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CGBCR

Discussion Paper Series

Learning in a Credit Economy

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January 2008
Number 100

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January 31, 2008

Abstract

In this work we analyze a credit economy à la Kiyotaki and Moore (JPE, 1997) enriched with learning dynamics. Both borrowers and lenders need to make expectations about the future price of the collateral, and under heterogeneous learning this can have interesting consequences for the economy when the possibility of bankruptcy is taken into consideration.

Key words: Credit Economy; Bankruptcy; Learning; Heterogeneity
JEL classification: D83; E44; G14.

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1 Introduction

"Bankruptcy – default – was at the center of the discussion. But in the IMF model – as in the models of most of the macroeconomics textbooks written two decades ago – bankruptcy plays no role. To discuss monetary policy and finance without bankruptcy is like Hamlet without the Prince of Denmark" (J. Stiglitz, "Globalization and its discontents", 2002).

The rational representative agent hypothesis is still the cornerstone of most of contemporary macroeconomics. However, the awareness of its limitations is spreading well beyond the circle of more or less dissenting economists. Even in mainstream macroeconomics, the representative agent is not as eagerly embraced as in the early years of the debate on micro-foundations in the remote '70s and is still adopted mainly for lack of a workable alternative.

In contrast, in behavioural finance, bounded rationality and heterogeneous agent models are becoming a serious alternative to the standard rational representative agent approach, as discussed e.g. in the extensive surveys of LeBaron (2006) and Hommes (2006). Moreover, in the last decade, bounded rationality and adaptive learning have become increasingly important in macroeconomics; see e.g. Evans and Honkapohja (2001) and (2006), Branch and Evans (2006), Branch and McGough (2006), Honkapohja and Mitra (2006) and Berardi (2007).

Learning can represent a form of bounded rationality that still maintains some rigor in the expectation formation process and for this reason has received increasing attention in the recent years, in particular in the fields of monetary economics and monetary policy. It has also quickly permeated other branches of macroeconomics, but to the best of our knowledge it has not yet been used to model and analyze information asymmetries in financial markets.

It is clear that people are different in many aspects (e.g., degree of rationality, computational capabilities, information set, financial conditions, etc.) and heterogeneity is a persistent and non-negligible part of any economic story. Imperfect information and information asymmetries are an important element in credit/debit relationships, and can not be properly addressed in a rational expectations framework.

Usually macroeconomic models that include an expectations formation process do not take into account the fact that agents may incur into errors in their forecast of economic variables and that these errors are usually costly and potentially even fatal. In other words, the existence of costly errors has consequences not only at an individual level but also on aggregate variables.

In our paper we investigate in a Kiyotaki and Moore (1997, KM hereafter) environment these events, in particular we consider an extreme consequence of errors of the decision making process: bankruptcy. In this framework, we substitute the equilibrium hypothesis of rational expectations with an expectation formation scheme based on learning dynamics.

The aim of our paper consists in taking into account the role of bankruptcy in a credit economy à la KM and to analyze credit/debit dynamics when the expected price of a collateral (land) is used by borrowers (farmers) and lenders (gatherers) to make their decisions, under the assumption of heterogeneity in the expectation formation process of the two type of agents.

In the original KM framework, given perfect foresight, if the farmer does not work, land will not yield fruit (due to the idiosyncratic nature of the farmer's technology) and he will be unable to reimburse debt. In the event of default, the gatherer can seize the farmer's land and sell it. By assumption, the value of the land will be exactly equal to the service of debt (principal and interest) so that the lender's balance sheet will not be affected by bankruptcy. In this framework, therefore, in principle the borrower can default but the gatherer is not bearing the risk of bankruptcy. Contrary to the KM framework where, given the structure of the model, bankruptcy does not play a role, we introduce uncertainty into the model and make agents learn the necessary information needed in order to make their economic decisions.

Assuming that agents are heterogeneous with respect to their learning process, borrowers can go bankrupt and bankruptcy will play an important role in the dynamic properties of the economy under scrutiny. We analyze the circumstances that lead to bankruptcy and the consequences of bankruptcy on the main variables of the model. We find that, in general, the economy is attracted (locally) towards an equilibrium, but heterogeneous learning dynamics, when coupled with the possibility of bankruptcy, can have important consequences for the economy, generating hysteresis and strong non-linearities.

The paper is organized as follows: in section 2 we briefly recall the benchmark KM model, in section 3 we consider the linearized version of the model and we study the properties of the dynamics of our economy introducing homogeneous learning. In section 4 we assume that farmers and gatherers possibly have heterogeneous expectations assuming that they learn independently from each other in two different scenarios: with constant and stochastic productivity respectively. Section 5 concludes.

2 The benchmark KM economy

A KM economy consists of two groups of agents: those who are financially constrained (farmers) and the unconstrained ones (gatherers). Agents in both groups produce a perishable good (fruit) by means of a technology that uses land and labor.

A farmer is an agent endowed with inalienable human capital. Therefore, he can get from lenders no more than the value of his collateralizable assets. This is the reason of the financing constraint.[1] A gatherer, on the contrary, does not face financing constraints.

KM assume *preference heterogeneity*: the farmers are less patient than gatherers, so that the former are also borrowers and the latter play the role of lenders. Moreover KM assume that there is perfect foresight on the future level of the price of land. An important consequence of the assumption of idiosyncratic farmer's technology is that the gatherer/lender bears the risk of default. If the farmer withdrew his labour, production would not be carried out, i.e. land would bear no fruit. As a consequence, if the farmer is indebted, he may have an incentive to threaten his creditor to withdraw his labour and repudiate debt. Lenders protect themselves against this threat by collateralizing the farmer's land. This is the reason why the farmer faces a financing constraint:

$$b_t = \frac{q_{t+1} K_t^F}{R} \tag{1}$$

i.e., the loan he gets (b_t) cannot exceed the value of his collateralizable assets $\left(\frac{q_{t+1} K_t^F}{R}\right)$ – the present value of his current landholding – which plays, in this framework, a role analogous to that of net worth or the equity base in Greenwald and Stiglitz (1993, 2003) and entrepreneurs' savings (internal finance) in Bernanke and Gertler (1989, 1990) and Bernanke, Gertler and Gilchrist (1999). As a consequence, also in a KM economy production depends upon net worth. In fact, the higher is net worth, the softer the borrowing constraint and the higher credit extended, investment and production.

There are two types of goods, output ("fruit") and a collateralizable, durable, non-reproducible asset ("land") whose total supply is fixed ($\bar{K}$). Output can be consumed or lent. If lent, each unit of output yields a constant return $R = 1 + r$ where r is the real interest rate. Output is produced by means of a technology which uses land and labour.

[1] On this issue see Hart and Moore (1994, 1998).

By assumption farmers and gatherers have access to different technologies.

The production function of each farmer is: $y_t^F = (a+\bar{c})K_{t-1}^F$ where y_t^F is output of the farmer in t, a and $\bar{c}$ are positive technological parameters and K_{t-1}^F is land of the farmer in $t-1$. $\bar{c}\,K_{t-1}^F$ is the output which deteriorates ("bruised fruit") and is therefore non-tradable.

According to (1), the maximum amount of debt a farmer succeeds to get "today" b_t is such that the sum of principal and interest Rb_t is equal to the value of the farmer's land when the debt is due, i.e. $q_{t+1}K_t^F$ where q_{t+1} is the (real) price of land at time $t+1$.

The farmer faces also a *flow-of-funds constraint*:

$$y_t^F + b_t = q_t(K_t^F - K_{t-1}^F) + Rb_{t-1} + c_t^F \tag{2}$$

where c_t^F is the farmer's consumption. Substituting (1) into (2) we get:

$$c_t^F = (a+\bar{c})K_{t-1}^F - \mu_t K_t^F \tag{3}$$

where $\mu_t = q_t - \dfrac{q_{t+1}}{R}$ is the *downpayment,* i.e. the amount the farmer has to put aside as internal finance to acquire one unit of land.

Preferences are modelled in such a way that farmers consume only non-tradable output, i.e. $c_t^F = \bar{c}\,K_{t-1}^F$. From (3) follows

$$\mu_t K_t^F = aK_{t-1}^F \tag{4}$$

i. e. the revenues obtained by selling (non-bruised) fruit (aK_{t-1}^F) are employed as downpayment ($\mu_t K_t^F$). The farmer's demand for land, therefore, is:

$$K_t^F = \frac{a}{\mu_t}K_{t-1}^F. \tag{5}$$

The production function of each gatherer is: $y_t^G = G(K_{t-1}^G)$ where y_t^G is output of the gatherer in t , $G(.)$ is a well behaved production function and K_{t-1}^G is land of the gatherer in $t-1$. The gatherer faces only a *flow-of-funds constraint*:

$$y_t^G + Rb_{t-1} = q_t(K_t^G - K_{t-1}^G) + b_t + c_t^G. \tag{6}$$

Substituting the production function of the gatherer and the financing constraint of the farmer into (6) and assuming, for the sake of simplicity and without loss of generality, that population consists only of one farmer and one gatherer so that $K_t^F = \bar{K} - K_t^G$ we get the constraint:

$$c_t^G = G(K_{t-1}^G) + \mu_t\left(\bar{K} - K_t^G\right). \tag{7}$$

From maximization of the utility of the gatherer one gets $G'(K_t^G) = R\mu_t$. Since the total amount of land is fixed by assumption, $K_t^F = \bar{K} - K_t^G$, $G'\left(K_t^G\right) = G'\left(\bar{K} - K_t^F\right)$. In the following, in order to save on notation, we will write $G'\left(\bar{K} - K_t^F\right) = g\left(K_t^F\right)$, where $g' = -G'' > 0$. Hence the condition above can be written as

$$\mu_t = \frac{g\left(K_t^F\right)}{R}. \tag{8}$$

Substituting this expression into (5) and rearranging we end up with:

$$K_t^F = \frac{Ra}{g\left(K_t^F\right)} K_{t-1}^F \tag{9}$$

which is a non-linear difference equation in the state variable K_t^F.

Denoting with a star the steady state value of a variable, plugging the steady state condition $K_t^F = K_{t-1}^F = K^{*F}$ into (5) we obtain $\mu^* = a$. But $\mu^* = q^*\left(1 - \frac{1}{R}\right)$ so that $q^* = a\frac{R}{R-1}$ and $b^* = \frac{q^* K^{*F}}{R}$ so that $b^* = a\frac{K^{*F}}{R-1}$. Substituting these steady state conditions into (8) we obtain $K^{*F} = g^{-1}(Ra)$. Hence $b^* = \frac{ag^{-1}(Ra)}{R-1}$ and $Rb^* = a\frac{RK^*}{R-1} = q^*K^*$.

As to the gatherer, from (7) it follows that $c_t^G = G(\bar{K} - K^{*F}) + aK^{*F}$.

KM log-linearize the economy in the neighborhood of the steady state and show that small shocks to the technological parameter a can produce large and persistent fluctuations in output and asset prices. In their model, in fact, the durable, non reproducible asset (land) plays the dual role of a factor of production for both constrained and unconstrained agents and of collateralizable wealth for financially constrained agents. Therefore the price of assets affects the borrowers' financing constraint and at the same time, the size of the borrowers' credit limits feeds back on asset prices.

3 Homogeneous learning

Starting from the economy described above, we now drop the rational expectations (perfect foresight) assumption and endow our agents with an adaptive scheme that they use in order to form expectations about the future price of the collateral, and we analyze whether they would be able to learn over time the correct value of the parameters and thus converge towards rationality. In order to carry out the learning analysis, we first need to linearize the above economy around its steady state.

Using a Cobb-Douglas specification for the production function of the gatherer ($G\left(K_t^G\right) = \sqrt{K_t^G}$), and starting from equations (8) and (9), the linearized system representing the dynamics for the economy under RE can be expressed as

$$
\begin{aligned}
K_t^F &= \Gamma_0 + \Gamma_1 K_{t-1}^F & (10)\\
q_t &= \Psi_0 + \Psi_1 q_{t-1} + \Psi_2 K_{t-1}^F. & (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma_0 &= \frac{\left[(2aR)^2\bar{K} - 1\right]^2}{(2aR)^2\left[(2aR)^2\bar{K} - 1\right]^2} & (12)\\
\Gamma_1 &= \frac{2}{1 + (2aR)^2\bar{K}} & (13)\\
\Psi_0 &= \frac{\bar{K}}{4(2aR)^3} - Ra - \frac{1}{4(2aR)^5} & (14)\\
\Psi 1 &= R, \quad \Psi 2 = -\frac{1}{4(2aR)^3}. & (15)
\end{aligned}
$$

If we consider instead agents as adaptive learners, we must write equation (8) in expectational form, and therefore replace the backward looking (11) with the following forward looking expression and consider explicitly the expectations formation mechanisms used by agents in order to forecast the values of q_{t+1} and K_t^F needed to make decisions at time t.

$$
q_t = -\frac{\Psi_0}{\Psi_1} + \frac{1}{\Psi_1} q_{t+1}^e - \frac{\Psi_2}{\Psi_1} K_t^F \tag{16}
$$

where q_{t+1}^e is the expectation in t on the level of the asset price in $t+1$.

We assume that agents (both farmers and gatherers) in their learning activity use a model compatible with the low of motion for the economy under RE; therefore they estimate the relationships (10) and (11) and use them to form their expectations which we then insert into the forward looking equation for q_t to obtain the actual law of motion (ALM) for the current value of land.

The estimated equations (also called perceived laws of motion - PLMs) are

$$
\begin{aligned}
q_t &= \phi_0 + \phi_1 q_{t-1} + \phi_2 K_{t-1}^F + e_t & (17)\\
K_t^F &= \theta_0 + \theta_1 K_{t-1}^F + e_t & (18)
\end{aligned}
$$

and we say that learning converges towards rational (perfect foresight) expectations if, over time, the parameter estimates converge to the corresponding values in (12) - (15).

Using (17) and (18) we can compute the expectations to be inserted into (16) and obtain the ALM for q_t:

$$q_t = -\frac{\Psi_0}{\Psi_1} + \frac{1}{\Psi_1}\left[\phi_0 + \phi_0\phi_1 + \phi_2\theta_0 + \phi_1^2 q_{t-1} + (\phi_1\phi_2 + \phi_2\theta_1)K_{t-1}^F\right] - \frac{\Psi_2}{\Psi_1}(\theta_0 + \theta_1 K_{t-1}^F) \tag{19}$$

Mapping the PLMs into the ALMs we obtain the ODEs for ϕ_0, ϕ_1, ϕ_2, θ_0, θ_1, whose fixed points represent equilibria for the economy under learning dynamics. It turns out that there are 2 fixed points for the system of ODEs for the $\phi' s$ parameters, giving the two solutions:

$$q_t = \Psi_0 + \Psi_1 q_{t-1} + \Psi_2 K_{t-1}^F \tag{20}$$

$$q_t = \frac{\Psi_0(\Gamma_1 - \Psi_1) - \Gamma_0\Psi_1\Psi_2}{(1-\Psi_1)(\Gamma_1 - \Psi_1)} + \frac{\Gamma_1\Psi_2}{(\Gamma_1 - \Psi_1)}K_{t-1}^F. \tag{21}$$

It is possible to show that (20) could be reduced to (21) provided that $\Psi_1 < 1$ and $\Gamma_1 > 1$. But since $\Psi_1 = R$, this parameter is always > 1. Moreover, in order for capital to be a stationary variable, we need the restriction $\Gamma_1 < 1$. Therefore the two solutions must be kept distinguished.

Analyzing the learnability of the two solutions, we find that the first fixed point for the system of ODEs, corresponding to the first solution, is always locally unstable (for sensible parameter values), while the second fixed point, giving equation (21), is stable for $\frac{\Gamma_1}{\Psi_1} < 1$. Notice that this last restriction is always satisfied for parameter values that respect the aforementioned restrictions on Γ_1 and Ψ_1. Therefore, for sensible parameter values, agents can learn the RE equilibrium represented by (21).

4 Heterogeneous learning

4.1 Constant productivity

Now we let farmers and gatherers learn independently from each other, and therefore possibly have heterogeneous expectations. There are a number of different ways in which heterogeneity in learning could be modelled. Agents could have different initial beliefs, they could use different models (PLMs) or different learning algorithms (and of course any combination of the three). We will consider only the last possibility, and in particular we will allow agents to use different gain parameters in their learning schemes.

Now that expectations could differ between farmer and gatherer, we need to take into account the possible consequences on financial incentives. We therefore introduce a voluntary bankruptcy condition for farmers, one that reflects the incentive for the borrower to pay back its debt to the lender. The intuition is simple: when the borrower needs to decide whether to pay back its debt, he compares the value of the debt with the expected value of the land (which stands as collateral) and decides to pay back the debt only if the first is smaller than the second. If instead the value of the collateral is expected to be smaller than the debt, the farmer will find it convenient to default on its debt and let the lender grab the collateral. And since the credit granted to the borrower depends on the lender's expectations on the value of the collateral,[2] the voluntary bankruptcy condition reduces to

$$q_{t+1}^{e,F} < q_{t+1}^{e,G}. \tag{22}$$

where $q_{t+1}^{e,F}$ and $q_{t+1}^{e,G}$ are respectively the expectations in t on the level of the asset price in $t+1$ for the farmer and the gatherer. If this condition holds, the value of the debt to be repaid at time $t+1$, i.e., $b_t = q_{t+1}^{e,G} K_t^F$, is higher than the farmer's expected value of the land ($q_{t+1}^{e,F} K_t^F$), and therefore he would decide to default on its debt (and this decision would be revealed to the lender only at time $t+1$)[3].

In addition to the voluntary bankruptcy there is of course the possibility of an involuntary bankruptcy, which can happen when there is a large negative shock to productivity, so that the price of land falls largely and unexpectedly. This is an additional constraint that we chose not to put into our economy. This bankruptcy condition would reduce in fact to a simple threshold for the productivity shock, which would not produce any interesting interaction with the expectation formation processes and would be highly sensitive to the parameterization of the stochastic process for productivity. So while the involuntary bankruptcy can surely represent an additional real-life reason why the economy might not converge towards an equilibrium, we will leave this case out of our analysis.

In order to rewrite the model under heterogeneous expectations, we need to start from the demand for land for farmers and gatherers. The farmers' (linearized) demand for land is

$$K_t^F = K_s^F + K_{t-1}^F - \frac{K_s^F}{a} \mu_t^{e,F} \tag{23}$$

[2] The implicit assumption here is that the farmer will accept any amount of funds that the gatherer is willing to lend him.

[3] The farmer needs to decide at time t whether or not to default at time $t+1$, since he must decide whether or not to cultivate the land.

where $\mu_t^{e,F}$ is the farmers' expected value for the downpayment μ_t and K_s^F is the steady state value for K^F. The gatherers' (linearized) demand for land is

$$K_t^G = \frac{3}{(2aR)^2} - \frac{1}{2a^3R^2}\mu_t^{e,G} \tag{24}$$

with $\mu_t^{e,G}$ representing the gatherers' expectations for μ_t.

To close the model we also use the equilibrium condition $K_t^F = \bar{K} - K_t^G$ and obtain the forward-looking equation for the price of land

$$q_t = c_0 + c_1 K_{t-1} + c_2 q_{t+1}^{e,F} + c_3 q_{t+1}^{e,G} \tag{25}$$

with

$$\begin{aligned} c_0 &= \frac{2a}{1+\bar{K}(2aR)^2}, \quad c_1 = \frac{4a^3R^2}{1+\bar{K}(2aR)^2} \\ c2 &= \frac{\bar{K}(2aR)^2 - 1}{1+\bar{K}(2aR)^2}, \quad c3 = \frac{2}{1+\bar{K}(2aR)^2} \end{aligned}$$

from which we can then obtain the ALM of the economy under learning. It is possible to see from (25) that the minimum state variables (MSV) REE for this economy is of the form

$$q_t = \phi_0 + \phi_1 K_{t-1}$$

which is the functional form we assume agents use in their expectations formation process. Farmers and gatherers therefore recursively estimate the parameters ϕ_0 and ϕ_1 and use these estimates to form their expectations about q_{t+1}. In order to do so they also need an estimate for K_t (as this variable is still to be determined at the time agents form their expectations for q_{t+1}), which is obtained by estimating an AR(1) equation for K_t (consistent with the REE law of motion for capital) with parameters θ_0 and θ_1. Inserting these expectations into the model we can obtain the ensuing ALMs for K_t and q_t and then derive the T-maps from the PLMs to the ALMs for the two agents, which give the system of ODEs governing the evolution of the estimated parameters in notional time.

$$\begin{aligned}
\dot{\phi}_0^F &= c_0 + c_2\phi_0^F + c_3\phi_0^G - \phi_0^F + c_2\phi_1^F\theta_0^F + c_3\phi_1^G\theta_0^G \\
\dot{\phi}_0^G &= c_0 + c_2\phi_0^F + c_3\phi_0^G - \phi_0^G + c_2\phi_1^F\theta_0^F + c_3\phi_1^G\theta_0^G \\
\dot{\phi}_1^F &= c_1 + c_2\phi_1^F\theta_1^F + c_3\phi_1^G\theta_1^G - \phi_1^F \\
\dot{\phi}_1^G &= c_1 + c_2\phi_1^F\theta_1^F + c_3\phi_1^G\theta_1^G - \phi_1^G \\
\dot{\theta}_0^F &= H_0 - \theta_0^F \\
\dot{\theta}_0^G &= H_0 - \theta_0^G \\
\dot{\theta}_1^F &= H_1 - \theta_1^F \\
\dot{\theta}_1^G &= H1 - \theta_1^G
\end{aligned}$$

where the superscript F refers to the farmer and G to the gatherer and

$$\begin{aligned}
H_0 &= K_s - \frac{K_s}{a}\left(c_0 + c_2\left(\phi_0^F + \phi_1^F\theta_0^F\right) + c_3\left(\phi_0^G + \phi_1^G\theta_0^G\right)\right) + \frac{K_s}{aR}\left(\phi_0^F + \phi_1^F\theta_0^F\right) \\
H_1 &= 1 - \frac{K_s}{a}\left(c_1 + c_2\phi_1^F\theta_1^F + c_3\phi_1^G\theta_1^G\right) + \frac{K_s}{aR}(\phi_1^F\theta_1^F)
\end{aligned}$$

There are two REE solutions to this system of non-linear ODEs, one that turns out to be E-stable and the other one E-unstable, for all (sensible) parameter values.

Since agents use the same learning algorithms and have the same initial beliefs by assumption, their expectations turn out to be always the same and the bankruptcy condition never becomes binding. Simulations in Figure 1 show that all agents actually learn (locally) the E-stable equilibrium.

4.2 Stochastic productivity and constant gain learning

Up to this point we have been working with a deterministic economy, where no intrinsic uncertainty was present. We now consider the more interesting case in which productivity is stochastic, so that one of the fundamental parameters of our economy keeps changing over time. In particular, we will consider the case in which productivity follows a stationary AR(1) process.

This change has important implications for the learning analysis. In an economy undergoing changes in its fundamentals, in fact, agents should use a learning scheme that allows for parameter drifts, such as a constant gain algorithm, which discounts past observations and gives relatively more importance to new data, thus keeping track of the structural changes in the economy. Therefore, they need to choose an appropriate value for the gain

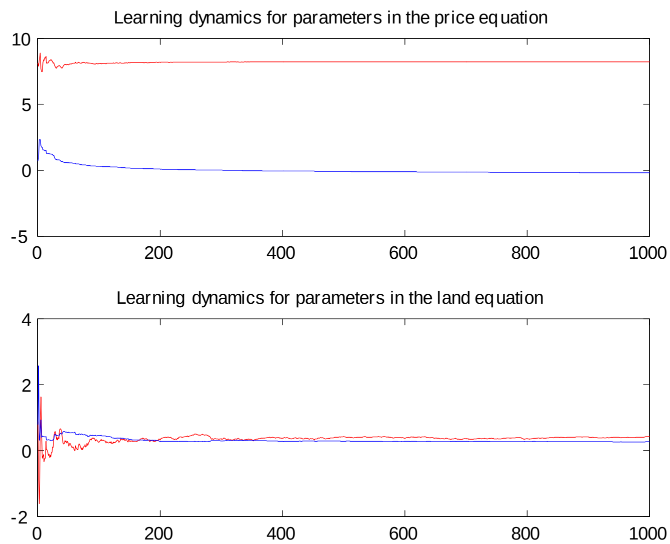

Figure 1: Learning dynamics for price and land equations.

parameter (or, equivalently, choose the length of the data windows in their regressions), and this is the route through which heterogeneity can enter into the expectation formation processes, since different agents could use different gain parameters.

Even with a time-invariant economy, parameter estimates coming from a constant gain algorithm could not point-converge to a single value, but they could still converge in distribution around the true value. But with a time-varying productivity the economic structure is evolving over time, and therefore no convergence at all can be expected. Agents can only hope to "follow" the economy with their (noisy) estimates. Simulations in Figure 2 show that this is what happens (for the E-stable REE) if gatherer and farmer use the same gain parameter.

But nothing ensures that farmer and gatherer will in fact choose the same gain parameter. The farmer could discount past data more or less heavily than the gatherer and in this case, even if the two agents start out with the same initial beliefs, their estimated parameters, and therefore their expectations, will sooner or later diverge.

The constant gain indicates how many data periods agents use in their estimates. A gain of .05, for example, means that agents are using 20 quar-

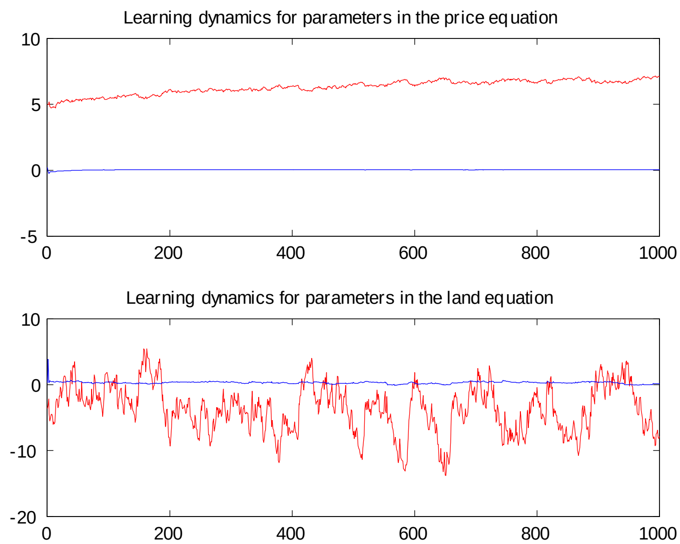

Figure 2: Learning dynamics. Agents' estimate follow (noisily) the structural evolution of the economy.

ters, while a gain of .055 corresponds roughly to 18 quarters. Even with such a small difference, simulations show that the expectations of the two groups diverge quickly and this can have drastic consequences for the economy by inducing the borrower to default. Figure 3 presents one such case: even though the learning algorithms are potentially able to keep track of the changes in the economy and the estimated parameters would follow (stochastically) the evolution of the true values, the borrower/lender relationship comes to an abrupt end after 35 periods, when the borrower decides to default on the basis of his expectations about the future price of the collateral.

The actual timing of the bankruptcy in the simulations we run depends critically on the difference in the gain parameters and on the size of the productivity shocks that displace the economy. The bigger is the difference in the gain parameters or the greater the producticity shocks, and the sooner bankruptcy arises. With different gains in the learning algorithms, in fact, one of the two agents is able to keep track of the changes in the economy faster than the other: therefore the greater is the difference in the gains or the bigger are the shocks that hit the economy, and the sooner the expectations of the two agents will diverge, thus opening the route to

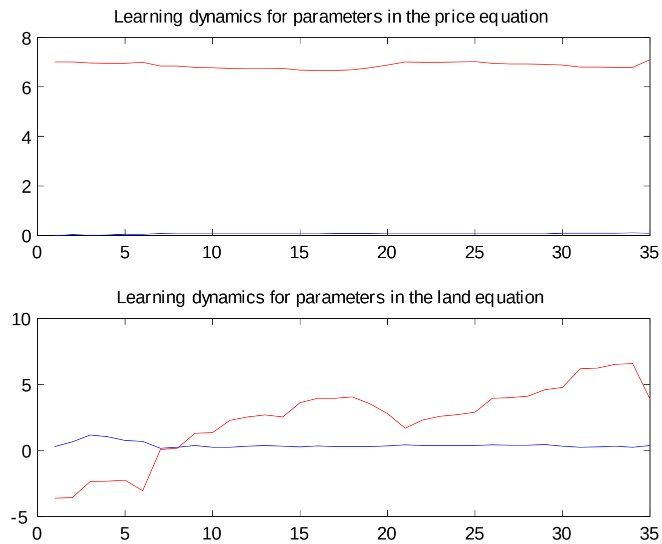

Figure 3: Learning dynamics. Agents' estimate tend to follow (noisily) the structural evolution of the economy, but the bankruptcy condition becomes binding and farmers go bankrupt after about 35 periods.

bankruptcy.

When a borrower decides to default, the relationship borrower/lender comes to an end, and in this economy, where all the borrowers are alike, this would also mean that the whole economy comes to an end. In a richer, and more realistic, setting there would be heterogeneity also among borrowers themselves, as well as new entries and exits of borrowers (and lenders) over time, so that the bankruptcy condition would realistically induce a turnover in the borrower/lender relationships. It is sensible to suppose that, under imperfect information, a borrower that has defaulted on a previous debt could maybe still manage to find a lender willing to grant him a new loan, but in a repeated game the reputation of the borrowers would soon become public information available to all the lenders and it would be extremely difficult for a "bad" borrower to find new lenders willing to engage in economic relations with him. We do not take these reputations considerations into account in our analysis here, but acknowledge their potential impact on the decision of the borrower to go bankrupt.

5 Conclusions

In this work we have analyzed a credit economy enriched with learning dynamics. We find that, though in general agents can learn the MSV REE equilibrium, farmers may be prevented from doing so by a bankruptcy condition becoming binding over the learning path towards rationality. This means that expectations formation processes and the heterogeneity of beliefs between lenders and borrowers can play an important role in a credit economy.

This work also shows that learning can introduce important hysteresis into an economy, even when the learning process would actually converge towards an equilibrium in the long run. Short run constraints, in fact, may drive economic agents out of the market while they are learning and before they have got the chance to fully understand the economic structure in which they operate. These phenomena introduce in the economy strong non-linearities and irreversibilities that are often neglected in RE models.

Further work will investigate a number of extensions to the present setting. First, the degree of heterogeneity in the learning schemes could be made endogenous, depending for example on the costs and benefits of using more data in the regressions. Also, a full, blown up analysis that takes into account long-run incentives and reputation effects on the part of the farmer could add useful insights to the findings of this paper.

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