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A New Keynesian Model with Heterogeneous Price Setting

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Abstract

The Calvo contract pricing mechanism has become the most widely accepted microfoundation to the NK Phillips curve but unfortunately predicts that all firms in the economy face the same probability of price change. To better explain the stylized fact this paper relaxes the homogeneous firm assumption in the Calvo contract to provide a multi Calvo macroeconomic explanation more consistent with recently available microeconomic evidence, that suggests firms face differing probabilities of price change. A simple New Keynesian dynamic stochastic general equilibrium (DSGE) model with nominal rigidities, habit in consumption and heterogeneity in price change is estimated using Bayesian techniques for the US and finds evidence of a flexible price sector of around 6%. Furthermore, consistent with recent findings, this work finds that accounting for heterogeneity in price setting allows the model to explain greater and more persistent real effects to a monetary shock, but less so to a technology shock.

KEYWORDS: heterogeneity, sticky prices, persistence, DSGE model, bayesian estimation

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1 Introduction

The Calvo contract pricing mechanism has become the most widely accepted microfoundation to the NK Phillips curve but unfortunately predicts that all firms in the economy face the same probability of price change. To better explain the stylized fact this paper relaxes the homogeneous firm assumption in the Calvo contract to provide a multi Calvo macroeconomic explanation more consistent with recently available microeconomic evidence, that suggests firms face differing probabilities of price change. A simple New Keynesian dynamic stochastic general equilibrium (DSGE) model with nominal rigidities, habit in consumption and heterogeneity in price change is estimated using Bayesian techniques for the US area. The model type is deliberately chosen for its economical use of theoretical innovation and introduces heterogeneity in price setting via a stylized aggregate pricing mechanism that accounts for firms facing differing probabilities of price change. The model incorporates a flexible price sector, a rule of thumb price sector and a traditional Calvo sector. A set of restricted models, with the habit parameter set to zero, are then estimated using Bayesian techniques and compared to the results obtained from an unrestricted set to measure the innovation's ability to better channel inertia displayed in the data.The results predict a flexible sector of around 6% and show that controlling for heterogeneity in price setting can improve the overall model fit. Furthermore, for the unrestricted case, the innovations substantially increase the size and persistence of real effects to monetary shocks, consistent with Carvalho (2006) but reduce the dependance on technology as an explanation of the business cycle.

The recent financial turmoil caused by the global financial crises has driven a requirement for macroeconomic models that can explain larger and more persistent movements of output from its so called steady state. As a result the accurate description and modeling of real and nominal inertia displayed in the aggregate data appears increasingly important. Relying on nominal rigidities to explain real inertia therefore risks a miss-specification of the model in its description of the data. Despite the recent debate on the source of rigidity, problems with the accurate channeling of inertia remain unresolved, see Riggi and Tancioni (2010) and Blanchard and Gali (2008). Criticisms of the standard New Keynesian (NK) model and its inability to generate as much inflation persistence as that displayed in the stylized fact are well documented. Numerous adjustments to the NK Phillips curve using competing micro foundations, such as the time and state dependent pricing mechanisms of Calvo (1983), Taylor (1980) and Rotemberg (1982) and the addition of a backward looking element to complement the rational expectations argument in the Phillips curve to increase price rigidity have done little to resolve this issue, see Gali and Gerlter

(1999) and Sheedy (2010) for a microfounded hybrid NK Phillips curve.

Evidence from the Inflation Persistence Network, IPN, using Euro area micro data, and recently many others have questioned the ability of the widely accepted structural models to explain the micro level behaviour at all, see Fougere (2004), Angeloni (2006), Baumgartner et al (2005), Campbell and Eden (2007), Dias et al (2005), Nakamura and Steinnsson (2008) and Vasquez-Ruiz (2011). According to this micro level evidence, unconditional hazard functions of price changes are either decreasing, increasing or non uniform in the duration of price contracts. Contrary to these findings the widely accepted Calvo contract pricing mechanism, whilst appealing in its tractability and aggregate approximation, predicts a flat hazard rate of price change, suggesting that all firms face the same probability of price change regardless of contract duration due to the assumption of homogeneous firm types. The growing volume of micro level literature challenges this prediction and provides the motivation for this research.

This paper introduces a simple stylization to the aggregate price mechanism to capture heterogeneity in price setting thus relaxing the homogeneous firm assumption in the Calvo contract in an attempt to provide a macroeconomic model more consistent with this recent evidence. For some time the business cycle literature has considered the modelling of persistence in the NK framework and its consistency with the micro fact, Aoki (2001), Dixon and Kara (2010) and Alvarez and Burriel (2010). In fact, the use of heterogeneity to account for such evidence is becoming increasingly popular, see Carvalho (2006), Shamloo and Silverman (2010), and Yao (2011) who all find an increased level of persistence and a better explanation for money non neutralities from doing so. The modern breed of macro models require, at least, some explanation of the existence of a flexible price sector, or description of firms that change prices more frequently, an issue discussed in Smets and Wouters (2007), and one that this paper attempts to address. Section 2 presents a baseline New Keynesian dynamic stochastic general equilibrium model, essentially a derivative of Gali (2002), with sticky prices and habit in consumption, for the US, which is estimated using Bayesian techniques with three key economic variables: output, prices and the nominal interest rate. To drive output from its natural rate, we introduce three shocks to monetary policy, productivity and government expenditure. The key innovation is the simple relaxation of Calvo's assumption of homogeneous firm types using a simple stylized aggregate price mechanism to account for heterogeneity in price setting using multi Calvo agents with three differing probabilities of price change.

To the baseline model, which relies solely on sticky prices to explain the

The Inflation Persistence Network, (IPN), is a team of Eurosytem economists undertaking joint research on inflation persistence. The IPN is chaired by Frank Smets, European Central Bank business cycle, the inclusion of a flexible price sector intuitively reduces the ability of that model to explain inertia displayed by the data and highlights the issue of just how that rigidity should be channeled in the modern breed of macro models. There is an emerging literature which concerns itself with the subject of the source, or channeling, of inertia. Such a recent argument concerns itself with whether or not we should rely on nominal or real rigidities to provide the bulk of inertia in modern macro models, see Blanchard and Gali (2010) and Riggi and Tancioni (2010) for an extensive discussion. This recent deliberation on the source of persistence, combined with the problems presented by the microevidence, motivates us to qualify any model enrichment. The Lucas critique demands a theoretical micro-foundation to the structural equations which represent the supply and demand sides of the economy, but is this enough? Micro-foundations to aggregate representations should also accurately describe the micro level evidence, else risk an over estimation of a rigidity that the model is designed to capture. The policy implications drawn from model enrichment and innovation could, perhaps, be better justified if motivated by the stylized fact. Accordingly we estimate two sets of models; a restricted set with habit parameter set to zero which relies solely on the Calvo explanation to explain persistence, outside of the monetary rule, and an unrestricted set with non zero habit consumption allowing output persistence to reduce the reliance on the Calvo parameter.

The preference for a Bayesian approach to our estimation of the model is that it can utilize prior information, or beliefs, to characterize the posterior distribution of the models structural estimated parameters, a distinct advantage over other methods of estimating these structural model parameters, and additionally provide us with a posterior odds analysis to imply probabilities that can be assigned to competing models, even where models are not nested, although in this analysis the models are nested. We find that a simple baseline model that incorporates heterogeneous price setting can improve overall fit, or the ability to describe the inertia within the data, depending on the model type that the innovation is nested within. Our estimates predict a flexible price sector in the US of around 6% and a rule of thumb sector of around 55-70%. Although a model without the flexible price sector is preferred initially over the baseline case, the inclusion of habit in consumption to our model reverses this result so that the model with a flexible price sector is preferred over the baseline with habit. In both cases the estimated size of the flexible price sector is around 6%. Furthermore, for the models with habit, the innovations substantially increase the amplitude and persistence of monetary shocks, a finding consistent with other recent works in this area such as Carvalho (2006), Shamloo and Silverman (2010) and Yao (2011). The estimated impulse response functions also highlight a reduced role for technology.

The rest of this section discusses the literature on price change and the innovation in our analysis. Section 2 sets out a simple baseline New Keynesian model with sticky prices and habit in consumption which is developed into a DSGE with shocks to productivity, demand and monetary policy. Section 3 describes our Bayesian methodology for estimation of the models structural parameters and comparison of two sets of competing models; The restricted set (Models 1 and 2), with habit parameter restricted to zero; Model 1 with a flexible and rule of thumb price sector and Model 2, a baseline without innovation. The unrestricted set (Models 1H and 2H); share the same specifications respectively but with the habit parameter non zero. The final section discusses the results of our estimation, model comparison and the estimated impulse response functions for the models with habit in consumption.

1.1 The Literature on Price Change

The recent micro level research results provide us with an informed direction to improve model building beyond the answers to the Lucas critique, a source already commonly employed by recent business cycle literature for the formation of priors and calibrations before estimation of competing closed macro models. Most of the micro literature on price change, to date, uses the focus of the slope of the hazard function in price change. The hazard function in price duration could be defined as the probability, at a particular time t of a firm resetting its price as a function of the time since its last price change. If prices become more likely to change, the older they become, then the hazard function of price change would become upward sloping, the latter outcome providing Sheedy (2010) with the motivation for a micro founded hybrid New Keynesian Phillips curve. Angeloni et al (2006) use this hazard function approach to suggest that new micro evidence collected from the Eurosystem via the IPN seriously challenges the most commonly utilized assumptions in the current micro founded macro models. Unconditional hazard functions of price changes are decreasing in the duration of price spells, a fact which poses problems for both the standard state and time dependent model explanations of price persistence. They find that these factors are all significant heterogeneous factors behind price rigidities. Instead of the model of monopolistic competition a la Dixit and Stiglitz, they call for more complex tractable market structures. Dias et al (2005) estimate a hazard function for Portugal, but find that the frequency of price change tends to depend on sectoral heterogeneity, as some firms depend on state dependent factors and some on time dependent. A simple time dependent rule cannot provide a reasonable approximation to the data and thus state dependent models are required to fully characterize price setting behaviour of Portuguese firms. They find that the significant state variables are inflation, demand and size of previous price change. Controlling for this heterogeneity is one way to tackle this bias when estimating the hazard function. Aucremanne and Dhyne (2005) show that these unconditional hazard functions become flatter when one controls for heterogeneity. Alvarez, Burriel and Hernando (2005) show that a mixture of pure Calvo with different probabilities of price adjustment provide a good estimation of declining hazard rates. They assume an economy made up of several types of Calvo agents; a flexible group of Calvo agents (price duration 1 month), an intermediate set of Calvo Agents (10 months), one group of sticky agents (3 years), and one group with an annual Calvo type price setting mechanism (18 months). The growing literature provides further microeconomic evidence on the slope of the hazard function, in duration of price change; Baumgartner et al (2005) and Fougere et al (2004) for the Euro area, Nakamura and Steinsson (2008) and Campbell and Eden (2007), for the US, all find in favour of a downward sloping hazard function. Baumgartner et al (2005) find, in an Austrian study, that the aggregate hazard function for all price spells is decreasing with time, although they also find strong evidence of state dependent or heterogeneous effects on price change, an issue addressed by Carvalho (2006). Fougere et al (2004) find evidence of decreasing hazard functions and that controlling for sectoral heterogeneity removes this bias, providing a hazard function in line with the baseline theoretical models. Nakamura and Steinsson (2008), for the US, find that hazard functions are predominantly downward sloping for the first few months with a significant twelve month spike, which is more pervasive in producer prices rather than consumer prices. Campbell and Eden (2007), using US scanner data, find that the longer a nominal price remains unchanged the less likely it is to change, a finding consistent with a downward sloping hazard function. Cecchetti (1986) finds that the frequency of price adjustment is endogenously determined by the change in the level of prices, so that higher inflation leads to less price stickiness. Gotte et al (2005) provide micro evidence which suggests that, not only is the probability of price change endogenous, but the hazard function is upward sloping for Switzerland. A study which compares the sectoral hazard functions with the aggregate hazards is Vasquez-Ruis (2011) who finds upward sloping hazard functions which become downward sloping when aggregated. The reason for this is that, as time passes, the proportion of flexible firms yet to change price decreases and thus the aggregate hazard function will always be decreasing. As far as the author is aware there are no findings of a purely horizontal hazard function of price change, predicted by the widely adopted Calvo contract. Controlling for heterogeneity therefore provides a means to capture the micro evidence discussed above. One such study that employs this measure is that of Carvalho

(2006) who shows that allowing for sectoral heterogeneity not only addresses the concerns of the micro level literature but also produces larger and more persistent effects from monetary shocks than would be the case in a homogeneous firm price setting economy. Accordingly, an identical firm model would require a price changing frequency of up to three times higher than the average heterogeneous economy to approximate these dynamics. This study offers a far more parsimonious approach than that of Carvalho (2006) and Alvarez and Burriel (2010) that is motivated by the suggestion of Angeloni et al (2006), namely that a Calvo model extended to allow for sectors with differing degrees of price stickiness would provide a good approximation of a model built to account for the new facts. The model introduced in the following section introduces the notion of flexi price firms mixed with rule of thumb prices and the standard Calvo agent. The adoption of this innovation leaves the prediction of the Calvo contract essentially unchanged but this is reconciled with the evidence above, that controlling for heterogeneity is consistent with flatter aggregated hazard functions of price change.

1.2 Relaxing the Assumption of Homogeneity

Drawing monetary policy conclusions from the presented micro evidence requires structural models consistent with that evidence, but these models should also be analytically tractable. Angeloni et al (2006) suggest a basic Calvo model extended to describe the heterogenous factors above would not be a bad approximation. The difficulty is highlighting the important micro features, or states, that affect the macro outcome. Alvarez et al (2005) show that a good description of the declining hazard rate can be achieved by mixing heterogeneous Calvo type agents. Carvalho (2006) suggests a model with multiple sectors with different degrees of stickiness.

The purpose of the innovation in this paper is to relax Calvo's assumption of the homogeneous firm through stylization of the aggregate price mechanism to include three firm types; the first type facing perfect price flexibility, a second with fixed or sluggish price change and the remainder awaiting the standard Calvo signal to change to their optimal price. By introducing heterogeneous firm types via the aggregate pricing mechanism we can not impose a non horizontal hazard function of price change, but can accommodate and estimate three sectors with different probabilities of price change, with the aim of encompassing the micro fact.

One such proposal to capture the differing probabilities of price change within the standard New Keynesian framework is presented below where a proportion ζ of firms face perfect price fluidity, a portion η face rule of thumb prices, and the remainder $(1-\zeta-\eta)$ follow a standard Calvo type price setting.

2 A Simple New Keynesian Model

Our model is essentially a derivative of Gali (2002), with no capital and a perfectly competitive labour market. A cashless economy where homogeneous goods are produced by a final goods sector using CES technology. Price inertia in our model is explained using nominal rigidities a la Calvo (1983) and output inertia by habit formation in consumption. In the model output is driven from its natural rate by shocks to monetary policy, productivity and government expenditure. Our innovation is the addition of flexible and rule of thumb price sectors, alongside the convenient Calvo explanation, to the aggregate price mechanism to control for the differing probabilities of price change highlighted by the recent evidence shown in the micro level literature on price change.

2.1 Preferences

The economy consists of a continuum of representative infinitely lived households whose instantaneous utility function is separable in consumption $C_t(i)$ and labour supply $N_t(i)$. As a result the first order condition for consumption growth will be independent of labour supply effects, as is consistent with the observed relative stability of labour supply in the US. We use habit formation in consumption as a real persistence mechanism and to reduce the reliance on the Calvo contract explanation of inertia displayed in the US data. Following the results of Levine, Pearlman and Yang (2008) the inclusion of persistence in labour supply is omitted to avoid over enrichment of the model.

The instantaneous utility function is given by

$$U(C_t, N_t) = \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

where σ is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution and φ is the elasticity of work effort with repect to the real wage or the inverse of the Frisch elasticity of labour supply. The parameter h represents the proportion of external habitual consumption or desire to herd.
Households seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

subject to an intertemporal budget constraint of the form

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{2}$$

where $P_t(i)$ is the price level $C_t(i)$ the consumption of differentiated good, i, respectively. Households hold their wealth in the form of securities and accordingly Q_t represents the price of the riskless, one period, bond; B_t is the holdings of risk free bonds at the beginning of period t. W_t is the nominal wage and N_t a measure of households employed. T_t is the lump sum component of net dividend income and taxation.

Maximising (1) subject to (2) yields the familiar Euler equations:

$$\frac{N_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma}} = \frac{W_t}{P_t} \tag{3}$$

$$Q_t = \beta E_t \left[\left(\frac{(C_{t+1} - hC_t)}{(C_t - hC_{t-1})} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{4}$$

The first equation represents the household's labour supply decision, equating the real wage with the marginal rate of substitution between consumption and work effort. The second is the familiar Keynes-Ramsey rule which relates the expected future path of consumption to the real interest rate.

2.2 Technology

Each firm produces a differentiated good i with labour the sole input and identical exogenous technology A_t assumed to evolve over time. Aggregate supply, then, evolves according to the product:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{5}$$

In this CES production function the parameter α is the elasticity of output with respect to labour. We also assume that the labour market is perfectly competitive and wages are fully flexible.

All firms face the isoelastic demand schedule

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

The parameter ε is the elasticity of substitution between differentiated goods, or the elasticity of demand and $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ is the Dixit-Stiglitz aggregate price index.

In an environment of monopolistic competition firms choose a price to maximise discounted future profits

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}(P_t^* Y_{t+k/t} - \Psi_{t+k}(Y_{t+k/t}))]$$

subject to the demand constraint

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

which is derived from the maximization of the Dixit Stiglitz consumption index subject to to any given level of expenditure. The expression $\Psi(\cdot)$ represents the implicit form of the firms's cost function, P_t^* the firm's target price and Q_t the discount factor for nominal payoffs. Note that the constraint faced by the firm is identical and therefore the price that they target will be the same.

The familiar solution to this problem can be expressed as:

$$\frac{P_t^*}{P_{t-1}} = \mathcal{M} \frac{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t} MC_{t+k|t} \Pi_{t-1,t+k}\right]}{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t}\right]} \tag{6}$$

Equation (6) expresses the relative target price for the optimizing firm as a weighted average of the expected path of current and future marginal cost.

Marginal cost can be defined as the ratio of the real wage to the marginal product of labour, written explicitly as:

$$MC_t = \frac{W_t}{(1-\alpha) A P_t N_t^{-\alpha}}$$

For later use it is convenient to rewrite this expression, using (3) and (5):

$$MC_t = \frac{N_t^{\varphi+\alpha}}{(1-\alpha) A (C_t - hC_{t-1})^{-\sigma}} \tag{7}$$

2.3 An Alternative Aggregate Price Level

Nominal rigidities are introduced in a manner consistent with much of the recent business cycle literature, using the model of Calvo (1983). Firms may reset their prices only when they receive a randomly set signal, generated with constant probability $1-\theta$ in any given period. Thus the remainder of firms that are unable to change price in any period can be considered the portion of sticky price firms in the economy. The aggregate price level thus satisfies

$$P_t = [\theta(P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{8}$$

For later use, dividing through by P_{t-1}, this expression becomes convenient to write as

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

The innovation in this paper is the relaxation of the assumption of homogeneous firms. By introducing a further two sectors to the baseline pure Calvo case we can accommodate the recently discussed evidence: imagine one sector of firms facing flexible price in all time periods and another with prices fixed to the previous period. The remainder face the typical Calvo type price mechanism, we can not impose a downward or upward sloping aggregate hazard function of price duration upon the model and so the probability of price change will remain constant, but now include three different probabilities. The notion of including different price sectors in a New Keynesian framework is not new. In fact, one such work involving the inclusion of a flexible and sticky price sector is described in Aoki (2001), who introduces the concept of a twin sector economy using a dynamic sticky price model in two sectors of production. To avoid model enrichment, this work takes a more parsimonious approach to encompass heterogeneous price setters. This delineation can be achieved through a simple stylization of the aggregate pricing mechanism. Gali and Gertler (1999) split the aggregate price mechanism in this manner to provide an explanation for a set of firms with backward looking behaviour to complement the standard Calvo agents in a model of this type.

To capture heterogeneity in price setting, imagine a portion (ζ) of firms outside the Calvo set up which face perfect price fluidity, and a further set of firms η that face fixed prices and the remaining proportion of firms facing the Calvo type price setting as $(1-\zeta-\eta)$. It is convenient to assume that $\zeta \in [0,1]$ and $\eta \in [0,1]$.

Allowing this framework to capture these micro facts at the aggregate level, the aggregate price mechanism now becomes:

$$P_t = \left[\zeta(P_t^*)^{1-\varepsilon} + \eta(P_{t-1})^{1-\varepsilon} + (1-\zeta-\eta)\left(\theta(P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right)\right]^{\frac{1}{1-\varepsilon}}$$

Note that in the trivial case that $\zeta = \eta = 0$, this price mechanism collapses down to the baseline, later referred to as Model 2 and 2H, which is the case that would result from the aggregate price mechanism given previously.
Rearranging and dividing through by $P_{t-1}^{1-\varepsilon}$ yields a similar expression for later use

$$\Pi_t^{1-\varepsilon} = (\eta + (1-\zeta-\eta)\theta) + (\zeta + (1-\zeta-\eta)(1-\theta))\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} \quad (9)$$

2.4 Equilibrium

The market clearing conditions in the goods market and labour market are given as:

$$Y_t = C_t + G_t$$

and

$$Y_t = \frac{A_t N_t}{D_t}$$

where G_t is government expenditure, also interpreted as the exogenous element of aggregate demand, and A_t productivity, the exogenous element of aggregate supply. In our model both variables are responsible for driving output away from its natural rate.

Finally market clearing in the labour market implies

$$N_t = \int_0^1 N_t(i)di$$

Assume further that government expenditure is met by lump sum taxes, then, by Walras' Law we can dispense with the government budget constraint or bond market equilibrium condition

2.5 The Log Linearized Model

A log linearized zero-inflation steady state, where lowercase variables describe proportional deviations from the deterministic steady state is given below, see appendix 4B for full derivation:

Combine the linearized versions of equation (6) and (9) to get the Phillips curve:

$$\pi_t = \frac{\beta\theta}{(\eta+\theta)(1-\zeta)}E_t(\pi_{t+1}) + \frac{(\zeta+(1-\zeta-\eta)(1-\theta))}{(\eta+\theta)(1-\zeta)}(1-\theta\beta)\Theta\hat{m}c_t$$

where $\Theta \equiv \frac{(1-\alpha)}{(1-\alpha+\alpha\varepsilon)} \leq 1$. The inflation equation shows how our parameters of interest ζ and η co-govern the sensitivity of inflation to changes in marginal cost alongside the price stickiness parameter in the Calvo contract θ. A larger value for ζ increases the slope of the Phillips curve whereas larger values of η decrease the slope, as we would expect given the nature of the sectors that the parameters represent. It is worth noting that when ζ and η are both equal to zero this inflation equation collapses down to the baseline New Keynesian Phillips Curve which would result from the standard aggregate pricing mechanism given by equation (8).

Marginal cost is given from the linearization of equation (7)

$$mc_t = \frac{\sigma}{(1-h)}(c_t - hc_{t-1}) + (\varphi+\alpha)n_t - a_t$$

and equation (5) for aggregate supply

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

See the appendix 4B for the derivation of the consumer's Euler equation from (4)

$$c_t = \frac{h}{(1-h)}c_{t-1} + \frac{1}{(1-h)}E_t[c_{t+1}] - \frac{1}{\sigma}\frac{(1-h)}{(1+h)}[i - E_t[\pi_{t+1}] - \rho]$$

and from the goods market clearing condition we obtain

$$y_t = c_y c_t + (1-c_y)g_t \qquad where \;\; c_y = \frac{C}{Y}$$

In order to introduce a simple analysis of monetary policy and to provide a channel for a monetary shock to drive output from its natural rate we use a simple Taylor rule. As is consistent with the literature and to generate persistence from monetary shocks we include a smoothing parameter ρ_e, following from Clarida et al (2000).

$$i_t = \rho_e i_{t-1} + (1-\rho_e)(\phi_\pi \pi_t + \phi_y \tilde{y}_t) + e_t$$

The linearized model is completed with three exogenous shocks: government expenditure, productivity and monetary. All processes follow first-order autoregressive processes with i.i.d normal disturbances:

$$g_{t+1} = \rho_g g_t + \epsilon_{g,t+1}$$
$$a_{t+1} = \rho_a a_t + \epsilon_{a,t+1}$$
$$e_{t+1} = \rho_e e_t + \epsilon_{e,t+1}$$

3 Bayesian Estimation

Bayesian estimation offers a useful tool to estimate and evaluate dynamic stochastic general equilibrium models. The aim of implementing this methodology is to characterize the posterior distribution of the models parameters conditional on prior beliefs of the estimated parameters, a distinct advantage over other methods of estimating these structural models.
The posterior distribution is obtained by employing the Bayes rule:

$$p(\theta/Y^T) = \frac{L(Y^T|\theta)p(\theta)}{\int L(Y^T|\theta)p(\theta)d\theta} \propto L(Y^T|\theta)p(\theta)$$

gives the Bayesian relationship between the posterior density, $p(\theta/Y^T)$, the unconditional sample density, $\int L(Y^T|\theta)p(\theta)d\theta$, and the prior density, $p(\theta)$. The posterior density evolves from a weighted average of prior non sample information and the conditional densities. These weights are related to the variances of the prior distributions and the data. A tighter prior, therefore, will result in a more constrained, and perhaps less informative, estimation. The parameters are estimated by maximizing the likelihood function and then combining with the prior distributions of the parameters in the model, to form the posterior density functions. The posterior distributions are then optimized using Monte-Carlo Markov Chain (MCMC) simulation techniques. Under the Bayesian perspective, both the posterior distribution and the likelihood function can be utilized to obtain a probabilistic interpretation of the estimated parameters. Another advantage of this methodology is the ability to make model comparisons, even where the models are not nested, using posterior odds analysis, conveying relative probabilities to competing models. We make use of log likelihood race statistics to compare our model fit with the case of the baseline model, where $\zeta = \eta = 0$.

A number of structural parameters are kept fixed during the estimation in order to identify them separately. Obviously our estimation results are sensitive to this calibration; but we justify this by assuming these values are estimated, equivalently, with a prior that exhibits a zero standard deviation. For our calibrated values we proceed in a manner which is consistent with quarterly data observations. For the Calvo parameter we use the consensus estimate of 0.68 consistent with an average price contract duration of three quarters. For the preference parameters in our model we assume values commonly found in the business cycle literature. The discount factor β is set to 0.99 which is congruous with a real interest rate of about 4%. The elasticity of labour supply φ is set to unity, following Christiano et al (2005), which is between the more commonly used values in DSGE models and those estimates in the micro labour literature. The elasticity of demand, ε, is a crucial parameter in our analysis as it primarily governs the sensitivity of inflation to marginal cost. Ellis (2006) provides empirical evidence of this parameter which is rather sensitive to model specification and remarks that assuming a constant value may be too restrictive, an observation addressed by Smets and Wouters (2003) who model the elasticity as a time varying stochastic process. However, our analysis assumes a constant markup and accordingly, we set this parameter to 6, following Blanchard and Gali (2010) although this is markedly lower than that used by Krause et al (2008) who set it high to address the sensitivity of inflation to marginal cost. The labour income share in the production function $(1 - \alpha)$ is set at 0.30. The distinguishing parameters of Models 1 and 1H are the proportion of flexi price firms, ζ, and the proportion of rule of thumb price firms, η. Both of these parameters are arbitrarily set with prior means of 0.30 and standard deviation 0.2 representing a relatively loose prior. The posterior mean of these parameters will provide us with an estimate of the size of the flexi and rule of thumb price sectors in the US.

The data used for the estimation is US quarterly macro economic time series: real GDP, GDP deflator and the nominal interest rate from 1970:1 to 2004:1. As the log linearised steady state solution represents deviations from their natural rate, time series are detrended using a linear trend and converted to quarterly rates. The choice of prior distributions for the Bayesian estimation of DSGE models matters both for posterior values and for model comparison. The views on priors varies considerably among commentators and, unfortunately, the facts described by the aggregate data are unable to discriminate amongst these views. One approach, suggested by Del Negro and Schorfheide (2008), to aid with this discrimination is the use of micro data studies, but work still needs to be done to show how the facts displayed by the micro data should relate to the macro picture, where much micro deviation is washed out in aggregation. As a result we are left to draw on the existing literature for the prior specification. The means and standard errors of the technology and government spending shocks are set with a mean of 0.85 and standard error 0.07, (monetary shock 0.75, 0.15). The corresponding innovations are harmonised, as in Smets and Wouters (2007) and consequently share a mean of 0.25 (monetary shock 0.05) and standard error of 2.00 representing fairly loose priors. The risk aversion parameter we also follow with a mean of 1.50 and a standard error of 0.375. For monetary policy we follow Levine, Pearlman and Yang (2008) so that the interest rate smoothing parameter has a mean of 0.75, an inflation feedback consistent with a robust FED response to inflation of 1.70 and an output feedback mean of 0.50.

We estimate the following model variants: The restricted models with $h = 0$ (Models 1 and 2), Model 1 with the parameters of interest, $\zeta > 0$ and $\eta > 0$, and the baseline case, Model 2, with $\zeta = \eta = 0$. The unrestricted models (Models 1H and 2H) follow the same set up, but with $h \neq 0$. The estimation is carried out in DYNARE (Matlab version) programme, see Juillard (2006) and the resulting posterior means and confidence internals can be found in Table 2. The estimated risk aversion parameter is greater than one in all unrestricted models, particularly Model 1H (1.558) as is consistent with empirical evidence. The estimate for the consumption habit parameter is high across both estimates of the unrestricted models which is most likely due to the simplicity of the model set up. The parameters characterizing monetary policy are stable across all models and close to the prior means; ρ_π in particular describing a predictably strong response from the Federal Reserve to the deviation of inflation from target. The interest rate smoothing parameter ρ_i is substantial as is the persistence of the productivity and demand shocks, consistent across all models. The monetary shocks are much less persistent than expected in both the restricted models, questioning their ability to contribute towards fluctuations in the business cycle, but considerably more persistent for the unrestricted model with innovation and habitual consumption, (Model 1H), a finding consistent with Carvalho (2006). The estimate for the size of the rule of thumb price sector is between 55% and 70% depending on model specification and the estimate for the flexible price sector around 6.5% consistently. The bottom line of Table 2 reports the log marginal density of the estimation of each model, indicating a preference for Model 1H, with habitual consumption and rule of thumb price sector. The most striking result, however, is given when comparing the Models with flexible price sector innovation and the baseline case. Without habit the baseline (Model 2) is preferred to the model with full innovation (Model 1). With habit this result is reversed and the model with full innovation is preferred (Model 1H). In summary, the innovation improves the model fit when provided with an alternative channel to emulate the inertia displayed in the data and also addresses the issue raised by the micro level literature, capturing different probabilities of price change.

3.1 Posterior Impulse Response Analysis

The impulse response functions (IRFs) implied by the model estimates outline the contribution to the aggregate dynamics of the three exogenous shocks in the unrestricted models with habit formation. Figure 1. plots the mean responses of the inflation rate, output and the interest rate to a one standard deviation shock in terms of percentage deviation from the steady state. Generally the responses depicted are consistent with a model based on the New Keynesian DSGE framework. A positive monetary policy shock has a contractionary effect on output and inflation and a positive technology shock has a negative effect on inflation due to the lowering of marginal cost. The responsibility for the hump shaped response of output lies with habit formation in consumption. A positive fiscal policy shock boosts output but causes the monetary authority to raise the interest rate, consistent with the familiar 'crowding out'. We now turn to the dynamics displayed by the innovations designed to capture the characteristics displayed by the heterogeneous economy. For monetary policy we see sizable differences in the real effects across model types. For the model with innovation, Model 1H, we see greater and more persistent real effects to ouput and a more muted response to inflation than to its baseline counterpart showing that heterogeneity delivers a source of rigidity through the distortion to the distribution of prices in the economy. Flexi price firms dominate the adjustment of the economy to a shock. Further we see that for a technology shock the impulse responses depict the opposite. The models with innovation deliver a more muted response to a technology shock which can be partly explained by the fact that technology is shared across sectors therefore has no effect on relative prices.

4 Conclusion

In this paper we have discussed the Calvo contract pricing mechanism, which has become the most widely accepted micro-foundation to the NK Phillips curve, and its prediction of a flat hazard rate caused by the assumption of homogeneous firm types. To better explain the stylized fact displayed in the micro data the aggregate price mechanism includes an innovation to relax the homogeneous firm assumption associated with the Calvo contract. With this aim in mind we have estimated a baseline New Keynesian DSGE model adapted to account for a flexible price sector, and a rule of thumb price sector to account for firms that are unable to change price in each period. The inclusion of these sectors is motivated by recent micro level literature that suggests that firms face differing probabilities of price change. The innovations to the aggregate price mechanism also allow for the control of heterogeneity in price setting implied by this recent microeconomic literature with the additional benefit of providing an estimate of the size of these sectors. The results predict the size of the flexible price sector to be around 6.5% and the size of the rule of thumb price sector to be around 55% to 70%. The estimate of the former being consistent across both model specifications.

The inclusion of a flexible price sector in the restricted model gives an entirely predictable outcome of a worsening of fit, when compared to the baseline case, as the model specification relies solely on sticky prices to explain the inertia displayed in the data. Most interestingly, this result is reversed when accounting for persistence in output and a model with both innovations is preferred over its baseline counterpart, suggesting an important role for the sector in the allocation of inertia in the model. Furthermore, controlling for heterogeneity in this manner, provides a result consistent with Carvalho (2006), who finds greater persistence in monetary shocks. Consequently, this paper also finds a reduced role for technology. The innovation outlined provides an alternative to the homogeneous firm type in the Calvo contract, capturing differing probabilities of price change reflected in the micro level literature. It is worth noting that the innovation also predicts a flat hazard rate of price change, but this can be reconciled with the micro evidence that suggests that controlling for heterogeneity in price change tends to flatten the aggregated hazard function. This study has shown that it is possible to capture the micro level behaviour in a parsimonious fashion and highlights the need for micro-foundations to better serve the fact they are introduced to represent.

References

[1] Alvarez, L., Burriel, P., Hernando, I. (2005). Do decreasing hazard functions for price changes make any sense?, ECB Working Paper Series, No. 461.

[2] Alvarez, L., Burriel, P., (2010). Is a Calvo Price Setting Model Consistent with Individual Price Data?, *The BE journal of Macroeconomics*, Vol. 10, Article 13.

[3] Angeloni, I., Aucremanne, L., Ehrmann, M., Gali, J., Levin, A., Smets, F. (2006). New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modelling. *Journal of the European Economic Association*, 4(2-3), 562-574.

[4] Aoki, K. (2001). Optimal monetary policy responses to relative-price changes. *Journal of Monetary Economics*, Vol. 48, 55-80.

[5] Aucremanne, L., Dhyne, E. (2005). Time-Dependent Versus State-Dependent Pricing A Panel Data Approach To The Determinants of Belgian Consumer Price Changes. ECB Working Paper Series, No. 462.

[6] Baumgartner, J., Glatzer, E., Rumler, F., and Stiglbauer, A. (2005). How frequently do consumer prices change in the Austria? Evidence from micro CPI data. ECB Working Paper, No. 523.

[7] Blanchard, O., Gali, J. (2008) A New Keynesian Model with Unemployment. CFS Working Paper No. 20007/08.

[8] Calvo, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, Vol. 12, 383-398.

[9] Campbell, J., Eden, B. (2007). Rigid prices: evidence from U.S. scanner data. Federal Reserve Bank of Chicago. Working Paper Series, No. WP-05-08.

[10] Carvalho, C. (2006). Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks. *The B.E Journal of Macroeconomics*, Vol. 2, Iss. 1, Article 1.

[11] Cecchetti, G., (1986). The frequency of price adjustment: A study of the news stand prices of magazines. *Journal of Econometrics*, Vol. 31, 255-274.

[12] Choudhary, A., Karlsson, T., Zoega, G. (2009). Survey Evidence on Customer Markets. Birkbeck Working Papers in Economics and Finance, 0916, Department of Economics, Mathematics and Statistics.

[13] Christiano, L., Eichenbaum, M., Evans, C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, Vol. 113(1), 1-45.

[14] Clarida, R., Gali, J., Gertler, M. (2000). Monetary rules and macroeconomic stability: evidence and some theory. *Quarterly Journal of Economics*, Vol. 115, 147-180.

[15] Dejong, D., Dave, C. (2007) *Structural Macroeconomics*, Princeton University Press.

[16] Dias, D., Robalo Marques, C., Santos Silva, J. (2005). Time or State Dependent Price Setting Rules? Evidence From Portuguese Micro Data. ECB Working Paper Series, No. 511.

[17] Dixon, H., Kara, E. (2010). Can we explain inflation persistence in a way that is consistent with the microevidence on nominal rigidity. *Journal of Money Credit and Banking*, Vol. 42, 151-170.

[18] Ellis, C. (2006). Elasticities, markups and technical progress: evidence from a state-space approach. Bank of England Working Paper, No. 300.

[19] Fougere, D., Le Bihan, H., Sevestre, P. (2004). Calvo, Taylor and the estimated hazard function for price changes. Banque de France Working Paper Series, Nov. 2004.

[20] Gali, J. (2002). New Perspectives on Monetary Policy, Inflation and the Business Cycle. CEPR Discussion Papers, No. 3210.

[21] Gali, J. (2008) *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press.

[22] Gali, J., Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, Vol. 44, 195-222.

[23] Gotte, L., Minsch, R., Tyran, J. (2005). Micro evidence on the adjusmtment of sticky- price goods: Its how often not how much. CEPR Discussion Paper, 5346.

[24] Juillard, M. (2006) *Dynare, a program for solving rational expectations models*, Edition 2.6.1.1 for Dynare version 2.6.1, August 2003; Dynare version 3.064, March 2006

[25] Kara, E., Dixon, H. (2005) How to Compare Taylor and Calvo Contracts: a comment on Michael Kiley. CDMA Working Paper Series, No. CDMA05/04

[26] Krause, M., Lopez-Salido, D., Lubik, T. (2008). Do search frictions matter for inflation dynamics? *European Economics Review*, Vol. 52, 1464-1479.

[27] Levine, P., Pearlman, J., Yang, B. (2008). The Credibility Problem Revisited: Thirty Years on from Kydland and Prescott. *Review of International Economics*, Volume 16 Issue 4, 728-746.

[28] Nakamura, E. and Steinsson, J. (2007). Five facts about prices: A reevaluation of menu cost models. Harvard University Working Paper.

[29] Riggi, M., Tancioni, M. (2010) Nominal vs Real Wage Rigidities in New Keynesian Models with Hiring Costs: a Bayesian Evaluation. *Journal of Economic Dynamics and Control*, Vol. 34, 1305-1324.

[30] Rotemberg, J. (1982) Monopolistic Price Adjustment and Aggregate Output. *Review of Economic Studies*, 49, 975-984.

[31] Shamloo, M., Silverman, C. (2010) Price setting in a model with production chains: Evidence from sectoral data. IMF Working Papers, 10/82.

[32] Sheedy, K. (2010). Intrinsic Inflation Persistence. *Journal of Monetary Economics*, 88(1), 1-23.

[33] Smets, F., Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association*, 1123-1175.

[34] Smets, F., Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, Vol. 97(3), 586-606.

[35] Taylor, J. (1979). Staggered Wage Setting in a Macro Model.*The American Economic Review*, Vol. 69.

[36] Taylor, J. (1980). Aggregate Dynamics and Staggered Contracts. *The Journal of Political Economy*, Vol. 88.

[37] Vasquez-Ruiz, H. (2011). Price stickiness and the hazard rate for price changes: evidence from qualitative survey data. Working Paper. Georgia State University.

[38] Yao, F. (2011). Price reset hazard function and inflation inertia. Working Paper, Universitat Erlangen-Nurnberg.

A Tables and Figures

Description	Notation	Prior Mean	Density	Std. Dev.
Preference Parameters				
Risk Aversion Parameter	σ	1.5	gamma	0.37
Innovations				
Proportion of Flexi Price Firms	ζ	0.3	beta	0.20
Proportion of Sticky Price Firms	η	0.3	beta	0.20
Habitual Consumption	h	0.7	beta	0.20
Shocks				
Persistence in Technology	ρ_a	0.85	beta	0.07
Persistence in Government Spending	ρ_g	0.85	beta	0.07
Persistence in Monetary Shock	ρ_e	0.75	beta	0.15
Sd of Technology Shock	ϵ_a	0.25	inv. gamma	2.00
Sd of Demand Shock	ϵ_g	0.25	inv. gamma	2.00
Sd of Monetary Shock	ϵ_e	0.05	inv. gamma	2.00
Monetary Policy				
Interest Rate Smoothing	ρ_i	0.75	beta	0.14
Output Feedback Parameter	ρ_y	0.5	normal	0.05
Inflation Feedback Parameter	ρ_π	1.7	normal	0.10

Table 1: Parameterisation of the Model

Description	Notation	Model 1	Model 2	Model 1H	Model 2H
Preference Parameters					
Risk Aversion Parameter	σ	0.987	0.685	1.5582	1.459
		[0.841-1.140]	[0.397-0.971]	[1.104-2.063]	[0.964-1.939]
Nominal Parameters					
Proportion of Flexi Price Firms	ζ	0.063	n/a	0.065	n/a
		[0.019-0.109]		[0.000-0.130]	
Proportion of Sticky Price Firms	η	0.547	n/a	0.707	n/a
		[0.486-0.600]		[0.617-0.799]	
Consumption Habit Parameter	h	n/a	n/a	0.960	0.940
				[0.914-0.999]	[0.875-0.996]
Shocks					
Persistence in Gov. Spending	ρ_g	0.853	0.808	0.815	0.767
Persistence in Technology	ρ_a	0.866	0.937	0.788	0.868
Persistence of Monetary Shock	ρ_e	0.262	0.221	0.470	0.290
Monetary Policy					
Interest Rate Smoothing	ρ_i	0.865	0.844	0.914	0.856
Output Feedback Parameter	ρ_y	0.551	0.552	0.505	0.523
Inflation Feedback Parameter	ρ_π	1.570	1.543	1.638	1.610
Log Marginal Density		-331.03	-327.05	-305.37	-310.80
Confidence Intervals in Parenthesis					

Table 2: Results from Bayesian Estimation

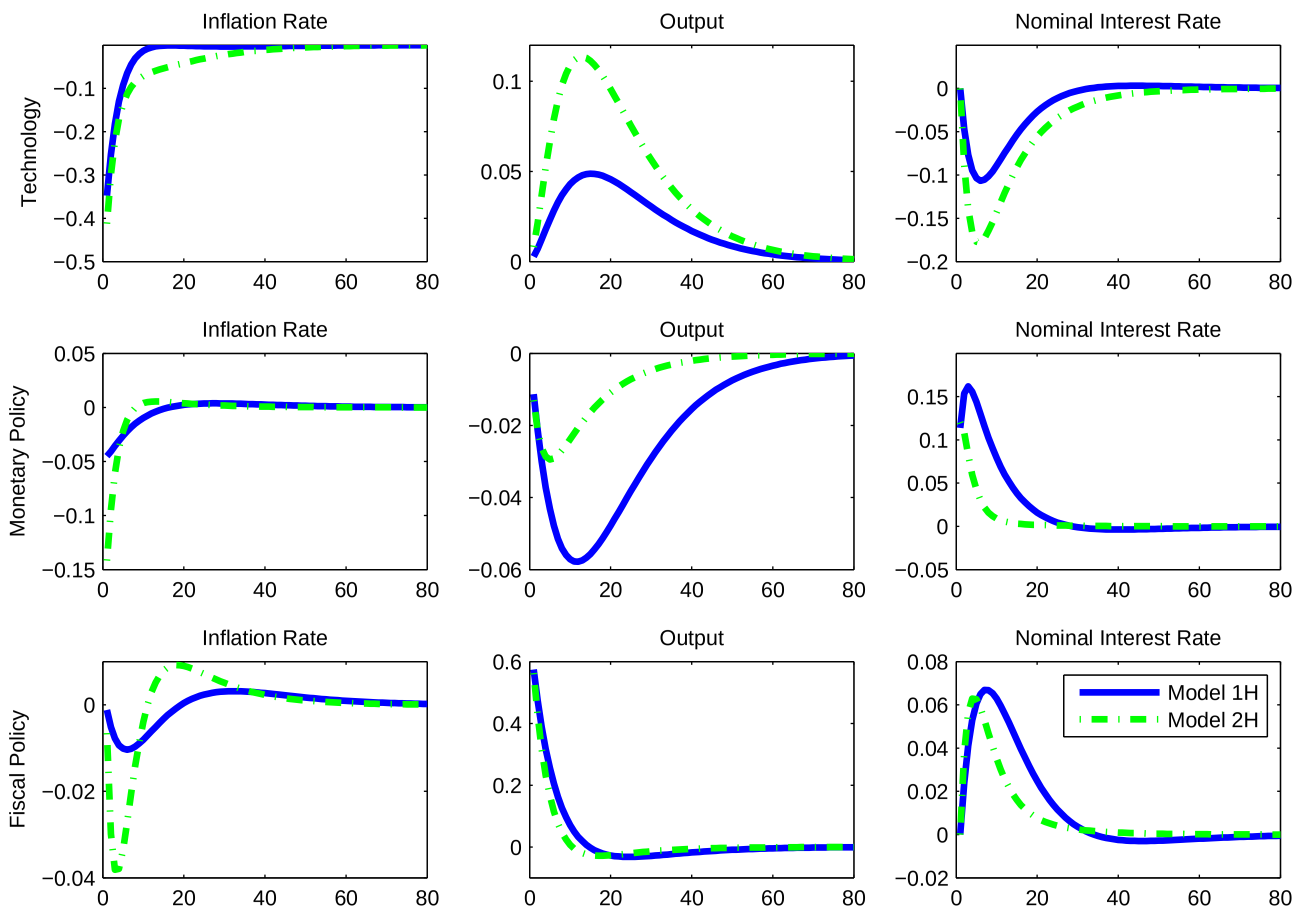

Figure 1: Estimated Impulse Responses

B Linerarisation of the Consumption Euler

To linearise the second optimality condition, first rearrange the second optimality condition:

$$\Pi_{t+1} = \frac{\beta}{Q_t} E_t \left[\left(\frac{(C_{t+1} - hC_t)}{(C_t - hC_{t-1})} \right)^{-\sigma} \right]$$

Taking logs of both sides

$$E_t[\pi_{t+1}] = log\beta - logQ_t - \sigma log E_t \left[(C_{t+1} - hC_t)\right] + \sigma log E_t \left[(C_t - hC_{t-1})\right]$$

If we write $z_t = E_t[\pi_{t+1}]$ then a simple Taylor first order approximation yields

$$\begin{aligned} z_t \;\cong\; & log\bar{\beta} - log\bar{Q} - \sigma log E_t \left[(\bar{C} - h\bar{C})\right] + \sigma log E_t \left[(\bar{C} - h\bar{C})\right] \\ & + E_t \frac{\partial z_t}{\partial logQ_t} \left[logQ_t - log\bar{Q}\right] \\ & + E_t \frac{\partial z_t}{\partial logC_{t+1}} \left[logC_{t+1} - log\bar{C}\right] \\ & + E_t \frac{\partial z_t}{\partial logC_t} \left[logC_t - log\bar{C}\right] \\ & + E_t \frac{\partial z_t}{\partial logC_{t-1}} \left[logC_{t-1} - log\bar{C}\right] \end{aligned}$$

Evaluating the derivatives at the steady state

$$\frac{\partial z_t}{\partial logQ_t} = \frac{\partial z_t}{\partial Q_t} \frac{\partial Q_t}{\partial logQ_t} = \left[-1 \frac{1}{\bar{Q}}\right] \bar{Q} = -1$$

$$\frac{\partial z_t}{\partial logC_{t+1}} = \frac{\partial z_t}{\partial C_{t+1}} \frac{\partial C_{t+1}}{\partial logC_{t+1}} = \left[-\sigma \frac{1}{(1-h)\bar{C}}\right] \bar{C} = -\sigma \frac{1}{1-h}$$

$$\frac{\partial z_t}{\partial logC_t} = \frac{\partial z_t}{\partial C_t} \frac{\partial C_t}{\partial logC_t} = \left[-\sigma \frac{-h}{(1-h)\bar{C}} + \sigma \frac{1}{(1-h)\bar{C}}\right] \bar{C} = \sigma \frac{1+h}{1-h}$$

$$\frac{\partial z_t}{\partial logC_{t-1}} = \frac{\partial z_t}{\partial C_{t-1}} \frac{\partial C_{t-1}}{\partial logC_{t-1}} = \left[\sigma \frac{-h}{(1-h)\bar{C}}\right] \bar{C} = -\sigma \frac{h}{1-h}$$

Substituting the derivatives into z_t, and noting that the third and fourth terms sum to zero, yields

$$\begin{aligned} z_t \ &\cong \ log\bar{\beta} - log\bar{Q} \\ &+E_t[-1]\left[logQ_t - log\bar{Q}\right] \\ &+E_t[-\sigma\frac{1}{1-h}]\left[logC_{t+1} - log\bar{C}\right] \\ &+E_t[\sigma\frac{1+h}{1-h}]\left[logC_t - log\bar{C}\right] \\ &+E_t[-\sigma\frac{h}{1-h}]\left[logC_{t-1} - log\bar{C}\right] \end{aligned}$$

Simplifying and noting that $E(c_t) = c_t$ and $E(c_{t-1}) = c_{t-1}$ at t+1

$$E_t[\pi_{t+1}] = -\rho + i - (\sigma\frac{1}{1-h})E_t[c_{t+1}] + (\sigma\frac{1+h}{1-h})c_t - (-\sigma\frac{h}{1-h})c_{t-1}$$

from which we can write the linearised consumers optimality condition.

$$c_t = \frac{h}{(1-h)}c_{t-1} + \frac{1}{(1-h)}E_t[c_{t+1}] - \frac{1}{\sigma}\frac{(1-h)}{(1+h)}[i - E_t[\pi_{t+1}] - \rho]$$