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## **Growth, Cycles and Stabilisation Policy**

By

Keith Blackburn<sup>†</sup> and Alessandra Pelloni<sup>\*</sup>

<sup>†</sup>Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, UK

<sup>\*</sup>University of Rome, Tor Vergata, Italy

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# Growth, Cycles and Stabilisation Policy\*

Keith Blackburn and Alessandra Pelloni  
Centre for Growth and Business Cycle Research  
School of Economic Studies, University of Manchester

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## Abstract

This paper presents an analysis of the joint determination of growth and business cycles with the view to studying the long-run implications of short-term monetary stabilisation policy. The analysis is based on a simple stochastic growth model in which both real and nominal shocks have permanent effects on output due to nominal rigidities (wage contracts) and an endogenous technology (learning-by-doing). It is shown that there is a negative correlation between the mean and variance of output growth irrespective of the source of fluctuations. It is also shown that, in spite of this, there may exist a conflict between short-term stabilisation and long-term growth depending on the type of disturbance. Finally, it is shown that, from a welfare perspective, the optimal monetary policy is that policy which maximises long-run growth to the exclusion of stabilisation considerations.

## 1 Introduction

A long-standing tradition in macroeconomics - at both theoretical and empirical levels - is the separation of the study of growth from the study of business cycles. The basis of this dichotomy is the presumption that aggregate time series can be decomposed into long-term trends and short-term fluctuations which are determined independently of each other. This presumption runs counter to the implications of endogenous growth theory, according to which any type of shock - be it temporary or permanent, real or nominal - can have

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a permanent effect on output so long as it changes the amount of resources on which productivity improvements depend (e.g., Fatas 2000; Pelloni 1997; Stadler 1990). Under such circumstances, there is no *a priori* reason to distinguish between low-frequency and high-frequency variations in economic activity, and the presence of stochastic trends is to be explained not by some arbitrary, exogenous impulse process (e.g., non-stationary productivity shocks), but rather by endogenous responses of technology to changes in the current state of the economy.

Recently, the question of precisely how cyclical fluctuations (booms and recessions) might affect secular trends (long-run growth) has been the subject of an expanding body of literature. Broadly speaking, one may distinguish between two contrasting approaches with the potential to generate different conclusions based on alternative assumptions about the mechanism responsible for engendering endogenous technological change.<sup>1</sup> According to one class of models - models based on re-allocation effects, whereby the mechanism entails deliberate actions (purposeful learning) which substitute for production activities - recessions are events which have a positive impact on growth by reducing the opportunity cost of diverting resources away from manufacturing towards productivity improvements (e.g., Aghion and Saint-Paul 1998a, 1998b). According to another class of models - models based on externality effects, whereby the mechanism reflects non-deliberate actions (serendipitous learning) which are complements to production activity - recessions are episodes which have a negative influence on growth by lowering factor employment through which expertise, knowledge and skills are acquired and disseminated (e.g., Martin and Rogers 1997, 2000). Within the context of each of these frameworks, attention has also been given to the question of how growth may be affected by the precise structure of business cycles in terms of their amplitude, frequency and persistence. Of particular interest has been the relationship between growth and volatility with different analyses reaching different conclusions (a positive or negative relationship) depending on what type of model is employed and what values for parameters are assumed (e.g., Aghion and Saint-Paul 1998a, 1998b; Canton 1996; de Hek 1999; Jones *et al.* 1999; Martin and Rogers 1997, 2000; Smith 1996).<sup>2</sup>

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<sup>1</sup>In addition to the references that follow, see Caballero and Hammour (1994), Hall (1991) and Ramey and Ramey (1991) for related contributions.

<sup>2</sup>This conflict in results at the theoretical level is matched by a similar conflict in evidence at the empirical level. In both cross-section and time series studies, the correlation between the average growth rate of output and the variability of output growth is found sometimes to be positive (e.g., Caporale and McKiernan 1996; Grier and Tullock 1989; Kormendi and Meguire 1985), sometimes to be negative (e.g., Kneller and Young 2001;

Significantly, all of the above analyses are based on purely real models of the economy and there are very few investigations that explore the role of monetary factors. An exception is the recent contribution by Dotsey and Sarte (2000) who develop a stochastic monetary growth model in which agents are subject to a cash-in-advance constraint, while operating a simple *AK* production technology with a fixed amount of labour. It is shown that an increase in the volatility of monetary growth, coupled with an increase in average monetary growth, may lead to either an increase or a decrease in average output growth due to offsetting effects through precautionary savings and inflation taxes.<sup>3</sup> One objective of the present paper is to conduct a similar analysis but within the context of a quite different monetary growth model that allows for endogenous labour, multiple shocks, learning-by-doing and nominal rigidities. The last of these features, encapsulated in the form of one-period wage contracts, does not appear in any of the above literature. Our analysis provides a further illustration of the joint determination of growth and business cycles. One of our main results is that an increase in the variance of any type of shock - real or nominal, demand or supply - generates a negative correlation between the mean and variance of output growth.

A second concern of the paper relates to an important, but largely neglected, issue in the formulation and evaluation of macroeconomic policy. This is the extent to which policies designed to stabilise short-run fluctuations might also affect the long-run performance of the economy. The existence of a relationship between growth and volatility has an obvious bearing on this issue: depending on whether this relationship is negative or positive, there is the presumption that successful stabilisation would also entail either an improvement or deterioration in growth prospects. The potential significance of this is self-evident, especially given the fact that it takes only small changes in the growth rate to produce substantial cumulative gains or losses in output. As yet, however, there are very few analyses that deal with the issue explicitly. Two recent contributions that do so are those of Blackburn (1999) and Martin and Rogers (1997). The former presents a model of imperfect competition with nominal rigidities in which monetary stabilisation policy has a negative effect on long-run growth. The latter consider a real model of the economy with perfect competition in which fiscal stabilisation policy has a positive effect on long-run growth. In both cases sustainable growth occurs due to learning-by-doing which provides the only source of

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Martin and Rogers 2000; Ramey and Ramey 1995), and sometimes to be zero (e.g., Dawson and Stephenson 1997; Grier and Perry 2000; Speight 1999).

<sup>3</sup>The assumption of a positive correlation between the mean and variance of monetary growth (or inflation) is justified by appealing to empirical evidence.

intrinsic dynamics, there being no capital accumulation or any other propagation mechanism under the direct control of agents. The models are also highly stylised in a number of other respects. In Blackburn (1999) there is no explicit optimisation by either agents or policy makers so that the normative aspects of policy are eschewed. In Martin and Rogers (1997) agents solve only a purely static optimisation problem from which employment is determined as a zero-one variable due to linear preferences. The analysis that follows is based on a more fully-specified dynamic general equilibrium model in which both capital accumulation and wage determination reflect the optimal decision rules of intertemporally maximising agents, and in which both the growth and welfare effects of monetary stabilisation policy may be evaluated explicitly. It is shown that, depending on the source of fluctuations, a policy that reduces volatility may either increase or decrease growth, even though volatility and growth are negatively correlated in terms of the underlying shocks. This means that the optimal policy that minimises volatility may not only be different from, but may also conflict with, the optimal policy that maximises growth, which is shown to be the optimal policy that maximises welfare as well.

Section 2 contains a description of the model. Section 3 presents the solution of the model. Section 4 turns to the analysis of growth, volatility and stabilisation policy. Section 5 concludes.

## 2 The Model

We consider an artificial economy in which there are constant populations (normalised to one) of identical, immortal households and identical, competitive firms. The basic structure of this economy is described by a standard stochastic, monetary growth model. Naturally, whilst the the model is more general than some others, it is constructed in such a way as to focus and simplify the analysis, and is not meant to provide a complete account of the mechanisms underlying aggregate fluctuations. In particular, since our intention is to illustrate without having to resort to numerical simulations, we adopt the usual specifications of preferences and technologies that admit closed-form solutions.<sup>4</sup> The exogenous shocks in the model are also chosen to provide clear and simple examples of how different conclusions may be reached on the basis of different assumptions about the sources of stochastic fluctuations.

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<sup>4</sup>That is, we assume logarithmic utility functions, Cobb-Douglas production functions and 100 percent rates of depreciation (e.g., Benassy 1995; Gali 1999).

## 2.1 Firms

The representative firm combines  $N_t$  units of labour with  $K_t$  units of capital to produce  $Y_t$  units of output according to

$$Y_t = \Psi_t (Z_t N_t)^\alpha K_t^{1-\alpha}, \quad (1)$$

$$\Psi_t = \Psi \exp(\psi_t), \quad (2)$$

$$\Psi > 0; \alpha \in (0, 1)$$

The term  $\psi_t$  represents a technology shock which is assumed to be identically, independently and normally distributed with mean zero and variance  $\sigma_\psi^2$ . The term  $Z_t$  denotes an index of knowledge which is freely available to all firms and which is acquired through serendipitous learning-by-doing. This provides the mechanism of endogenous growth in the model. Following convention, we approximate the stock of disembodied knowledge by the aggregate stock of capital which is taken as given by each firm so that learning takes the form of a pure externality.

Labour and capital are hired from households at the real wage rate  $\frac{W_t}{P_t}$  and real rental rate  $R_t$ , respectively, where  $W_t$  is the nominal wage and  $P_t$  is the price of output. Profit maximisation implies

$$\frac{W_t}{P_t} = \alpha \Psi_t Z_t^\alpha N_t^{\alpha-1} K_t^{1-\alpha}, \quad (3)$$

$$R_t = (1 - \alpha) \Psi_t Z_t^\alpha N_t^\alpha K_t^{-\alpha}. \quad (4)$$

## 2.2 Households

The representative household derives lifetime utility,  $U$ , according to

$$U = \sum_{t=0}^{\infty} \beta^t \left[ \Gamma \log(C_t) + \Theta \log\left(\frac{M_{t-1} \Phi_t}{P_t}\right) - \Lambda_t L_t^\eta \right], \quad (5)$$

$$\Lambda_t = \Lambda \exp(\lambda_t), \quad (6)$$

$$\beta \in (0, 1); \Gamma, \Theta, \Lambda > 0; \eta > 1,$$

where  $C_t$  denotes consumption,  $\frac{M_{t-1} \Phi_t}{P_t}$  denotes real money balances and  $L_t$  denotes labour. To generate a demand function for money, we adopt the familiar short-cut device of introducing money directly into the utility function, rather than specifying explicitly a separate transactions technology. The quantity  $M_{t-1}$  is understood to represent nominal cash balances at the

beginning of period  $t$  (or end of period  $t - 1$ ) which are augmented by a proportional monetary transfer,  $\Phi_t$ .<sup>5</sup> The specification of the labour term is another feature that our model shares with several others and encompasses the linear case ( $\eta = 1$ ) associated either with the assumption of constant marginal utility of leisure, or with a reduced-form preference ordering under circumstances where labour is indivisible and individuals choose employment lotteries in the manner of Hansen (1985) and Rogerson (1988). The quantity  $\lambda_t$  represents a preference (or taste) shock, being an identically and independently distributed normal random variable with mean zero and variance  $\sigma_\lambda^2$ .<sup>6</sup>

The budget constraint facing the household is given by

$$C_t + \frac{M_t}{P_t} + A_{t+1} = \frac{W_t}{P_t} L_t + \frac{M_{t-1} \Phi_t}{P_t} + R_t A_t, \quad (7)$$

where  $A_t$  denotes real asset holdings. The right-hand-side of this expression gives the household's total real resources in each period, being comprised of labour income, previously accumulated money balances (augmented by the nominal transfer) and previously accumulated assets (augmented by interest payments). The left-hand-side shows the allocation of these resources between consumption, further additions to money holdings and further additions to asset holdings.

Each household confronts the problem of maximising the expected value of intertemporal utility in (5) subject to the sequence of budget constraints in (7). This problem is solved, in part, by choosing plans for consumption, money balances and asset holdings that satisfy the following conditions:

$$\frac{1}{C_t} = \beta E_t \left( \frac{R_{t+1}}{C_{t+1}} \right), \quad (8)$$

$$\frac{1}{P_t C_t} = \frac{\beta \Theta}{\Gamma M_t} + \beta E_t \left( \frac{\Phi_{t+1}}{P_{t+1} C_{t+1}} \right), \quad (9)$$

where  $E_t$  denotes the expectations operator.<sup>7</sup> Each of these conditions has the usual interpretation of equating the current marginal costs and (expected)

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<sup>5</sup>In some models, it is end-of-period (rather than beginning-of-period) money holdings that serve as the reference point. To the extent that money yields utility by facilitating transactions, it seems more reasonable to adopt the present formulation. The assumption that monetary transfers are proportional (rather than lump-sum) is made largely for analytical convenience (e.g., Benassy 1995).

<sup>6</sup>As in some other analyses, the model could be extended to allow the other preference parameters,  $\Gamma$  and  $\Theta$ , to be stochastic as well (e.g., Ireland 1997; McCallum and Nelson 1999). We choose not to do so for the sake of simplicity and brevity.

<sup>7</sup>Multiplying these expressions through by  $\Gamma$  produces the term  $\frac{\Gamma}{C_t}$  which is understood to be the marginal utility of consumption, or shadow value of wealth, being equal to the Lagrange multiplier attached to (7).

future marginal benefits of foregoing a unit of consumption in favour of an additional unit of savings (either assets or money). Plans for the number of hours to work are governed by circumstances in the labour market which we treat as being imperfectly competitive and imperfectly flexible. We adopt a standard monopoly union model of wage determination, whereby households (or unions) set nominal wages and firms determine the level of employment. We also make the assumption, familiar in business cycle analysis but less so in growth theory, that wage setting takes place prior to the realisations of shocks on the basis of one-period contracts. Accordingly, the economy displays nominal rigidities, as in the early contracting models of Fischer (1977) and Gray (1976), as well as those of a more recent vintage (e.g., Benassy 1995; Cho and Cooley 1995). In contrast to these models, however, we suppose that the contract wage is chosen so as to maximise households' expected utility (e.g., Hairault and Portier 1993; Rankin 1998), rather than to satisfy some *ad hoc* criterion, such as the maximisation of other union objectives or the requirement that the labour market is expected to clear. When making this choice, workers take account of the response of labour demand, as expressed in (3). Given this, the optimal wage set at the end of period  $t - 1$  for period  $t$  is found to satisfy<sup>8</sup>

$$\eta E_{t-1}(\Lambda_t N_t^\eta) = \alpha \Gamma W_t E_{t-1} \left( \frac{N_t}{P_t C_t} \right) \quad (10)$$

The maximising behaviour of the representative household is now characterised fully by the first-order conditions in (8), (9) and (10), the budget constraint in (7) and the transversality conditions  $\lim_{\tau \rightarrow \infty} \beta^\tau E_t \left( \frac{M_{t+\tau}}{P_{t+\tau} C_{t+\tau}} \right) = \lim_{\tau \rightarrow \infty} \beta^\tau E_t \left( \frac{A_{t+\tau+1}}{C_{t+\tau}} \right) = 0$ .

## 2.3 Monetary Policy

We assume that monetary policy is governed by a feedback rule through which the central bank exercises imperfect control over the aggregate money supply. The feedback rule dictates how the central bank responds to the occurrence of exogenous shocks in the economy. The imprecision in monetary control reflects the assumption that the central bank's policy instrument is the growth rate of the monetary base to which the growth rate of the money supply is imperfectly (randomly) related.<sup>9</sup> Formally, we have

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<sup>8</sup>That is,  $W_t$  is chosen so as to maximise the expected value of (5) subject to (7) and the condition that  $L_t = N_t = \left( \frac{W_t}{P_t \alpha \Psi_t Z_t^\alpha K_t^{1-\alpha}} \right)^{\frac{1}{\alpha-1}}$  from (3).

<sup>9</sup>Aside from its realism, this assumption serves as a convenient device for incorporating nominal shocks into the model. An alternative approach which yields identical results is to introduce money demand (or velocity) shocks by allowing the preference parameter  $\Theta$  to



$$H_t = H_{t-1}\Phi_t, \quad (11)$$

$$\Phi_t = F_t \exp(\phi_t), \quad (12)$$

$$F_t = F \exp(\rho_\phi \phi_t + \rho_\lambda \lambda_t + \rho_\psi \psi_t), \quad (13)$$

$$F > 1; \rho_\phi, \rho_\lambda, \rho_\psi \stackrel{\geq}{\leq} 0,$$

where  $H_t$  denotes the money supply,  $F_t$  denotes the feedback rule and  $\phi_t$  is the control error which we assume to be identically, independently and normally distributed with mean zero and variance  $\sigma_\phi^2$ . If  $\rho_\phi = \rho_\lambda = \rho_\psi = 0$ , then monetary policy is completely unresponsive to changes in the state of the economy and the money supply grows at the exogenous, stochastic rate  $F \exp(\phi_t)$ . If any of these feedback parameters are non-zero, however, then monetary policy is determined endogenously with the central bank reacting in a systematic fashion to the realisations of the shocks.<sup>10</sup>

### 3 General Equilibrium

The solution of the model is a stochastic dynamic general equilibrium which describes the aggregate behaviour of the economy based on the optimal decision rules that solve firms' and households' maximisation problems. The equilibrium is computed by combining the relationships obtained so far with the market clearing conditions  $C_t + K_{t+1} = Y_t$  (for goods),  $K_t = A_t$  (for capital),  $M_t = H_t$  (for money) and  $N_t = L_t$  (for labour). Given the structure of the model, we may proceed in two stages, the first of which entails determining the solutions for consumption, capital and money holdings for a given level of employment, and the second of which involves establishing the solution for employment, itself. Details of the derivations are relegated to an Appendix.

#### 3.1 Consumption, Capital Accumulation and Cash Balances

After appropriate substitutions, we are able to write (8) and (9) as

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be stochastic (e.g., Ireland 1997).

<sup>10</sup>As in other models, we suppose that the monetary authority is able to respond to the contemporaneous realisations of disturbances so that monetary policy can have real effects in the presence of one period wage contracts (e.g., Galí 1999; Ireland 1997). More generally, we would assume that the reaction lag in monetary policy is shorter than the length of contracts.

$$\frac{K_{t+1}}{C_t} = \beta(1 - \alpha) + \beta(1 - \alpha)E_t\left(\frac{K_{t+1}}{C_t}\right), \quad (14)$$

$$\frac{M_t}{P_t C_t} = \frac{\beta\Theta}{\Gamma} + \beta E_t\left(\frac{M_{t+1}}{P_{t+1} C_{t+1}}\right). \quad (15)$$

Each of these expressions defines a stochastic expectations difference equation which is solved by imposing the relevant transversality condition. Doing this, and exploiting our other relationships, we obtain the following results:

$$C_t = (1 - \omega)Y_t, \quad (16)$$

$$K_{t+1} = \omega Y_t, \quad (17)$$

$$\frac{M_t}{P_t} = \Omega Y_t, \quad (18)$$

where  $\omega = \beta(1 - \alpha)$  and  $\Omega = \frac{(1-\omega)\beta\Theta}{(1-\beta)\Gamma}$ . These expressions show that the equilibrium levels of consumption, capital and real money balances are all proportional to the level of output. This is a direct consequence of our assumptions about preferences and technologies which allow us to obtain simple, closed-form solutions. The solutions imply that, for a given level of employment, consumption and capital (as well as output) are independent of monetary factors. As we shall now see, however, employment is not invariant with respect to changes in nominal variables, meaning that monetary shocks and monetary policy can have real effects in the economy.

### 3.2 Employment

According to our description of the labour market, households supply whatever labour is demanded by firms at the optimally chosen contract wage implied by (10). After various manipulations, a precise expression for this wage may be obtained as

$$W_t = \left(\frac{\alpha}{\Omega}\right) \left[\frac{\eta(1 - \omega)}{\alpha^2 \Gamma}\right]^{\frac{1}{\eta}} [E_{t-1}(M_t^\eta \Lambda_t)]^{\frac{1}{\eta}} \quad (19)$$

Thus the nominal wage depends on expectations about the money supply,  $M_t$ , and the preference shock,  $\Lambda_t$ . Given the processes governing these variables, it is possible to compute the value of expectations as  $E_{t-1}(M_t^\eta \Lambda_t) = (M_{t-1} F)^\eta \Lambda \exp\{\frac{1}{2}[\eta^2(\rho_\phi + 1)^2 \sigma_\phi^2 + (\eta\rho_\lambda + 1)^2 \sigma_\lambda^2 + \eta^2 \rho_\psi^2 \sigma_\psi^2]\}$ .<sup>11</sup> In general, therefore, the nominal wage increases with an increase in the variance of each of the shocks.

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<sup>11</sup>That is, we combine the expressions in (11), (12) and (13), substitute for  $\Lambda_t = \Lambda \exp(\lambda_t)$  and exploit the fact that  $E[\exp(x)] = \exp(\frac{1}{2}\sigma^2)$  if  $x$  is a normally distributed random variable with mean zero and variance  $\sigma^2$ .

Having established the above, we may now proceed to derive an expression for the equilibrium level of employment. This turns out to be

$$N_t = \left[ \frac{\alpha^2 \Gamma}{\eta(1-\omega)} \right]^{\frac{1}{\eta}} \frac{M_t}{[E_{t-1}(M_t^\eta \Lambda_t)]^{\frac{1}{\eta}}} = \left[ \frac{\alpha^2 \Gamma}{\eta \Lambda(1-\omega)} \right]^{\frac{1}{\eta}} \frac{\exp(\epsilon_t)}{\exp(\sigma^2)}, \quad (20)$$

$$\epsilon_t = (\rho_\phi + 1)\phi_t + \rho_\lambda \lambda_t + \rho_\psi \psi_t, \quad (21)$$

$$\sigma^2 = \frac{\eta(\rho_\phi + 1)^2}{2} \sigma_\phi^2 + \frac{(\eta\rho_\lambda + 1)^2}{2\eta} \sigma_\lambda^2 + \frac{\eta\rho_\psi^2}{2} \sigma_\psi^2, \quad (22)$$

Accordingly, we arrive at the well-known result that, in the presence of nominal rigidities, fluctuations in employment can occur because of (unanticipated) fluctuations in money. This result would not arise were we to abandon the notion of wage contracts and assume, instead, that wages are chosen contingent on the realisation of  $M_t$ . Under such circumstances, this variable would vanish from (20) and employment would deviate from its perfectly competitive level only by a constant factor of proportionality,  $\alpha$ , reflecting the pure inefficiency effect of monopolistic wage setting (i.e., a downward bias to employment associated with an upward bias to wages). As it is, the existence of pre-determined wages in our model means that monetary fluctuations are non-neutral, having real effects on the economy through variations in employment, consumption, investment and output. The final expression for employment is seen to depend on both the realisations and variances of the exogenous shocks. Note that the preference shock,  $\lambda_t$ , and the technology shock,  $\psi_t$ , disappear from this expression if monetary policy is unresponsive to them (i.e., if  $\rho_\lambda = \rho_\psi = 0$ ). In the case of the former this is because of the fact that employment is completely demand-determined under our assumption of wage contracting. In the case of the latter, it is because of offsetting income and substitution effects under our specifications of preferences and technologies. Notwithstanding these observations, a positive realisation of any of the shocks leads to an increase in the money supply, an increase in the demand for goods and an increase in the demand for labour. Conversely, larger variances of the shocks lead to higher nominal wages and a lower demand for labour.<sup>12</sup>

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<sup>12</sup>To the extent that larger variances of the shocks lead to greater volatility in the money supply, this result is consistent with the findings obtained from some other models (which provide different explanations) and may be interpreted as saying that employment is negatively related to the degree of nominal uncertainty (e.g., Evans 1989; Rankin 1998; Sorensen 1992).

## 4 Stochastic Endogenous Growth and Monetary Policy

We are now in a position to address the main issues of interest to us - namely, the extent to which there are linkages between the cyclical and secular properties of aggregate fluctuations, and the implications of such linkages for monetary policy. We do this by solving for the growth rate of output, from which the growth rates of other non-stationary variables (consumption and capital) may be inferred. These growth rates are both stochastic and endogenous. It is recalled that we account for the latter property on the basis of learning-by-doing, formalised by approximating the stock of disembodied knowledge available to firms by the aggregate stock of capital: that is,  $Z_t = K_t$  in (1). As shown by others, the main implication of this is to make it possible for the level of output (and, with it, the levels of other variables) to depend on the accumulated realisations of any type of shock, whether real or nominal, temporary or permanent. We show this to be true in the present framework. More significantly, we establish the result that the average rate of growth of output is a function of the variances of the shocks, implying a relationship between secular growth and cyclical volatility from which we draw further implications regarding the growth and welfare effects of monetary stabilisation policy.

### 4.1 The Output Process

Substitution of (17) and (20) into (1) yields

$$\frac{Y_t}{Y_{t-1}} = \Psi\omega \left[ \frac{\alpha^2\Gamma}{\eta\Lambda(1-\omega)} \right]^{\frac{\alpha}{\eta}} \frac{\exp(\alpha\epsilon_t + \psi_t)}{\exp(\alpha\sigma^2)} \quad (23)$$

Defining  $y_t = \log(Y_t)$ , we then have

$$y_t - y_{t-1} = \delta - \alpha\sigma^2 + \alpha\epsilon_t + \psi_t = g + \varepsilon_t, \quad (24)$$

where  $\delta = \log(\Psi\omega) + \frac{\alpha}{\eta} \log \left[ \frac{\alpha^2\Gamma}{\eta\Lambda(1-\omega)} \right]$ . Hence

$$y_t = y_0 + gt + \sum_{j=1}^t \varepsilon_j. \quad (25)$$

Expression (24) shows that output follows a non-stationary stochastic process that is integrated of order one. This process is described by a random walk with drift, implying that the economy displays stochastic and sustainable growth. In turn, this means that the level of output is permanently affected by the occurrence of exogenous shocks, as indicated by (25). Accordingly, the model provides another example of how endogenous technological change (in our case, learning-by-doing) can generate unit roots and stochastic trends in

macroeconomic time series without having to assume some arbitrary, exogenous process governing productivity improvements. Since the growth rate of technology is  $z_t - z_{t-1} = y_t - y_{t-1}$ , we have the standard result of learning-by-doing models that technological change is pro-cyclical. In addition, since output (like employment) depends positively on monetary surprises, we have the other standard result of such models that positive (negative) demand shocks have positive (negative) effects on the growth rate. Of greater interest to us, however, is the fact that the drift term in (24) depends on the variances of all of the underlying disturbances, as reflected in the term  $\sigma^2$ . This is indicative of a relationship between growth and volatility, a matter to which we now turn.

Given (24), together with the definitions of  $\varepsilon_t$  and  $\sigma^2$ , we may compute the mean and variance of output growth as, respectively,

$$Mean(y_t - y_{t-1}) = \delta - \frac{\alpha\eta(\rho_\phi + 1)^2}{2}\sigma_\phi^2 - \frac{\alpha(\eta\rho_\lambda + 1)^2}{2\eta}\sigma_\lambda^2 - \frac{\alpha\eta\rho_\psi^2}{2}\sigma_\psi^2, \quad (26)$$

$$Var(y_t - y_{t-1}) = \alpha^2(\rho_\phi + 1)^2\sigma_\phi^2 + \alpha^2\rho_\lambda^2\sigma_\lambda^2 + (\alpha\rho_\psi + 1)^2\sigma_\psi^2. \quad (27)$$

These expressions show that, in general, an increase in the variance of any of the shocks causes an increase in  $Var(y_t - y_{t-1})$  but a decrease in  $Mean(y_t - y_{t-1})$ . This is the sense in which the model generates a negative correlation between long-run (secular) growth and short-run (cyclical) volatility. As we have seen, a larger variance of each shock is associated with a higher average wage and a lower average level of employment. The latter produces a lower average level of output and, with it, a lower average rate of capital accumulation. This general reduction in real economic activity is translated into a reduction in average growth by virtue of a fall in the rate of technological progress via the process of learning-by-doing. The resulting negative correlation between  $Mean(y_t - y_{t-1})$  and  $Var(y_t - y_{t-1})$  is consistent with the predictions of some other models, though the precise mechanism at work is different (e.g., de Hek 1999; Jones *et al.* 1999; Martin and Rogers 1997, 2000). Beyond the theoretical level, our analysis finds support in a number of empirical studies which, collectively, suggest that output growth is negatively related both to the amount of output variability and the degree of nominal uncertainty (e.g., Grier and Perry 2000; Grier and Tullock 1989; Judson and Orphanides 1996; Kneller and Young 2001; Kormendi and Meguire 1985; Martin and Rogers 2000; Ramey and Ramey 1995).

## 4.2 Growth, Stabilisation and Welfare

The existence of a relationship between growth and volatility adds a new dimension to the design and evaluation of macroeconomic policies aimed

at stabilising fluctuations. As indicated earlier, however, there are very few analyses that have so far attended to this issue. In the present framework stabilisation policy is modelled explicitly in (13) as a feedback rule for monetary policy through which the central bank responds endogenously and systematically to realisations of each of the disturbances. The precise nature of the response in each case is summarised by the relevant feedback parameter -  $\rho_\phi$ ,  $\rho_\lambda$  or  $\rho_\psi$  - which may be thought of as being chosen optimally by the bank according to its particular objectives.

Suppose that the central bank is concerned with both reducing short-term volatility, as given in (27), and enhancing long-term growth, as expressed in (26). In the case of the nominal shock,  $\phi_t$ , there is no conflict between these objectives: from the perspective of either minimising  $Var(y_t - y_{t-1})$  or maximising  $Mean(y_t - y_{t-1})$ , the optimal policy is the same, being to set  $\rho_\phi = -1$ . Such a policy eliminates completely the fluctuations and uncertainty that would otherwise arise from this disturbance, causing a higher level of real activity (because of lower nominal wages) and a higher rate of technological progress (via learning-by-doing). This is an example of how monetary stabilisation policy can be complementary to the promotion of growth. A counter-example is provided by consideration of the preference shock,  $\lambda_t$ : in this case it is optimal to set  $\rho_\lambda = 0$  for minimising  $Var(y_t - y_{t-1})$ , but  $\rho_\lambda = -\frac{1}{\eta}$  for maximising  $Mean(y_t - y_{t-1})$ . These results are explained by our earlier observation that employment depends only on the expectation, and not the realisation, of this shock if monetary policy does not respond to it; but a negative response is called for if one wants to ensure that expectations remain low so that, on average, wages remain low and employment, output and output growth remain high. The opposite situation arises with respect to the technology shock,  $\psi_t$ : it is now the case that minimising  $Var(y_t - y_{t-1})$  entails setting  $\rho_\psi = -\frac{1}{\alpha}$ , while maximising  $Mean(y_t - y_{t-1})$  requires  $\rho_\psi = 0$ . By conditioning monetary policy on the realisation of this shock, fluctuations in output can be stabilised, but only at the cost of raising expectations about the money supply (through greater nominal uncertainty) so that, on average, wages remain high and real activity remains low. Thus the model provides a simple illustration of how different scenarios may lead to different conclusions about the extent to which there may exist a trade-off between short-term stabilisation and long-term growth. In terms of optimising such a trade-off, one might view the policy maker as maximising the objective function

$$V = Mean(y_t - y_{t-1}) - \mu Var(y_t - y_{t-1}), \quad (28)$$

where  $\mu \geq 0$  is the weight assigned to the stabilisation objective relative to the growth objective. Under such circumstances, the optimal values for the

feedback parameters are  $\rho_\phi = -1$ ,  $\rho_\lambda = -\frac{1}{\eta+2\mu\alpha}$  and  $\rho_\psi = -\frac{2\mu}{\eta+2\mu\alpha}$ , from which the values above may be obtained by considering the limiting cases of  $\mu = 0$  and  $\mu = \infty$ .

Of course, minimising volatility or maximising growth may not be the same as maximising welfare. To study the welfare effects of monetary policy, we compute the unconditional expected value of utility in (5), arriving at the expression

$$E(U) = U_0 - U_1 \left[ \frac{\alpha\eta(\rho_\phi + 1)^2}{2} \sigma_\phi^2 + \frac{\alpha(\eta\rho_\lambda + 1)^2}{2\eta} \sigma_\lambda^2 + \frac{\alpha\eta\rho_\psi^2}{2} \sigma_\psi^2 \right], \quad (29)$$

where  $U_0$  and  $U_1$  are composite parameter terms, independent of monetary policy. A comparison of (29) with (26) reveals that welfare is maximised by choosing values for the feedback parameters which maximise growth: that is,  $\rho_\phi = -1$ ,  $\rho_\lambda = -\frac{1}{\eta}$  and  $\rho_\psi = 0$ . This result lends support to the views that, from a pure welfare perspective, it is growth, not volatility, that matters the most, and that it is growth, not volatility, to which stabilisation policy should be directed. Indeed, our analysis suggests that policy makers may do rather well in relinquishing any concern about volatility and focusing, instead, on a simple growth objective.

## 5 Conclusions

Two of the most long-standing traditions in macroeconomics are the study of growth and the study of business cycles. Until recently, these traditions have been largely divorced from each other with little cross-fertilisation of ideas between them. With the emergence of endogenous growth theory, however, economists have begun to question the validity of this dichotomy and there is now a growing body of research that seeks to explore the potential linkages between secular and cyclical activity. The present paper is intended as a further contribution to this new and important area research.

Unlike most other contributions, our analysis allows a role for nominal factors - nominal shocks and nominal rigidities - in the joint determination of growth and business cycles. Together with an endogenous technology based on learning-by-doing, these factors are responsible for generating a stochastic and sustainable growth rate of output, the mean and variance of which are both dependent on the variances of both real and nominal shocks. In this way, the model is able to account for a negative relationship between long-run growth and short-run volatility in accordance with the findings of several empirical studies.

Another distinguishing feature of our analysis is the attention given to the role of policy - in particular, stabilisation policy. Indeed, it is one of

only a very few investigations that provide fully worked-out examples of how policies designed to mitigate the impact of exogenous shocks can have consequences for the long-run performance of the economy. The possibility of this adds a new dimension to the design and evaluation of such policies, the most important aspect of which may not be their stabilisation qualities *per se*, but rather their potential to influence long-term growth prospects. According to our own investigation, monetary stabilisation policy aimed at reducing stochastic fluctuations may work either for or against the promotion of long-run growth depending on the source of the fluctuations. This result is notable in itself and is made more notable by the fact that the optimal policy which maximises welfare is not the policy which minimises volatility but the policy which maximises growth.

Our analysis is intended to be illustrative, being based deliberately on an analytically tractable framework for which closed-form solutions can be obtained from appropriate assumptions about preferences and technologies. The alternative approach would have been to use a more complicated model under more general assumptions and to conduct the analysis via numerical simulations. We have no reason to believe that the basic message of the paper would have been different had we followed this alternative which could have led one to lose sight of the precise objectives of the exercise and the intuition underlying the results. Nevertheless, it would be interesting to acquire an idea of the orders of magnitude of involved, and we intend to pursue this in later work by conducting a quantitative analysis of a more general, calibrated version of the model.



## Appendix

The results in (16), (17) and (18) may be computed as follows. Substitution of (4) into (8) delivers

$$\frac{1}{C_t} = \beta(1 - \alpha)E_t\left(\frac{Y_{t+1}}{C_{t+1}K_{t+1}}\right), \quad (\text{A1})$$

which may be transformed into (14) by exploiting  $Y_t = C_t + K_{t+1}$ . Given the transversality condition  $\lim_{\tau \rightarrow \infty} \beta^\tau E_t\left(\frac{K_{t+\tau+1}}{C_{t+\tau}}\right) = 0$ , the solution to (14) is

$$\frac{K_{t+1}}{C_t} = \frac{\beta(1 - \alpha)}{1 - \beta(1 - \alpha)}. \quad (\text{A2})$$

Combining (A2) with  $Y_t = C_t + K_{t+1}$  gives the expressions in (16) and (17). Similarly, (11) in conjunction with  $H_t = M_t$  implies that (9) may be converted to (15), the solution to which follows by imposing the transversality condition  $\lim_{\tau \rightarrow \infty} \beta^\tau E_t\left(\frac{M_{t+\tau}}{P_{t+\tau}C_{t+\tau}}\right) = 0$ . Thus

$$\frac{M_t}{P_t C_t} = \frac{\beta\Theta}{(1 - \beta)\Gamma}. \quad (\text{A3})$$

Together with (16), (A3) yields the expression in (18).

The results in (19) and (20) may be derived in the following manner. Substitution of (3) and (16) into (10) gives

$$\eta E_{t-1}(\Lambda_t N_t^\eta) = \frac{\alpha^2 \Gamma}{1 - \omega}. \quad (\text{A4})$$

In turn, (3) and (18) may be combined to obtain

$$N_t = \left(\frac{\alpha}{\Omega}\right) \left(\frac{M_t}{W_t}\right), \quad (\text{A5})$$

which implies

$$\eta E_{t-1}(\Lambda_t N_t^\eta) = \eta \left(\frac{\alpha}{\Omega}\right)^\eta \frac{E_{t-1}(M_t^\eta \Lambda_t)}{W_t^\eta}. \quad (\text{A6})$$

Equating (A6) with (A4) yields the expression in (19) which may be substituted back into (A5) to arrive at the expression in (20). From (11), (12) and (13),  $M_t = M_{t-1} F \exp(\epsilon_t)$ , where  $\epsilon_t$  is defined in (21), and  $[E_{t-1}(M_t^\eta \Lambda_t)]^{\frac{1}{\eta}} = M_{t-1} F \Lambda^{\frac{1}{\eta}} \{E_{t-1}[\exp(\eta(\rho_\phi + 1)\phi_t + (\eta\rho_\lambda + 1)\lambda_t + \eta\rho_\psi\psi_t)]\}^{\frac{1}{\eta}} = M_{t-1} F \Lambda^{\frac{1}{\eta}} \exp(\sigma^2)$ , where  $\sigma^2$  is defined in (22).

The result in (29) may be established along the following lines. Let  $c_t = \log(C_t)$ ,  $m_t = \log\left(\frac{M_t}{P_t}\right)$ ,  $n_t = \log(N_t)$  and  $k_t = \log(K_t)$ . From (5), using the fact that  $M_t = M_{t-1}\Phi_t$  in equilibrium, the unconditional expected value of utility may be written as

$$\begin{aligned}
E(U) &= E\left\{\sum_{t=0}^{\infty} \beta^t [\Gamma c_t + \Theta m_t - \Lambda_t N_t^\eta]\right\} \\
&= \Gamma \sum_{t=0}^{\infty} \beta^t E(c_t) + \Theta \sum_{t=0}^{\infty} \beta^t E(m_t) + \sum_{t=0}^{\infty} \beta^t E(\Lambda_t N_t^\eta) \quad (\text{A7})
\end{aligned}$$

By virtue of (16), (18) and (24),  $c_t - c_{t-1} = m_t - m_{t-1} = g + \varepsilon_t$ . Hence,

$$c_t = c_0 + gt + \sum_{j=1}^t \varepsilon_j, \quad m_t = m_0 + gt + \sum_{j=1}^t \varepsilon_j, \quad t \geq 1, \quad (\text{A8})$$

$$c_0 = \log[(1 - \omega)\Psi] + \alpha n_0 + k_0, \quad m_0 = \log(\Omega\Psi) + \alpha n_0 + k_0, \quad (\text{A9})$$

with  $k_0$  given. By virtue of (20) (using  $\Lambda_t = \Lambda \exp(\lambda_t)$ ),

$$\Lambda_t N_t^\eta = \left[ \frac{\alpha^2 \Gamma}{\eta(1 - \omega)} \right] \frac{\exp(\eta \epsilon_t + \lambda_t)}{\exp[\eta \sigma^2]}, \quad (\text{A10})$$

$$n_t = \frac{1}{\eta} \log \left[ \frac{\alpha^2 \Gamma}{\eta \Lambda (1 - \omega)} \right] - \sigma^2 + \epsilon_t. \quad (\text{A11})$$

It follows from (A8), (A9), (A10) and (A11) that

$$E(c_t) = E(c_0) + gt, \quad (\text{A12})$$

$$E(m_t) = E(m_0) + gt, \quad (\text{A13})$$

$$E(c_0) = \log[(1 - \omega)\Omega] + \alpha E(n_0) + k_0, \quad (\text{A14})$$

$$E(m_0) = \log(\Omega\Psi) + \alpha E(n_0) + k_0, \quad (\text{A15})$$

$$E(\Lambda_t N_t^\eta) = \frac{\alpha^2 \Gamma}{\eta(1 - \omega)}, \quad (\text{A16})$$

$$E(n_0) = \frac{1}{\eta} \log \left[ \frac{\alpha^2 \Gamma}{\eta \Lambda (1 - \omega)} \right] - \sigma^2, \quad (\text{A17})$$

These expressions provide all the necessary information by which to evaluate (A7). After some tedious algebra, the final result is (29).

## References

- [1] Aghion, P. and G. Saint-Paul, 1998a. On the virtue of bad times: an analysis of the interaction between economic fluctuations and productivity growth. *Macroeconomic Dynamics*, 2, 322-344.
- [2] Aghion, P. and G. Saint-Paul, 1998b. Uncovering some causal relationships between productivity growth and the structure of economic fluctuations: a tentative survey. *Labour*, 12, 279-303.
- [3] Benassy, J-P., 1995. Money and wage contracts in an optimising model of the business cycle. *Journal of Monetary Economics*, 35, 303-315.
- [4] Blackburn, K., 1999. Can stabilisation policy reduce long-run growth? *Economic Journal*, 109, 67-77.
- [5] Caballero, R. and M. Hammour, 1994. The cleansing effect of recessions. *American Economic Review*, 84, 350-368.
- [6] Canton, E., 1996. Business cycles in an two sector model of endogenous growth. Mimeo, Centre for Economic Research, University of Tilburg.
- [7] Caporale, T. and B. McKiernan, 1996. The relationship between output variability and growth: evidence from post-war UK data. *Scottish Journal of Political Economy*, 43, 229-236.
- [8] Cho, J.O. and T.F. Cooley, 1995. The business cycle with nominal contracts. *Economic Theory*, 6, 13-33.
- [9] Dawson, J.W. and E.F. Stephenson, 1997. The link between volatility and growth: evidence from the States. *Economics Letters*, 55, 365-369.
- [10] de Hek, P.A., 1999. On endogenous growth under uncertainty. *International Economic Review*, 40, 727-744.
- [11] Evans, G.W., 1989. The conduct of monetary policy and the natural rate of unemployment. *Journal of Money, Credit and Banking*, 21, 498-507.
- [12] Fatas, A., 2000. Endogenous growth and stochastic trends. *Journal of Monetary Economics*, 45, 107-128.
- [13] Fischer, S., 1977. Long-term contracts, rational expectations and the optimal money supply rule. *Journal of Political Economy*, 85, 163-190.

- [14] Gali, J., 1999. Technology, employment and the business cycle: do technology shocks explain aggregate fluctuations? *American Economic Review*, 89, 249-271.
- [15] Gray, J., 1976. Wage indexation: a macroeconomic approach. *Journal of Monetary Economics*, 2, 221-235.
- [16] Grier, K.B. and M.J. Perry, 2000. The effects of real and nominal uncertainty on inflation and output growth: some Garch-M evidence. *Journal of Applied Econometrics*, 15, 45-58.
- [17] Grier, K.B. and G. Tullock, 1989. An empirical analysis of cross-national economic growth, 1951-1980. *Journal of Monetary Economics*, 24, 259-276.
- [18] Hairault, J-O. and F. Portier, 1993. Money, new-Keynesian macroeconomics and the business cycle. *European Economic Review*, 37, 1533-1568.
- [19] Hall, R.E., 1991. Recessions as reorganisations. *NBER Macroeconomics Annual*.
- [20] Hansen, G.D., 1985. Indivisible labour and the business cycle. *Journal of Monetary Economics*, 16, 309-328.
- [21] Ireland, P.N., 1997. A small structural model quarterly model for monetary policy evaluation. *Carnegie-Rochester Conference Series on Public Policy*, 47, 83-108.
- [22] Jones, L.E., R.E. Manuelli and E. Stacchetti, 1999. Technology (and policy) shocks in models of endogenous growth. NBER Working Paper No.7063.
- [23] Judson, R. and A. Orphanides, 1996. Inflation, volatility and growth. Mimeo, Board of Governors of the Federal Reserve System.
- [24] Kormendi, R. and P. Meguire, 1985. Macroeconomic determinants of growth: cross-country evidence. *Journal of Monetary Economics*, 16, 141-163.
- [25] Kneller, R. and G. Young, 2001. Business cycle volatility, uncertainty and long-run growth. *Manchester School* (Papers and Proceedings of the Conference on Growth and Business Cycles in Theory and Practice), forthcoming.

- [26] Martin, P. and C.A. Rogers, 1997. Stabilisation policy, learning-by-doing and economic growth. *Oxford Economic Papers*, 49, 152-166.
- [27] Martin, P. and C.A. Rogers, 2000. Long-term growth and short-term economic instability. *European Economic Review*, 44, 359-381.
- [28] McCallum, B.T. and E. Nelson, 1999. An optimising IS-LM specification for monetary policy and business cycle analysis. *Journal of Money, Credit and Banking*, 31, 296-316.
- [29] Pelloni, A., 1997. Nominal shocks, endogenous growth and the business cycle. *Economic Journal*, 107, 467-474.
- [30] Ramey, G. and V.A. Ramey, 1991. Technology commitment and the cost of economic fluctuations. NBER Working Paper No.3755.
- [31] Ramey, G. and V.A. Ramey, 1995. Cross-country evidence on the link between volatility and growth. *American Economic Review*, 85, 1138-1152.
- [32] Rankin, N., 1998. Nominal rigidity and monetary uncertainty. *European Economic Review*, 42, 185-199.
- [33] Rogerson, R., 1988. Indivisible labour, lotteries and equilibrium. *Journal of Monetary Economics*, 21, 3-16.
- [34] Smith, R.T., 1996. Cyclical uncertainty, precautionary saving and economic growth. *Economica*, 63, 477-494.
- [35] Sorensen, J.R., 1992. Uncertainty and monetary policy in a wage bargaining model. *Scandinavian Journal of Economics*, 94, 443-455.
- [36] Speight, A.E.H., 1999. UK output variability and growth: some further evidence. *Scottish Journal of Political Economy*, 46, 175-181.
- [37] Stadler, G.W., 1990. Business cycle models with endogenous technology. *American Economic Review*, 80, 150-167.