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Public and Private Maintenance Expenditure in a Growing Economy

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Abstract

This paper studies the optimal taxation structure and public maintenance spending in an endogenous growth model where public maintenance spending effects the efficiency of public and private capital. Private firms also spend on maintenance, which increases the effieicieny of private capital and reduces its depreciation. The growth-maximizing tax rate and maintenance spending are derived in this baseline setting, and the first-best welfare maximizing solution is also considered. The model is then extended to analyze the congestion effects of private usage on the efficiency of public and private capital, as well as the impact of maintenance spending tax refunds received by the private sector on the optimal tax rate.

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1 Introduction

The effect of public infrastructure on economic growth has received much attention both in academic research and policy debates recently. Following early papers by Aschauer (1989) and Barro (1990), several studies have analyzed the link between public investment in infrastructure capital and economic growth both empirically and theoretically. Among others, Calderón and Servén (2004), Loayza, Fajnzylber, and Caldéron (2004) and Bose, Haque, and Osborn (2007) have all confirmed a positive relationship between public infrastructure and economic growth empirically. At the theoretical level, Rioja (1999), Turnovsky (2000), Eicher and Turnovsky (2000), Agénor (2005) and Agénor (2008a) have provided insights into the determination of growth and welfare-maximizing share of public investment in infrastructure, establishing a key role for public capital in spurring growth.

An equally important consideration for policy-makers is undoubtedly the preservation and effective provision of infrastructure services through maintenance. As McGrattan and Schmitz (1999) argues, maintenance activity can be considered both as a complement and a substitute for new investment. Often, policymakers face a choice between building new infrastructure projects or increasing the durability of the existing stock of public capital through maintenance spending. As documented by Agénor (2008c), smooth functioning of infrastructure services such as power and water supplies or phone lines through appropriate maintenance may be vital for the efficiency of stock of public capital and may have significant growth effects. However, in general, flashy new projects are favoured over the enhancement of the services provided by the existing stock of public capital. In an empirical study on developing countries for example, Devarajan et al. (1996) find that the governments have spent too much on new projects at the expense of maintaining the existing capital and increasing the level of maintenance expenditures could be more productive than investing in new projects.

Despite its importance, the effect of public maintenance spending has not received much attention in the endogenous growth literature. Among few studies, Rioja (2003a) presents a model where increases in public maintenance spending increases the efficiency of all inputs and depreciation rate of public capital is constant. On the other hand, Rioja (2003b) sets up a growth model where new investment is financed by aid from international donors whereas maintenance spending is financed through domestic tax revenues, and increases the durability of private capital by reducing its depreciation rate. The paper shows that optimal maintenance spending as a share of GDP depends on several parameters and presents calibration results for Latin American countries. Kalaitzidakis and Kalyvitis (2004) develops a growth model where the depreciation rate of public capital depends on public maintenance spending and its usage by the private sector, and derives the growth-maximizing tax rate and share of public maintenance spending. And finally, in a more recent study, Agénor (2008c) presents an endogenous growth model where public maintenance spending increases both the durability of public capital -by reducing the depreciation rate- and its efficiency, as well as the depreciation rate of private capital.

However, although public maintenance spending effects both the durability and efficiency of public capital and the depreciation rate of private capital in Agénor (2008c), the impact of public maintenance spending on the efficiency of private capital is not analyzed. As argued above, a well-functioning network of public infrastructure is vital for the efficient use of private capital as well as public capital itself. Neglecting this relationship may result in an underestimation of potential benefits of public maintenance spending and hamper growth by reducing the productivity of private capital.

Further, maintenance spending by the private sector has not been paid attention in any of these models and the depreciation rate of private capital is assumed to be either constant, or dependent on public maintenance spending as in Agénor (2008c). However, confirming earlier results by Griliches(1970) and Cagan (1971), Nelson et al (1997) found that the assumption that private capital depreciates at a constant geometric rate is not realistic and depreciation rates must be endogenously determined. As Licandro and Luch (2000) show, incorporating the expenditures on maintenance and repair might change some of the key results regarding aggregate economic activity in growth models. Although several authors such as Feldstein and Rothschild (1974), Su (1975) and Parks (1979) have investigated the link between private maintenance spending and maintenance-dependent depreciation rates in asset-pricing models, there has been no systematic study of this relation in endogenous growth literature. Among the very few studies addressing this issue at the macro level are Licandro et al (2001), which employs a simple neoclassical growth model with private maintenance spending effecting the depreciation rate of private capital and Guo et al (2006), which presents a two-period one-sector real business cycle model with maintenance-dependent depreciation rate of private capital and capacity utilization rate under increasing returns to scale. While none of these models incorporate productive government spending as a flow or public capital as a stock in the production of goods, they also neglect the fact that private maintenance spending will increase the efficient use of private capital and therefore might have significant growth effects.

This paper will therefore contribute to the existing literature on maintenance spending in several ways, by developing an endogenous growth framework where public maintenance expenditure not only effects the efficiency of public capital as in Hulden (1996) and Agénor (2008c) but also the efficiency of private capital in production, as well as reducing the depreciation rate of public capital as in Rioja (2003b), Kalaitzidakis and Kalyvitis (2004) and Agénor (2008c). On the other hand, we will explicitly introduce endogenous private maintenance spending by private firms, which will reduce the depreciation rate of private capital as in Licandro et al (2001) and also increase its efficiency. In order to be able to analyze the effect of public maintenance spending on infrastructure, we will assume that the flow of services derived from public capital is proportional to the existing level of capital stock as in Turnovsky (1997, 2000), Fisher and Turnovsky (1998), Dasgupta (1999), Chen (2006), Agénor and Yilmaz (2008) and Agénor (2008c).

The rest of the paper is organized as follows. Section 2 presents the outline of the model and Section 3 solves for the decentralized equilibrium. Section 4 analyzes the existence and stability properties of the model. While Section 5 analyzes the dynamics of the model when depreciation rate of private capital becomes more sensitive to private maintenance spending, Section 6 and 7 derive the growth-maximizing tax rate and share of public maintenance spending under different specifications of efficiency of public and private capital. Section 8 derives the first-best welfare-maximizing solution and Section 9 studies the case where private sector receives part of its maintenance expenditures as tax refunds. Section 10 compares the results with previous studies and the last part concludes.

2 Basic Framework

We consider an economy populated by an infinitely-lived representative household, which produces and consumes a single traded good. The good can be used for consumption or investment. The government invests in infrastructure and spends on maintenance. It provides infrastructure services free of charge to the representative household and balances its budget continuously, by levying a flat tax rate on output.

2.1 Production Structure

Output, Y, is produced with effective stock of private capital, $e_P K_P$, and the effective stock of public infrastructure capital, $e_G K_G$, using a Cobb-Douglas technology:[1]

$$Y = (e_G K_G)^{\alpha}(e_P K_P)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0,1)$, K_G is the physical stock of public capital, and $e_G > 0$ its efficiency, K_P is the broad measure of physical and human capital and $e_P > 0$ its efficiency. Thus, production exhibits constant returns to scale in both factors. As in Chatterjee and Turnovsky (2005), Chen (2006) and Agénor (2008c), K_P should be interpreted as a broad measure of (physical and human) capital, since the model does not explicitly account for labour. For simplicity, the flow of infrastructure services is assumed to be directly proportional to the effective stock of public capital, which is non-rival and non-excludable. Similarly, K_P denotes both the stock of private capital and the flow of services that it provides.

As in Agénor (2008c), we initially assume that efficiency of public capital is a concave function of the ratio of public spending on maintenance, M_G, to the stock of public capital:

$$e_G = (\frac{M_G}{K_G})^{\chi}, \tag{2}$$

where $\chi \in (0,1)$.[2] On the other hand, efficiency of private capital is a composite function of ratio of public maintenance spending to public capital and of private maintenance spending, M_P, to private capital stock:

$$e_P = (\frac{M_G}{K_G})^{\kappa}(\frac{M_P}{K_P})^{\mu}, \tag{3}$$

where $\kappa, \mu \in (0,1)$. As argued in the introduction, maintenance spending by public sector is essential for the efficient use of private capital, particularly when one considers for instance the importance of proper power supply and

[1] In what folllows, time subscripts are omitted for simplicity and a dot over a variable is used to denote its time derivative.

[2] In principle, to ensure that $e > 0$ when $M = 0$, a constant term should be added in (2). However, this would needlessly complicate the solution of the model, without any qualitative impact on the results.

communication services available to private firms. Similarly, maintenance spending on machinery by the private sector will also have a key role in determining the efficiency of private capital.

We assume that the depreciation rate of private capital is endogenous and depends linearly on the ratio of private maintenance spending over private capital stock.

$$\delta_P = 1 - d_P \ - \theta_P(\frac{M_P}{K_P}), \tag{4}$$

where $\theta_P, d_P \in (0,1)$.Thus, private maintenance expenditure enhances the durability of private capital. Here, $1 - d_P$ represents the autonomous rate of depreciation for private capital when private maintenance spending is zero.[3]

Private capital K_P evolves according to

$$\dot{K}_P = I_P - \delta_P K_P, \tag{5}$$

which, using (4), can be written as

$$\dot{K}_P = I_P + \theta_P(\frac{M_P}{K_P})K_P - 1, \tag{6}$$

where I_P denotes gross private investment.

2.2 Household

The infinitely-lived representative household-producer maximizes the discounted stream of future utility

$$\max_C U = \int_0^\infty \ln C \exp(-\rho t) dt, \tag{7}$$

where C is consumption and $\rho > 0$ the discount rate.[4]

The household's budget constraint is

$$C + I_P + M_P = (1 - \tau)Y, \tag{8}$$

[3] See below for the boundedness of the depreciation rate of private capital.

[4] The log-specification is adopted only for simplicity. A more general CRRA specification as in Agenor (2008b) or utility-enhancing public services as in the second chapter could also be used.

where $\tau \in (0, 1)$ is the tax rate on output. We assume that the total maintenance spending of the household is a fraction of total private capital stock:

$$M_P = \upsilon_P K_P, \tag{9}$$

where $\upsilon_P \in (0, 1)$. As in Licandro et al (2001), this specification captures the fact that private maintenance spending depends on the total capital stock rather than physical depreciation in every period. For example, tyres of cars need to replaced after a certain period although they are not completely worn out, or computers need to be updated regularly regardless of depreciation. The cost of maintenance spending in such cases will depend on the total capital stock rather than physical depreciation.

The household chooses its consumption path and the optimal level of private maintenance expenditure so as to maximize the present value of utility, taking as given the impact of public maintenance spending on the efficiency of private capital stock, the tax rate, and the effective stock of public capital as given. Using (1), (2), (3), (6), (8) and (9), the current-value Hamiltonian for problem (7) can be written as

$$H = \ln C + \lambda \left[\begin{array}{c} (1-\tau)(e_G K_G)^{\alpha} (\frac{M_G}{K_G})^{(1-\alpha)\kappa} \upsilon_P^{\mu(1-\alpha)} {K_P}^{1-\alpha} - \upsilon_P K_P \\ -C - (1 - d_P - \theta_P \upsilon_P) K_P \end{array} \right],$$

where λ is the co-state variable associated with constraint (8). From the first-order conditions $dH/dC = 0, dH/d\upsilon_P = 0$ and the co-state condition $-dH/dK_P = \rho\lambda - \dot{\lambda}$, optimality conditions for this problem are given by

$$1/C = \lambda, \tag{10}$$

$$(1-\tau)\mu(1-\alpha)\frac{Y}{\upsilon_P} = (1-\theta_P)K_P, \tag{11}$$

$$\dot{\lambda} = \lambda[\rho + (1 - d_P - \theta_P \upsilon_P) + \upsilon_P - s\frac{Y}{K_P}], \tag{12}$$

together with the budget constraint (8) and the transversality condition

$$\lim_{t\to\infty} \lambda K_P \exp(-\rho t) = 0. \tag{13}$$

Rearranging (11) gives the optimal share of private maintenance spending as

$$v_P^* = \frac{(1-\tau)\mu}{1-\theta_P}(1-\alpha)(\frac{Y}{K_P}), \tag{14}$$

Equation (14) shows that v_P^* varies over time as the output-capital ratio changes. In the steady state, output and private capital grow at the same rate so $\frac{Y}{K_P}$ is constant, implying that v_P^* is constant as well. Assuming that θ_P is not too large, and output-private capital ratio is less than one in the steady-state ensures that v_P^* remains bounded between zero and one.[5] It is important to note here that an increase in the tax rate reduces v_P^* as a fraction of marginal return to private capital, $(1-\alpha)(\frac{Y}{K_P})$.

From (4) and (14),

$$\delta_P = 1 - d_P \ - \theta_P \frac{(1-\tau)\mu(1-\alpha)}{[1-\theta_P]}(\frac{Y}{K_P}), \tag{15}$$

The assumption that d_P and θ_P are not too large are sufficient to ensure that the depreciation rate of private capital remains positive in the steady-state, as long as the steady-state value of output-private capital ratio is less than one, as stressed above.

Totally differentiating equation (10) and using (12), we can obtain the growth rate of consumption as

$$\frac{\dot{C}}{C} = (1-\tau)(1-\alpha)\frac{Y}{K_P} - [1-\theta_P]\, v_P - 1 + d_P - \rho, \tag{16}$$

which, using (14), can be rewritten as

$$\frac{\dot{C}}{C} = s_1(\frac{Y}{K_P}) - 1 + d_P - \rho. \tag{17}$$

where $s_1 \equiv (1-\alpha)(1-\tau)(1-\mu) \in (0,1)$.

[5] From (1), we have $\frac{Y}{K_P} = k_G^{\alpha}(\frac{e_G}{e_P})^{\alpha} e_P$. From (3), $\tilde{e}_P < 1$ when $\frac{M_G}{K_G} < 1$ and $\frac{M_P}{K_P} < 1$. As long as this condition holds, the restriction on the steady-state value of output-private capital ratio requires that private sector accumulates more capital than public sector and utilizes it with more efficiency, which is likely to hold in practice.

2.3 Government and Public Capital

We assume that the government invests in infrastructure capital, I_G, and spends on maintenance. In order to finance these expenditures, it collects a proportional tax on output. Thus, the government budget constraint is given by

$$I_G + M_G = \tau Y. \tag{18}$$

Investment in infrastructure and spending on maintenance are both constant fractions of tax revenue, υ_G and υ_M:

$$I_G = \upsilon_G \tau Y, \qquad M_G = \upsilon_M \tau Y, \tag{19}$$

with $\upsilon_G, \upsilon_M \in (0,1)$. The government budget constraint can thus be rewritten as

$$\upsilon_G + \upsilon_M = 1. \tag{20}$$

Using (19), the stock of public capital in infrastructure evolves over time according to

$$\dot{K}_G = I_G - \delta_G K_G = \upsilon_G \tau Y - \delta_G K_G, \tag{21}$$

where δ_G is the rate of depreciation of public capital in infrastructure, which is taken to depend negatively and linearly on the ratio of maintenance expenditure, M, to the public stock of capital, K_G:

$$\delta_G = 1 - d_G - \theta_G(\frac{M_G}{K_G}), \tag{22}$$

where $d_G, \theta_G \in (0,1)$.[6] Thus, public maintenance expenditure enhances the durability of public infrastructure capital. This specification is similar to Agénor (2008c) but it differs from the formulation there as $d_G \in (0,1)$ implies that public capital does not depreciate fully every period even if the government does not spend on maintenance at all. It is also different from Rioja (2003*b*) and Kalaitzidakis and Kalyvitis (2004), which assume that depreciation rate of public capital depends on maintenance spending over output rather than public capital. As argued by Agénor (2008c), the specification used here is both analytically convenient and captures the fact that maintenance spending would depend up to a degree to the existing stock of

[6] This restriction on θ_G is sufficient to ensure that $\delta_G \in (0,1)$, as long as d_G is not large and $M_G/K_G < 1$.

public capital regardless of usage.

3 The Balanced Growth Path

The balanced growth path (BGP) can be determined as follows. First, using (1), (2), (3) and (9), production of output can be written as

$$Y = (\frac{M_G}{K_G})^{\alpha\chi+\kappa(1-\alpha)} \upsilon_P{}^{\mu(1-\alpha)} K_G{}^{\alpha} K_P{}^{1-\alpha}. \tag{23}$$

From (14), (19), and using $Y/K_G = \dfrac{Y}{K_P}\dfrac{K_P}{K_G}$,

$$Y = (\tau\upsilon_M)^{\Gamma}(\frac{Y}{K_P})^{\alpha\chi+(\kappa+\mu)(1-\alpha)} A(1-\tau)^{\mu(1-\alpha)}(\frac{K_P}{K_G})^{\Gamma} K_G{}^{\alpha} K_P^{1-\alpha}, \tag{24}$$

where A= $(\frac{\mu(1-\alpha)}{1-\theta_P})^{\mu(1-\alpha)/\Sigma}$, and $\Gamma = \alpha\chi + \kappa(1-\alpha)$. Dividing by K_P and rearranging yields

$$\frac{Y}{K_P} = A(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma} k_G{}^{\eta/\Sigma}, \tag{25}$$

where $\eta \equiv \alpha(1-\chi) - \kappa(1-\alpha)$, $\Sigma = 1 - \alpha\chi - (\kappa+\mu)(1-\alpha)$, and $k_G = K_G/K_P$.

Assuming that $\kappa + \mu \leq 1$ so that there are no increasing returns to the efficiency of private capital and $\chi < 1$ ensures that $\Sigma > 0$. However, the sign of η depends on the values of χ and κ even when $\kappa, \chi \in (0,1)$.In essence, $\alpha > \frac{\kappa}{1-\chi+\kappa}$ must hold for $\eta > 0$. It can easily be verified that a higher κ and χ require a higher α for η to be positive.

The household budget constraint (equation (6)) can be rewritten as, using (8) and (9),

$$\dot{K}_P = (1-\tau)Y - M_P - C - (1 - d_P - \theta_P v_P)K_P. \tag{26}$$

Dividing (26) by K_P gives

$$\frac{\dot{K}_P}{K_P} = (1-\tau)\frac{Y}{K_P} - v_P + \theta_P v_P - 1 + d_P - c,$$

where $c = C/K_P$. Substituting (14) in this expression yields

$$\frac{\dot{K}_P}{K_P} = (1-\tau)\left[1-\mu(1-\alpha)\right]\frac{Y}{K_P} - 1 + d_P - c,$$

which, using (25), becomes

$$\frac{\dot{K}_P}{K_P} = As_2(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Sigma}{k_G}^{\eta/\Sigma} - 1 + d_P - c, \tag{27}$$

where $s_2 = (1-\tau)\left[1-\mu(1-\alpha)\right] \in (0,1)$.

Similarly, using (25), equation (17) can be rewritten as

$$\frac{\dot{C}}{C} = As_1(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma} - \rho - 1 + d_P. \tag{28}$$

From (21), and (22), and noting that $M/K_G = \tau\upsilon_M Y/K_G$,

$$\frac{\dot{K}_G}{K_G} = \frac{\upsilon_G\tau Y}{K_G} - \delta_G = \tau(\upsilon_G + \theta_G\upsilon_M)(\frac{Y}{K_G}) - 1 + d_G, \tag{29}$$

that is, given again that $Y/K_G = (Y/K_P)k_G^{-1}$ and using (25),

$$\frac{\dot{K}_G}{K_G} = A\tau(\upsilon_G + \theta_G\upsilon_M)(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma-1} - 1 + d_G. \tag{30}$$

Combining equations (27), (28), and (29) yields

$$\frac{\dot{c}}{c} = -\alpha A(1-\tau)^{[1-\alpha\chi-\kappa(1-\alpha)]/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma} + c - \rho, \tag{31}$$

$$\frac{\dot{k}_G}{k_G} = \left\{\tau\upsilon {k_G}^{-1} - s_2\right\} A(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma} + c + (d_G - d_P). \tag{32}$$

where $\upsilon = (\upsilon_G + \theta_G\upsilon_M)$. These two nonlinear differential equations in c and k_G, together with the initial condition $k_{G,0} = K_{G0}/K_{P0} > 0$, and the transversality condition (13), rewritten as

$$\lim_{t\to\infty} c^{-1}\exp(-\rho t) = 0, \tag{33}$$

characterize the dynamics of the economy. The BGP is a set of functions $\{c, k_G\}_{t=0}^{\infty}$ such that equations (31) and (32), the budget constraint (8), and the transversality condition (13), are satisfied, and consumption and the stocks of public and private capital, all grow at the same constant rate γ. This is also the rate of growth of output, given the assumption of constant returns to scale. Because consumption and the stock of private capital grow at the same constant rate, the ratio $c = C/K_P$ is also constant in the steady state; the transversality condition (13) is thus always satisfied along any interior BGP equilibrium.

From (28) and (30), the steady-state growth rate γ is given by the equivalent forms

$$\gamma = As_1(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma} - \rho - 1 + d_P, \tag{34}$$

$$\gamma = A\tau(\upsilon_G + \theta_G\upsilon_M)(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma-1} - 1 + d_G, \tag{35}$$

where $\tilde{k}_G$ denotes the stationary value of k_G.[7]

4 Stability and Uniqueness

To show that the BGP is unique, note first Setting $\dot{c} = 0$ in (31) and $\dot{k}_G = 0$ (32) yields the implicit function

$$F(\tilde{k}_G) = As_1(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma}$$

$$-A\tau\upsilon(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-1} + (d_P - d_G) - \rho = 0 \tag{36}$$

where $\upsilon = (\upsilon_G + \theta_G\upsilon_M)$. Remember that $\eta \equiv \alpha(1-\chi) - \kappa(1-\alpha)$ and $\Sigma = 1 - \alpha\chi - (\kappa+\mu)(1-\alpha)$. Let us first assume $\eta > 0$. From (36), $\lim_{\tilde{k}_G\to 0}F(\tilde{k}_G) = -\infty$ and $\lim_{\tilde{k}_G\to\infty}F(\tilde{k}_G) = +\infty$. Further,

$$F_{\tilde{k}_G} = (\eta/\Sigma)As_1(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-1}$$

$$-(\eta/\Sigma - 1)A\tau\upsilon(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-2} > 0. \tag{37}$$

[7] From (38), there is a third equivalent form, $\gamma = As_2(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Sigma}{k_G}^{\eta/\Sigma} - 1 + d_P - c$. However, given equation (38) below, this expression is identical to (28).

Thus, $F(\tilde{k}_G)$ is a monotonically increasing function of $\tilde{k}_G$ and there is a unique equilibrium value of public-private capital ratio that satisfies (36). Setting $\frac{\dot{c}}{c} = 0$ in (31) gives

$$\tilde{c} = \alpha A(1-\tau)^{[1-\alpha\chi-\kappa(1-\alpha)]/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma} + \rho. \tag{38}$$

Therefore, there is also a unique positive value of $\tilde{c}$ that corresponds to $\tilde{k}_G$ and the equilibrium is unique. To investigate the dynamics in the vicinity of the steady state, this system can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{k}_G \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c - \tilde{c} \\ k_G - \tilde{k}_G \end{bmatrix}, \tag{39}$$

with the a_{ij} given by $a_{11} = \tilde{c}$, $a_{21} = \tilde{k}_G$, and

$$a_{12} = -\tilde{c}(\eta/\Sigma)\alpha A(1-\tau)^{[1-\alpha\chi-\kappa(1-\alpha)]/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-1} < 0,$$

which using (27) can be written as

$$a_{12} = -(\eta/\Sigma)(\frac{\alpha}{[1-\mu(1-\alpha)]})\frac{(\gamma+\tilde{c}+1-d_P)\tilde{c}}{\tilde{k}_G} < 0. \tag{40}$$

Similarly,

$$\begin{aligned} a_{22} &= (\eta/\Sigma - 1)A\tau\upsilon(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-1} \\ &\quad -(\eta/\Sigma)A(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma} < 0, \end{aligned}$$

which again using (27) and (30) can be arranged as

$$a_{22} = (\eta/\Sigma - 1)(\gamma + 1 - d_G) - (\eta/\Sigma)(\gamma + \tilde{c} + 1 - d_P) < 0. \tag{41}$$

Here, $\tilde{x}$ denotes the stationary value of x. In the above system, c is a jump variable, whereas k_G is predetermined. Saddlepath stability requires one unstable (positive) root. To ensure that this condition holds, the determinant of the Jacobian matrix of partial derivatives of the dynamic system (39) must be negative, that is, $a_{11}a_{22} - a_{12}a_{21} < 0$, or equivalently, $-a_{12}/a_{11} < -a_{22}/a_{21}$. Let us examine if this condition holds. From (40) and (41) and using $a_{11} = \tilde{c}$, $a_{21} = \tilde{k}_G$

$$a_{11}a_{22} = (\eta/\Sigma - 1)(\gamma + 1 - d_G)\tilde{c} - (\eta/\Sigma)(\gamma + \tilde{c} + 1 - d_P)\tilde{c} < 0. \qquad (42)$$

$$a_{12}a_{21} = -(\eta/\Sigma)(\frac{\alpha}{[1 - \mu(1 - \alpha)]})(\gamma + \tilde{c} + 1 - d_P)\tilde{c} < 0. \qquad (43)$$

The determinant of the Jacobian matrix, A, is therefore given by

$$DET(A) = (\eta/\Sigma - 1)(\gamma + 1 - d_G)\tilde{c} - (\eta/\Sigma)(\gamma + \tilde{c} + 1 - d_P)\tilde{c}\left[1 - \frac{\alpha}{1-\mu(1-\alpha)}\right] < 0. \qquad (44)$$

Equation (44) implies that the determinant of the system will be negative as long as $\alpha < 1 - \mu(1 - \alpha)$. This condition is always satisfied as long as $\mu < 1$ and therefore the system is always stable.

The phase diagram in Figure 1 is similar to Agénor (2008c) and illustrates the adjustment process to the steady state in this case. The phase curve CC represents the combinations of c and k_G for which the consumption-private capital stock ratio is constant ($\dot{c} = 0$), whereas the phase curve KK represents the combinations of c and k_G for which the public-private capital stock ratio is constant ($\dot{k}_G = 0$). Both curves are strictly increasing and strictly concave, but saddlepath stability requires that the slope of curve KK be steeper than the slope of curve CC . The saddlepath is denoted SS and the initial equilibrium is obtained at point A.

On the other hand, if $\eta < 0$, the existence properties of the system are altered. From (36), it can be inferred that in this case $\lim_{\tilde{k}_G \to 0} F(\tilde{k}_G) = -\infty$ and $\lim_{\tilde{k}_G \to \infty} F(\tilde{k}_G) = d_P - d_G - \rho$.From (37), the sign of $F_{\tilde{k}_G}$ is now ambiguous and $F_{\tilde{k}_G} > 0$ requires $0 < \tilde{k}_G < \dfrac{\tau\upsilon(\eta - \Sigma)}{s_1\eta}$. Therefore, $F(\tilde{k}_G)$ first increases as $\tilde{k}_G$ increases and then decreases monotonically to $d_P - d_G - \rho$ as $\tilde{k}_G \to \infty$. If $d_P - d_G - \rho > 0$, then $F(\tilde{k}_G)$ will cross the horizontal axis once from below and the equilibrium will be unique. Figure 2A shows the phase diagram in this case. If $d_P - d_G - \rho < 0$, $F_{\tilde{k}_G}$ may not cross the horizontal axis, it may be tangent to the horizontal axis or it may cross the horizontal axis twice before it converges to $d_P - d_G - \rho$. Therefore, there may be one, two or no equilibrium. Figures 2B-2C present the phase diagrams for multiple equilibrium and no equilibrium [8]. If there are two equilibria, the second one

[8] As argued above, $F(k_G)$ may be tangent to the horizontal axis and there may only be

has a higher steady-state value of public-private capital ratio and therefore has lower growth than the first one.

Let us first consider the case when $d_P - d_G - \rho < 0$ and there are multiple equilibria. As (44) implies with $\eta < 0$, $DET(A) < 0$ no longer holds unambiguously. Consider the first equilibrium initially. Since the slope of KK curve is positive here, we know that $-a_{22}/a_{21} > 0$ must hold. $a_{21} = \tilde{k}_G > 0$ implies $a_{22} < 0$. On the other hand, CC is monotonically decreasing so $-a_{12}/a_{11} < 0$ must also hold, which implies that $a_{12} > 0$ since $a_{11} = \tilde{c} > 0$. Therefore, $a_{11}a_{22} - a_{12}a_{21} < 0$ will always hold and the first equilibrium is stable. However, for the second equilibrium, KK is decreasing as well so $-a_{22}/a_{21} < 0$, implying that $a_{22} > 0$.Now, $a_{11}a_{22} - a_{12}a_{21} < 0$ requires $-a_{12}/a_{11} < -a_{22}/a_{21}$, which means that KK should be less steep than CC. However, as the phase diagram shows, KK is steeper than CC and therefore $-a_{12}/a_{11} > -a_{22}/a_{21}$, meaning that $DET(A) > 0$. So the sign of the eigenvalues of the system will now depend on the trace of the Jacobian matrix. From $a_{11} = \tilde{c}$ and (41), the trace of this matrix can be written as

$$TR(A) = \tilde{c} + (\eta/\Sigma - 1)(\gamma + 1 - d_G) - (\eta/\Sigma)(\gamma + \tilde{c} + 1 - d_P), \tag{45}$$

which after some manipulations can be rearranged as

$$TR(A) = c + (\eta/\Sigma)(d_P - d_G - \tilde{c}). \tag{46}$$

Remember that multiple equilibrium requires $d_P - d_G < \rho$, and from (38) $\tilde{c} = \alpha A(1-\tau)^{[1-\alpha\chi-\kappa(1-\alpha)]/\Sigma}(\tau\upsilon_M)^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma} + \rho$.Thus, $d_P - d_G - \tilde{c} < 0$ and with $\eta < 0$ as well, the trace is always positive. In such a case, $DET(A) > 0$ and $TR(A) > 0$, and the Jacobian matrix has two positive eigenvalues. Therefore, the second equilibrium is a sink and is not stable.

Analogous to this result, if $\eta < 0$ and $d_P - d_G - \rho > 0$ holds, there is a unique value of $\tilde{k}_G$ satisfying $F(\tilde{k}_G) = 0$ as shown before and the equilibrium occurs in the region where KK is upward sloping. As shown above, $DET(A) < 0$ in this case and the equilibrium is stable. Table 1 below summarizes these results.

one equilibrium in principle. In this case, KK will be tangent to CC at the steady-state value of k_G. However, we ignore this corner case.

Table 1

$\eta > 0$	$\eta < 0$			
	$d_P - d_G - \rho > 0$	$d_P - d_G - \rho < 0$		
Unique Eq	Unique Eq	No Eq	Unique Eq	Two Eq
Stable	Stable	-	Sink	Stable/Sink

5 An Increase in θ_P

Let us now analyze the impact of an increase in θ_P,the sensitivity of depreciation of private capital to private maintenance spending, on the steady state equilibrium under the assumption that $\eta > 0$. From (31) and (32) and using the fact that A= $(\frac{\mu(1-\alpha)}{1-\theta_P})^{\mu(1-\alpha)/\Sigma}$, it is easy to verify that both CC and KK curves shift left. Therefore, the net effect on the steady state value of c and $\tilde{k}_G$ are ambiguous in general. However, it can be easily verified that the magnitude of the shift in the curves will depend on the wedge between $d_P - d_G$ and ρ. If $d_P - d_G - \rho > 0$ holds, then KK will shift less than CC, and therefore both $\tilde{c}$ and $\tilde{k}_G$ will increase (Figure 3A). On the other hand, if $d_P - d_G - \rho < 0$, KK will shift more than CC and thus the equilibrium value of $\tilde{k}_G$ will fall while the net effect on $\tilde{c}$ will be ambiguous in general (Figure 3B). Intuitively, if depreciation rate of private capital is more sensitive to private maintenance spending, firms increase the share of maintenance spending and v_P^* rises. While the increase in maintenance spending increases costs for private firms and has a negative effect on accumulation of private capital, a higher share of maintenance spending also reduces the depreciation rate of private capital and these two effects cancel each other out. As (23) shows, a higher private maintenance spending also increases the efficiency of private capital and therefore increases output. The growth rate of consumption, private capital and public capital all rise. If $d_P > d_G$, and therefore public capital depreciates more than private capital without any maintenance spending, the positive impact of higher output will be more for private capital accumulation than for public capital accumulation. On the other hand, as can be inferred from (26), a higher growth rate of consumption will reduce the accumulation of private capital. If the rate of time preference is low, the increase in consumption will be high and the negative effect of the increase in consumption on private capital accumulation will outweigh the difference between $d_P - d_G$. In such a case, public-private capital ratio and consumption-private capital ratio will both unambiguously increase, and from (34), the growth rate will be higher. If the rate of time preference is high or $d_P - d_G$ is low, the net effect of the increase in output on capital accumulation will be higher for private capital than public capital as opposed to above and $\tilde{k}_G$ will fall. From (34), the net effect on the growth rate, as well as $\tilde{c}$, will thus be ambiguous.

6 Growth-Maximizing Rules

Before studying the optimal allocation of spending, let us derive the growth-maximizing tax rate, taking the allocation of spending as given (that is, $d\upsilon_M = d\upsilon_G = 0$). From (28) and (30), setting $d\gamma/d\tau = 0$ and solving yields the following result:

Proposition 1. *The growth-maximizing tax rate is given by $\tau^* = \alpha$, as predicted by the Barro rule.*

Thus, even if public maintenance spending effects the efficiency of both private and public capital, and a higher tax rate reduces private maintenance spending, the growth-maximizing tax rate depends on the elasticity of effective public capital as predicted by Barro (1990) and also found by Agénor (2008c) in a similar model without private maintenance spending or efficiency of private capital.[9] Despite the fact that a higher tax rate reduces private maintenance spending as a fraction of marginal product of private capital, the increase in output through productive maintenance spending offsets this negative effect and Barro-rule remains optimal.

Let us now consider the case where the government sets optimally the share of revenue allocated to maintenance, for a given tax rate (that is, $d\tau = 0$). From the budget constraint (20), $d\upsilon_G = -d\upsilon_M$. Using this restriction, and setting $d\gamma/d\upsilon_M = 0$ in (28) and (30) yields the following result:

Proposition 2. *The growth-maximizing share of spending on maintenance is given by*

$$\upsilon_M^* = \frac{\alpha\chi + \kappa(1-\alpha)}{\alpha(1-\theta_G)}. \tag{47}$$

[9] It must be noted that Barro-rule is optimal in Agénor (2008) when depreciation of private capital is exogenous but becomes sub-optimal if it depends on public maintenance spending.

This solution is admissible as long as $\chi + \theta_G + \dfrac{\kappa(1-\alpha)}{\alpha} < 1$. Assuming that this condition holds, Proposition 2 shows that the growth-maximizing share of spending on maintenance is positively related (as could be expected) to χ, the elasticity of the efficiency function with respect to maintenance If $\kappa = 0$ and public maintenance spending does not effect the productivity of private capital, the optimal maintenance spending becomes $\frac{\chi}{(1-\theta_G)}$ as obtained by Agénor (2008c). However, in general, if $\kappa > 0$, it can be easily verified that growth maximizing share of maintenance spending is higher than this rate since υ_M^* in Proposition 2 can be written as $\frac{\chi}{(1-\theta_G)} + \frac{\kappa(1-\alpha)}{\alpha(1-\theta_G)}$. Intuitively, the term $\frac{\kappa(1-\alpha)}{\alpha(1-\theta_G)}$ reflects the positive effect of public maintenance spending on the efficiency of private capital, $\kappa(1-\alpha)$, discounted by the negative effect of an increase in public maintenance spending on public capital, α, since $d\upsilon_G = -d\upsilon_M$. At the same time, a higher response of the depreciation rate to spending on maintenance (that is, an increase in θ_G) tends to raise the share of spending on that category. Put differently, the more "effective" public spending on maintenance is in terms of raising the durability of public capital, the higher should be the share of tax revenues allocated to it.

On the other hand, differentiating υ_M^* with respect to α gives $\dfrac{d\upsilon_M^*}{d\alpha} = -\dfrac{\kappa}{\alpha^2(1-\theta_G)} < 0$ and thus optimal share maintenance spending falls as α increases. Intuitively, an increase in α reduces the positive effect of public maintenance spending on the efficiency of private capital while increasing the negative effect on the accumulation of public capital, and therefore υ_M^* falls. From $d\upsilon_G = -d\upsilon_M$, optimal share of spending on pubic investment υ_G^* increases as α rises. This is in line with earlier result by Agénor (2008a), but in contrast with Agénor (2008c), which finds that the optimal υ_M (and therefore υ_G) does not depend on the elasticity of output with respect to public capital.

7 Efficiency of Public and Private Capital

The foregoing analysis has shown that the introduction of private maintenance spending or efficiency of private capital does not alter the results regarding the growth-maximizing tax rate obtained by Agénor (2008c) in a similar model and the Barro-rule remains optimal with regards to optimal taxation. However, although the preceding model has assumed that efficiency of public and private capital depends on the ratio of public maintenance spending to the stock of public capital, one might argue that private usage also matters for efficiency. In order to capture this, I first assume that efficiency of public capital is a function of the ratio of public maintenance spending to output. Formally, (2) is now replaced by

$$e_G = (\frac{M_G}{Y})^{\chi}, \tag{48}$$

where $\chi \in (0,1)$. Solving as before, we get

$$\frac{\dot{C}}{C} = As(1-\tau)^{\mu(1-\alpha)/\Theta}(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Theta}k_G{}^{\eta/\Theta} - \rho - 1, \tag{49}$$

$$\frac{\dot{K}_G}{K_G} = A\tau(\upsilon_G + \theta_G\upsilon_M)(1-\tau)^{\mu(1-\alpha)/\Sigma}(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Theta}k_G{}^{\eta/\Theta-1} - 1, \tag{50}$$

$$\frac{\dot{K}_P}{K_P} = As_2(1-\tau)^{\mu(1-\alpha)/\Theta}(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Theta}k_G{}^{\eta/\Theta} - 1 - c, \tag{51}$$

where $\eta \equiv \alpha - \kappa(1-\alpha) > 0$ this time, and $\Theta = 1-(\kappa+\mu)(1-\alpha) > 0$ as long as $(\kappa+\mu) < 1$. These equations can be reduced to a system of differential equations in c and k_I as before. Direct inspection shows that $\eta > 0$ if $\kappa < 1$. Thus, the stability properties of the model are the same with those derived earlier for $\eta > 0$ and the system is stable. Let us calculate the growth maximizing tax rate and share of public maintenance spending in this case. Taking the allocation of spending as given (that is, $d\upsilon_M = d\upsilon_G = 0$), setting $d\gamma/d\tau = 0$ and solving as before yields the following result:

Proposition 3. *If efficiency of public capital depends on the ratio of public maintenance spending over output, the growth-maximizing tax rate is given by*

$$\tau^* = \frac{\alpha + \alpha\chi}{1+\alpha\chi} > \alpha, \tag{52}$$

and the growth-maximizing share of maintenance spending[10]

$$v_M^* = \frac{\alpha\chi + \kappa(1-\alpha)}{(\alpha + \alpha\chi)(1-\theta_G)}. \tag{53}$$

Proposition 3 states that as opposed to the benchmark case, the growth-maximizing tax rate is higher than the elasticity of output with respect to public capital and Barro-rule is sub-optimal. It can be shown that $\frac{d\tau^*}{d\chi} > 0$ and the optimal tax rate increases as the effect of public maintenance spending on the efficiency of public capital increases. This is because when efficiency of public capital depends on the ratio of public maintenance spending over output, public capital becomes more productive relative to private capital when compared with the benchmark economy and therefore the growth-maximizing tax rate is higher than the Barro-rule.

On the other hand, $\frac{dv_M^*}{d\alpha} = -\frac{\kappa(1+\chi)}{(\alpha+\alpha\chi)^2(1-\theta_G)} < 0$ and as before, optimal maintenance spending depends negatively on the elasticity of output with respect to public capital. The negative effect is stronger than the benchmark case, and comparing (47) and (53) shows that optimal share of maintenance spending is lower than the benchmark economy. With the efficiency of public capital being a function of $\frac{M_G}{Y}$, maintenance spending is now less productive and the growth maximizing share of maintenance is lower, implying that the growth-maximizing share of investment in public capital is higher.

While the foregoing discussion has assumed that the efficiency of public capital may be congested by private usage, the same argument can also be raised for the effect of public maintenance spending on the efficiency of private capital. Formally, efficiency of private capital may depend on the ratio of public maintenance spending to output and of private maintenance spending, M_P, to private capital stock as:

$$e_P = (\frac{M_G}{Y})^\kappa (\frac{M_P}{K_P})^\mu. \tag{54}$$

Assuming that $e_G = (\frac{M_G}{K_G})^\chi$ as in the benchmark equilibrium and solving

[10] In Proposition 3, Proposition 4 and Lemma 1, a new condition must be imposed to ensure that v_M is bounded. However, as shown below, v_M^* is lower than the benchmark value in every case so $\chi + \theta_G + \dfrac{\kappa(1-\alpha)}{\alpha} < 1$ is sufficient (though not necessary) to ensure the boundedness of v_M^*

for the optimal tax rate and share of maintenance spending as before gives Proposition 4:

Proposition 4 *If efficiency of private capital depends on the ratio of public maintenance spending over output, the growth-maximizing tax rate is given by*

$$\tau^* = \frac{\alpha + \kappa(1-\alpha)}{1 + \kappa(1-\alpha)} > \alpha, \tag{55}$$

and the growth-maximizing share of maintenance spending by

$$v_M^* = \frac{\alpha\chi + \kappa(1-\alpha)}{[\alpha + \kappa(1-\alpha)]\,(1-\theta_G)}. \tag{56}$$

Proposition 4 shows that the growth-maximizing tax rate is higher than the Barro-rule in this case as well. Analogous to the previous result, $\frac{d\tau^*}{d\kappa} > 0$, and the optimal tax rate increases as the effect of public maintenance spending on the efficiency of private capital, κ, increases. Intuitively, congestion reduces the productivity of private capital relative to public capital and increases the growth-maximizing tax rate. With respect to the optimal share of public maintenance spending, v_M^* depends positively on χ and κ and negatively on α as before, and it's lower than the benchmark growth-maximizing share. The intuition behind this result is the same as before. Congestion by private usage reduces the productivity of public maintenance spending, resulting in a fall in the optimal share.

Lemma 1 follows directly from Proposition 3 and Proposition 4:

Lemma 1 *If efficiency of private and public capital both depend on the ratio of public maintenance spending over output, the growth-maximizing tax rate is given by*

$$\tau^* = \frac{\alpha + \alpha\chi + \kappa(1-\alpha)}{1 + \alpha\chi + \kappa(1-\alpha)} > \alpha, \tag{57}$$

and the growth-maximizing share of maintenance spending by

$$v_M^* = \frac{\alpha\chi + \kappa(1-\alpha)}{[\alpha + \alpha\chi + \kappa(1-\alpha)]\,(1-\theta_G)}. \tag{58}$$

8 Welfare-Maximizing Rule

Let us now consider the first-best scenario, which involves a planner choosing optimally all quantities and policy instruments so as to maximize (subject to appropriate constraints) the household's discounted lifetime utility. Because the government budget constraint must hold at all times, one of the spending shares must again be determined residually. In what follows, I will assume that the government determines optimally υ_M, with υ_G determined through (20).

To begin with, note that from (8), (19) and (20) the economy's aggregate resource constraint is

$$Y = C + I_P + I_G + M_G + M_P.$$

This condition can be rewritten as, using (21) and (26), and noting that $M_G/K_G = \tau\upsilon_M Y/K_G$,

$$\dot{K}_P+\dot{K}_G = (1-\tau\upsilon_M)Y-C-\{(1-\theta_P)\upsilon_P+1-d_P\}K_P-\{1-d_G-\theta_G\tau\upsilon_M(\frac{Y}{K_G})\}K_G. \tag{59}$$

which can be rearranged to give

$$\dot{K}_P+\dot{K}_G = (1-(1-\theta_G)\tau\upsilon_M)Y-C-\{(1-\theta_P\upsilon_P)+1-d_P\}K_P-K_G. \tag{60}$$

The social planner's problem is thus to maximize (7), subject to (1) and (60), with respect to C, τ, υ_M, υ_P, K_P, K_G and Y. After substituting (2) and (3) in (1), the Hamiltonian can be written as

$$H = \ln C + \lambda\{(1-(1-\theta_G)\tau\upsilon_M)Y - C - \{(1-\theta_P\upsilon_P)+1-d_P\}K_P - K_G\}$$
$$+\varsigma\left\{(\tau\upsilon_M)^{[\alpha\chi+\kappa(1-\alpha)]/\Delta}{\upsilon_P}^{\mu(1-\alpha)/\Delta}{K_G}^{\eta/\Delta}K_P^{(1-\alpha)/\Delta} - Y\right\},$$

where $\Delta = 1-\alpha\chi-\ \kappa(1-\alpha)$, and λ denotes the co-state variable associated with constraint (60) and ς the Lagrange multiplier associated with the production equation (1). Optimality conditions for this problem are given by

$$\frac{\partial H}{\partial C} = 0 \Rightarrow 1/C = \lambda, \tag{61}$$

$$\frac{\partial H}{\partial \tau} = 0 \Rightarrow \lambda \left\{-(1-\theta_G)\tau \upsilon_M\right\} Y = -\varsigma \left\{\left[\alpha\chi + \kappa(1-\alpha)\right]/\Delta\right\}(Y/\tau), \tag{62}$$

$$\frac{\partial H}{\partial \upsilon_M} = 0 \Rightarrow \lambda \left\{-(1-\theta_G)\tau\right\} Y = -\varsigma \left\{\left[\alpha\chi + \kappa(1-\alpha)\right]/\Delta\right\}(Y/\upsilon_M), \tag{63}$$

$$\frac{\partial H}{\partial \upsilon_P} = 0 \Rightarrow \lambda \left\{-(1-\theta_P)\right\} K_P = -\varsigma \left[\mu(1-\alpha)/\Delta\right](Y/\upsilon_P), \tag{64}$$

$$\frac{\partial H}{\partial Y} = 0 \Rightarrow \lambda[(1-(1-\theta_G)\tau\upsilon_M] = \varsigma, \tag{65}$$

$$\frac{\partial H}{\partial K_P} = -\lambda\left[(1-\theta_P)\upsilon_P + 1 - d_P\right] + \varsigma\left[(1-\alpha)/\Delta\right](Y/K_P) = -\dot{\lambda} + \rho\lambda, \tag{66}$$

$$\frac{\partial H}{\partial K_G} = -\lambda(1-d_G) + \varsigma(\eta/\Delta)(Y/K_G) = -\dot{\lambda} + \rho\lambda, \tag{67}$$

together with (1), (60), and the transversality conditions

$$\lim_{t\to\infty} \lambda K_P \exp(-\rho t) = \lim_{t\to\infty} \lambda K_G \exp(-\rho t) = 0. \tag{68}$$

From (64) and (65), the optimal share of private maintenance spending is determined by

$$\upsilon_P^{**} = \frac{\mu(1-\alpha)(1-(1-\theta_G)\tau\upsilon_M)}{\Delta(1-\theta_P)}(\frac{Y}{K_P}). \tag{69}$$

Further, from (63) and (65),

$$\frac{\tau(1-\theta_G)}{1-\tau\upsilon_M(1-\theta_G)} = \frac{\alpha\chi + \kappa(1-\alpha)}{\Delta\upsilon_M},$$

which can be rearranged to give[11]

$$\upsilon_M^{**}\tau^{**} = \frac{\alpha\chi + \kappa(1-\alpha)}{(1-\theta_G)}. \tag{70}$$

[11] The same result can be obtained from (62) and (64).

Substituting this result in (69) gives

$$\upsilon_P^{**} = \frac{\mu(1-\alpha)}{(1-\theta_P)}(\frac{Y}{K_P}). \tag{71}$$

From (65), we have

$$\frac{\varsigma}{\lambda} = [(1-(1-\theta_G)\tau\upsilon_M], \tag{72}$$

which, using (70) and the fact that $\Delta = 1-\alpha\chi-\ \kappa(1-\alpha)$ can be written as

$$\frac{\varsigma}{\lambda} = \Delta. \tag{73}$$

From (66) and (67), we can write

$$-\lambda\left[(1-\theta_P)\upsilon_P + 1 - d_P\right] + \varsigma\left[(1-\alpha)/\Delta\right](Y/K_P) = -\lambda(1-d_G) + \varsigma(\eta/\Delta)(Y/K_G). \tag{74}$$

Dividing by λ and using (71) and (73) gives

$$(1-\mu)(1-\alpha)(Y/K_P) = (d_G - d_P) + \eta(Y/K_G). \tag{75}$$

Let us first assume that $d_G = d_P$. Equation (75) shows that in this case,

$$\tilde{k}_G = \frac{\eta}{(1-\mu)(1-\alpha)}. \tag{76}$$

In general, however, if $d_G \neq d_P$, Equation (75) takes the form

$$(1-\mu)(1-\alpha)(Y/K_P) - \eta(Y/K_G) - (d_G - d_P) = 0. \tag{77}$$

Dividing (23) with K_P and using (71), we have

$$\frac{Y}{K_P} = A(\upsilon_M^{**}\tau^{**})^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma}. \tag{78}$$

Substituting this result and (70) into (77) and using the fact that $Y/K_G = \frac{Y}{K_P}\frac{K_P}{K_G}$ gives

$$G(\tilde{k}_G) = (1-\mu)(1-\alpha)A(\frac{\alpha\chi+\kappa(1-\alpha)}{(1-\theta_G)})^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma}$$

$$-\eta A(\frac{\alpha\chi+\kappa(1-\alpha)}{(1-\theta_G)})^{\Gamma/\Sigma}\tilde{k}_G{}^{\eta/\Sigma-1}-(d_G-d_P)=0. \qquad (79)$$

It can be easily verified that if $\eta>0$, $\lim_{\tilde{k}_G\to 0}G(\tilde{k}_G)=-\infty$, $\lim_{\tilde{k}_G\to\infty}G(\tilde{k}_G)=+\infty$, and $G_{\tilde{k}_G}>0$ so there is a unique value of k_G that satisfies equation (79) and therefore a unique welfare maximizing solution. If on the other hand, $\eta<0$ holds, $\lim_{\tilde{k}_G\to 0}G(\tilde{k}_G)=+\infty$ and $\lim_{\tilde{k}_G\to\infty}G(\tilde{k}_G)=d_P-d_G$ and therefore whether or not there will be a welfare-maximizing equilibrium will depend on the value of d_P-d_G. If $d_P-d_G<0$, there will be a unique welfare-maximizing solution whereas a positive value of d_P-d_G will imply that there is no equilibrium. This is because when $\eta<0$, the marginal product of private capital depends negatively to public-private capital ratio. If $d_P-d_G>0$ as well, autonomous depreciation of public capital is also higher than private capital, implying that there is no incentive for the social planner to accumulate public capital. Therefore, there is no welfare maximizing equilibrium.

As before, it is possible to reduce the solution to a system of differential equations in c and k_G. However, equations (76) and n (79) imply that the value of $\tilde{k}_G$ will be constant and the economy will always be on the balanced growth path. Equation (76) shows that if $d_G=d_P$ (and therefore the autonomous depreciation rates are same), the social planner always sets the public-private capital ratio equal to the ratio of their elasticities with respect to output, taking maintenance spending by both sectors as given. In order to see this, remember that $\eta=\alpha-\alpha\chi-\kappa(1-\alpha)$. The production function in (1) can be written as

$$Y=(\frac{M_G}{K_G})^{\alpha\chi+\kappa(1-\alpha)}(\frac{M_P}{K_P})^{\mu(1-\alpha)}K_G{}^{\alpha}K_P{}^{1-\alpha}, \qquad (80)$$

which can be rearranged as

$$Y=M_G^{\alpha\chi+\kappa(1-\alpha)}M_P{}^{\mu(1-\alpha)}K_G{}^{\eta}K_P{}^{(1-\mu)(1-\alpha)}. \qquad (81)$$

If on the other hand, $d_G\neq d_P$, there may or may not be a welfare maximizing equilibrium as shown above. Let us assume that $\eta>0$, so that there is always a unique welfare maximizing solution, and let us take

$d_P - d_G > 0$. From (73), this condition implies that

$$\tilde{k}_G < \frac{\eta}{(1-\mu)(1-\alpha)}. \tag{82}$$

Therefore, if the autonomous depreciation of public capital is higher than the autonomous depreciation of private capital, the social planner will take this into account when setting the optimal value of k_G, and the steady state value of public-private capital ratio will be lower than the ratio of their elasticities with respect to output in this case.

Moreover, from (70), the planner can set one of the two policy instruments, τ or υ_M, arbitrarily, as it is the total share of resources allocated to maintenance rather than its composition that matters to the planner. The planner could achieve the same level of total spending on maintenance by a low (high) tax rate and a high (low) share of maintenance spending in total tax revenues. Comparing (70) with Proposition 1 and Proposition 2 shows that if the tax rate is set at the growth-maximizing rate, α here, the welfare maximizing share of maintenance spending is equal to the growth maximizing rate, $\frac{\alpha\chi+\kappa(1-\alpha)}{\alpha(1-\theta_G)}$ here, as found by Agénor (2008c).[12]

9 Maintenance Spending Tax Refunds

The previous discussion has mainly focused on the growth and welfare maximizing tax rate and share of maintenance spending when private maintenance spending is not subject to taxation or associated with refunds. However, in some cases, private firms receive part of their maintenance spending back as tax refunds from the government. Therefore, we know assume that a fraction ψ of total private maintenance expenditure is deducted from the total income tax paid by the household. Formally, the household budget constraint now takes the form

$$C + I_P + (1-\psi\tau)M_P = (1-\tau)Y, \tag{83}$$

Let us assume, without loss of generality, that $d_P = d_G = 0$. Using (6), the household budget constraint now becomes

[12] Results regarding different specifications of the efficiency of public and private capital can be directly obtained from Proposition 3, Proposition 4 and Lemma 1. Remember that $\eta > 0$ holds in all other specifications so there is a unique welfare-maximizing equilibrium.

$$\dot{K}_P = (1-\tau)Y - (1-\psi\tau)v_P K_P - C - (1-\theta_P v_P)K_P \tag{84}$$

As before, the household maximizes the discounted stream of utility given by (7) subject to the budget constraint (84). Using (23), the current value Hamiltonian for this problem becomes

$$H = \ln C + \lambda \left[\begin{array}{c} (1-\tau)(e_G K_G)^{\alpha}(\frac{M_G}{K_G})^{(1-\alpha)\kappa} v_P^{\mu(1-\alpha)} {K_P}^{1-\alpha} \\ -(1-\psi\tau)v_P K_P - C - (1-\theta_P v_P)K_P \end{array} \right],$$

where λ is the co-state variable associated with constraint (84). From the first-order conditions $dH/dC = 0, dH/dv_P = 0$ and the co-state condition $-dH/dK_P = \rho\lambda - \dot{\lambda}$, optimality conditions for this problem are given by

$$1/C = \lambda, \tag{85}$$

$$(1-\tau)\mu(1-\alpha)\frac{Y}{v_P} = (1-\psi\tau-\theta_P)K_P, \tag{86}$$

$$\dot{\lambda} = \lambda[\rho + (1-\theta_P v_P) + (1-\psi\tau)v_P - s_1\frac{Y}{K_P}], \tag{87}$$

together with the budget constraint (83) and as before the transversality condition

$$\lim_{t\to\infty} \lambda K_P \exp(-\rho t) = 0. \tag{88}$$

Rearranging (86) gives the optimal share of private maintenance spending

as

$$v_P^* = s_3(\frac{Y}{K_P}), \tag{89}$$

where $s_3 = \frac{(1-\tau)\mu(1-\alpha)}{(1-\psi\tau-\theta_P)}$.[13] Totally differentiating equation (85) and using (87), we can obtain the growth rate of consumption as

$$\frac{\dot{C}}{C} = (1-\tau)(1-\alpha)\frac{Y}{K_P} - (1-\psi\tau-\theta_P)v_P - 1 - \rho, \tag{90}$$

which can be rearranged to give

[13] As before, a sufficient condition for $\mathrm{v}_P \in (0,1)$ is that $\mathrm{s}_3 < 1$ which requires ψ and θ_P are not too large, assuming that output-private capital ratio is less than one.

$$\frac{\dot{C}}{C} = s_1 \frac{Y}{K_P} - 1 - \rho. \tag{91}$$

The government budget constraint now takes the form

$$I_G + M_G = \tau(Y - \psi M_P). \tag{92}$$

We continue to assume that investment in infrastructure and spending on maintenance are both constant fractions of tax revenue, υ_G and υ_M:

$$I_G = \upsilon_G \tau (Y - \psi M_P), \qquad M_G = \upsilon_M \tau (Y - \psi M_P). \tag{93}$$

Using (9) and (89),

$$I_G = \upsilon_G \tau (1 - \psi s_3) Y, \qquad M_G = \upsilon_M \tau (1 - \psi s_3) Y. \tag{94}$$

.

Equation (21) can be combined with (94) to give

$$\frac{\dot{K}_G}{K_G} = \tau \upsilon (1 - \psi s_3)(\frac{Y}{K_P}) k_G^{-1} - 1. \tag{95}$$

Substituting (89) in (23), we have

$$Y = [(1 - \psi s_3)\tau \upsilon_M]^{\alpha\chi + \kappa(1-\alpha)} (\frac{Y}{K_P})^{\alpha\chi + (\kappa+\mu)(1-\alpha)} {s_3}^{\mu(1-\alpha)} (\frac{K_P}{K_G})^{\alpha\chi + \kappa(1-\alpha)} {K_G}^{\alpha} K_P^{1-\alpha} \tag{96}$$

which, dividing by K_P and using (94), can be rearranged as

$$\frac{Y}{K_P} = {s_3}^{\mu(1-\alpha)/\Sigma} (1 - \psi s_3)^{\Gamma/\Sigma} (\tau \upsilon_M)^{\Gamma/\Sigma} {k_G}^{\eta/\Sigma}. \tag{97}$$

Using (97), equations (91) and (95) can be written as

$$\frac{\dot{C}}{C} = s_1 {s_3}^{\mu(1-\alpha)/\Sigma} (1 - \psi s_3)^{\Gamma/\Sigma} (\tau \upsilon_M)^{\Gamma/\Sigma} {k_G}^{\eta/\Sigma} - 1 - \rho. \tag{98}$$

$$\frac{\dot{K}_G}{K_G} = \tau \upsilon (1 - \psi s_3)^{[1-\mu(1-\alpha)]/\Sigma} {s_3}^{\mu(1-\alpha)/\Sigma} (\tau \upsilon_M)^{\Gamma/\Sigma} {k_G}^{\eta/\Sigma - 1} - 1. \tag{99}$$

And finally, dividing (84) with Kp and substituting (97) gives

$$\frac{\dot{K}_P}{K_P} = s_2 {s_3}^{\mu(1-\alpha)/\Sigma}(1-\psi s_3)^{\Gamma/\Sigma}(\tau \upsilon_M)^{\Gamma/\Sigma}{k_G}^{\eta/\Sigma} - 1 - c. \tag{100}$$

As before, equations (98), (99) and (100) could be reduced to a system of differential equations in c and k_I and the stability of the steady state equilibrium could be investigated. However, it can be easily verified that as long as $(1-\psi s_3) > 0$, the stability properties of the system is identical to the benchmark equilibrium analyzed above. The condition $(1-\psi s_3) > 0$ is always satisfied as long as $s_3 < 1$ so we proceed to the determination of the growth-maximizing tax rate. From (98) and (99), setting $d\gamma/d\tau = 0$ and solving as before yields the following equation:

$$\tfrac{\tau}{1-\tau} = \tfrac{\alpha}{1-\alpha-\mu(1-\alpha)} + \varepsilon_{s_3|\tau}\tfrac{\mu(1-\alpha)}{1-\alpha-\mu(1-\alpha)}\left[1 - \tfrac{\alpha\psi(1-\tau)}{1-\psi\tau-\theta_P-\psi\mu(1-\alpha)(1-\tau)}\right], \tag{101}$$

where

$$\varepsilon_{s_3|\tau} = \frac{(\psi+\theta_P-1)\tau}{(1-\tau)(1-\psi\tau-\theta_P)} \tag{102}$$

It can be easily verified that setting $\psi = 0$ gives the optimal tax rate as $\tau = \alpha$ as in Proposition 1. However, in general if $\psi \neq 0$, determining the optimal tax rate requires the solution of a third-order equation defined by (101). When $\psi + \theta_P = 1$, $\varepsilon_{s_3|\tau} = 0$ and the second term in equation (101) disappears. In this case, the growth maximizing tax rate can be determined through a linear equation and Proposition 5 can be obtained.

Proposition 5 *If* $\psi + \theta_P = 1$ holds*, the growth-maximizing tax rate is given by*

$$\tau^* = \frac{\alpha}{1-\mu(1-\alpha)} > \alpha. \tag{103}$$

The condition $\psi + \theta_P = 1$ arises because of the linear specification of the depreciation of private capital in (4). In general, the condition could be written as $\psi + (-\frac{d\delta_P}{d\upsilon_P}) = 1$. In words, if the rate of tax refund and the marginal effect of share of private maintenance spending on the depreciation rate of private capital add up to one, the growth maximizing tax rate is larger than the elasticity of public capital in the production of output by the coefficient $\frac{1}{1-\mu(1-\alpha)}$.In such a case, s_3 will not depend on the tax rate, and optimal private maintenance spending will be a constant fraction of the marginal return to private capital. Compared to the benchmark case considered above, private capital will be less productive relative to public capital by the coefficient $\mu(1-\alpha)$ and therefore an increase in the impact of private maintenance spending on the efficiency of private capital, μ, will increase the growth-maximizing tax rate. Similarly, an increase in α will have a positive effect on the growth-maximizing tax rate as long as $\mu < 1$.

In general, though, if $\psi + \theta_P = 1$ does not hold, it is not possible to get an analytical solution to equation (101). Although we could use implicit function theorem to analyze the impact of a change in ψ, θ_P, μ and α on the growth-maximizing tax-rate in principal, the calculations do not yield any unambiguous results. Therefore, we resort to the numerical calibration of the elasticities α and μ, the sensitivity of depreciation rate of private capital to private maintenance spending, θ_P and the rate of tax refund, ψ. For the elasticity of output with respect to public capital, α, Calderón and Servén (2004), and Suescun (2005) estimate values very close to 0.15, which is also employed by Agénor (2008c), while Chatterjee and Turnovsky (2005) uses a value of 0.2. Therefore, we take α to range between 0.1 and 0.2. Similarly, we choose the effect of private maintenance spending on the efficiency of private capital to be between 0.1 and 0.4 and perform sensitivity analysis.

Table 2 presents the growth maximizing tax rates obtained by solving equation (101).[14] In Table 2A, we assume that $\psi$and θ_P are not high and set $\psi = 0.3$, $\theta_P = 0.3$. The results indicate that the growth maximizing tax rate is larger than α in all cases, and it depends positively on α and μ. Similarly, setting $\psi = 0.6$, $\theta_P = 0.6$ in Table 2B yields exactly the same results even if $\psi + \theta_P > 1$. On the other hand, Table 2C shows that if μ is low, the growth-maximizing tax rate will increase as the tax refund ψ, or θ_P increases as long as the sum of the two are not too high, and it will remain above α. However, if both μ and $\psi+\theta_P$ are very high, the growth-maximizing tax rate may be lower than the elasticity of output with respect to public capital, α

[14] Although (101) is a third-order equation, numerical solution of the equation yields only one root. In all the solutions, s_3 remains less than 1, ensuring that $v_P^* < 1$ as long as $Y/K_P < 1$. $v_P^* < 1$ also ensures a positive depreciation rate for private capital.

(Table 2D).

Table 2A
Growth-Maximizing Tax Rates ($\psi = 0.3$, $\theta_P = 0.3$)

	$\mu = 0.1$	$\mu = 0.2$	$\mu = 0.3$	$\mu = 0.4$
$\alpha = 0.1$	0.106	0.113	0.121	0.131
$\alpha = 0.15$	0.159	0.169	0.180	0.193
$\alpha = 0.2$	0.211	0.223	0.237	0.253

Table 2B
Growth-Maximizing Tax Rates ($\psi = 0.6$, $\theta_P = 0.6$)

	$\mu = 0.1$	$\mu = 0.2$	$\mu = 0.3$	$\mu = 0.4$
$\alpha = 0.1$	0.115	0.136	0.165	0.201
$\alpha = 0.15$	0.171	0.199	0.233	0.258
$\alpha = 0.2$	0.226	0.257	0.288	0.297

Table 2C
Growth-Maximizing Tax Rates ($\alpha = 0.15$, $\mu = 0.1$)

	$\theta_P = 0.3$	$\theta_P = 0.4$	$\theta_P = 0.5$	$\theta_P = 0.6$
$\psi = 0.25$	0.154	0.155	0.156	0.158
$\psi = 0.5$	0.159	0.161	0.163	0.167
$\psi = 0.75$	0.165	0.167	0.171	0.176
$\psi = 1$	0.170	0.173	0.176	0.172

Table 2D
Growth-Maximizing Tax Rates ($\alpha = 0.15$, $\mu = 0.4$)

	$\theta_P = 0.3$	$\theta_P = 0.4$	$\theta_P = 0.5$	$\theta_P = 0.6$
$\psi = 0.25$	0.170	0.174	0.180	0.190
$\psi = 0.5$	0.197	0.209	0.227	0.257
$\psi = 0.75$	0.235	0.257	0.258	0.168
$\psi = 1$	0.265	0.223	0.138	0.048

Intuitively, when $\psi + \theta_P < 1$, a higher tax rate reduces s_3, implying that optimal private maintenance spending as a fraction of marginal return to private capital falls. This leads to an increase in the fraction of income tax revenues available to the government, $(1-\psi s_3)$, meaning a twofold positive effect on public investment and maintenance spending from (94). The increase in productive public spending increases output and therefore output-private capital ratio, compensating for the negative effect of the fall in s_3 on the efficiency of private capital. The net effect on output is always positive, and the growth-maximizing tax rate is therefore higher than α. In this case, if θ_P is low, the effect of private maintenance spending on the depreciation rate of private capital is weak, and the tax refunds do not have a strong effect on the accumulation of private capital or increasing its efficiency. Since the efficiency of private capital also depends on output-private capital ratio, a higher μ implies that tax revenues get more productive relative to private capital and increases the growth-maximizing tax rate. In other words, the government has to tax more to compensate for the tax refunds in order to maximize the growth rate as μ increases.

When $\psi + \theta_P > 1$, a higher tax rate increases s_3 and reduces the fraction of income tax revenues available to the government, $(1 - \psi s_3)$.[15] So the net effect on total public investment and maintenance spending, and thus the total effect on output is ambiguous. Table 2C shows that even for some values of $\psi + \theta_P > 1$, the optimal tax rate will still increase as ψ and θ_P increases if μ is low. Intuitively, if $\psi + \theta_P$ are very high, the impact of tax refunds on the accumulation and efficiency of private capital is strong and an increase in the tax rate reduces $(1 - \psi s_3)$ significantly. This has a negative effect on output and therefore on the efficiency and depreciation rate of private capital, as they both depend on output-private capital ratio. If μ is high as well, the negative effect of the fall in output-private capital ratio on the efficiency of private capital is stronger, and therefore a higher tax rate might lower the growth rate overall. This is the case in Table 2D with $\theta_P = 0.5$ and $\psi = 1$ for instance, where the optimal tax rate is below α.

10 Comparison with other Results

The preceding discussion have first presented that the Barro (1990) rule, which states that growth-maximizing tax rate depends only on the elasticity of output with respect to public capital α, is optimal if both the efficiency of public and private capital depend on the ratio of the level of public mainte-

[15] Note that an increase in s_3 implies that optimal private maintenance spending increases as a fraction of marginal return to private capital when the tax rate increases. This is quite counter-intuitive, considering the distortionary effects of taxation but the condition $\psi + \theta_P > 1$ is very unlikely to hold for realistic values of ψ and θ_P. As argued by Agénor (2008), part of private capital such as human capital is insensitive to maintenance spending and θ_P cannot be very high.

nance spending over the stock of public capital. This is also in line with the result established by Glomm and Ravikumar (1994), Devarajan et al. (1998) in earlier models and by Agénor (2008c) in a similar model without private maintenance spending or the efficiency of private capital. On the other hand, it is in contrast with the results of Kalaitzidakis and Kalyvitis (2004), who find that growth-maximizing tax rate is higher than the Barro-rule due to the positive effect of maintenance spending on the accumulation of public capital, which raises the benefits of taxation. Further, the authors also find that optimal tax rate depends on the share of public maintenance spending while in the present setting the composition of public spending has no direct effect on the growth-maximizing tax rate. Second, the inclusion of private maintenance spending as a determinant of the efficiency and depreciation of private capital does not alter the results on the growth-maximizing tax rate per se. Although the direct effect of a higher tax rate is to reduce private maintenance spending by reducing the returns to private capital, productive usage of tax revenues increases output-private capital ratio indirectly by increasing public maintenance spending and investment in public capital (and therefore output), which offsets this negative effect of a higher tax rate and the Barro-rule remains optimal.

On the other hand, if we assume public capital is congested by private usage and it is the ratio of public maintenance spending over output that effects the efficiency of public and/or private capital, the Barro-rule becomes sub-optimal and growth maximizing tax rate is higher than α. The wedge between the elasticity of output with respect to infrastructure and the growth maximizing tax rate now depends positively on the elasticity of efficiency of public and/or private capital with respect to public maintenance. More precisely, if the efficiency of public capital depends on the ratio of public maintenance spending over output, then optimal tax rate depends positively on the elasticity of efficiency of public capital with respect to public maintenance spending, χ, as opposed to Agénor (2008c). If the efficiency of private capital depends on the ratio of public maintenance spending over output, the growth-maximizing tax rate depends positively on the elasticity of efficiency of private capital with respect to public maintenance spending, κ, this time. In both cases, public maintenance becomes more productive relative to private capital than the benchmark case considered and therefore the benefits from taxation are higher than the elasticity of output with respect to public capital, as in Kalaitzidakis and Kalyvitis (2004).

As argued above, the presence of private maintenance spending does not have any effect on the growth-maximizing tax rate in the benchmark economy or the aforementioned extensions of the model. However, if we assume that private firms receive part of their spending on maintenance expenditures as tax refunds from the government, the optimal tax rate then depends in a very complex way on the elasticity of efficiency of private capital with respect to private maintenance spending, μ, the elasticity of output with respect to public capital, α, and the sensitivity of the depreciation rate of private capital to private maintenance spending, θ_P. In particular, it is possible to show analytically that if the rate of tax refund, ψ, and the marginal effect of private maintenance spending on the depreciation rate of private capital, θ_P, add up to one, Barro-rule is sub-optimal and growth-maximizing tax rate is higher than α. If this condition does not hold, numerical solution of the growth-maximizing tax rate shows that Barro-rule is always sub-optimal if ψ and θ_P are not too large, and the optimal tax rate increases as α and μ increases. The growth-maximizing tax rate becomes less than α only if ψ, θ_P and μ are very large. In such a case, optimal private maintenance spending as a fraction of marginal return to private capital depends positively on the tax rate. Private capital and private maintenance spending becomes very productive compared to public capital and growth-maximizing tax rate calls for a lower rate than the elasticity of output with respect to infrastructure.

With regards to optimal composition of government spending, the present study finds that the growth-maximizing share of public maintenance spending found by Agénor (2008c) is sub-optimal if the efficiency of both public and private capital depend on the ratio of public maintenance spending over public capital. The optimal share now includes the effect of public maintenance spending on the efficiency of private capital, κ, as well as the efficiency parameter χ, and depends also on the elasticities of output with respect to private and public capital, in line with Rioja (2003*b*) but in contrast to Agénor (2008c). A higher elasticity of output with respect to public capital, α, implies that the effect of public maintenance spending on the efficiency of private capital is lower and the negative effect of maintenance spending on the accumulation of public capital and therefore growth of output is higher. As a result, the optimal share of maintenance spending falls and from $\upsilon_G^* = 1 - \upsilon_M^*$, the optimal share of infrastructure investment increases as α rises. This positive relation between α and optimal share of public investment is also found by Agénor (2008a) in a model where public capital also effects the supply of health services.

Extension of the benchmark model to assume that efficiency of public capital depends on the ratio of public maintenance spending to output shows that the optimal spending on maintenance is lower in this case. The same result is obtained if the efficiency of private capital depends on the ratio of public maintenance spending over output. In both cases, the effect of public maintenance spending on the efficiency of public or private capital (and therefore output) is smaller than the benchmark model so optimal spending on maintenance is lower.

11 Conclusion

This paper has investigated the optimal taxation and allocation of public and private maintenance expenditure in an endogenous growth setting. We have shown that even if public maintenance spending effects the efficiency of public and/or private capital, and private sector also spends on maintenance, the growth-maximizing tax rate will be equal to the Barro-rule if there is no congestion. However, if the effect of public maintenance spending on the efficiency of either type of capital is congested by private usage, Barro-rule is sub-optimal and growth-maximizing tax rate is higher than the elasticity of output with respect to public capital. Similarly, if private firms receive part of their maintenance expenditure as tax refunds and the depreciation rate of private capital is not very sensitive to private maintenance spending, optimal tax rate is higher than predicted by the Barro-rule. Ignoring the congestion effects or tax refunds may therefore result in inefficient taxation and inability to utilize the full potentials of productive government spending.

On the other hand, the presence of congestion has the opposite effect on the optimal share of public maintenance spending. If the effect of public maintenance spending on the efficiencies of public or private capital is congested by usage by private sector, the growth-maximizing share of public maintenance spending falls. As argued above, this is because the presence of congestion reduces the productivity of public maintenance spending. As implied by the government budget constraint, this increases the optimal share of investment in public capital. As opposed to optimal taxation, neglecting the negative effect of congestion may result in an overestimation of benefits of public maintenance spending at the expense of investment in new capital projects.

A natural extension of the present study is to assume that public and private maintenance spending effect the depreciation rate of private capital in a complementary way. In other words, public maintenance spending may increase the quality of the existing public capital stock, thus reducing the depreciation of private capital and the resources allocated to private maintenance spending. Better roads and energy supplies for instance might reduce the depreciation of the private capital that uses public infrastructure and might have an additional channel through which public maintenance spending creates growth. Accounting for this effect might increase the benefits of public maintenance spending, as well as altering the optimal taxation structure.

And finally, the results highlight the necessity for further empirical study on the determinants of depreciation rates and efficiencies of public and private capital. As summarized in the preceding section, growth and welfare-maximizing tax rate and share of maintenance spending depend crucially on the specifications adopted for the efficiency and depreciation of public and private capital, namely, whether maintenance spending should be measured in proportion of output or the stock of physical public capital. In theory, there are arguments to support both formulations. Therefore, identifying the robustness of these specifications through empirical estimation is vital for assessing the true benefits of taxation and optimal composition of government spending.

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Figure 1

$\eta > 0$

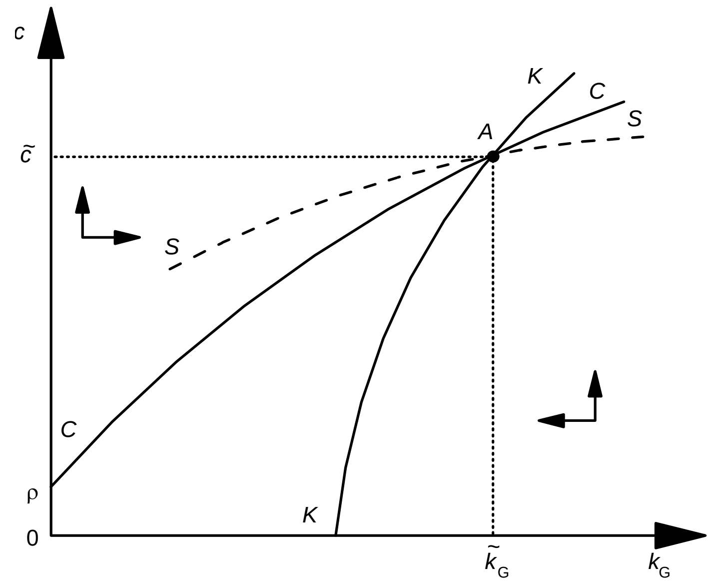

Figure 2A

$\eta < 0 \quad d_P - d_G > \rho$

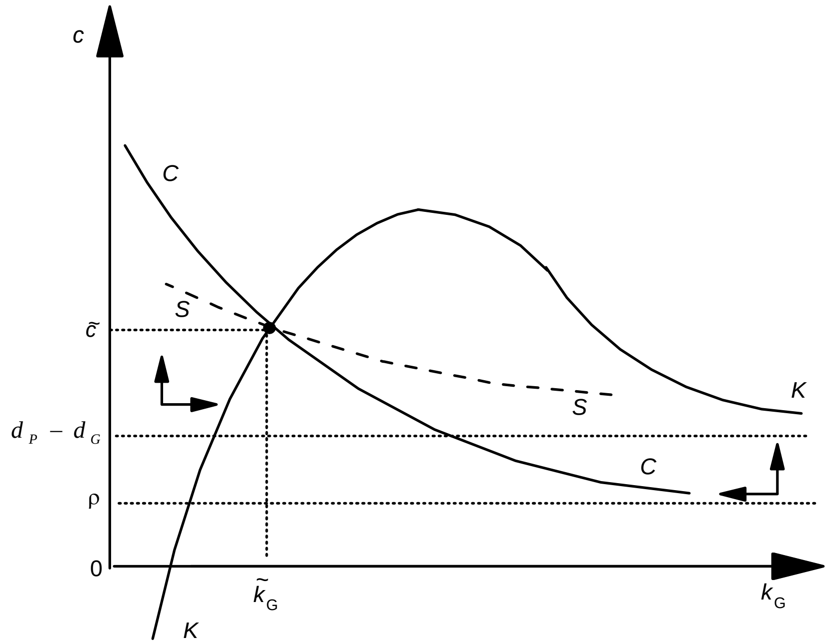

Figure 2B

(Multiple Equilibrium)

$\eta < 0 \quad d_P - d_G < \rho$

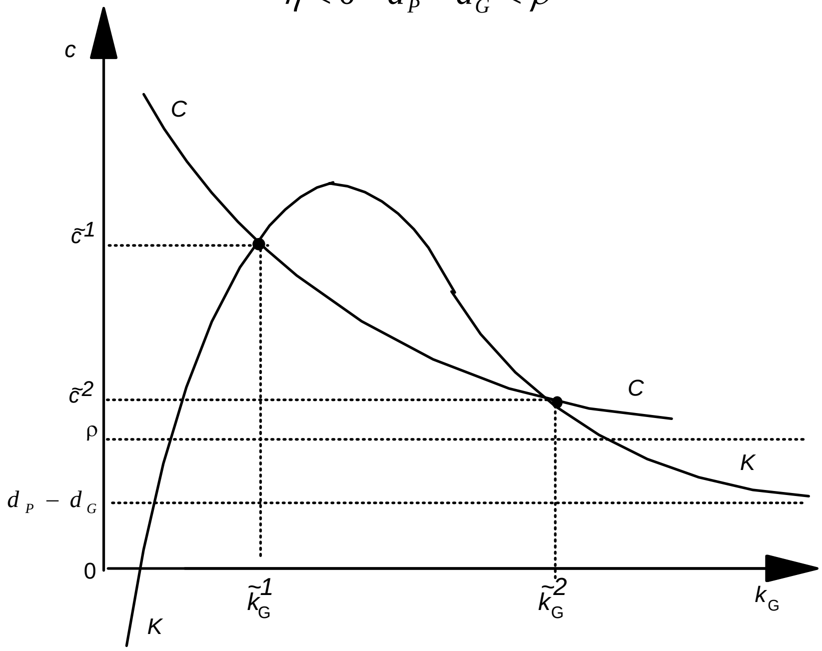

Figure 2C

(No Equilibrium)

$$\eta < 0, \quad d_P - d_G < \rho$$

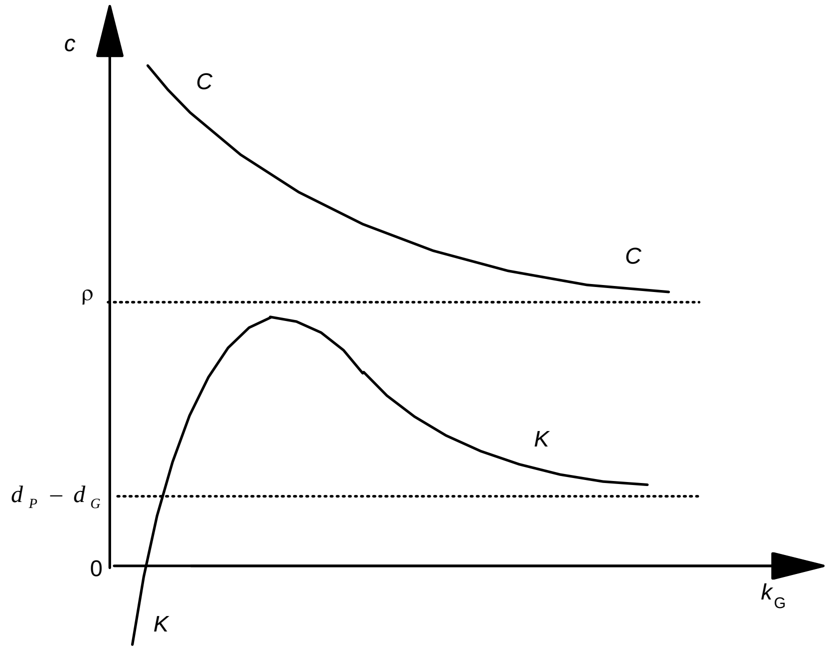

Figure 3A

An Increase in θ_P ($d_P - d_G > \rho$)

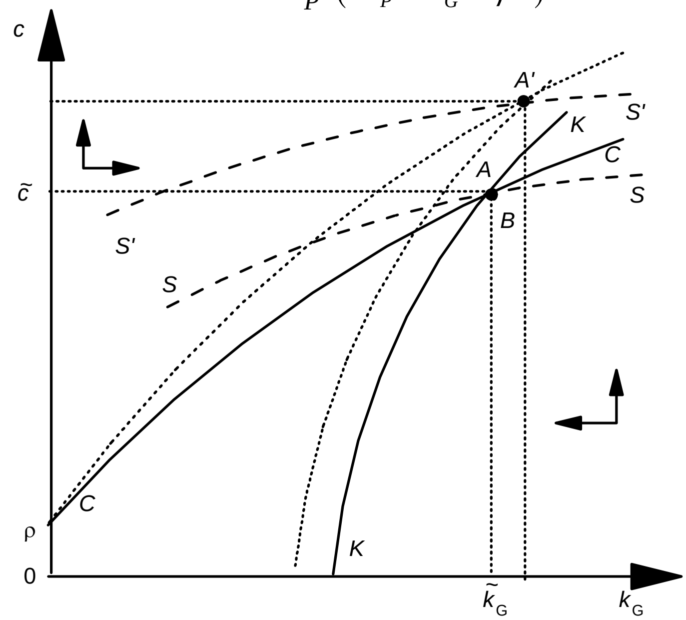

Figure 3B

An Increase in θ_P $(\; d_P - d_G < \rho \;)$