Status, hyperbolic discounting, growth, and distribution

By

Christopher Tsoukis*

London Metropolitan University, London, UK

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Abstract:

We analyse hyperbolic discounting together with status in a standard model of intertemporal optimisation. Status reduces the additional discount rate that hyperbolic discounting introduces. Full commitment is assumed to deal with time inconsistency. While the steady state is standard, both aggregate and individual transitional dynamics are affected by hyperbolic discounting and status. The latter delivers more and possibly stronger implications than the generic short-termism of hyperbolic discounting, including a rise in the variance of consumption relative to that of income; greater co-movement between consumption and predictable income changes; and saving rates that vary in the cross-section.

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***Address:** Economics, FBL, London Metropolitan University, Calcutta House, Old Castle Street, London E1 7NT, UK. E-mail: c.tsoukis@londonmet.ac.uk

‘It is a sign of the times. There is absolutely no patience in the world now.’
Sir Alex Ferguson, former manager of Manchester United FC

1. Introduction

The aim of this paper is to combine hyperbolic discounting with status seeking, in the form of relative consumption affecting the discounting of utility, and to analyse the consequences in terms of growth, income and wealth distribution. Hyperbolic discounting is gaining traction as a way of capturing various insights of psychology and experimental/behavioural economics about attitudes to time. The key idea of hyperbolic discounting is that individuals are more impatient if a dilemma involving intertemporal choices emerged now than if the same dilemma presented itself now but involving future choices (Laibson, 1997, 2001; see Angeletos *et al*., 2001; and Frederick *et al*., 2002, for reviews). This is because the rate of time preference is high in the short run but lower in the future, as viewed from today's perspective. Strotz (1956) may have been among the first to recognise that short- and long-term discounting differ; that this creates time consistency issues and therefore problems of self-control; and to finally relate that to questions and mechanisms of commitment. These attitudes to time are formalised by Lowenstein and Prelec (1992) in postulating a psychological discount rate that declines with time. This is often assumed to apply over and above the constant discount rate that is involved in exponential or geometric discounting. In other words, asymptotically, hyperbolic discounting disappears, leaving only the exponential discounting of standard theory. In refining Lowenstein and Prelec’s (1992) postulates, al-Nowaihi and Dhami (2006) confirm that the implied discount factor takes the form of a generalised hyperbola, hence the designation ‘hyperbolic discounting’ (HD).

Early evidence of hyperbolically discounting behaviour is offered by Ainslie (1992) from psychological studies among animals and humans. Frederick *et al.* (2002) review evidence that seems to ‘overwhelmingly favour hyperbolic discounting over the exponential alternative’ (p. 361). Imputed discount rates seem to fall with the horizon over which the relevant decisions are to take effect, and hyperbolically-based discount factors fit the data better. Laibson *et al*. (2007) find considerable divergence between discount rates over the short- and long-run within lifetimes. Starz and Tsang (2012) offer evidence in favour of non-exponential discounting by exploring the implication of discount functions for term premia. Apart from formal evidence, anecdotal evidence of short termism also abounds (some is reviewed in Laibson, 1997; Ariely, 2008, Chapter 6). The quote from the subtitles, by Manchester United’s manager Sir Alex Ferguson, reflects the perceived short-termism in football; this is evident in the frequent sacking of managers. This occurs particularly under external pressure, e.g. under the influence of bad results.[1] Such external influence on short-termist behaviour leads us to introduce our contribution.

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[1] Ferguson made that remark on 9-2-2009 upon learning the news that his colleague Luiz Felipe Scolari had been sacked after only seven months at the helm of Chelsea. Whatever the immediate reasons, the sacking was widely attributed to the competitive pressures facing Premier League football in England, and to the pressure on club managers (by fans and high-spending owners alike) to produce good results immediately. Generalising, a world where there is a lot of pressure on agents for status becomes less patient. See: http://www.guardian.co.uk/football/2009/feb/09/ferguson-shocked-scolari-sacking

Our specific innovation is to introduce an interaction between 'status' and hyperbolic discounting. Status-seeking is the idea that individuals derive utility not only from their individual consumption, as standard theory suggests, but also from comparisons with 'the Joneses'. It is widely assumed that 'the Joneses' is society at large, so that relative (in addition to absolute) consumption confers utility as well (see Frey and Stutzer, 2002; Clark *et al*., 2008; for evidence and reviews). Such ideas have been widely investigated in macroeconomics; among a voluminous literature, one may cite Abel (1990), Gali (1994), Carroll, Overland and Weil (1997), Futagami and Shibata (1998), Corneo and Jeanne (2001), Ljungqvist and Uhlig (2000), Tsoukis (2007), Tournemaine and Tsoukis (2008, 2009), Tsoukis and Tournemaine (2012). Implications for growth and dynamics have been highlighted, among others, by Alonso-Carrera, Caballé and Raurich (2004) and Alvarez-Cuadrado, Monteiro and Turnovsky (2004).

Here, we incorporate the insights of hyperbolic discounting combined with status into a standard growth model. Following Barro's (1999) lead, we let the heterogeneous but infinitely-lived agents discount hyperbolically in the manner of Lowenstein and Prelec (1992) as well as exponentially. Additionally, we introduce status, defined in terms of relative consumption, in conjunction with the discount rate related to hyperbolic discounting. Specifically, we argue that those with lower status are subject to a higher discount factor, all else equal. The reasons why status may interact with our attitudes to time is that such attitudes involve the conflict between two aims, immediate self-gratification (which discounts heavily the future) and longer-term goals (which takes the future into account). For longer-term goals to prevail, one needs to exercise discipline over the tendency, or instinct, of immediate self-gratification, something that may be more difficult in a less favourable context such as less status. Banerjee and Duflo (2012) provides evidence of short-termism among the poor, as the poor have less inner incentive to fight the instinct of self-gratification. This is partly due to neurological reasons (the disproportionately high levels of cortisol – the hormone produced by stress) and partly due to psychological reasons (the will to discipline oneself weakens when the chances of success in the task for which discipline is required are minimal). As a result, poor people are likely to exercise less self-control and may be more present-biased.

Further evidence on how socio-economic status may affect reasoning and choices, including intertemporal ones, is provided by Wilkinson and Pickett (2009a); they survey studies that relate various measures of health and social problems and dysfunctions, including obesity, drug abuse and drug abuse-related deaths to income inequality within each country or US state; in other words, income inequality is positively correlated with symptoms of lack of ability to pursue long-term goals, even vital ones like maintenance of a healthy lifestyle. They also review studies of experiments with non-human primates which show that cortisol is related to lack of status. From these arguments and findings, the following logical chain emerges: low status produces cortisol, resulting in low self-control, therefore less ability to sacrifice current gratification for longer term goals. Furthermore, they argue, '[e]xposure to chronic stress shifts physiological priorities: Processes that are not essential when responding to immediate threat or danger— such as tissue maintenance and repair, digestion, growth, and reproductive functions—are all downregulated in favor of processes that improve reaction times and provide energy for muscular activity'

(Wilkinson and Pickett, 2009a, p. 505). The authors argue that these physiological implications of stress are an important intermediary mechanism by which the social environment has serious consequences for health. For our line of reasoning, this argument is important because those deprived of status are less capable of functions that contribute to longer-term well-being.

An additional plausible chain link is, from socio-economic status to health (as per the evidence in Wilkinson and Pickett, 2009a, 2009b), to higher future discounting due to lower probability of survival (as in Chakraborty, 2004). Naturally, the causal direction may be two-way, as has been pointed out in the health economics literature (e.g., Ikeda, Kang, and Ohtake, 2010, who show that temporal decision biases result in obesity and lower health), but this 'reverse causality' (from discounting to health) is beyond our scope. The bottomline is, there are a variety of channels through which socio-economic status may affect future discounting. Georgarakos, Haliassos and Pasini (2013) provide empirical support for this thesis: Using data from a Dutch household survey, they find robust social effects on borrowing and indebtedness, particularly among those considering themselves poorer than their peers.

Thus, our contribution may be seen as combining two deviations from the paradigm of exponentially-discounted intertemporal optimisation which epitomises standard notions of rationality. The deviations are hyperbolic discounting, which involves time inconsistency and regret as will be discussed shortly, and status, which, though not necessarily incompatible with rationality as a methodical pursuit of well-defined means, is a deviation of rationality as crystallised in the standard model. As Frederick et al. (2002, p. 377) argue, combining insights from more than one behavioural models may improve the intertemporally optimising model's normative and predictive appeal. This is what we do in this paper, and show that indeed there are great gains in the ability of the model to meet stylized macroeconomic facts.

As Barro (1999) notes, the main reason for the general reliance on exponential discounting is mainly analytical convenience: Exponential discounting is time consistent, whilst any other discounting scheme, including hyperbolic, is time inconsistent (see also Angeletos *et al*., 2001; and Thaler, 1981, for early evidence on dynamic inconsistency): Except in exponential discounting, any plan made now for action at any time t in the future generally differs from the actual actions taken when time t arrives. Consistency through time has the very convenient implication that any course of action determined now will be validated at any point in the future, so the decision time need only be once. In contrast, time inconsistency presents the modeller with some rather difficult choices not only because no course of action will be sustained but also because one would need to have a succession, potentially infinite, of decision times. To bypass these difficulties, one may assume commitment, whereby one course of action decided now will be sustained, although it may not be deemed optimal at any point in the future; this may happen if the individual by appropriate purchases (e.g. a pension plan, or housing) or loans commit themselves to future paths of consumption. If so, the individual will display the short-termism inherent in hyperbolic discounting, but over time they will revert to the course of action implied by exponential discounting alone – the 'rational' course of action which is free of short-termism and regrets. The alternative would be to assume that the 'long run' never arrives, that a decision taken now only holds until the next decision time (potentially infinitely closely), in other words to view the long run as a succession of short runs. Barro (1999) considers both alternatives.

The strategy taken in this paper is to assume full commitment. This argument faces two difficulties: firstly, lack of realism in assuming that mechanisms that ensure full, commitment exist; secondly, the idea that the commitment may have been undertaken in the past, perhaps the infinite past, so that we are now away from the short-termist end of the course of action and closer to the exponential-discounting, rational end. But there are also a number of arguments in favour of the case of full commitment: Full commitment is Euler-equation based, therefore it is intuitive and more familiar than the alternative of no commitment (which involves a completely different approach, see Barro, 1999). Moreover, one may interpret the short-termist end of the model as that which pertains more to the real world, and juxtapose its properties to those of the rational end emphasised by standard theory. In this paper, we shall indeed make this interpretation and follow this path that makes some sacrifices in realism in order to make tractable progress.

By taking the full commitment modelling route, our approach abstracts from problems of games between the two ‘selves’, the current self vying for immediate gratification while the future self taking a long view of benefit (see Laibson, 1997). We also abstract from the degree of sophistication or naivete that may characterise the individual with self-control problems – the degree of awareness that a postponed decision will be subject to the same problems of self-control in the future (Pollak, 1968; O’Donoghue and Rabin, 2000, 2001). The only (and strong) assumption we are making is that the individual finds ways to commit themselves to their initial decisions and plans via such mechanism is like pensions plans or the purchase of such illiquid assets as housing. It could be argued that these self-imposed mechanisms implicitly presuppose awareness by the individual of their self-control problems, hence they individual should be plausibly thought of as a ‘sophisticate’.

Our basic argument that poorer individuals, who are under more pressure from the status motive, are more prone to impatience and short-termism has important implications. Heterogeneity in terms of individual attitude to time results in different saving rates across the income distribution, with the richer individuals having lower discount rates and therefore exercising more patience and saving; an argument in line with the finding that the rich save proportionately more, which is not easily explained along more traditional lines of reasoning, see Dynan, Skinner, and Zeldes (2004). Moreover, it is well known that wealth inequality is considerably higher than income or earnings inequality (Wolff, 2010; Allegretto, 2012; Banks, Blundell, and Smith, 2000; Kennickell, 2009; Díaz-Giménez, Quadrini and Ríos-Rull, 1997). Again, this finding is a challenge to more conventional models of intertemporal choice as in those models wealth distribution is closely linked to that of income or earnings (De Nardi, 2004; Francis, 2009; Krusell and Smith, 1998). With attitudes to time and saving linked to the income distribution, models with hyperbolic discounting may be better placed to account for the gap in wealth and income inequality. In this respect, this analysis could reinforce the findings of Hendricks (2007) on the importance of discount factor heterogeneity for wealth inequality. Futhermore, there is evidence of co-movement between consumption and predictable income evolution that standard optimisation models find difficult to explain (Attanasio and Weber, 2010; Frederick *et al*., 2002). We show that the introduction of status in HD enables the model to better account for the excess co-movement of income and consumption or for under-saving (see O'Donoghue and Rabin, 2001). More generally, we show that the status effect on discounting delivers possibilities beyond those implied by non-geometric discounting *per se.*

The paper is structured as follows: Section 2 introduces the way we model the interaction between hyperbolic discounting and status; Section 3 explores the implications for a standard model of the individual's intertemporal optimization; in particular, we derive aggregate and individual dynamics, and show the effects of status on them. In Section 4 analyses more specific issues like the relative variance of income and wealth, the excess smoothness of consumption, and saving rates in the cross section and shows the effect of status. Section 4 concludes.

2. Hyperbolic discounting and status.

2.a: Hyperbolic discounting and status

As in Barro (1999), the individual is assumed to maximise:

$$U_\tau = \int_\tau^\infty u(C_{it})\exp\{-\rho(t-\tau) - \Phi^i_{\tau,t}\}\, dt, \tag{1}$$

subject to a standard budget constraint specified below. The non-standard part of the discount factor $\exp\{-\Phi^i_{\tau,t}\}$ introduces hyperbolic discounting (HD) in addition to exponential discounting; this additional discounting is individual-specific (as indicated by the superscript i). Our departure from Lowenstein and Prelec (1992), Barro (1999) and related analyses is that this 'customised' hyperbolic discount factor is affected by the individual's 'status', i.e. their relative consumption, as discussed in the Introduction. In other words, under pressure from lower status, lower-ranking individuals (in the consumption distribution) are more short-termist than the better off individuals who are closer to the benchmark exponential discounting.

More formally, we postulate the following discount factor for individual i:

$$\Phi^i_{\tau,t} \equiv \phi_{\tau,t}\left(\frac{C_{it}}{C_t}\right)^{-\theta}, \qquad \phi_{\tau,t} \equiv \int_\tau^t \varphi_{s-\tau} ds \tag{2}$$

This formulation decomposes the (log) discount factor into two parts: $\phi_{\tau,t}$ is the pure hyperbolic portion that regulates the extent of short-termism (see below); secondly, there is the effect of status, captured by relative consumption. The pure discounting portion $\phi_{\tau,t}$ has the following properties for $t \geq \tau$:[2]

[2] A couple of important clarifications are in order. Though we talk about 'hyperbolic discounting', we follow Barro (1999) in postulating a more general functional form for $\phi_{\tau,t}$ than a generalised hyperbola; this is both for more generality and because the underlying postulates for the latter appear somewhat specific (see e.g. al-Nowaihi and Dhami, 2006, Axioms A3 and A3a). Furthermore, we separate, as does Barro, the 'hyperbolic' part of the discount factor from the pure exponential, although the latter may be thought of as a special case of the former. This allows us to separate the component of the discount factor that is affected by status and that (i.e. the exponential one) that is not.

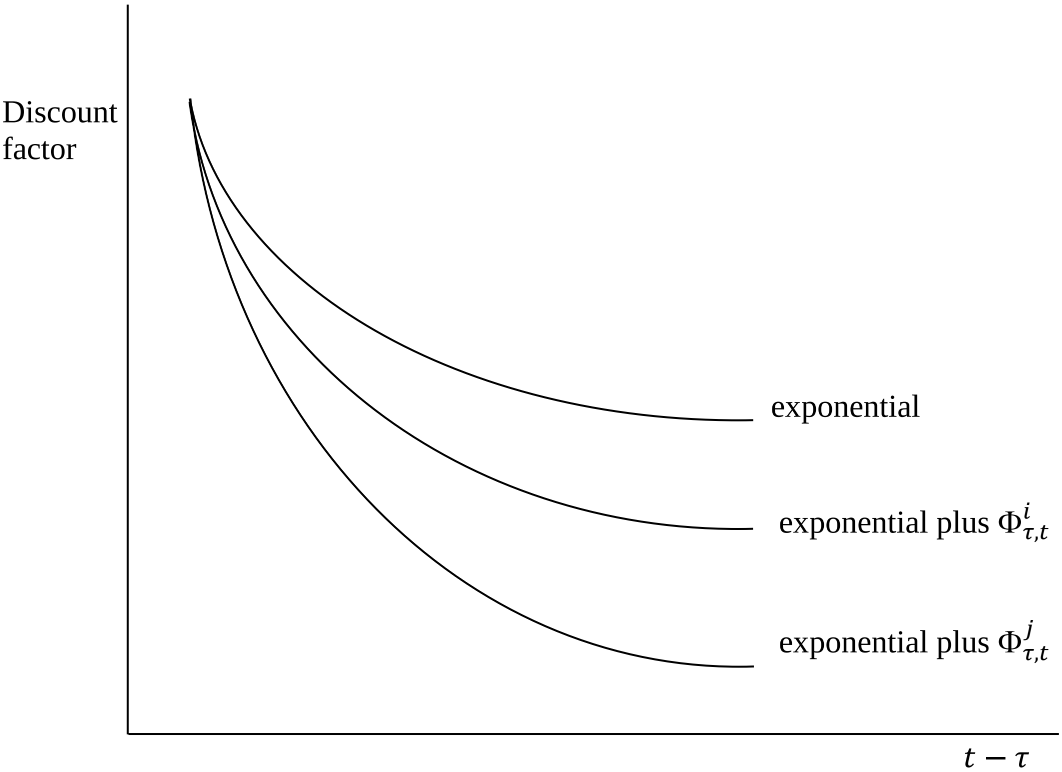
$$\phi_{\tau,t} \geq 0, \qquad \phi_{\tau,\tau} = 0, \qquad \frac{d\phi_{\tau,t}}{dt} = \varphi_{t-\tau} > 0, \tag{3a}$$

With the implied discount rate $\varphi_{t-\tau}$ being:

$$\varphi_0 > 0, \qquad d\varphi_{t-\tau}/dt \leq 0, \qquad \lim_{t\to\infty} \varphi_{t-\tau} = 0 \tag{3b}$$

The introduction of status affects the discount rate, an effect parameterised by $\theta \geq 0$. Figure 1 below graphically represents discounting in the presence of HD and status:

Figure 1: The total discount factor (geometric plus hyperbolic)



Three discount factors are shown: the purely exponential; and the exponential plus hyperbolic for two individuals, i, j, with $C_{it}>C_{jt}$. In all cases, current consumption ($t=\tau$) receives no discounting; when hyperbolic discounting is added, the individual with less status discounts the future more heavily. In the latter two cases, the curvature of the discount factor gives the degree of short-termism inherent in HD, where a sharper early decline implies a greater present-bias. We also see that for both individuals, the short-termism inherent in HD is evident in the short run; asymptotically, the discount rate goes to zero irrespective of status.

2.b: A parametric example

In order to obtain sharper results at a later stage, we shall use the following parametric discount rate:

$$\varphi \geq \varphi_{\tau-s} = \frac{\varphi}{(1+\varphi(t-\tau))^2} > 0 \ , \qquad \varphi \geq 0 \qquad (4a)$$

The constant φ regulates the curvature of the discount factor; a rise in φ increases the discount rate but more over shorter than longer horizons, hence it gives the degree of short termism inherent in HD. φ=0 is the benchmark case of no HD discounting.

Therefore, the hyperbolic discount factor is:

$$0 \leq \phi_{\tau,t} = \left[1 - \frac{1}{1+\varphi(t-\tau)}\right] < 1 \qquad (4b)$$

It is readily obvious that (4a, b) satisfy all the properties in (3a, b).

Therefore, we shall use:

- $\phi_{\tau,t}$ or equivalently the parameter that controls its curvature (φ) as an index of short-termism; the higher is $\phi_{\tau,t}$ for any t>τ the greater the short-termism and the relevance of HD;
- θ as an index of status in HD. A rise in θ amplifies the effects of status on the HD discount factor; e.g., in terms of Figure 1, it would increase the distance between the bottom two lines.

The focus of this paper is mainly on θ. We aim to show the effects obtainable when we allow for the possibility of θ>0 (while the standard HD model assumes θ=0).

3. The model under full commitment

3.1: Preliminaries: A 'yeoman farmer's' problem

The individual is assumed to be a 'yeoman' farmer who finances his/her consumption and capital accumulation out of current production; no recourse is made to capital markets. This implies that the marginal products of capital of the individual (r_{it}) are not equalised across agents. We choose this setup as it is the most fruitful in developing an endogenous distribution of individual consumption and capital. The alternative, of assuming perfect capital markets whereby marginal products are equalised to the (notional) real interest rate, would have provided too rigid a relation between individual and aggregate variables.

The budget constraint of the individual alluded to above is:

$$\dot{K}_{it} = Y_{it} - C_{it} \qquad (5)$$

Production is based on a standard 'learning by investing' production function:

$$Y_{it} = B_i K_{it}^{\beta} K_t^{1-\beta} \qquad (6)$$

Endogenous growth is guaranteed by constant returns to scale, due to the external, learning-by-investing effects (externalities derived from aggregate capital).

The individual's marginal product of capital is:

$$r_{it} = \beta B_i (K_t/K_{it})^{1-\beta} \tag{7}$$

Dividing the individual budget constraint (5) by individual capital, we get:

$$\dot{K}_{it}/K_{it} = Y_{it}/K_{it} - C_{it}/K_{it}$$

Aggregating over (6), taking a geometric-mean approximation to the true arithmetic mean of capital, we get an expression for aggregate output:

$$Y_t = \bar{B} K_t \tag{8}$$

Where $\bar{B}$≡exp{E$_i$log(B_i)} is the geometric mean of B_i; likewise, geometric means will be considered throughout, e.g. K_t≡exp{E$_i$log(K_{it})}. This approximation bypasses the difficulty that would arise with an arithmetic mean which would involve the variance of relative capital, which is time-dependent. Likewise, the aggregate over all individual marginal products of capital (7) is:

$$r_t = \beta \bar{B} \tag{9}$$

Note that this an aggregate defined in a similar manner (as a geometric mean over all individual marginal products), and is not a common interest rate to which all marginal products should be equalised.

Aggregate output is proportional to aggregate capital. Aggregating the individual budget constraints (5), we readily have the aggregate resource constraint:

$$\dot{K}_t = Y_t - C_t \tag{10}$$

Or, dividing through by the aggregate capital stock:

$$\frac{\dot{K}_t}{K_t} = \bar{B} - \frac{C_t}{K_t} \tag{10'}$$

(There is no depreciation.)

3.2: Optimisation

As mentioned, the individual maximises utility (1) subject to the budget constraint (5). Hereafter, the beginning of the planning period is set to τ=0. Standard FOC yield:

$$exp\{-\rho t - \Phi_{0,t}^i\} u_c(C_{it}) \left[1 + \theta \Phi_{0,t}^i \frac{u(.)}{u_c(.) C_{it}}\right] = \mu_t, \tag{11a}$$

where μ_t is the dynamic Lagrange multiplier. Furthermore,

$$\dot{\mu}_t = -r_{it} \mu_t \tag{11b}$$

Note that there will be no requirement for such marginal products to be equalised across agents except in the steady state (cf. the next sub-Section).

For tractability, we shall use iso-elastic utility,

$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-1/\sigma} \ , \tag{12}$$

where σ is the intertemporal elasticity of substitution. Thus, (11a) becomes:

$$exp\{-\rho t - \Phi_{0,t}^{i}\} u_C(C_{it}) \left[1 + \theta \Phi_{0,t}^{i} \frac{\sigma}{\sigma-1}\right] = \mu_t \tag{11a'}$$

Taking time derivatives of both sides and re-arranging, noting (2), we get:

$$\frac{1}{\sigma}\frac{\dot{C_{it}}}{C_{it}} - \theta \Phi_{0,t}^{i} \left[\frac{\dot{C_{it}}}{C_{it}} - \frac{\dot{C_t}}{C_t}\right] \left\{1 - \frac{\theta \frac{\sigma}{\sigma-1}}{1 + \theta \Phi_{0,t}^{i} \frac{\sigma}{\sigma-1}}\right\} =$$

$$= r_{it} - \rho - \frac{\partial \Phi_{0,t}^{i}}{\partial t} \left\{1 - \frac{\theta \frac{\sigma}{\sigma-1}}{1 + \theta \Phi_{0,t}^{i} \frac{\sigma}{\sigma-1}}\right\} \tag{13}$$

Furthermore, linearising the curly brackets around θ=0, we have, $1 - \frac{\theta \frac{\sigma}{\sigma-1}}{1+\theta \Phi_{\tau,t}^{i} \frac{\sigma}{\sigma-1}} \approx 1 - \theta \frac{\sigma}{\sigma-1}$. We shall assume that:

$$\Theta \equiv 1 - \theta \frac{\sigma}{\sigma-1} \geq 1$$

The presumed inequality follows from the intertemporal elasticity of substitution σ<1 in the data (see Hall, 1988). Hence, Θ=1 (θ=0) is the special case of HD but without the status effect. This quantity rises with θ; this parameter introduces the effect of time on the discount factor via the status (relative consumption) term.

Equation (13) represents the main effect of HD and status in our framework; it replaces the conventional Euler equation. HD increases the rate of time preference, but more so in the near future (low t) than later on; this effect is amplified by the association of status with discounting (the θ term in Θ). From now on, the subscript indicating the beginning of the planning period will be dropped (i.e., $\Phi_t^i \equiv \Phi_{0,t}^i$).

3.3: Steady state:

In the balanced-growth steady state, $lim_{t\to\infty}\varphi_t = 0$, so that by (2) and (3), $lim_{t\to\infty} \frac{\partial \Phi_t^i}{\partial t} = 0$. Moreover, all quantities, individual and aggregate, grow at the same rate g, therefore (13) gives:

$$g_i = g = \sigma(r - \rho) \tag{14a}$$

Where $g_i \equiv \frac{\dot{C_{it}}}{C_{it}}$ is the growth rate of individual consumption, and likewise for the aggregate growth rate g. In other words, in the steady state, we obtain the standard 'Keynes-Ramsey rule' of consumption growth of the conventional model, essentially because asymptotically, discounting reverts to the exponential form; under balanced growth the growth of relative consumption (the square brackets) is zero. Thus, relative positions are constant and depend on history. To avoid cluttering notation further, we shall drop time subscripts from the constant ratios and relative positions in the steady state, although individual variables do grow over time.

From production (8), marginal product (9) and national income accounting (11'), the steady-state investment-capital and consumption-capital ratios become:

$$\frac{I}{K} = g = \sigma(\beta\bar{B} - \rho) \tag{14b}$$

$$\frac{C}{K} = \bar{B} - \frac{I}{K} = \bar{B} - \sigma(\beta\bar{B} - \rho) \tag{14c}$$

The steady state is not affected by hyperbolic discounting, as in the steady state we have reverted to the standard setup of exponential discounting. This structure is akin to an 'AK model', which represents therefore the benchmark case of no HD.

Furthermore, manipulating the individual (5, 6, 7) and aggregate (8, 9, 10) production function, marginal product and budget constraints appropriately, noting that marginal products are equalised in the steady state, we get the following relation characterising relative consumption and capital:

$$\frac{C_i}{C} = \frac{K_i}{K} = (B_i/\bar{B})^{1/(1-\beta)} \tag{15}$$

Relative consumption and capital are equal and rigidly tied (by technical parameters) to the exogenous skills or productivity heterogeneity. Thus, from the point of view of understanding income distribution, the model without hyperbolic discounting, or indeed the steady-state version of the model with discounting, does not offer a rich framework of analysis.

3.4: Aggregate dynamics:

The key equation is the individual Euler equation (13). We linearise as follows:

$$(C_{it}/C_t)^{-\theta} \approx [1 - \theta\widehat{c_{it}}]$$

This represents a linearization around unity (the steady-state mean relative consumption), as is standard. The lower-case c is defined as relative consumption:

$$c_{it} \equiv C_{it}/C_t,$$

and hats indicate log-deviations from the steady state, e.g.$\widehat{c_{it}} \equiv logc_{it} - logc_i$. .

Therefore, ignoring a $\theta^2\widehat{c_{it}}(\frac{\dot{c_{it}}}{c_{it}})$ product, equation (13) simplifies to:

$$\frac{1}{\sigma}\frac{\dot{C_{\iota t}}}{C_{it}} - \theta\Theta\phi_t\ \left[\frac{\dot{C_{\iota t}}}{C_{it}} - \frac{\dot{C}_t}{C_t}\right] = r_{it} - \rho - \Theta\frac{\partial\phi_t}{\partial t}[1 - \theta\widehat{C_{\iota t}}] \tag{13'}$$

Under the assumed geometric mean,[3] we can readily integrate over all i to get the aggregate Euler equation:

$$\frac{\dot{C}_t}{C_t} = \sigma\left(\beta\bar{B} - \rho - \Theta\frac{\partial\phi_t}{\partial t}\right) \tag{16}$$

In sharp contrast to the standard 'AK' model, hyperbolic discounting introduces extraneous short-run dynamics that reduces the growth rate, as noted by Barro (1999). Under the linearisations we have carried out, the status effect embedded in hyperbolic discounting is present only in a multiplicative factor, therefore observationally equivalent to the short-termism introduced by pure HD. Thus, important implications for aggregate transitional dynamics stem from both HD and status.

The dynamics of capital may be obtained implicitly by combining (10') with (16) above and linearising to get:

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t} = \bar{B}(\sigma\beta - 1) + \frac{C}{K}(1 + \widehat{\left(\frac{C_t}{K_t}\right)}) - \sigma\left(\rho + \Theta\frac{\partial\phi_t}{\partial t}\right) \tag{17}$$

Where, as before, hatted values are log-deviations from steady-state ones, and C/K (without time subscripts) is the steady-state consumption-capital ratio. Note that in the balanced steady state the growth rate of aggregate consumption and capital are equal, so the LHS readily gives the growth rate of the consumption-capital ratio as a deviation from the steady state.

The dynamic solution of the aggregate consumption equation (16) is:

$$C_t = C_{0^+} exp\left\{\int_0^t \sigma\left(\beta\bar{B} - \rho - \Theta\frac{\partial\phi_s}{\partial s}\right) ds\right\} \tag{18}$$

Aggregate initial consumption C_0^+ is not the one given by endowment but one that is established one moment after the 'big bang' (t=0), as consumption is a 'jump' variable. To simplify, we assume that the only endowment received by individuals at the beginning of history is capital, (a 'predetermined' variable), with $K_{i0}>0$ differing across i. This is immediately put to production; the proceeds can either be consumed or invested. Hence, C_{0^+} represents the jump in consumption at the beginning of time; the initial jump of consumption at $t=0^+$ will be definitised next from the aggregate resource constraint.

The dynamic solution of the consumption-capital ratio (17), which is unstable therefore needs to be solved forward, in deviations form (hats) is:

$$\widehat{\left(\frac{C_t}{K_t}\right)} = \sigma\Theta\int_t^\infty \frac{\partial\phi_{t,s}}{\partial s} exp\left\{-\left(\frac{C}{K}\right)(s-t)\right\} ds \tag{19}$$

[3] Formally, C (without the individual-specific subscript i) is the reference standard to which individual i compare themselves in consumption; as such, it may well be the geometric mean.

Since the HD-related discount rate is positive, $\frac{\partial \phi_s}{\partial s} > 0$, it follows that $sgn\left\{\widehat{\frac{C_t}{K_t}}\right\} > 0$. The consumption-capital ratio attains higher levels earlier on in history and declines gradually towards its steady-state value; this is because, as will be shown shortly, consumption jumps at the beginning of history. This is important, as it effectively gives the consumption-income correlation over time, since aggregate output is of the 'AK' form (cf. 9); more on this below. Noting (10'), the growth rate of capital is inversely related to the consumption-capital ratio.

From (19), we can easily check the effect of the parameters of interest. The degree of short-termism, φ (embodied in the discount rate $\frac{\partial \phi_s}{\partial s}$), and the relevance of status (θ and Θ), raise the consumption-capital ratio and therefore decrease the growth rate of capital. In other words, we see that status enhances the short-termist features of HD (higher initial consumption, lower transitional growth) in the aggregate model. In a nutshell, consumption is more 'front-loaded', the more intensive is HD, and the status effect related to it. This is of interest as one of the stylised facts, and puzzles, related to consumption is that its growth is too sensitive to predictable changes in income (see e.g. Attanasio and Weber, 2010). We may check that the correlation between growth rates also rise with the parameters of interest, noting from (18) and (10') with (16) that the correlation between the growth rates of consumption and capital are given by the correlation between

$$exp\left\{\int_0^t \sigma\left(\beta\bar{B} - \rho - \Theta\frac{\partial \phi_s}{\partial s}\right)ds\right\} \quad \text{and} \quad -\sigma\Theta\int_t^\infty \frac{\partial \phi_{t,s}}{\partial s} exp\left\{-\left(\frac{C}{K}\right)(s-t)\right\}ds\ .$$

Thus, all parameters of interest raise this correlation. Hence, hyperbolic discounting, and status in it, help explain this puzzle.

Finally, to construct the full time profiles, we start from time t=0 when we have:

$$\widehat{\left(\frac{C_{0^+}}{K_0}\right)} = \sigma\Theta\int_0^\infty \frac{\partial \phi_s}{\partial s} exp\left\{-\left(\frac{C}{K}\right)s\right\}ds \qquad (19')$$

Since K_0 is predetermined, and given the steady-state ratio, this determines the jump in consumption one instant after the dawn of history. Both parameters of interest positively affect this jump. We can summarise as follows:

Proposition 1: Properties of the aggregate model under full commitment:

a) In the steady-state, the model with hyperbolic discounting is identical to a standard 'AK' model. Consumption and capital distribution are determined by the exogenous skills distribution and technological parameters.
b) In terms of transitional dynamics:
    i. The consumption-capital ratio at time $t=0^+$ rises with the short-termism embedded in HD (φ) and the relevance of status (θ); since capital is predetermined at t=0, this implies that consumption at the beginning of history rises with both parameters;
    ii. Correspondingly, the rate of growth of both consumption and capital from $t>0^+$ declines with both parameters of interest;

iii. Both parameters also raise the consumption-income ratio and the correlation between the respective growth rates.

3.5: Individual dynamics

The key equation is the Euler equation of individual consumption (13'), repeated below for convenience:

$$\frac{1}{\sigma}\frac{\dot{C_{it}}}{C_{it}} - \theta\Theta\phi_t \left[\frac{\dot{C_{it}}}{C_{it}} - \frac{\dot{C_t}}{C_t}\right] = r_{it} - \rho - \Theta\frac{\partial\phi_t}{\partial t}\left[1 - \theta\widehat{c_{it}}\right] \qquad (13')$$

Where $k_{it} \equiv \frac{K_{it}}{K_t}$ and $c_{it} \equiv \frac{C_{it}}{C_t}$, and where hatted variables indicate log-deviations from the steady state, e.g. $\widehat{k_{it}} \equiv logk_{it} - logk_i$. We also use a linearised version of the individual marginal product (7); since in the steady state we have $K_i = K(B_i/\bar{B})^{1/(1-\beta)}$, this implies in deviations form:

$$r_{it} \approx \beta\bar{B}\left[1 - (1-\beta)\widehat{k_{it}}\right] \qquad (7')$$

Multiplying (13') by σ, subtracting aggregate consumption dynamics (16) and using (7'), we get:

$$\left(1 - \sigma\theta\Theta\phi_t\right)\left[\frac{\dot{c}_{it}}{c_{it}}\right] = -\sigma\beta\bar{B}(1-\beta)\widehat{k_{it}} + \sigma\theta\Theta\frac{\partial\phi_t}{\partial t}\widehat{c_{it}} \qquad (20)$$

Note that (20) can be re-written using the transformed variable

$$\widetilde{c_{it}} \equiv \left(1 - \sigma\theta\Theta\phi_t\right)\widehat{c_{it}} \qquad (21)$$

as follows:

$$\dot{\widetilde{c_{it}}} = -\sigma\beta\bar{B}(1-\beta)\widehat{k_{it}} \qquad (20')$$

Finally, the equation describing individual capital dynamics is derived from the individual and aggregate budget constraints (5) and (10) and individual production function (6). After linearisations, these yield:

$$\frac{\dot{k}_{it}}{k_{it}} = -(1-\beta)\bar{B}\widehat{k_{it}} - \frac{C}{K}\left(\frac{\widetilde{c_{it}}}{1-\sigma\theta\Theta} - \widehat{k_{it}}\right) \qquad (22)$$

Individual consumption (20') and capital (22) dynamics can be represented in the following 2x2 system in more compact notation:

$$\begin{bmatrix}\dot{\widetilde{c_{it}}}\\ \dot{\widehat{k_{it}}}\end{bmatrix} = A\begin{bmatrix}\widetilde{c_{it}}\\ \widehat{k_{it}}\end{bmatrix} \qquad (23)$$

Where: $$A \equiv \begin{bmatrix} 0 & -\sigma\beta\bar{B}(1-\beta) \\ -\frac{C}{K}/(1-\sigma\theta\Theta) & \frac{C}{K} - (1-\beta)\bar{B}\end{bmatrix}$$

The system is saddle point-stable since $|A|<0$. Moreover, as the Appendix shows in more detail, with $\psi_1<0$ being the stable eigenvalue and $[q_1, 1]$ being the associated eigenvector, we are able to establish that:

$$0 < q_1 = -\frac{\sigma\beta\bar{B}(1-\beta)}{\psi_1} < 1 \qquad (24a)$$

In terms of Figure 2, the slope of the stable arm (q_1) is less than that of the 45^0 degree line. Furthermore (see Appendix 2):

$$\frac{\partial\psi_1}{\partial\theta} < 0 , \qquad \frac{\partial\psi_1}{\partial\varphi} = 0 \qquad (24b)$$

Since $|\psi_1|$ gives the speed of adjustment, the effect of status (θ) is to increase this speed. Moreover,

$$sgn\left\{\frac{\partial q_1}{\partial z}\right\} = sgn\left\{\frac{\partial\psi_1}{\partial z}\right\} \qquad (24c)$$

The effects of the parameters of interest, $z=\theta,\varphi$, on the slope of the stable arm (q_1) inherit the same signs as the effects on the stable eigenvalue. Therefore, the effect of status (θ) is to make the stable arm flatter. It is worth emphasising also that the degree of short-termism and index of HD (φ) does not affect the transitional dynamics in (23), either speed or the slope of the stable arm.

More formally, the trajectory of actual consumption can be obtained from the solution of (23):

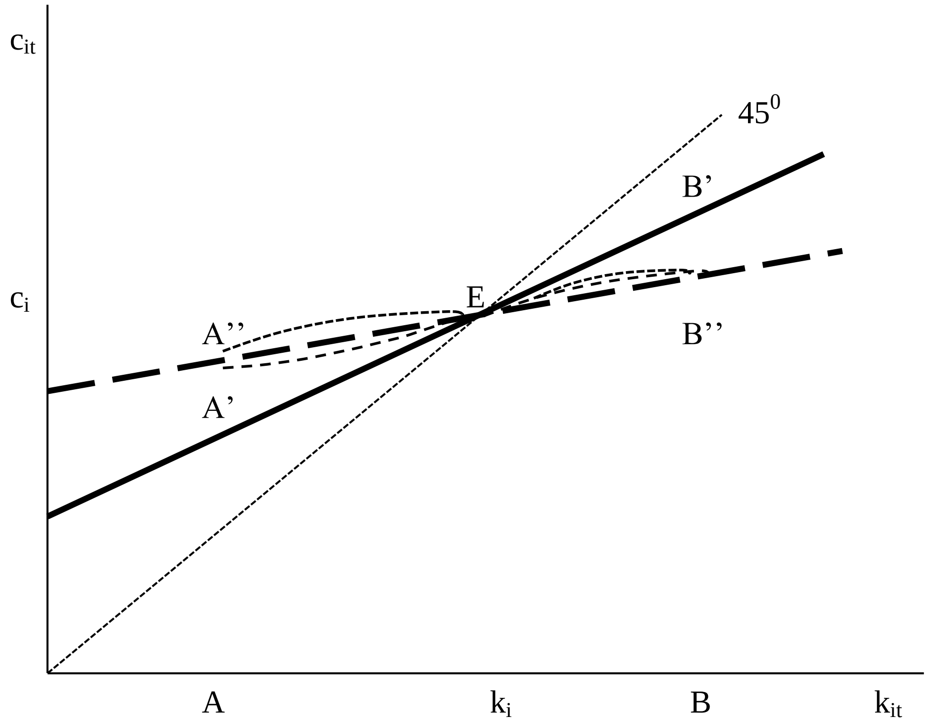
$$\begin{bmatrix}\widetilde{c_{\iota t}} \\ \widehat{k_{\iota t}}\end{bmatrix} = \begin{bmatrix}\widehat{k_{\iota 0}}q_1\exp\{\psi_1 t\} \\ \widehat{k_{\iota 0}}\exp\{\psi_1 t\}\end{bmatrix} \qquad (25)$$

Original consumption can be recovered by transformation (21), therefore:

$$\widehat{c_{\iota t}} = \frac{\widehat{k_{\iota 0}}q_1\exp\{\psi_1 t\}}{(1-\sigma\theta\Theta\phi_t\ )} \qquad (21')$$

Figure 2 illustrates the transitional dynamic paths of the individual system (25):

Figure 2: Transitory individual dynamics



The solid thick line is the trajectory that would apply if there were no HD or status in the system ($\theta=\varphi=0$) and we used the transformed variable $\widetilde{c_{it}}$. (Note that the graph is shown in terms of c_{it}, k_{it}, rather than their deviations from the steady state.) The steady state, represented by point E, is on the 45^0 line emanating from the origin, as $c_i=k_i$. The graph shows that in the case of an individual that starts out as poor, in the sense that their relative capital endowment is below their steady-state relative capital ($k_{i0}<k_i$, point A), their consumption will instantaneously jump upwards to point A' (to a level that will be indicated as c_{i0^+}), thereafter rising gradually to $c_i=k_i$ (point E). For a rich individual, in the relative sense, starting out at a point like B, the opposite process takes place (via B').

The broken thick line represents the trajectory of the variables with status in HD ($\theta>0$) evaluated at the asymptotic HD factor ($\phi_\infty=1$). The line is flatter (while still positive), as discussed above. As the final point is the same with or without HD (relative consumption as well as capital is exogenously determined by skills), it follows that the jump of a poor individual (who starts at A) is to have a higher initial jump in consumption (to A'') motivated by the higher discount rate (due to lower status) of this individual. Correspondingly, the relatively rich individual starting at B will have a lower initial jump (to B''). On these grounds, we expect the variance of early consumption to fall in relation to that of both steady-state consumption and, importantly, to that of contemporaneous capital; we return to this issue in the next Section.

Finally, the thin broken line represents the original consumption variable $\widehat{c_{it}} = \frac{\widehat{k_{i0}} q_1 \exp\{\psi_1 t\}}{(1-\sigma\theta\Theta\phi_t)}$ in (21'). The extra term represented by the denominator increases in principle the slope of this line in relation to $\widetilde{c_{it}}$, the broken thick line for t>0; but the two coincide at t=0 (as $\phi_0 = 0$). We need therefore to determine more formally the position of this line and we do so by examining its slope $q_1^t \equiv \frac{q_1}{(1-\sigma\theta\Theta\phi_t\ )}$ (for this, note 25 and 21') and how this responds to the parameters of interest:

$$\frac{\partial q_1^t}{\partial \theta} = \frac{\partial q_1/\partial \theta}{\left(1-\sigma\theta\Theta\phi_t\ \right)} + \frac{q_1 \sigma \phi_t}{\left(1-\sigma\theta\Theta\phi_t\ \right)^2} \frac{d(\theta\Theta)}{d\theta}$$

This derivative gives us the divergence of the slope of the consumption line from the solid thick line when the importance of status (θ) rises. Consider first the situation for low enough t (low enough ϕ_t); as $\frac{\partial q_1}{\partial \theta} < 0$, we get:

$$\frac{\partial q_1^t}{\partial \theta} < 0$$

(When θ=0, this derivative boils down to $\partial q_1/\partial \theta$, the change in slope between the solid and broken thick lines.) Thus, the early paths for individuals that begin with capital endowments either below or above their steady states (i.e., either from point A or B) are flatter than the solid thick line.

The slope however increases for high enough t. To this end, note that from the second line of matrix A, we get $q_1 = (1-\sigma\theta\Theta)\frac{\frac{C}{K}-(1-\beta)\bar{B}-\psi_1}{\frac{C}{K}}$, therefore the derivative can be expressed as:

$$\frac{\partial q_1^t}{\partial \theta} = q_1^t \frac{\sigma(\phi_t\ -1)}{(1-\sigma\theta\Theta)\left(1-\sigma\theta\Theta\phi_t\ \right)} \frac{d(\theta\Theta)}{d\theta} - q_1^t \left(\frac{1}{\frac{C}{K}-(1-\beta)\bar{B}-\psi_1} \right) \frac{\partial \psi_1}{\partial \theta}$$

with $\frac{d(\theta\Theta)}{d\theta} > 0$ and $\frac{\partial \psi_1}{\partial \theta} < 0$. Therefore, as ϕ_t rises with time, the positive sign dominates; the line becomes steeper than the solid thick line, as shown in the Figure.

On the other hand, it is clear that:

$$\frac{\partial q_1^t}{\partial \phi_t} > 0$$

Therefore, the degree of short termism (φ) increases the slope of this line. Note however that this effect applies only in association with the status effect, and it vanishes when θ=0; this is because the latter case implies that discounting is common to all individuals, therefore it 'washes out' of individual dynamics and remains only in the aggregate system. This confirms the importance of the interaction between the discount rate and status. Graphically, this effect is shown by the dotted (thin) lines: They are both above the broken thick line (as the slope is higher), thus showing higher consumption for both those that begin below and those above their steady state. In this respect, there is a significant difference with the effects of a rise in status, which affects differently those above and those below their steady state

In this regard, we should note that the approximation used in (22), $\widehat{c_{it}} \approx \frac{\widetilde{c_{it}}}{1-\sigma\theta\Theta}$, is taken at the asymptotic value of ϕ_∞=1; more generally, the difference ϕ_t-1<0 for finite t leaves a second-order term that has been ignored so far and will be ignored in what follows for tractability; Appendix B incorporates this term into the solution and outlines the ways in which it affects the results we obtain in the main text.

We may summarise as follows:

Proposition 2: On individual dynamics under full commitment:

a) A rise in the status effect in HD (θ):
   i. Decreases the speed of adjustment ($|\psi_1|$);
   ii. Increases consumption and decreases the transitional growth rate for those individuals with endowed relative capital less than the steady-state and conversely for those with endowed relative capital more than steady-state.

b) A rise in short-termism and generic HD (φ):
   i. Does not affect the speed of adjustment;
   ii. Increases consumption for all individuals, irrespective of whether their endowed relative capital is less or more than their steady-state values;
   iii. The effect in (ii) above only works in the presence of the status effect (θ>0).

4. Further results

To proceed, noting that the steady-state values of consumption and capital are identical (cf. 15), (25) with (21') and the definition $\widehat{x_{it}} \equiv logx_{it} - logx_i$ imply that the full time profiles are given by:

$$\begin{bmatrix} logc_{it} \\ logk_{it} \end{bmatrix} = \begin{bmatrix} logk_i\left(1 - \frac{q_1\exp\{\psi_1 t\}}{(1-\sigma\theta\Theta\phi_t)}\right) + \frac{q_1\exp\{\psi_1 t\}}{(1-\sigma\theta\Theta\phi_t)} logk_{i0} \\ logk_i(1 - \exp\{\psi_1 t\}) + \exp\{\psi_1 t\} logk_{i0} \end{bmatrix} \tag{26}$$

4.1: Cross-sectional variances and the income-wealth relative variance:

We now use this framework to derive results related to the variances of the key variables. Using the production function (6), the solution for individual capital in (26) and the steady-state relative capital (15), we get (relative) income dynamics:

$$logy_{it} = logk_i(1 - \beta\exp\{\psi_1 t\}) + \beta\exp\{\psi_1 t\} logk_{i0} \tag{27}$$

Assuming that the initial and steady-state relative capitals are independently distributed, an assumption which we shall maintain from here onwards, the variance of relative income is:

$$Var(y_{it}) = Var(logk_i)[1 - \beta\exp\{\psi_1 t\}]^2 + Var(logk_{i0})\beta^2[\exp\{\psi_1 t\}]^2 \tag{28a}$$

Furthermore, the variances of consumption and capital are given by:

$$Var(c_{it}) = Var(logk_i)[1-\Psi_t]^2 + Var(logk_{i0})[\Psi_t]^2 \tag{28b}$$

Where $\Psi_t \equiv \frac{q_1 \exp\{\psi_1 t\}}{(1-\sigma\theta\Theta\phi_t)}$, and:

$$Var(k_{it}) = Var(logk_i)[1-\exp\{\psi_1 t\}]^2 + Var(logk_{i0})[\exp\{\psi_1 t\}]^2 \tag{28c}$$

From (28a-c), we get the following effect of θ:

$$\frac{\partial Var(logy_{it})}{\partial\theta} = 2t\exp\{\psi_1 t\}\frac{\partial\psi_1}{\partial\theta}x$$

$$x\,\{-\beta[1-\beta\exp\{\psi_1 t\}]Var(logk_i) + \beta^2\exp\{\psi_1 t\}Var(logk_{i0})\} \tag{29a}$$

$$\frac{\partial Var(logc_{it})}{\partial\theta} = 2\frac{\partial\Psi_t}{\partial\theta}[-(1-\Psi_t)Var(logk_i) + \Psi_t Var(logk_{i0})] \tag{29b}$$

$$\frac{\partial Var(logk_{it})}{\partial\theta} = 2t\exp\{\psi_1 t\}\frac{\partial\psi_1}{\partial\theta}x$$

$$x\,[-(1-\exp\{\psi_1 t\})Var(logk_i) + \exp\{\psi_1 t\}Var(logk_{i0})] \tag{29c}$$

We furthermore note that the degree of short-termism and generic HD (φ) only affects consumption and its variance, and not income or capital; we defer a discussion of its effects till later.

We first consider the relation between income and capital variance and the effect that HD and status have on this. Taking the ratio between (29a) and (29c):

$$\frac{\frac{\partial Var(logy_{it})}{\partial\theta}}{\frac{\partial Var(logk_{it})}{\partial\theta}} = \frac{-\beta[1-\beta\exp\{\psi_1 t\}]Var(logk_i) + \exp\{\psi_1 t\}\beta^2 Var(logk_{i0})}{-(1-\exp\{\psi_1 t\})Var(logk_i) + \exp\{\psi_1 t\}Var(logk_{i0})}$$

In the benchmark case of equal initial and steady-state capital variances, this becomes:

$$\frac{\frac{\partial Var(logy_{it})}{\partial\theta}}{\frac{\partial Var(logk_{it})}{\partial\theta}} = \frac{-\beta + 2\beta^2\exp\{\psi_1 t\}}{-1+2\exp\{\psi_1 t\}} = \beta + \frac{\beta(1-\beta)2\exp\{\psi_1 t\}}{1-2\exp\{\psi_1 t\}}$$

For low enough t, $1\text{-}2\exp\{\psi_1 t\}<0$, so the above will be less than $\beta<1$ at least for that time. Hence, an increase in status (θ) will reduce the variance of income in relation to that of capital – the only measure of wealth in this model. Thus, the introduction of status in HD can help meet one major challenge in intertemporal macroeconomics, as argued in the Introduction. This is under a number of assumptions (independent distributions and equal variances for endowment and steady-state capital, low enough t), but this line of argument shows the possibilities; and the fact that t should be low enough underscores the idea that these effects are possible in the 'behavioural' (low t) and not the 'rational' (big t) end of the horizon. It is worth pointing out that the extent of short-termism (φ) cannot have this effect as it does not affect either of these variances.

Furthermore, the effect of status on the consumption variance is:

$$\frac{\partial Var(logc_{it})}{\partial \theta} = 2\frac{\partial \Psi_t}{\partial \theta}[-(1-\Psi_t)Var(logk_i) + \Psi_t Var(logk_{i0})]$$

Again, we can investigate some informative special cases:

At the beginning of time (t=0), when there is no discounting ($\phi_0 = 0$), we have:

$$0 < \Psi_0 \equiv \mathrm{q}_1 = -\frac{\sigma\beta\bar{B}(1-\beta)}{\psi_1} < 1$$

so that that $sgn\left\{\frac{\partial \Psi_0}{\partial \theta}\right\} < 0$ under the maintained assumptions.

$$sgn\left\{\frac{\partial Var(logc_{i0})}{\partial \theta}\right\} = -sgn\{-(1-\Psi_0)Var(logk_i) + \Psi_0 Var(logk_{i0})\}$$

Thus, a lot depends on the RHS of the above expression. We can distinguish two cases:

- High social mobility: $-(1-\Psi_0)Var(logk_i) + \Psi_0 Var(logk_{i0}) > 0$: This is the case of the variance of endowment capital variance being sufficiently higher than steady-state capital. In this case, the variance of early consumption will fall with the θ (relevance of status in HD);
- Low social mobility: $-(1-\Psi_0)Var(logk_i) + \Psi_0 Var(logk_{i0}) < 0$. In this case, the variance of early consumption will rise with status.

Again, the degree of short-termism in generic HD (φ) does not have any of these effects as it does not affect Ψ_0.

4.2: The consumption and income growth rates and 'excess co-movement' of consumption and income

Furthermore, from system (26) and (27), we get the growth rates of consumption and income during transition:

$$\frac{dlogc_{it}}{dt} = -\left(\psi_1 + \sigma\theta\Theta\frac{\mathrm{d}\phi_t}{dt}\frac{1}{1-\sigma\theta\Theta\phi_t}\right)\frac{\mathrm{q}_1\exp\{\psi_1 \mathrm{t}\}}{1-\sigma\theta\Theta\phi_t}(logk_i - logk_{i0}) \qquad (30a)$$

For consistency, we need to impose the restriction:

$$\psi_1 + \sigma\theta\Theta\frac{\mathrm{d}\phi_t}{dt}\frac{1}{1-\sigma\theta\Theta\phi_t} < 0 \qquad (31)$$

So that the consumption growth of an individual that is born 'poor' ($logk_i - logk_{i0}$) is positive. This is a rather mild restriction as can be established by the use of the functional form (4):

$$\psi_1 + \frac{\sigma\theta\Theta\varphi}{1+\varphi t}\frac{1}{1+(1-\sigma\theta\Theta)\varphi t} \leq \psi_1 + \sigma\theta\Theta\varphi \tag{31'}$$

The mild restriction $\psi_1 + \sigma\theta\Theta\varphi < 0$ (i.e. that the intensity of HD and status in it is not too pronounced) guarantees (31). Income growth, on the other hand, is:

$$\frac{dlogy_{it}}{dt} = -\beta\psi_1 \exp\{\psi_1 t\}(logk_i - logk_{i0}) \tag{30b}$$

Again, recourse should be made to (4), in which case the ratio between (30a) and (30b) becomes:

$$z_{it} \equiv \frac{dlogc_{it}}{dt} - \frac{dlogy_{it}}{dt} = \psi_1\left(\frac{(1+\varphi t)q_1}{1+(1-\sigma\theta\Theta)\varphi t} - \beta\right) + \frac{\sigma\theta\Theta\varphi q_1}{(1+(1-\sigma\theta\Theta)\varphi t)^2} > 0$$

For tractability, we consider only this difference at the beginning of history (t=0):

$$z_{i0} = \psi_1(q_1 - \beta) + \sigma\theta\Theta\varphi q_1$$

The effect of the parameters of interest is given by:

$$\frac{dz_{i0}}{d\theta} = -\beta\frac{\partial\psi_1}{\partial\theta} + \frac{\partial q_1}{\partial\theta}\sigma\theta\Theta\varphi + \frac{d(\theta\Theta)}{d\theta}\sigma\varphi q_1 > 0$$

And

$$\frac{dz_{i0}}{d\varphi} = \sigma\theta\Theta q_1 > 0$$

To understand these, we note that $\psi_1 q_1$ is free of any parameters of interest, while ψ_1 and q_1 are not affected by φ. Both parameters work to increase the 'sensitivity' of the consumption growth rate to predictable income growth rate, thus in principle going some way to meeting the relevant stylised fact at the individual level and one of the major challenges in the theory of consumption (see e.g. stylise fact 3 in Attanasio and Weber, 2010, Section 3). But status may hold a bigger potential in this respect: With $\frac{d(\theta\Theta)}{d\theta} = \Theta - \theta\frac{\sigma}{\sigma-1} > \theta\Theta > 1$ under maintained assumptions, the effect of status (θ) will be bigger than the effect of generic HD (φ) if

$$-\beta\frac{\partial\psi_1}{\partial\theta} + \frac{\partial q_1}{\partial\theta}\sigma\theta\Theta\varphi + \sigma q_1\left[(\varphi-\theta)\Theta - \theta\frac{\sigma}{\sigma-1}\right] > 0$$

This will be satisfied at least for the status motive being strong enough in relation to the generic short-termism embedded in HD as captured by φ. While of course this result is quite specific as it derived only for t=0, it is indicative of the status motive in the context of HD to increase the prescriptive appeal of the model.

4.3: Saving rates in the cross section:

Finally, in this sub-Section, we enquire about the saving rate, closely linked but not identical to the growth rate. From the individual budget constraint (5) and (6) production function, the saving rate may be written as:

$$s_{it} = \frac{Y_{it} - C_{it}}{Y_{it}} = \frac{\frac{\dot{K}_{it}}{K_{it}}}{\frac{Y_{it}}{K_{it}}} = \frac{\dot{K}_{it}}{K_{it}} \frac{1}{B_i} \left(\frac{K_{it}}{K_t}\right)^{1-\beta}$$

From the evolution of capital given above, this is:

$$s_{it} = \frac{dlogk_{it}}{dt} \frac{1}{B_i} \exp\{(1-\beta)logk_{it}\} = \frac{d\exp\{\psi_1 \mathrm{t}\}}{dt} (logk_{i0} - logk_i)\, x$$

$$x\ \frac{1}{B_i} \exp\{(1-\beta)[(1-\exp\{\psi_1 \mathrm{t}\})logk_i + \exp\{\psi_1 \mathrm{t}\} logk_{i0}]\}$$

Since $\frac{d\exp\{\psi_1 \mathrm{t}\}}{dt} < 0$, there is dis-saving from those that start with a capital endowment above that of the steady state, but positive saving from those that start below. We now consider how these results are affected by the parameters of interest.

We have:

$$\frac{ds_{it}}{d\theta} = \frac{s_{it}}{\psi_1} \frac{d\psi_1}{d\theta} [1 + t\exp\{\psi_1 \mathrm{t}\}\{1 + (1-\beta)(logk_{i0} - logk_i)\}]$$

Considering the situation at t=0, we have:

$$sgn\left(\frac{ds_{i0}}{d\theta}\right) = sgn\left(\frac{s_{i0}}{\psi_1} \frac{d\psi_1}{d\theta}\right)$$

Since $\frac{d\psi_1}{d\theta}$<0, 'poor' individuals, in the sense of $s_{it} < 0$ (borrowers) will lower their saving early on, as they raise their initial consumption due to their low status and high discount rate – vis. Figure 2. In contrast, those that start with endowments above the steady state (savers) will raise their saving further because of their higher status and lower discount rates.

In the general case, for a borrower $(logk_{i0} - logk_i < 0)$, then $1 + t(1 - \beta)(logk_{i0} - logk_i)$ changes sign at high enough t, so that the effect of status on individual saving reverses sign: status in HD will increase the saving of a borrower for a high enough t. Conversely, the 'savers' (those with endowment relative capital above steady-state) will experience a rise in saving early on and decline later as HD and status in it are intensified. So, we have a different regime for the 'hyperbolic' regime (low t) and the 'rational' regime (high t). We summarise as follows:

Proposition 3: Under full commitment:

a) With a rise in status in HD (θ):
   i. under the stylised assumptions of equally and independently distributed endowment and steady-state capital, and for a low enough t, the variance of income relative to that capital increases;
   ii. at the beginning of history (t=0), the variance of consumption increases under an additional assumption termed 'low social mobility' in the text;
   iii. for a low enough t, the saving of an individual that is 'poor' in the sense that their endowment relative capital is lower than their steady-state one (and is thus a borrower) decreases;
   iv. at the beginning of history (t=0), and under the parametric example of the HD discount factor (3a), the growth rate of consumption rises in relation to that of income.

b) With a rise in the degree of short-termism in generic HD (φ):
   i. at the beginning of history (t=0), and under the parametric example (3a), the growth rate of consumption rises in relation to that of income.
   ii. But under mild assumptions, the effect of b) (i) is less than the effect in a) (iv) above.

Thus, the introduction of status in HD has a number of implications, a) (i), (ii), and (iii), that are not implied by generic HD, while the one that is, a) (iv), is arguably a stronger effect than the one obtained by generic HD-related short-termism. Status in HD delivers an outcome whereby poorer households save less and richer ones more; poor and rich households are not scaled up (or down) factors of one another. This meets a major puzzle in macroeconomics, as argued in the Introduction. Note also that the short-termism of HD per se (φ) does not affect saving, so that it is only status in HD that presents these interesting possibilities.

Furthermore, the potential that a borrower's (negative) saving declines further with status in HD may lead to excessive indebtedness and high interest payments for the poorer individuals(see Angeletos *et al.*, 2001; Georgarakos *et al.*, 2013) or poverty traps (Kumhof and Ranciere, 2010). Still wider implications are also possible: Kumhof and Rancière (2010), and Rajan (2011) attribute the latest crisis to the weak financial markets that arose as those lower down the income ladder got pressured to catch up in housing by unaffordable borrowing. Debt and these are not modelled here, but these are conceptually straightforward but important extensions of the present arguments.

5. Conclusions

This paper has sought to analyse the interaction of the status-seeking motive and hyperbolic discounting in the context of an otherwise standard intertemporally optimising macro model. Both hyperbolic discounting and status have compelling evidence to back them up, and quite serious implications. The point of this paper is that their interaction is also important. Our specific contribution is to argue that status decreases the discount rate introduced by hyperbolic discounting (additional to that introduced by exponential discounting). The model is closed by assuming a 'yeoman-farmer' agent who splits their produce, derived from a learning-by-investing production function, into current consumption and investment for growth. The aim is to analyse the effects of both status and short-termism, and of the mode of their interaction, on aggregate growth, but also individual dynamics, distribution, and saving.

Full commitment is assumed to deal with the problem of time inconsistency inherent in any non-exponential scheme of discounting. We argue that, though its realism may be questioned, this model offers an interesting analytical device for juxtaposing the short-run, real-world aspects of the model, to its 'full-rationality' implications arrived at in the steady state when all hyperbolic discounting has evaporated and only exponential discounting remains. As the model reverts to standard optimisation in the long run, its steady-state properties are standard. But both hyperbolic discounting and status, and their interaction, matter in the short run in important ways.

We show that the aggregate growth rate declines during the transition because of the extra discount rate of hyperbolic discounting (φ), and this effect is enhanced by status (θ). *Ceteris paribus*, early consumption rises with both parameters, and consumption-income co-movement increases; since all income movement is predictable here, this increases the ability of the model to account for the relevant puzzle (see e.g. Attanasio and Weber, 2010). In terms of the individual dynamics, we generally find that the status effect in HD (θ) has more effects and stronger implications that our proxy for short-termism and generic HD intensity (φ). For instance, it decreases the speed of adjustment and increases (decreases) consumption for those individuals with endowed relative capital less (more) than the steady-state; of these, only the latter is affected by short-termism and generic HD intensity (φ) – in the same direction.

Other effects of status in HD include a positive effect on the variance of income relative to that capital; an increase of the variance of consumption; a decrease (increase) of saving by an individual whose endowment relative capital is lower (higher) than their steady-state one and who is thus a borrower (lender); and rise in the growth rate of consumption in relation to that of income. Of these effects, only the last only is triggered by a rise in the degree of short-termism in generic HD (φ), and arguably in a weaker degree. While these effects have been derived under some more or less special assumptions for tractability, they do indicate the potential of status in HD to increase the predictive appeal of the model in relation to well-known stylised facts and associated puzzles in macroeconomics and to deliver a number of interesting possibilities. Thus, the interaction introduced here between hyperbolic discounting and the status motive should be an interesting avenue for further research. The case of no commitment suggests itself as the next step.

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Appendix A

In this Appendix, we show the derivations underlying the results (24a-d). The eigenvalues of the time-invariant (22) are given from

$$\begin{vmatrix} -\psi_{\mathrm{i}} & -\sigma\beta\bar{B}(1-\beta) \\ -\frac{C}{K}/(1-\sigma\theta\Theta) & \frac{C}{K}-(1-\beta)\bar{B}-\psi_{\mathrm{i}} \end{vmatrix} = 0,$$

so that,

$$2\psi_{1,2} = \mathrm{trA} \mp \sqrt{\Delta},$$

where $\Delta \equiv (\mathrm{trA})^2 - 4|\mathrm{A}| > (\mathrm{trA})^2 > 0$. Therefore, the stable root is given by

$$2\psi_1 = C/K - (1-\beta)\bar{B} - \sqrt{(C/K-(1-\beta)\bar{B})^2 + 4\frac{C}{K}\sigma\beta\bar{B}(1-\beta)/(1-\sigma\theta\lambda\Theta)},$$

We note that under the (weak) assumption that $\frac{\sigma\beta}{(1-\sigma\theta\Theta)} < 1$, we have the following restriction on the stable eigenvalue:

$$-(1-\beta)\bar{B} < \psi_1 < 0$$

As can be verified from the fact that in this case $(C/K-(1-\beta)\bar{B})^2 < \Delta < \left(\frac{C}{K}+(1-\beta)\bar{B}\right)^2$. The eigenvectors are given from:

$$\begin{bmatrix} -\psi_{\mathrm{i}} & -\sigma\beta\bar{B}(1-\beta) \\ -\dfrac{C}{K}/(1-\sigma\theta\Theta) & \dfrac{C}{K}-(1-\beta)\bar{B}-\psi_{\mathrm{i}} \end{bmatrix} \begin{bmatrix} q_i \\ 1 \end{bmatrix} = 0$$

For each i=1,2. For i=1, which corresponds to the stable root, and from the 2nd row, we get:

$$q_1 = (1-\sigma\theta\Theta)\frac{\frac{C}{K}-(1-\beta)\bar{B}-\psi_1}{\frac{C}{K}} \qquad \text{(A1)}$$

i.e. the slope of the stable arm is positive. Moreover, since $-(1-\beta)\bar{B} < \psi_1 < 0$, we have:

$$0 < q_1 < 1 \qquad \text{(A2)}$$

Furthermore, from the 1st row, we get

$$q_1 = -\frac{\sigma\beta\bar{B}(1-\beta)}{\psi_1} > 0 \qquad \text{(A3)}$$

So that the effects of the parameters of interest are:

$$\frac{dq_1}{d\theta} = \frac{d\psi_1}{d\theta}\frac{\sigma\beta\bar{B}(1-\beta)}{(\psi_1)^2} < 0 \tag{A4}$$

We note that the hyperbolic discount rate plays no role here; we thus have (24c).

Furthermore, we have:

$$\frac{\partial 2\psi_1}{\partial\theta} = \frac{\partial \mathrm{trA}}{\partial\theta}\left(1 - \frac{\mathrm{trA}}{\sqrt{\Delta}}\right) + 2\frac{\partial|A_t|}{\partial\theta}\frac{1}{\sqrt{\Delta}},$$

and $0 < 1 - \frac{\mathrm{trA}}{\sqrt{\Delta}} < 1$. Collecting all the info we have:

$\mathrm{trA} = \frac{C}{K} - (1-\beta)\bar{B}$, therefore $\frac{\partial \mathrm{trA}}{\partial\theta} = 0$ and $\frac{\partial \mathrm{trA}}{\partial\varphi} = 0$;

$|A| = -\sigma\beta\bar{B}(1-\beta)\frac{C}{K}/(1-\sigma\theta\Theta)$, so $\frac{\partial|A|}{\partial\theta} = -\frac{\sigma\beta\bar{B}(1-\beta)\frac{C}{K}}{(1-\sigma\theta\Theta)^2}\frac{\mathrm{d}(\sigma\theta\Theta)}{\mathrm{d}\theta} < 0, \frac{\partial|\mathrm{A}|}{\partial\varphi} = 0$;

With $\Theta \equiv 1 - \theta\frac{\sigma}{\sigma-1}$, so that $\frac{\mathrm{d}\theta\Theta}{\mathrm{d}\theta} = \Theta - \theta\frac{\sigma}{\sigma-1}(>0)$ which can be plausibly thought to be positive (hence the inequality in brackets) under low enough σ and/or θ.

Therefore, we finally have:

$$\frac{\partial\psi_1}{\partial\theta} < 0, \qquad \frac{\partial\psi_1}{\partial\varphi} = 0 \tag{24b}$$

$$\frac{\partial q_1}{\partial\theta} < 0, \qquad \frac{\partial q_1}{\partial\varphi} = 0 \tag{24d}$$

Appendix B

Transformation (21), $\widehat{c_{\iota t}} = \frac{\widetilde{c_{\iota t}}}{(1-\sigma\theta\Theta\phi_t)}$, has been approximated as $\widehat{c_{\iota t}} \approx \frac{\widetilde{c_{\iota t}}}{1-\sigma\theta\Theta}$ in (22) (i.e., around ϕ_∞=1). In this Appendix, we derive a more precise approximation, and show the solution in the presence of the omitted second-order term; furthermore, we outline what effect this term will have on the results of the main text.

A more precise approximation is as follows:

$$\frac{\widetilde{c_{\iota t}}}{1-\sigma\theta\Theta\phi_t} \approx \frac{\widetilde{c_{\iota t}}}{1-\sigma\theta\Theta} + \sigma\theta\Theta(\phi_t - 1)\frac{\widetilde{c_{\iota 0^+}}}{2(1-\sigma\theta\Theta)^2}$$

This approximates ϕ_t around its steady-state value of ϕ_∞=1 and $\widetilde{c_{\iota t}} \approx \frac{(\widetilde{c_{\iota 0^+}}+\widetilde{c_{\iota\infty}})}{2} = \frac{\widetilde{c_{\iota 0^+}}}{2}$; in other words, we do not approximate $\widetilde{c_{\iota t}}$ around $\widetilde{c_{\iota\infty}} = 0$ as that would imply that we lose information. So that:

$$\frac{\dot{k}_{it}}{k_{it}} = -(1-\beta)\bar{B}\widehat{k_{\iota t}} - \frac{C}{K}\left(\frac{\widetilde{c_{\iota t}}}{1-\sigma\theta\Theta} + \sigma\theta\Theta(\phi_t \ -1)\frac{\widetilde{c_{\iota 0^+}}}{2(1-\sigma\theta\Theta)^2} - \widehat{k_{\iota t}}\right)$$

Thus, the system is:

$$\begin{bmatrix} \dot{\widetilde{c_{\iota t}}} \\ \dot{\widehat{k_{\iota t}}} \end{bmatrix} = A\begin{bmatrix} \widetilde{c_{\iota t}} \\ \widehat{k_{\iota t}} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{C}{K}\sigma\theta\Theta(\phi_t \ -1)\frac{\widetilde{c_{\iota 0^+}}}{2(1-\sigma\theta\Theta)^2} \end{bmatrix} \tag{23}$$

Where: $$A \equiv \begin{bmatrix} 0 & -\sigma\beta\bar{B}(1-\beta) \\ -\frac{C}{K}/(1-\sigma\theta\Theta) & \frac{C}{K}-(1-\beta)\bar{B} \end{bmatrix}$$

Following e.g. Turnovsky (1995, p. 165n), one can show that the solution of this system is:

$$\begin{bmatrix} \widetilde{c_{\iota 0^+}} \\ \widehat{k_{\iota t}} \end{bmatrix} = \begin{bmatrix} \mathrm{q}_1 \\ 1 \end{bmatrix}(p + H_t)\exp\{\psi_1 \mathrm{t}\}$$

Where

$$H_t \equiv -\int_0^{\mathrm{t}} \mathrm{Q}\frac{C}{K}\sigma\theta\Theta(\phi_s \ -1)\frac{\widetilde{c_{\iota 0^+}}}{2(1-\sigma\theta\Theta)^2}\exp\{-\psi_1 \mathrm{s}\}\mathrm{ds}$$

and $Q \equiv \frac{\mathrm{q}_2}{\mathrm{q}_2-\mathrm{q}_1} > 0$. Obviously, $H_0 = 0$; we proceed now to check that $\lim_{t\to\infty} H_t \exp\{\psi_1 \mathrm{t}\} = 0$, as required since the variables are deviations from the steady state therefore disappear asymptotically. We have:

$$H_t\exp\{\psi_1 \mathrm{t}\} = -\int_0^{\mathrm{t}} \mathrm{Q}\frac{C}{K}\sigma\theta\Theta(\phi_s \ -1)\frac{\widetilde{c_{\iota 0^+}}}{2(1-\sigma\theta\Theta)^2}\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds}$$

The behaviour of this depends on the behaviour of $\int_0^{\mathrm{t}}(\phi_s \ -1)\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds}$. We first note that $-1 \leq (\phi_s \ -1)\exp\{-\psi_1 \mathrm{s}\} \leq 0$, as $\phi_s \ -1 = -1$ for s=0, rising to 0; moreover, this rise is monotonic as $\frac{d(\phi_s \ -1)\exp\{-\psi_1 \mathrm{s}\}}{ds} = \phi_s'\exp\{-\psi_1 \mathrm{s}\} - \frac{1}{\psi_1}(\phi_s \ -1)\exp\{\psi_1 - \mathrm{s}\} > 0$. Thus, $(\phi_s \ -1)\exp\{-\psi_1 \mathrm{s}\}$ is bounded by -1 and 0.

We now need to check the asymptotic behaviour of $\int_0^{\mathrm{t}}(\phi_s \ -1)\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds}$. By the previous arguments, we have that:

$$\exp\{\psi_1 t\}\int_0^t(-1\mathrm{d})\mathrm{s} \le \int_0^t\left(\phi_s\ -1\right)\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds} \le \exp\{\psi_1 t\}\int_0^t 0\mathrm{ds} = 0$$

The lower bound is developed as $\exp\{\psi_1 \mathrm{t}\}\int_0^t(-1)\mathrm{ds} = -t\exp\{\psi_1 \mathrm{t}\}$, therefore

$$lim_{t\to\infty}\exp\{\psi_1 \mathrm{t}\}\int_0^t(-1\mathrm{d})\mathrm{s} = -lim_{t\to\infty}t\exp\{\psi_1 \mathrm{t}\} = 0$$

as $\exp\{|\psi_1|\mathrm{t}\}$ grows faster than t. Therefore

$$0 \le \int_0^t\left(\phi_s\ -1\right)\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds} \le 0$$

i.e., $\int_0^t\left(\phi_s\ -1\right)\exp\{\psi_1(\mathrm{t}-\mathrm{s})\}\mathrm{ds} = 0$ as required.

Thus, consumption and capital move along a saddle path with a constant slope ($0<q_1<1$) but their levels evolve not only by the standard dynamics that would be exhibited by HD (i.e., by ψ_1) but also by the exogenous common forcing dynamics that is generated by HD ($v_t\exp\{\psi_1 \mathrm{t}\}$). Note that this extra dynamics would vanish without the status effect associated with it (i.e., if $\theta=0$). This is because this dynamics would in that case be common to all agents (irrespective of relative consumption) thus would be only manifested in the aggregate system and not the individual one.

From system (23) and matrix A, we have that:

$$0 < q_1 = -\frac{\sigma\beta\bar{B}(1-\beta)}{\psi_1} < 1 \qquad (24a)$$

$$q_2 = -\frac{\sigma\beta\bar{B}(1-\beta)}{\psi_2} < 0 \qquad (24a)$$

Hence the sign Q>0 shown above.

For $t=0^+$, the system becomes:

$$\begin{bmatrix}\widetilde{c_{\iota 0^+}}\\ \widehat{k_{\iota 0}}\end{bmatrix} = p\begin{bmatrix}\mathrm{q}_1\\ 1\end{bmatrix}$$

Since capital is predetermined, we have that $\widehat{k_{\iota 0}} = \mathrm{p}$. Thus, we get:

$$\widetilde{c_{\iota 0^+}} = p\mathrm{q}_1 = \mathrm{q}_1\widehat{k_{\iota 0}}$$

For general t, the system (25) becomes:

$$\begin{bmatrix}\widetilde{c_{\iota t}}\\ \widehat{k_{\iota t}}\end{bmatrix} = \begin{bmatrix}\mathrm{q}_1\\ 1\end{bmatrix}(\widehat{k_{\iota 0}} + H_t)\exp\{\psi_1 \mathrm{t}\} \qquad (25')$$

We ay conveniently define:

$$h_t \equiv \frac{H_t}{\widetilde{c_{\iota 0^+}}} = -\int_0^{\mathrm{t}} \mathrm{Q}\frac{C}{K}\sigma\theta\Theta(\phi_s \ -1)\frac{1}{2(1-\sigma\theta\Theta)^2}\exp\{-\psi_1 \mathrm{s}\}\mathrm{ds} > 0$$

The positive sign follows as $\phi_t \ < 1$ for a finite t. We get equivalently:

$$\begin{bmatrix}\widetilde{c_{\iota t}} \\ \widehat{k_{\iota t}}\end{bmatrix} = \begin{bmatrix}\mathrm{q}_1 \\ 1\end{bmatrix}\widehat{k_{\iota 0}}(1+\mathrm{q}_1 h_t)\exp\{\psi_1 \mathrm{t}\}$$

The first result we obtain is that the slope of the stable arm is unaffected by the omitted term in the approximation (i.e., the exogenous forcing variable h_t in 25'). Since $h_t > 0$, the true path of consumption is amplified compared to the approximate one in the homogeneous system, so that there is more variance in consumption between those that begin below and those that begin above their steady-state relative capital. Furthermore, capital is also affected by this extra term: Taking first those individuals that begin below their steady state, as they try to consume more to catch up, they accumulate less capital; this lead them to consume; thus, a kind of poverty trap emerges. The opposite happens with those that begin above their steady state: they accumulate more capital, thus their consumption ends up being higher, too.

Furthermore, as $\frac{\partial h_t}{\partial \varphi} < 0$, increasing short-termism increases the true variables for those that begin below their steady state; and decreases them for those individuals that begin above their steady state. Thus, once again, increasing short-termism amplifies the cross-sectional variance in both capital and consumption; and since q_1, this variance amplification effect is higher for capital than consumption, hence going some way towards answering one of the main puzzles highlighted in the Introduction. The effect of rising importance of status is unclear here as it affects the eigenvalues and h_t with different signs; but it should be emphasised that the effects of short termism highlighted before would not be present without a status effect ((i.e., if θ=0), thus confirming once again the significance of the interaction between short-termism and status.