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Intergenerational Transfers and Demographic Transition

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Intergenerational Transfers and Demographic Transition*

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Abstract

This paper presents an analysis of demographic transition based on the endogenous evolution of intergenerational transfers along an economy's endogenous path of development. Two-period-lived agents belonging to overlapping generations choose optimally their desired levels of consumption and fertility, together with their desired sizes of transfers to both parents and children. Parents are more efficient than children in producing output, but some parental time must be devoted to child-rearing. At low levels of development, fertility is high and the flow of net intergenerational transfers is from the young to the old. At high levels of development, fertility is low and the flow of net transfers is from the old to the young. These results accord strongly with empirical observations and the analysis may be seen as formalising, for the first time, a long-standing and well-respected hypothesis in the demographic transition literature.

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1 Introduction

It is now widely recognised that most, if not all, societies undergo a process of demographic transition during the course of their economic development. Broadly speaking, this process describes the evolution of an economy from a pre-modern (technologically-primitive) state of low per capita income with high rates of fertility and mortality to a post-modern (technologically-advanced) state of high per capita income with low rates of fertility and mortality. Explaining precisely how this process operates is one of the most fundamental objectives of demographic transition theory. Unsurprisingly, since reductions in mortality are relatively easy to explain (through improvements in hygiene, sanitation, medical knowledge and the like), it is the decline in fertility that has attracted the most attention and which occupies our interest in the present paper.[1]

Naturally, any theory of fertility behaviour rests on an underlying theory of the demand for children. In the literature on economic demography, such a demand is typically motivated in one or more of the following ways: individuals derive utility from the number of children they have (e.g., Blackburn and Cipriani 2002; Eckstein and Wolpin 1985; Zhang and Zhang 1998, 2001); individuals derive utility from the utility of their children (e.g., Barro and Becker 1989; Becker and Barro 1988; Zhang 1995); individuals derive utility from the consumption, or the characteristics of consumption, of their children (e.g., Bernheim and Ray 1987; Fan 2001; Kohlberg 1976); or individuals expect to receive old-age support from their children (e.g., Morand 1999; Raut 1991; Srinivasan 1988; Willis 1980). At the same time, child-bearing is not costless, but rather competes with other aims and objectives of individuals. In particular, child-rearing is viewed as a time-intensive household activity to which productivity improvements through capital accumulation and technological progress do not apply very much compared to market activity. This leads to the principal economic explanation for declining fertility rates as the increasing opportunity cost of child-rearing as development takes place. In some models this is also accompanied by the substitution of child quality for child quantity as parents prefer to invest more in the education (human capital) of a smaller number of offspring.

The demographic transition of an economy from high to low rates of fertility is typically associated with other notable trends. One of these is the shift in the direction of the flow of private intergenerational transfers of wealth

[1] Other stylised facts of demographic transition include greater life expectancy, later timing of births, greater clustering of births, higher levels of education and greater female participation in the workforce. For surveys of the demographic transition literature, see Ehrlich and Lui (1997) and Kirk (1996).

within families: at low levels of development, when family size is large, this flow of lineage wealth tends to run from the young to the old; at higher levels of development, when family size is smaller, the flow tends to be reversed, running from the old to the young.[2] To some observers, these events are more than a coincidence and provide strong support for the hypothesis that changes in fertility behaviour are inextricably linked to changes in the pattern of intergenerational transfers associated with various socio-economic developments along an economy's growth path. The first, most fully-articulated, account of this hypothesis is credited to Caldwell (1976, 1978, 1982). Intuitively, given that the flow of net transfers is from children to parents, then it may be rational for parents to have large numbers of children, especially if the costs of child-rearing are low, as they are in traditional societies. Conversely, given that the net transfer flow is from parents to children, then parents may find it optimal to have small numbers of children, particularly if child-rearing costs are high, as they are in advanced societies.

Like fertility behaviour, intergenerational transfers - be they gifts from children to parents or bequests from parents to children - have been modelled in a number of different ways which may or may not reflect some degree of altruism on the part of individuals. In some models individuals are treated as being members of infinitely-lived dynastic households, having fully altruistic utility functions which ultimately depend on the welfare of all of their descendants and/or the welfare of all of their ancestors (e.g., Abel 1987; Barro and Becker 1989; Becker and Barro 1988). In other models it is assumed that individuals are only semi-altruistic, deriving utility from the consumption of their offspring and/or the consumption of their parents, or even simply from the sizes of transfers they make (e.g., Andreoni 1989; Bernheim and Ray 1987; Kohlberg 1987; Nishimura and Zhang 1995; Zhang and Nishimura 1993). From a non-altruistic perspective, transfers may arise out of pure self-interest because of perceived mutual benefits of exchange between progenitors and progeny (e.g., Bernheim *et al.* 1985; Cox 1987). Alternatively, the existence of bequests may simply be an accident of precautionary savings

[2] There are, of course, other types of intergenerational transfer, including transfers of space (e.g., coresidency) and transfers of time (e.g., education), and other ways in which transfers of wealth may be executed. In particular, unfunded social security systems are a means of reallocating wealth from the young to the old and there is a large literature on the implications of such systems for fertility and growth (e.g., Cigno and Rosati 1992; Laitner 1988; Raut 1991; Zhang and Nishimura 1993; Zhang 1995; Zhang and Zhang 1998, 2001). The focus of the present paper is different, being concerned with the implications of privately-motivated transfers that are chosen voluntarily by individuals. These types of transfer are particular relevant for developing countries (which lack well-established social security systems) and may become increasingly important for developed countries (as the sustainability of such systems is threatened by an ageing population).

behaviour in an environment of uncertainty (e.g., Abel 1985), while the existence of gifts may be the result of social norms or customs which obligate children to provide some form of material support to their parents during old-age (e.g., Morand 1999; Raut 1991; Srinivasan 1988). While the evidence on each of these possibilities is mixed, there is no doubt that intergenerational transfers represent sizeable financial flows which account for a significant part of aggregate savings and wealth accumulation (e.g., Gale and Scholz 1994; Kotlikoff 1988; Kotlikoff and Summers 1981).[3]

The Caldwell hypothesis is one of the most well-known and well-respected hypotheses in the literature on demographic transition. As far as we know, however, it has never been formalised within the context of a specific analytical framework. The objective of the present paper is to do this. We develop an overlapping generations model in which two-period lived agents produce and consume output, bear and rear children, and receive and donate intergenerational transfers. In common with most of the literature, we assume that transfers from parents to children (i.e., bequests) are motivated by altruism (specifically, semi, or paternalistic, altruism) on the part of the former towards the latter. As regards transfers from children to parents (i.e., gifts), we consider two different motives that have been popularised in the literature: the first is simply duty or obligation on the part of children towards parents, while the second is (semi) altruism on the part of children towards parents. Accordingly, we study two types of scenario, one of which (the simplest) involves only one-sided altruism and the other of which (a generalisation) entails two-sided altruism. There are very few existing analyses that consider the possible co-existence of both gifts and bequests, and even fewer investigations that confront the problem of determining these transfers jointly and optimally. In attending to these issues, our analysis continues the tradition of the literature on economic demography in its progressive treatment of demographic and economic outcomes as mutually-dependent

[3]Original estimates suggested that intergenerational transfers account for as much as 80 percent of observed aggregate wealth (Kotlikoff and Summers 1981). Even the most conservative estimates (20-30 percent) are of non-trivial magintudes, while it has also been found that intra-family wealth transfers and other types of family support mechanisms are especially important for low income households and low income countries (e.g., Bloom *et al.* 2000; Brown and Weisbenner 2002). Indirect evidence of transfers, particularly those from children to parents as a means of providing old-age support, is available from studies in which the implications of theoretical models are tested (e.g., Jensen 1990; Ehrlich and Lui 1994). Some of these studies also provide evidence that the importance of old-age support declines with income, as do a number of case studies. An example of the latter, drawn from Japan, is the fall in the percentage of women who expect to rely on their children during old-age, and the fall in the percentage of elderly who live with their children (e.g., Ogawa and Retherford 1993).

phenomena which vary along an economy's path of development.

A key feature of the model is that, as development takes place, parental income increases relative to child income. This is because agents have access to a more advanced production technology (i.e., are more skilled) when they are mature than when they are young due to the training and education that they receive when young. This technology allows for endogenous improvements in the productivity of mature agents through a process of learning-by-doing. To this extent, the model is consistent with the empirical evidence on the widening earnings differentials between the well-educated and less well-educated members of society: that is, individuals with more human capital tend to have steeper age-earnings profiles, which is a major reason why more recent generations have a more rapid growth of earnings than earlier generations (e.g., Hanoch and Honig 1985; Juhn *et al.* 1993). At the same time, it is the mature agents in our model who bear children and who incur child-rearing costs which reduce the amount of labour available for producing output.

The way that we study the Caldwell hypothesis is by analysing the conditions under which the non-negativity constraints on gifts and bequests are either binding or non-binding. Significantly, these conditions depend on the level of development of the economy. Thus, as the economy evolves along its growth path, the motive for each type of transfer changes from being operative to inoperative, or inoperative to operative. This leads to multiple development regimes, each of which is characterised by its own patterns of transfers, fertility and growth, and between which the economy switches endogenously as part of its transition towards a steady state equilibrium. In the case of one-sided altruism there are two regimes - a low development regime in which fertility, although declining, is high and the flow of net transfers is from children to parents, and a high development regime in which fertility, still declining, is low and the flow of net transfers is from parents to children. In the case of two-sided altruism there is an additional third (intermediate) regime in which fertility is constant and net transfers are zero. Naturally, the model is deliberately stylised in order to focus and simplify the analysis, and is not meant to provide a complete account of the mechanisms underlying the process of demographic transition. In particular, it's primary objective is to illustrate formally, and in the simplest way possible, the Caldwell hypothesis by assigning a major role in the transition process to intergenerational transfers. If these transfers were absent from the model, then there would be no changes in fertility, no switches in regime and no demographic transition.

The model is set out in Section 2. In Section 3 we analyse the case of one-sided altruism. In Section 4 we extend the analysis to the case of two-sided altruism. In Section 5 we make some concluding remarks. As well as its specific focus on demographic transition and intergenerational transfers, the paper may be viewed within the broader context of the growing body of literature on the development of economies over the very long-run and the transition from pre-industrial to post-industrial societies (e.g., Galor and Weil 1998; Kremer 1993; Jones 1999; Tamura 1999).

2 A Framework

We consider an artificial economy in which there is an endogenous population of two-period-lived agents belonging to overlapping generations connected by altruism. Each agent is a bearer and rearer of children, and a producer and consumer of output.[4] Child-bearing and child-rearing take place in the second period of life, while consumption and production occur in both periods. Intergenerational transfers within the family occur in both periods as well and may flow in both directions - that is, as gifts from children to parents and as bequests from parents to children.

As indicated earlier, we are motivated to consider two types of scenario in which fertility behaviour and intergenerational wealth flows are determined jointly. In both cases agents derive utility from their own consumption and the number of offspring they have (e.g., Blackburn and Cipriani 2002; Eckstein and Wolpin 1985; Zhang and Zhang 1998, 2001). By virtue of the latter, children are demanded partly for selfish reasons. At the same time, parents are also semi-altruistic towards their progeny, deriving utility from their children's consumption (e.g., Bernheim and Ray, 1987; Kohlberg 1976). This provides another motive for child-bearing and explains why parents may leave bequests to their offspring. The difference between the two scenarios lies in the explanation of why children may donate gifts to their parents. In one case the reason is due solely to a child's filial association with her parent, as might arise from some social norm or custom which compels the child to provide her parent with some fixed amount of material support (e.g., Morand 1999; Raut 1991; Srinivasan 1988). In the other case the reason is down to semi-altruistic motives on the part of children who derive direct gratification from parental consumption (e.g., Nishimura and Zhang 1995; Zhang and Nishimura 1993).

Given the above, we specify two alternative forms of utility function that may apply to a representative agent of generation t. The selfish aspects of preferences are summarised in each case by a subutility function,

[4] As usual, we abstract from complications of marriage and integer constraints by assuming that an agent is able to bear children on her own and that the number of children is a continuous, non-random variable.

$$U^t = \alpha \log(c_t^t) + \beta \log(c_{t+1}^t) + \eta \log(n_{t+1}), \quad \alpha, \beta, \eta > 0 \tag{1}$$

where c_t^t denotes consumption when young, c_{t+1}^t denotes consumption when old and n_{t+1} denotes the number of children. Depending on whether altruism is one-sided or two-sided, the lifetime utility of an agent is then given by either of the following expressions:

$$V^t = U^t + \gamma \log(c_{t+1}^{t+1}), \quad \gamma > 0 \tag{2}$$

$$V^t = U^t + \gamma \log(c_{t+1}^{t+1}) + \delta \log(c_t^{t-1}), \quad \delta > 0 \tag{3}$$

As usual, logarithmic specifications are chosen in order to maintain tractability and provide analytical solutions. In some parts of our analysis we impose other restrictions on preferences in order to ensure the existence of different types of equilibria with positive rates of fertility.

In the first period of life an agent is able to produce a fixed (subsistence) amount of output, $q > 0$, while being reared by her parent. We think of an agent as being born with certain innate abilities that allows her to operate an elementary production technology (or to work as unskilled labour). We interpret child-rearing in a general sense as covering anything (from nurturing, nursing and nourishing to educating, informing and instructing) that ensures a child's survival to the second period and that endows a child with the ability to take on more advanced tasks in that period. In addition to her own production and consumption of output, a young agent may receive a bequest from her parent of the amount b_t and may make a gift to her parent of the amount g_t. The first period budget constraint of an agent is therefore given as

$$c_t^t + g_t = q + b_t. \tag{4}$$

In the second period of life an agent has access to a more advanced production technology (or is able to work as skilled labour). This technology is given by $Ql_{t+1}z_{t+1}$ $(Q > q)$, where l_{t+1} denotes labour and z_{t+1} is a productivity shift factor. At the same time, the agent bears children and must spend resources on costly child-rearing activities. We make the common assumption that it takes a fixed amount, $s \in (0, 1)$, of a parent's total time (normalised to one) to raise each child. Accordingly, time spent on producing output is $l_{t+1} = 1 - sn_{t+1}$. A parent may also receive gifts from her children, totalling $n_{t+1}g_{t+1}$, and may also leave bequests to her children, totalling, $n_{t+1}b_{t+1}$. The second period budget constraint of an agent is therefore given by

$$c_{t+1}^t + n_{t+1}b_{t+1} = Q(1 - sn_{t+1})z_{t+1} + n_{t+1}g_{t+1}. \tag{5}$$

The economy displays positive (and sustainable) growth due to endogenous technological progress based on serendipitous learning-by-doing in the production of output by the more mature members of the population. This is captured by the following process governing the evolution of the technology parameter z_t which may be used to define the level of development of the economy:

$$z_{t+1} = (1 + \theta L_t)z_t, \quad \theta > 0 \tag{6}$$

where L_t denotes the average labour supply of mature agents. As in other models of endogenous growth, the interpretation of z_t is that it represents accumulated, disembodied knowledge which is gained from experience during productive activity (e.g., Martin and Rogers 2000; Stadler 1990). In particular, higher average employment is presumed to offer a greater range of opportunities for learning and acquiring new skills. The assumption that it is the average (rather than an individual's) supply of labour that determines learning is meant to capture the notion that there are spillovers of knowledge among the parent population whose productivity increases from one generation to the next as a result of the dispersion of ideas, information and expertise. This means that the process of learning is external to an individual who rationally treats z_{t+1} as given and beyond her control. Of course, in equilibrium $L_t = 1 - sn_t$ and the growth rate of technology is

$$x_{t+1} = \frac{z_{t+1}}{z_t} = 1 + \theta(1 - sn_t). \tag{7}$$

Before turning to the maximisation problem of agents, it is worth remarking on certain features of the model that could be modified without altering the analysis. One of our assumptions is that the altruism of an agent extends for no longer than the period in which she co-exists with the older and younger members of her family. While concern for one's parent may well be limited in this way (since a parent's behaviour prior to one being born is history), concern for one's children may endure for a longer period of their lifetime and beyond one's own demise. This could be incorporated by adding a term such as $\mu \log(c_{t+2}^{t+1})$ ($\mu > 0$) to (2) and (3). This would be irrelevant, however, since a parent has no influence over the behaviour of her offspring during the second period of their lives. Another of our assumptions, reflected in the term $\gamma \log(c_{t+1}^{t+1})$ in (2) and (3), is that an agent derives utility from the average consumption of her children. An alternative formulation might postulate $\gamma \log(n_{t+1}c_{t+1}^{t+1})$ as a replacement for this term, implying that utility is derived from the total consumption of children. Under such circumstances, the utility weight on $\log(n_{t+1})$ would simply become $\eta + \gamma$ and the analysis would remain unchanged.[5] Finally, we have assumed that agents are able to produce only a fixed amount of output, q in (4), during the first period of

[5] A questionable aspect of this formulation is that agents would be indifferent between having a small number of well-nourished children and a large number of emaciated children.

their lives. The analysis could be extended to the case in which young agents have access to a similar technology to that of mature agents, but one that is less efficient and that is not directly proportional to z_t.[6]

The decision problem of an agent is to maximise her lifetime utility subject to her budget constraints, together with the appropriate non-negativity restrictions on variables. In solving this problem, we follow the common practice of assuming that agents take the decisions of both previous and future generations as given. This Nash-type assumption rules out any strategic aspects of decision making which may be relevant in certain types of environment where parents may anticipate that changes in their behaviour will instigate reactions in the behaviour of their children. This arises in some models of exchange-motivated transfers and in other models where parents have an incentive to exploit the altruism of their children by deliberately undersaving when young in order to extract more gifts when old (e.g., Bernheim *et al.* 1985; Laitner 1988; O'Connell and Zeldes 1993). Since we abstract from both exchange motives and savings opportunities, such considerations do not arise in our case.[7]

[6]For example, one might have qz_t^{ϕ} or $\phi z_t + q$ ($\phi \in (0,1)$), reflecting some diffusion of technology from skilled to unskilled occupations (with $\phi = 0$ corresponding to our simpler specification). What is important for the analysis is that the productivity of skilled (mature) agents increases relative to that of unskilled (young) agents.

[7]In addition, it is the altruistic (rather than the exchange or strategic) motive for transfers that remains the null hypothesis in most of the emprical literature. The absence of savings and capital markets - a feature of several other models - is intended primarily to simplify the analysis. Essentially, our assumption that agents are able to produce output in each period of their lives serves as a convenient substitute for the case in which agents use savings to finance their old-age consumption. Of course, a stylised fact of economic development is the emergence and expansion of capital markets which may compete with other ways of allocating wealth between the present and the future. On the other hand, such markets have a generally ambiguous effect on fertility, are unlikely to be that important for poor countries and do not necessarily mean that intergenerational transfers are redundant. Some recent empirical evidence does, indeed, seem to suggest that the existence of such transfers and other types of family support mechanisms reduces significantly the life-cycle savings of low income households (e.g., Bloom *et al.* 2000; Brown and Weisbenner 2002). For further discussion, see Ehrlich and Lui (1997).

3 One-sided Altruism

The first scenario that we consider is the one in which agents are altruistic towards only their children and not towards their parents. Under such circumstances, an agent's gift to her parent is fixed at some exogenous amount, $g_t = g < q$, and the agent's problem is to choose her own consumption, number of children and bequests to her children in order to maximise (2) subject to (3), (4) and $b_{t+1} \geq 0$.[8] The first-order conditions for this problem are

$$\gamma c_{t+1}^{t} \leqslant \beta n_{t+1} c_{t+1}^{t+1}, \tag{8}$$

$$\eta c_{t+1}^{t} = \beta n_{t+1}(Qsz_{t+1} - g + b_{t+1}), \tag{9}$$

where the condition in (8) holds with strict equality if $b_{t+1} > 0$. To allow for the possibility of both zero and positive bequests, we impose the following restrictions on parameter values and initial conditions: $\gamma < \eta$ and $g < \min[\frac{\beta Q s z_0}{\beta+\eta}, \frac{\beta q}{\beta+\gamma}]$, where z_0 is the initial state of technology (or initial level of development). Together, these restrictions ensure that the two cases are admissible as alternative solutions with positive levels of fertility that satisfy the constraint on parental time.[9] We consider each case in turn.

When the bequest motive of agents is inoperative ($b_{t+1} = 0$), one needs to solve for only the optimal level of fertility and endogenous state of technology. Doing this leads to the following result.

Proposition 1 *Suppose that $b_{t+1} = 0$. Then the equilibrium of the economy is given by the couple $\{n_{t+1}, z_{t+2}\}$ such that*

$$n_{t+1} = \widehat{N}(z_{t+1}) = \frac{\eta Q z_{t+1}}{(\beta+\eta)(Qsz_{t+1} - g)}, \tag{10}$$

$$z_{t+2} = \widehat{Z}(z_{t+1}) = \left\{1 + \frac{\theta[\beta Q s z_{t+1} - (\beta+\eta)g]}{(\beta+\eta)(Qsz_{t+1} - g)}\right\} z_{t+1}, \tag{11}$$

where $\widehat{N}'(\cdot) < 0$ and $\widehat{Z}'(\cdot) > 0$. In addition, $\widehat{N}''(\cdot) > 0$ and $\widehat{Z}''(\cdot) < 0$.

Proof. Substituting (5), with $b_{t+1} = 0$, into (9) gives the result in (10). Substituting (10) into (7) gives the result in (11). ■

According to (10), fertility is decreasing in the level of development and increasing in the size of gifts. In both instances substitution effects dominate

[8] The non-negativity constraints on other variables are implicit and are satisfied under the parameter restrictions stated below.

[9] That is, $n_{t+1} > 0$ and $sn_{t+1} \in (0,1)$. If, for reasons given earlier, η is replaced by $\eta + \gamma$, then the first restriction is satisfied automatically.

income effects: the greater is the productivity of an agent during her fertile years, the less attractive is child-bearing because the greater is the opportunity cost of child-rearing; by contrast, the greater is the transfer of wealth to an agent from her offspring, the more attractive it is to bear children because the greater are the returns from doing so. According to (11), development takes place for sure as the state of technology is always improving. The growth rate of technology is $x_{t+2} = \frac{z_{t+2}}{z_{t+1}} = \widehat{X}(z_{t+1})$, where $\widehat{X}'(\cdot) > 0$ and $\widehat{X}''(\cdot) < 0$.

When the bequest motive of agents is operative ($b_{t+1} > 0$), the optimal size of bequests is an additional consideration. From (8) (holding with equality) and (9), together with (4) and (5), we have

$$b_{t+1} = g + \frac{\gamma Q(1 - sn_{t+1})z_{t+1}}{(\beta+\gamma)n_{t+1}} - \frac{\beta q}{\beta+\gamma}, \tag{12}$$

$$n_{t+1} = \frac{\eta Q z_{t+1}}{(\beta+\eta)(Qsz_{t+1} - g + b_{t+1})}. \tag{13}$$

Expression (12) shows that, *ceteris paribus*, the size of an agent's bequest increases with her own income from production (and therefore the level of development), decreases with the production income of her offspring and decreases with her number of offspring. Expression (13) shows that, *ceteris paribus,* the demand for children increases with the level of development (because income effects now dominate substitution effects) and decreases with the size of bequests. The complete solutions in this case are given as follows.

Proposition 2 *Suppose that $b_{t+1} > 0$. Then the equilibrium of the economy is given by the triple $\{b_{t+1}, n_{t+1}, z_{t+2}\}$ such that*

$$b_{t+1} = \widetilde{B}(z_{t+1}) = g + \frac{\gamma Qsz_{t+1} - \eta q}{\eta - \gamma}, \tag{14}$$

$$n_{t+1} = \widetilde{N}(z_{t+1}) = \frac{(\eta-\gamma)Qz_{t+1}}{(\beta+\eta)(Qsz_{t+1} - q)}, \tag{15}$$

$$z_{t+2} = \widetilde{Z}(z_{t+1}) = \left\{1 + \frac{\theta[(\beta+\gamma)Qsz_{t+1} - (\beta+\eta)q]}{(\beta+\eta)(Qsz_{t+1} - q)}\right\} z_{t+1}, \tag{16}$$

where $\widetilde{B}'(\cdot) > 0$, $\widetilde{N}'(\cdot) < 0$ and $\widetilde{Z}'(\cdot) > 0$. In addition, $\widetilde{B}''(\cdot) = 0$, $\widetilde{N}''(\cdot) > 0$ and $\widetilde{Z}''(\cdot) < 0$.

Proof. Solving (12) and (13) simultaneously gives the results in (14) and (15). Substituting (15) into (7) gives the result in (16). ■

The solution in (14) preserves the aforementioned property that bequests are positively related to the state of technology. This relationship is strong enough to produce a solution for fertility in (15) that depends negatively (not positively) on the state of technology. The solution in (16) verifies that technological progress does take place, and does so at the rate $x_{t+2} = \frac{z_{t+2}}{z_{t+1}} = \widetilde{X}(z_{t+1})$, where $\widetilde{X}'(\cdot) > 0$ and $\widetilde{X}''(\cdot) < 0$.

Propositions 1 and 2 describe two alternative regimes for the economy, each of which is characterised by its own pattern of transfers, own pattern of fertility behaviour and own pattern of technological change. In both regimes fertility is decreasing (at a decreasing rate), implying that successive generations of mature agents spend an increasing amount of their time in production activity which causes technology to increase (at a decreasing rate) through the process of learning-by-doing. We now establish the following result which demonstrates that the prevalence of one regime over the other depends on the level of development, itself.

Proposition 3 *There exists a critical state of technology,* $z^c = \frac{\eta(q-g)+\gamma g}{\gamma Q s}$, *such that (i)* $b_{t+1} = 0$ *for* $z_{t+1} \leq z^c$, *and (ii)* $b_{t+1} > 0$ *for* $z_{t+1} > z^c$.

Proof. Consider the case in which $b_{t+1} = 0$ and the condition in (8) holds with weak inequality. Using (4) and (5), together with (10), this condition implies $z_{t+1} \leq \frac{\eta(q-g)+\gamma g}{\gamma Q s} = z^c$. If $z_{t+1} > z^c$, then $b_{t+1} > 0$ and the condition in (8) holds with strict equality to yield $b_{t+1} = \widetilde{B}(\cdot)$ in (14). ■

Thus, the economy exhibits a threshold level of development, below which the bequest motive is inoperative and the equilibrium is characterised as in Proposition 1, while above which the bequest motive is operative and the equilibrium conforms to the description in Proposition 2.

Given that technological progress takes place, then the economy is always destined to cross the development threshold and to undergo transition from one regime to the other. This is illustrated in Figure 1. Panel A depicts the evolution of technology, as determined by (11) and (16), where $\widehat{Z}(z^c) = \widetilde{Z}(z^c)$ and $\lim_{z\to\infty} \widetilde{Z}'(\cdot) = 1 + \frac{\theta(\beta+\gamma)}{\beta+\eta}$. Panel B shows the growth rate of technology, as also determined by (11) and (16), with $\widehat{X}(z^c) = \widetilde{X}(z^c)$ and $\lim_{x\to\infty} \widetilde{X}'(\cdot) = 0$ such that $\lim_{z\to\infty} \widetilde{X}(\cdot) = 1 + \frac{\theta(\beta+\gamma)}{\beta+\eta} = x^*$. Panel C portrays the behaviour of fertility in accordance with (10) and (15) which imply $\widehat{N}(z^c) = \widetilde{N}(z^c)$ and $\lim_{z\to\infty} \widetilde{N}'(\cdot) = 0$ such that $\lim_{z\to\infty} \widetilde{N}(\cdot) = \frac{\eta-\gamma}{(\beta+\eta)s} = n^*$. And Panel D illustrates the profile of net transfers, $f_{t+1} = g - b_{t+1}$, where b_{t+1} is either zero or determined by (14) with $\widetilde{B}(z^c) = 0$. Starting from a low state of technology, z_0, the economy evolves along the low development paths, $\widehat{Z}(\cdot)$ and $\widehat{X}(\cdot)$, with fertility declining along $\widehat{N}(\cdot)$ and with the flow of net transfers running from the young to the old at the exogenously given level of gifts, g, in the absence of any bequests. On reaching the threshold level, z^c, the economy underdoes a switch in regime as agents begin to leave positive bequests. Thereafter, the economy proceeds along the high development paths, $\widetilde{Z}(\cdot)$ and $\widetilde{X}(\cdot)$, with fertility continuing to decline along $\widetilde{N}(\cdot)$ and with the flow of net transfers eventually changing direction, running from the old to the young, as bequests increase along $\widetilde{B}(\cdot)$. In the long-run, as development continues unbounded, the economy converges to a steady state equilibrium in which technology grows at the constant rate x^* and population grows at the constant rate n^*.

The foregoing description of events accords well with the Caldwell hypothesis on demographic transition and the stylised facts that support this hypothesis. Changes in fertility are associated with changes in the direction of the flow of intergenerational transfers during the course of economic development: at low levels of development, fertility is high and the flow of net transfers is from children to parents; at high levels of development, fertility is low and parents make net transfers to their children. As indicated earlier, we are unaware of any other analysis that gives a formal account of these observations. Partly for this reason, we have chosen to provide such an account in the simplest way possible that deliberately puts intergenerational transfers at the centre of the stage. To be sure, note that the existence of transfers is essential for any amount of demographic transition to occur in the model. More specifically, there must be a non-zero net flow of wealth between progenitors and progeny if fertility is to change with economic development. This is evident from (10) and (13), each of which implies $n_{t+1} = \overline{n} = \frac{\eta}{(\beta+\eta)s}$ when the net wealth flow (g or $g - b_{t+1}$) is zero, in which case growth is also constant, occuring at the rate $\overline{x} = 1 + \frac{\theta\beta}{\beta+\eta}$. Given that net transfers are non-zero, however, then the model generates transitional dynamics for fertility and growth towards a steady state equilibrium. Along the transition path, there is an endogenous switch in regime which reinforces the fact that economic and demographic outcomes are determined jointly in a relationship that is fundamentally two-way causal. The change in regime occurs because the bequest motive of agents eventually becomes operative which is crucial for instigating a new phase in the development process. In the absence of any bequest motive, fertility (growth) would still fall (rise) in accordance with (10) ((11)) but would never fall (rise) below (above) the level $\overline{n}$ ($\overline{x}$). In our model the onset of positive bequests changes the decision rule for child-bearing to (15) (the process governing technological change to (16)) such that fertility (growth) declines (increases) with additional momentum and converges to the lower (higher) steady state value $n^* < \overline{n}$ ($x^* > \overline{x}$).

4 Two-sided Altruism

The second scenario that we contemplate is the one in which agents are altruistic towards not only their children but also their parents. In this case both gifts and bequests, together with consumption and fertility, are chosen voluntarily by an agent in order to maximise (3) subject to (4), (5) and $g_t, b_{t+1} \geq 0$.[10] The first-order conditions for this problem are

$$\alpha c_t^{t-1} \geq \delta n_t c_t^t, \tag{17}$$

$$\gamma c_{t+1}^t \leq \beta n_{t+1} c_{t+1}^{t+1}, \tag{18}$$

$$\eta c_{t+1}^t = \beta n_{t+1}(Qsz_{t+1} - g_{t+1} + b_{t+1}), \tag{19}$$

where the condition in (17) ((18)) holds with strict equality if $g_t > 0$ ($b_{t+1} > 0$). The restrictions on parameter values and initial conditions that guarantee the existence of alternative, meaningful equilibria are $\gamma\delta < \alpha\beta < \delta\eta$ and $Qsz_0 \in \left(\frac{\delta(\beta+\eta)q}{\beta(\alpha+\delta)}, \frac{\delta\eta q}{\alpha\beta}\right)$. The first of these restrictions encompasses our earlier requirement that $\gamma < \eta$. The inequality $\gamma\delta < \alpha\beta$ is the condition for intergenerational consistency of family decisions when there is two-sided altruism: it ensures that the first-order conditions of parents and children do not contradict each other (e.g., Abel 1987).[11] Given these restrictions, then there are three possible solutions to the agent's maximisation problem: the first is when gifts are positive and bequests are zero; the second is when gifts are zero and bequests are positive; and the third is when both gifts

[10] There is a subtle issue concerning how an agent is assumed to interact with her siblings when deciding on her gift strategy. In common with several other analyses (e.g., Nishimura and Zhang 1995; Zhang and Nishimura 1993) we make the implicit assumption that children act cooperatively in the sense that each child understands that her own choice of gift is replicated by the choices made by her siblings. Formally, each agent chooses g_t jointly with her siblings in the knowledge that this affects c_t^{t-1} by the multiple n_t. The alternative approach is to assume that children act non-cooperatively, each one choosing her own size of gift while taking the gift decisions of her siblings as given. For further discussion, see, for example, Abel (1987).

[11] For example, the first-order condition characterising the optimal bequest of a parent at time t is $\gamma c_t^{t-1} \leq \beta n_t c_t^t$ (i.e., the one-period lagged version of (18)). Together with (17), this implies that $\frac{\gamma}{\beta} \leq \frac{n_t c_t^t}{c_t^{t-1}} \leq \frac{\alpha}{\delta}$ which requires that $\gamma\delta \leq \alpha\beta$. For the sake of brevity, we abstract from the knife-edge scenario in which this condition holds with equality. In this case both gifts and bequests may be positive at the same time, though the net flow of intergenerational transfers is still determinate. The resulting pattern of demographic transition has the same essential features as those that we establish below.

and bequests are zero. Since we are interested in the net flow of transfers between adjacent generations, we express our results in terms of g_{t+1} (rather than g_t) and b_{t+1} by dealing with the one-period lead version of (17) (i.e., the first-order condition with respect to g_{t+1} of an agent of generation $t+1$). As before, we consider each of the possible cases in turn.

If the gift motive is operative ($g_{t+1} > 0$), but the bequest motive is inoperative ($b_{t+1} = 0$), then (17) (holding with equality) and (19), together with (4) and (5), imply

$$g_{t+1} = \frac{\delta q}{\alpha+\delta} - \frac{\alpha Q(1-sn_{t+1})z_{t+1}}{(\alpha+\delta)n_{t+1}}, \tag{20}$$

$$n_{t+1} = \frac{\eta Q z_{t+1}}{(\beta+\eta)(Qsz_{t+1}-g_{t+1})}. \tag{21}$$

Expression (20) shows that, *ceteris paribus*, an agent makes larger gifts to her parent the greater is her own income from production, the lower is her parent's income from production and the greater is the number of her siblings. Expression (21) shows that, *ceteris paribus*, an agent has fewer children the higher is the level of development and the smaller is the gift that she receives from her children. We can now establish the following result.

Proposition 4 *Suppose that $g_{t+1} > 0$ and $b_{t+1} = 0$. Then the equilibrium of the economy is given by the triple $\{g_{t+1}, n_{t+1}, z_{t+2}\}$ such that*

$$g_{t+1} = \widehat{G}(z_{t+1}) = \frac{\delta\eta q - \alpha\beta Qsz_{t+1}}{\delta\eta - \alpha\beta}, \tag{22}$$

$$n_{t+1} = \widehat{N}(z_{t+1}) = \frac{(\delta\eta-\alpha\beta)Qz_{t+1}}{\delta(\beta+\eta)(Qsz_{t+1}-q)}, \tag{23}$$

$$z_{t+2} = \widehat{Z}(z_{t+1}) = \left\{1 + \frac{\theta[\beta(\alpha+\delta)Qsz_{t+1} - \delta(\beta+\eta)q]}{\delta(\beta+\eta)(Qsz_{t+1}-q)}\right\} z_{t+1}, \tag{24}$$

where $\widehat{G}'(\cdot) < 0$, $\widehat{N}'(\cdot) < 0$ and $\widehat{Z}'(\cdot) > 0$. In addition, $\widehat{G}''(\cdot) = 0$, $\widehat{N}''(\cdot) > 0$ and $\widehat{Z}''(\cdot) < 0$.

Proof. Solving (20) and (21) simultaneously gives the results (22) and (23). Substituting (23) into (7) gives the result in (24). ■

According to (22), gifts are decreasing in the level of development. According to (23), fertility is also decreasing in the level of development. And according to (24), development takes place at a positive rate equal to $x_{t+2} = \frac{z_{t+2}}{z_{t+1}} = \widehat{X}(z_{t+1})$.

For the case in which the gift motive is inoperative ($g_{t+1} = 0$), but the bequest motive is operative ($b_{t+1} > 0$), the conditions in (18) (holding with equality) and (19) may be combined with (4) and (5) to obtain

$$b_{t+1} = \frac{\gamma Q(1 - sn_{t+1})z_{t+1}}{(\beta + \gamma)n_{t+1}} - \frac{\beta q}{\beta + \gamma}, \tag{25}$$

$$n_{t+1} = \frac{\eta Q z_{t+1}}{(\beta + \eta)(Qsz_{t+1} + b_{t+1})}. \tag{26}$$

These are simply the expressions in (12) and (13) with $g = 0$. In accordance with those expressions, (25) shows that an agent makes larger bequests to her children the greater is her own income from production, the lower is her children's income from production and the lower is her number of children, while (26) shows that an agent has a higher demand for children the greater is the level of development and the lower is the size of her bequest. The complete solutions in this case are given as follows.

Proposition 5 *Suppose that $g_{t+1} = 0$ and $b_{t+1} > 0$. Then the equilibrium of the economy is given by the triple $\{b_{t+1}, n_{t+1}, z_{t+2}\}$ such that*

$$b_{t+1} = \widetilde{B}(z_{t+1}) = \frac{\gamma Qsz_{t+1} - \eta q}{\eta - \gamma}, \tag{27}$$

$$n_{t+1} = \widetilde{N}(z_{t+1}) = \frac{(\eta - \gamma)Qz_{t+1}}{(\beta + \eta)(Qsz_{t+1} - q)}, \tag{28}$$

$$z_{t+2} = \widetilde{Z}(z_{t+1}) = \left\{1 + \frac{\theta[(\beta + \gamma)Qsz_{t+1} - (\beta + \eta)q]}{(\beta + \eta)(Qsz_{t+1} - q)}\right\} z_{t+1}, \tag{29}$$

where $\widetilde{B}'(\cdot) > 0$, $\widetilde{N}'(\cdot) < 0$ and $\widetilde{Z}'(\cdot) > 0$. In addition, $\widetilde{B}''(\cdot) = 0$, $\widetilde{N}''(\cdot) > 0$ and $\widetilde{Z}''(\cdot) < 0$.

Proof. Solving (25) and (26) simultaneously gives the results in (27) and (28). Substituting (28) into (7) gives the result in (29). ■

These solutions are the same as those given in (14), (15) and (16) with $g = 0$. Thus, the solution in (27) reveals bequests to be an increasing function of the level of development, while the solution in (28) shows fertility to be a decreasing function of the level of development. The rate at which development takes place is established from (29) as $x_{t+2} = \frac{z_{t+2}}{z_{t+1}} = \widetilde{X}(z_{t+1})$.

The final case to consider is when neither the gift motive nor bequest motive is operative ($g_{t+1} = b_{t+1} = 0$). The outcomes in this case are summarised as follows.

Proposition 6 *Suppose that* $g_{t+1} = b_{t+1} = 0$*. Then the equilibrium of the economy is given by the couple* $\{n_{t+1}, z_{t+2}\}$ *such that*

$$n_{t+1} = \overline{n} = \frac{\eta}{(\beta + \eta)s}, \tag{30}$$

$$z_{t+2} = \overline{Z}(z_{t+1}) = \left(1 + \frac{\theta\beta}{\beta + \eta}\right) z_{t+1}. \tag{31}$$

Proof. Substituting (5), with $g_{t+1} = b_{t+1} = 0$, into (19) gives the result in (30). Substituting (30) into (7) gives the result in (31). ■

According to (30), fertility is constant. Since time devoted to production activity is also constant, then so too is the rate of learning-by-doing and so too is the rate of technological progress, $x_{t+2} = \frac{z_{t+2}}{z_{t+1}} = 1 + \frac{\theta\beta}{\beta+\eta} = \overline{x}$.

Propositions 4, 5 and 6 describe three possible regimes for the economy. The following result establishes the conditions under which each of these regimes will prevail.

Proposition 7 *There exist two critical states of technology,* $z_1^c = \frac{\delta\eta q}{\alpha\beta Q s}$ *and* $z_2^c = \frac{\eta q}{\gamma Q s}$*, such that (i)* $g_{t+1} > 0$ *and* $b_{t+1} = 0$ *for* $z_{t+1} < z_1^c$*, (ii)* $g_{t+1} = b_{t+1} = 0$ *for* $z_{t+1} \in (z_1^c, z_2^c)$*, and (iii)* $g_{t+1} = 0$ *and* $b_{t+1} > 0$ *for* $z_{t+1} > z_2^c$*.*

Proof. Consider the case in which $g_{t+1} = b_{t+1} = 0$ and the conditions in (17) and (18) hold with weak inequality. Using (4) and (5), together with (30), these conditions imply $z_{t+1} \geq \frac{\delta\eta q}{\alpha\beta Q s} = z_1^c$ and $z_{t+1} \leq \frac{\eta q}{\gamma Q s} = z_2^c$, where $z_1^c < z_2^c$. If $z_{t+1} < z_1^c$, then $g_{t+1} > 0$ and the condition in (17) holds with strict equality to give $g_{t+1} = \widehat{G}(\cdot)$ in (22). If $z_{t+1} > z_2^c$, then $b_{t+1} > 0$ and the condition in (18) holds with strict equality to give $b_{t+1} = \widetilde{B}(\cdot)$ in (27). ■

Thus, the economy now exhibits two threshold levels of development: below the lowest one, z_1^c, only the gift motive is operative and the equilibrium is characterised as in Proposition 4; above the highest one, z_2^c, only the bequest motive is operative and the equilibrium conforms to the description in Proposition 5; and in between the two thresholds, neither transfer motive is operative and the equilibrium is given by Proposition 6.

The transition of the economy from one regime to another is depicted in Figure 2. The evolution of technology, shown in Panel A, is derived from (24), (29) and (31), where $\widehat{Z}(z_1^c) = \overline{Z}(z_1^c)$, $\overline{Z}(z_2^c) = \widetilde{Z}(z_2^c)$ and $\lim_{z\to\infty} \widetilde{Z}'(\cdot) = 1 + \frac{\theta(\beta+\gamma)}{\beta+\eta}$. The growth rate of technology, displayed in Panel B, is derived from the same expressions with $\widehat{X}(z_1^c) = \overline{X}(z_1^c)$, $\overline{X}(z_2^c) = \widetilde{X}(z_2^c)$ and

$\lim_{z\to\infty} \widetilde{X}'(\cdot) = 0$ such that $\lim_{z\to\infty} \widetilde{X}(\cdot) = 1 + \frac{\theta(\beta+\gamma)}{\beta+\eta} = x^*$. Fertility behaviour, illustrated in Panel C, is obtained from (23), (28) and (30) which imply $\widehat{N}(z_1^c) = \overline{N}(z_1^c)$, $\overline{N}(z_2^c) = \widetilde{N}(z_2^c)$ and $\lim_{z\to\infty} \widetilde{N}'(\cdot) = 0$ such that $\lim_{z\to\infty} \widetilde{N}(\cdot) = \frac{\eta-\gamma}{(\beta+\eta)s} = n^*$. And the flow of net intergenerational transfers, profiled in Panel D as $f_{t+1} = g_{t+1} - b_{t+1}$, is deduced from (22) and (27) which satisfy $\widehat{G}(z_1^c) = \widetilde{B}(z_2^c) = 0$. In the early stages of transition, only the gift motive is operative and the economy progresses along the low development paths, $\widehat{Z}(\cdot)$ and $\widehat{X}(\cdot)$. Fertility declines along $\widehat{N}(\cdot)$ and the flow of (positive) net transfers (i.e., the transfers from children to parents) declines along $\widehat{G}(\cdot)$. On crossing the first threshold level, z_1^c, the gift motive ceases to be operative, net transfers turn to zero and fertility becomes constant at $\overline{n}$. Technology improves along $\overline{Z}(\cdot)$ at the constant rate $\overline{x}$. On crossing the second threshold level, z_2^c, the bequest motive becomes operative and the economy moves onto the high development paths, $\widetilde{Z}(\cdot)$ and $\widetilde{X}(\cdot)$. Fertility starts falling again along $\widetilde{N}(\cdot)$ with the flow of (negative) net transfers (i.e., the transfers from parents to children) increasing along $-\widetilde{B}(\cdot)$. In the limit, the economy converges to a steady state equilibrium in which technology grows at the constant rate x^* and population grows at the constant rate n^*.[12]

The present analysis produces results which generalise and extend those obtained previously. In support of the Caldwell hypothesis, the analysis demonstrates how demographic transition is fundamentally linked to changes in the flow of intergenerational wealth during the course of economic development. As technology improves, an economy evolves from a low state of development, in which fertility is high and children make net transfers to their parents, to a high state of development, in which fertility is low and parents make net transfers to their children. As before, the possibility that intergenerational transfers may flow in both directions is crucial for this transition to take place. If there was neither a gift motive nor bequest motive, then the economy would start and remain in the equilibrium of Proposition 6, where fertility is constant at $\overline{n}$ and growth is constant at $\overline{x}$. Similarly, if only a gift motive ever existed, then transition would progress no further than this situation: starting in the regime of Proposition 4, fertility (growth) would first decline (increase), but would eventually become constant at $\overline{n}$ ($\overline{x}$) once the gift motive ceases to be operative at the lower threshold level, z_1^c. Only if there is also a bequest motive will the economy undergo the final stage of transition to the regime of Proposition 5. As soon as this motive

[12]The period during which fertility is constant and net transfers are zero may be relatively short, depending on parameter values. For the knife-edge case in which $\gamma\delta = \alpha\beta$, transition takes place as a smooth, continuous process with both fertility and net transfers declining monotonically throughout.

becomes operative at the higher threshold level, z_2^c, fertility (growth) starts to decrease (increase) again and converges to the lower (higher) steady state value of $n^* < \overline{n}$ ($x^* > \overline{x}$).

A quantitative illustration of demographic transition under two-sided altruism is presented in Figure 3 which is based on numerical simulations of a calibrated version of the model.[13] In addition to confirming our analytical results, these simulations can be used to generate artificial time series of variables which depict the model's stylised account of the transition process in real time. Treating each period as 35 years, our baseline set of parameter values is $\{\alpha = 1.00,\ \beta = 0.50,\ \gamma = 0.60,\ \delta = 0.60,\ \eta = 2.50,\ \theta = 3.75,\ q = 0.95,\ Q = 5.00,\ s = 0.45,\ z_0 = 1.00\}$. As well as satisfying the appropriate restrictions, these values imply an annual discount factor of 0.98, a steady state annual population growth rate of 1 percent, a steady state annual productivity growth rate of 2.5 percent and a steady state proportion of time spent working of 30 percent. For the first two periods, the economy is in the regime of positive (but declining) gifts and zero bequests. During the third period, after reaching the first threshold level, it is in the regime where both gifts and bequests are zero. Subsequently, after crossing the second threshold level, it evolves within the regime of zero gifts and positive (and increasing) bequests. Changes in parameter values cause changes in the transitional dynamics and steady state values of variables, together with changes in the threshold levels of technology and the time at which these thresholds are reached. For example, an increase (decrease) in the value of δ or η (γ or θ) produces one or more of the following changes: an increase in steady state fertility, a decrease in steady state growth, an increase in the threshold levels, an increase in the date at which the thresholds are reached and an increase in the length of time between the thresholds. In all cases there is the same essential process of demographic transition which involves a decline in fertility and a shift in the direction of intergenerational transfers as the economy develops over time.

5 Conclusions

Of the many stylised facts of demographic change, the most well-known and widely-studied is the decline in fertility. The principal economic explanation for this is the increasing opportunity costs of child-rearing as development takes place. While such forces are clearly important, they are not the only reason why individuals may choose to have fewer children. Changes in cul-

[13]The case of one-sided altruism may be illustrated in a similar way, though we refrain from doing so for the sake of brevity.

ture, ideology and public policy, to say nothing of shifts in mortality and life expectancy, have a prominent role to play as well. Another factor, recognised for some time but never formally articulated, is the motivation of individuals to alter their pattern of intergenerational transfers *vis-a-vis* their progenitors and progeny. Our objective in the present paper has been to develop the first analytical model of demographic transition which takes explicit account of this factor in a simple and tractable way.

Our results accord strongly with the following observations that form the basis of the Caldwell hypothesis. At low levels of development, fertility is high and the flow of net intergenerational transfers is from children to parents. At high levels of development, fertility is low and the flow of net transfers runs from parents to children. The model that we have used to establish these results has been constructed in such a way as to provide the clearest illustration of how demographic transition is fundamentally linked to changes in intra-family wealth flows: if there were no such flows whatsoever, then there would be no transition whatsoever, while if there were flows from only children to parents, transition would cease at a relatively early stage. For transition to take place and to continue unheeded, it is essential that individuals have the motives and opportunities for making transfers not only to their parents but also to their children. Given this, then there is a critical stage of development at which individuals switch from the former type of behaviour to the latter type of behaviour such that the transition process regains its momentum. Based on these results, we view our analysis as a promising first step in modelling formally the endogenous co-evolution of intergenerational transfers, demographic outcomes and economic activity.

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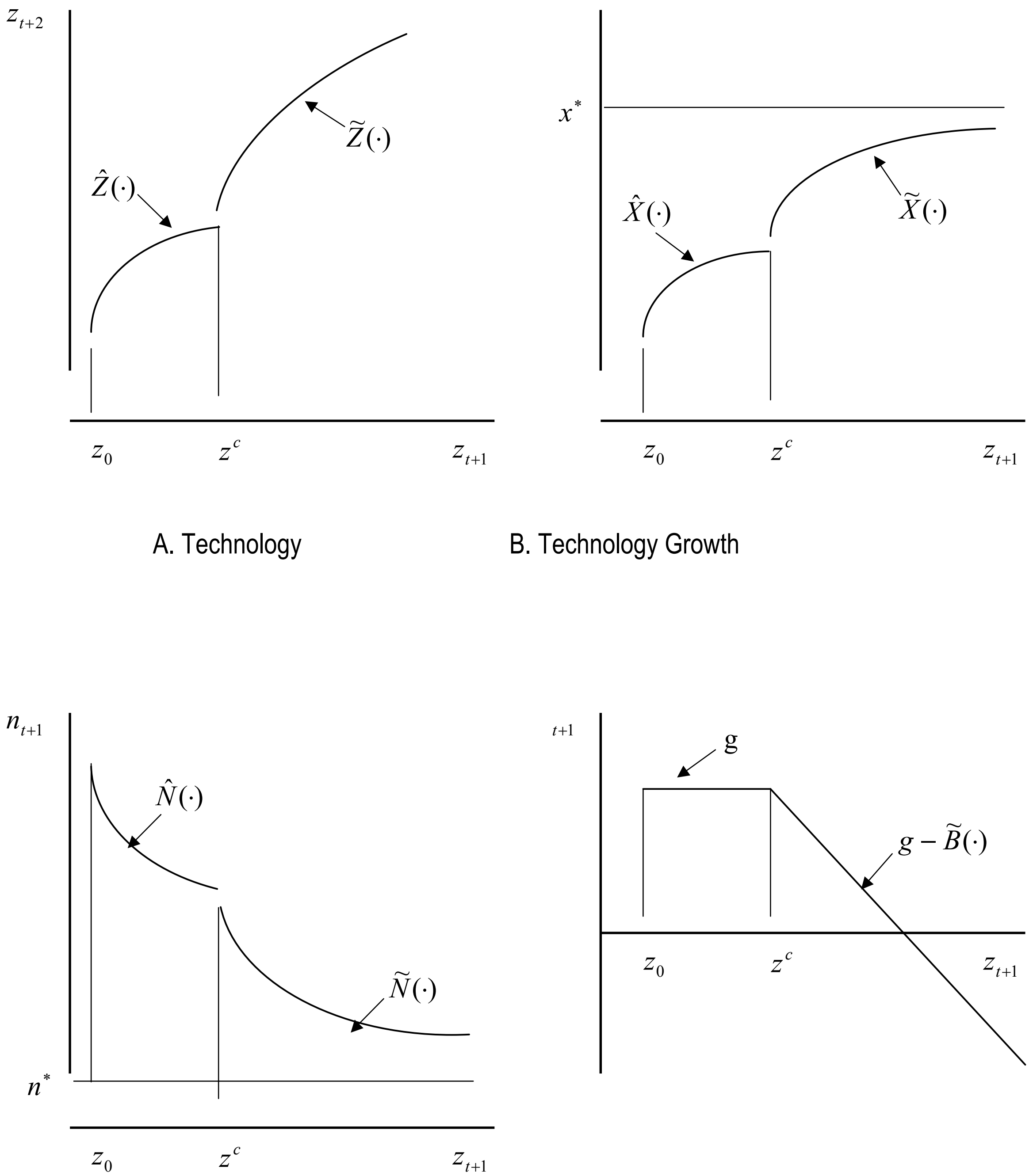
Figure 1
Demographic Transition with One-sided Altruism



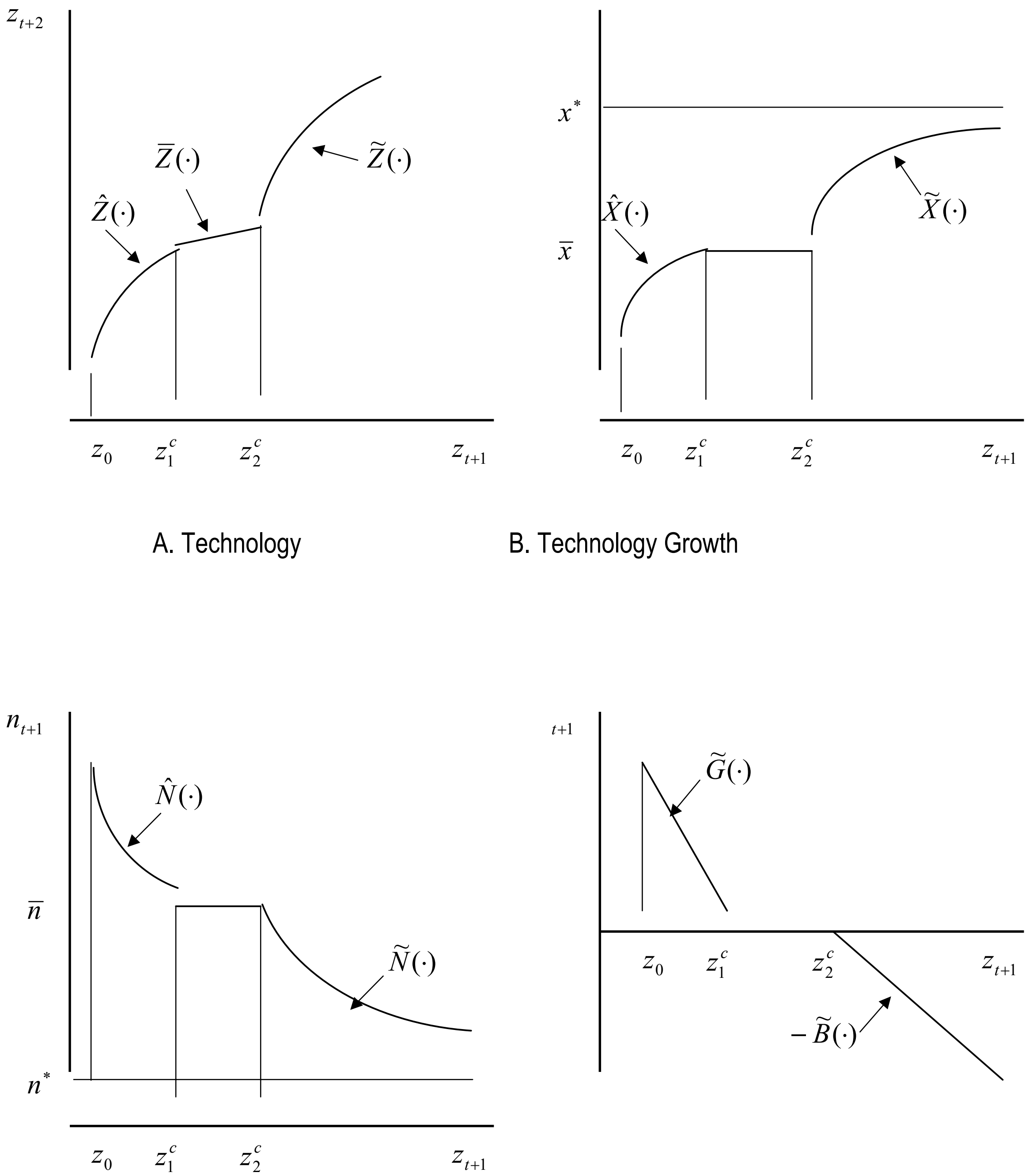
A. Technology

B. Technology Growth

C. Fertility

D. Net Transfers

Figure 2
Demographic Transition with Two-sided Altruism



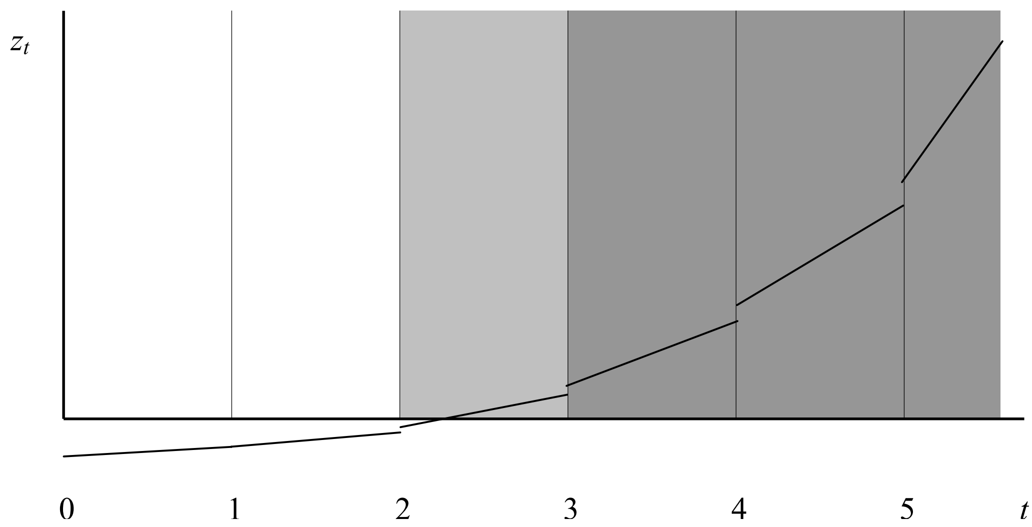
A. Technology

B. Technology Growth

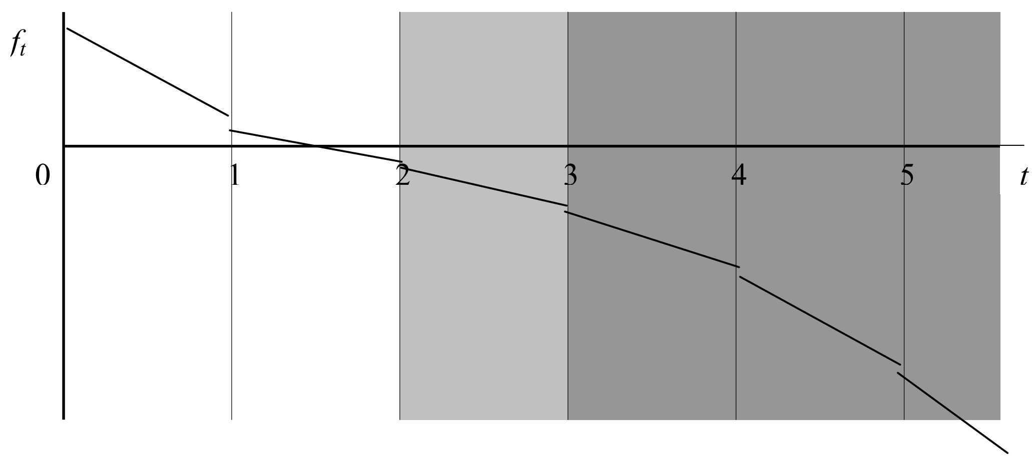
C. Fertility

D. Net Transfers

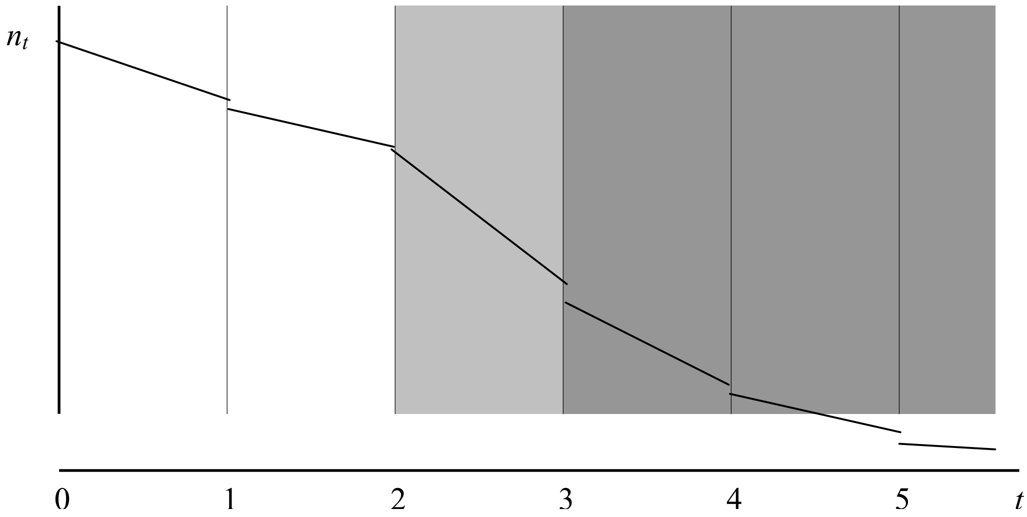
Figure 3
Numerical Simulations



A. Technology



B. Net Transfers



C. Fertility