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Infrastructure, Women's Time Allocation, and Economic Development

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Infrastructure, Women's Time Allocation, and Economic Development

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Abstract

This paper develops a gender-based OLG model of endogenous growth to analyze the impact of infrastructure on women's time allocation between market work, raising children, own health care, home production, and leisure. Gender bias occurs as a result of firms discriminating between men and women, and of mothers devoting relatively more time to rearing their sons. Women's health status in adulthood, which affects productivity and wages, depends on their health status in childhood. A stagnation equilibrium and multiple development regimes are derived. An increase in productive government spending may shift the economy to a high-growth equilibrium, in a process involving changes in life expectancy, fertility, and a reallocation of women's time.

JEL Classification Numbers: I18, I21, O41.

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[In Africa] we see women carrying products from the farm on their head... Now imagine the acceleration of productivity if they did not have to carry these heavy loads on their heads, if they had the necessary infrastructure with which to make agriculture truly work for African society. Every time we see women incapacitated by the unavailability of infrastructure, they are foregoing very important activities that they should otherwise be applying themselves to, like going to school or engaging in income-earning activities that would improve their lives and those of their families.

Obiageli K. Ezekwesili, World Bank, opening remarks at conference on *The Role of Infrastructure in Women's Economic Empowerment*, Japan, May 28-30, 2008.

1 Introduction

The role of infrastructure in the process of economic development has received renewed attention in the ongoing debate on how to promote growth in low-income countries. In addition to the conventional positive effects on factor productivity and private investment, more recent evidence suggests that infrastructure may have a significant impact on health and education outcomes. Moreover, this impact tends to be magnified through interactions between health and education themselves. Better health has been shown to have a large impact on the ability to learn and study; conversely, more educated parents tend to take better care of themselves and their children.

Since the seminal contribution of Boserup (1970), the role of women in promoting growth and development has also occupied centre stage in policy debates.[1] For instance, Coulombe and Tremblay (2006) found that investment in the human capital of women is more important for growth in today's industrialized countries than investment in the human capital of men. At the

[1] See Momsen (2003) and Kevane (2004) for in-depth discussions of the links between gender and development.

same time, however, it has been well documented that women trail men in a number of dimensions—formal labor force participation, access to credit and infrastructure, entrepreneurship rates, and income levels. In low- and middle-income countries for instance, the labor force participation rate for women is only 57 percent, compared to 85 percent for men. On average, women workers earn about three-quarters of what men earn. Gender differences in education, work experience and job characteristics explain some fraction of this gap; In sub-Saharan Africa, 54 percent of girls do not complete even a primary school education (Herz and Sperling (2004)). This is important also because it has been found that illiterate women have more children and that mothers' education has a positive effect on child survival, education and nutritional status. Because of better nutritional knowledge, more educated mothers tend to adopt safer health and hygiene practices, which improve their children's health and survival (see Glewwe (1999) and Morrison, Raju, and Sinha (2007)). In Africa, children of mothers who receive five years of primary education are 40 percent more likely to live beyond age five, and educated mothers are about 50 percent more likely to immunize their children than uneducated mothers. As documented by Blackden et al. (2006), gender inequality (particularly with respect to education and access to formal sector employment) remains a significant constraint to growth in sub-Saharan Africa.[2]

The key issue that this paper addresses is at the intersection of these two agendas: it focuses on how improved access to infrastructure affects women's time allocation and how, in turn, changes in such allocation affect the process of growth and economic development. The impact of infrastruc-

[2] See Herz and Sperling (2004) for a broad review of the various channels through which better education for women may help to stimulate growth.

ture on women's time allocation has been documented in a number of empirical studies. As suggested by the opening quote of this paper, better access to infrastructure may enable women to devote more time to market activity, thereby promoting growth (see also Mehra and Rojas (2008)). At the same time, it may lead to improved learning monitoring (in the case of electricity, for instance) as well as improved child care practices (including breast feeding), which may strengthen the health status of children and their ability to learn.[3] There are therefore opportunity costs to market activity, and they must be taken into account when assessing growth effects.

Accordingly, this paper develops a three-period, gender-based overlapping generations (OLG) model with public capital to explore the implications of public infrastructure on women's time allocation and growth. Gender-based OLG models are relatively few; among the notable exceptions are Ehrlich and Lui (1991), Galor and Weil (1996, 2000), Zhang, Zhang, and Li (1999), Lagerlöf (2003), and de la Croix and Vander Donckt (2008). None of these contributions, however, accounts explicitly for the impact of public capital on growth, either directly through production or indirectly through women's time allocation.

Our model differs from the existing literature in several other ways as well. First, women's time in the model is endogenously allocated not only to market work and leisure, but also to home production, child rearing, and own health care. In most of the relevant literature, rearing time is typically considered exogenous, whereas time devoted to own health care is generally ignored—even in OLG models where interactions between health and growth take center stage, as for instance in Chakraborty (2004), Hashimoto and

[3] For evidence that parental tutoring is important for children in developing countries, see Glewwe and Kremer (2006).

Tabata (2005), Finlay (2006), Bhattacharya and Qiao (2007), and Tang and Zhang (2007). Endogenizing mothers' rearing time is important because in the model such time is productive—it helps to enhance education and health outcomes.

Second, the model accounts not only for the productivity effects of public infrastructure (as in many other contributions) but also for its effects on the production of health services, human capital, and the efficiency of mothers' time allocation. Thus, in contrast to existing studies, but in line with the evidence alluded to earlier, we are able to highlight the key role that infrastructure may play in affecting education and health outcomes, both directly and indirectly. To the extent that access to infrastructure affects the time devoted by mothers to caring for their children, it will also exert an indirect effect on long-run growth. In addition, we also account for the possibility that female labor allocated to home production may exhibit low productivity levels if access to infrastructure is limited.

Third, human capital accumulation depends directly on mothers' level of education; this specification is consistent with the evidence provided by Filmer (2000), according to which women's education generally has more impact than men's education on children's schooling.[4] Fourth, health outcomes exhibit serial dependence, in the sense that health status (and productivity) in adulthood depends to a significant extent on health outcomes in childhood. This is consistent with a number of studies, such as Case, Fertig, and Paxson (2005), Schady and Paxson (2007), and Smith (2008). Such dependence gives a crucial role to mothers' time allocation in shaping the future

[4] In the same vein, Behrman et al. (1999) found that in India children of educated women study two extra hours per day. However, there is also evidence that better-educated women marry better-educated husbands. So it is possible that the observed effect of women's education might also reflect unobserved preferences of their husbands for healthier or better-educated children (see Breierova and Duflo (2004)).

of their children.

Fifth, we introduce gender bias both in the work place and in the home. Women workers earn less than men, and mothers allocate more of their available rearing time to their sons than daughters. This differs from the treatment in Zhang, Zhang, and Li (1999) in which parents allocate the same amount of time to boys and girls and gender bias takes the form of parents directly choosing more boys than girls. In our analysis, however, gender gaps are not "locked in"; as mothers' health improves, the health status of their daughters also improves, thereby increasing their productivity (and wages) in adulthood. Because access to infrastructure affects the efficiency of time allocated to children, increases in public capital may therefore mitigate over time initial gender gaps in income.

The paper is organized as follows. Section II provides an overview of the literature on the impact of infrastructure on women, particularly with respect to their time allocation. Section III presents the model. Section IV derives the steady-state growth rate and examines the properties of the model. In particular, we examine how limited access to infrastructure may contribute to the economy being "trapped" in a low-level equilibrium with large gender gaps in education and earnings, high fertility rates, poor health outcomes, and low levels of per capita incomes. Section V examines the possibility of multiple development regimes resulting from threshold effects associated with female life expectancy. Section VI discusses whether an increase in public spending on health or investment in infrastructure may allow a country to escape from a low-growth trap. The last section provides some concluding remarks.

2 Gender Dimension of Infrastructure

The effects of public infrastructure on productivity are well documented. If, as it is normally the case, production factors are gross complements, a higher stock of public capital in infrastructure would tend to raise the productivity of private inputs—including female labor—thereby reducing unit production costs. Given decreasing returns, the magnitude of this effect may be substantial in low-income countries. As background motivation to the model developed in this paper, this section provides instead a brief review of the evidence on the links between public infrastructure and health and education outcomes. It then examines the implications of these links for women's time allocation.[5]

2.1 Effects on Education and Health

2.1.1 Education

A number of studies have found a direct positive impact of various types of infrastructure services (namely, roads, electricity, water and sanitation, and telecommunications) on learning indicators for boys and girls alike. A better transportation system and a safer road network (particularly in rural areas) helps to raise school attendance. In a study of Bangladesh, Khandker et al. (2006) found that improved rural roads lead not only to lower poverty (through higher agricultural production, higher wages, lower input and transportation costs, and higher output prices) but also to an increase in schooling. The quality of education may also improve, as greater accessibility makes it easier to hire teachers and facilitate commuting between rural and urban areas (see Levy (2004)).

[5] For a more detailed account of the literature, together with extensive references, see P.-R. Agénor (2009*b*).

Similarly, greater access to safe water and sanitation in schools tends to raise attendance rates (particularly for girls) and the ability of children to learn, by improving their health. Access to electricity helps also to improve the learning process, by allowing children to spend more time studying and by providing more opportunities to use electronic equipment, such as computers.

2.1.2 Health

Infrastructure may have a sizable impact on health outcomes as well. As documented in the various microeconomic studies summarized by P.-R. Agénor (2009*b*), access to safe water and sanitation helps to improve health, particularly among children. Surveys indicate that in several Sub-Saharan African cities, the death rate of children under five is about twice as high in slums (where water and sanitation services are poor, if not inexistent), compared to other urban communities. More formal studies (including Wagstaff and Claeson (2004)) have also found that access to clean water and sanitation infrastructure helps to reduce infant mortality.

Access to electricity, by reducing the cost of boiling water, helps to improve hygiene and health as well. Availability of electricity is essential for the functioning of health care facilities and the delivery of health services; vaccines, for instance, require continuous and reliable refrigeration to retain their effectiveness (see World Bank (2008)). Getting access to clean energy for cooking in people's homes (as opposed to smoky traditional fuels, such as wood, crop residues, dung, and charcoal) improves health outcomes by reducing indoor air pollution and the incidence of not only respiratory illnesses (such as asthma and tuberculosis), but also low birth weight and infant mortality (see World Bank (2008)). According to some estimates, indoor air pollution from the burning of solid fuels kills over 1.6 million people (predominantly women and children) a year. More efficient electric stoves would reduce this death toll, which is almost as great at that caused by unsafe water and sanitation, and greater than that cause by malaria.

Better transportation networks contribute to easier access to health care, particularly in rural areas. In Morocco, a program developed in the mid-1990s to expand the network of rural roads led—in addition to reducing production costs and improving access to markets—to a sizable increase in visits to primary health care facilities and clinics (see Levy (2004)). In Malaysia and Sri Lanka, the World Bank found that the dramatic drop in the maternal mortality ratio (from 2,136 in 1930 to 24 in 1996 in Sri Lanka, and from 1,085 in 1933 to 19 in 1997 in Malaysia) was due not only to a sharp increase in medical workers in rural and disadvantaged communities, but also to improved communication and transportation services—which helped to reduce geographic barriers. At a more formal level, Wagstaff and Claeson (2004, pp. 170-74) found, using cross-section regressions, that road infrastructure (as measured by the length of the paved road network) had a significant effect on a number of health indicators, such as infant and female mortality rates.

2.2 Impact on Women's Time Use

While the aforementioned effects of infrastructure on health impact all poor individuals in developing countries (especially those living in rural areas), they disproportionately affect poor women.[6] The reason is that women tend to devote considerably more time to household production activities than

[6]Most research on gender and growth in developing countries has focused on women's differential access to education, formal sector employment, assets, technology, health care, and social institutions (see Blackden et al. (2006))—as well as how the relationship between gender and growth is mediated by women's labor force participation, productivity, and earnings (see Morrison et al. (2007))—without exploring the role that women's access to infrastructure plays in the gender-growth pathway.

men (see Ilahi and Grimard (2000)). In what follows we examine briefly how access (or lack thereof) to transportation, water and sanitation, and electricity, affect women's time allocation.

2.2.1 Transportation

A large fraction of total transport activity in Sub-Saharan Africa is related to women's travel needs (see Weiss (1999)). Women travel—and thus rely on roads and other transport infrastructure—for a multitude of reasons, including household production activities, health care, education, and income-generating activities. As documented by Riverson et al. (2006) for instance, in Ethiopia, 73 percent of women's trips and 61 percent of their travel time is dedicated to meeting their household's energy, water, and food needs. Malmberg-Calvo (1994) found that, in Zambia, women spend over 800 hours per year gathering and transporting firewood while their male counterparts spend no more than 50 hours per year, whereas Semu and Mawaya (1999) found that, in Malawi, 91.5 percent of respondents identified firewood collection as a woman's task. More generally, available data suggest that, on average, women in rural Sub-Saharan Africa spend between .9 and 2.2 hours per day on transporting water and firewood (see Weiss (1999)).

Women also depend on transportation for health care. Data compiled by national Demographic and Health Surveys (DHS) in Sub-Saharan Africa show that a majority of women in rural areas rank distance and inadequate transportation as major obstacles in accessing health services (see African Union (2005)). Moreover, because of scarce or inexistent modes of public transportation and a lack of access to private transportation (such as bicycles, two or four-wheel motor vehicles, and carts), poor women in developing countries tend to travel on foot (Riverson et al. (2006)). For instance, Malmberg-Calvo (1996) found that in rural areas of Sub-Saharan Africa, 87 percent of women's travel occurs on foot and that women in these households are more likely to walk to their destination than their male counterparts. In addition, World Bank data indicate that rural Sub-Saharan African women travel over 1 to 5km per day on foot for 2.5 hours while carrying a load of about 20kg (see Riverson et al. (2006)). This not only constrains time available for other activities but also has adverse health implications.

2.2.2 Water and Sanitation

Women in low-income countries allocate a considerable amount of time to collecting water for household production (see Isha (2007) for an overview). Available data show that women in Benin, Madagascar, and South Africa spend 273 hours per year, 164 hours per year, and 48 hours per year, respectively, collecting water (Wodon and Blackden (2006)). In addition, using the 1991 Pakistan Integrated Household Survey, Ilahi and Grimard (2000) found that, in Pakistan, women who reported spending some time collecting water allocate an average of 27 hours per month—or approximately 15 percent of their monthly work time—to this task. In South Africa, 90 percent of the households in a survey reported that women were the primary collectors of water (Aggarwal et al. (2001)).

Furthermore, a number of studies have shown that if clean water were more accessible to women, they would save a notable amount of time, which they could in turn allocate to other activities—including leisure, child rearing, education, or health care. For instance, Blackwell (1996) found that if a source of clean water were located within 400 meters of all households in rural areas of Burkina Faso, Uganda, and Zambia, every household would save between 125 and 664 hours per year. While these data are not specific to women, it is reasonable to assume that women were the primary water collectors in the households sampled in these studies as well. In the same vein, Ilahi and Grimard (2000) found that in rural Pakistan, as access to public water infrastructure improves, the amount of time that women allocate to water collection decreases, thereby freeing up time to engage in income-earning activities.

2.2.3 Electricity

A number of studies have shown that access to electricity significantly improves women's (and, by implication, children's) health outcomes. For instance, the World Bank (2008) found that rural electrification has a positive impact on fertility reduction among women in low-income countries by increasing their access to television and, in turn, their exposure to health and family planning information. In addition, access to electricity can also decrease the amount of time that women spend on household production activities such as cooking and collecting water and firewood. For instance, a study found that women in the Philippines spent one less hour per day on domestic tasks as a result of electrification (World Bank (2008)). Moreover, Ilahi (2001) found that women living in rural Peru who rely on firewood or coal as a source of energy tend to allocate a smaller proportion of their time to self-employment activities and a greater proportion to housework than their counterparts who use gas or electricity. As a result, women who have access to electricity can devote more time to income-generating activities, rearing children, furthering their education, and accessing health care than those who rely on fossil fuels.

Moreover, using DHS data, Wang (2003) found that access to electricity had the greatest impact on decreasing infant mortality compared to other significant variables, namely income, access to water and sanitation, vaccination in the first year of life, and the share of health expenditures to GDP. Similarly, access to electricity explained 64 percent of the variation in mortality among children under five in low-income countries. Electricity may improve infants' and children's health by decreasing their exposure to indoor air pollution produced by the burning of fossil fuels, as well as decreasing their exposure to bacteria and parasites by facilitating the refrigeration of food and the boiling of water (see Wang (2003)). More importantly for our purpose, access to electricity may also improve child health outcomes by decreasing the amount of time that women allocate to home production activities and increasing the amount of time that they can devote to raising their children and acquiring further education.

In sum, when women lack access to infrastructure such as clean water and sanitation, roads and transportation, and electricity, they must allocate a greater proportion of their time to household activities (including home production and child rearing) than if they had access to sound infrastructure. The opportunity costs of poor infrastructure for women include leisure, wage labor, acquiring an education, and investing in their own health.[7] The key issue to address therefore is how an improvement in the quality and quantity of infrastructure affects, both directly and indirectly, the time women allocate to these various activities and how, in turn, changes in women's time allocation affect economic growth.

[7] Infrastructure can also have unintended or unexpectedly negative implications for women's health. One example is the increased risk of HIV infection among male migrant workers and their female sexual partners with the construction of migration routes as part of the development of transportation infrastructure in various regions of Sub-Saharan Africa (see Lurie, Hintzen, and Lowe (2004)).

3 The Model

We consider an OLG economy where two goods are produced, a marketed commodity and a home good, and individuals live for (at most) three periods: childhood, adulthood (or middle age) and retirement. The marketed commodity can be either consumed in the period it is produced or stored to yield capital at the beginning of the following period. Each individual is either male or female, and is endowed with one unit of time in childhood and adulthood, and zero units when they are old. Schooling is mandatory, so children devote all their time to education. They depend on their parents for consumption and any spending associated with schooling and health care. All individuals, males and females, work in middle age; the only source of income is therefore wages in the second period of life, which serve to finance family consumption in adulthood and old age. Savings can be held only in the form of physical capital. Agents have no other endowments, except for an initial stock of physical capital at $t = 0$, which is the endowment of an initial old generation.

In adulthood, individuals match randomly into couples with someone of the opposite sex to form a family. We abstract from intra-family distribution of assets and resources by assuming that all income is pooled; couples therefore become joint decision makers. For simplicity, once married, individuals do not divorce; couples retire together (if they survive to old age) and die together.[8] Parents have ready access to gender selection techniques; each couple decides on the (even) number of children to have, n_t, with half of them

[8] Given that we focus later on only on the link between women's health status and survival rate, we implicitly assume that in adulthood husbands do not survive their wives and die of sorrow soon after the passing of their spouse. This simplifies matters considerably by allowing us to use the same survival rate for men and women and to keep the gender composition of the population constant.

being daughters, and half of them sons. Boys and girls have the same innate abilities and thus the same intrinsic capacity to acquire human capital.

The cost of rearing children involves the cost of schooling and the cost of keeping them healthy. In turn, these costs involve both parental time and spending on marketed commodities (school supplies, medicines, etc.). As a result of biological differences (women are the ones who actually bear children and are capable of breast feeding) or social norms, mothers incur the whole time cost involved in rearing children. Thus, women "specialize" in that activity within the family—even though there are no innate gender differences in home production skills. Male spouses are not involved in child rearing and face in principle two alternative uses of their time, market work or leisure, whereas females spouses must consider five alternatives: market work, raising children, taking care of one's health, home production, and leisure. To simplify the analysis, and given the focus of this study on women's time allocation, adult males are assumed to devote inelastically all their time to market work.

The health status of children and adults are determined in different ways. The former depends on the time mothers allocate to rearing their brood. We account for gender bias in the allocation of mothers' time by assuming that they devote relatively more time to their sons. This is consistent with the evidence of Duryea et al. (2007) for instance, who found that in Bolivia, Guatemala, Mexico, and Peru, there is a clear preference for boys in low-income groups, and that such bias explains in part the persistence of gender gaps in adult schooling attainment.[9] Crucially, we also assume that health

[9] As noted by Zhang, Zhang, and Li (1999), there are two possible causes for gender bias: "pure" sex preference (linked to social norms, religion, or culture) and differences in earnings opportunities (related for instance to labor market discrimination). We account for both here. A third reason could be social norms about provision of old age support; mothers might favor boys in settings where sons provide old age support and favor girls

status in adulthood, in addition to the time spent caring about one's health and access to publicly-provided health services, depends on health status in childhood. There is therefore "state dependence" in health outcomes. This specification is consistent with the results of Case, Fertig and Paxson (2005), according to which children who experience poor health have on average significantly lower educational attainment, and significantly poorer health and lower earnings, as adults.[10]

At the beginning of the first period of life and the end of the second, there is a non-zero probability of dying. Survival rates from childhood to adulthood, as well as from adulthood to old age, are treated as distinct; this is consistent with the evidence suggesting that the determinants of adult and child mortality are in general different (see Cutler, Deaton, and Lleras-Muney (2006)). At the same time, to simplify the analysis we take the survival probabilities for men and women, in both childhood and adulthood, to be the same, despite the fact that there are good biological and socioeconomic reasons to suggest that adult mortality rates differ by gender.

In addition to individuals, the economy is populated by firms and an infinitely-lived government. Firms produce marketed commodities using public capital in infrastructure as an input, in addition to male and female labor and private capital. Home production also requires access to infrastructure. These features of the production side of the model therefore bring to the fore the productivity effects of infrastructure, as discussed earlier. In addition, home production (which affects positively utility) is a linear function of the time allocated by women to that activity. The government invests in in-

in settings where daughters provide such support (see Schultz (1997)). We ignore this last possibility, given that we do not model explicitly intergenerational bequests.

[10]In particular, hunger and infections in early childhood result in stunting, which in adulthood often brings about substantial income losses.

frastructure and spends on education, health, and some unproductive items. It taxes the wage income of adults (males and females), but not the interest income of retirees. It cannot borrow and therefore must run a balanced budget in each period. Finally, all markets clear and there are no debts or bequests between generations.

3.1 Family's Utility and Income

At the beginning of adulthood in $t+1$, all men and women are randomly matched into married couples. Each couple produces n_{t+1} children, half of whom are girls, and half are boys.

A mother raising a child faces two types of costs. First, she must spend $\varepsilon^R_{t+1} \in (0,1)$ units of time on each of them, because she provides tutoring or "home schooling" and takes care of the child's health (going to the hospital for checkups and vaccination, etc.). Each mother therefore allocates $\varepsilon^R_{t+1} n_{t+1}$ units of time to that activity. Second, raising children involves fixed costs in terms of marketed commodities. Specifically, it entails a loss per child (regardless of gender) equal to a fraction $\theta^R \in (0,1)$ of the family's net income. This loss is related to sending children to school and educating them at home (which involves buying school supplies, etc.), and to taking care of their health needs (buying medicines).[11] Thus, although access to "out of home" schooling and health services *per se* is free, families face a cost in terms of foregone wage income and foregone consumption. Let $(1-\tau)w^T_{t+1}$ denote the family's net wage income in $t+1$, with w^T_{t+1} denoting gross income and $\tau \in (0,1)$ the tax rate; the total cost of raising n_{t+1} children is thus given by the sum of the opportunity cost in terms of foregone wage

[11] These two types of costs could be separated by introducing different spending shares for the schooling and health components.This, however, would mainly add notational clutter and produce little value added to the analysis.

earnings, and the opportunity cost in terms of foregone consumption, that is, $(\varepsilon_{t+1}^{f,R}+\theta^R)n_{t+1}(1-\tau)w_{t+1}^T$. Thus, because both $\varepsilon_{t+1}^{f,R}$ and θ^R affect a child's health status (as discussed later), the existence of these costs creates a trade-off between the quality and quantity of children, in the tradition of Barro and Becker (1989).

In addition to raising children, mothers allocate time to market activity (in proportion $\varepsilon_{t+1}^{f,W}$) and to taking care of their own health needs (in proportion $\varepsilon_{t+1}^{f,H}$); this involves seeking medical treatment, personal hygiene, and exercise. Doing so involves also a loss equal to a fraction $\theta^H\varepsilon_{t+1}^{f,H}$, where $\theta^H \in (0,1)$, of the family's net income. Let $\varepsilon_{t+1}^{f,P}$ denote the time women allocate to home production (which includes time spent collecting water and firewood, for instance); leisure time is thus measured as $1-\varepsilon_{t+1}^{f,H}-\varepsilon_{t+1}^{f,P}-n_{t+1}\varepsilon_{t+1}^{f,R}-\varepsilon_{t+1}^{f,W}$.

The probability of survival from childhood to adulthood (at the beginning of period $t+1$) is denoted by $p_m^C \in (0,1)$, whereas the probability of survival from adulthood to old age is denoted by $p_m^A \in (0,1)$. Both rates are taken to be independent of gender and both are constant for the moment. Given the deterministic nature of the model, the actual number of survivors in each age group is simply given by the expected number of survivors. To avoid convergence of population size toward zero, we also assume that $p_m^C n_{t+1} \geq 1$.

There is an actuarially fair annuity market that channels savings to investment in physical capital, for production in the next period. With the annuity market, old-age survivors share the savings plus interest left by savers who die in adulthood.[12] The rate of return to saving is thus r_{t+2}/p_m^A.

The good consumed at home is a "composite" good produced by combining (using a Cobb-Douglas technology) marketed commodities and the good

[12] Alternatively, it could be assumed that the saving left by agents who do not survive to old age is "confiscated" by the government, who spends it for unproductive purposes.

produced at home. Assuming that consumption of children is subsumed in their parents' consumption, the family's lifetime utility takes the form

$$U = \ln[(c_{t+1}^t)^\omega q_{t+1}^{1-\omega}] + \eta_N \ln p_m^C h_{t+1}^C n_{t+1} \tag{1}$$

$$+\eta_L \ln(1 - \varepsilon_{t+1}^{f,H} - \varepsilon_{t+1}^{f,P} - p_m^C n_{t+1}\varepsilon_{t+1}^{f,R} - \varepsilon_{t+1}^{f,W}) + \frac{p_m^A}{1+\rho} \ln c_{t+2}^t,$$

where c_{t+1}^t (c_{t+2}^t), is the family's total consumption in adulthood (old age), q_{t+1} production of home goods, h_{t+1}^C health status of a child, $\rho > 0$ the discount rate, and $\omega \in (0,1)$.[13] Actual family size is $p^C n_{t+1}$, which differs from fertility, n_{t+1}, because the child survival rate is less than unity. The term $p_{t+1}^C h_{t+1}^C n_{t+1}$ is thus the actual number of healthy children. Coefficients η_N and η_L measure the family's relative preference for surviving healthy children and leisure. For simplicity, only the marketed commodity is consumed in old age.

A male (female) adult in period $t+1$ is endowed with e_{t+1}^m (e_{t+1}^f) units of human capital. Each unit of human capital earns a market wage, w_{t+1}^m for men and w_{t+1}^f for women, per unit of time worked.

As in P.-R. Agénor (2009*a*), child mortality occurs only at the beginning of the period; parents therefore incur rearing costs only on their children who survive into adulthood.[14] We also assume that giving birth involves no

[13] It could also be assumed that utility is positively affected by the time allocated by mothers to rearing children and to home production, which would lead to the addition of terms like $\eta_P \ln \varepsilon_{t+1}^{f,P}$ and $\eta_R \ln \varepsilon_{t+1}^{f,R}$ in (1). However, assuming that rearing children and home production also provide utility would not affect qualitatively the analysis, as long as η_P and η_R are not too large relative to η_L. Note also that studies for industrial countries do not suggest that time allocated to child care is a direct source of utility; see, for instance, Kimmel and Connelly (2006).

[14] If rearing costs are incurred for all births, the term $\theta^R p_m^C n_{t+1}$ in (2) should be replaced by $\theta^R n_{t+1}$. However, the assumption in the text is more natural, given that infant mortality in developing countries tends indeed to occur very early in life. Indeed, in these countries newborn mortality accounts for about 40 percent of the mortality of children under five years of age, and more than half of infant mortality (World Health Organization (2005, p. 9))

time cost (or, equivalently, that the time involved is fixed and normalized to zero). There is no budget constraint in the first period of life, as children's consumption needs are taken care of by their parents. The family's budget constraints for period $t+1$ and $t+2$ are given by[15]

$$c^t_{t+1} + s_{t+1} = (1 - \theta^H \varepsilon^{f,H}_{t+1} - \theta^R p^C_m n_{t+1})(1-\tau) w^T_{t+1}, \tag{2}$$

$$c^t_{t+2} = (1 + r_{t+2}) s_{t+1} / p^A_m, \tag{3}$$

where s_{t+1} is saving and gross wage income is defined as

$$w^T_{t+1} = e^m_{t+1} w^m_{t+1} + e^f_{t+1} \varepsilon^{f,W}_{t+1} a^f_{t+1} w^f_{t+1}. \tag{4}$$

In this expression, a^f_{t+1} is female labor productivity. As noted earlier, husbands supply inelastically to paid work the unit of time that they have available; for simplicity, we also assume that male productivity is constant and normalized to unity.

The family's consolidated budget constraint is thus

$$c^t_{t+1} + \frac{p^A_m c^t_{t+2}}{1 + r_{t+2}} = (1 - \theta^H \varepsilon^{f,H}_{t+1} - \theta^R p^C_m n_{t+1})(1-\tau) w^T_{t+1}. \tag{5}$$

3.2 Home Production

Home production (which includes cooking dinner, doing laundry, washing the kitchen floor, cleaning the house, etc.) depends linearly on the "effective" amount of women's time allocated to that activity:[16]

$$q_t = \zeta_t \varepsilon^{f,P}_t, \tag{6}$$

[15] With different rearing costs for boys and girls the term $\theta^R p^C_m n_{t+1}$ in (2) would be replaced by $(\theta^{R,m} + \theta^{R,f}) 0.5 p^C_m n_{t+1}$.

[16] It could be assumed that home production requires also the use of marketed commodities. We abstract from this complication, as it would not bring additional insight for the purpose at hand. We also do not consider child labor—as in Moe (1998), for instance—given our assumption that children allocate all their time to schooling.

where $\zeta_t > 0$ is an efficiency parameter, which is assumed to depend on access to public infrastructure:

$$\zeta_t = (\frac{K_t^I}{\bar{K}_t^P})^{\pi^Q}, \tag{7}$$

where K_t^I is the stock of public capital in infrastructure, $\bar{K}_t^P$ the aggregate stock of private capital, and $\pi^Q \in (0,1)$. Thus, greater access to roads or electricity allows mothers to devote less "raw" time to home production, while providing the same effective time. Access to infrastructure is assumed subject to congestion, as discussed next.

3.3 Market Production

Firms are identical and their number is normalized to unity. They produce a single nonstorable commodity, using male effective labor, $L_t^{m,i}$, and female effective labor, defined as $A_t^f \varepsilon_t^{f,W} L_t^{f,i}$, where A_t^f is economy-wide female labor productivity, and $L_t^{j,i} = E_t^{t-1,j} N_t^{j,i}$ (where $E_t^{t-1,j}$ is average human capital for $j = m, f$), private capital, $K_t^{P,i}$, and public infrastructure. Although public capital is nonexcludable, it is partially rival (use of it by one firm partly precludes its use by another firm) because of congestion effects; for simplicity, congestion is taken to be proportional to the aggregate private capital stock, $\bar{K}_t^P = \int_0^1 K_t^{P,i}$.[17]

We capture the "gender gap" in the firm in a very simple way: in each firm, men have privileged access to technology—perhaps as a result of "glass ceiling", which limits the ability of women to rise above certain ranks. A productivity gap therefore emerges between men and women, leading employers to pay men (for a given ratio of human capital) relatively more.

[17]Given the linearity of aggregate output in K_t^P, as shown later, using the former as the congestion factor in (7) and elsewhere would not alter the results in any fundamental way.

The production function of individual firm i takes the form

$$Y_t^i = (\frac{K_t^I}{\bar{K}_t^P})^{\alpha}(L_t^{m,i})^{\beta}(bA_t^f \varepsilon_t^{f,W} L_t^{f,i})^{\beta}(K_t^{P,i})^{1-\alpha-2\beta}, \tag{8}$$

where $\alpha, \beta \in (0,1)$. The parameter $b \in (0,1)$ captures the fact that women's labor productivity is lower than men's.[18] For simplicity, the elasticity of output with respect to male and female labor is assumed to be the same.

Profit maximization with respect to private inputs yields

$$w_t^m = \frac{\beta Y_t^i}{L_t^{m,i}}, \quad w_t^f = b\frac{\beta Y_t^i}{A_t^f \varepsilon_t^{f,W} L_t^{f,i}}, \quad r_t = (1-2\beta)\frac{Y_t^i}{K_t^{P,i}}, \tag{9}$$

where r_t is the rental rate of private capital.

In equilibrium, given that men and women are in equal numbers in the adult population ($N_t^m = N_t^f$),

$$w_t^m = b^{-1}(\frac{A_t^f \varepsilon_t^{f,W} E_t^{t-1,f}}{E_t^{t-1,m}})w_t^f. \tag{10}$$

To examine the properties of this arbitrage condition, consider first the case where men and women have identical average human capital ($E_t^{t-1,m} = E_t^{t-1,f}$), equal productivity ($A_t^f = 1$), and devote all their time to work ($\varepsilon_t^{f,W} = 1$). Then $w_t^m = b^{-1}w_t^f > w_t^f$. The wage differential between men and women is therefore due entirely to the fact that men have access to a more productive technology than women—a direct reflection of discrimination in the workplace. In general, however, relatively lower wages for women may also result from differences in relative educational levels, $E_t^{t-1,m}/E_t^{t-1,f}$, relative productivity, $1/A_t^f$, and the relative allocation of time, $1/\varepsilon_t^{f,W}$. Put differently, women may have lower wages than men also because men attain higher educational levels (as discussed next), because they are not as healthy

[18]Note that b could be related to the level of technology, which could itself be endogenously related to the economy's (average) level of human capital.

as men—which, as shown later, has an adverse effect on their productivity—or because they cannot devote as much time as they would like to market activity, as a result of social norms or inadequate access to infrastructure , which requires them to allocate relatively more of their available time to child rearing and home production.

Given that all firms are identical, and that their number if normalized to 1, $\bar{K}_t^P = K_t^{P,i}\ \forall i$, and aggregate output is

$$Y_t = \int_0^1 Y_t^i = (\frac{K_t^I}{K_t^P})^\alpha (\frac{L_t^m}{K_t^P})^\beta (\frac{bA_t^f \varepsilon_t^{f,W} L_t^f}{K_t^P})^\beta K_t^P. \tag{11}$$

Private capital accumulation is driven by, assuming full depreciation for simplicity,

$$K_{t+1}^P = I_t, \tag{12}$$

where I_t is private investment.

3.4 Human Capital Accumulation

As noted earlier, schooling is mandatory so children allocate all of their time to education. Boys and girls have identical innate abilities and have access to the same "out of home" learning technology. However, each group's education outcomes depend also on the amount of time that parents devote to tutoring them at home.

Let $e_{t+1}^{t,j}$, $j = m, f$ be the human capital of men and women born in period t and used in period $t+1$. The production of either type of human capital requires several inputs. First, it depends on the time allocated to education in childhood, which (as noted earlier) is normalized to unity. Second, it depends on the time that mothers allocate to tutoring their children.[19] We consider

[19] This effect is consistent with studies showing that measures of maternal time input make mothers' IQ and parents' education insignificant in explaining verbal skills. This

a sequential process, whereby mothers determine first the total amount of time allocated to rearing children, $\varepsilon_t^{f,R}$, and then subdivide that time into a fraction $\chi \in (0,1)$ allocated to sons and $1-\chi$ allocated to daughters. To reflect bias in parental preferences toward boys (as mentioned earlier), we assume that $\chi > 0.5$.[20]

Third, the production of human capital depends on the stock of public infrastructure, taking into account a congestion effect measured again by the private capital stock. This effect captures the importance of infrastructure for education outcomes, as discussed in Section II.

Fourth, knowledge accumulation depends on government spending on education per (surviving) child, $G_t^E/p_m^C n_t N_t$, where N_t is the number of adults alive in period t, itself given by

$$N_t = p_m^C n_{t-1} N_{t-1}, \tag{13}$$

that is, the number of children born in $t-1$, $n_{t-1}N_{t-1}$, who survived to period t.

Finally, and in line with the empirical evidence discussed earlier, human capital accumulation depends on a mother's human capital. Because individuals are identical within a generation, a mother's human capital at t is equal to the average human capital of the previous generation.

Thus, abstracting from gender-based discrimination in the public education system itself, and assuming no depreciation for simplicity, the human

suggests that the mechanism generating the correlation between parents' and children's education is related to the greater time invested by educated mothers in the quality of their children (see Moav (2005)).

[20]Note that gender bias could also take the form of differences in spending the fraction θ^R of net income between boys and girls; there is indeed some evidence suggesting that expressed gender preferences translate into discrimination in health care and nutrition—e.g., less food being given to girls (see Walker (1997)).

capital that men and women have in the second period of life is

$$e_{t+1}^{j} = \varepsilon_t^{f,R}(\frac{K_t^I}{K_t^P})^{\nu_1}(\frac{G_t^E}{p_m^C n_t N_t})^{\nu_2}(E_t^{t-1,f})^{1-\nu_2} \times \begin{cases} \chi & \text{for } j = m \\ 1-\chi & \text{for } j = f \end{cases}, \tag{14}$$

where $\nu_1 > 0$ and $\nu_2 \in (0, 1)$.[21]

Equations (14) also show that public infrastructure has a direct effect on the rate of human capital accumulation. The complementarity between $\varepsilon_t^{f,R}$ and K_t^I in the production of human capital implies that the effectiveness of mothers' time spent in raising their children depends on access to roads, electricity, etc.—critical constraints in poor countries, as documented earlier. For tractability, the education technology is taken to be linear in ε_t^R and to exhibit constant returns to scale in government spending and the average human capital of mothers.

Combining equations (14) yields

$$\frac{e_{t+1}^m}{e_{t+1}^f} = \frac{\chi}{1-\chi} > 1, \tag{15}$$

which implies that, as long as $\chi > 0.5$, a boy's human capital will exceed systematically a girl's human capital—as a result solely of the greater time that mothers allocate to rearing their sons. In turn, the fact that women in period-t couples have lower human capital may explain not only why their wages are lower (as discussed earlier) but also the fact they "specialize" in rearing children, as assumed in the model, because *ceteris paribus* the opportunity cost of not working is not as high.[22]

[21] It could be assumed that parental spending on rearing each child, θ^R, affects the quality of schooling. For simplicity, and given that θ^R is constant, we abstract from real resources as an input in human capital formation. We could also assume that the production of human capital of either sort depends on the child's health status. However, given that we model only the health status of girls, not boys, this would significantly complicate the model.

[22] This is consistent with various studies for developing countries; see for instance Ilahi (2001) for Peru.

Note also that the difference between the human capital stocks of sons and daughters is constant over time because it is determined only by the difference in their mother's allocation of time—which is itself exogenously given.[23] However, even though the human capital stocks of sons and daughters may differ systematically in the long run, the wage gap in this model does not necessarily perpetuate itself: from (10), as long as the health status of women (and therefore their productivity) improves over time, the wage differential will narrow as well—despite persistent discrimination in the work place (as measured by b). The same result would obtain if the time women allocate to marketed work increases over time.[24]

For simplicity, the parameters characterizing the production of human capital (coefficients ν_h) have been kept the same. Assuming that they differ by gender would complicate the analysis considerably, given that the male-female human capital ratio would no longer be constant over time, as implied by (15). Note finally that, given (15), it does not really matter whether it is the average human capital of mothers that enters in the education technology, or instead the average human capital of *both* parents, $E_t^{t-1,m} + E_t^{t-1,f}$, as in some studies focusing on cross-gender intergenerational effects—of mothers on sons and fathers on daughters (see for instance Zhang, Zhang, and Li (1999)).

[23] A gender gap in human capital could also arise as a result of a parental bias in the amount of resources spent on rearing children of a specific gender group; this would occur, for instance, if we had assumed $\theta^{R,m} > \theta^{R,f}$ instead of a uniform θ^R, and that rearing expenditure affects the human capital technology.

[24] Of course, both results come out in a stark fashion because we have not endogenized the time allocation and health status of adult males.

3.5 Health Status and Productivity

Health status in childhood, h_t^C, depends on the fraction of net family income spent on each child, the effective amount of time allocated by the child's mother to rearing their brood, and the provision of public services provided by the government, H_{t+1}^G, which is also subject to congestion:[25]

$$h_t^C = \theta^R(\zeta_t^R \varepsilon_t^{f,R})^{\nu_C}(\frac{H_t^G}{K_t^P})^{1-\nu_C}, \tag{16}$$

where $\nu_C \in (0,1)$ and ζ_t^R is an efficiency parameter. Thus spending on children helps to improve their health and nutrition, thereby reducing their vulnerability to disease, as documented in various studies (see for instance Pelletier et al. (2003) and Caulfield et al. (2004)).[26] The efficiency of mothers' time is assumed to depend on access to infrastructure:

$$\zeta_t^R = (\frac{K_t^I}{K_t^P})^{\pi^R}, \tag{17}$$

where $\pi^R \in (0,1)$. Thus, greater access to roads or electricity allows mothers to devote less "raw" time to child care, while providing the same effective time. Access to infrastructure is assumed subject to congestion, as measured again by the private capital stock.

As noted earlier, given the focus of this paper on women's time allocation, we do not model explicitly the health status of males and their productivity, and assume instead that both are constant and normalized to unity. By

[25] We could also assume that the health status of children depends on their mother's health status, as in P.-R. Agénor (2009*a*), or on consumption of the home produced good—and therefore on time allocated by mothers to that activity; such time would also become directly productive.

[26] Gender bias in resources allocated to taking care of girls—with more health care being sought for boys, as documented by Walker (1997))—can be captured in a simple way by multiplying θ^R in (16) by a coefficient lower than unity. This would not affect qualitatively our results.

contrast, we assume that the health status of females in adulthood, h_{t+1}^f, is determined by two factors: their health status in childhood and the time spent taking care of their own health, $\varepsilon_{t+1}^{f,H}$:

$$h_{t+1}^f = h_t^C(\varepsilon_{t+1}^{f,H})^{\nu_A}, \tag{18}$$

where $\nu_A \in (0,1)$. For simplicity, health status is assumed linear in h_t^C. Given that health status at t depends on health status at $t-1$, our specification is consistent with the evidence suggesting that early childhood health affects cognitive and physical development, which in turn affects health outcomes later in life.[27]

Female productivity, a_{t+1}^f , is simply a linear function of health status:

$$a_{t+1}^f = a_m h_{t+1}^f, \tag{19}$$

where $a_m > 0$. Substituting (16) and (18) in (19) yields

$$a_{t+1}^f = a_m \theta(\varepsilon_t^{f,R})^{\nu_C}(\varepsilon_{t+1}^{f,H})^{\nu_A}(\frac{K_t^I}{K_t^P})^{\pi^R \nu_C}(\frac{H_t^G}{K_t^P})^{1-\nu_C}. \tag{20}$$

3.6 Government

As noted earlier, the government taxes only the wage income of adults. It spends a total of G_t^I on infrastructure investment, G_t^E on education, G_t^H on health, and G_t^U on unproductive items. All its services are provided free of charge. It cannot issue bonds and must therefore run a balanced budget:

$$G_t = \sum G_t^h = \tau(w_t^m L_t^m + w_t^f A_t^f \varepsilon_t^{f,W} L_t^f). \tag{21}$$

[27] This specification could be extended to account for the possibility that consumption of public health services affects health, and that more educated individuals tend to take better care of their health. Indeed, in specifications (16) and (18), "raw" time could be replaced by "effective" time, $\varepsilon_t^{f,R} e_t^f$ and $\varepsilon_{t+1}^{f,H} e_{t+1}^f$, to capture in the first case the idea that the productivity of the time allocated to child rearing by a mother depends on her level of education, and in the second the idea that more educated individuals tend to take better care of their own health.

Shares of spending are all assumed to be constant fractions of government revenues:

$$G_t^h = \upsilon_h \tau (w_t^m L_t^m + w_t^f A_t^f \varepsilon_t^{f,W} L_t^f), \qquad h = E, H, I, U \tag{22}$$

Combining (21) and (22) therefore yields

$$\sum \upsilon_h = 1. \tag{23}$$

Assuming again full depreciation for simplicity, public capital in infrastructure evolves according to

$$K_{t+1}^I = G_t^I. \tag{24}$$

The production of health services by the government is assumed to exhibit constant returns to scale with respect to the stock of public capital in infrastructure, K_t^I, and government spending on health services, G_t^H:

$$H_t^G = (K_t^I)^\mu (G_t^H)^{1-\mu}, \tag{25}$$

where $\mu \in (0,1)$. This captures, as discussed earlier, the fact that access to infrastructure is essential to the production of health services.[28]

3.7 Market-Clearing Conditions

With full depreciation, the market-clearing condition for the goods market is again

$$Y_t = C_t + G_t + K_{t+1}^I + K_{t+1}^P, \tag{26}$$

where $C_t = 0.5N_t[c_t^t + (\theta^H \varepsilon_t^H + \theta^R p_m^C n_t)(1-\tau)w_{t+1}^T] + 0.5p_m^A N_{t-1}c_t^{t-1}$ is total consumption at t, with $p_m^A N_{t-1}$ denoting the number of (surviving) retirees at period t.

[28]We abstract from the possibility that the production of health services may also require labor (medical workers); assuming that this is a fixed fraction of the adult effective supply of labor would not alter qualitatively our results.

The asset market clearing condition requires tomorrow's private capital stock to be equal to savings in period t by individuals born in $t-1$. Given that s_t is savings per family, and that the number of families is $(N_t^m + N_t^f)/2$, we have

$$K_{t+1}^P = 0.5(N_t^m + N_t^f)s_t = N_t^f s_t. \tag{27}$$

4 Equilibrium and Growth

Given the initial capital stocks K_0^P and $K_0^I > 0$, A competitive equilibrium for this economy is a sequence of prices $\{w_t^m, w_t^f, r_t\}_{t=0}^{\infty}$, allocations $\{c_t^t, c_{t+1}^t, s_t\}_{t=0}^{\infty}$, physical capital stocks $\{K_t^P, K_t^I\}_{t=0}^{\infty}$, human capital stocks $\{E_t^m, E_t^f\}_{t=0}^{\infty}$, a constant tax rate, and constant spending shares such that, given initial stocks $K_0^P, K_0^I > 0$ and $E_0^m, E_0^f > 0$, individuals maximize utility, firms maximize profits, markets clear, and the government budget is balanced. In equilibrium, we also have $e_t^j = E_t^j$, for $j = m, f$, and $A_t^f = a_t^f$.

A balanced growth equilibrium is a competitive equilibrium in which c_t^t, c_{t+1}^t, K_{t+1}^P, K_{t+1}^I, E_{t+1}^m , E_{t+1}^f grow at the constant, endogenous rate γ, the rate of return on private capital r_t is constant, and health status of both children and adults, h_t^C and h_t^f, are constant.

4.1 Women's Time Allocation and Fertility

To simplify calculations, and without loss of generality, we assume in what follows that $\theta^H = 0$. As shown in the Appendix, solving the family's optimization problem leads to the following solutions for women's time allocation and the fertility rate:[29]

$$\tilde{\varepsilon}^{f,H} = \frac{1}{1+\Lambda_1(1+\Lambda_3)}, \tag{28}$$

$$\tilde{\varepsilon}^{f,W} = (\frac{\Lambda_1}{\Lambda_2})\tilde{\varepsilon}^{f,H}, \tag{29}$$

$$\tilde{\varepsilon}^{f,R} = (\frac{\nu_C\Lambda_1}{\eta_L})(\frac{\theta^R[1+\eta_N(1-\nu_C)(1-\sigma)]}{(1-\nu_C)(1-\sigma)})\tilde{\varepsilon}^{f,H}, \tag{30}$$

$$\tilde{\varepsilon}^{f,P} = [\frac{(1-\omega)\Lambda_1}{\eta_L}]\tilde{\varepsilon}^{f,H}, \tag{31}$$

$$\tilde{n} = \frac{\eta_N(1-\nu_C)(1-\sigma)}{\theta^R p_m^C[\omega+\eta_N(1-\nu_C)(1-\sigma)]} > 0, \tag{32}$$

where $\sigma \equiv p_m^A/[\omega(1+\rho)+p_m^A] < 1$ is the family's marginal propensity to save, and

$$\Lambda_1 = \frac{\eta_L}{\nu_A}(\frac{1-\sigma}{\omega}) > 0, \quad \Lambda_2 = \nu_A\Lambda_1,$$

$$\Lambda_3 \equiv \frac{1}{\eta_L}\left\{1-\omega+\eta_N\nu_C+(\frac{\omega}{1-\sigma})\right\} > 0.$$

The allocation of time is also such that women's leisure is positive in equilibrium, that is, $\sum_{h=H,P,W}\tilde{\varepsilon}^{f,h} + p_m^C\tilde{n}\tilde{\varepsilon}^{f,R} < 1$. From these solutions, it is straightforward to show that an increase in the survival probability from adulthood to old age, p_m^A, increases the savings rate and reduces the fertility rate, as is standard in the literature (see for instance Blackburn and Cipriani (2002)): higher longevity dictates a need for higher savings to finance future consumption, and thereby has a positive effect, *ceteris paribus*, on savings in adulthood. More importantly for our purpose, we can establish the following proposition:

[29] To ensure that $p_m^C\tilde{n} \geq 1$ requires imposing $\theta^R \leq \eta_N(1-\nu_C)(1-\sigma)/p_m^C[\omega+\eta_N(1-\nu_C)(1-\sigma)]$, thaht is, the share of spending on marketed commodities per child cannot be too high. Note also that (32) guarantees that $\theta^R p_m^C\tilde{n} < 1$, as long as $\omega > 0$.

Proposition 1. *An increase in the survival probability from adulthood to old age, p_m^A, increases both the time allocated to market work and the time women allocate to their own health. It also reduces both the amount of time allocated to home production and total time spent caring for surviving children.*

The effect of the survival rate on women's time allocation operates essentially through the change in the saving rate. The increase in time allocated to work is also part of the life-cycle effect associated with greater longevity. At the same time, an increase in the survival rate to old age leads to more time being allocated to one's health (because it affects productivity and income), and to less time being allocated to home production and to caring for surviving children, $p_m^C \tilde{n} \tilde{\varepsilon}^{f,R}$. As shown in the Appendix, it can also be established that leisure time falls. Thus, the increase in the survival rate leads to both intertemporal arbitrage (between less leisure today and more consumption tomorrow) and intratemporal substitution in mothers' time (between home production and child rearing, on the one hand, and working time and time allocated to own health, on the other).

The effect on "raw time" $\tilde{\varepsilon}^{f,R}$ itself is in general ambiguous and depends on the structure of preferences and the parameter that measures the response of health status in childhood to mothers' time, ν_C. In particular, the higher ν_C is, and the higher the ratio η_N/η_L is (that is, the more mothers value their surviving children, relative to their own leisure), the more likely it is that time allocated to each surviving child will increase—despite the fact that total time allocated to child rearing falls unambiguously. If so then there is substitution between "quantity" (as reflected in the drop of the fertility rate) and "quality."

Another useful proposition, which can be established directly from (32), is as follows:

Proposition 2. *An increase in the survival probability from childhood to adulthood, p_m^C, reduces pari passu the fertility rate and has no effect on women's time allocation.*

Thus, parents fully internalize an improvement in the survival rate of their offsprings by reducing the number of children. Total time allocated to child care, $p_m^C\tilde{n}\tilde{\varepsilon}^R$, therefore does not change.

Yet another interesting result—although somewhat tangential to the main issue at stake—that one may infer from the solutions (28)-(32) is the possibility of a *home-bias equilibrium*, in which women allocate relatively more of their time to domestic activities (home production and child rearing) compared to market work; this requires therefore $\tilde{\varepsilon}^{f,P} + p_m^C\tilde{n}\tilde{\varepsilon}^{f,R} > \tilde{\varepsilon}^{f,W}$. Inspection of this condition shows that it does not depend on the "discrimination" parameter, b. This yields the following proposition, regarding the relative allocation of women's time between market work and home activities:

Proposition 3. *A home-bias equilibrium obtains if $\tilde{\varepsilon}^{f,P} + p_m^C\tilde{n}\tilde{\varepsilon}^{f,R} > \tilde{\varepsilon}^{f,W}$. This condition does not depend on the extent of gender bias in the workplace.*

This result is of course related to our assumption that husbands and wives pool all their resources when taking family decisions. As discussed further in the conclusion, it would not necessarily hold if we were to depart from this "unitary framework" and account for bargaining power between spouses in the family decision process.

4.2 Growth and Stagnation

The balanced growth rate of the economy is derived in the Appendix. The public-private capital ratio is shown to be given by

$$k_t^I = \frac{K_t^I}{K_t^P} = \frac{\upsilon_I \tau}{\sigma(1-\tau)(1-\theta^R p_m^C \tilde{n})} \equiv J, \quad \forall t \tag{33}$$

which is constant as long as the savings rate is constant.

As also shown in the Appendix, the system boils down to an autonomous, first-order linear difference equation system in $\hat{h}_t^f = \ln h_t^f$ and $\hat{x}_t^f = \ln x_t^f$, where h_t^f is female health status and $x^f = e_t^f N_t^f / K_t^P$ the effective female labor-capital ratio. The steady-state growth rate per worker is given as

$$1+\gamma = \frac{J^\alpha}{p_m^C \tilde{n}} (\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1-\chi})^\beta b\beta\Phi\sigma(1-\theta^R p_m^C \tilde{n})(\tilde{h}^f)^\beta (\tilde{x}^f)^{-2\beta}, \qquad (34)$$

where $\Phi \equiv (1-\tau)(b^{-1}+1)$ and $\tilde{h}^f$ and $\tilde{x}^f$ are steady-state solutions determined by

$$\tilde{h}^f = [\theta^R (\tilde{\varepsilon}^{f,R})^{\nu_C} (\tilde{\varepsilon}^{f,H})^{\nu_A} J^{\Omega_1}]^{1/(1-\Omega_2)} \qquad (35)$$

$$\times \left\{ [\upsilon_H \tau (1+b)\beta]^{(1-\mu)(1-\nu_C)} (\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1-\chi})^{\Omega_2} \right\}^{1/(1-\Omega_2)} (\tilde{x}^f)^{-2\Omega_2/(1-\Omega_2)}.$$

$$\tilde{x}^f = \left\{ \Gamma \{ J^\alpha [\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1-\chi}]^\beta \}^{1-\nu_2} \right\}^{1/\Omega_3} (\tilde{h}^f)^{\beta(1-\nu_2)/\Omega_3}, \qquad (36)$$

with

$$\Omega_1 \equiv \pi^R \nu_C + (1-\nu_C)[\mu + \alpha(1-\mu)] > 0,$$

$$\Omega_2 \equiv \beta(1-\mu)(1-\nu_C) \in (0,1),$$

$$\Omega_3 \equiv 1 - (1-2\beta)(1-\nu_2) > 0,$$

$$\Gamma \equiv [\frac{b\beta\Phi\sigma(1-\theta^R p_m^C \tilde{n})}{(1-\chi)\tilde{\varepsilon}^{f,R} (p_m^C \tilde{n})^{1-\nu_2} (0.5)^{\nu_2}}] J^{-\nu_1} [\upsilon_E \tau (1+b)\beta]^{-\nu_2}.$$

The steady-state relationship (35) is shown as the decreasing convex curve HH in Figure 1, whereas the relationship (36) is depicted as the upward-sloping concave curve XX. It is immediately clear from the diagram that there is a unique non-trivial equilibrium, located at Point E. As shown in the Appendix, the equilibrium is stable as long as ν_2 is not too large. Depending on the initial values, however, the economy may converge either monotonically or with cycles.

The following proposition is immediately clear from the previous results:

Proposition 4. *A low public-private capital ratio is associated with a stagnation equilibrium, characterized by a low steady-state level of growth in income per worker, poor health status for women, and high fertility.*

From (33), a low public-private capital ratio may result, in particular, from a small share of public spending allocated to investment in infrastructure or a high saving rate—which itself may be due to a high survival probability from adulthood to old age, as noted earlier. Thus, although an increase in life expectancy may promote growth directly (as implied by (34)), its overall effect can be negative, in contrast to the literature (see Blackburn and Cipriani (2002)). One reason is that higher savings translate into a higher stock of private capital accumulation and this tends to increase, all else equal, congestion effects on public infrastructure. Another is that higher life expectancy may, as discussed earlier, lower raw time allocated by mothers to child care.

The impact of women's time allocation on growth can be summarized in the following proposition:

Proposition 5. *Women's time allocated to home production has no effect on steady-state growth. An autonomous increase in time allocated to own care or child rearing improves female health status and raises growth; an autonomous increase in time allocated to market work raises the capital-female effective labor ratio but has an ambiguous impact on female health status and steady-state growth.*

This proposition illustrates well the importance of accounting for the productive effects of non-market work. The positive effect of an increase in child rearing time is a direct consequence of the "serial dependence" in female health status introduced in the model; this makes rearing time productive, unlike most of the existing literature. Time allocated by women to their own health is also productive and growth-promoting. The intuition behind the third result is that an increase in time devoted to market work raises family income, savings, and investment in physical capital; this, in turn, reduces the output-private capital ratio and the supply of public health services as a result of congestion. The net effect on female health status is thus ambiguous. Graphically, an autonomous increase in $\tilde{\varepsilon}^{f,R}$ or $\tilde{\varepsilon}^{f,H}$ leads to an upward shift in HH and no change in XX, as in Figure 1. The new equilibrium point E' is located to the Northeast of E. An autonomous increase in $\tilde{\varepsilon}^{f,W}$ by contrast, leads to an upward shift in both HH and XX; the new equilibrium can be located either to the Northeast or the Northwest of E.

Note that the reason why time allocated to home production does not affect long-run growth is because we have assumed that production of home goods only affects utility. If we were to assume that home production affects children's health as well (because a cleaner environment reduces the risks of respiratory illnesses, for instance), an autonomous increase in $\tilde{\varepsilon}^{f,P}$ would also promote growth. However, any reallocation that leaves leisure unchanged would have ambiguous effects on growth and health outcomes.

Equations (28) to (36) can be used as well to examine the impact of specific parameters on long-run growth. In particular, the following result can be established:

Proposition 6. *The weaker the effect of public infrastructure on the efficiency of women's time allocated to child rearing (the lower π^R), the smaller the steady-state growth rate of output per worker.*

The impact of an increase in π^R is also illustrated in Figure 1. Curve HH shifts upward whereas XX does not change; the outcome is both an improvement in health status and a higher private capital-labor ratio in the new long-run equilibrium.

Note also that π^Q (the parameter measuring the efficiency of time allocated to home production) has no direct effect on growth; again, this is because we have assumed that production of home goods is only utility-enhancing. If we were to assume that home production affects directly children's health, we could also show that public infrastructure, by making domestic activity more efficient, allows women to reallocate their time toward other uses—including not only to market work but also to taking care of their own health. In turn, this would help to promote growth.

5 Multiple Development Regimes

In the foregoing discussion, we have assumed that survival rates for both children and adults are constant over time. We now consider the case where there are threshold effects associated with changes in women's health status. Doing so implies that the model can easily generate multiple development regimes.

To illustrate this result as simply as possible, suppose that the autonomous component of women's productivity, a, can take on two values, such that

$$a = \begin{cases} a_m & \text{If } h_t^f < h_m^f \\ a_M > a_m & \text{If } h_t^f \geq h_m^f \end{cases} . \tag{37}$$

Thus, if health status is below h_m^f, a is constant at a_m (as before); as health status improves above that threshold, a increases to $a_M > a_m$. This specification provides a simple way to capture the link between nutrition and work effort, as emphasized in the early development literature. The increase in productivity beyond the threshold point shifts both XX and HH upward. The model may therefore display two development regimes, as illustrated in Figure 2. For $h_0^f < h_m^f$, the economy will converge to the low-growth equilibrium point A, whereas for $h_0^f \geq h_m^f$, it will converge to the high-growth equilibrium point B. The low-growth equilibrium is characterized also by low productivity, low savings, and poor health outcomes.

A similar result may also be obtained by endogenizing life expectancy. Suppose now that the survival rate in childhood (which has no effect on long-run growth, as shown earlier) remains constant at p_m^C and that the adult survival probability depends on *average* female health status in the economy—which, in equilibrium, is of course the same for all women.[30] Specifically, suppose that the adult survival rate is now a piece-wise function defined in a manner similar to (37), that is,

$$p_t^A = \begin{cases} p_m^A & \text{If } h_t^f < h_m^f \\ p_M^A > p_m^A & \text{If } h_t^f \geq h_m^f \end{cases}, \tag{38}$$

where $p_M^A \in (0,1)$. If the net effect of an increase in the survival rate is positive, then two development regimes may emerge, just as in Figure 2. Note, however, that in contrast to the previous case, a high-growth equilibrium is not a necessary outcome, due to the time reallocation effect discussed earlier.

6 Public Policy

The model can also be used to examine the impact of a variety of public policy variables on long-run growth. A budget-neutral increase in the share of spending on health financed by a cut in unproductive spending (that is, an increase in υ_H, offset by a reduction in υ_U) leads to an upward shift in HH with no change in XX, as in Figure 1; thus, women's health status unambiguously increases. Whether the steady-state growth rate increases

[30] An alternative and perhaps more natural assumption would be to assume that survival rates are related to the individual's *own* health status. However, because these effects must be internalized in solving the family's optimization problem, explicit analytical solutions cannot be obtained. Our formulation, despite its simplicity, is sufficient to convey the main point of our analysis.

depends on whether $1-2\beta(1-\nu_2)/[1-(1-2\beta)(1-\nu_2)]>0$, which always holds; growth therefore increases as well.

Likewise, a budget-neutral increase in the share of spending on education (that is, an increase such that $d\upsilon_E + d\upsilon_U = 0$) translates into a downward shift in XX with no change in HH; this corresponds to a point like C' in Figure 3. Again, women's health status unambiguously improves, whereas the capital-effective female labor ratio falls. The net effect on growth, nevertheless, remains positive.

Finally, a budget-neutral increase in the share of public spending on infrastructure (such that $d\upsilon_I + d\upsilon_U = 0$) results in an increase in the steady-state public-private capital ratio. This exerts not only productivity and time efficiency effects, but also a positive effect on women's human capital; in turn, this tends to reduce the private capital-effective female labor ratio—and possibly women's health status and growth. Which effect dominates cannot be determined a priori. As shown in Figure 3, the increase in υ_I leads to an upward shift in HH but XX can shift either up or down. The new equilibrium can be either at A, B, C, or D. If the human capital effect (as measured by ν_2) is not strong, XX shifts downward, and women's health status unambiguously improves, whereas the capital-effective female labor ratio may either increase (Point C) or fall (Point D). If, on the contrary, the human capital effect dominates, XX shifts upward, and women's health status may either deteriorate (Point A) or improve (Point B), depending on the magnitude of the shift.[31]

The thrust of the foregoing analysis is that the mechanism through which an increase in productive spending in health, education, or infrastructure *may*

[31] Of course, spending reallocations between components of productive spending (an increase in the share of spending on education financed by, say, a cut in spending on infrastructure) will also generate ambiguous aggregate effects.

induce a shift to a high-growth equilibrium involves significant changes in women's health status, productivity, and savings. Threshold effects induced by improvements in women's health status may also involve changes in life expectancy as well as changes in women's time allocation and fertility. However, these changes are not necessarily all conducive to higher growth. As noted in Proposition 1, an increase in adult life expectancy affects women's time allocation in opposite directions: while time devoted to market work and caring for own health increases, time allocated to caring for surviving children may fall; this, in turn, may have an adverse effect on growth. Our analysis therefore adds an important note of caution to Big Push theories—unrelated to financing constraints and debt sustainability considerations, given the assumption above that budget neutrality is maintained by cuts in unproductive outlays—based on a large expansion of productive government expenditure.

7 Concluding Remarks

The purpose of this paper was to develop a gender-based OLG model of endogenous growth to analyze interactions between public capital in infrastructure, women's time allocation (between market work, home production, raising children, own health care, and leisure), and economic development. A "gender bias" was accounted for by assuming that employers discriminate between men and women and that mothers devote relatively more time to rearing their sons. In addition, women were assumed to bear the brunt of domestic tasks (processing food crops, providing water and firewood, caring for children, etc.), in line with the evidence for developing countries. Health status in adulthood (which affects productivity and wages) was assumed to depend on health status in childhood as well as time allocated to own health care. The analysis showed that the economy may be stuck in a low-growth equilibrium, characterized by poor health and education outcomes, as well as high fertility. Multiple development regimes may emerge if changes in health status generate threshold effects on productivity or life expectancy. A reallocation of government spending toward productive outlays (especially education and health) may shift the economy to a high-growth equilibrium with low fertility. If the increase in public spending is large enough for threshold effects to kick in, the transition to a high growth rate may involve changes in life expectancy, women's time allocation, and fertility. However, because all of these changes do not necessarily promote growth, a Big Push in productive government expenditure may not necessarily help a country to escape from a low-growth equilibrium. This is an important note of caution for the ongoing debate on ways to spur growth in poor countries.

Two broad messages emerge from our analysis. First, although improved access to infrastructure (water and sanitation services, rural electrification, and transport) represent interventions that may reduce the amount of time that women spend doing unpaid work, it is important to consider also other aspects of women's time allocation—especially the time devoted to their own health and the health of their children. Moreover, "unpaid" work is not synonymous to "unproductive" work; home production may contribute importantly to children's health and thereby affect their future productivity and contribution to economic activity. In fact, better access to roads, for instance, may have a greater long-run effect on growth by allowing mothers to take regularly their children to hospitals for preventive care than by allowing them to engage in market activities. Second, it is commonly argued that women face a "vicious cycle of deprivation": they do not get formal education because their earnings are low, and their earnings are low because they possess little human capital. The analysis in this paper suggests that this is only part of the story. In addition to gender bias within the family (resulting from social and institutional factors), poor access to infrastructure may act as a major constraint on the time allocation of women—especially in rural areas. Because of the multitude of tasks that they must perform (including home production and rearing children), women may not be able to devote as much time as they should to their health, implying that their productivity and income may eventually suffer. The main lesson for public policy is thus that it is crucial to invest in areas which reduce women's excessive time burden.[32] Greater priority to water supply and sanitation, energy for household needs, and access to appropriate means of transport, may all be critical to enhance the role of women in helping poor countries to escape from an underdevelopment trap. At the same time, however, because nonmarket time may also be productive, it is also important to accompany policies aimed at promoting market participation for women with measures aimed at providing affordable access to child care.

As it stands, the model accounts for a number of well-documented facts about infrastructure, women's time allocation, and its implications for children's health and education. In doing so it highlights the economic constraints women face in their productive activities and how they differ from those faced by men—an issue that has received only limited attention in growth theory. However, it can be adapted to explore the implications of several other empirical regularities observed in developing countries. First, as noted earlier, the efficiency of the time spent rearing (surviving) children could be taken to depend also on the level of mothers' education; an im-

[32] The World Bank has developed an initiative on gender and infrastructure housed under its Gender Action Plan (GAP), *Gender Equality as Smart economics*, which places emphasis on women's access to infrastructure as a key to their economic empowerment.

provement in women's literacy rate could therefore translate into more time allocated to market activities or more time spent on own health care, in both cases with positive effects on growth. It may also raise the (future) productivity of children, through an improvement in the quality of human capital, which would also help to promote growth. Related to that, it could be assumed that the ability to educate children differs among mothers. Some mothers may be more skilled than others at educating their children; this would translate into heterogeneity in levels of human capital among individuals belonging to the same cohort, even though these individuals are *ex ante* perfectly identical and endowed with the same skills.

Second, it could be assumed that a mother's health affects her children's ability to learn in school. This is consistent with the evidence suggesting that cognitive and physical impairments of children may begin *in utero* due to inadequate nutrition and poor health of the mother.[33] During the early childhood years, a mother with poor health may also be unable to devote sufficient time to taking care of her children, thereby increasing their exposure to disease.

Third, it could be assumed that the share of income allocated to rearing children is chosen endogenously by the family. This could be useful to study how access to infrastructure affects the conventional Barro-Becker trade-off between "quality and quantity" of children. Although a fall in fertility may still give parents greater incentives to invest in the human capital of (fewer) children, this effect may be mitigated by poor access to time-burden-reducing public goods (such as standpipes).

Finally, by using a "unitary" framework (that is, by treating a family

[33] According to estimates reported by Bloom and Canning (2007), 30 million infants are born each year in developing countries with impaired growth due to poor nutrition during fetal life. See also Field, Robles, and Torero (2008) for evidence on Tanzania.

as a single decision-making unit that pools all the resources of individual members), we did not address issues associated with intra-household allocation. The evidence on the unitary framework is actually mixed for developing countries.[34] Nevertheless, it would be worth exploring issues associated with bargaining power between husbands and wives in the family decision process, and the possible influence of the relative level of education among spouses (see Echevarria and Merlo (1999), Vermeulen (2002), and de la Croix and Donckt (2008)). For instance, many observers have argued that with greater control over household resources, women would be more likely to invest in their children's health, nutrition, and education. Changes in the intra-household division of labor may also have important implications for the gender gap.

[34] In a study of Indonesia, for instance, Park (2007) found that, with respect to children's nutritional status, the resource pooling hypothesis can be rejected and that parental household bargaining has an important impact on outcomes. However, with respect to investment in children's education, results are mixed. The implication is that the process of intrahousehold resource allocation may differ according to the type of decisions being made; thus, it is possible that no single model can explain all these decisions.

Technical Appendix

Consider first the family's optimization problem. Substituting (6) in (1) yields

$$U = \ln[(c^t_{t+1})^{\omega}(\zeta_{t+1}\varepsilon^{f,P}_{t+1})^{1-\omega}] + \eta_H \ln h^f_{t+1} + \eta_N \ln p^C_m h^C_{t+1} n_{t+1}, \tag{A1}$$

$$+\eta_L \ln(1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}) + \frac{p^A_m}{1+\rho}\ln c^t_{t+2},$$

where the term $\eta_H \ln h^f_{t+1}$, with $\eta_H > 0$ measuring the family's relative preference for the mother's health, is added for a more general specification.

From equation (10),

$$e^m_t w^m_t = b^{-1} a^f_t e^f_t \varepsilon^{f,W}_t w^f_t, \tag{A2}$$

which can be substituted in (4) to give

$$w^T_t = e^m_t w^m_t + a^f_t e^f_t \varepsilon^{f,W}_t w^f_t = (b^{-1}+1) a^f_t e^f_t \varepsilon^{f,W}_t w^f_t. \tag{A3}$$

In turn, this expression can be substituted in (5) to give, using (19) to replace a^f_t,

$$\frac{a_m(1 - \theta^H \varepsilon^H_{t+1} - \theta^R p^C_m n_{t+1})}{[(1-\tau)(b^{-1}+1)]^{-1}} h^f_{t+1} e^f_{t+1} \varepsilon^{f,W}_{t+1} w^f_{t+1} - c^t_{t+1} - \frac{p^A_m c^t_{t+2}}{1+r_{t+2}} = 0. \tag{A4}$$

Writing (16) for $t+1$ using (17), and repeating (20) for convenience, yields

$$h^C_{t+1} = \theta^R (\varepsilon^{f,R}_{t+1})^{\nu_C} (\frac{K^I_{t+1}}{K^P_{t+1}})^{\pi^R \nu_C} (\frac{H^G_{t+1}}{K^P_{t+1}})^{1-\nu_C}, \tag{A5}$$

$$h^f_{t+1} = \theta^R (\varepsilon^{f,R}_t)^{\nu_C} (\varepsilon^{f,H}_{t+1})^{\nu_A} (\frac{K^I_t}{K^P_t})^{\pi^R \nu_C} (\frac{H^G_t}{K^P_t})^{1-\nu_C}. \tag{A6}$$

Families maximize (A1) subject to (A4), (A5), and (A6), with respect to c^t_{t+1}, c^t_{t+2}, $\varepsilon^{f,H}_{t+1}$, $\varepsilon^{f,R}_{t+1}$, $\varepsilon^{f,W}_{t+1}$, $\varepsilon^{f,P}_{t+1}$, and n_{t+1}, taking as given period-t variables (namely, $\varepsilon^{f,R}_t$), H^G_{t+1}/K^P_{t+1}, e^f_{t+1}, w^f_t, and the public-private capital ratio. First-order conditions yield the familiar Euler equation

$$\frac{c^t_{t+2}}{c^t_{t+1}} = \frac{1+r_{t+2}}{\omega(1+\rho)}, \tag{A7}$$

together with, setting $\theta^H = 0$,

$$\frac{\eta_H \nu_A}{\varepsilon^{f,H}_{t+1}} - \frac{\eta_L}{1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}} = \tag{A8}$$

$$-\frac{\omega a_m (1 - \theta^R p^C_m n_{t+1})\nu_A}{\Phi^{-1} c^t_{t+1} \varepsilon^H_{t+1}} h^f_{t+1} e^f_{t+1} \varepsilon^{f,W}_{t+1} w^f_{t+1},$$

$$\frac{\eta_N \nu_C}{\varepsilon^{f,R}_{t+1}} - \frac{\eta_L p^C_m n_{t+1}}{1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}} = 0, \tag{A9}$$

$$\frac{\eta_L}{1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}} = \frac{\omega a_m (1 - \theta^R p^C_m n_{t+1})}{\Phi^{-1} c^t_{t+1}} h^f_{t+1} e^f_{t+1} w^f_{t+1}, \tag{A10}$$

$$\frac{1-\omega}{\varepsilon^{f,P}_{t+1}} - \frac{\eta_L}{1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}} = 0, \tag{A11}$$

$$\frac{\eta_N}{n_{t+1}} - \frac{\eta_L p^C_m \varepsilon^{f,R}_{t+1}}{1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1}} = \frac{\omega a_m \theta^R p^C_m}{\Phi^{-1} c^t_{t+1}} h^f_{t+1} e^f_{t+1} \varepsilon^{f,W}_{t+1} w^f_{t+1}, \tag{A12}$$

where $\Phi \equiv (1-\tau)(b^{-1}+1)$.

Substituting (A7) in the intertemporal budget constraint (A4) yields

$$c^t_{t+1} = [\frac{\omega(1+\rho)}{\omega(1+\rho) + p^A_m}](1 - \theta^R p^C_m n_{t+1})\Phi a_m h^f_{t+1} e^f_{t+1} \varepsilon^{f,W}_{t+1} w^f_{t+1}, \tag{A13}$$

Thus, family savings, s_{t+1}, is equal to

$$s_{t+1} = \sigma(1 - \theta p^C_m n_{t+1})\Phi a_m h^f_{t+1} e^f_{t+1} \varepsilon^{f,W}_{t+1} w^f_{t+1}, \quad \sigma \equiv \frac{p^A_m}{\omega(1+\rho) + p^A_m} < 1. \tag{A14}$$

Equations (A9) and (A11) can be rewritten as

$$1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_{tm} n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1} = \frac{\eta_L p^C_m n_{t+1} \varepsilon^{f,R}_{t+1}}{\eta_N \nu_C}, \tag{A15}$$

$$1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1} = \frac{\eta_L \varepsilon^{f,P}_{t+1}}{1-\omega}. \tag{A16}$$

Substituting (A13) in (A8), (A10), and (A12) yields

$$1 - \varepsilon^{f,H}_{t+1} - \varepsilon^{f,P}_{t+1} - p^C_m n_{t+1}\varepsilon^{f,R}_{t+1} - \varepsilon^{f,W}_{t+1} = \Lambda_1 \varepsilon^{f,H}_{t+1}, \tag{A17}$$

$$1 - \varepsilon_{t+1}^{f,H} - \varepsilon_{t+1}^{f,P} - p_m^C n_{t+1}\varepsilon_{t+1}^{f,R} - \varepsilon_{t+1}^{f,W} = \Lambda_2 \varepsilon_{t+1}^{f,W}, \tag{A18}$$

$$\frac{\eta_L p_m^C \varepsilon_{t+1}^{f,R}}{1 - \varepsilon_{t+1}^{f,H} - \varepsilon_{t+1}^{f,P} - p_m^C n_{t+1}\varepsilon_{t+1}^{f,R} - \varepsilon_{t+1}^{f,W}} - \frac{\eta_N}{n_{t+1}} = -\frac{\omega\theta^R p_m^C}{(1-\sigma)(1-\theta^R p_m^C n_{t+1})}, \tag{A19}$$

where

$$\Lambda_1 = \frac{\eta_L}{\nu_A}[\eta_H + (\frac{\omega}{1-\sigma})]^{-1} > 0,$$

$$\Lambda_2 = \eta_L(1-\sigma)/\omega > 0.$$

From (A17) and (A18),

$$\varepsilon_{t+1}^{f,W} = (\frac{\Lambda_1}{\Lambda_2})\varepsilon_{t+1}^{f,H}, \tag{A20}$$

whereas substituting (A17) in (A15), (A16) yields

$$p_m^C n_{t+1}\varepsilon_{t+1}^{f,R} = (\frac{\eta_N \nu_C \Lambda_1}{\eta_L})\varepsilon_{t+1}^{f,H}, \tag{A21}$$

$$\varepsilon_{t+1}^{f,P} = [\frac{(1-\omega)\Lambda_1}{\eta_L}]\varepsilon_{t+1}^{f,H}. \tag{A22}$$

Equation (A9) can be rewritten as

$$\frac{p_m^C n_{t+1}\varepsilon_{t+1}^{f,R}}{1 - \varepsilon_{t+1}^{f,H} - \varepsilon_{t+1}^{f,P} - p_m^C n_{t+1}\varepsilon_{t+1}^{f,R} - \varepsilon_{t+1}^{f,W}} = \frac{\eta_N \nu_C}{\eta_L}.$$

Substituting (A20), (A21), and (A22) in this expression yields

$$\frac{\Lambda_1 \varepsilon_{t+1}^{f,H}}{1 - (1 + \Lambda_1\Lambda_3)\varepsilon_{t+1}^{f,H}} = 1, \tag{A23}$$

where

$$\Lambda_3 \equiv \frac{1}{\eta_L}\left\{1 - \omega + \eta_N \nu_C + (\frac{\omega}{1-\sigma})\right\} > 0.$$

This equation can be solved for the optimal value of $\varepsilon_{t+1}^{f,H}$:

$$\tilde{\varepsilon}^{f,H} = \frac{1}{1 + \Lambda_1(1+\Lambda_3)} < 1. \tag{A24}$$

Substituting (A20), (A21), and (A22) in (A19) yields

$$\frac{\eta_N \nu_C \Lambda_1 \tilde{\varepsilon}^{f,H}}{n_{t+1}[1-(1+\Lambda_1\Lambda_3)\tilde{\varepsilon}^{f,H}]} - \frac{\eta_N}{n_{t+1}} = -\frac{\omega\theta^R p_m^C}{(1-\sigma)(1-\theta^R p_m^C n_{t+1})}.$$

Using (A23), this equation gives

$$\frac{-\eta_N(1-\nu_C)}{n_{t+1}} = -\frac{\omega\theta^R p_m^C}{(1-\sigma)(1-\theta^R p_m^C n_{t+1})},$$

which can be solved for n_{t+1}:

$$\tilde{n} = \frac{\eta_N(1-\nu_C)(1-\sigma)}{\theta^R p_m^C[\omega+\eta_N(1-\nu_C)(1-\sigma)]} > 0. \tag{A25}$$

Given that (A24) implies a constant $\tilde{\varepsilon}^{f,H}$, and using (A25), equations (A20), (A21), and (A22) yield

$$\tilde{\varepsilon}^{f,W} = (\frac{\Lambda_1}{\Lambda_2})\tilde{\varepsilon}^{f,H}, \tag{A26}$$

$$\tilde{\varepsilon}^{f,R} = (\frac{\nu_C\Lambda_1}{\eta_L})(\frac{\theta^R[\omega+\eta_N(1-\nu_C)(1-\sigma)]}{(1-\nu_C)(1-\sigma)})\tilde{\varepsilon}^{f,H}, \tag{A27}$$

$$\tilde{\varepsilon}^{f,P} = [\frac{(1-\omega)\Lambda_1}{\eta_L}]\tilde{\varepsilon}^{f,H}. \tag{A28}$$

Using (A17) leisure is

$$1-\tilde{\varepsilon}^{f,H}-\tilde{\varepsilon}^{f,P}-p_m^C\tilde{n}\tilde{\varepsilon}^{f,R}-\tilde{\varepsilon}^{f,W} = \Lambda_1\tilde{\varepsilon}^{f,H},$$

that is, using (A24),

$$\tilde{\varepsilon}^{f,H}+\tilde{\varepsilon}^{f,P}+p_m^C\tilde{n}\tilde{\varepsilon}^{f,R}+\tilde{\varepsilon}^{f,W} = 1-\frac{\Lambda_1}{1+\Lambda_1(1+\Lambda_3)} = \frac{1+\Lambda_1\Lambda_3}{1+\Lambda_1(1+\Lambda_3)}. \tag{A29}$$

Thus, for $\tilde{\varepsilon}^{f,H}+\tilde{\varepsilon}^{f,P}+p_m^C\tilde{n}\tilde{\varepsilon}^{f,R}+\tilde{\varepsilon}^{f,W} < 1$ (that is, for for the time constraint to be satisfied and for leisure to be nonnegative in equilibrium) we must have $(1+\Lambda_1\Lambda_3)/[1+\Lambda_1(1+\Lambda_3)] < 1$, or equivalently $\Lambda_1 > 0$. This condition is always satisfied. Leisure being positive also implies that $\tilde{\varepsilon}^{f,P}$, $p_m^C\tilde{n}\tilde{\varepsilon}^{f,R}$, $\tilde{\varepsilon}^{f,W}$ are all less than unity. From (A25), it can be directly established that $\theta^R p_m^C\tilde{n} < 1$.

From (A14), $d\sigma/dp_m^A > 0$. From (A24), with $\eta_H = 0$, so that $\Lambda_1 = \eta_L(1-\sigma)/\nu_A\omega$,

$$\tilde{\varepsilon}^{f,H} = \frac{1}{1+\nu_A^{-1}\{1+(1-\sigma)\omega^{-1}[\eta_L+1-\omega+\eta_N\nu_C]\}}, \tag{A30}$$

from which it can be established that $d\tilde{\varepsilon}^{f,H}/dp_m^A > 0$. From (A26), with $\eta_H = 0$, so that $\Lambda_2 = \nu_A\Lambda_1$,

$$\frac{d\tilde{\varepsilon}^{f,W}}{dp_m^A} = \nu_A\frac{d\tilde{\varepsilon}^{f,H}}{dp_m^A} > 0.$$

From (A21) and (A22), and with $\eta_H = 0$,

$$p_m^C\tilde{n}\tilde{\varepsilon}^{f,R} = (\frac{\eta_N\nu_C}{\nu_A\omega})(1-\sigma)\tilde{\varepsilon}^{f,H},$$

$$\tilde{\varepsilon}^{f,P} = (\frac{1-\omega}{\nu_A\omega})(1-\sigma)\tilde{\varepsilon}^{f,H}.$$

Using (A30), it can be established from these expressions that

$$\frac{d(p_m^C\tilde{n}\tilde{\varepsilon}^{f,R})}{dp_m^A} < 0, \quad \frac{d\tilde{\varepsilon}^{f,P}}{dp_m^A} < 0.$$

From (A25) and (A29),

$$\frac{d\tilde{n}}{dp_m^A} < 0, \quad \frac{d(\tilde{\varepsilon}^{f,H}+\tilde{\varepsilon}^{f,P}+p_m^C\tilde{n}\tilde{\varepsilon}^{f,R}+\tilde{\varepsilon}^{f,W})}{dp_m^A} > 0,$$

where the last result indicates that leisure falls following an increase in p_m^A.

To study the dynamics in this economy, substitute first (A14) in (27) with $n_t = \tilde{n}\ \forall t$, to give

$$K_{t+1}^P = N_t^f s_t = N_t^f\Phi\sigma(1-\theta^R p_m^C\tilde{n})a_m h_{t+1}^f e_t^f\tilde{\varepsilon}^{f,W}w_{t+1}^f,$$

that is, substituting for w_{t+1}^f from (9) with $A_{t+1}^f = a_m h_{t+1}^f$,

$$K_{t+1}^P = b\beta\Phi\sigma(1-\theta^R p_m^C\tilde{n})Y_t. \tag{A31}$$

Equations (22) can be rewritten as, given that $L_t^j = e_t^{t-1,j}N_t^j$ and $N_t^m = N_t^f$,

$$G_t^h = \upsilon_h\tau(e_t^m w_t^m + h_t^f e_t^f\tilde{\varepsilon}^{f,W}w_t^f)N_t^f,$$

that is, using (A3),

$$G_t^h = \upsilon_h \tau (b^{-1} + 1) h_t^f e_t^f \tilde{\varepsilon}^{f,W} w_t^f N_t^f. \tag{A32}$$

Substituting for w_t^f from (9) gives

$$G_t^h = \upsilon_h \tau (1 + b) \beta Y_t, \tag{A33}$$

which can be substituted for $h = I$ in (24) to give

$$K_{t+1}^I = \upsilon_I \tau (1 + b) \beta Y_t. \tag{A34}$$

Combining (A31) and (A34), and noting that $\Phi \equiv (1 - \tau)(b^{-1} + 1)$, yields

$$k_{t+1}^I = \frac{K_{t+1}^I}{K_{t+1}^P} = \frac{\upsilon_I \tau}{\sigma(1 - \tau)(1 - \theta^R p_m^C \tilde{n})} \equiv J, \quad \forall t, \tag{A35}$$

which is independent of b, as well as θ^R and p_m^C given (A25).

The next step is to calculate H_t^G / K_t^P to substitute in (A6) and obtain a dynamic equation for h_{t+1}^f. From (25),

$$\frac{H_t^G}{K_t^P} = (\frac{K_t^I}{K_t^P})^{\mu} (\frac{G_t^H}{K_t^P})^{1-\mu},$$

that is, using (A32) with $h = H$ and (A35),

$$\frac{H_t^G}{K_t^P} = J^{\mu} [\upsilon_H \tau (1 + b) \beta]^{1-\mu} (\frac{Y_t}{K_t^P})^{1-\mu}. \tag{A36}$$

We therefore need to solve now for Y_t / K_t^P. To do so, note that from (11), (19), and (A35), given again that $L_t^j = e_t^j N_t^j$,

$$\frac{Y_t}{K_t^P} = J^{\alpha} (a_m b \tilde{\varepsilon}^{f,W})^{\beta} (\frac{1}{x_t^m})^{\beta} (h_t^f)^{\beta} (\frac{e_t^f N_t^f}{K_t^P})^{\beta},$$

where $x_t^m = K_t^P / e_t^m N_t^m$ is the capital-male effective labor ratio. The term $e_t^f N_t^f / K_t^P$ is equal to $1/x_t^f$, where x_t^f is the capital-female effective labor ratio.

Because $N_t^f = N_t^m$, and given that from (15) $e_t^m = \chi e_t^f / (1 - \chi)$, we have $x_t^m = (1 - \chi) x_t^f / \chi$. Substituting these results in the above expression yields

$$\frac{Y_t}{K_t^P} = J^{\alpha} (\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1 - \chi})^{\beta} (h_t^f)^{\beta} (x_t^f)^{-2\beta}. \tag{A37}$$

Substituting (A37) in (A36) gives

$$\frac{H_t^G}{K_t^P} = J^{\mu+\alpha(1-\mu)}[\upsilon_H\tau(1+b)\beta]^{1-\mu}(\frac{a_m\chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^{\beta(1-\mu)}\frac{(h_t^f)^{\beta(1-\mu)}}{(x_t^f)^{2\beta(1-\mu)}}.$$

In turn, substituting this expression in (A6) gives, together with (A35),

$$h_{t+1}^f = \theta^R(\tilde{\varepsilon}^{f,R})^{\nu_C}(\tilde{\varepsilon}^{f,H})^{\nu_A}J^{\Omega_1} \tag{A38}$$

$$\times[\upsilon_H\tau(1+b)\beta]^{(1-\mu)(1-\nu_C)}(\frac{a_m\chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^{\Omega_2}(h_t^f)^{\Omega_2}(x_t^f)^{-2\Omega_2},$$

where

$$\Omega_1 \equiv \pi^R\nu_C + (1-\nu_C)[\mu+\alpha(1-\mu)] > 0,$$

$$\Omega_2 \equiv \beta(1-\mu)(1-\nu_C) \in (0,1).$$

Equivalently,

$$h_{t+1}^f = g_1(h_t^f, x_t^f), \tag{A39}$$

where $\partial\ln g_1/\partial\ln h_t^f = \Omega_2$ and $\partial\ln g_2/\partial\ln x_t^f = -2\Omega_2$.

We now derive the dynamic equation for x_{t+1}^f. From (A33),

$$\frac{G_t^E}{N_t} = \upsilon_E\tau(1+b)\beta(\frac{Y_t}{N_t}).$$

Substituting this result in (14) for $j=f$ and using (A35) yields

$$e_{t+1}^f = (1-\chi)\tilde{\varepsilon}^{f,R}J^{\nu_1}\frac{[\upsilon_E\tau(1+b)\beta]^{\nu_2}}{(p_m^C\tilde{n})^{\nu_2}}(\frac{Y_t}{N_t})^{\nu_2}(e_t^f)^{1-\nu_2}. \tag{A40}$$

From (13), (A31), and (A40), given that $N_{t+1}^f = 0.5N_{t+1}$,

$$x_{t+1}^f = \frac{K_{t+1}^P}{e_{t+1}^f N_{t+1}^f} = \Gamma(\frac{Y_t}{0.5e_t^f N_t})^{1-\nu_2}, \tag{A41}$$

where

$$\Gamma \equiv [\frac{b\beta\Phi\sigma(1-\theta^R p_m^C\tilde{n})}{(1-\chi)\tilde{\varepsilon}^{f,R}(p_m^C\tilde{n})^{1-\nu_2}(0.5)^{\nu_2}}]J^{-\nu_1}[\upsilon_E\tau(1+b)\beta]^{-\nu_2}.$$

By definition, $Y_t/0.5e_t^f N_t = (Y_t/K_t^P)x_t^f$. Using (A37) to substitute for Y_t/K_t^P yields therefore

$$\frac{Y_t}{0.5e_t^f N_t} = J^\alpha[\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi}]^\beta (h_t^f)^\beta (x_t^f)^{1-2\beta}.$$

Substituting this result in (A41) yields

$$x_{t+1}^f = \Gamma \left\{ J^\alpha[\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi}]^\beta \right\}^{1-\nu_2} (h_t^f)^{\beta(1-\nu_2)} (x_t^f)^{(1-2\beta)(1-\nu_2)}, \tag{A42}$$

or equivalently

$$x_{t+1}^f = g_2(h_t^f, x_t^f), \tag{A43}$$

with $\partial \ln g_2/\partial \ln h_t^f = \beta(1-\nu_2) > 0$ and $\partial \ln g_2/\partial \ln x_t^f = (1-2\beta)(1-\nu_2) > 0$.

To determine the growth rate of output per worker, it is convenient to note first that $Y_{t+1}/N_{t+1} = (Y_{t+1}/K_{t+1}^P)(K_{t+1}^P/N_{t+1})$. Now, using (13), (A31), and (A37) for $t+1$ yields

$$\frac{Y_{t+1}}{N_{t+1}} = J^\alpha(\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^\beta (h_{t+1}^f)^\beta (x_{t+1}^f)^{-2\beta} b\beta\Phi\sigma(1-\theta^R p_m^C \tilde{n})(\frac{Y_t}{p_m^C \tilde{n} N_t}).$$

The balanced-growth rate of output per worker is thus, substituting out for Φ,

$$1+\gamma = \frac{J^\alpha}{p_m^C \tilde{n}}(\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^\beta \frac{\beta\sigma(1-\theta^R p_m^C \tilde{n})}{[(1-\tau)(1+b)]^{-1}} (\tilde{h}^f)^\beta (\tilde{x}^f)^{-2\beta}, \tag{A44}$$

where $\tilde{h}^f$ and $\tilde{x}^f$ are the steady-state solutions obtained by setting $\Delta h_{t+1}^f = \Delta x_{t+1}^f = 0$ in (A38) and (A42):

$$\tilde{h}^f = [\theta^R (\tilde{\varepsilon}^{f,R})^{\nu_C} (\tilde{\varepsilon}^{f,H})^{\nu_A} J^{\Omega_1}]^{1/(1-\Omega_2)} \tag{A45}$$

$$\times \left\{ [\upsilon_H \tau(1+b)\beta]^{(1-\mu)(1-\nu_C)} (\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^{\Omega_2} \right\}^{1/(1-\Omega_2)} (\tilde{x}^f)^{-2\Omega_2/(1-\Omega_2)},$$

$$\tilde{x}^f = \left\{ \Gamma[J^\alpha(\frac{a_m \chi b\tilde{\varepsilon}^{f,W}}{1-\chi})^\beta]^{1-\nu_2} \right\}^{1/\Omega_3} (\tilde{h}^f)^{\beta(1-\nu_2)/\Omega_3}, \tag{A46}$$

where

$$\Omega_3 \equiv 1 - (1-2\beta)(1-\nu_2) > 0.$$

These equations define the steady-state relationships between h_t^f and x_t^f. Equation (A45) defines a curve whose slope in the $\tilde{h}^f$-$\tilde{x}^f$ plane is given by $-(1-\Omega_2)/2\Omega_2$. Given that $\Omega_2 < 1$, this slope is negative. This curve is defined as curve HH in Figure 1, and it is convex. Equation (A46) defines a curve depicted as XX in Figure 1. Its slope is $\beta(1-\nu_2)/\Omega_3$, which is positive and can be shown to be less than unity. Thus, curve XX is concave. There is therefore a unique equilibrium.

To examine stability in the vicinity of that equilibrium, note that equations (A38) and (A42) form a first-order linear difference equation system in $\hat{h}_t^f = \ln h_t^f$ and $\hat{x}_t^f = \ln x_t^f$ which can be written as

$$\begin{bmatrix} \hat{x}_{t+1}^f \\ \hat{h}_{t+1}^f \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_t^f \\ \hat{h}_t^f \end{bmatrix}, \tag{A47}$$

where

$$a_{10} = \ln\left\{\Gamma[J^{\alpha}(\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1-\chi})^{\beta}]^{1-\nu_2}\right\},$$

$$a_{20} = \frac{1}{1-\Omega_2}\ln\left\{[\frac{\theta^R(\tilde{\varepsilon}^{f,R})^{\nu_C}}{(\tilde{\varepsilon}^{f,H})^{-\nu_A}J^{-\Omega_1}}][\frac{\upsilon_H \tau \beta}{(1+b)^{-1}}]^{(1-\mu)(1-\nu_C)}(\frac{a_m \chi b \tilde{\varepsilon}^{f,W}}{1-\chi})^{\Omega_2}\right\},$$

$$a_{11} = (1-2\beta)(1-\nu_2) > 0, \quad a_{12} = \beta(1-\nu_2) > 0,$$

$$a_{21} = -2\Omega_2 < 0, \quad a_{22} = \Omega_2 > 0.$$

Let $\mathbf{A}$ denote the matrix of coefficients in (A47) and let $\det \mathbf{A}$ denote its determinant and $\text{tr}\mathbf{A}$ its trace. Let λ_j, $j = 1, 2$ denote the eigenvalues of $\mathbf{A}$; the characteristic polynomial is thus $p(\lambda) = \lambda^2 - \lambda\text{tr}\mathbf{A} + \det \mathbf{A}$. Thus, $p(1) = 1 - \text{tr}\mathbf{A} + \det \mathbf{A}$, whereas $p(-1) = 1 + \text{tr}\mathbf{A} + \det \mathbf{A}$.

From the above definitions,

$$\text{tr}\mathbf{A} = (1-2\beta)(1-\nu_2) + \Omega_2 > 0,$$

$$\det \mathbf{A} = (1-2\beta)(1-\nu_2)\Omega_2 + 2\Omega_2\beta(1-\nu_2) > 0.$$

Given the signs of $\text{tr}\mathbf{A} = \lambda_1 + \lambda_2$ and $\det \mathbf{A} = \lambda_1\lambda_2$, it is clear that $p(-1) > 0$. We also have

$$p(1) = 1 - (1-2\beta)(1-\nu_2)$$

$$+\Omega_2[-1 + (1-2\beta)(1-\nu_2) + 2\beta(1-\nu_2)]$$

so that

$$p(1) = 1 - (1 - 2\beta)(1 - \nu_2) - \Omega_2 \nu_2.$$

If ν_2 is not too large, then $p(1) > 0$ and the steady state is a sink (see Azariadis (1993, p. 65)).

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Figure 1
Equilibrium and Effect of an Increase
in Efficiency of Women's Time Allocated to Child Rearing

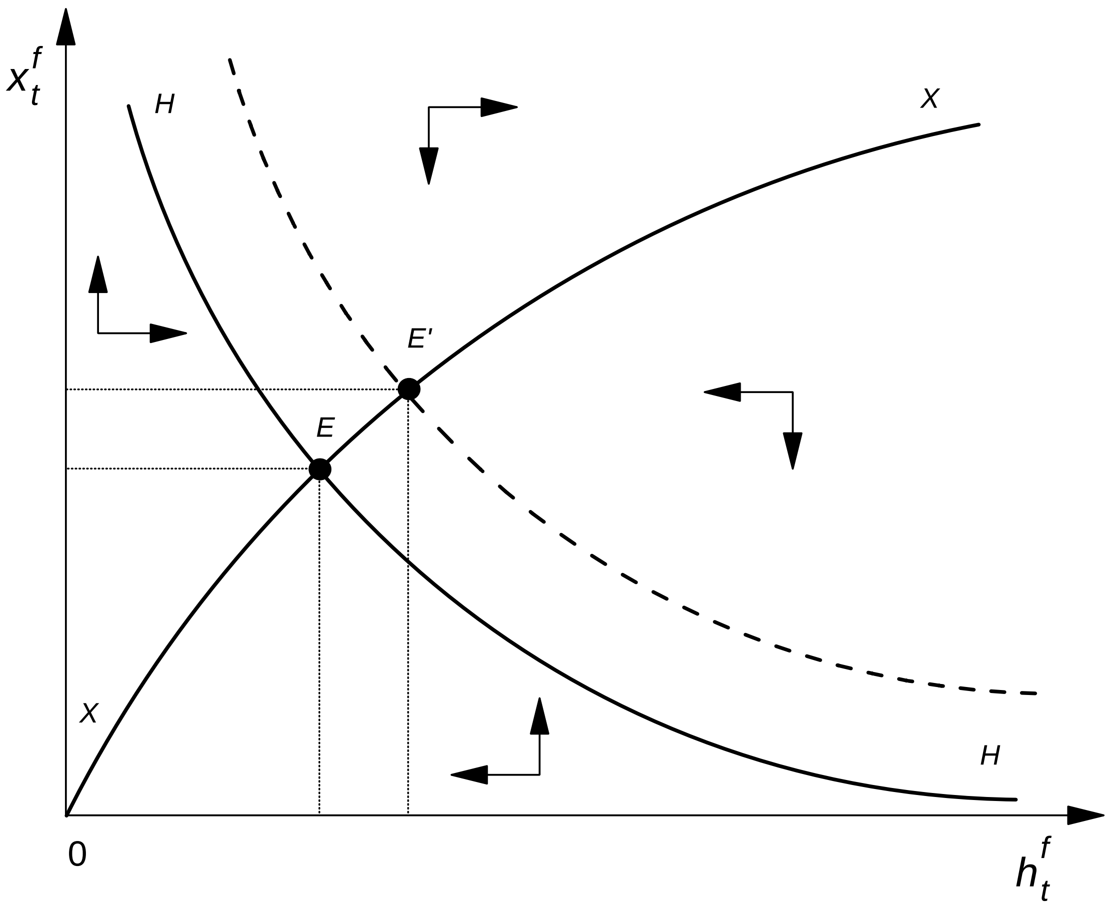

Figure 2
Multiple Development Regimes
with Threshold Effects of Health on Women's Productivity

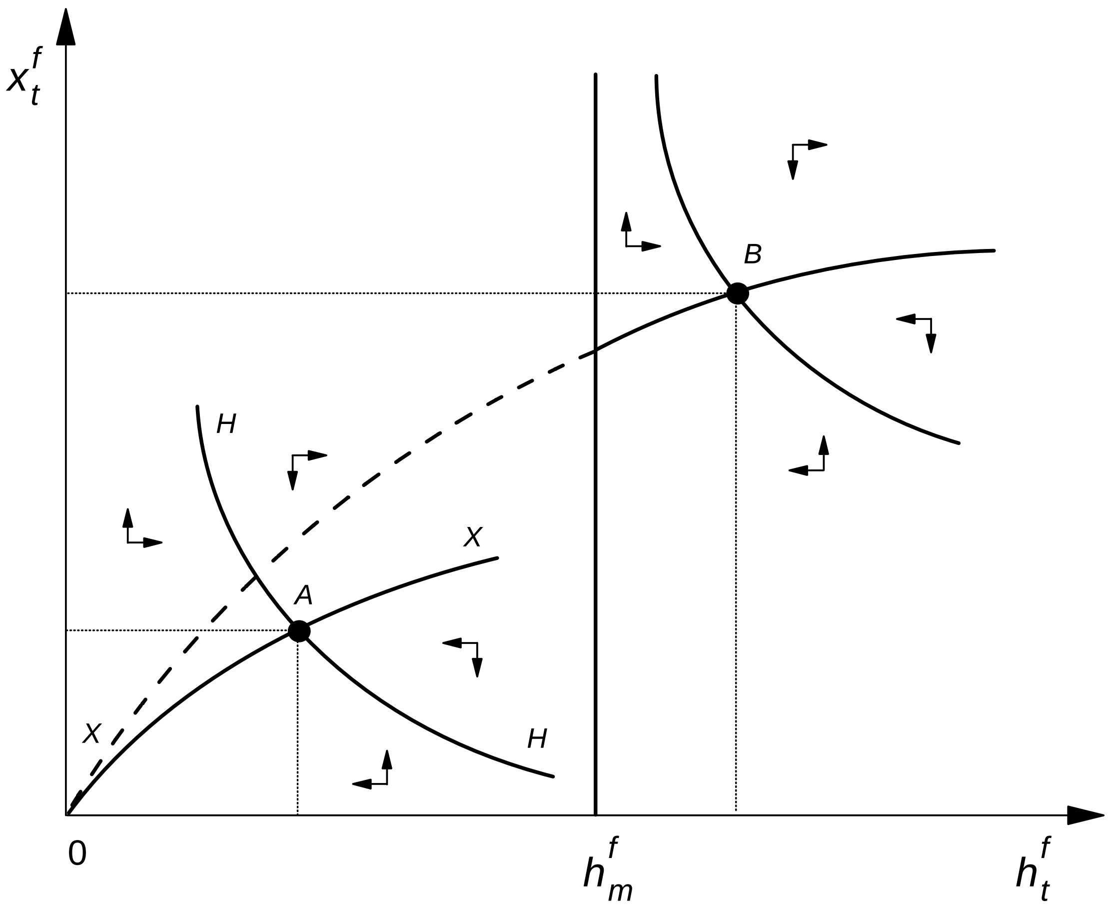

Figure 3
Budget-Neutral Increase
in the Share of Public Expenditure on Infrastructure