\dot{b}_2 = (\mu - 1) b_2 + \mu\varphi a_2 - \varphi\gamma_g^R + 1. \tag{45}$$

The above system of ODEs can be divided in three subsystems: $(\dot{a}_0,\ \dot{b}_0)^T$, $(\dot{a}_1,\ \dot{b}_1)^T$, $(\dot{a}_2,\ \dot{b}_2)^T$. Denoting

$$\Omega = \begin{bmatrix} \beta + \varphi\lambda & \lambda \\ \varphi & 1 \end{bmatrix}, \tag{46}$$

stability of the three subsystems (and therefore E-stability), is governed, respectively, by the matrices $(\Omega - I)$, $(\rho\Omega - I)$ and $(\mu\Omega - I)$: for E-stability to obtain, we need the real part of the eigenvalues of these three matrices to be negative. For the first matrix, $(\Omega - I)$, this corresponds to having $\mid \Omega - I \mid > 0$ and $\mathrm{tr}(\Omega - I) < 0$. Since clearly $\mid \Omega - I \mid < 0$, the condition is not satisfied and the equilibrium is E-unstable.

On the contrary, if the policy is derived taking into account the expectations formation process of agents, as the EEFB policy rule (21) does, then the policymaker can guide the learning process of agents towards rationality. In policy rule (21), in fact, γ_c^{EE} is conditioned on a_0 and b_0, γ_u^{EE} on a_1 and b_1, and γ_g^{EE} on a_2 and b_2: this dependence crucially changes the maps from PLMs to ALM and is able to generate E-stability. Going through the same steps as before, the new system

[12] For details about the techniques used, see Evans and Honkapohja (2001).

of ODEs becomes

$$\dot{a}_0 = (\beta + \varphi\lambda - 1)\, a_0 + \lambda b_0 - \varphi\lambda\gamma_c^{EE} \quad (47)$$

$$\dot{a}_1 = (\beta\rho + \lambda\rho - 1)\, a_1 + \rho\lambda b_1 - \varphi\lambda\gamma_u^{EE} + 1 \quad (48)$$

$$\dot{a}_2 = (\varphi\mu\lambda + \beta\mu - 1)\, a_2 + \mu\lambda b_2 - \varphi\lambda\gamma_g^{EE} + \lambda \quad (49)$$

$$\dot{b}_0 = \varphi a_0 - b_0 - \varphi\gamma_c^{EE} \quad (50)$$

$$\dot{b}_1 = (\rho - 1)\, b_1 + \rho\varphi a_1 - \varphi\gamma_u^{EE} \quad (51)$$

$$\dot{b}_2 = (\mu - 1) b_2 + \mu\varphi a_2 - \varphi\gamma_g^{EE} + 1 \quad (52)$$

and after substituting for the policy parameters γ_c^{EE}, γ_u^{EE} and γ_g^{EE}, we have that E-stability is governed by the eigenvalues of the matrices $(\Omega' - I)$, $(\rho\Omega' - I)$ and $(\mu\Omega' - I)$, where now

$$\Omega' = \begin{bmatrix} \frac{\alpha\beta}{\alpha+\lambda^2} & 0 \\ -\frac{\lambda\beta}{\alpha+\lambda^2} & 0 \end{bmatrix}. \quad (53)$$

For E-stability to obtain, the real part of the eigenvalues of these three matrices must be negative. It can easily be seen that all three matrices have positive determinant and negative trace. The EEFB policy rule (21) therefore generates E-stability.

Proposition 1 *In an economy represented by (1)-(2), policy rule (21) makes the MSV REE E-stable.*

Our results so far show that the EEFB rule is just as good, in terms of equilibrium stability under learning, as an expectations based rule, and it therefore provides a plausible alternative for the policymaker. We will now argue that the EEFB rule is superior to an expectations based rule in one important aspect, in that it is learnable by the policymaker and does not require external information about agents' expectations.

3 Learnability of EEFB rule

Evans and Honkapohja (2003) show that when implementing an expectations based rule, the CB can adaptively learn the structural parameters of the model, but the policymaker would still need to observe private sector's expectations in order to implement such rule. We show here how the CB could instead implement an EEFB rule simply relying on its learning activity and without having to observe or guess private sector's expectations.

When setting the policy instrument, the interest rate, the CB is trying to implement the optimality condition (6): in order to do so, they need to have information about what would be the outcome in terms of current output and inflation, given the current shocks and their policy rate. It is therefore natural for the CB to estimate relationships (PLMs) of the form

$$\pi_t = \phi_c^\pi + \phi_u^\pi u_t + \phi_g^\pi g_t + \phi_i^\pi i_t \quad (54)$$

$$x_t = \phi_c^x + \phi_u^x u_t + \phi_g^x g_t + \phi_i^x i_t, \quad (55)$$

using data up to time $t-1$, which, once current shocks u_t and g_t are observed, would allow them to implement the optimality condition (6) by setting the interest rate equal to

$$i_t = \gamma_c^L + \gamma_u^L u_t + \gamma_g^L g_t + v_t, \quad (56)$$

where

$$\gamma_c^L = -\frac{\lambda\phi_c^\pi + \alpha\phi_c^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} \tag{57}$$
$$\gamma_u^L = -\frac{\lambda\phi_u^\pi + \alpha\phi_u^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} \tag{58}$$
$$\gamma_g^L = -\frac{\lambda\phi_g^\pi + \alpha\phi_g^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} \tag{59}$$

and v_t is a zero mean i.i.d. noise term introduced to allow identification in the CB learning regressions. Clearly, parameters in this rule evolve over time as the CB updates its beliefs in terms of the coefficients in (54)-(55). The question now is: for a given set of private sector's beliefs, would parameters in policy rule (56) converge towards γ_c^{EE}, γ_u^{EE}, γ_g^{EE}, so that the policymaker can learn to implement an EEFB rule?

Given the structural equations (1)-(2) and the expectations formation rules (22)-(23) for private agents, we can derive the ALMs consistent with the PLMs used by the CB:

$$\pi_t = [(\varphi\lambda + \beta)\, a_0 + \lambda b_0] + [(\varphi\lambda + \beta)\, \rho a_1 + \lambda\rho b_1 + 1]\, u_t + [(\varphi\lambda + \beta)\, \mu a_2 + \lambda\mu b_2 + \lambda]\, g_t - \lambda\varphi i_t \tag{60}$$
$$x_t = [\varphi a_0 + b_0] + [\varphi\rho a_1 + \rho b_1]\, u_t + [\varphi\mu a_2 + \mu b_2 + 1]\, g_t - \varphi i_t. \tag{61}$$

Mapping the PLMs (54)-(55) into the ALMs (60)-(61) we can obtain the T-maps for the evolution of the estimated parameters and from there the set of ODEs governing stability of the CB's learning activity

$$\dot{\phi}_c^\pi = (\varphi\lambda + \beta)\, a_0 + \lambda b_0 - \phi_c^\pi \tag{62}$$
$$\dot{\phi}_u^\pi = (\varphi\lambda + \beta)\, \rho a_1 + \lambda\rho b_1 + 1 - \phi_u^\pi \tag{63}$$
$$\dot{\phi}_g^\pi = (\varphi\lambda + \beta)\, \mu a_2 + \lambda\mu b_2 + \lambda - \phi_g^\pi \tag{64}$$
$$\dot{\phi}_i^\pi = -\lambda\varphi - \phi_i^\pi \tag{65}$$
$$\dot{\phi}_c^x = \varphi a_0 + b_0 - \phi_c^x \tag{66}$$
$$\dot{\phi}_u^x = \varphi\rho a_1 + \rho b_1 - \phi_u^x \tag{67}$$
$$\dot{\phi}_g^x = \varphi\mu a_2 + \mu b_2 + 1 - \phi_g^x \tag{68}$$
$$\dot{\phi}_i^x = -\varphi - \phi_i^x. \tag{69}$$

It is clear that these ODEs are stable and the expectational parameters for the CB converge towards the fixed values

$$\phi_c^\pi = (\varphi\lambda + \beta)\, a_0 + \lambda b_0 \tag{70}$$
$$\phi_u^\pi = (\varphi\lambda + \beta)\, \rho a_1 + \lambda\rho b_1 + 1 \tag{71}$$
$$\phi_g^\pi = (\varphi\lambda + \beta)\, \mu a_2 + \lambda\mu b_2 + \lambda \tag{72}$$
$$\phi_i^\pi = -\lambda\varphi \tag{73}$$
$$\phi_c^x = \varphi a_0 + b_0 \tag{74}$$
$$\phi_u^x = \varphi\rho a_1 + \rho b_1 \tag{75}$$
$$\phi_g^x = \varphi\mu a_2 + \mu b_2 + 1 \tag{76}$$
$$\phi_i^x = -\varphi, \tag{77}$$

which in turn imply that policy parameters will converge towards the values

$$\gamma_c^L = -\frac{\lambda\phi_c^\pi + \alpha\phi_c^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} = \left(1 + \frac{\lambda\beta}{\varphi\left(\alpha + \lambda^2\right)}\right) a_0 + \frac{1}{\varphi} b_0 \tag{78}$$

$$\gamma_u^L = -\frac{\lambda\phi_u^\pi + \alpha\phi_u^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} = \frac{\lambda}{\varphi\left(\alpha + \lambda^2\right)} + \left(\rho + \frac{\lambda\beta\rho}{\varphi\left(\alpha + \lambda^2\right)}\right) a_1 + \frac{\rho}{\varphi} b_1 \tag{79}$$

$$\gamma_g^L = -\frac{\lambda\phi_g^\pi + \alpha\phi_g^x}{\lambda\phi_i^\pi + \alpha\phi_i^x} = \frac{1}{\varphi} + \left(\mu + \frac{\lambda\beta\mu}{\varphi\left(\alpha + \lambda^2\right)}\right) a_2 + \frac{\mu}{\varphi} b_2. \tag{80}$$

These are exactly the values in the EEFB rule as reported in (24)-(26), which means that such a rule is adaptively learnable by the CB.

Proposition 2 *The EEFB rule (21) is adaptively learnable by the CB, in the sense that when the CB adaptively learns parameters in the PLMs (54)-(55) and used them to implement a policy rule of the form (56), such a policy will converge to the EEFB rule.*

The above results is very important, as it shows that the CB can learn to implement a policy rule that is equivalent to an expectations based rule and shares its desirable properties in terms of stability of equilibrium.

4 Combined learning

Results of E-stability of the EEFB rule in the previous section were derived keeping fixed the parameters in private sector's PLMs. In order to assess whether combined learning dynamics of the private sector and the CB would make the economy converge towards the RE solution, we need to analyze simultaneously the local stability properties, at the MSV REE, of the two systems of learning represented by (62)-(69) and (47)-(52), with the modification that all γ^{EE} parameters in this last set of ODEs are substituted by the equivalent γ^L parameter from (57)-(59).

This new enlarged system can be divided in three subsystems: $(\dot{a}_0, \dot{b}_0, \phi_c^\pi, \phi_c^x, \phi_i^\pi, \phi_i^x)^T$, $(\dot{a}_1, \dot{b}_1, \phi_u^\pi, \phi_u^x, \phi_i^\pi, \phi_i^x)^T$, $(\dot{a}_2, \dot{b}_2, \phi_g^\pi, \phi_g^x, \phi_i^\pi, \phi_i^x)^T$: it turns out that each subsystem, evaluated at the equilibrium point, has two eigenvalues equal to -1 (corresponding to the dynamics of ϕ_i^π and ϕ_i^x), and the remaining eigenvalues come from matrices

$$A = \begin{bmatrix} \Omega & \Pi \\ \Theta & 0 \end{bmatrix} - I \tag{81}$$

$$B = \begin{bmatrix} \rho\Omega & \Pi \\ \rho\Theta & 0 \end{bmatrix} - I \tag{82}$$

$$C = \begin{bmatrix} \mu\Omega & \Pi \\ \mu\Theta & 0 \end{bmatrix} - I \tag{83}$$

where Ω is as defined in (46), 0 is a two by two matrix of zeros, and

$$\Pi = -\begin{bmatrix} \frac{\lambda^2}{\lambda^2+\alpha} & \frac{\alpha\lambda}{\lambda^2+\alpha} \\ \frac{\lambda}{\lambda^2+\alpha} & \frac{\alpha}{\lambda^2+\alpha} \end{bmatrix} \tag{84}$$

$$\Theta = \begin{bmatrix} \varphi\lambda + \beta & \lambda \\ \varphi & 1 \end{bmatrix}. \tag{85}$$

E-stability requires the real part of the eigenvalues of matrices A, B and C to be negative. While one eigenvalue of each matrix is always equal to -1, we can not say anything analytically about the values of the others, and we resort to numerical calculations for specific parameterizations.

There is no agreement in the literature about the values to assign to parameters in structural equations (1)-(2). Prominent examples of values that have been used are: Clarida, Gali and Gertler (2000): $\varphi = 1$, $\lambda = .3$, $\beta = .99$; McCallum and Nelson (1999): $\varphi = .164$, $\lambda = .3$, $\beta = .99$; and Woodford (1999): $\varphi = 1/.157$, $\lambda = .024$, $\beta = .99$. We computed the eigenvalues under all these parameterization, and for the remaining parameters α, ρ and μ we experimented with a large variety of values. In all cases, the real part of all eigenvalues was found to be negative, meaning that combined learning dynamics for the private sector and CB are stable and converge to their equilibrium values.

5 Policy errors

Evans and Honkapohja (2003) show that an expectations based rule is robust to observational errors in expectations. We ask here a similar question for our EEFB rule and investigate whether such rule is robust to policy errors, in the sense that it would still be able to induce E-stability of equilibrium even if policy parameters were not precisely tuned as required in (21).

Specifically, suppose that the policymaker does not implement the EEFB rule perfectly but incurs some errors in its response to shocks. The policy rule can then be expressed as

$$i = (\gamma_c^{EE} + err_c) + (\gamma_u^{EE} + err_u)u_t + (\gamma_g^{EE} + err_g)g_t + v_t, \tag{86}$$

where γ_c^{EE}, γ_u^{EE} and γ_g^{EE} are the policy parameters in (21) and err_c, err_u and err_g are the deviations of the actual policy parameters from the optimal ones.

By looking and the E-stability analysis in Section 2, it should result clear that these error terms do not affect the E-stability properties of the equilibrium as long as they are not correlated with the expectational parameters of private agents: it is in fact possible to separate the policy errors from the rest of the system, that remains unchanged, and carry out the E-stability analysis as before, obtaining the same positive results.

We can consider two cases, one in which policy errors are constant over time, and one in which they are random variables taken from an i.i.d. distribution with zero mean and constant variance. In the first case policy errors, while not affecting the E-stability properties of equilibrium, would nevertheless change the values for the reduced form parameters in the MSV solution. To derive the new values, we compute the new MSV REE through the undetermined coefficient procedure closing the model with policy rule (86):

$$a_0 = err_c \tag{87}$$

$$a_1 = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} - \frac{(\alpha+\lambda^2)\lambda\varphi}{\lambda^2 + \alpha(1-\beta\rho)} err_u, \tag{88}$$

$$a_2 = -\frac{(\alpha+\lambda^2)\lambda\varphi}{\lambda^2 + \alpha(1-\beta\mu)} err_g, \tag{89}$$

$$b_0 = (1 - \beta + \varphi(1-\lambda))\, err_c \tag{90}$$

$$b_1 = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)} + \left(\frac{\lambda^2\beta\rho\varphi}{\lambda^2 + \alpha(1-\beta\rho)} - \varphi\right) err_u, \tag{91}$$

$$b_2 = \left(\frac{\lambda^2\beta\mu\varphi}{\lambda^2 + \alpha(1-\beta\mu)} - \varphi\right) err_g. \tag{92}$$

Note that when $err_c = err_u = err_g = 0$, we obtain again (27)-(28).

In the second case instead, with zero mean uncorrelated random errors, the MSV solution would not be affected at all by policy errors and equilibrium values would still be as shown in (27)-(28).

6 Real time dynamics

We now present some stochastic simulations of our model (1)-(4) when alternatively closed with different policy rules. Agents recurrently estimate parameters a_i and b_i in the MSV solution (17) and (18) using the algorithm (19) and (20), and form their expectations according to (22) and (23). The authority implements its policy rule ((7), (21), (56) or (86)) in order to stabilize the economy: when implementing rule (56), the CB recurrently estimates through RLS the parameters in (54)-(55) and use those estimates to compute (57)-(59).

We will report the outcome of our simulations using the Clarida, Gali and Gertler (2000) parameterization but qualitative results do not change under the alternative two calibrations. In addition, we set $\alpha = .1$, the two AR parameters for the fundamental shocks (ρ and μ) equal to .8, and the gain parameter ζ_t in the learning algorithm is set to $1/t$ in order to implement RLS. The mean squared error for all shocks in the model, drawn from a normal distribution with zero mean, is set equal to .25. Initial beliefs parameters are chosen to be 10% off their rational values.

6.1 Dynamics under learning

We start by showing the evolution of parameter estimates for the private sector under different policy rules. Figure 1 shows that when the policymaker implements the fundamentals based policy derived under the wrong assumption of rationality from the part of agents, i.e., when he uses policy rule (7), private beliefs do not converge. This result is due to the fact that the MSV REE is E-unstable under this policy rule: if beliefs happen to be off from full rationality, they are led further apart and will never reach their RE values. Dotted lines in the graph represent the REE values for each parameter in the PLMs (19) and (20). Note that the RE values for a_0, b_0, a_2 and b_2 are zero.

Figure 2 represents instead the evolution of beliefs in an economy where the authority takes into account the fact that agents are learning and implements the EEFB policy rule (21). We can see that in this case private beliefs converge towards their RE values: the REE is now E-stable, and agents are able to learn it.[13]

The two pictures confirm our analytical results in Section 2, and clearly show the importance of the policy implemented by the central bank in determining the economic dynamics. Under policy rule (7), the authority wrongly believes that agents are rational, and therefore implements a policy that responds in a constant way to fundamental shocks: in this way the policy rule is not able to adapt to changes in the economy introduced by the learning activity of private agents, and the equilibrium results unstable. Under policy rule (21), instead, the response of the authority to fundamental shocks changes over time, as private beliefs are being modified through learning: in this way the policymaker is able to guide private expectations towards rationality.

We also investigate numerically what are the effects introduced by the learning activity of agents on the optimal response to shocks for the policymaker. Figure 3 shows a comparison of the interest rate, the policy instrument, under the two different policy specifications (7) and (21). As private sector's beliefs converge towards rationality, the responses prescribed by the two policies become more and more similar, until perfect overlapping is reached once beliefs have fully converged to rationality. But while beliefs are off from rationality, the optimal responses prescribed by the two rules are quantitatively very different, though qualitatively similar since they both ultimately depend on the shocks that constantly hit the economy. The exact way in which the interest rates

[13] Convergence is faster when the variance for the shocks is made larger.

called for by the two policies differ from each other, as we have seen in Section 2, depends on how agents' beliefs depart from rationality, and in particular whether the perceived impact of shocks on output and inflation is milder or stronger than in the REE.

Figure 4 shows the policy parameters in the two cases: under policy rule (7) the response to shocks is constant (dotted line), while under policy rule (21) it varies over time (continuous line) and converges towards the optimal response under RE as agents learn the equilibrium value of the parameters in their PLMs. We have seen that the evolution of policy parameters over time is due to the fact that the CB is now taking into account, in its response to shocks, the perceived effect of those shocks on inflation and output according to private sector's beliefs.

An interesting feature of the EEFB rule that makes it superior to other rules is that it can be implemented by the CB without direct knowledge of the economic structure or of agents' expectations: the policymaker can simply rely on its learning activity to implement such policy, and deliver determinacy and E-stability of equilibrium. We have already shown in Sections 3 and 4 the theoretical analysis: we now present numerical simulations that confirm our previous results. Figure 5 shows the evolution of policy coefficients when the CB is learning, while Figures 6 and 7 show the simultaneous evolution of agents beliefs: we can clearly see that over time the CB learns to implement the EEFB rule, and that at the same time agents' beliefs converge towards rationality. As the CB learns its policy parameters, private agents learn parameters in their forecasting models, and the economy converges towards the MSV REE.

As we have said before, we want to stress that in order to induce convergence of private beliefs towards rationality, the policy parameters need not be exactly as prescribed in (21). Figure 8 shows the evolution of private sector's beliefs when the CB implements policy rule (86), where policy errors are zero mean, i.i.d., random variables drawn from a normal distribution. We can see that even after introducing an error term in each policy parameter, convergence still obtains. Moreover, it is interesting to note that the size of the error is not important for convergence to obtain: we have experimented with errors two orders of magnitude larger than the parameter values in the model, and still convergence obtains when the CB implements the EEFB policy rule. This means that stochastic policy errors can not destabilize the stochastic recursive algorithm that governs real time learning dynamics.

6.2 Some measures of relative performance

We now compare the relative performance of the economy when alternative policy specifications, under the assumption that agents learn through RLS. Figure 9 shows the patterns for inflation and output gap under policy rules (7) and (21): we can see that under rule (7) both variables diverge away and present a wide and increasing variability, as a consequence of the fact that private beliefs diverge, while under policy (21) both inflation and output fluctuate steadily around zero, their perfect foresight equilibrium value in the deterministic model. These results show that the policymaker, when implementing policy rule (7) because of the wrong assumption of rationality from the part of the agents, is not able to stabilize inflation and output gap around the desired values: as time passes, private beliefs diverge and this has a negative impact on the outcomes that can be achieved. Under policy rule (21), instead, the CB is able to lead private sector's expectations towards rationality, and thus maintain inflation and output dynamics under control.

7 Conclusions

Using a policy rule that assumes rationality from the part of the agents when they in fact are not rational and are instead forming their beliefs by adaptive learning, generates instability for the economy: in particular, such a policy prevents agents from learning the reduced form solution for the endogenous variables and the resulting macroeconomic outcomes are subject to large and increasing volatility. It follows that a different approach to monetary policy is needed, one that takes explicitly into account the fact that people are learning from data and thus modifying their beliefs over time.

Since private agents have beliefs that evolve as they learn, their behavior in response to fundamental shocks changes over time: in order to account for and compensate these movements in the transmission from shocks to aggregate endogenous variables, the policymaker must change his own response to shocks. In this way the CB can guide the evolution of private sector's beliefs and allow agents to learn the relevant equilibrium. If the policymaker instead neglects the fact that private expectations are off from rationality and evolving over time, agents are not able to adaptively learn the fundamental RE solution.

It has been suggested in the literature that the CB should directly respond to private sector's expectations in order to account for the endogenous changes in the economy due to time evolving beliefs. We have instead proposed in this work an alternative way by which the CB can achieve the same result, that is, by responding to shocks with time evolving coefficients. Our results show in fact that a CB can make the equilibrium for the economy learnable for agents by responding solely to shocks, as in the fundamentals based RE rule, but with policy parameters that evolve over time. Through what we have called an EEFB rule, the policymaker is in fact responding to the endogenous evolution of the economy brought about by changing beliefs, and can thus avoid the instability associated with fundamentals based rules with constant coefficients.

We have also shown that an EEFB rule can be learned by the CB in a straightforward way, thus making it implementable without the need to obtain additional information about agents' expectations. This feature, we have argued, makes it superior to an expectations based rule. Combined learning dynamics of private sector and CB also converge, so that both sides can learn simultaneously the REE.

Our EEFB rule has also allowed us to understand better what are the necessary modifications in CB's responses to shocks called for by the non rationality of private sector's expectations: it turns out that the required modifications of the policy responses to shocks depend on the relative position of private sector's beliefs compared to their RE values, and that the policymaker in implementing his policy must prevent those beliefs from becoming self fulfilling.

Since beliefs are part of the structural model for an economy, having expectational parameters that evolve over time is like having continuous structural changes in the economy, which the policymaker should take into account and deal with. This phenomenon is more likely to happen after main changes in underlying factors have occurred, and less likely in periods of stability, when expectational parameters may not be far from rationality and relatively stable.

The necessary adaptation of the policy rule to changing beliefs of private agents can be done by directly incorporating private expectations into the policy rule, as suggested by Evans and Honkapohja (2003), or it can be done by responding solely to the exogenous variables, with policy parameters that evolve over time. In this second case, there is no need for the policymaker to obtain accurate information on private expectations and he can rely solely on past data to learn to implement an optimal policy and thus guide the economy towards equilibrium.

8 Figures

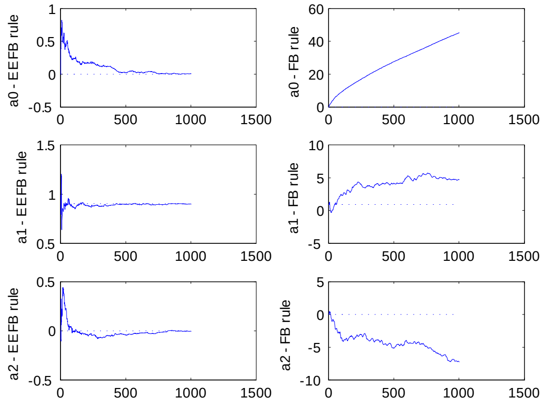

Figure 1: Evolution of beliefs when the CB implements the fundamentals based rule (7).

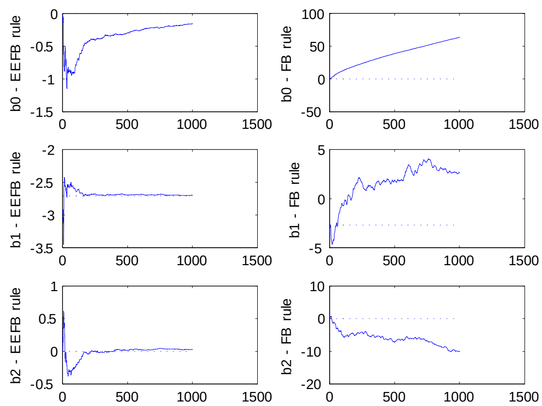

Figure 2: Evolution of beliefs when the CB implements the EEFB rule (21).

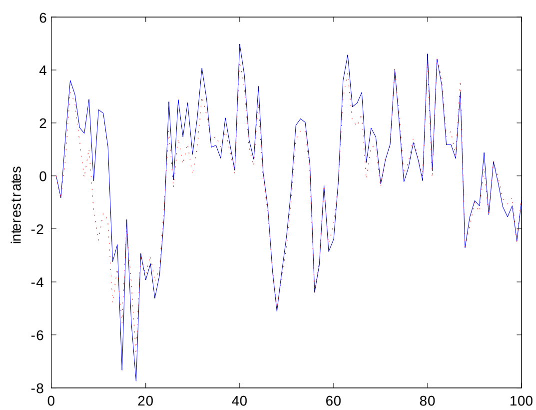

Figure 3: Interest rates implied by fundamentals based rule (dotted line) and EEFB rule (solid line).

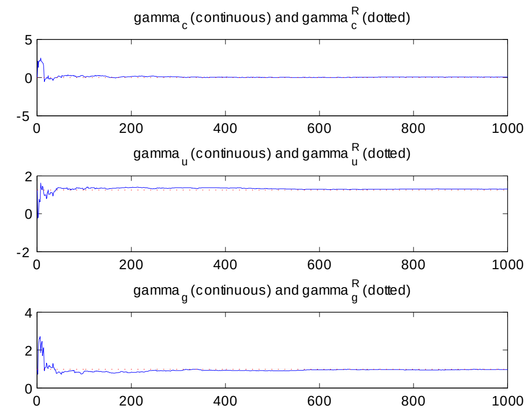

Figure 4: Policy parameters in the EEFB rule (continuous line) and fundamentals based rule (dotted line).

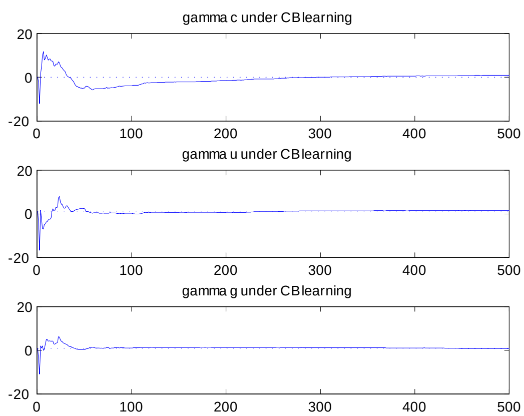

Figure 5: Policy parameters under CB learning.

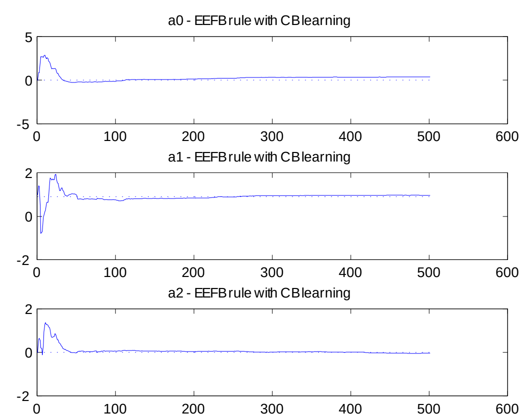

Figure 6: Private sector's beliefs (inflation equation) under CB learning.

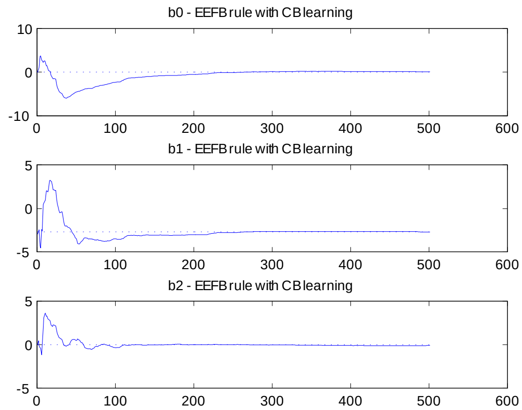

Figure 7: Private sector's beliefs (output equation) under CB learning.

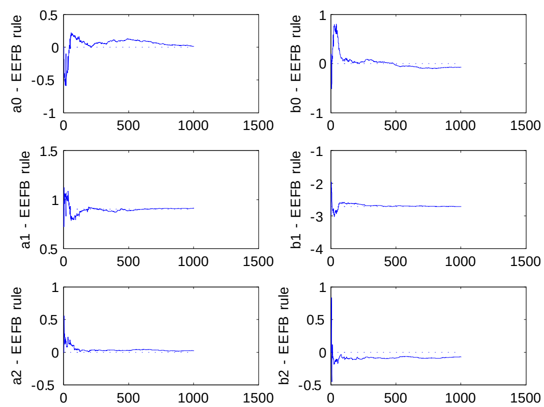

Figure 8: Evolution of private sector beliefs when CB is implementing policy rule (86) with random errors.

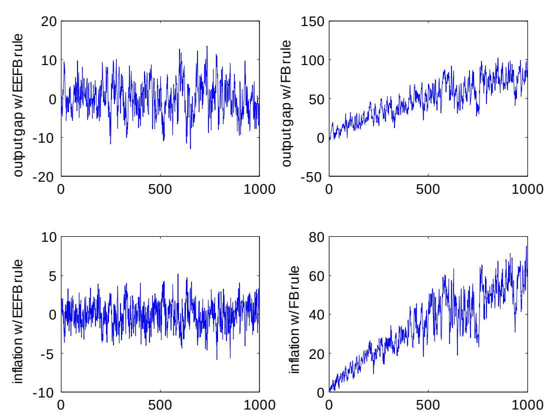

Figure 9: Inflation and output gap under the EEFB rule (left panles) and under the fundamentals based rule (right panels) when agents are learning.

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