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Expectations, learning and policy rule

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Abstract

The optimal discretionary policy rule in the New Keynesian forward-looking model under the hypothesis of rational expectations responds only to fundamental shocks. This leads to indeterminacy of equilibria and E-unstability of the MSV REE. The outcome can be improved by responding to private expectations. This requires the Central Bank to be able to observe those expectations, or to precisely estimate them. It has also been shown in the literature that when the private sector doesn't have RE and instead is trying to learn the structure of the economy from data, the policymaker should implement a more aggressive policy. In light of these considerations, we ask how a policymaker that responds only to fundamental shocks should change its response when private expectations depart from rationality. In addition, we show that a policy rule that adeguately takes into account the learning process of agents while responding only to fundamentals can obtain the same results as an expectations based policy rule.

Key words: Monetary policy, expectations, learning, determinacy, E-stability.

JEL classification: E52, E58, D84.

1 Introduction

The optimal discretionary policy rule in a standard New Keynesian forward-looking model under the hypothesis of rational expectations responds only to fundamental shocks. The resulting system is characterized by indeterminacy

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of equilibria and E-instability[1] of the minimum state variable (MSV) rational expectations equilibrium (REE).[2] E-instability depends on the fact that in deriving the optimal policy rule, perfect rationality from the part of the agents is assumed. But if this assumption doesn't hold, then expectations represent an important factor for determining the system dynamics, and should be explicitly considered when deciding the policy rule. Evans and Honkapohja (2003a) show that in this case the outcome can be improved by responding to private expectations. In particular, not only the system has a unique REE in this case, but the equilibrium is also E-stable, which means that it can be learned in real time by agents.[3] But the implementation of this type of policy requires the policymaker to be able to observe private expectations, or at least to be able to precisely estimate them.

Orphanides and Williams (2003), in a different setting, show that when the private sector doesn't have RE and instead is trying to learn the structure of the economy from data, the central bank should implement a more aggressive policy. The policy rule that would be efficient under RE performs poorly when expectations are not rational and agents form their beliefs by implementing an adaptive learning scheme.

In light of these considerations, we ask how a policymaker that responds only to fundamental shocks should change his response when private expectations depart from rationality. We use a standard forward-looking New Keynesian model. A significant difference with the Orphanides and William (2003) setting regards the expectations timing in the Phillips curve: in their model past expectations of current inflation affect current inflation, while in the New Keynesian model it is current expectations of future inflation that affect current outcomes. Moreover, while in Orphanides and William (2003) the policymaker responds to current inflation, here he responds to the fundamental shocks that hit the economy. This difference is important since, as argued for example by Taylor (1993) and McCallum and Nelson (1999), a policy rule that responds to current endogenous variables is not operational.

The idea is to try to achieve the same results that an expectations based policy rule can achieve, but by responding only to fundamental shocks. In order to do that, the central bank must change its response to those shocks when private expectations are not rational. Actually, given the indetermi-

[1] For the relevance of the E-stability concept, and for an exaustive guide to its analysis, see Evans and Honkapohja (2001).

[2] The concept of MSV was first introduced by McCallum (1983). For a discussion of the role of this solution concept in rational expectations models, see McCallum (1999).

[3] This link relies on the E-stability principle. Again, see Evans and Honkapohja (2001).

nacy result, there would be good reasons to change the policy response even if expectations are rational, in order to try to achieve determinacy and thus exclude a possible source of fluctuations, but we refer mainly to the non-rationality case since stress is placed here on the learnability of equilibria more than on their determinacy. This reflects the idea that, even if there is a multiplicity of solutions, this may not constitute a problem as long as one of them, possibly the MSV, is learnable by agents.

We might wonder how could the authority know that private expectations are off from rationality, and thus decide to change its policy response. The answer is that the central bank can realize that private expectations are not rational when, implementing its optimal policy (i.e., optimal under the assumption of RE), it does not get what is expected to see under the rationality hypothesis, i.e., inflation and output in our model do not conform to the MSV solution. If actual outcomes depart from the expected ones, it must be that private expectations are affecting those outcomes in a way that is different from what was assumed, given that the central bank is assumed to have perfect knowledge of all the remaining structure of the economy.

In this case, the policymaker should change his response to fundamental shocks. We show that under discretionary policy (so that the policy effect on expectations is not taken into account), the optimal response to fundamental shocks depends on the parameter values in the expectations formation equations. If the authority could observe those parameters, it would be able to optimally tune its fundamentals based policy. But such a precision is not actually needed to get convergence of beliefs toward rationality. Simulations show that there is convergence even when policy parameters are not exactly tuned, even though in this case convergence is not toward the same solution: it still is the MSV REE, but is different from the one obtained under perfect knowledge, as the system representing the economic dynamics is different (because of the different policy rule).

We note that for responding to shocks instead of to expectations to make sense it must be that shocks and expectations are correlated. We know that when expectations are rational they are functions of the shocks, so there is the necessary correlation. But when expectations are not rational, can they still be considered functions (only) of fundamental shocks? The answer is yes under our hypothesis of learning, which assumes that people recurrently estimate a correctly specified model, thus one in which only fundamental shocks enter. We leave for future work to consider the case in which agents learn by estimating a misspecified model, one that possibly includes also sunspot components.

The rational expectations hypothesis allows a direct map from shocks to current endogenous variables. If expectations are not rational, the map should be from shocks and expectations to endogenous variables. But expectations, on their part, are also function of these shocks. Thus a direct map from shocks to endogenous variables is still possible, even though it will change over time as expectational parameters are updated through learning. We are therefore able to rewrite the optimal expectations based policy rule as a fundamentals based rule with time varying parameters, and this allows us to clearly see how the response of the CB should change when agents are learning.

The structure of the paper is as follows: Section 2 presents the full knowledge benchmark; Section 3 introduces imperfect knowledge from the part of the public: 3.1 presents an expectations based policy rule, while 3.2 shows how this rule can be equivalently expressed in terms of fundamentals only, when a specific form of expectation formation scheme is assumed. Section 4 presents some simulations and the main results, while Section 5 concludes.

2 Full knowledge

The model we use is a standard forward-looking New Keynesian framework, as presented in Clarida, Gali and Gertler (1999). The equations describing the economic system are:

$$x_t = -\varphi\left(i_t - E_t\pi_{t+1}\right) + E_t x_{t+1} + g_t \quad (1)$$

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t. \quad (2)$$

Equation (1) is a forward-looking IS equation, obtained by log-linearizing the consumption Euler equation that arises from the household's optimal saving decision. x_t is the output gap, the deviation of the output from its potential level, i_t is the interest rate, which is here taken to be the policy instrument, and π_t is the inflation rate. E indicates expectations, for now taken to be rational.[4]

Equation (2) is a forward-looking Phillips curve derived under the assumption of staggered nominal price setting by optimizing monopolistically

[4]Preston (2005) points out that when expectations are not rational, equations (1) and (2) do not actually represent optimality conditions for the houlehold and firm's problems. In this case, he claims, the forecast of the whole future paths for inflation and output gap should be considered. Honkapohja, Mitra and Evans (2002) defend instead this formulation as a sensible way to represent behavioral rules under adaptive learning.

competitive firms. Individual pricing decisions are aggregated and the ensuing relation is log-linearized about the steady state.

There is no agreement in the literature about the values to assigned to the parameters of these structural equations. Prominent examples of values that has been used are: Clarida, Gali and Gertler (2000): $\varphi = 1$, $\lambda = .3$, $\beta = .99$; McCallum and Nelson (1999): $\varphi = .164$, $\lambda = .3$, $\beta = .99$; Woodford (1999): $\varphi = 1/.157$, $\lambda = .024$, $\beta = .99$. In our simulations we will use the Clarida, Gali and Gertler (2000) calibration, since different parameter values don't affect our qualitative results.

The shocks follow AR(1) processes:

$$g_t = \mu g_{t-1} + \tilde{g}_t \tag{3}$$
$$u_t = \rho u_{t-1} + \tilde{u}_t \tag{4}$$

with damping coefficients μ and ρ between 0 and 1. g_t represents a demand shock coming from potential output or government expenses, while u_t summarizes any cost push shock to marginal costs not entering the Phillips curve through y_t. $\tilde{g}_t$ and $\tilde{u}_t$ are white noise processes with zero mean and variances $\sigma^2_{\tilde{g}}$ and $\sigma^2_{\tilde{u}}$ respectively.

2.1 Fundamentals based policy rule

The policy problem is to minimize expected deviations of output gap and inflation from their target levels. Here for simplicity we assume that both those targets are zero, which means that the policymaker aims to reach the potential output while driving inflation to zero. The policy objective function is then

$$\min E_t \sum_{i=0}^{\infty} \beta^i \left(\alpha x^2_{t+i} + \pi^2_{t+i} \right). \tag{5}$$

The parameter α weighs the relative importance of the two target variables in the loss function. When $\alpha = 0$ we have pure inflation targeting. β is the discount factor for the policymaker, here taken to be the same as the private sector's in equation (2).

We will consider here only the case of discretionary policy. In this case the first order condition for optimal policy is

$$\lambda \pi_t + \alpha x_t = 0. \tag{6}$$

The system (1), (2) and (6) under the RE assumption gives the reaction function for the interest rate:

$$i_t = \frac{\lambda(1-\rho)+\varphi\alpha\rho}{\varphi(\lambda^2+\alpha(1-\beta\rho))}u_t + \frac{1}{\varphi}g_t{}^5 \tag{7}$$

As showed in different works (see e.g. Evans and Honkapohja, 2003) the New Keynesian model when closed with this policy rule is indeterminate, i.e. there are multiple REE. Moreover, the MSV solution, based only on fundamentals, is E-unstable and thus can not be adaptively learned in real time by agents.

3 Imperfect knowledge

3.1 Expectations based policy rule

We now consider an economy where agents' expectations depart from rationality. The system is described by equations (1), (2) and (6), where now E is replaced by $\hat{E}$, indicating that expectations are not constrained to be rational. From (2) and (6) we get

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\hat{E}_t\pi_{t+1} + \frac{\alpha}{\alpha+\lambda^2}u_t \tag{8}$$

and combining (1) with (6) results

$$\pi_t = \frac{\alpha\varphi}{\lambda}i_t - \frac{\alpha\varphi}{\lambda}\hat{E}_t\pi_{t+1} - \frac{\alpha}{\lambda}\hat{E}_t x_{t+1} - \frac{\alpha}{\lambda}g_t. \tag{9}$$

Equations (8) and (9) give

$$i_t = \left(1+\frac{\lambda\beta}{\varphi(\alpha+\lambda^2)}\right)\hat{E}_t\pi_{t+1} + \frac{1}{\varphi}\hat{E}_t x_{t+1} + \frac{\lambda}{\varphi(\alpha+\lambda^2)}u_t + \frac{1}{\varphi}g_t, \tag{10}$$

the expectations based policy rule.

Equations (1), (2) and (10) represent the new system describing the evolution of the economy, given private sector expectations, when the central bank implements policy rule (10). In Evans and Honkapohja (2003a) it is showed that this system is determinate under RE and moreover the (unique) equilibrium is E-stable. This means that there is only one RE equilibrium and this is learnable by agents using econometrics techniques.

[5] This appears to differ but fundamentally agrees with the form derived in Evans and Honkapohja (2003), which includes a constant. The difference is due to the fact that we have set to zero the target values for inflation and output gap. The same form as ours is derived in Clarida, Gali and Gertler (1999).

3.2 Policy rule and expectations formation scheme

By responding to private expectations the policymaker can obtain determinacy and E-stability of equilibrium, so that if agents happen to be temporarily off from rationality, they are able to learn the REE and conform to it. But this requires the central bank to observe private expectations.

Private expectations are functions of the fundamental shocks, and possibly of other extraneous components. We assume here that only fundamentals affect private expectations, leaving aside the problem of sunspot equilibria. We want now see how a policymaker should change its response to fundamental shocks when people are not rational, in order to get convergence of beliefs towards rationality.

We assume, as it is common in recent literature on monetary policy, that private agents learn from data by using recursive least square (RLS) to estimate a (correctly specified) reduced form model. Their perceived laws of motion (PLMs) for inflation and output are thus

$$\pi_t = a_1 u_t + a_2 g_t \tag{11}$$

$$x_t = b_1 u_t + b_2 g_t \tag{12}$$

which they estimate recursively using data that become available as time moves on. In doing this they can use a constant gain or a decreasing gain algorithm: the choice affects convergence of beliefs to the RE values. With a decreasing gain, agents have infinite memory and don't discard past data as time goes on: when the value of $1/t$ is chosen for the gain, and appropriate initial conditions are specified, this procedure corresponds to the standard least square. In this case, beliefs converge towards the RE values. Constant gain instead is equivalent to rolling windows regression, and is more appropriate when the economy is perceived to be subject to structural changes. In this case the parameter values, and thus the policy coefficients, remain noisy at the limit. They can not converge towards their REE values, but can converge towards a stationary distribution around those values.

The learning algorithms are of the form

$$\theta_t = \theta_{t-1} + \zeta_t R_t^{-1} z_{t-1}(y_{t-1} - \theta'_{t-1} z_{t-1}) \tag{13}$$

$$R_t = R_{t-1} + \zeta_t (z_{t-1} z'_{t-1} - R_{t-1}), \tag{14}$$

where θ_t is the vector of parameters to be estimated, ζ_t is the gain, and z_{t-1} is the vector of exogenous variables determining the endogenous variable y_{t-1}. In our case, θ corresponds either to the vectors of a_i or b_i, z_{t-1} is the vector $[u_{t-1}\ g_{t-1}]'$ and y_{t-1} is either π_{t-1} (when estimating a_i) or x_{t-1} (when θ corresponds to b_i).[6]

The policymaker thus solves the maximization problem (5) with respect to γ_u and γ_g, subject to (1), (2), a policy rule of the form

$$i = \gamma_g g_t + \gamma_u u_t \tag{15}$$

and expectations formation equations

$$\hat{E}_t \pi_{t+1} = a_1 \rho u_t + a_2 \mu g_t \tag{16}$$
$$\hat{E}_t x_{t+1} = b_1 \rho u_t + b_2 \mu g_t \tag{17}$$

that follow from the PLMs (11) and (12) and from the AR form for the shocks (3) and (4). Expectations are indicated by $\hat{E}$, meaning that they are not fully rational, since parameters a_i and b_i are recursively estimated from data. We assume that ρ and μ are known: otherwise they could be consistently estimated, given that the shocks are observable. We thus get

$$i_t = \left(\frac{\lambda(a_{1,t}\beta\rho + 1)}{\varphi\left(\alpha + \lambda^2\right)} + \frac{\rho}{\varphi} b_{1,t} + \rho a_{1,t} \right) u_t + \left(\frac{a_{2,t}\lambda\beta\mu}{\varphi\left(\alpha + \lambda^2\right)} + \frac{\mu}{\varphi} b_{2,t} + \mu a_{2,t} + \frac{1}{\varphi} \right) g_t, \tag{18}$$

which is the optimal fundamentals based policy rule when people are not rational but form instead their expectations by estimating the parameters in correctly specified PLMs (in short: expectations equivalent fundamentals based - EEFB - policy rule).[7] Note that the optimal policy parameters vary over time, as people learn. This time dependence has been made explicit here by adding a subscript t to the coefficients a and b. This is the crucial difference with the fundamentals based policy rule derived under the perfect knowledge assumption. If the policy rule parameters are time-invariant, when people are learning the movements in their estimated parameters is transmitted on the macroeconomic outcomes. In order to dampen this effect, the policy rule must change over time, in order to offset, at least partially, movements in expectations.

By noting that private expectations under this learning scheme are given by (16) and (17), we can rearrange terms in (18) and get again policy rule (10). Thus this policy achieves both determinacy and E-stability, even

[6] It is common practice to use data only through time $t-1$ when estimating parameters at time t used to form expectations for time $t+1$ endogenous variables. This avoids a simultaneity problem between parameter estimates and current endogenous variables.

[7] This policy is optimal in a restricted sense, since the policymaker does not try to exploit actively the learning mechanism of agents.

though it responds only to fundamentals. This happens because it takes advantage of the fact that also expectations are functions of the fundamentals.

Under RE, the expectational parameters a_i and b_i are

$$a_1 = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)}, \qquad a_2 = 0 \tag{19}$$

$$b_1 = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)}, \qquad b_2 = 0 \tag{20}$$

and policy parameters in (18) coincide with their RE values in (7).

We can see that out of the RE equilibrium the policy coefficient for g_t, γ_g, in (18) is larger than its counterpart in (7), since a_2 and b_2, if not at their RE value of zero, are both supposed to be positive: a positive shock to demand should be perceived to increase output and inflation. This implies that the stronger g_t is perceived to affect output and inflation (i.e., the larger are a_2 and b_2), the stronger should be the reaction of the CB to the shock.

With regard to the policy coefficient for u_t, γ_u, one would expect a_1 to be positive and b_1 negative (a cost-pushing shock should be perceived to increase inflation and reduce output): it follows that when either a_1 or b_1 are greater than their RE values,[8] γ_u in (18) is larger than the correspondent value in (7). Thus, the stronger the shock u_t is perceived by agents to affect inflation, the stronger should be the response by the policymaker to that shock; and the milder is the perceived negative effect of this shock on output, the stronger again should be the policy response to it, since in this case expectations of future output are higher, and this negatively affects current inflation.

3.3 Determinacy and E-stability analysis

As anticipated above, in the NK model under analysis, policy rule (18) is able to induce the same properties in terms of determinacy and E-stability that policy rule (10).

The policymaker can affect determinacy iff he is able to affect the parameters on the expectational terms in the semi-reduced form (1)-(2). Therefore, the CB must condition its policy rule on expectations, either directly or through the EEFB policy presented above. The two are in fact equivalent and implement the same equilibrium, since it is always possible to

[8] Given that b_1 is negative, 'greater' means less negative, thus corresponding to output expectations less sensitive to the shock u_t. On the other hand, since a_1 is positive, 'greater' in this case means that inflation expectations are more sensitive to the shock.

rewrite the latter in the form of the former. If the response to expectations is optimally tuned, the policymaker is able to generate determinacy in the economy, as shown by Evans and Honkapohja (2003).

As for E-stability, we show now why also this property can be achieved by the CB by conditioning its rule on the time varying belies of agents. Using (1), (2), (15), (16), (17) we obtain the ALMs

$$x_t = \left[\rho\left(a_1+b_1\right)-\varphi\gamma_u\right]u_t+\left[\mu\left(a_2\varphi+b_2\right)-\varphi\gamma_g+1\right]g_t \tag{21}$$

$$\begin{aligned}\pi_t &= \left[\left(\rho\left(a_1+b_1\right)-\varphi\gamma_u\right)\lambda+\beta\rho a_1+1\right]u_t+ \\ &\quad +\left[\left(\mu\left(a_2\varphi+b_2\right)-\varphi\gamma_g+1\right)\lambda+\beta\mu a_2\right]g_t\end{aligned} \tag{22}$$

from which follow the T-maps and the ensuing ODEs

$$\dot{a}_1 = \left(\beta\rho+\lambda\rho-1\right)a_1+\rho b_1\lambda-\varphi\lambda\gamma_u+1 \tag{23}$$

$$\dot{a}_2 = \left(\varphi\mu\lambda+\beta\mu-1\right)a_2+\mu\lambda b_2-\varphi\lambda\gamma_g+\lambda \tag{24}$$

$$\dot{b}_1 = \left(\rho-1\right)b_1+\rho a_1-\varphi\gamma_u \tag{25}$$

$$\dot{b}_2 = (\mu-1)b_2+\mu a_2\varphi-\varphi\gamma_g+1 \tag{26}$$

As it can be seen from the ODEs above, the policy parameters do not affect E-stability (which obtains iff the system of ODE is locally asymptotically stable) unless they depend on the time-varying parameters in the PLMs. This is what happens with the EEFB policy rule (18), where γ_u is conditioned on a_1 and b_1, and γ_g on a_2 and b_2: this dependence changes the map from the PLMs to the ALMs and is able to generate E-stability.

3.4 Policy mistakes and their effects

It is interesting to note that, in order to achieve superior outcomes in terms of determinacy and E-stability, it is not necessary that policy parameters be precisely tuned as in (18). Suppose, in fact, that the policymaker incur some errors in its response to shocks. The policy rule can then be expressed as

$$i = (\gamma_u + err1)u_t + (\gamma_g + err2)g_t, \tag{27}$$

where γ_u and γ_g are the policy parameters in (18) and err1 and err2 are the deviations of the actual policy parameters from the optimal ones.

It results clear from the analysis in section 3.3 that the error terms do not affect determinacy and E-stability of equilibrium, as it is always possible to rewrite the system as before, plus the policy error terms, that clearly do not change the properties of the system in terms of determinacy and E-stability.

What do change, though, are the values for the reduced form parameters in the MSV REE.

To derive these values, we must find the MSV REE solution through the undetermined coefficient procedure using policy rule (18) modified to account for the errors. We thus get

$$a_1 = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} - \frac{(\alpha+\lambda^2)\lambda\varphi}{\lambda^2 + \alpha(1-\beta\rho)} err1, \quad (28)$$

$$a_2 = -\frac{(\alpha+\lambda^2)\lambda\varphi}{\lambda^2 + \alpha(1-\beta\mu)} err2, \quad (29)$$

$$b_1 = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)} + \left(\frac{\lambda^2\beta\rho\varphi}{\lambda^2 + \alpha(1-\beta\rho)} - \varphi\right) err1, \quad (30)$$

$$b_2 = \left(\frac{\lambda^2\beta\mu\varphi}{\lambda^2 + \alpha(1-\beta\mu)} - \varphi\right) err2. \quad (31)$$

Note that when err1=err2=0, we obtain again (19)-(20).

4 Simulations

We now present some stochastic simulations of our model (1)-(4) when alternatively closed with policy rule (7) or (18). Agents recurrently estimate the parameters a_i and b_i in the MSV solution (11) and (12) using the algorithm constituted by (13) and (14), and form their expectations according to (16) and (17); the authority implements either policy rule (7) or (18) in order to stabilize the economy. We consider both constant and decreasing gain learning from the part of the agents. All simulations are run with the Clarida, Gali and Gertler (2000) parametrization presented above, while α is set to 0.1. The two AR parameters for the fundamental shocks are both set equal to .8, while ζ_t, the gain in the learning algorithm, is set to $1/t$. The mean squared error for all the shocks in the model, drawn from a normal distribution with zero mean, is set equal to .25. Initial beliefs are chosen to be non-rational, but with the "right" sign: people may not know the exact effect of exogenous shocks on endogenous variables, but it seems sensible to suppose that they can at least understand the correct direction of influence.[9]

[9] The specific values adopted are a_1=6, a_2=6, b_1=-3, b_2=6.

4.1 Dynamics under learning

We start by setting the gain to $1/t$, which corresponds to infinite memory learning. Figure 1 shows that when the policymaker implements the fundamentals based policy derived under the wrong assumption of RE, i.e., when he uses policy rule (7), private beliefs do not converge. This result is due to the fact that the MSV REE is E-unstable under this policy rule, as said before: if beliefs happen to be off from full rationality, they are led further apart and will never reach the RE values. Dotted lines in the graph represent the REE values for each belief parameter in the PLMs (13) and (14). Note that the RE values for a_2 and b_2 are zero.

Figure 2, instead, represents the evolution of beliefs in an economy where the authority takes into account the fact that agents are learning and implements policy rule (18). We can see that in this case private beliefs converge towards their RE values: the REE is now E-stable, and agents are able to learn it.[10]

The two pictures clearly show the importance of the policy implemented by the central bank in determining the economic dynamics. Under policy rule (7) the authority wrongly believes that agents are rational, and therefore implements a policy that responds in a constant way to fundamental shocks: in this way the policy rule is not able to adapt to changes in the structure of the economy introduced by the learning activity of private agents, and the equilibrium results unstable. Under policy rule (18), instead, the response of the authority to fundamental shocks changes over time, as private beliefs are being modified through learning: in this way the policymaker is able to guide private expectations towards rationality. Figure 3 shows a comparison of the interest rate, the policy instrument, under the two different policy specifications. As private sector beliefs converge towards rationality, the responses prescribed by the two policies become more similar, until perfect overlapping is reached once beliefs have fully converged to rationality. But while beliefs are off from rationality, the optimal responses prescribed by the two rules are quantitatively very different, though qualitatively similar since both ultimately depend on the shocks that constantly hit the economy. We can see that policy rule (18) (continuous line) prescribes a stronger response to shocks, i.e., the interest rate is always bigger in absolute value. This means that when the economic situation calls for a restrictive policy, the policymaker has to raise the interest rate to an higher level, since it must not only offset the shocks that have hit the economy but also the private expectations that by those shocks have been spurred. By the same argument,

[10] Convergence is faster when the variance for the shocks is made larger.

when the economy goes through negative shocks, the policymaker has to decrease the interest rate to a lower point, in order to compensate both the exogenous shocks and the ensuing negative expectations.

Figure 4 shows the policy parameters in the two cases: under policy rule (7) the response to shocks is constant (continuous line), while under policy rule (18) it varies over time (dashed line) and converges towards the optimal response under RE as agents learn the equilibrium value of the parameters in their PLMs.

As we have said before, we want to stress that in order to induce convergence of private beliefs towards rationality, the policy parameters need not be exactly as prescribed in (18). In fact, even introducing an error in the policy parameters, convergence still obtains. We can consider two cases: one in which the error is a constant, and the other where the error is a zero mean, i.i.d., random variable, In Figure 5 the error introduced is constant and equal to 1 in each period, while in Figure 6 it is drawn randomly from a normal distribution with zero mean and is thus different in each period. We can see that in both cases convergence still obtains. But in the case of a fixed error, convergence of beliefs is not towards the same RE values as before, since the error made by the policymaker in implementing his policy enters into the solution equations, as can be seen from (28)-(31) with constant err1 and err2.[11] It is interesting to note that the size of the error is not important for convergence to obtain. We experimented with errors two orders of magnitude larger than the parameter values in the model, and still convergence obtains when the CB implements the EEFB policy rule. For the case of random errors, we set for example the variance of err1 and err2 equal to 10^2 and show the resulting dynamics for the reduced form parameters in Fig. 7: as it can be seen, despite the large volatility in the period by period reduced form parameters (green dash-dotted line) generated by the policy errors, agents are able to (approximately) learn the equilibrium value for the reduced form parameters (red dotted line) that would prevail if the CB was not introducing policy mistakes and was implementing exactly policy rule (18). Interestingly, even if the equilibrium values for $a_{1/2}$ and $b_{1/2}$ change over time with random err1 and err2, agents can still (approximately) learn the MSV REE values that would prevail without errors, i.e., (19) (20).

[11] In Figure 5, dotted lines show the RE values of beliefs for the model without policy errorr, while dashdot lines show RE values of beliefs when policy errors (of magnitude 1) are introduced.

4.2 Some measures of relative performance

We now compare the relative performance of the economy when the two different policy rules (7) and (18) are implemented, under the assumption that agents learn through RLS. Figure 8 shows the output gap under the two policy rules (7) and (18): we can see that under rule (7) the output gap presents a wide and increasing variability, as a consequence of the fact that private beliefs diverge, while under policy (18) the output gap floats steadily around 0, the perfect foresight equilibrium of the deterministic model. Figure 9 shows the pattern for inflation under the two alternative policies. Again, inflation shows a stationary distribution under policy rule (18) while its variance is greater and increases over time when policy rule (7) is implemented. Thus the policymaker, when implementing policy rule (7) because of the wrong assumption of rationality from the part of the agents, is not able to stabilize inflation and output gap around the desired values. As time passes, private beliefs diverge and this has a negative impact on the outcomes that can be achieved.

The same happens under the assumption that agents learn through a constant gain learning: when policy rule (18) is implemented, output and inflation are kept around the equilibrium values, while with policy rule (7) their variance increases over time.

5 Conclusions

Using a policy rule that assumes rationality from the part of the agents when they in fact are not rational and are instead forming their beliefs by adaptive learning, generates instability for the economy: in particular, such a policy prevents agents from learning the reduced form solution for the endogenous variables and the resulting macroeconomic outcomes are subject to large and increasing volatility. It follows that a different approach to monetary policy is needed, one that takes explicitly into account the fact that people are learning from data, and thus modifying their beliefs over time.

Since private agents have beliefs that evolve as they learn, the way in which agents respond to fundamental shocks changes over time: to account for these movements in the transmission channel, the policymaker must change his own response to shocks, so as to compensate for the evolution of private beliefs and allow agents to learn the relevant equilibrium. If instead the policymaker neglects the fact that private expectations are off rationality and evolving over time, agents are not able to adaptively learn the fundamental RE solution.

We have shown in this work that a central bank can obtain E-stability (and determinacy) of equilibrium by responding solely to shocks, as in the fundamentals based RE rule, but with policy parameters that are more aggressive and change over time. The fact that agents are not rational but instead are adaptively learning from data calls for a more activist monetary policy. The stronger the shocks are perceived by agents to affect the economy, and the stronger should be the policy response to them.

These results have been derived in a New Keynesian model, which is indeterminate and its MSV solution is E-unstable under the standard fundamentals based policy rule. A similar analysis should be carried out in different settings, to check for the robustness of these conclusions. Orphanides and Williams (2003), in a different model, consider the implications of private sector learning for the policymaker and they find that when people are learning the policymaker should respond more strongly to inflation. But they don't consider the possibility of changing that response over time, as people modify their beliefs.

Since beliefs are part of the structural model of the economy, having beliefs that evolve over time is like having continuous structural changes in the economy, which the policymaker should take into account and deal with. This phenomenon is more likely to happen after main changes in underlying factors have occurred, and less likely in periods of stability, when expectational parameters (that depend on the structural parameters and on the policy implemented by the policymaker) may not be far from rationality and relatively stable.

The necessary adaptation of the policy rule to changing beliefs of private agents can be done by directly incorporating private expectations into policy rule, as suggested by Evans and Honkapohja (2003). But if for some reason this is not feasible and the policymaker can respond only to exogenous variables, then this response must evolve over time to account for the fact that the way in which these same exogenous variables affect the economy is perceived to change over time by private agents.

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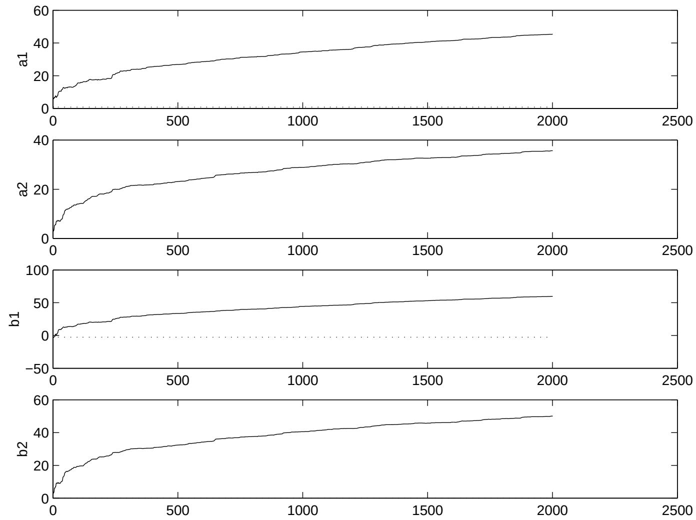

Fig. 1: Evolution of beliefs when policymaker implements policy rule (7).

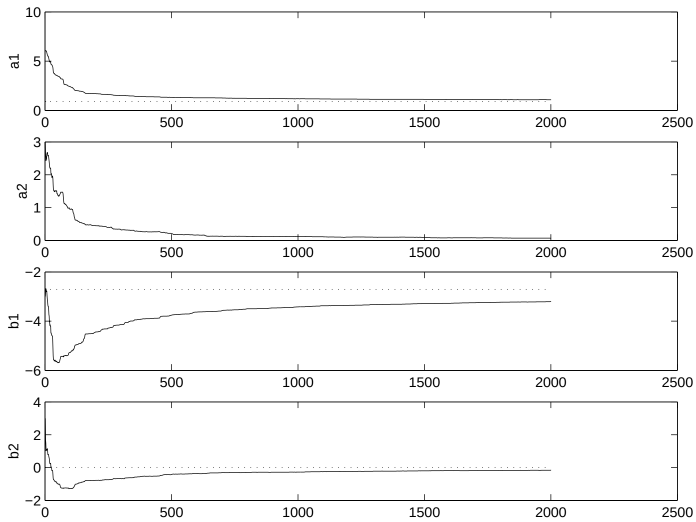

Fig. 2: Convergence of beliefs under policy rule (18).

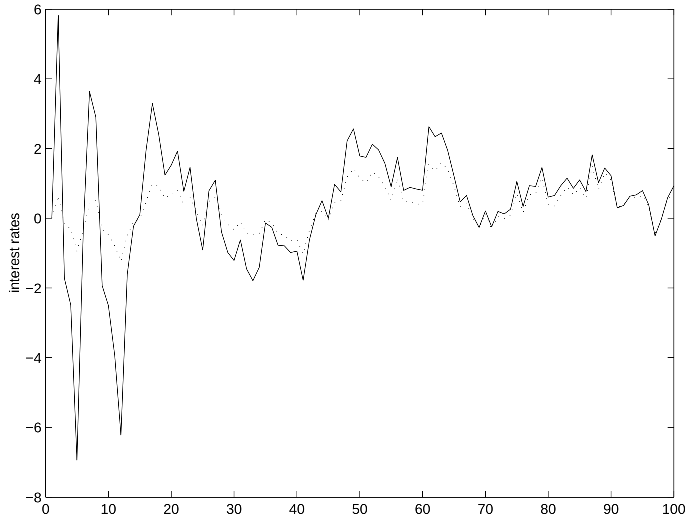

Fig. 3: Interest rates under the two policy specifications. Dotted line represents interest rate under policy rule (7), solid line under (18).

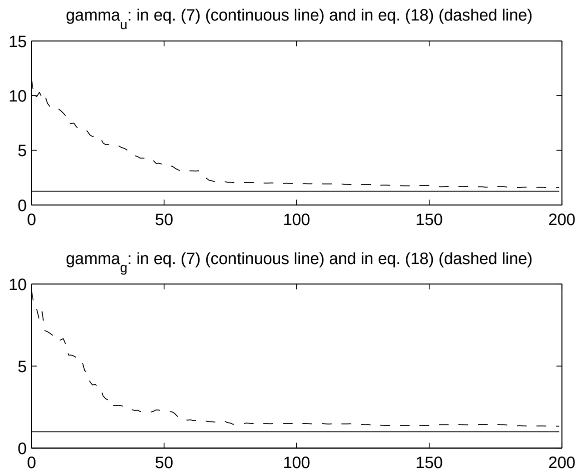

Figure 4: Policy response to u(t) and g(t) for policy rule (7) (continuous line) and (18) (dashed line).

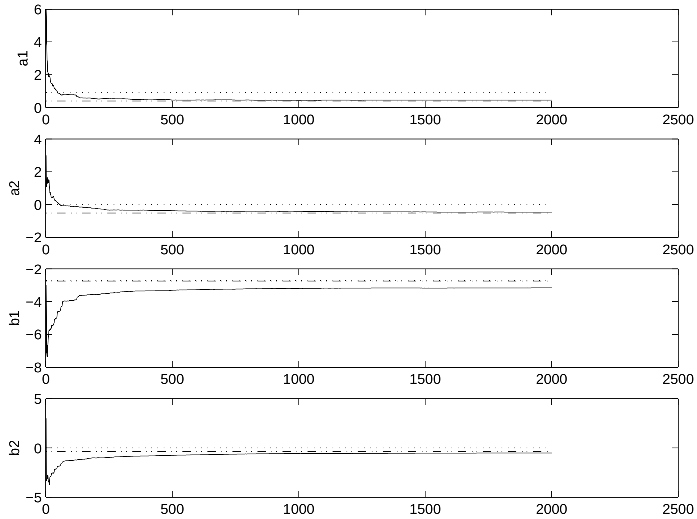

Fig. 5: Evolution of private beliefs when a constant error is introduced in the parameters of policy rule (18).

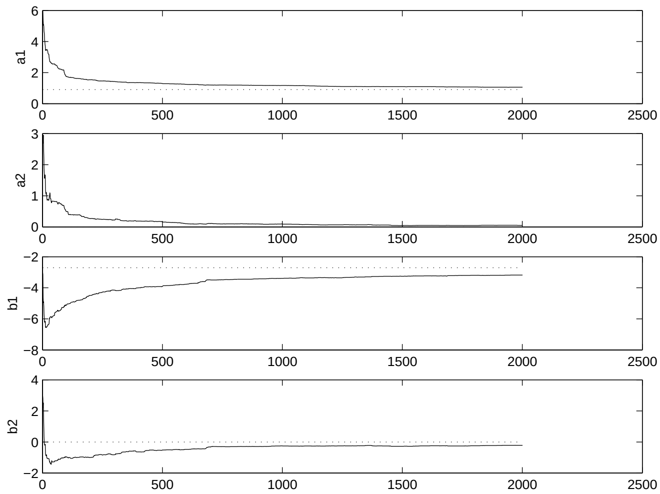

Fig. 6: Evolution of private beliefs when a random error is introduced in the parameters of policy rule (18).

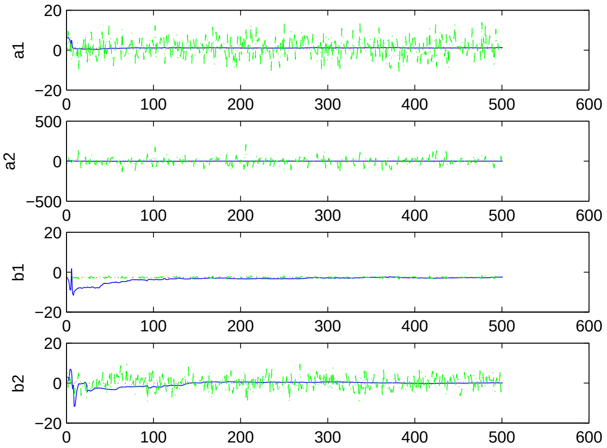

Figure 7: Evolution of reduced form parameters when the CB implements an EEFB policy with large errors.

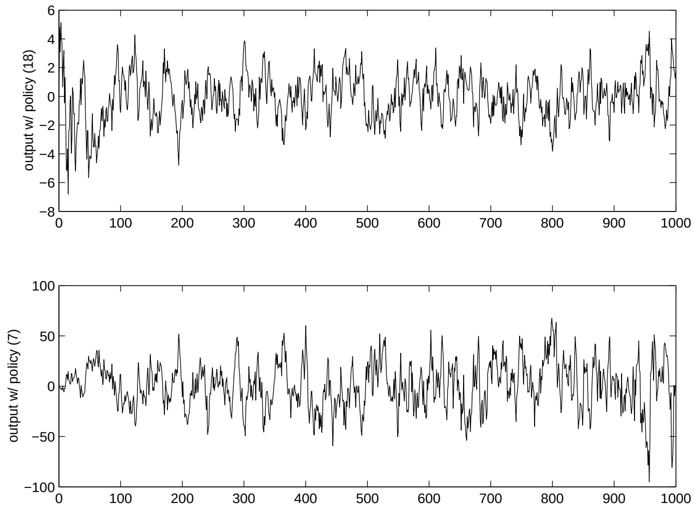

Fig. 8: Output gap under the two policy rules (7) and (18).

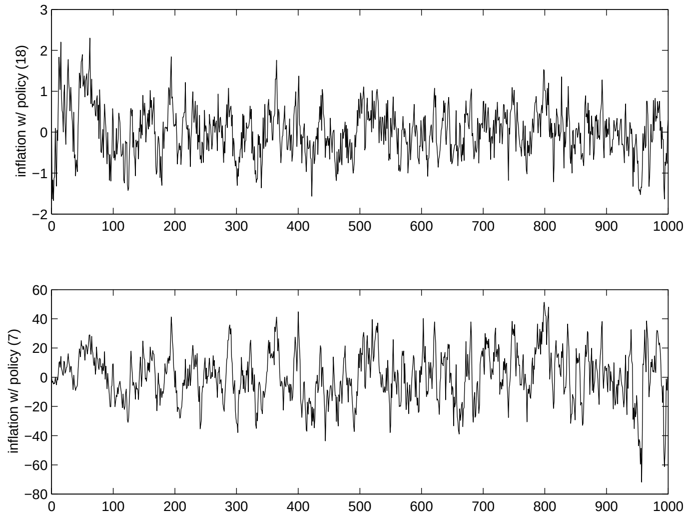

Fig. 9: Inflation fluctuations under the two policy rules (7) and (18).