The University *of* Manchester

Discussion Paper Series

On the Relationship Between Growth and Volatility in Learning-by-Doing Economies

By

Keith Blackburn and Alessandra Pelloni

Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, UK

January 2001
Number 001

On the Relationship Between Growth and Volatility in Learning-by-Doing Economies[¤]

Keith Blackburn and Alessandra Pelloni
Centre for Growth and Business Cycle Research
School of Economic Studies, University of Manchester

January 10, 2001

Abstract

This paper contains an investigation into the potential linkages between the short-run (cyclical) and long-run (secular) movements in economic activity. The investigation is based on an analytically solvable stochastic monetary growth model in which learning-by-doing accounts for endogenous technological change. The dynamic general equilibrium of this model implies that both the …rst and second moments of disturbances have …rst-order e¤ects on both the …rst and second moments of variables. Given this, it is shown that the correlation between the mean and variance of output growth depends fundamentally on two main factors - the source of stochastic ‡uctuations (real shocks or nominal shocks) and the functioning of the labour market (wage ‡exibility or wage rigidity). These results contradict certain common presumptions and may help to explain certain empirical evidence.

1 Introduction

Since the seminal contribution by Nelson and Plosser (1982), it has become customary to treat most macroeconomic time series as containing stochastic trends. Until lately, these trends were typically associated with the occurrence of exogenous technology shocks that follow a unit root process, a perspective exempli…ed in early real business cycle models. Such a perspective

[¤]The authors are grateful for the …nancial support of the Bank of England and the Leverhulme Trust. The usual disclamer applies.

is based either implicitly or explicitly on the traditional dichotomy between business cycles and growth. Yet modern dynamic general equilibrium analysis does not support this dichotomy: on the contrary, a key implication of stochastic endogenous growth theory is that any temporary disturbance can have a permanent e¤ect on output so long as it changes the amount of resources on which productivity improvements depend. Moreover, this is true not only of temporary real shocks, as in the models of Bean (1990), Fatas (2000) and King et al. (1988), but also of transitory nominal shocks in the presence of nominal rigidities, as in the models of Blackburn (1999), Pelloni (1997) and Stadler (1990). In all of these cases, stochastic trends are generated not by some arbitrary, exogenous impulse process, but rather by endogenous responses of technology to changes in the current state of the economy.[1]

Recently, the question of how the structure of the business cycle (in particular, the volatility of ‡uctuations) might a¤ect long-term growth has been the subject of more detailed consideration. Broadly speaking, one may distinguish between two contrasting approaches with the potential to generate di¤erent conclusions based on alternative assumptions about the mechanism responsible for engendering endogenous technological change. According to one class of models - models in the spirit of Schumpeter (1942), where the mechansim takes the form creative destruction - recessions are events which have a positive impact on growth by reducing the opportunity cost of diverting resources away from manufacturing towards productivity improvements (e.g., Aghion and Saint-Paul 1998a, 1998b). According to another class of models - models in the spirit of Arrow (1962), where the mechansim is based on learning-by-doing - recessions are episodes which have a negative e¤ect on growth by lowering factor employment through which expertise, knowledge and skills are acquired and disseminated (e.g., Martin and Rogers 1997, 2000). Depending on which of these perspectives is taken, it is possible to draw very di¤erent implications concerning the overall relationship between short-term volatility and long-term growth - that is, a positive relationship in the case of the former, but a negative relationship in the case of the latter.[2]

Signi…cantly, none of the above analyses take into account the role of savings behaviour in determining growth. By contrast, de Hek (2000) develops a model of cyclical-secular interactions in which learning-by-doing is com-

[1] This view of aggregate ‡uctuations raises questions not only about the traditional decomposition of time series into cyclical and secular components, but also about the more recent practice of conditioning this decomposition on the assumption that demand shocks have no long-run e¤ects (e.g., Blanchard and Quah 1989).

[2] For related contributions on the subject, see Caballero and Hammour (1994), Hall (1991) and Ramey and Ramey (1991).

bined with the accumulation of physical capital, and the source of stochastic fluctuations is a random productivity parameter in the learning process itself. The main finding is that the correlation between growth and volatility depends acutely on the nature of agents' preferences, as reflected in the elasticity of intertemporal substitution: the higher (lower) is this elasticity, the more likely it is that an increase in uncertainty will lower (raise) long-run growth. A similar result is established by Jones et al. (1999) in a model of stochastic growth based on the private accumulation of physical and human capital (rather than the public accumulation of non-rival knowledge) into which final output is converted subsequent to the realisation of technology shocks.[3]

Empirically, the sign of the relationship between growth and volatility remains inconclusive. For example, there are a number of studies based on cross-country or cross-regional comparisons in which the correlation between the average growth of output and the variability of output growth is found sometimes to be positive (e.g., Grier and Tullock 1989; Kormendi and Meguire 1985), sometimes to be negative (e.g., Martin and Rogers 2000; Ramey and Ramey 1995) and sometimes to be zero (e.g., Dawson and Stephenson 1997). Similarly, evidence from the fewer time series investigations of individual countries is also mixed, suggesting a correlation that is positive in some instances (e.g., Caporale and McKiernan 1996), but statistically insignificant in others (e.g., Grier and Perry 2000; Speight 1999).

This paper is intended as a further contribution to the integration of business cycle analysis with growth theory. Like the contributions by de Hek (2000) and Jones et al. (1999), it may also be viewed within a broader context as an extension of the literature on optimal savings and growth under uncertainty (e.g., Brock and Mirman 1972; Leland 1968; Levhari and Srinivasan 1969; Mirman 1971; Rothschild and Stiglitz 1971). Our preoccupation in the paper involves taking a closer look at the non-stationary time series properties of a typical stochastic growth model (with the inclusion of money) in which learning-by-doing accounts for linkages between the short-run (cyclical) and long-run (secular) movements in economic activity.[4] The model is deliberately constructed so as to enable us to establish our results

[3] de Hek (2000) also considers the case of physical and human capital accumulation, but in a two-sector model where uncertainty derives from randomness in the return on human capital investment. While there is the prediction of a negative correlation between growth and volatility, the analysis is restricted to the case of logarithmic utility for reasons of tractability.

[4] For a recent evaluation of learning-by-doing, including the estimation of an explicit learning-by-doing function and a quantitative investigation of the role of learning-by-doing in propagating fluctuations, see Cooper and Johri (1999).

analytically on the basis of closed-form representations for the stochastic decision rules that solve agents' intertemporal optimisation problems. The key feature of the model is that the means and variances of variables are determined jointly as part of the dynamic general equilibrium. As a consequence, both the …rst and second moments of disturbances have …rst-order e¤ects on both the …rst and second moments of the growth rate of output.

The main insight of our analysis is that, given learning-by-doing, the relationship between cyclical volatility and secular growth depends fundamentally on two main factors - the source of stochastic ‡uctuations and the functioning of the labour market. As regards the former, we consider three di¤erent varieties of exogenous shock, two of which are real (a preference shock and a government spending shock) and one of which is nominal (a monetary growth shock).[5] As regards the latter, we distinguish between two types of labour market environment according to whether wages are ‡exible (as in a pure real business cycle model) or predeterimed by contracts (as in a new-Keynesian-type model).[6] Depending on both the nature of the disturbance and the circumstances in the labour market, the correlation between short-term volatilty and long-term growth may be either positive or negative. As in de Hek (2000), this contradicts the normal presumption of a singularly negative correlation in situations (like ours) where the engine of growth is learning-by-doing. On the contrary, our analysis yields the unequivocal results that long-run growth is negatively related to the volatility of (non-neutral) nominal shocks, but positively related to the volatility of real shocks. From an empirical standpoint, these results are consistent with the observations of a generally ambiguous relationship between output growth and output variability, but a systematically negative relationship between output growth and nominal variability (e.g., Grier and Tullock 1989; Grier and Perry 2000; Judson and Orphanides 1996; Kormendi and Meguire 1985).

The remainder of the paper is organised as follows. In Section 2 we present a description of the model. In Section 3 we solve for the stochastic dynamic general equilibrium. In Section 4 we establish our main results.

[5] Each of these shocks has been incorporated previously into real business cycle-type models (e.g., Stockman and Tesar 1995; Christiano and Eichenbaum 1992; Cooley and Hansen 1997). We abstract from technology shocks solely for simplicity since the e¤ects of these shocks are subdued in the usual way (and rendered inconsequential for our principle concern with the relationship between growth and volatility) by the assumptions needed to ensure closed-form solutions. For related analyses that deal with such disturbances, see Benavie et al. (1996) and Jones et al. (1999).

[6] Alternatively, or simultaneously, one might wish to consider similar di¤erences in the structure of the goods market (i.e., price ‡exibility or price rigidity). Our presumption is that one would obtain the same basic results as those established here. For recent evidence on nominal wage and price stickiness, see Kahn (1997) and Taylor (1998).

And in Section 5 we offer a few concluding remarks.

2 The Model

Time is disrete and indexed by $t = 0, \ldots, \infty$. We consider an artificial economy in which there are constant populations (normalised to one) of identical, immortal households and identical, competitive firms. This economy has the familiar basic structure of a standard stochastic growth model. It is deliberately stylised in order to focus and simplify the analysis, and is not meant to provide a complete account of the mechanisms underlying aggregate fluctuations. In particular, since our intention is to illustrate without having to resort to numerical simulations, we adopt the usual specifications of preferences and technologies that admit closed-form solutions. That is, we assume logarithmic utility functions, Cobb-Douglas production functions and 100 percent rates of depreciation. We have no reason to believe that our main results would not survive under more general specifications that would be unlikely to yield significant additional insights from the loss of analytical tractability. What is important for obtaining our results is that one preserves the intrinsic dependence of the equilibrium decision rules on the variance-covariance structure of the random disturbances. It is worth noting that this aspect is often lost - albeit for excusable purposes - due to the common practice (especially in the absence of closed-form solutions) of deriving such rules by taking linear (e.g., log-linear) approximations which bestow the convenient property of certainty equivalence at the cost of sacrificing the higher order moments of the stochastic processes and missing out potentially important effects of uncertainty.

2.1 Firms

The representative firm combines N_t units of labour with K_t units of capital to produce Y_t units of output according to

$$Y_t = (Z_t N_t)^{\alpha} K_t^{1-\alpha}; \quad \alpha \in (0, 1). \tag{1}$$

The term Z_t represents an index of knowledge which is freely available to all firms and which is acquired through serendipitous learning-by-doing. As in Romer (1986), this provides the mechanism of endogenous growth in the model. Following convention, we approximate the stock of disembodied knowledge by the aggregate stock of capital which is taken as given by each firm so that learning takes the form of a pure externality.

Labour and capital are hired from households at the real wage rate $\frac{W_t}{P_t}$ and real rental rate R_t, respectively, where W_t is the nominal wage and P_t is the price of output. Profit maximisation implies

$$\frac{W_t}{P_t} = \alpha Z_t^{\alpha} N_t^{\alpha - 1} K_t^{1-\alpha} = \frac{\alpha Y_t}{N_t}; \tag{2}$$

$$R_t = (1-\alpha) Z_t^{\alpha} N_t^{\alpha} K_t^{-\alpha} = \frac{(1-\alpha) Y_t}{K_t}. \tag{3}$$

2.2 Households

The representative household derives lifetime utility, U, according to

$$U = \sum_{t=0}^{\infty} \beta^t \left[\gamma_t \log(C_t) + \theta \log\left(\frac{M_{t-1}\phi_t}{P_t}\right) - \lambda L_t\right]; \quad \beta \in (0,1); \theta, \lambda > 0; \tag{4}$$

where C_t denotes consumption, $\frac{M_{t-1}\phi_t}{P_t}$ denotes real money balances and L_t denotes labour. To generate a demand function for money, we adopt the familiar short-cut device of introducing money directly into the utility function, rather than specifying explicitly a separate transactions technology. The quantity M_{t-1} is understood to denote beginning-of-period t (i.e., end-of-period $t-1$) nominal cash balances which are augmented by a proportional monetary transfer, ϕ_t.[7] This transfer is an exogenous random variable, as is the term γ_t which represents a preference a shock (or a shock in the underlying transactions process). The specification of the labour term, λL_t, is another feature that our model shares with several others and may be interpreted in one of two ways - either as a simple assumption that individuals derive linear utility from leisure, or as a reduced-form preference ordering under circumstances where labour is indivisible and individuals choose employment lotteries in the manner of Hansen (1985) and Rogerson (1988).[8] Defining A_t as real assets and S_t as real lump-sum taxes, the budget constraint for the household is given by

$$C_t + \frac{M_t}{P_t} + A_{t+1} = \frac{W_t}{P_t} L_t + \frac{M_{t-1}\phi_t}{P_t} + R_t A_t - S_t. \tag{5}$$

[7] In some models, it is end-of-period (rather than beginning-of-period) money holdings that serve as the reference point. To the extent that money yields utility by facilitating transactions, it seems more reasonable to adopt the present formulation. The assumption that monetary transfers are proportional (rather than lump-sum) is made largely for analytical convenience, as in other investigations (e.g., Benassy 1995).

[8] This term could be generalised to λL_t^{η} $(\eta > 1)$ without affecting our main results.

Each household confronts the problem of maximising the expected value of intertemporal utility in (4) subject to the sequence of budget constraints in (5). The information set conditioning expectations consists of the values of all parameters, the current and past values of all variables and the probability distributions of all shocks. The problem is solved, in part, by choosing plans for consumption, money balances and asset holdings that satisfy the following conditions:

$$\frac{\gamma_t}{C_t} = \beta E_t(\gamma_{t+1} R_{t+1} C_{t+1}); \tag{6}$$

$$\frac{\gamma_t}{P_t C_t} = \beta\theta \frac{1}{M_t} + \beta E_t(\gamma_{t+1} \phi_{t+1} P_{t+1} C_{t+1}); \tag{7}$$

where E_t denotes expectations.[9] Plans for the number of hours to work are governed by circumstances in the labour market, about which we make two alternative assumptions. The first is that this market is competitive with households being free to choose their desired labour supplies, given perfectly flexible wages. This implies the condition

$$\lambda = \frac{\gamma_t W_t}{P_t C_t}. \tag{8}$$

The second assumption is that the labour market is non-competitive, but characterised, instead, by monopolistic unions that choose a nominal wage at which households supply whatever labour is demanded by firms. We assume that wage setting takes place prior to the realisations of shocks on the basis of one-period contracts. In this case, therefore, the economy displays nominal rigidities, as in the early contracting models of Gray (1976) and Fischer (1977), as well as those of a more recent vintage (e.g., Benassy 1995; Cho and Cooley 1995; Cooley and Hansen 1995). In contrast to these models, however, we suppose that the contract wage is chosen so as to maximise households' expected utility (e.g., Hairault and Portier 1993; Rankin 1998; Soskice and Iverson 2000), rather than to satisfy some ad hoc criterion, such as the maximisation of other union objectives or the requirement that the labour market is expected to clear. When making this choice, workers take account of the response of labour demand, as expressed in (2). Given this, the optimal wage set at the end of period $t-1$ for period t is found to satisfy[10]

$$\lambda E_{t-1}(N_t) = \alpha W_t E_{t-1}(\gamma_t N_t P_t C_t). \tag{8'}$$

[9] The term $\frac{\gamma_t}{C_t}$ is understood to be the marginal utility of consumption, or shadow value of wealth, being equal to the Lagrange multiplier attached to (5).

[10] That is, W_t is chosen so as to maximise the expected value of (4) subject to (5) and the condition that $L_t = N_t = (W_t P_t \alpha Z_t^{\alpha} K_t^{1-\alpha})^{\frac{1}{\alpha-1}}$ from (2).

The equilibrium of the household is now characterised completely by the first-order conditions in (6), (7) and either (8) or (8′), the budget constraint in (5) and the transversality conditions $\lim_{\tau\to\infty}\beta^{\tau}E_t()\gamma_{t+\tau}M_{t+\tau-1}\phi_{t+\tau}P_{t+\tau}C_{t+\tau} = \lim_{\tau\to\infty}\beta^{\tau}E_t()\gamma_{t+\tau}A_{t+\tau+1}C_{t+\tau} = 0$.

2.3 Stochastic Processes

There are three types of random disturbance in the model, two of which are real and one of which is nominal. The two real disturbances are the preference (or transactions) shock, γ_t, and a fiscal shock, ψ_t. In order to focus on pure fiscal disturbances, we assume that the government runs a continuously balanced budget so that its total expenditures, G_t, are financed exclusively by its total tax receipts, S_t. In addition, we require that these disturbances are introduced in such a way as to be consistent with a steady state balanced growth equilibrium in which the share of government spending is constant. The simplest means of achieving this is to define ψ_t as the stochastically-determined (stationary) ratio of public expenditures, or taxes, to private consumption.[11] Accordingly,

$$G_t = S_t = \psi_t C_t. \tag{9}$$

The nominal disturbance is the random monetary transfer, ϕ_t.[12] This is understood to be a shock to the monetary growth rate, causing fluctuations in the money supply, H_t, in compliance with

$$H_t = \phi_t H_{t-1}. \tag{10}$$

We assume that the disturbances are governed by independent, bounded, stationary stochastic processes with constant first and second moments. The realisations of the disturbances, $\{\gamma_t; \psi_t; \phi_t\}$, are confined to lie within non-negative intervals, $\{(\underline{\gamma}; \bar{\gamma}); (\underline{\psi}; \bar{\psi}); (\underline{\phi}; \bar{\phi})\}$, that satisfy the bounds on employment. The unconditional means and variances of the disturbances are denoted, respectively, by $\{\mu_\gamma; \mu_\psi; \mu_\phi\}$ and $\{\sigma^2_\gamma; \sigma^2_\psi; \sigma^2_\phi\}$. Other than these properties, we place no restrictions on probability distributions.

[11] Alternatively, we could define ψ_t as the ratio of government expenditures to output (or investment), though the analysis is a little less straightforward in this case. The same type of approach to modelling fiscal shocks has been followed by others (e.g., Turnovsky 1993). Similarly, we assume for simplicity that government expenditures do not enter the utility or production functions of agents.

[12] The model could be extended to include another type of nominal shock - a velocity shock - by allowing the parameter θ in (4) to be stochastic. Since the effects of such a shock are essentially the same as the effects of ϕ_t, we focus on the latter for convenience.

3 General Equilibrium

The solution of the model is a stochastic dynamic general equilibrium which describes the aggregate behaviour of the economy based on the optimal decision rules that solve firms' and households' maximisation problems. The equilibrium is computed by combining the relationships obtained so far with the market clearing conditions $C_t + K_{t+1} + G_t = Y_t$ (for goods), $K_t = A_t$ (for capital), $M_t = H_t$ (for money) and $N_t = L_t$ (for labour). Given the structure of the model, we may proceed in two stages, the first of which entails determining the solutions for consumption, capital and money holdings for a given level of employment, and the second of which involves establishing the solution for employment, itself. Details of the derivations are relegated to an Appendix.

3.1 Consumption, Capital Accumulation and Cash Balances

After appropriate substitutions, we are able to write (6) and (7) as

$$\frac{\gamma_t K_{t+1}}{C_t} = \beta(1-\alpha)\mu\nu(1+\mu_\psi) + \beta(1-\alpha)E_t()\gamma_{t+1}K_{t+2}C_{t+1}; \tag{11}$$

$$\frac{\gamma_t M_t}{P_t C_t} = \beta\theta + \beta E_t()\gamma_{t+1}M_{t+1}P_{t+1}C_{t+1}. \tag{12}$$

Each of these expressions defines a stochastic expectations difference equation which is solved by imposing the relevant transversality condition. Doing this, and exploiting our other relationships, we obtain the following results:

$$C_t = \frac{(1-a)\gamma_t}{(1-a)\gamma_t(1+\psi_t) + a\mu\nu(1+\mu_\psi)}Y_t; \tag{13}$$

$$K_{t+1} = \frac{a\mu\nu(1+\mu_\psi)}{(1-a)\gamma_t(1+\psi_t) + a\mu\nu(1+\mu_\psi)}Y_t; \tag{14}$$

$$\frac{M_t}{P_t} = \frac{(1-a)\beta\theta}{(1-\beta)[(1-a)\gamma_t(1+\psi_t) + a\mu\nu(1+\mu_\psi)]}Y_t; \tag{15}$$

where $a = (1-\alpha)\beta$.

According to (13), (14) and (15), the equilibrium levels of consumption, capital and real money balances are stochastically proportional to the level of output.[13] For a given level of employment (on which output depends), each

[13] In the absence of preference and fiscal shocks, (13) and (14) would reduce to the standard expressions obtained in the simplest type of real business cycle model, namely $C_t = aY_t$ and $K_{t+1} = (1-a)Y_t$.

of these variables exhibits the usual response to each type of real disturbance: consumption increases, investment decreases and money demand decreases with higher realisations of the preference shock, γ_t; and consumption decreases, investment decreases and money demand decreases with higher realisations of the fiscal shock, ψ_t. The fact that these responses are non-linear plays a crucial role in our subsequent analysis. In particular, we note that the average output share of investment is an increasing function of the variances of the shocks. This is due to the positive effect of uncertainty on the precautionary demand for savings and can be established straightforwardly by appealing to the well-known result of Rothschild and Stiglitz (1970) that the expected value of a concave (convex) function of a variable is decreased (increased) by a mean-preserving spread of that variable. As indicated earlier, this result would not have survived had we followed the common practice (at least in the absence of closed-form solutions) of linearising the model at the point of establishing the first-order conditions for agents - a case of premature approximation that would have led to certainty equivalence which is not strictly satisfied in the model. By preserving the result, we keep sight of the true properties of the equilibrium decision rules which imply that both the first and second moments of disturbances have first-order effects on the average levels of variables. As we shall see, these effects are transmitted to the average growth rates of variables as well.

3.2 Employment

The solution for employment depends on the assumption about the structure of the labour market - that is, whether this market is treated as being either perfectly competitive with fully flexible wages, or imperfectly competitive with temporarily (one-period) fixed wages. We consider each case in turn.

In a perfectly competitive environment, households choose their labour supplies so as to satisfy the condition in (8). Together with our other relationships, this impies an equilibrium level of employment equal to

$$N_t = \frac{\alpha[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]}{\lambda(1-a)}. \tag{16}$$

In this case, therefore, employment depends only real disturbances in the usual manner, being positively related to both the preference shock, γ_t, and the fiscal shock, ψ_t. Evidently, since employment is invariant to the nominal disturbance, then so too are consumption, capital and output. Thus the economy exhibits the properties of a pure real business cycle model in which monetary fluctuations are neutral, having no real effects.

In an imperfectly competitive environment with one-period wage contracts, households supply labour on demand from firms, given the optimally chosen wage implied by (8′). After various manipulations, we can determine a precise expression for this wage which depends on expectations about the money supply during the course of a contract.[14] The equilibrium level of employment at this wage is found to be

$$N_t = \frac{\alpha^2[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]\phi_t}{\lambda(1-a)\mu_\phi}. \tag{16′}$$

Accordingly, employment now depends on the realisations of both real and nominal shocks. The consequence of each real shock is the same as above, while the effect of a random increase in the monetary growth rate, ϕ_t, is to raise employment. This effect is transmitted to consumption, capital and output, and the economy now displays the features of a new-Keynesian-type model in which (unanticipated) monetary fluctuations are no longer neutral.

The absence or presence of nominal factors is one way in which the levels of employment in (16) and (16′) differ. These factors would vanish from the latter were we to abandon the notion of contracts and assume, instead, that wages were chosen contingent on the realisations of shocks. Under such circumstances, the two levels of employment would deviate from each other only by a constant factor of proportionality, α.[15] This is the pure inefficiency effect of monopoly power which creates an upward bias to wages and a downward bias to employment relative to the case of perfect competition.

4 Growth and Volatility

We are now in a position to address the main issue of interest to us - namely, the extent to which there are linkages between the cyclical and secular properties of aggregate fluctuations. We do this by solving for the growth rate of output, from which the growth rates of other non-stationary variables (consumption and capital) may be inferred. These growth rates are both stochastic and endogenous. It is recalled that we account for the latter property on the basis of learning-by-doing, formalised by approximating the stock of disembodied knowledge available to firms by the aggregate stock of capital: that is, $Z_t = K_t$ in (1). As shown by others, the main implication of this is to make it possible for the level of output (and, with it, the levels of other

[14] This expression is given by (A10) in the Appendix.

[15] To be sure, note that $\mu_\phi = E_{t-1}(\phi_t)$ in (16′). In the case of contingent wage-setting, we would have $E_t(\phi_t) = \phi_t$ so that ϕ_t would vanish from this expression.

variables) to depend on the accumulated realisations of any type of shock, whether real or nominal, temporary or permanent. The significant additional insight obtained from the present analysis is that the average rate at which output grows is a function of the variances of the shocks, implying a non-trivial relationship between secular growth and cyclical volatility. The precise nature of this relationship depends acutely on both the source of volatility and the operation of the labour market. These results are demonstrated as follows.

4.1 The Economy Without Rigidities

With perfect competition in all markets and complete flexibility of all prices, the economy attains an equilibrium in which the level of employment is given by (16). Substituting this, together with (14), into (1), we obtain

$$\frac{Y_{t+1}}{Y_t} = \frac{\alpha^\alpha a\mu_\gamma(1+\mu_\psi)[(1-a)\gamma_{t+1}(1+\psi_{t+1}) + a\mu_\gamma(1+\mu_\psi)]^\alpha}{\lambda^\alpha(1-a)^\alpha[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]}; \qquad (17)$$

Naturally, the growth rate of output depends only on real disturbances in this type of economy. Using standard formulae, the mean and variance of this growth rate can be approximated, respectively, as[16]

$$\mathrm{Mean}(Y_{t+1}/Y_t) \simeq A(1 + m_\gamma\sigma_\gamma^2 + m_\psi\sigma_\psi^2); \qquad (18)$$

$$\mathrm{Var}(Y_{t+1}/Y_t) \simeq A^2(v_\gamma\sigma_\gamma^2 + v_\psi\sigma_\psi^2); \qquad (19)$$

where $A = a[\alpha\mu_\gamma(1+\mu_\psi)\lambda(1-a)]^\alpha$, $m_\gamma = \frac{(1-a)^2[\alpha(\alpha-1)+2]}{2\mu_\gamma^2}$, $m_\psi = \frac{(1-a)^2[\alpha(\alpha-1)+2]}{2(1+\mu_\psi)^2}$, $v_\gamma = \frac{(1-a)^2(\alpha^2+1)}{\mu_\gamma^2}$ and $v_\psi = \frac{(1-a)^2(\alpha^2+1)}{(1+\mu_\psi)^2}$. A casual glance at these expressions reveals that both $\mathrm{Mean}(Y_{t+1}/Y_t)$ and $\mathrm{Var}(Y_{t+1}/Y_t)$ are increasing in both σ_γ^2 (the variance of the preference shock) and σ_ψ^2 (the variance of the fiscal shock). In terms of (17), the actual growth rate of output is convex in the lagged realisations of shocks, but concave in the current realisations of shocks. The former property is a direct reflection of the convexity in savings behaviour, alluded to earlier, which is transmitted linearly to production via the process of learning-by-doing. The latter property is a symptom of diminishing returns to labour which convert linear employment responses into non-linear output effects. An increase in either σ_γ^2 or σ_ψ^2 has a positive impact on $\mathrm{Mean}(Y_{t+1}/Y_t)$ through the savings channel which more than offsets the negative impact on $\mathrm{Mean}(Y_{t+1}/Y_t)$ through the employment channel. The net result, therefore, is that the average growth rate of output increases with

[16] See, for example, Mood et al. (1974).

the variances of both types of disturbance. Since the variance of the growth rate displays the same property, one is led to conclude that there is a positive correlation between long-term growth and short-term volatility. What is notable about this conclusion is that it runs counter to the normal presumption that growth and volatility are negatively correlated in models of endogenous technological change based solely on learning-by-doing (as our model is). Indeed, some authors have contended that any model in which learning is a concave function of economic activity (taken to be the most plausible case) is sure to produce such a correlation (e.g., Martin and Rogers 2000). Yet this property is just as true in our model which yields the unambiguously opposite result that long-run growth is positively related to the volatility of real shocks. This suggests that the set of (reasonable) circumstances under which smoother cyclical fluctuations might be associated with flatter secular trends is wider than has hitherto been thought, and that empirical findings of a positive correlation between growth and volatility do not militate against the hyposthesis of learning-by-doing as a mechanism of growth.[17]

4.2 The Economy With Wage Contracts

When the labour market is imperfectly competitive and wages are predetermined by one-period contracts, the equilibrium of the economy entails a level of employment given by (16′). Substituting this, together with (14), into (1) yields

$$\frac{Y_{t+1}}{Y_t} = \frac{\alpha^{2\alpha} a\mu_\epsilon(1+\mu_\psi)[(1-a)\gamma_{t+1}(1+\psi_{t+1}) + a\mu_\epsilon(1+\mu_\psi)]^\alpha \phi^\alpha_{t+1}}{\lambda^\alpha(1-a)^\alpha \mu^\alpha_\psi[(1-a)\gamma_t(1+\psi_t) + a\mu_\epsilon(1+\mu_\psi)]}. \qquad (17')$$

In accordance with our previous results, the growth rate of output is now dependent on the realisations of both real and nominal shocks. Proceeding as above, the mean and variance of this growth rate are approximated, respectively, by

$$\mathrm{Mean}\left(\frac{Y_{t+1}}{Y_t}\right) \simeq \alpha^\alpha A(1 + m_\epsilon\sigma^2_\epsilon + m_\psi\sigma^2_\psi + m_\phi\sigma^2_\phi); \qquad (18')$$

$$\mathrm{Var}\left(\frac{Y_{t+1}}{Y_t}\right) \simeq (\alpha^\alpha A)^2(v_\epsilon\sigma^2_\epsilon + v_\psi\sigma^2_\psi + v_\phi\sigma^2_\phi); \qquad (19')$$

[17] As mentioned previously, de Hek (2000) is also able to obtain a positive correlation in a model of (stochastic) learning-by-doing for a sufficiently high degree of risk aversion. The argument of Martin and Rogers (2000) is based on their result of a negative correlation between growth and the variance of real (technology) shocks in a model without savings. This result is explained in terms of the concavity of the learning function, which implies that the loss of learning during recessions more than offsets the gain in learning during expansions. In our model, this effect is dominated by the effects on growth of changes in the savings behaviour of agents.

where $m_\phi = \frac{\alpha(\alpha-1)}{2\mu_\phi^2}$, $v_\phi = \frac{\alpha^2}{\mu_\phi^2}$ and σ_ϕ^2 is recalled to be the variance of the nominal shock. Expression ($18'$) reveals that output grows on average at a lower rate in this case than in the previous case. This is due partly to the inefficiency introduced by monopolistic wage setting and partly, and more significantly, to the effect of nominal volatility arising from monetary non-neutrality when wage setting takes place according to non-contingent contracts. For the same reasons, expression ($19'$) shows that the variance of the growth rate is also different in this case from what it was previously. Evidently, both $Mean\left(\frac{Y_{t+1}}{Y_t}\right)$ and $Var\left(\frac{Y_{t+1}}{Y_t}\right)$ depend positively on the variances of the real shocks in the same way as before. By contrast, average growth falls, while the volatility of growth rises, with an increase in the variance of the nominal shock. This type of disturbance impacts on growth through its (linear) effect on employment, of which output is a concave function by virtue of diminishing returns to labour. Given an increase in the variance of this shock, one is confronted with a negative, not positive, correlation between long-term growth and short-term volatility so that smoother cyclical fluctuations are associated with steeper, not flatter, secular trends. These are the results that one typically encounters in other models of learning-by-doing. Compared to the present framework, however, those models are based either on technology shocks with a dubious absence of savings behaviour (e.g., Martin and Rogers 2000), or on shocks to the learning process, itself, with a questionably high elasticity of intertemporal substitution (e.g., de Hek 2000).

5 Concluding Remarks

Collecting our results together, we arrive at the main verdict of this paper, which is that the correlation between long-term growth and short-term volatility depends fundamentally on two main factors - the source of stochastic fluctuations and the functioning of the labour market. As regards the former, we predict this correlation to be positive if real shocks predominate, but negative if nominal shocks predominate. As regards the latter, we predict the correlation to be positive in the absence of nominal rigidities, but either positive or negative in the presence of such rigidities. These results may help to explain why it has been difficult to find systematic empirical evidence in favour of one type of relationship over the over. By the same token, if such evidence becomes available in the future, then the results may be used as additional information with which to evaluate competing theories.

It is worth re-emphasising that our analysis has been based on an entirely standard model under an entirely conventional set of assumptions. Even within the confines of this model, our results do not exhaust the full range of potential interactions between growth and volatility. For example, suppose that the probability distributions of shocks are of a type for which the first and second moments are not independent of each other (as we have otherwise assumed). One such type of distribution is the frequently-used log-normal distribution, reflecting the specification of a shock as $x_t = \exp(X_t)$, where X_t is normally distributed with mean μ_X and variance σ_X^2: hence x_t, itself, is log-normally distributed with mean $\mu_x = \exp(\frac{1}{2}\sigma_X^2)$ and variance $\sigma_x^2 = \exp[\frac{1}{2}\sigma_X^2(\sigma_X^2 - 1)]$. In this case both the actual and average values of variables will be functions of σ_X^2 due to additional expectations effects through μ_x.[18] This serves to reinforce the point that linkages between growth and volatility are not difficult to establish, but rather arise naturally, and ought to be treated more as the rule, rather than the exception.

From a policy perspective, our analysis has been kept simple by the assumptions that both monetary growth and fiscal expenditures are determined by exogenous stochastic processes. At the same time, this allows one to draw clear and notable implications concerning the effects of policy variability or uncertainty. These implications are that monetary uncertainty is bad for growth, while fiscal uncertainty is good for growth.[19] By thinking slightly beyond the present confines of our model, one will also begin to appreciate some wholly new considerations of equal importance. The most conspicuous of these relates to the evaluation of stabilisation policies - that is, policies designed to smooth fluctuations by mitigating the impact of exogenous shocks. Given our results, it is possible to speculate about other consequences of such policies, consequences that are typically ignored but that may actually be the most significant. Our results suggest that policies aimed at stabilising nominal shocks would have the added bonus of delivering higher growth, while policies directed towards stabilising real shocks would have the adverse effect of reducing long-term growth. At present, there are very few existing analyses that provide fully worked-out examples of the potential long-run implications of short-term stabilisation policy. Two notable exceptions that reach different conclusions are the recent contributions by Blackburn (1999) and Martin and Rogers (1997). Providing a further contribution within the

[18] To be sure, let $x_t = \{\gamma_t; \psi_t; \phi_t\}$ and $X_t = \{\Gamma_t; \Psi_t; \Phi_t\}$. It is straightforward to verify that the solutions for all of our variables remain unchanged, except for the fact that $\{\mu_\gamma; \mu_\psi; \mu_\phi\} = \{\exp(\frac{1}{2}\sigma_\Gamma^2); \exp(\frac{1}{2}\sigma_\Psi^2); \exp(\frac{1}{2}\sigma_\Phi^2)\}$.

[19] These results are consistent with those established by Benavie et al. (1996), Grinols and Turnovsky (1993) and Turnovsky (1993) in different models and for different reasons (based on portfolio substitution effects). Aizenman and Marion (1993) present evidence which suggests that the effects of policy are highly sensitive to the type of policy, the sample period and the geographical region.

context of the current paradigm would appear to be an avenue of research worth pursuing.

Our analysis has been based deliberately on an analytically tractable framework for which closed-form solutions could be obtained from appropriate assumptions about preferences and technologies. The alternative approach would have been to use a more complicated model under more general assumptions and to conduct the analysis via numerical simulations. We have no reason to believe that our main results would have been different had we followed this alternative. Nevertheless, it would be interesting to acquire an idea of the orders of magnitude of our results, together with their implications for welfare, especially since it takes only small changes in the growth rate to produce substantial cumulative gains or losses in output. We intend to pursue this in later work by conducting a quantitative analysis of a more general, calibrated version of the model.

Appendix

The results in (13), (14) and (15) may be computed as follows. Substitution of (3) into (6) delivers

$$\frac{\gamma_t}{C_t} = \beta(1-\alpha)E_t\left(\frac{\gamma_{t+1}Y_{t+1}}{C_{t+1}K_{t+1}}\right). \tag{A1}$$

Exploiting $Y_t = C_t + K_{t+1} + G_t$, where G_t is given by (9), (A1) may be transformed into

$$\frac{\gamma_t K_{t+1}}{C_t} = \beta(1-\alpha)E_t[\gamma_{t+1}(1+\psi_{t+1})] + \beta(1-\alpha)E_t\left(\frac{\gamma_{t+1}K_{t+2}}{C_{t+1}}\right), \tag{A2}$$

which, in turn, may be converted to (11) by substitution of $E_t(\gamma_{t+1}) = \mu_\gamma$ and $E_t(\psi_{t+1}) = \mu_\psi$. Given the transversality condition $\lim_{\tau\to\infty}\beta^\tau E_t\left(\frac{\gamma_{t+\tau}K_{t+\tau+1}}{C_{t+\tau}}\right) = 0$, the solution to (11) is

$$\frac{\gamma_t K_{t+1}}{C_t} = \frac{\beta(1-\alpha)\mu_\gamma(1+\mu_\psi)}{1-\beta(1-\alpha)}. \tag{A3}$$

Combining (A3) with $Y_t = C_t + K_{t+1} + G_t$ gives the results in (13) and (14). Similarly, (10) in conjunction with $H_t = M_t$ implies that (7) may be converted to (12), the solution to which follows by imposing the transversality condition $\lim_{\tau\to\infty}\beta^\tau E_t\left(\frac{\gamma_{t+\tau}M_{t+\tau}}{P_{t+\tau}C_{t+\tau}}\right) = 0$ to obtain

$$\frac{\gamma_t M_t}{P_t C_t} = \frac{\beta\theta}{1-\beta}. \tag{A4}$$

Together with (13), (A4) yields the result in (15).

The results in (16) and (16′) may be derived in the following manner. Substitution of (13) into (8) yields

$$\lambda = \frac{[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]W_t}{(1-a)P_tY_t}. \tag{A5}$$

The condition in (2) may then be used to obtain (16). Substitution of (13) into (8′) yields

$$\lambda E_{t-1}(N_t) = \alpha E_{t-1}\left\{\frac{[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]W_tN_t}{(1-a)P_tY_t}\right\}. \tag{A6}$$

Exploiting (2), together with $E_{t-1}(\gamma_t) = \mu_\gamma$ and $E_{t-1}(\psi_t) = \mu_\psi$, (A6) can be reduced to

$$E_{t-1}(N_t) = \frac{\alpha^2\mu_\gamma(1+\mu_\psi)}{\lambda(1-a)}. \qquad \text{(A7)}$$

In turn, (2) and (15) may be combined to obtain

$$N_t = \frac{\alpha(1-\beta)[(1-a)\gamma_t(1+\psi_t) + a\mu_\gamma(1+\mu_\psi)]M_t}{\beta\theta(1-a)W_t}, \qquad \text{(A8)}$$

which, on taking expectations, delivers

$$E_{t-1}(N_t) = \frac{\alpha(1-\beta)\mu_\gamma(1+\mu_\psi)E_{t-1}(M_t)}{\beta\theta(1-a)W_t}. \qquad \text{(A9)}$$

Equating (A7) with (A9) yields the following expression for the contract nominal wage:

$$W_t = \frac{\lambda(1-\beta)E_{t-1}(M_t)}{\alpha\beta\theta}. \qquad \text{(A10)}$$

Substituting (A10) into (A8) produces the result in (16′) after making use of (10) and the fact that $E_{t-1}(M_t) = M_{t-1}\mu_\phi$.

References

[1] Aghion, P. and G. Saint-Paul, 1998a. On the virtue of bad times: an analysis of the interaction between economic ‡uctuations and productivity growth. Macroeconomic Dynamcs, 2, 322-344.

[2] Aghion, P. and G. Saint-Paul, 1998b. Uncovering some causal relationships between productivity growth and the structure of economic ‡uctuations: a tentative survey. Labour, 12, 279-303.

[3] Aizenman, J. and N. Marion, 1993. Policy uncertainty, persistence and growth. Review of International Economics, 1, 145-163.

[4] Arrow, K.J., 1962. The economic implications of learning-by-doing. Review of Economic Studies, 29, 155-173.

[5] Bean, C., 1990. Endogenous growth and the pro-cyclical behaviour of productivity. European Economic Review, 34, 355-363.

[6] Benassy, J-P., 1995. Money and wage contracts in an optimising model of the business cycle. Journal of Monetary Economics, 35, 303-315.

[7] Benavie, A., E. Grinols and S.J. Turnovsky, 1996. Adjustment costs and investment in a stochastic endogenous growth model. Journal of Monetary Economics, 38, 77-100.

[8] Blackburn, K., 1999. Can stabilisation policy reduce long-run growth? Economic Journal, 109, 67-77.

[9] Blanchard, O.J. and D. Quah, 1989. The dynamic e¤ects of aggregate demand and supply disturbances. American Economic Review, 79, 655-673.

[10] Brock, W.A. and L.J. Mirman, 1972. Optimal economic growth and uncertainty: the discounted case. Journal of Economic Theory, 4, 479-513.

[11] Caballero, R. and M. Hammour, 1994. The cleansing e¤ect of recessions. American Economic Review, 84, 350-368.

[12] Caporale, T. and B. McKiernan, 1996. The relationship between output variabiity and growth: evidence from post-war UK data. Scottish Journal of Political Economy, 43, 229-236.

[13] Cho, J.O. and T.F. Cooley, 1995. The business cycle with nominal contracts. Economic Theory, 6, 13-33.

[14] Cooley, T.F. and G.D. Hansen, 1995. Money and the business cycle. In T.F. Cooley (ed.), Frontiers of Business Cycle Research. Princeton University Press: Princeton.

[15] Christiano, L.J. and M. Eichenbaum, 1992. Current real business cycle theories and aggregate ‡uctuations. American Economic Review, 82, 430-450.

[16] Cooper, R. and A. Johri, 1999. Learning-by-doing and aggregate ‡uctuations. NBER Working Paper No.6898.

[17] Dawson, J.W. and E.F. Stephenson, 1997. The link between volatility and growth: evidence from the States. Economics Letters, 55, 365-369.

[18] de Hek, P.A., 1999. On endogenous growth under uncertainty. International Economic Review, 40, 727-744.

[19] Fatas, A., 2000. Endogenous growth and stochastic trends. Journal of Monetary Economics, 45, 107-128.

[20] Fischer, S., 1977. Long-term contracts, rational expectations and the optimal money supply rule. Journal of Political Economy, 85, 163-190.

[21] Gray, J., 1976. Wage indexation: a maroeconomic approach. Journal of Monetary Economics, 2, 221-235.

[22] Grier, K.B. and M.J. Perry, 2000. The e¤ects of real and nominal uncertainty on in‡ation and output growth: some Garch-M evidence. Journal of Applied Econometrics, 15, 45-58.

[23] Grier, K.B. and G. Tullock, 1989. An empirical analysis of cross-national economic growth, 1951-1980. Journal of Monetary Economics, 24, 259-276.

[24] Grinols, E. and S.J. Turnovsky, 1993. Risky, the …nancial market and macroeconomic equilibrium. Journal of Economic Dynamics and Control, 17, 1-36.

[25] Hairault, J-O. and F. Portier, 1993. Money, new-Keynesian macroeconomics and the business cycle. European Economic Review, 37, 1533-1568.

[26] Hall, R.E., 1991. Recessions as reorganisations. NBER Macroeconomics Annual.

[27] Hansen, G.D., 1985. Indivisible labour and the business cycle. Journal of Monetary Economics, 16, 309-328.

[28] Jones, L.E., R.E. Manuelli and E. Stacchetti, 1999. Technology (and policy) shocks in models of endogenous growth. NBER Working Paper No.7093.

[29] Judson, R. and A. Orphanides, 1996. In‡ation, volatility and growth. Mimeo, Board of Governers of the Federal Reserve System.

[30] Kahn, S. 1997. Evidence of nominal wage stickiness from micro data. American Economic Review, 87, 993-1008.

[31] King, R., C.I. Plosser and S. Rebelo, 1988. Production, growth and business cycles II: new directions. Journal of Monetary Economics, 21, 195-232.

[32] Kormendi, R. and P. Meguire, 1985. Macroeconomic determinants of growth: cross-country evidence. Journal of Monetary Economics, 16, 141-163.

[33] Leland, H.E., 1968. Saving and uncertainty: the precautionary demand for saving. Quarterly Journal of Economics, 82, 465-473.

[34] Levhari, D. and T.N. Srinivasan, 1969. Optimal savings under uncertainty. Review of Economic Studies, 36, 153-163.

[35] Martin, P. and C.A. Rogers, 1997. Stabilisation policy, learning-by-doing and economic growth. Oxford Economic Papers, 49, 152-166.

[36] Martin, P. and C.A. Rogers, 2000. Long-term growth and short-term economic instability. European Economic Review, 44, 359-381.

[37] Mirman, L.J., 1971. Uncertainty and optimal consumption decisions. Econometrica, 39, 179-185.

[38] Mood, A.M., F.A. Graybill and D.C. Boes, 1975. Introduction to the Theory of Statistics, McGraw-Hill: London.

[39] Nelson, C.R. and C.I. Plosser, 1982. Trends and random walks in macroeconomic time series: some evidence and implications. Journal of Monetary Economics, 10, 139-162.

[40] Pelloni, A., 1997. Nominal shocks, endogenous growth and the business cycle. Economic Journal, 107, 467-474.

[41] Ramey, G. and V.A. Ramey, 1991. Technology commitment and the cost of economic ‡uctuations. NBER Working Paper No.3755.

[42] Ramey, G. and V.A. Ramey, 1995. Cross-country evidence on the link between volatility and growth. American Economic Review, 85, 1138-1152.

[43] Rankin, N., 1998. Nominal rigidity and monetary uncertainty. European Economic Review, 42, 185-199.

[44] Rogerson, R., 1988. Indivisible labour, lotteries and equilibrium. Journal of Monetary Economics, 21, 3-16.

[45] Romer, P.M., 1986. Increasing returns and long-run growth. Journal of Political Economy, 94, 1002-1037.

[46] Rothschild, M.R. and J.E. Stiglitz, 1970. Increasing risk I: a de…nition. Journal of Economic Theory, 2, 225-243.

[47] Rothschild, M.R. and J.E. Stiglitz, 1971. Increasing risk II: its economic consequences. Journal of Economic Theory, 3, 66-84.

[48] Schumpeter, J., 1942. Capitalism, Socialism and Democracy. Harper and Brothers: New York.

[49] Soskice, D. and T. Iverson, 2000. The non-neutrality of monetary policy with large price or wage setters. Quarterly Journal of Economics, 115, 265-284.

[50] Speight, A.E.H., 1999. UK output variability and growth: some further evidence. Scottish Journal of Political Economy, 46, 175-181.

[51] Stadler, G.W., 1990. Business cycle models with endogenous technology. American Economic Review, 80, 150-167.

[52] Taylor, J.B., 1998. Staggered price and wage setting in macroeconomics. NBER Working Paper No.6754.

[53] Turnovsky, S.J., 1993. Macroeconomic policies, growth and welfare in a stochastic economy. International Economic Review, 34, 953-981.