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Cross Price Effects, Nominal Rigidity and Endogenous Persistence

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Abstract

This paper shows that in macroeconomic models of product differentiation that are built on CES utility specifications, the widely used assumption of approximating cross price effects to zero, (since Dixit-Stiglitz 1979), plays indeed no crucial role. This is true not only when a large number of agents is assumed, but also at the flexible symmetric macro equilibrium where such effects are shown to cancel out regardless of the number of agents. We then show that this latter result is no longer true in the presence of nominal rigidities, where the ratio of *cross* to *own price elasticities*, (typically absent in recent New Keynesian models), is shown to be the key determinant of the coefficient of wage and inflation persistence.

JEL classification: D4; E30; E50.

Keywords: Cross Price Effects; CES Utility; Product differentiation; New Keynesian; Endogenous Persistence.

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1. Introduction

The recent New Keynesian literature places particular emphasis on the importance of microfoundations for explaining nominal persistence. Following Dixit and Stiglitz (1979), the standard New Keynesian literature, that uses largely CES utility function models of product differentiation, approximates for simplicity cross price effects to zero.[1] This implies that in models where constant returns are also assumed, the *own* and *cross price elasticities of demand* (hence the degree of gross substitutability/complementarity in the products market) are fully eliminated. The latter explains why despite the presence of product differentiation these price elasticities are typically absent from the New Keynesian Phillips Curve.[2]

This paper emphasizes the importance of cross price effects for nominal persistence in models of oligopolistic competition with sticky prices. We first show that neglecting cross price effects in macro models that use CES utility functions plays indeed no crucial role, not only when a large number of agents is assumed, but also at the flexible symmetric aggregate equilibrium where such effects are shown to cancel out regardless of the number of oligopolistic competitors. We then show that in models with nominal rigidities the ratio of *cross* to *own price elasticities*, (that is typically absent in the key dynamic equations of the standard New Keynesian model), not only does not cancel out but instead it is shown to be the key determinant of the coefficient of wage and inflation persistence. For transparency, this demonstrated through nominal wage rigidity where for simplicity each differentiated industry pays a different wage in its own sector.

2. The Model

We consider a simple economy consisting of a fixed number, *N*, of imperfectly competitive firms indexed by *j=1,2..N*, each producing a differentiated good. For simplicity, and with no loss in generality, we assume that there is a representative household who supplies one type of labour to all firms and receives the average wage in the economy. In each sector, firms demand labor and the household sets the wage. The representative household, consumes goods from all industries, receives a monetary transfer in the beginning of each period and receives profits from all sectors. The household's utility is,

$$(1) \qquad U_t = C_t^{\gamma}\left(\frac{M_t}{P}\right)^{1-\gamma} - L_t^{\theta}, \qquad \gamma \in (0,1), \quad \theta = (\eta+1)/\eta; \quad \eta > 0$$

$$(2) \qquad \text{where} \quad C_t = N^{\frac{1}{1-\sigma}}\left(\sum_{j=1}^{N} C_{j,t}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}, \qquad \sigma > 1.$$

[1] This is based on the assumption that at the macro level there is an infinitely large number of competitors driving these effects to a negligible size.
[2] In fact since most New Keynesian models are based on Calvo type contracts, constant returns and also constant mark-ups that are eliminated upon log-linearization price elasticities disappear completely from the key equations in such models.

(3) and $$P_t = \frac{1}{N}\left(\sum_{j=1}^{N} P_{j,t}^{(1-\sigma)}\right)^{1/(1-\sigma)},$$

C_j and C are the consumption of each product j and the total consumption basket of the typical household; η measures the labour supply elasticity, and so $\theta - 1$ is the marginal disutility of labour; σ is the elasticity of substitution between consumption goods in a typical household's utility. For simplicity, as widely employed, all consumption goods enter the utility function symmetrically.

The household maximises (1) subject to the following budget constraint,

(4) $$\sum_{j=1}^{N} P_{j,t} C_t + M_t = W_t L_t + M_{t-1} + \Pi_t \equiv I_t ,$$

where M_{t-1}, M_t, Π_t and W_t are initial and desired money holdings, profits from firms and the average nominal wage to the representative household from all employment services L_t ; I_t is the household's total income.

From the maximisation problem described by equation (1)-(4), the typical household, chooses the desired levels of desired money balances and consumption for each commodity j.

(5) $$M_t = (1-\gamma) I_t ,$$

(6) $$C_{i,t} = \frac{\gamma I_t P_{i,t}^{-\sigma}}{N}\left\{\frac{1}{N}\left(\sum_{j=1}^{N} P_{j,t}^{(1-\sigma)}\right)^{1/(1-\sigma)}\right\}^{\sigma-1} .$$

Upon aggregation over all sectors, assuming money market equilibrium and equilibrium in the goods markets implies, $C_i = Y_i$, and using (3), we obtain the total demand for each product i, ($i \in j$),

(7) $$Y_{i,t} = \frac{\gamma M_t}{(1-\gamma)N} P_{i,t}^{-\sigma}\left\{\frac{1}{N}\left(\sum_{j=1}^{N} P_{j,t}^{(1-\sigma)}\right)^{1/(1-\sigma)}\right\}^{\sigma-1} .$$

Equation (7) represents the conventional product demand function in a macro model of differentiated goods, with unit income elasticity of demand. Log linearizing this for more transparency with the elasticities, we obtain,

(8) $$lnY_{i,t} = \ln\left(\gamma/(1-\gamma)N\right) + \ln M_t - \varepsilon_{ii} \ln P_{i,t} + \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \varepsilon_{ij} \ln P_{j,t} ,$$

where, $\varepsilon_{ii} \equiv -\frac{d \ln Y_i}{d \ln P_i} = \sigma - (\sigma-1)S_j$ and $\varepsilon_{ij} \equiv \frac{d \ln Y_i}{d \ln P_j} = (\sigma-1)S_j$ are the *own* and *cross price elasticities of demand* respectively and $S_j \equiv P_{i,t}^{1-\sigma} / \sum_{j=1}^{N} P_{j,t}^{1-\sigma}$ is the budget share of product j, as determined by its relative price; at the symmetric equilibrium we obtain

$S_j = S_i = 1/N$.[3]

2.1 Cross Price Effects under Flexible Prices and Wages

Firms use a linear production in labour, $Y_{i,t} = L_{i,t}$.[4] From a standard profit maximisation function, $V_{i,t} = P_{i,t}Y_{i,t} - W_{i,t}L_{i,t}$, and using the above information the optimal nominal price of firm *i* is,

(9) $$P^*_{i,t} = \mu_i W_{i,t}, \quad \text{where} \quad \mu_i = \left(\frac{1}{1-1/\varepsilon_{ii}}\right) > 1,$$

With constant return to scale, $\ln P_i = \ln W_i$ (from the log-linearization of 9), and using this into (8) the effective labour demand in each industry is,[5]

(10) $$\ln L_{i,t} = \ln M_t - (\varepsilon_{ii} \ln W_{i,t} - \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \varepsilon_{ij} \ln W_{j,t}).$$

From (10) employment in each industry is shown to depend on relative wages. In the absence of nominal rigidities (i.e. with synchronised wages here), symmetric equilibrium implies that $W_{i,t} = W_{j,t} = W_t$ and the employment equation reduces to $\ln L_{i,t} = \ln L_t = \ln M_t - \ln W_t$ and $\ln P_t = \ln W_t$, thus eliminating at the aggregate symmetric equilibrium any *cross price (wage) elasticity* effects. This is because having assumed a homothetic CES utility function the *own* and *cross price elasticities of demand* reduce to unity $-(\varepsilon_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \varepsilon_{ij}) = 1$, thus cancelling out any cross price effects at the symmetric equilibrium regardless of the number of oligopolistic competitors.[6]

Next we demonstrate that this is no longer true in the presence of nominal rigidity, where such effects can crucially determine not only the degree of nominal persistence but also the direction of wage and price dynamics.

[3] The standard approximated product demand that neglects cross price effects (i.e. because of a large *N*) is, $lnY_{i,t} = \ln\left(\gamma/(1-\gamma)N\right) + \ln M_t - \tilde{\varepsilon}_{ii}(\ln P_{i,t} - \ln P_{j,t})$, where $\tilde{\varepsilon}_{ii} = \sigma$.

[4] We purposely choose constant returns to scale, because we want to show that even under this assumption, as assumed widely in the recent New Keynesian literature, accounting for cross price effects results in the *own* and *cross price elasticities* entering endogenously the coefficient of nominal persistence.

[5] Constants are removed in the log-linearization.

[6] A unity income elasticity of demand is the direct result of using a homothetic CES utility function. This is a feature shared by all models based on Dixit-Stiglitz preferences and hence by most of the bulk of the macro literature where such preferences are used widely (see Blanchard and Kiyotaki, 1987). Bergin and Feenstra (2003) avoid the limitations of the linear homothetic CES by employing a symmetric translog expenditure function. This latter function implies a product demand system that has unitary income elasticity but non-constant price elasticities, as result the optimal price of each industry results in being a function of its own marginal costs but also of all its competitor's prices. Using similar specifications, Bergin and Feenstra (1989) show that translog expenditure functions result in cross price effects affecting nominal persistence.

2.2 Cross Price Effects under Nominal Wage Rigidity

Let us assume for simplicity that the household provides the same type of labour services to three different industries, hence *N=3*. Each industry sets the demand for employment whereas the household sets the wage.[7] For transparency we assume that at time *t* the household sets wage $W_{i,t}$ for industry *i*, sets expected wage $W_{j,t+1}$ for another industry *j*, but has already set wages, $W_{j,t-1}$, for a third industry.[8] Using this information and given symmetric cross price (wage) elasticities, equation (10) becomes,

$$\ln L_{i,t} = \ln M - \left[\varepsilon_{ii} \ln W_{i,t} - \tfrac{1}{2}(\varepsilon_{ij} \ln W_{j,t-1} + \varepsilon_{ij} E_{t-1} \ln W_{j,t+1}) \right]. \tag{11}$$

For simplicity, we assume that in each period the household wants to minimize deviations of *r*eal wages and employment from some target levels $\overline{W}^*$ and $L*$ respectively, based on a standard wage setting loss function with preferences ψ on employment,[9]

$$\Lambda_{i,t} = E_{t-1}\left\{ (\ln W_{i,t} - \ln P_t - \ln \bar{W}*) + \psi(\ln L_{i,t} - \ln L^*)^2 \right\}, \tag{12}$$

Using the fact that $\ln P_i = \ln W_i$ and $\ln P_t = \ln W_t = \frac{1}{3}(\ln W_{j,t-1} + \ln W_{i,t} + \ln E_{t-1} W_{j,t+1})$, and setting, $\partial \Lambda_{i,t} / \partial \ln W_{i,t} = 0$ the optimal wage chosen in any period *t* is,

$$\ln W_{i,t} = c_w + E_{t-1}\left\{ \rho(\ln W_{j,t-1} + \ln W_{j,t+1}) + \varepsilon_{ii}^{-1} \ln M_t \right\}, \tag{13}$$

where $c_w[L^*, \psi, \varepsilon_{ii}]$ is a constant (in log deviation) and given symmetric cross price elasticities the coefficient of wage correlation is, $\rho = \frac{1}{2}\sum_{\substack{j=1 \\ j \neq i}}^{2} \varepsilon_{ij} / \varepsilon_{ii}$. The rational expectation solution to the second order difference equation in (13) is,

$$\ln W_t = \lambda_1 \ln W_{t-1} + \left(\frac{1}{\rho \lambda_2} \right) \sum_{s=0}^{\infty} \frac{\lambda_2^{-s}}{\varepsilon_{ii}} E_{t-1}(\ln M_{t+s}). \tag{14}$$

Using $\ln P = \ln W$ and taking a first difference in (14) gives,

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[7] Assuming other wage bargaining process does not alter the main result in this paper.

[8] This results in the average wage being, $\ln W_t = \ln P_t = (1/3) E_{t-1}(\ln W_{t-1} + \ln W_t + \ln W_{t+1})$, as found in many staggered wage or price models (see Blanchard 1983). A number of other more sophisticated types of staggered wage setting can also be assumed but they should not alter the point made in this paper.

[9] The household could also derive wages from maximizing their own utility function without affecting our main result, but for algebraic transparency the use of a standard wage setting loss function (as in the literature of monopolistic unions) is preferred.

(10) $$\pi_t = \lambda_1 \pi_{t-1} + \left(\frac{1}{\rho\lambda_2}\right)\sum_{s=0}^{\infty}\frac{\lambda_2^{-s}}{\varepsilon_{ii}}E_{t-1}(\ln\Delta M_{t+s}).$$

where $\pi_t \equiv \ln P_t - \ln P_{t-1}$ is the inflation rate and $\lambda_1, \lambda_2 = 0.5/\rho \pm 0.5(\sqrt{(1/\rho)^2 - 4})$ are the two roots in the dynamic equation above. From this, the degree of nominal persistence, λ_1, (i.e. the small root, λ_1<1) is shown to be determined purely by the relative ratio of *cross* to *own price elasticities*, $\rho = \frac{1}{2}\sum_{\substack{j=1 \\ j\neq i}}^{2}\varepsilon_{ij}/\varepsilon_{ii}$.[10]

Higher nominal correlation (ρ) results in higher persistence (λ_1). Both ρ and λ_1 are higher the higher is the substitutability between the products produced by firms. An exogenous rise in the price of one industry (i.e. due to a temporary increase in the money supply), will result in higher demand for its substitutes and hence in a positive wage correlation between i and j if these are *gross substitutes* (i.e. when $\sigma > 1$, $\varepsilon_{ij} > 0$ and $\rho > 0$) and this is shown to amplify inflation persistence λ_1. Yet, the presence of cross price elasticities in the coefficient of persistence also implies that the nature of goods dominating the production in this economy may also determine the direction of dynamics. For example if i and j are *gross complements* (i.e. $\sigma < 1$, implying $\varepsilon_{ij} < 0$ and $\rho < 0$), then a price increase in product i will result in a lower product demand for this product but also of its complements, thus resulting in a negative price and wage correlation here.[11]

In general, the size and direction of dynamics in this simple model are shown to depend on the ratio of *cross* to *own price elasticities* and the number of oligopolistic competitors.

3. Concluding Remarks

Although the bulk of New Keynesian models are based on product differentiation, for simplicity they assume an infinitely large number of symmetric competitors and approximate cross price effects to zero. This together with the widely used assumption of constant returns to scale in such models, results in the key equations in these models, including the much celebrated New Keynesian Phillips curve, being independent of the price elasticities of demand.

This paper shows that regardless of the number of oligopolistic competitors, the widely used assumption of neglecting cross price effects plays indeed no crucial role for the bulk of flexible price macroeconomic models that are based on homothetic CES utility specifications. This is because at the symmetric flexible price aggregate

[10] Note in this simple setup and in the presence of constant returns to scale, this result relies on preferences being non-linear in employment; or alternatively if wages were derived from the household's utility that θ≠1.

[11] Note that in this paper we make our point by using for simplicity symmetry, but for three or more goods the signs of the cross price effects do not have to be restricted to be either symmetric or of the same sign, (see Weber 2002).

equilibrium, the sum of *own* and *cross price elasticities* reduces to unity, regardless of the number of oligopolistic competitors. The paper then shows that in the presence of nominal rigidities the latter effect is no longer true and the ratio of *cross* to *own price elasticities* becomes the key determinant of inflation persistence.

The intuition here is that each industry's product demand depends negatively on its own price elasticity of demand, ε_{ii}, but positively on the cross price elasticities of demand ε_{ij}. The higher is the cross price effect in relation to the own price effect (hence the ratio $\varepsilon_{ij} / \varepsilon_{ii}$) the higher is the demand for each good with respect to changes in the price of other goods. Hence the presence of cross price effects is shown to offset part of the negative own price effect. This implies that in dynamic models of sticky prices, a higher ratio of cross price to own price elasticity will also imply a higher level of nominal persistence.

In effect, this paper suggests that unless we consider oligopolistic structures where the number of competitors is indeed very large, neglecting cross price effects may eliminate oligopolistic interactions that may be important for the degree of wage and inflation persistence. Relaxing, for example, the symmetry between firms, or assuming that a few large oligopolistic competitors or unions operate in the economy,[12] or considering economic structures where a few particular industries dominate the nature of the production of an economy, then even if many other smaller competitors are assumed in such models cross price or wage effects will not be negligible and as demonstrated here they will endogenously affect the rate of wage and inflation persistence.

References

Bergin P. R. and Feenstra R.C., (1998). Staggered Price Setting and Endogenous Persistence, NBER Working Paper No. 6492**.**

Bergin P. R. and Feenstra R.C., (2003). **"**A homothetic Utility Function for Monopolistic Competition Models, Without Constant Price Elasticity", *Economics Letters* 78, (1) pp. 79–86.

Blanchard 0. J., (1983). Price Asynchronization and Price Level Inertia. In Dornbusch, R., Simonsen M.H. (Eds), Inflation Debt and Indexation. MIT Press, Cambridge, MA.

Blanchard, O. J. and Kiyotaki, N., (1987). "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review*, 77, (4), pp. 647-666.

Bratsiotis G. J. (2008). "Influential price and wage setters, monetary policy and real effects", *European Journal of Political Economy*, 24, (2), pp. 503-517.

[12] In macro models where a few large firms or unions operate such cross price and wage effects can lead to a number of non-negligible real effects arising from the interaction between price and wage setters but also between the latter two and monetary policy even in the absence of nominal rigidities (for a recent paper see Bratsiotis 2008).

Dixit, A. K. and Stiglitz, J. E., (1977). "Monopolistic Competition and Optimum Product Diversity", *American Economic Review,* 67(3), pp. 297-308.

Weber, C. E. (2002). "On the Hicks-Allen Definition of Complements and Substitutes with Discrete Changes in Prices", *Metroeconomica,* 53 (1), pp. 37-45.