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A Fair Wage Model of Unemployment with Inertia in Fairness Perceptions*

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Abstract

Theories of psychology and empirical evidence suggest that the reference transactions against which workers judge fairness exhibit inertia. This paper shows that a fair-wage model with inertia in fairness perceptions provides a plausible explanation for the observed negative correlation between changes in productivity growth and equilibrium unemployment over the medium run, a stylized fact that remains elusive to most other classes of models. It also shows that skill-biased productivity shocks and shocks to workers’ taste for equal pay have permanent effects on unemployment and the skill premium. Our quantitative results suggest that the effect of these shocks can be sizeable.

\textit{Keywords:} Efficiency Wages; Fair Wage-Effort Hypothesis; Unemployment; Fairness; Personnel Management

\textit{JEL Classification:} D03; E24; J31; M12

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1. Introduction

Efficiency wage models of the labour market have long established the importance of fairness considerations in explaining a number of key labour market stylized facts such as real wage rigidity, involuntary unemployment as an equilibrium phenomenon, wage compression, the inverse correlation between unemployment and skills, and the existence of wage distributions for workers of identical characteristics.¹

The fundamental assumption of fair-wage models is that workers may withhold effort if their actual wage falls short of what they perceive to be a fair wage rate. The fair wage can be defined in a number of ways but often depends on the wage of other workers at the same and other firms as well as on past wages and on labour market conditions.

In their seminal 1990 paper, Akerlof and Yellen develop a fair-wage model with two groups of labour: a skilled and an unskilled group. The fair wage of the unskilled group is the average of the wage of the skilled group in the same firm and the wage they would be paid if the market cleared. The unskilled group would supply the optimal amount of effort only if it were paid the wage it considers fair. Profit maximizing employers have therefore a strong motive to pay the fair wage. As, by assumption, the fair wage exceeds the market-clearing wage there is involuntary unemployment among unskilled workers.

Despite the presence of involuntary unemployment, employers are unwilling to hire underbidders, i.e. unemployed workers who are willing to work for less, because of the lack of a credible commitment mechanism whereby, if hired, underbidders would eschew fairness considerations. In Solow’s words, “if I were to hire them after a slight wage cut to replace some of my current workers, the newcomers will pretty soon supply less than the optimal amount of effort because they would feel aggrieved at getting less than a fair wage. They are, after all, exactly like their currently employed sisters-in-law, by definition”.² As a result, unemployment in equilibrium is, to a good extent, involuntary.

Experimental evidence and survey research have borne out the main predictions of Akerlof and Yellen’s fair wage hypothesis and leave little doubt about the importance of fairness concerns in determining labour market outcomes. Fehr, Kirchsteiger, and Riedl (1993) tested the fair wage hypothesis in the context of a competitive market experiment where market prices were determined

¹See Akerlof and Yellen (1988).
²Solow (1990) p. 36.
in an auction with buyers as price makers. Upon acceptance of an offer, sellers determined the quality of the good. The experiment showed that most buyers offered prices that were, on average, substantially above the market-clearing level on the expectation that sellers would reciprocate, i.e., they would respond by supplying higher quality levels. This expectation was, on average, confirmed and, as the experiment was repeated, there was no tendency for prices to converge toward the market-clearing level.

Blinder and Choi (1990) interviewed managers of nineteen firms in New Jersey and Pennsylvania in an attempt to elicit their reaction to the central ideas of competing theories of wage stickiness. A significant majority of respondents (79 percent) indicated that taking advantage of labour market slack to reduce wages would be considered by their workers as unfair or very unfair while a vast majority (95 percent) agreed that an unfair wage policy would cause a fall in morale and be reciprocated by a decline in work effort.

Bewley (1998, 1999) carried out a ground-breaking survey among managers, union leaders, business consultants and counsellors of unemployed people in the Northeast of the United States in order to find out why wages failed to decline during the recession of the early nineties. The answers were clear and unequivocal: employers were reluctant to cut wages because they believed that doing so would hurt employee morale and lead to lower productivity, lending, thus, further support to the Akerlof-Yellen fair wage hypothesis.3

Further survey evidence is given by Campbell and Kamlani (1997) who also investigated the reasons for wage rigidity on the basis of a sample of 184 American firms. Respondents identified fear of reduced effort as a major reason behind firms’ reluctance to reduce wages while they considered wage reductions to be particularly harmful to the morale and effort levels of low-skilled workers.4

Despite the evident contribution of the Akerlof-Yellen fair wage model in simultaneously explaining a number of labour market puzzles, an important limitation of the model lies in its lack of fair wage dynamics as workers’ perception of fairness exhibits no inertia. Yet, there are good grounds to believe that past interactions have a significant and lasting influence on the reference transaction against which workers judge fairness. In an important contribution, Kahneman, Knetsch and Thaler (1986) provide evidence suggesting that incumbent workers assess the fairness of proposed wage changes in their ongoing employment relative to the status quo. In particular, they show that

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3Note that Bewley (1999) presents evidence in favour of a fair wage specification that includes past wages and wages of other employees in the same work site but excludes labour market conditions.

4The other major reason respondents identify is fear that the more productive workers would quit.
an incumbent worker’s reference standard is last period’s contract, influencing, thus, future wage outcomes independently from the prevailing market conditions.\(^5\) In the context of a laboratory experiment, Falk, Fehr and Zehnder (2006) show that the temporary introduction of a minimum wage has long-term effects on workers’ perception of what is a fair transaction by generating feelings of entitlement which persist even after the removal of the minimum wage. In their own words “once workers have been exposed to a minimum wage, they become used to receiving a relatively high wage. This experience may create entitlements, i.e., workers think they have a right to receive high wages and are willing to defend them. As a consequence, they set relatively high reservation wages even after the elimination of the minimum wage”.\(^6\)

The idea that the dynamics of workers’ perceptions of fairness exhibit inertia is consistent with the view that reference transactions are largely shaped by norm. Kahneman, Knetch and Thaler (1986) suggest that the reference transaction “provides a basis for fairness judgements because it is normal, not necessarily because it is just”.\(^7\) Hicks (1975) argues that “the system of wages should be well established, so that it has the sanction of custom. It then becomes what is expected and what is expected is fair”.\(^8\) As shaped by norms, reference transactions are historically rooted and evolve slowly.\(^9\) Alternatively, perceptions of fairness may exhibit inertia because information about the current wage of the reference group and the own market-clearing wage may diffuse slowly across workers. The sources of slow diffusion could well be formal or informal restrictions on wage disclosure or signal extraction costs regarding the prevailing macroeconomic conditions. Overall, taking into account the dynamic behaviour of workers’ perceptions of fairness could enhance the realism of the fair wage model and expand the range of stylized facts that this model can explain. Importantly, as this paper shows, it enables the fair wage model to account for one of the significant empirical regularities of the past forty years, i.e. the well-documented negative correlation between equilibrium unemployment and the change in productivity growth. On the observed correlation between changes in productivity growth and equilibrium unemployment, see Grubb, Jackman and Layard (1982), Lynch and Nickell (2001), Staiger, Stock and Watson (2001), Ball and Mankiw (2002), and Hatton (2007).

\(^5\) Note that a similar effect can be observed in the fairness judgement of price changes. Bolton, Warlop and Alba (2003) find that, in repeated transactions, the price that the firm charged last was the relevant reference price, much more so than the price the competitors were offering.


\(^7\) Kahneman, Knetch and Thaler (1986) p. 730.

\(^8\) Hicks (1975) p.65.

\(^9\) Norms are defined as entitlements and obligations as in Schlicht (1998).
Our goal in this paper is to contribute to the development of the fair wage hypothesis by reconsidering the dynamic behaviour of workers’ perceptions of fairness and, thus, to explore the labour market implications of such inertial behaviour. More particularly, we extend the seminal work of Akerlof and Yellen (1990) to allow workers to update their perception of fair wages following a Poisson process.10 Thus, in each period, a fraction of workers updates their reference transactions and recalculates optimal plans according to the current state of their reference standards. Other workers, however, continue to formulate their respective fair wages based on past standards. Inertia is therefore present up until the entire workforce adapts to the new reference transactions. The model that we develop leads to a number of novel and appealing predictions. It can account for the medium-run and long-run effects of aggregate and skill-biased productivity shocks, labour supply shocks, and shocks to workers’ taste for equal pay on equilibrium unemployment and the wage premium. For a plausible set of parameter values, the model simulations generate results that conform to reality.

The remainder of the paper is organised as follows. Section 2 introduces a fair wage model of the labour market with heterogeneous workers and no inertia in fairness perceptions. In the context of this model, Section 3 explores the effects of various shocks on the equilibrium rate of unemployment and the wage premium. Section 4 extends the model by allowing for inertia in the evolution of what workers consider a fair wage and then explores the dynamic adjustment of the labour market in response to a number of technology and preference shocks. Section 5 concludes.

2. A fair-wage labour market model

2.1. Assumptions

Heterogeneous labour

We consider an economy with two types of labour, type 1 and type 2.11 Both types of labour are supplied inelastically, as denoted by the pair of labour force values \( \{L_{1t}, L_{2t}\} \), and behave according to the fair wage-effort hypothesis (Akerlof and Yellen, 1990). In particular, the effort levels \( \{e_{1t}, e_{2t}\} \) that the respective labour types may exert in the workplace are given, at any discrete time \( t \), by:

\[
\begin{align*}
e_{1t} &= \min\{w_{1t}/w_{1t}^*, 1\}, \\
e_{2t} &= \min\{w_{2t}/w_{2t}^*, 1\},
\end{align*}
\]

10 Akin to the sticky-information rule in Mankiw and Reis (2002, 2003).
11 Throughout the analysis, subscripts 1 and 2 refer, respectively, to type 1 and type 2 labour market variables.
where \( \{w_{1t}, w_{2t}\} \) is the pair of actual wages received by type 1 and type 2 labour, and where \( \{w_{1*}, w_{2*}\} \) is the pair of endogenously determined fair wages of type 1 and type 2 labour. In Eqs. (1) and (2), the fair wage-effort hypothesis states that if the actual wage paid to a worker of a given type is at least equal to her fair wage, the worker contributes the maximum effort level of 1. However, if the actual wage received by the worker falls below what she considers fair, the worker reduces effort in proportion.

Type 1, or high-skilled, workers are defined as the labour group that receives a higher actual wage in equilibrium. Type 2, or low-skilled, workers are defined as the labour group that receives a lower actual wage in equilibrium.

The fairness rule

What does a worker consider fair to receive at any date \( t \)? We assume that the fair wage, or reference transaction, of a worker of type 1 at date \( t \) is given by:

\[
 w_{1*} = (w_{2t})^{\beta} (w_{1t}^{c})^{1-\beta},
\]

where \( w_{2t} \) is the actual wage received by type 2 workers of the same firm, \( w_{1t}^{c} \) is the market-clearing wage of type 1 labour, and the parameter \( \beta \in (0,1) \) captures the relative weight attached to each argument in the determination of the fair wage. A symmetric specification to Eq. (3) corresponds to the fair wage of a worker of type 2 at date \( t \).

As in Akerlof and Yellen (1990), Eq. (3) states that, on the one hand, fairness is influenced by the wage of the peers of the other labour type. In this context, \( \beta \) can be interpreted as a parameter that measures workers’ preference for equal pay. On the other hand, the fair wage is influenced by the ‘own’ market-clearing wage which reflects the current state of the ‘own’ labour market. A rule with a high value of \( \beta \) can be thought of as being informed by sociological theories of equity. Likewise, a rule with a low value of \( \beta \) can be thought of as being informed by market-clearing theories.

Firms’ environment

Firms operate in a perfectly competitive product-market environment and they set wages in order to maximise profits subject to the effort functions (1) and (2). Each firm has a production
function of the Cobb-Douglas form in the two types of effective labour input:

\[ y_t = (A_{1t}e_{1t}L_{1t})^\alpha (A_{2t}e_{2t}L_{2t})^{1-\alpha}, \]  

(4)

where \( y_t \) denotes final output, \( \{e_{1t}L_{1t}, e_{2t}L_{2t}\} \) are units of effective labour input of type 1 and type 2, \( \{A_{1t}, A_{2t}\} \) are positive coefficients, and the parameter \( \alpha \in (0,1) \) measures the relative weight attached to each type of labour input in production. Eq. (4) is a simplified version of the constant-elasticity-of-substitution (CES) aggregate of skilled and unskilled labour used, among others, by Katz and Autor (1999), Hornstein et al. (2005), Caselli and Coleman (2006), and Autor et al. (2008). Unlike Akerlof and Yellen (1990) and Agénor and Aizenman (1997), that use a quadratic, additive production technology in the two labour types, Eq. (4) allows a simpler parameterization of productivity shocks and allows the log-linearization of the model. The specification in Eq. (4) imposes an elasticity of substitution between type 1 and type 2 labour equal to one. It also allows the two types of labour to be gross complements in the sense that the cross-input effect on the marginal product is positive.\(^{12}\) As Agénor and Aizenman (1997), we consider the (gross) complementarity case of labour types in the production of the final good. Throughout the analysis, we impose the parameter restrictions \( A_{1t} > A_{2t} \) and \( \alpha > 1 - \alpha \), such that the contribution of type 1 labour to the level of technology is greater than the contribution of type 2 labour.\(^{13}\) In the spirit of Autor et al. (2008), the parameter \( \alpha \) can be interpreted as the share of activity allocated to high-skilled workers.

The production function in Eq. (4) lets us parameterize productivity along two dimensions. Changes in the coefficients \( \{A_{1t}, A_{2t}\} \) capture skill-neutral technological progress. Specifically, an increase in either or both coefficients has a positive effect on the marginal products of both types of labour, leaving the relative productivity of the two labour types unaffected.\(^{14}\) On the other hand, skill-biased technical change is captured by exogenous increases in the parameter \( \alpha \). That is, we assume that the exogenous variables and the parameters of the model are such that the productivity of type 1 labour increases in \( \alpha \) and the productivity of type 2 labour decreases in \( \alpha \).\(^{15}\) Note that

\(^{12}\)In particular, the cross-input effect on the marginal product is given by: \( \partial^2 y_t / \partial e_{2t}L_{2t}\partial e_{1t}L_{1t} = \alpha(1 - \alpha)A_{1t}^{1-\alpha}(e_{1t}L_{1t})^{\alpha-1}(e_{2t}L_{2t})^{-\alpha} > 0 \).

\(^{13}\)A straightforward way to notice these contributions amounts to writing the production function in Eq. (4) as \( y_t = A_t(e_{1t}L_{1t})^\alpha(e_{2t}L_{2t})^{1-\alpha} \), where the index of technology is \( A_t = A_{1t}^{\alpha-1}A_{2t}^{1-\alpha} \).

\(^{14}\)The relative productivity of effective labour of type 1 is simply \( \alpha/(1 - \alpha)(e_{2t}L_{2t}/e_{1t}L_{1t}) \).

\(^{15}\)The productivity of effective labour of type 1 increases in parameter \( \alpha \) under the restriction \( 1 + \alpha \ln(A_{1t}/A_{2t}) - \alpha \ln(e_{2t}L_{2t}/e_{1t}L_{1t}) > 0 \). Similarly, the productivity of effective labour of type 2 decreases in parameter \( \alpha \) under the restriction \( 1 - (1 - \alpha) \ln(A_{1t}/A_{2t}) - (1 - \alpha) \ln(e_{1t}L_{1t}/e_{2t}L_{2t}) > 0 \).
in a two-sector economy with heterogenous labour and shirking à la Shapiro-Stiglitz, Agénor and Aizenman (1997) also model a skill-biased technological shock as a simultaneous pro-skilled, anti-unskilled change in technology.

Let us define the pair of market-clearing wages \( \{ w_{1t}^c, w_{2t}^c \} \) as the wages that clear the market for labour of a given type in a neoclassical economy where all workers elicit maximum effort regardless of the actual wage they receive (Akerlof and Yellen, 1990). When effort is maximum, profit optimization yields the standard labour demand functions:

\[
\begin{align*}
    w_{1t} &= \left( \frac{L_{2t}}{L_{1t}} \right)^{1-\alpha} \alpha A_{1t}^\alpha A_{2t}^{1-\alpha}, \\
    w_{2t} &= \left( \frac{L_{1t}}{L_{2t}} \right)^{\alpha} (1-\alpha) A_{1t}^\alpha A_{2t}^{1-\alpha};
\end{align*}
\]

notice that, in a neoclassical equilibrium with maximum effort, the actual wage of type 1 labour will be higher than the actual wage of type 2 labour if and only if the inequality restriction \( \alpha/(1-\alpha) > L_{1t}/L_{2t} \) holds.

The market-clearing wage of type 1 labour, \( w_{1t}^c \), is that wage which is sufficiently low to deliver full employment of type 1 labour, holding constant the level of employment of type 2 labour. And vice versa for the market-clearing wage of type 2 labour. Then, the pair of market-clearing wages at date \( t \) can be also written as:

\[
\begin{align*}
    w_{1t}^c &= w_{1t}(1-u_{1t})^{1-\alpha}, \\
    w_{2t}^c &= w_{2t}(1-u_{2t})^\alpha,
\end{align*}
\]

where \( u_{1t} = (T_{1t} - L_{1t})/T_{1t} \) is the unemployment rate of type 1 labour and \( u_{2t} = (T_{2t} - L_{2t})/T_{2t} \) is the unemployment rate of type 2 labour. Eqs. (7) and (8) simply state that the larger the deviation of the actual wage from the market-clearing wage of a given labour type, the higher the rate of unemployment of that type.

We finally assume that the firm has a preference to pay fair wages when profits are invariant to the firm’s wage choice, i.e. when the minimum of the marginal cost per unit of effective labour is not unique.

2.2. Equilibrium

The equilibrium of the model is a symmetric and integrated equilibrium, in which firms pursue
identical hiring strategies given the complementarity of labour types in production. In equilibrium all firms hire workers of both types.¹⁶ Yet, labour market outcomes differ between groups. In particular, type 1 labour is fully employed while type 2 labour experiences some unemployment. Workers of type 1 are paid their market-clearing wage, which is above their fair wage, and workers of type 2 are paid their fair wage, which is above their market-clearing wage.

The above characterization of the integrated equilibrium solution is easily justified along the lines in Akerlof and Yellen (1990).¹⁷ Thus, a hypothetical equilibrium solution in which both types of labour are fully employed and elicit maximum effort is not an integrated equilibrium solution. In such a hypothetical solution, workers of type 2 would receive an actual wage that would be below what they would consider fair. As a result, their effort level would fall, thus raising the marginal cost of effective labour above the marginal product and creating positive unemployment of type 2 workers. Similarly, a hypothetical equilibrium solution in which both types of labour experience some unemployment and elicit maximum effort is not an integrated equilibrium solution. In such a hypothetical solution, firms would always be better-off by cutting the actual wage of type 1 labour, hence by eliminating unemployment of type 1 workers, while maintaining their effort at maximum level. Overall, the integrated equilibrium solution combines full employment of type 1 labour and some unemployment of type 2 labour. In the integrated equilibrium outcome, the actual wage of type 1 workers is equal to their market-clearing wage and the actual wage of type 2 workers is equal to their fair wage. Both types of labour elicit the maximum level of effort in equilibrium.

The next section derives the integrated equilibrium solution of the labour market and explores the comparative static effects of technology, fairness, and labour supply shocks when there is no inertia in fairness perceptions.

3. Technology, fairness, and labour supply shocks in the absence of inertia in perceptions of fairness

In an economy in which workers’ reference transactions evolve quickly, the labour market equilibrium is given by the solution to the system of the three conditions described by the integrated equilibrium. As it shall become clear later, it will be useful to work with the following log-linear

¹⁶ Asymmetric and segregated equilibria are likely to occur when labour types are substitutes in production. In a partially segregated outcome, some firms hire both types of labour while other firms hire only type 2 labour. In a fully segregated outcome, each firm may hire either type 1 labour or type 2 labour.

¹⁷ See Akerlof and Yellen (1990) for a detailed discussion.
approximation of the three equilibrium conditions:

\[
\ln w_{1t} = \ln \alpha + \alpha \ln A_{1t} + (1 - \alpha) \ln A_{2t} + (1 - \alpha) \ln \frac{L_{2t}}{L_{1t}} - (1 - \alpha)w_{2t}; \tag{9}
\]

\[
\ln w_{2t} = \ln w_{1t} - \frac{\alpha(1 - \beta)}{\beta}w_{2t}; \tag{10}
\]

\[
\ln w_{2t} = \ln (1 - \alpha) + \alpha \ln A_{1t} + (1 - \alpha) \ln A_{2t} - \alpha \ln \frac{L_{2t}}{L_{1t}} + \alpha w_{2t}; \tag{11}
\]

where Eq. (9) is the market-clearing wage of type 1 workers, Eq. (10) is the fair wage of type 2 workers, or, using Akerlof and Yellen’s (1990) terminology, the fair wage constraint (FWC), and Eq. (11) is the labour demand schedule for type 2 workers. More particularly, Eq. (9) is obtained from Eq. (5), once the right-hand-side of the latter is re-written in terms of \(w_{2t}\) and the full employment outcome of type 1 labour is introduced. Similarly, Eq. (10) is obtained by introducing Eq. (8) into the symmetric specification of the fair wage rule in Eq. (3) and by writing the left-hand-side of the resulting expression in terms of \(w_{2t}\). Finally, Eq. (11) follows from Eq. (6) once the right-hand-side of the latter is re-written in terms of \(w_{2t}\) and the full employment outcome of type 1 labour is introduced. The resulting system of equations is log-linearized.

Note that introducing \(u_{2t} = (L_{2t} - L_{1t})/L_{1t}\) in conditions (10) and (11) yields, respectively, the standard definitions of the fair wage constraint, or labour supply schedule, and the labour demand schedule in the employment-wage space for type 2 labour. A graphical illustration of the equilibrium conditions is given below.

\[\text{Fig. 1. Labour market equilibrium}\]

Eqs. (9)-(11) form a system of structural equations whose solution yields the equilibrium rate of
unemployment of type 2 labour and the pair of equilibrium wages of type 1 and type 2 labour written as a function of the exogenous variables and the parameters of the model. In particular, the equilibrium rate of unemployment is given by:18

\[
u_{2t} = \frac{\beta}{\beta + \alpha(1 - \beta)} \left( \ln \alpha - \ln(1 - \alpha) + \ln \frac{L_{2t}}{L_{1t}} \right).
\] (12)

The comparative static effects of technology, fairness, and labour supply shocks on the equilibrium rate of unemployment are summarised in Proposition 1 next.

**Proposition 1.** The equilibrium rate of unemployment of low-skilled labour, \(\nu_{2t}\), increases with:

(i) an increase in the technology parameter \(\alpha\); (ii) an increase in the fairness parameter \(\beta\); and (iii) an increase in the relative supply of low-skilled workers. Note that the change in the equilibrium rate of unemployment is positive, \(\Delta \nu_{2t} > 0\), if the growth rate of the relative supply of low-skilled labour is positive, and vice versa.

**Proof.** See Appendix A.

The results established in Proposition 1 are intuitive. Thus, a productivity shock biased in favour of high-skilled workers raises the unemployment rate of low-skilled workers. The skill-biased shock raises the productivity of type 1 labour, hence it raises the actual wage received by type 1 workers in equilibrium. The effect of the shock on the labour market for type 2 workers is twofold. On the one hand, type 2 workers’ labour demand schedule shifts downwards, as the productivity of type 2 labour falls. On the other hand, the fair wage constraint shifts upwards, as type 2 workers consider it fair to receive a higher wage in the presence of a wage rise for type 1 labour, and it becomes steeper, as the fair wage is now more sensitive to changes in unemployment. Overall, the equilibrium rate of unemployment of low-skilled labour increases as a result of the skill-biased productivity shock.

In Eq. (12), we also notice that the unemployment rate of low-skilled workers does not depend upon the coefficients \(\{A_{1t}, A_{2t}\}\). That is, productivity growth does not affect the equilibrium rate of unemployment, unless it is biased.19

A fairness shock that raises the workers’ preference for equal pay shifts the fair wage constraint of type 2 labour upwards. This is because low-skilled workers consider it fair to receive a wage

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18 Notice that the unemployment rate is nonnegative given that the inequality restriction \(\alpha/(1 - \alpha) > L_{1t}/L_{2t}\) holds.

19 Note that the parameter \(\alpha\) is time-invariant and only subject to one-off exogenous changes.
closer to the wage received by high-skilled workers. In addition, the fair wage constraint flattens, as the fair wage becomes less responsive to changes in unemployment. Overall, the shift in the fair wage constraint induced by the shock results in a rise in the equilibrium rate of unemployment of low-skilled workers.

The model predicts that an increase in the supply of high-skilled workers reduces unemployment among the low-skilled, and that an increase in the supply of low-skilled workers increases unemployment among the low-skilled. Intuitively, we note that an increase in the supply of type 1 workers reduces their productivity in equilibrium while it increases the productivity of type 2 workers. The fair wage constraint shifts downwards, as type 2 workers adjust their fair wage in the presence of a cut in the wage of type 1 labour. The overall effect of the shock brings about a reduction in equilibrium unemployment of type 2 labour. The opposite effects are played out following a rise in the supply of type 2 workers.

Regarding the determination of wages, the relative wage of type 1 labour in equilibrium is given by:

$$\ln \frac{w_{1t}}{w_{2t}} = \frac{\alpha(1 - \beta)}{\beta + \alpha(1 - \beta)} \left( \ln \alpha - \ln(1 - \alpha) + \ln \frac{T_{2t}}{L_{1t}} \right).$$

(13)

Note that expression (13) can be interpreted as the skilled-unskilled wage premium or, simply, the skill premium. Appendix A reports the expressions for the wage of type 1 labour and the wage of type 2 labour in equilibrium. Proposition 2 establishes the effects of technology, fairness, and labour supply shocks on the relative wage of type 1 labour.

**Proposition 2.** In equilibrium, the relative wage of high-skilled labour, or skill premium, increases with: (i) an increase in the technology parameter $\alpha$; (ii) a decrease in the fairness parameter $\beta$; and (iii) an increase in the relative supply of low-skilled workers. In equilibrium, the growth rate of the relative wage of high-skilled labour is positive if the growth rate of the relative supply of low-skilled labour is positive, and vice versa.

**Proof.** See Appendix A.

The intuitive explanations underpinning the results summarised in Proposition 2 follow easily from the discussion of Proposition 1. A technology shock that works in favour of the high-skilled raises the skill premium. In addition, the model predicts that, other things being equal, an increase in the workers’ preference for wage equality would be successful in achieving a lower degree of wage dispersion at the cost of higher equilibrium unemployment among the low-skilled. Finally, Eq. (13)
shows that the wage gap does not depend on skill-neutral productivity growth, thus it does not change in response to aggregate productivity shocks.

We next investigate the dynamic adjustment of the labour market to various shocks as predicted by our fairness model when workers’ perceptions of fairness exhibit inertia.

4. The dynamic adjustment of unemployment and wages when fairness perceptions exhibit inertia

This section explores the dynamic effects of technology, fairness, and labour supply shocks on equilibrium unemployment and the skill premium under the assumption that perceptions of fairness in the labour market change slowly. In particular, we assume that firms’ relevant reference transactions exhibit no inertia and set wages as dictated by the integrated equilibrium conditions while workers may set their fair wage based on past reference standards. Workers may do so because past interactions have a significant and lasting influence on the reference transaction against which they judge fairness or because collecting and processing information about the wage of their reference group or about prevailing macroeconomic conditions involves significant costs. For these reasons, it is fair to assume that, at each date $t$, only a fraction of workers formulate their respective fair wages based on current reference standards while the rest formulate their respective fair wages based on past standards. In the long-run, the whole of the workforce will update the perceptions of what is considered to be fair and, in the absence of further shocks, the model will converge to that of Section 3.20

In particular, the updating of fairness perceptions is an event governed by an independent Poisson process with arrival rate $\lambda : \lambda \in (0, 1)$. In each period, a fraction $\lambda$ of the workforce updates the determinants of the fair wage and the fraction $(1 - \lambda)$ continues formulating the fair wage based on past reference standards. We assume that updating to the new reference standards occurs simultaneously in workers of a given type who belong to the same firm and that the arrival rate $\lambda$ is time-invariant. Thus, the fair wage of type 2 labour working for a competitive firm takes,
in period $t$, the log-linear form:

$$\ln w_{2t}^* = E_{t-j} \left( \beta \ln w_{1t} + (1 - \beta) \ln w_{2t}^\circ \right),$$

(14)

where the fair wage of type 2 labour is formulated as the expected value of the fair wage in period $t$ based on reference transactions set $j$ periods ago, $j \in [0, \infty)$. A symmetric expression to (14) describes the fair wage of type 1 labour in period $t$. In Eq. (14), we notice that, out of steady-state, the integrated equilibrium solution delivers wage dispersion among low-skilled workers who belong to different firms.

The aggregate fair wage of type 2 labour is the average of fair wages of type 2 labour in the economy, i.e.:

$$\ln w_{2t}^* = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} \left( \beta \ln w_{1t} + (1 - \beta) \ln w_{2t}^\circ \right).$$

(15)

Suppose now that there is a shock at $t = 0$. The aggregate fair wage of type 2 labour at any date $t \geq 0$ can be written as:

$$\ln w_{2t}^* = (1 - (1 - \lambda)^{t+1}) \left( \beta \ln w_{1t} + (1 - \beta) \ln w_{2t}^\circ \right) + (1 - \lambda)^{t+1} \ln \tilde{w}_{2t},$$

(16)

where $\tilde{w}_{2t}$ is the actual wage of type 2 labour in the initial (pre-shock) steady-state. In Eq. (16), we note that the higher the probability per unit of time of updating the fair wage now, that is the higher the value of $\lambda$, the smaller the weight of the initial steady-state in the determination of the current (post-shock) aggregate fair wage. Thus, a lower degree of inertia in the labour market is associated with fast evolving fairness perceptions.

Introducing the log-linear definition of the competitive wage, i.e. $w_{2t}^\circ = \ln w_{2t}^* - \alpha w_{2t}$, in Eq. (16) and solving yields the dynamic fair wage constraint of type 2 labour at any date $t \geq 0$:

$$\ln w_{2t}^* = \frac{(1 - (1 - \lambda)^{t+1})}{(1 - \beta)(1 - \lambda)^{t+1} + \beta} \left( \beta \ln w_{1t} - \alpha(1 - \beta)w_{2t} \right) + \frac{(1 - \lambda)^{t+1}}{(1 - \beta)(1 - \lambda)^{t+1} + \beta} \ln \tilde{w}_{2t}^*. $$

(17)

Overall, the dynamic adjustment path of labour market variables following a shock at date $t = 0$ is given by the solution to the system of equilibrium conditions formed by Eq. (9), Eq. (11), and Eq. (17).\textsuperscript{21} In particular, the time path of equilibrium unemployment of low-skilled labour is

\textsuperscript{21} Note that, in the integrated equilibrium solution, it must also hold that $w_{1t}^* > w_{2t}^j$ and that $w_{1t}^* > w_{1t}^j$, $\forall t \geq 0$. 

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described by the following expression:

\[ u_{2t} = \frac{1}{\alpha + \beta(1 - \alpha)(1 - (1 - \lambda)^{t+1})} \left[ \beta(1 - (1 - \lambda)^{t+1}) \ln \alpha - ((1 - \beta)(1 - \lambda)^{t+1} + \beta) \ln(1 - \alpha) \right. \\
- \alpha(1 - \lambda)^{t+1} \ln A_{1t} - (1 - \alpha)(1 - \lambda)^{t+1} \ln A_{2t} + ((\alpha - \beta)(1 - \lambda)^{t+1} + \beta) \ln \frac{T_{2t}}{L_{1t}} + (1 - \lambda)^{t+1} \ln \tilde{w}^*_2 \right], \]

where \( t \geq 0 \). In the long-run, i.e. when \( t \to \infty \), the total of the labour force will adapt to the new reference standards and the equilibrium path of unemployment will converge to its steady-state value given by Eq. (12). In the interest of brevity, we omit the expressions for the dynamic adjustment path of wages. These are easily derived by introducing the path of equilibrium unemployment in the solution to the system of equations formed by Eq. (9), Eq. (11), and Eq. (17).

4.1. Calibration

The solution to the system of equilibrium conditions given by Eq. (9), Eq. (11), and Eq. (17) allows us to examine the dynamic adjustment paths of equilibrium unemployment and the skill premium following a shock at date \( t = 0 \). In order to proceed, one has to choose plausible baseline values for the exogenous variables and the parameters of the model. In particular, we choose the efficiency levels of skilled and unskilled labour computed by Caselli and Coleman (2002) for the U.S. economy in the year 1992. Thus, we set \( \ln \bar{\ell}_1 = 3.32 \) and \( \ln \bar{\ell}_2 = 2.80 \). Similarly, the labour supplies of skilled and unskilled labour recorded in the U.S. in the year 1992 are taken from Krusell et al. (2000). Hence, we set \( \ln(\bar{L}_2/\bar{L}_1) = -0.55 \). Skilled labour encompasses those individuals with at least a college degree while unskilled labour encompasses the rest of the labour force. To ensure consistency, note that Krusell et al. (2000) is the source used by Caselli and Coleman (2002) in the identification of efficiency levels.

The parameter \( \lambda \) is the rate at which workers’ perceptions of fairness change. In order to proxy for it, we use the arrival rate of information to workers that Mankiw and Reis (2007) estimate using U.S. quarterly data for the period 1954Q3–2006Q4. In particular, their estimate of \( \lambda = 0.195 \) indicates that workers update their information about every five quarters or, in other words, that a fifth of workers update their information sets every quarter. Thus, adopting Mankiw and Reis’ (2007) terminology, workers are ‘quite inattentive’ or, in terms of fairness considerations, workers

\[ \forall j: j \in [0, \infty). \]
are ‘quite conservative’ in that their reference transactions evolve relatively slowly over time.

Potential baseline values for the remaining two parameters, $\alpha$ and $\beta$, are less well-documented in the literature. The parameter $\alpha$ can be interpreted as the share of work activities allocated to skilled labour (Autor et al., 2008). In the numerical simulations, we set the baseline value of $\alpha = 0.7$. Similarly, we consider appropriate to choose a low value for the taste for equal pay parameter $\beta$ in the case of the U.S. labour market. In particular, we work with the baseline value of $\beta = 0.1$. Robustness checks of the main results are carried out for alternative values of these parameters.$^{22}$ Finally, note that the assumed baseline values meet the restrictions of the model. For detailed analytical descriptions of the shocks, see Appendix B.

4.2. The dynamic adjustment of labour market variables to technology, fairness, and labour supply shocks

We next explore the dynamic responses of the equilibrium unemployment rate of low-skilled labour and the relative wage of high-skilled labour to various exogenous shocks. In each case, the shocks are assumed to be sudden, unexpected, and permanent. The period in the model is taken to equal one quarter.

Given the baseline values and the simulation assumptions, the steady-state equilibrium rate of unemployment in this model, i.e. the rate of unemployment that would prevail if fairness perceptions exhibited no inertia, is equal to 4 percent. Similarly, the skill-unskilled wage premium in steady-state equilibrium is equal to 29 percent.

An aggregate productivity slowdown

Suppose that, at date $t = 0$, the annualised rate of growth of trend productivity falls by 3 percentage points, or 0.75 percentage points per quarter. That is, the exogenous shock induces a permanent deceleration in trend productivity growth. Furthermore, assume that the aggregate productivity slowdown is skill-neutral, i.e. the parameter $\alpha$ is unaffected by the shock.

Fig. 2 shows the adjustment paths followed by equilibrium unemployment and the skill premium as predicted by the model.

$^{22}$Robustness results are available from the authors upon request.
Fig. 2. Equilibrium unemployment and the skill premium after a skill-neutral trend productivity slowdown

There are three stylized facts that become apparent in Fig. 2. These are summarised in the following Proposition.

**Proposition 3.** An aggregate skill-neutral productivity slowdown raises temporarily the equilibrium rate of unemployment of low-skilled labour above its steady-state value. The adjustment lags indicate that there is a medium-run effect on equilibrium unemployment of a skill-neutral shock to productivity growth. The relative wage of high-skilled labour, or skill premium, falls temporarily as a result of the productivity slowdown before it converges back to its long-run trend.

In Fig. 2, the maximum impact of the productivity slowdown on equilibrium unemployment occurs at about four quarters, by which date the equilibrium unemployment rate of low-skilled labour has reached 5.8 percent starting from an initial steady-state value of 4 percent. The increase in equilibrium unemployment gradually dissipates thus \(w_{2t}\) converges back to steady-state after thirty quarters. The dynamic profile of the skill premium is similar to that of equilibrium unemployment, although with the opposite sign. Note that the relative wage converges to its initial steady-state value after thirty quarters.\(^{23}\)

The key intuitive explanation that underpins the patterns of adjustment depicted in Fig. 2 becomes apparent by inspecting the dynamic fair wage constraint given by Eq. (17). In the aftermath of the shock, the growth rate of the aggregate wage of type 2 labour is above its new warranted growth rate. This is because the productivity slowdown takes time to feed into the

\(^{23}\)Throughout the dynamic adjustment path of wages, we have also checked that the two further conditions for integrated equilibrium are not violated, namely \(w_{1t} > w_{2t}^*\) and \(w_{1t} > w_{2t}^*, \forall t: t \geq 0, \forall j: j \in [0, \infty).\)
fair wages of type 2 labour. As a result of the marginal cost raising above the marginal product, the equilibrium rate of unemployment of type 2 labour increases. Thus, at the firm level, while some firms optimally adjust their labour market outcomes for type 1 and type 2 workers to the productivity shock, others take optimal decisions facing a fair wage that behaves according to the initial (pre-shock) steady-state. In the latter group of firms, and for as long as their workforce does not adapt to the new reference transaction, the growth rate of the wage of low-skilled labour is above the growth rate of the wage of high-skilled labour, which induces the dynamic adjustment path of the skill premium depicted in Fig. 2. The medium-run effects of the shock disappear as low-skilled workers update their perceptions of fairness reflecting the true state of the components of their respective fair wage, thus equilibrium unemployment and the skill premium return gradually to steady-state.

A skill-biased productivity shock

Suppose that, in period $t = 0$, the parameter $\alpha$ increases exogenously while the coefficients $\{A_{1t}, A_{2t}\}$ stay constant at their respective baseline values. Therefore, the shock has an impact on the level of technology, $A_t$, and not on its trend growth rate. In the context of our model, the skill-biased productivity shock captures an increase in the absolute efficiency of high-skilled labour and a reduction in the absolute efficiency of low-skilled labour. In particular, suppose that the parameter $\alpha$ increases from the baseline value $\alpha = 0.7$ to the value $\alpha = 0.72$ at date $t = 0$.

Fig. 3 illustrates the dynamic convergence paths of equilibrium unemployment and the relative wage to the new steady-state equilibrium.

![Fig. 3. Equilibrium unemployment and the skill premium after a skill-biased productivity shock](image)

Proposition 4 next outlines the stylized facts that are easily discernible in Fig. 3.
Proposition 4. A skill-biased productivity shock causes an instantaneous increase in the equilibrium rate of unemployment of low-skilled labour that is sufficiently large as to overshoot its new, higher, steady-state equilibrium value. Subsequently, the equilibrium rate of unemployment converges monotonically from above to the new steady-state. The skill-biased productivity shock produces an immediate jump in the relative wage of high-skilled labour, which then converges monotonically from below to its new steady-state equilibrium value.

Fig. 3 shows that the skill-biased productivity shock has a large, instant impact on the equilibrium rate of unemployment of low-skilled labour, which jumps from an initial steady-state value of 4 percent to a value of 9.5 percent at date $t = 0$. Thereafter, the unemployment rate converges steadily to its new steady-state equilibrium value of 5.2 percent. The rise in the relative wage of high-skilled labour yields a 40 percent wage gap in the new steady-state equilibrium, thus 11 percentage points up from the initial steady-state value.\(^{24}\)

The large overshooting impact of the skill-biased productivity shock on the equilibrium rate of unemployment of type 2 labour is caused by the time the fair wages of type 2 labour take to reflect the effects of the skill-biased change in productivity. Thus, firms whose workers have not updated their reference standards face a fair wage that is constant at the pre-shock steady-state value while the labour demand schedule has shifted downwards, which induces the overshooting of unemployment. The whole of the workforce will eventually adapt to the new reference standards and equilibrium unemployment will adjust gradually down towards the new, higher, steady-state level. The same explanation underpins the adjustment process of the skill premium to the new steady-state equilibrium.

A change in workers’ taste for fairness

Suppose that, in period $t = 0$, workers’ taste for equal pay changes and this change spreads gradually across the workforce at arrival rate $\lambda$. One might like to think of the shock as a result of a shift towards greater equality in labour market outcomes. Assume, in particular, that the parameter $\beta$ increases from the baseline value $\beta = 0.1$ to the value $\beta = 0.2$.

Fig. 4 represents the convergence paths followed by equilibrium unemployment and the relative wage as predicted by the model.

\(^{24}\)Note that, throughout the dynamic adjustment path of wages, we have also checked that the two further conditions for integrated equilibrium hold, that is $w_{it}^{t2} > w_{it}^{*2}$ and $w_{it}^{t1} > w_{it}^{*1}$, $\forall t: t \geq 0$, $\forall j : j \in [0, \infty)$.  

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Proposition 5 summarises two results that are apparent in Fig. 4.

**Proposition 5.** A fairness shock that raises the workers’ taste for equal pay yields an instant increase in the equilibrium rate of unemployment of low-skilled labour. Thereafter, the equilibrium rate of unemployment converges monotonically from below to the new steady-state equilibrium value. The fairness shock reduces steadily the relative wage of the high-skilled until the new steady-state equilibrium value is reached.

In Fig. 4, the fairness shock has a contemporaneous effect on the equilibrium rate of unemployment of low-skilled labour of about 0.8 percentage points. At five quarters, the equilibrium rate of unemployment is roughly 2.8 percentage points above the initial steady-state value of 4 percent. Or, in other words, it is more than half-way through the total adjustment induced by the shock, which amounts to 3.8 percentage points. Thus, the dynamic profile depicted in Fig. 4 suggests that equilibrium unemployment would converge fairly fast and smoothly to the new, substantially higher, steady-state equilibrium. A similar conclusion is obtained from the dynamic profile of the skill premium. That is, the fairness shock would promptly and steadily reduce the relative wage of the high-skilled. In our example, the steady-state wage gap falls 5 percentage points, i.e. from 29 to 24 percent, and the shock is mostly felt within the first five quarters.\(^{25}\)

Intuitively, the dynamic responses of equilibrium unemployment and the relative wage are explained by the gradual spread of the change in workers’ taste for equal pay. In the context of inertia in fairness perceptions, the fairness shock shifts the dynamic fair wage constraint in Eq.\(^{25}\)

\(^{25}\)We have checked that the two further conditions for integrated equilibrium hold throughout the dynamic adjustment path of wages.
(17) upwards, but at a pace dictated by the rate at which workers’ perception of fairness change.

**A labour supply shock**

Finally, suppose that, in period \( t = 0 \), the rate of growth of the labour supply rises exogenously. The exogenous labour supply shock may be caused by a permanent change in either demographic or migration trends. Consider, in particular, an initial steady-state situation in which the supply of labour, both high-skilled and low-skilled, was growing at an annualised rate of 1 percent, or 0.25 percent per quarter. At date \( t = 0 \), the quarterly growth rate of the supply of low-skilled labour rises to 0.3 percent while the growth of the supply of high-skilled labour remains at 0.25 percent. Therefore, the exogenous labour supply shock induces a permanent rise in the growth rate of the relative supply of low-skilled workers\(^{26}\).

Fig. 5 depicts the convergence paths of equilibrium unemployment and the relative wage to the new steady-state situation.

![Fig. 5. Equilibrium unemployment and the skill premium after a relative labour supply shock](image)

Proposition 6 outlines a number of stylized facts that follow from Fig. 5.

**Proposition 6.** A permanent rise in the growth rate of the relative supply of low-skilled labour increases the equilibrium rate of unemployment of low-skilled labour and the skill premium. At steady state, ceteris paribus, the rates of change of equilibrium unemployment and the skill premium are positive.

Fig. 5 shows small effects of the relative labour supply shock on equilibrium unemployment, i.e. at five quarters the equilibrium rate of unemployment is just 0.18 percentage points above the

\(^{26}\)See Appendix B for a detailed description of the shock.
initial steady-state value.\textsuperscript{27} Intuitively, a permanent acceleration in the growth rate of the relative supply of type 2 workers produces an upward shift in the demand for type 1 labour and a downward shift in the demand for type 2 labour. The dynamic $FWC$ shifts upwards, as type 2 workers adjust their fair wage claims in the presence of a rise in the wage of type 1 labour. The small overshooting impact of the relative labour supply shock on equilibrium unemployment is caused by the share of low-skilled workers who have not yet updated their perceptions with regard to the overall adverse effects of the shock on their fair wage. So it is explained the initial sluggish response of the skill premium. In the new steady-state situation, all workers in the economy adapt to the positive growth rate of the relative supply of labour and a steady increase in unemployment and the skill premium would characterize the labour market.

5. Conclusion

Theories of psychology, theories of the formation and evolution of norms, and a good deal of empirical evidence suggest that the reference transactions against which workers judge fairness exhibit inertia. This paper shows that a fair-wage model with inertia in fairness perceptions can explain a number of stylized facts that remain elusive to most other classes of models. In particular, it provides a plausible explanation for the observed negative correlation between changes in productivity growth and equilibrium unemployment over the medium run, a well-documented stylized fact that partly underpins the rise in European and US unemployment during the seventies and the fall in US unemployment during the late nineties. The model also shows that skill-biased productivity shocks and shocks to workers’ taste for equal pay have permanent effects on the equilibrium unemployment rate of the unskilled and the skill premium. The quantitative results of the paper suggest that the effect of these shocks can be sizeable. In particular, a skill-biased productivity shock may have a contemporaneous impact on the unemployment rate of the unskilled that is sufficiently large as to overshoot its new, higher steady-state value. Similarly, a shift towards equal pay may be successful in achieving more wage compression at the cost of a large increase in unemployment among the low-skilled.

\textsuperscript{27}Note that, throughout the dynamic adjustment path of wages, we have also checked that the two further conditions for integrated equilibrium are not violated, namely $w_{i1}^{c} > w_{2}^{*}j$ and $w_{ij}^{j} > w_{ij}^{a}j$, $\forall t \geq 0$, $\forall j : j \in [0, \infty)$. 

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Appendix A.

Proof of Propositions 1 and 2

The partial derivative of equilibrium unemployment in Eq. (12) with respect to $\alpha$ is given by:

$$\frac{\partial u_{2t}}{\partial \alpha} = \frac{\beta}{\beta + \alpha (1 - \beta)} \left( \frac{1}{\alpha (1 - \alpha)} \right) - \frac{\beta (1 - \beta)}{(\beta + \alpha (1 - \beta))^2} \left( \ln \alpha - \ln (1 - \alpha) + \ln \frac{L_{2t}}{L_{1t}} \right),$$

which is positive under the parameter assumptions of the model. In particular, the shock is a pro-skilled, anti-unskilled technology shift such that the marginal product of type 1 labour increases in $\alpha$, the marginal product of type 2 labour decreases in $\alpha$, and the fair wage constraint shifts upwards in the employment-wage space.

Similarly, the partial derivative of equilibrium unemployment with respect to parameter $\beta$ is given by:

$$\frac{\partial u_{2t}}{\partial \beta} = \frac{\alpha}{(\beta + \alpha (1 - \beta))^2} \left( \ln \alpha - \ln (1 - \alpha) + \ln \frac{L_{2t}}{L_{1t}} \right),$$

which is positive under the parameter assumptions of the model. In particular, the shock shifts the fair wage constraint upwards in the employment-wage space. Finally, the positive relationship between equilibrium unemployment and the relative supply of low-skilled labour is easily inferred from Eq. (12).

The partial derivative of the skill premium in Eq. (13) with respect to parameter $\alpha$ is given by:

$$\frac{\partial \ln \frac{w_{1t}}{w_{2t}}}{\partial \alpha} = \frac{(1 - \beta)}{(\beta + \alpha (1 - \beta))^2} \left( \beta \left( \ln \alpha - \ln (1 - \alpha) + \ln \frac{L_{2t}}{L_{1t}} \right) + \alpha (1 - \beta) \left( \frac{1}{\alpha} + \frac{1}{1 - \alpha} \right) \right),$$

which is positive under the parameter assumptions of the model.

Similarly, the partial derivative of the skill premium in Eq. (13) with respect to parameter $\beta$ is given by:

$$\frac{\partial \ln \frac{w_{1t}}{w_{2t}}}{\partial \beta} = -\frac{\alpha}{(\beta + \alpha (1 - \beta))^2} \left( \ln \alpha - \ln (1 - \alpha) + \ln \frac{L_{2t}}{L_{1t}} \right),$$

which is negative under the parameter assumptions of the model. The positive relationship between the skill premium and the relative supply of low-skilled labour is easy to infer from Eq. (12).

List of equilibrium outcomes

The expressions for the wage of type 1 labour and the wage of type 2 labour in equilibrium are
given by:

\[
\ln w_{1t} = \frac{1}{\beta + \alpha(1 - \beta)} \left( \alpha \ln \alpha + \beta(1 - \alpha) \ln (1 - \alpha) + \alpha(1 - \beta)(1 - \alpha) \ln \frac{T_{2t}}{L_{1t}} \right) + \alpha \ln A_{1t} + (1 - \alpha) \ln A_{2t};
\]

\[
\ln w_{2t} = \frac{1}{\beta + \alpha(1 - \beta)} \left( \beta \alpha \ln \alpha + (\beta(1 - \alpha) + \alpha(1 - \beta)) \ln (1 - \alpha) - \alpha^2(1 - \beta) \ln \frac{T_{2t}}{L_{1t}} \right) + \alpha \ln A_{1t} + (1 - \alpha) \ln A_{2t}.
\]

Note that wage growth depends upon the parameters \( \{\alpha, \beta\} \), skill-neutral productivity growth, and relative labour supply growth.

**Appendix B.**

**Description of shocks**

*An aggregate productivity slowdown:* Consider an initial steady-state situation in which productivity was growing quarterly at a rate of 0.75 percent. At date \( t = 0 \), an unexpected and permanent negative productivity shock reduces the quarterly growth rate of productivity to 0 percent. In order to introduce the shock in the model, we set \( \ln A_{1t} = 3.3275 + 0.0075t \) and \( \ln A_{2t} = 2.8075 + 0.0075t \) for \( t \leq -1 \), and \( \ln A_{1t} = 3.32 \) and \( \ln A_{2t} = 2.80 \) for \( t \geq 0 \). The rest of the exogenous variables and parameters of the model take their respective baseline values. Thus, the dynamic path of equilibrium unemployment is given by:

\[
u_{2t} = \frac{0.029 + 0.805'(0.005 + 0.006t)}{0.73 - 0.024(0.805)^t} \quad \text{where } t \geq 0 ;
\]

and the dynamic response of the relative wage follows immediately from the solution of the model.

*A skill-biased productivity shock:* Consider an initial steady-state situation in which productivity was experiencing zero growth, i.e. the coefficients \( \{A_{1t}, A_{2t}\} \) were constant at their respective baseline values. In period \( t = 0 \), an unexpected, one-off, permanent increase in the parameter \( \alpha \) raises the level of aggregate efficiency or productivity, defined as \( \ln A_{t} = \alpha \ln A_{1t} + (1 - \alpha) \ln A_{2t} \), by increasing (respectively, reducing) the contribution of high-skilled (low-skilled) labour. In particular,
suppose that parameter $\alpha$ increases from a value of $\alpha = 0.7$ to a value of $\alpha = 0.72$ in period $t = 0$. The rest of the exogenous variables and parameters of the model take their respective baseline values. The convergence path of equilibrium unemployment to its new steady-state value is given by:

$$u_{2t} = -1.309 + \frac{1}{0.734 - 0.022(0.805)^t} \text{ where } t \geq 0 ;$$

and the convergence path of the relative wage follows immediately from the solution of the model.

A fairness shock: Consider an initial steady-state situation in which the exogenous variables and parameters of the model were taking their respective baseline values. In period $t = 0$, an unexpected, one-off, permanent shock to the fairness parameter $\beta$ raises its value from $\beta = 0.1$ to $\beta = 0.2$. The rest of the exogenous variables and parameters of the model are unaffected by the shock and stay constant at their respective baseline values. The convergence path of equilibrium unemployment to its new steady-state equilibrium is:

$$u_{2t} = 0.516 + \frac{1}{-2.282 + 0.145(0.805)^t} \text{ where } t \geq 0 ;$$

and the convergence path of the relative wage follows immediately from the solution of the model.

A labour supply shock: Consider an initial steady-state situation in which the supply of labour, both high-skilled and low-skilled, was growing quarterly at a rate of 0.25 percent, or 1 percent annualised growth. That is, in the initial steady-state situation, the relative supply of low-skilled workers was constant. At date $t = 0$, a permanent labour supply shock increases the quarterly growth rate of the supply of low-skilled workers to 0.3 percent while the supply of high-skilled workers grows still at 0.25 percent. In order to introduce the shock, we set $\ln(T_{2t}/T_{1t}) = -0.55$ for $t \leq -1$ and $\ln(T_{2t}/T_{1t}) = -0.55 + 0.0005t$ for $t \geq 0$. The rest of the exogenous variables and parameters of the model take their respective baseline values throughout. The dynamic path of equilibrium unemployment is given by:

$$u_{2t} = 0.03 + \frac{-0.304 - 0.304t}{-30.227 + (0.805)^t} - 0.01t \text{ where } t \geq 0 ;$$

and the dynamic response of the relative wage follows immediately from the solution of the model.
References


