Banks Paying for Banks
A Dynamic General Equilibrium Perspective*

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Abstract

This paper presents a dynamic general equilibrium approach to analysing recent policy proposals on a resolution fund for direct recapitalisation of banks financed by banks. The model incorporates endogenous risk inherent in any economic activity due to imperfect information and incomplete financial markets giving rise to dependence on banking finance and the possibility of default. Financial intermediaries could assume this risk ex-ante and support economic activity in expectation of ex-post monetary refinancing and recapitalisation. The downside of such policies and practices would be the resulting financial repression, i.e. erosion of household savings, and the installment of moral hazard incentives in the economic decisions of banks and enterprises. In this setting we explore the short and long-term consequences of a burden-sharing scheme whereby banks contribute towards their own refinancing and recapitalisation.

JEL classification: E5, D5

Keywords: bail-outs, bail-ins, burden sharing, partial default, uncertainty, financial intermediation, imperfect information and incomplete markets.

1 Introduction

The beginning of the recent financial and economic crisis was marked with a diverse mix of bold policy actions of monetary and fiscal nature designed to mitigate the adverse consequences on real economic activity from tighter financial constraints in the presence of rising uncertainty.

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Implemented policies include ample liquidity provision to the banking system, fiscal spending programmes and bank recapitalisations. While the crisis turned out to be long-lasting and resilient, the policies proved expensive, unexplored and therefore potentially dangerous. Consequently, the issue of a fare sharing of the financial burden of policies arose naturally.

Alternative policy strategies were put forward, promoted under the technical term 'bail-in' and catchy phrases like 'banks paying for banks', which essentially aim at the creation of a resolution fund for direct recapitalisation of banks funded by banks.[1] This approach is generally implemented though imposing a levy on banking liabilities and targets fiscal neutrality in the long-run to alleviate the financial burden on households both in their role as tax-payers and consumers whose savings in the form of future consumption could be eroded by potential inflationary pressures due to ample liquidity provision.

This paper is contributing towards the enlightenment of a number of theoretical and conceptual issues related to this policy proposal. For example, what would be the short- and long-term consequences of the new burden-sharing arrangement on economic activity given the presence of endogenous default risk and dependence on external banking finance? Given the role of banks as financial intermediaries, where would risk accumulate eventually in a general equilibrium setting? In other words, could the assertion "what happens in Vegas stays in Vegas" really hold in a dynamic general equilibrium environment? What could be the optimal mix of bank refinancing and policies and banking levies that capture the price of risk and ensure dynamic stability?

The paper analyses these issues in a dynamic general equilibrium context where incomplete information on the product markets and financial markets imperfections underly the presence of uninsurable risk to economic activity. Banks as financial intermediaries could assume a share of the ex-ante risk in expectation of ex-post refinancing, thus reducing default risk, increasing the risk appetite of otherwise risk neutral companies and stimulating economic activity. Ample provision of liquidity and banks' refinancing could negatively affect households' savings (and thus current and future consumption) through the potential for inflation in a low-interest environment, a phenomenon also known as financial repression. Banks are levied in order to partially compensate the households for the latter, while at the same time produce stable and sustainable outcome in the short- and long-run. The model considers ex-ante incentives on decisions of rational, but heterogeneous economic agents (banks, households and enterprises) operating under perfect competition (as an important benchmark for further exploration).

[1]The approach in general and phrase in particular were coined by Michel Barnier, European Union's Commissioner on Financial Affairs, stating "What I'm proposing is logical – banks paying for banks, not taxpayers", Interview at France 24 International News, "EU Commission Wants Bank Levy to Pay for Future Crises", 26 May 2010.

The dynamic general equilibrium approach is employed in order to take into account the complex spillovers and feedback loops between various economic agents and institutions, such as between borrowers and savers, banks and the state, consumers and producers. The fundamental structure of the model incorporates economic uncertainty, default risk and ex-ante incentives in economic decision-making.

The model builds on the dynamic micro-found literature that incorporates incomplete information, financial markets' imperfections and endogenous default inside the equilibrium fundamentals, such as Greenwald and Stiglitz (1987, 1990, 1993a and 1993b), Gatti and Galegati (1996 and 1997). The model also relates to the literature on financial intermediation, incomplete markets and endogenous default in a finite horizon optimisation setting, such as Tsomocos (1993), Tsomocos and Zicchino (2005), Goodhart at. al. (2005 and 2006) and Tsenova (2013a and 2013b). Evidence for the importance of information imperfections, imperfect knowledge and learning by economic agents is presented in a rapidly growing literature, which includes Orphanides and Williams (2008) and Tsenova (2012).

Another important pillar is the literature on incentives and moral hazard under uncertainty and financing constraints, including Goldfeld and Quandt (1988 and 1992), Dewatripont and Maskin (1995) and Kornai (1992). Furthermore, the analysis carries the spirit of public sector economics and the importance of tax incidence in prospective policy analysis, for the entity that pays a levy does not necessarily bear the full cost. The ultimate burden sharing is determined by the market, economic interaction and complex feedback loops captured only in a general equilibrium context.

2 Model Setup and Equilibrium Analysis on Different Markets

2.1 Firms

In the model the productive sector is composed of a continuum of firms each using a productive technology exhibiting decreasing returns to scale. The production function $y = f(x)$ depends on labour x and possesses the following properties: (1) differentiability and decreasing marginal product $f^{'}(x) > 0$ and $f^{''}(x) < 0$; (2) $f(x) \geq 0$ and $f(0) = 0$; (3) the Inada conditions are satisfied $\lim\limits_{x \to 0} f^{'}(x) = \infty$, $\lim\limits_{x \to \infty} f^{'}(x) = 0$.[2]

[2]Capital in the production function is normalized to unity with labour being the only variable factor.

Firms operate in an environment of perfect competition and rational expectations, but incomplete markets and information imperfections. Production, employment and borrowing decisions are made one period before output is sold and at firm's level price uncertainty exists. Each individual firm is a price taker on its own product market, but due to product markets separability and information imperfections, at the time of production a firm has incomplete information as to the pricing of its product relative to the general price level.

$$y_t^i = f(x_{t-1}^i)$$

An individual firm's price is a random variable p_i with mean equal to the general price level - P, and finite variance. The relation between the individual firm's price and the general price level $\frac{p^i}{P} = \theta^i$ is also a random variable with mean 1 and finite variance.[3] $G(\theta)$ is the distribution function of θ and $g(\theta)$ is its density function.

Firms are liquidity constrained. To finance their operating costs in production, firms use bank loans in addition to own liquid buffers generated through accumulating profit from previous periods, referred to as financial asset. A firm's real borrowing from the banks b at time t is determined by:

$$b_t = w_t x_t - a_t$$

where w is the real wage; x - labour input; a - real asset, i.e. real asset, i.e. financial result from the previous period; nominal variables are divided by the general price level P_t.

At time t, when production takes place, firms make production and borrowing decisions based on their expectations of prices they would face on their product market at time $t+1$ being equal to the general (average) price level P, i.e. $E_t[p_{t+1}^i] = E_t[P_{t+1}]$. However, actual revenues and profit of firms when their products are sold at time $t+1$ would depend on the sectorial prices they face on their particular goods' market represented by the realisations of the random variable p_{t+1}^i falling both above and below the general price level in the economy P_{t+1}. Under rational expectations the actual general price level turns out to be the expected price level, i. e. $E_t[P_{t+1}] = P_{t+1}$. However, the individual producer faces price uncertainty related to the realization of its sectorial price p_{t+1}^i and resulting revenues.

A firm's realised profit in real terms would be expressed as:

[3] This implies that the general price level P and the individual price p_i are independent of each other. It is a direct consequence of the assumption of perfect competition, which suggests the presence of a large number of firms and product prices none of which can individually influence the general price index P, because the weight of p^i in P is negligible.

$$\pi^i_{t+1} = \theta^i_{t+1} f(x^i_t) - (1+i_t)\frac{P_t}{P_{t+1}} b^i_t$$

$$\pi^i_{t+1} = \theta^i_{t+1} f(x^i_t) - (1+i_t)\frac{P_t}{P_{t+1}} (w_t x^i_t - a^i_t)$$

where π_{t+1} real financial profit realised at $t+1$; θ^i_{t+1} - relative sectorial price realisations; i_t - nominal interest rate on loans from the bank; w_t - real wage rate; a^i_t - real financial asset; $\dfrac{P_t}{P_{t+1}}$ - ratio between the current $t+1$ and previous prices t.

Because firms are functioning under both price uncertainty and borrowing to finance production, they constantly face the possibility of getting bankrupt. It is assumed that in every period the bankruptcy condition holds: if a firm realizes a negative profit (a firm's revenues after the sale of its output are less than the firm's obligations towards its bank) it is either liquidated or bailed out by the state[4]. For a firm to realize profit, it is essential that its released price exceeds the general price level.

The bankruptcy threshold $\widehat{\theta}$ is defined as the relative price ratio θ below which a firm receives negative profit. It is formally derived as:

$$\widehat{\theta}_{t+1} = (1+i_t)\frac{P_t}{P_{t+1}} \frac{(w_t x_t - a_t)}{f(x_t)} \tag{1}$$

The macro-economic meaning of the bankruptcy mechanism is that after uncertainty is resolved, different firms receive different prices along the distribution of the random variable p_i which form different relative price ratios $\theta^i = \frac{p_i}{P}$. The bankruptcy threshold $\widehat{\theta}$ is unique for the whole economy, and firms that obtain relative price ratios above the economy's bankruptcy threshold $\left(\theta^i_{t+1} > \widehat{\theta}_{t+1}\right)$ realize profits, firms that obtain relative price ratios below the bankruptcy threshold $\left(\theta^i_{t+1} < \widehat{\theta}_{t+1}\right)$ realize losses. We assume that firms must satisfy a *bankruptcy condition* that holds every period: if a firm realizes a loss, it must enter a bankruptcy procedure - either receive a bail-out or get liquidated - because of this individual asset is non-negative $a_t \geq 0$. New firms might enter to replace the bankrupt ones, but have to start with zero asset level. Negative profits cannot be accumulated.

In Equation 1 the bankruptcy threshold is derived as a ratio between enterprize nominal debt and nominal output in the economy. At the same time, the bankruptcy threshold is a realization of the random variable θ that is a price ratio and as such its value must be positive

[4]The methods of financing of the bail-out by the state are discussed later in the section modelling the behaviour of the state.

$(0 < \theta)$ implying that also the value of the bankruptcy threshold will be positive. From Equation 1, the bankruptcy threshold is increasing with respect to labour input, output, nominal interest rate and decreasing with inflation (defined as $\frac{P_{t+1}-P_t}{P_{t+1}} = 1 - \frac{P_t}{P_{t+1}} = 1 - \varphi_{t+1}$) and asset level.

$$\widehat{\theta}_{t+1} = \left(\underset{+}{x_t}, \underset{+}{y_t}, \underset{+}{w_t}, \underset{+}{i_t}, \underset{-}{a_t}, \underset{+}{\varphi_{t+1}} \right) \tag{2}$$

The modelled firms are risk neutral and myopic and maximise their expected profit in period $t+1$, looking forward from period t, in order to decide what amount of resources to employ in period t. When bailing out of unprofitable enterprises exists, the firms profit expectations will include this possibility.

$$\begin{aligned} E\left[\pi_{t+1}\right] &= E\left[\theta_{t+1} f\left(x_t\right) - \left(1+i_t\right) \frac{P_t}{P_{t+1}} \left(w_t x_t - a_t\right)\right] = \\ &= f\left(x_t\right) - \left(1+i_t\right) \frac{P_t}{P_{t+1}^e} \left(w_t x_t - a_t\right) \\ &- f\left(x_t\right) s \int_0^{\widehat{\theta}_{t+1}} \theta g(\theta) d\theta + \left(1+i_t\right) \frac{P_t}{P_{t+1}^e} \left(w_t x_t - a_t\right) s \int_0^{\widehat{\theta}_{t+1}} g(\theta) d\theta \end{aligned}$$

where s is a ratio between 0 and 1 reflecting the perceived probability in the economy of a firm to be bailed out; s is a constant - a parameter characterising the economic environment and its financial softness; s is common knowledge and is perceived by all economic agents.

$G\left(\theta_{t+1}\right) = \int_0^{\widehat{\theta}_{t+1}} g(\theta) d\theta$ is the probability of a firm to obtain a loss and

$sG\left(\theta_{t+1}\right) = s \int_0^{\widehat{\theta}_{t+1}} g(\theta) d\theta$ is the probability being bailed out after receiving a loss.

The firms profit maximising level of labour input and output is chosen in order to equalize the expected marginal product of labour with the expected marginal factor cost.

$$\frac{d}{dx_t} f\left(x_t\right) = \left(1+i_t\right) \frac{P_t}{P_{t+1}^e} w_t \left(\frac{1 - s \int_0^{\widehat{\theta}_{t+1}^e} g(\theta) d\theta}{1 - s \int_0^{\widehat{\theta}_{t+1}^e} \theta g(\theta) d\theta} \right)$$

The effect of the bailing out policy on the firms production decisions is captured by the term $k\left(\widehat{\theta}^e_{t+1}\right) = \left(\dfrac{1 - s\int\limits_0^{\widehat{\theta}^e_{t+1}} g(\theta)d\theta}{1 - s\int\limits_0^{\widehat{\theta}^e_{t+1}} \theta g(\theta)d\theta}\right)$, where $k\left(\widehat{\theta}^e_{t+1}\right)$ is a function of the bankruptcy threshold.

Because $\int\limits_0^{\widehat{\theta}^e_{t+1}} g(\theta)d\theta > \int\limits_0^{\widehat{\theta}^e_{t+1}} \theta g(\theta)d\theta$ this term $k\left(\widehat{\theta}^e_{t+1}\right)$ is smaller than unity for any $\widehat{\theta}^e_{t+1} > 0$. This and the assumed decreasing returns to scale of the production technology, leads the firms to choose to employ more labour and financial resources and produce more than if bailing out policy did not exist $(0 < s \leq 1)$. This is a conclusion already well established in the micro literature on soft budget constraints (for example, Goldfeld and Quandt (1988), Goldfeld and Quandt (1992), Prell (1996)) and also known as the First Kornai Effect.

Rewriting the output profit maximising decision of firms,

$$x_t = \frac{d}{dx_t} f^{-1}\left[(1 + i_t)\frac{P_t}{P^e_{t+1}} w_t k\left(\widehat{\theta}^e_{t+1}\right)\right] \qquad (3)$$

nominal interest rates, inflation and real wages have a direct effect on output: optimum labour input and output are increasing with nominal interest rate and real wage decreasing and inflation rising.

$$y_t = \left(\underset{-}{i_t}, \underset{-}{w_t}, \underset{-}{\varphi^e_{t+1}}, \widehat{\theta}^e_{t+1}\right)$$

However, nominal interest rates, inflation and real wages can also affect output through their influence on the bankruptcy threshold (Equation 2). Because $k\left(\widehat{\theta}^e_{t+1}\right)$ is not a monotonic function for any value of s within its range $(0 < s \leq 1)$ and any $\widehat{\theta}^e_{t+1}$ within its range $(0, +\infty)$, the relationship between the maximising level of labour input (output) and the bankruptcy threshold is dubious. The relationship is proportionate for values of the bankruptcy threshold in the interval $(0, 1]$ and for a larger or smaller interval above 1 depending on the form of distribution for θ and the value of s. If the bankruptcy threshold is within these intervals, the indirect effects nominal interest rate, inflation and real wages are in the opposite direction compared to the direct effects of these variables on output. For example, an increase in the nominal interest rate will discourage output because the costs of borrowing will increase and at the same time will stimulate production because due to the higher interest rate, the bankruptcy threshold will rise increasing the chances of being bailed out after receiving a loss. A firm level analysis (partial equilibrium analysis) cannot draw a definite conclusion on how output will react on changes in variables in the model.

$$y_t = \left(\underset{+/-}{i_t}, \underset{+/-}{w_t}, \underset{+/-}{\varphi^e_{t+1}}, \underset{+/-}{a_t}\right)$$

That is why, a macroeconomic analysis is necessary to determine the effects of the Soft Budget Constraints on output.

In our model the labour supply is assumed to be perfectly elastic and wages are fixed at their market clearing level. There is no involuntary unemployment and labour is perfectly mobile across firms. This simplifying assumption about the labour markets is made in order to isolate the effect of financial market imperfections and soft budget constraints on macro level. Thus, for equilibrium on the labour market $w_t^d = \overline{w} = w_t^s$.

2.2 Banks

The role of the banks is to supply the firms with credit and determine the nominal interest rate. The banks are interested in their expected nominal revenues on lend funds $E\left[1+\rho_t\right]$ (where ρ is the bank's nominal return on loans) which is formed by the following three sources weighed by their probabilities: the successful (returned) loans plus the interest rate, the output revenues of all enterprises that have realised a loss but were not bailed out by the state, and state finds for the loss making firms which are bailed out by the state.

$$E\left[1+\rho_t\right] = (1+i_t)\left(1 - \int_0^{\widehat{\theta}_{t+1}^e} g(\theta)d\theta + s\int_0^{\widehat{\theta}_{t+1}^e} g(\theta)d\theta\right) + (1-s)\,\frac{P_{t+1}^e}{P_t}\,\frac{f\left(x_t\right)}{\left(\overline{w}x_t - a_t\right)}\int_0^{\widehat{\theta}_{t+1}^e} \theta g(\theta)d\theta$$

Reorganising the above equation and assuming for simplicity that the banks nominal return on lended funds is 0 we receive the equilibrium condition for the nominal interest rate as a function of the bankruptcy threshold.

$$i_t = \frac{(1-s)\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t - \theta\right) g(\theta)d\theta}{\widehat{\theta}_{t+1}^e - (1-s)\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t - \theta\right) g(\theta)d\theta} \tag{4}$$

The nominal interest rate is increasing with the bankruptcy threshold rising. The higher is the bankruptcy threshold in the economy, more will be the bankruptcies, and the banks will be asking for a higher interest rate in order to retrieve their lended funds.

$$i_t = \left(\underset{+}{\widehat{\theta}^e_{t+1}} \right)$$

The higher is the number of firms to be bailed out in case of bankruptcy (higher s),the less the banks will care about the level of the bankruptcy threshold. In the extreme case when the $s = 1$ the interest rate i will be zero.

2.3 The State

For simplicity, the model assumes that both monetary and fiscal policies are executed by the state. In practice monetary policy, i.e. the price and quantity of money, is decided on by an independent monetary authority, while fiscal policy, taxation and spending policies are set the government's fiscal authority. The assumed theoretical simplification is necessary, because here the aim is to explore the effects of nonstandard policies exploring and endogenising the links between monetary and fiscal policies. Furthermore, money is used as a store of value akin to non-interest bearing deposits, rather than fiat money.

The bankruptcy condition always holds (firms that obtain negative profit are either bailed out or get liquidated). That is why a monetary refinancing cannot be partial for the individual firm: it is either zero or full. On aggregate, however, the bail-out funds are equal to a fraction of the negative profit in the economy. The bail-out funds are payed after the price uncertainty about the current period is resolved.

$$\frac{S_t}{P_t} = s \int_0^{\widehat{\theta}_t} \pi_t g\left(\theta\right) d\theta = f\left(x_{t-1}\right) s \int_0^{\widehat{\theta}_t} \left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta \tag{5}$$

S denotes the amount of bail-out funds in nominal terms. The choice of a firm to be bailed out is random and determined by s. As noted from the above equation, the bail-out is a function of the output, the bankruptcy threshold and the probability of a firm to be bailed out.

We assume that the state does not have expenditures other than the grants dedicated to save loss making firms from liquidation. We will omit taxation issues by assuming that there are no state revenues other than money creation. Thus, for balanced budget, the funds granted to banks to save them from decartelization and enterprises from liquidation, are financed by money creation. The mechanism is that money is created and transferred to the banks in order to cover a fraction of firms' bad debts.

$$\frac{M_t^s - M_{t-1}^s}{P_t} = \frac{S_t}{P_t} \tag{6}$$

2.4 Aggregate Asset

Here we use the concept of Greenwald and Stiglitz (1993) about the retained earnings dynamics. According to it, the firms on aggregate sell their output at the general price level and the aggregate level of assets is determined as the difference between the mean of the firms' revenues plus other incomes (here, grants covering proportion of negative profits), and firms' payments (expenditures and financial leakages from enterprize profits). In Greenwald and Stiglitz (1993), the financial levies on enterprise profits are taxes collected by the state and then wasted since no agent in the economy receives them. In our model, the levies on enterprize profits are paid to households to share the burden of monetary stimulus on economy through the banking system. These levies are called here bail-in.

Because bail-out of losses exists, the aggregate level of asset will increase with the amount of the bail-out funds. At the same time, the levies on banks, ultimately paid by profitable enterprises would decrease the amount of net asset. Aggregating across firms,

$$A_t = E\left[A_t^i\right] = E\left[p_t^i\right] f\left(x_{t-1}\right) - \left(1 + i_{t-1}\right) P_{t-1}\left(\overline{w}x_{t-1} - a_{t-1}\right) + S_t - T_t P_t$$

$$A_t = P_t f\left(x_{t-1}\right) - \left(1 + i_{t-1}\right) P_{t-1}\left(\overline{w}x_{t-1} - a_{t-1}\right) + S_t - T_t P_t$$

where T_t is the amount of bail-in levies (in real terms) from the firms' asset towards households. Substituting for the value of state support provided to provided to the enterprises in the form of soft bank loans **??**.

$$a_t = f\left(x_{t-1}\right) - f\left(x_{t-1}\right)\widehat{\theta}_t - T_t + f\left(x_{t-1}\right) s \int_0^{\widehat{\theta}_t} \left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta$$

Real bail-in levies could also be expressed as a proportion τ_t of firms' gross profits since the higher the aggregate gross asset, the higher the leakages from firms' gross asset are expected to be.

$$T_t = \tau_t f\left(x_{t-1}\right)\left(1 - \widehat{\theta}_t + s\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta\right) \tag{7}$$

$$a_t = \left(1 - \tau_t\right) f\left(x_{t-1}\right)\left(1 - \widehat{\theta}_t + s\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta\right) \tag{8}$$

The assets in the current period are the current profit (revenues from the sale of the previous period production minus costs) that is not transferred to households. The assets are used to finance the current production expenses and the rest of the necessary financing is obtained by the banks in terms of loans. From the assets accumulation Equation 8, the aggregate assets are decreasing with the bail-in levy rate and with the bankruptcy threshold.

2.5 Households

The households obtain utility from consumption and holding money. To explore the dynamics of households savings and the banks lending policy separately and at the same time retain model simplicity, we assume that no bonds exist in the economy, and the only form of savings are the money holdings carried over by the households from one period to another. In other words, The only way to postpone consumption for the future and engage in consumption smoothing is through holding money, which in the model resemble non-interest bearing household deposits. Inflation is able to erode the value of money and household's future consumption.

We assume the following form of the intertemporal utility function of the representative household:

$$U\left(c_t, m_t\right) = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[c_t + v\left(m_t\right)\right]$$

c_t - consumption in real terms;

m_t - real money balances held until the end of period t and carried over to the beginning of period $t+1$;

β - a discount factor s. t. $0 < \beta < 1$;

v - utility of holding real money balances defined in $(0, \infty)$; it is increasing, strictly concave, continuously differentiable;

utility of consumption is linear with respect to c (to avoid having a second order difference equation on production and consumption);

utility of consumption and utility of holding money are separable, implying money superneutrality.

The representative household's budget constraint in real terms:

$$\frac{M_t^d - M_{t-1}^d}{P_t} = \overline{w} x_t - c_t + T_t$$

$$c_t = T_t + \overline{w} x_t - m_t + \frac{P_{t-1}}{P_t} m_{t-1}$$

The maximization problem involves choosing an optimal quantity of m_t and c_t;

$$\max \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[c_t + v\left(m_t\right)\right]$$

$$s.t. c_t = \overline{w}x_t + T_t - m_t + \frac{P_{t-1}}{P_t} m_{t-1}$$

$$\max_{c_t,\lambda_t,m_t} L = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ c_t + v\left(m_t\right) - \lambda_t \left[c_t - \overline{w}x_t - T_t + m_t - \frac{P_{t-1}}{P_t} m_{t-1} \right] \right\}$$

$$\lambda_t = 1 \tag{9}$$

$$v^{'}\left(m_t\right) = 1 - \lambda_{t+1} \beta \frac{P_t}{P_{t+1}^e} \tag{10}$$

From the FOC of the consumer maximization problem, we derive the households optimum choice of money holding. Optimum money holdings fall with the expected inflation rising.

Let the inflation factor between periods $t-1$ and t $\frac{P_{t-1}}{P_t}$ be represented by the symbol φ_t ($\frac{P_{t-1}}{P_t} = \varphi_t$ and $\frac{P_t}{P_{t+1}} = \varphi_{t+1}$). Rewritten with the new notation, the households optimal choice and budget constraint will be

$$v^{'}\left(m_t\right) = 1 - \beta \varphi_{t+1}^e \tag{11}$$

$$c_t = T_t + \overline{w}x_t - m_t + \varphi_t m_{t-1} \tag{12}$$

Concerning the money supply process, in the section dealing with Government behavior, we have already assumed that the state is committed to provide an increased money supply sufficient to finance its bail out policy. The money supply growth is μ_t and is defined as $M_t^s = M_{t-1}^s \left(1 + \mu_t\right)$. The growth of money supply is variable and depends on the funds granted to banks with the purpose of cancelling banks' bad debts.

$$\frac{M_t^s - M_{t-1}^s}{P_t} = \frac{S_t}{P_t}$$

Reorganizing the money supply side in two alternative ways:

$$\frac{M_t^s - M_{t-1}^s}{P_t} = \frac{\left(1 + \mu_t\right) M_{t-1}^s - M_{t-1}^s}{P_t} = \frac{M_{t-1}^s \mu_t}{P_t} = \frac{M_{t-1}^s}{P_{t-1}} \mu_t \frac{P_{t-1}}{P_t} \tag{13}$$

and

$$\frac{M_t^s - M_{t-1}^s}{P_t} = \frac{M_t^s - \frac{1}{\left(1+\mu_t\right)} M_t^s}{P_t} = \frac{M_t^s \frac{\mu_t}{\left(1+\mu_t\right)}}{P_t} \tag{14}$$

and taking into account that for equilibrium on the money market

$$M_t^s = M_t^d$$

we derive a rule for money growth, where money growth increase with the amount of current soft loans and inflation in the economy, and decrease with the previous period real money balances (From Equation 13).

$$\mu_t = \frac{1}{\varphi_t m_{t-1}} \frac{S_t}{P_t} \tag{15}$$

From Equation 14 we derive the rate of money growth $\frac{\mu_t}{(1+\mu_t)}$ which increases with the amount of soft loans and decreases with the level of real money balances currently in the economy.

$$\frac{\mu_t}{(1+\mu_t)} = \frac{1}{m_t} \frac{S_t}{P_t} \tag{16}$$

Also, to have equilibrium on the goods market, consumption in period t has to equal to production in the previous period $t-1$ due to the one period lag in production:

$$c_t = f(x_{t-1}) \tag{17}$$

For the goods markets to clear, current consumption is equal to the previous period production.

2.6 Summary of Equilibrium Conditions

Because firms are operating in an environment of relative price uncertainty and borrowing from banks to finance their production, there is possibility for the firms to get bankrupt. The economy's behaviour is characterised by the bankruptcy threshold which is the relative price at which a firm will receive zero profit. The bankruptcy threshold increases with the nominal interest rate, inflation factor, labour input (x) and output ($y = x^{\alpha}$), and decreases with inflation and aggregate assets (Equation 1).

$$\widehat{\theta}_{t+1} = (1+i_t)\,\varphi_{t+1}^{e} \frac{(w_t x_t - a_t)}{f(x_t)}$$

Firms decide how much to produce every period by equalizing their marginal productivity of labour to their marginal costs. In our model (unlike in Greenwald and Stiglitz (1993), firms are not penalized if they get bankrupt. Neither the firm nor the managers of the firm incur any additional cost in case of bankruptcy. On the opposite, they might get rescued by the state. Depending on how soft is the policy of the state in this respect, a proportion of firms chosen randomly would get their unserviceable obligations to their banks (negative profits) cancelled and banks will receive a corresponding proportion of their bad loans recapitalized. The higher the softness of the financial environment s, the more individual firms will be encouraged to produce because the bail-out policy increases the expected resources available and decreases marginal costs (Equation 3).

$$x_t = \frac{d}{dx_t} f^{-1}\left[(1+i_t)\,\varphi_{t+1}^{e} w_t k\left(\widehat{\theta}_{t+1}^{e}\right)\right]$$

Even though the possibility of getting bankrupt has some positive consequences for the output individual firms due to the possibility of being bailed out by the state, in a dynamic macro framework, there are forces also in the opposite direction. There is one period lag between production and sale of output. Firms' asset are formed as retained profits after the sale of what has been produced in the previous period and bail-in levies. Aggregate asset are formed as retained profits on aggregate - these are net earnings from the sale of output (sale of output minus costs plus bail-out funds obtained), minus bail-in levies to households (Equation 8).

$$a_t = (1-\tau_t)\, f(x_{t-1})\left(1-\widehat{\theta}_t + s\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t-\theta\right) g(\theta)d\theta\right)$$

Other things being equal, the higher the bankruptcy threshold in period t, the larger number of firms will get bankrupt, the lower will be the aggregate assets, the less assets will be available to finance production, and for the bankruptcy threshold not to soar to even higher levels in period $t+1$, output will decrease in period t.

Because the interest rate is formed as to bring a non-negative return to banks, the higher is the bankruptcy threshold, more firms will be expected to default on their loans and the higher will be the interest rate. Higher degree of financial softness in the economy will bring lower interest rates because a larger proportion of the unserved loans will be payed back by the state (Equation 4).

$$i_t = \frac{(1-s)\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t-\theta\right) g(\theta)d\theta}{\widehat{\theta}_{t+1}^{e} - (1-s)\int_0^{\widehat{\theta}_t}\left(\widehat{\theta}_t-\theta\right) g(\theta)d\theta}$$

The bailing out policy of the state is financed by money creation (Equation 6).

$$\frac{M_t - M_{t-1}}{P_t} = \frac{S_t}{P_t}$$

The money supply growth is increasing with the funds spend on bailing out enterprises and the current value of the previous period money balances (Equation 15).

$$\mu_t = \frac{1}{\varphi_t m_{t-1}}\frac{S_t}{P_t}$$

The bailing out funds themselves are equal to the proportion of negative profits that have been covered by the state and are increasing with the financial softness of the economy, previous period's output and the bankruptcy threshold (Equation **??**).

$$\frac{S_t}{P_t} = f\left(x_{t-1}\right) s \int_0^{\widehat{\theta}_t} \left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta$$

The households aiming to maximise their consumption and real money balances, choose to hold lower real money balances, the lower the inflation rate and the higher the inflation factor (Equation 11).

$$v^{'}\left(m_t\right) = 1 - \beta \varphi_{t+1}^{e}$$

According to the households' budget constraint, consumption is higher the higher are the incomes of the households' incomes - labour incomes and from firms' net revenue earnings, and the lower the change in money bail-in levies demand/supply in real terms (Equation 12).

$$c_t = T_t + \overline{w} x_t - \frac{M_t - M_{t-1}}{P_t}$$

The bail-in levies are the proportion of the firms' earnings that become transferred to households (Equation 7).

$$T_t = \tau_t f\left(x_{t-1}\right) \left(1 - \widehat{\theta}_t + s \int_0^{\widehat{\theta}_t} \left(\widehat{\theta}_t - \theta\right) g(\theta) d\theta\right)$$

The bail-in levies cannot be arbitrarily high because large leakages from the firms' earnings could bring low level of aggregate assets which will be able to finance low level of output. Lower level of current output will bring lower level of labour income to households and lower current consumption. Also, because equilibrium requires that consumption is equal to the previous period production, (Equation 17)

$$c_t = f\left(x_{t-1}\right)$$

in the long run consumption will be equal to production and lower labour input will bring lower consumption. The households will choose a level of bail-in levies and levy rates, such that consumption is optimal.

3 Model Evaluation

3.0.1 Steady state analysis

Steady states are estimated for $t+1=t$ and expected values equal to real values. Under these conditions Equation **??** becomes

$$\left[\frac{\alpha\left[4-(1-s)\,\widehat{\theta}\right]}{4\varphi k\left(\widehat{\theta}\right)}\right]\frac{1}{\alpha}\widehat{\theta}k\left(\widehat{\theta}\right)-\widehat{\theta}=0$$

and because $\widehat{\theta}\neq 0$, we find that at the steady state

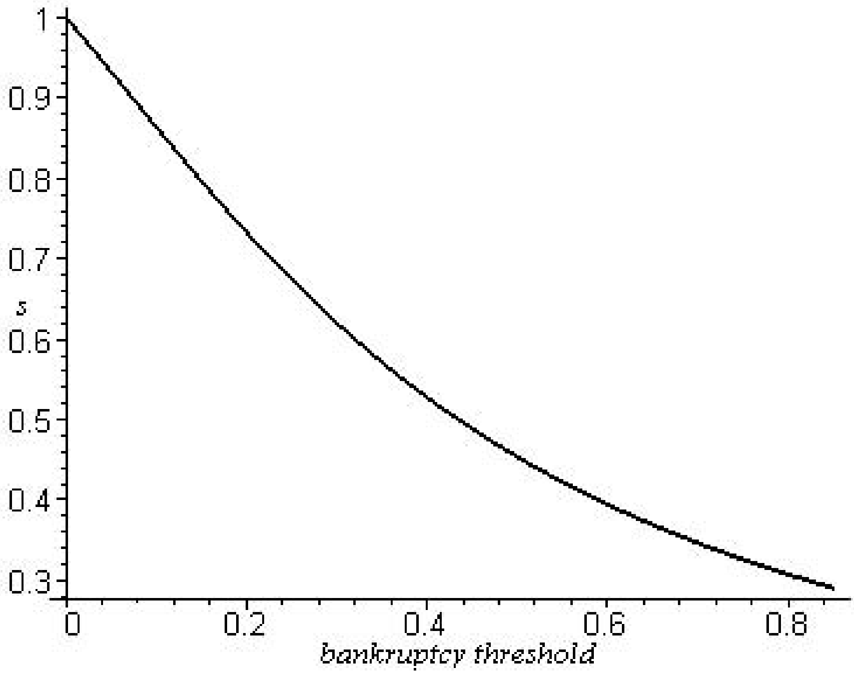
$$\varphi=\left[1-\frac{1}{4}(1-s)\,\widehat{\theta}\right] \tag{18}$$

and $\widehat{\theta}$ is a non-zero root of the equation

$$\frac{(1-s)\,\widehat{\theta}\frac{1}{4}}{1-\beta+\beta(1-s)\,\widehat{\theta}\frac{1}{4}}-\frac{1}{4}s\widehat{\theta}^{2}\left(\frac{\alpha}{k\left(\widehat{\theta}\right)\overline{w}}\right)^{\frac{\alpha}{1-\alpha}}=0 \tag{19}$$

The above equation is obtained by imposing the steady state conditions on Equation **??** and substituting for the steady state of the inverse inflation factor φ Equation 18).

We numerically evaluate the steady states of the bankruptcy threshold and inflation (the inverse of the inflation ratio) for a range of values of financial softness in the economy $(0<s\leq 0)$. All numerical evaluations are carried out assuming the following values of the structural parameters: $\overline{w}=0.5$, $\alpha=0.8$, $\beta=0.8$. The the steady state general equilibrium values of the bankruptcy threshold $\widehat{\theta}$ are plotted on the figure below and show how the bankruptcy threshold decreases with the financial softness in the economy.



Steady states of the bankruptcy threshold for the full range of degrees of financial softness in the economy.

The roots for $\widehat{\theta}$ of Equation 19 for values of s between 0 and 0.28 are not equilibrium steady state values in our model and are not plotted on the graph. The reason is that when $0 < s \leq 0.28$ (for low values of financial softness in the economy), the values of the bankruptcy threshold are so high that lead to negative aggregate value of assets. The latter can finance a negative amount of output.

At the same time, the equilibrium steady state of inflation $\left(\frac{P_{t+1}-P_t}{P_t} = \frac{1}{\varphi} - 1\right)$ is also decreasing with the possibility of firms being bailed out and does not reach very high values. From the steady state of the inflation factor (Equation 18), we derive the steady state level of inflation.

$$\frac{1}{\varphi} - 1 = \frac{1}{\left[1 - \frac{1}{4}\left(1-s\right)\widehat{\theta}\right]} - 1$$

Comparing the above expression with Equation **??**, we can see that at the steady state the inflation rate is equal to the interest rate. The reason for this could be that the households' savings are in the form of money. Inflation is a cost on real money holdings of households in the same way that the interest rate is a cost on firms. When the bail-out policy is financed by money creation, the costs on firms decrease at the expense of the households. The possibility of bail-in levies from firms to households in our model, allow for the two costs to equalise. Figure below evaluates the steady state of inflation and interest rate for the corresponding values of the

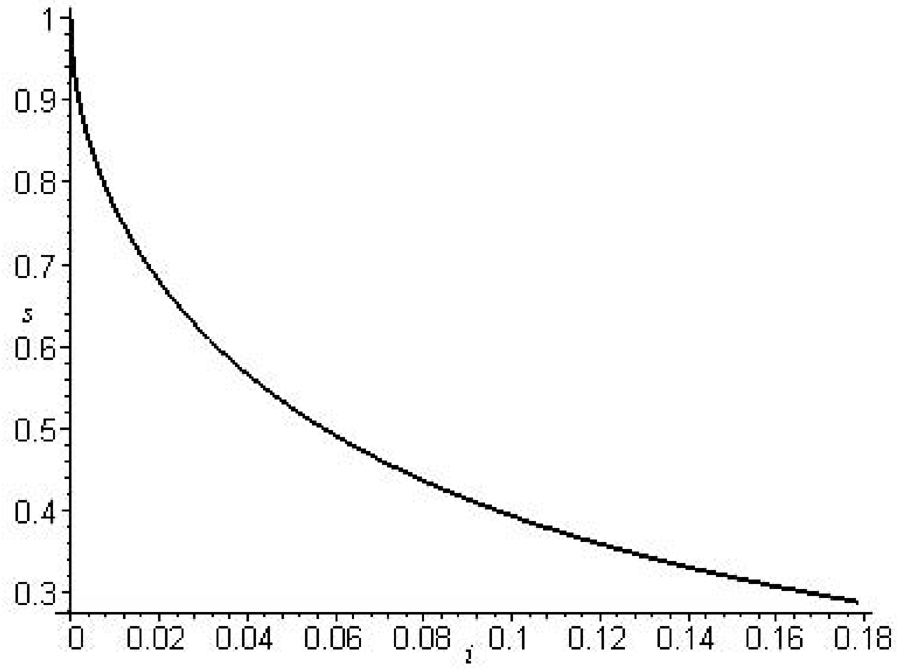


Figure 1: Steady states of the inflation rate, interest rate and money growth rate for the full range of degrees of financial softness in the economy.

steady state bankruptcy threshold. The steady state of the inflation and interest rates decrease with the financial softness.

Also, money growth μ is equal to inflation at the steady state. This is a standard consequence in a flexible prices monetary model. Due to the money creation and money demand processes described in the section analysing the behaviour of the households in this paper, and more specifically, from Equations 16 and 15, we find that

$$\frac{\mu}{1+\mu} = \frac{1}{m} m\varphi\mu$$

Rearranging the above:

$$\mu = \frac{1}{\varphi} - 1$$

The right hand side is the steady state inflation, which according to the above equation is equal to the steady state money growth rate μ.

The next figure plots the steady states for aggregate assets and output. The thinner line on the Figure represents the steady states of the aggregate assets, and the thicker line, the equilibrium values of output. A permanently higher probability of receiving a bail-out increases

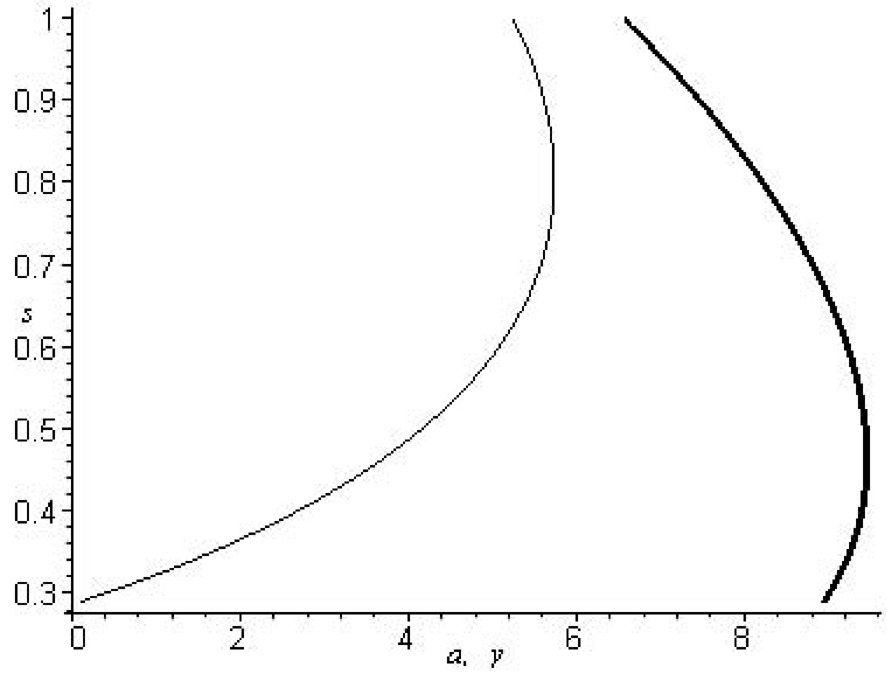


Figure 2: Steady states of the output and aggregate assets for the full range of degrees of financial softness in the economy.

the steady state of output only up to a point ($s = 0.48$ and $y = 9.46$), after which it starts decreasing. The aggregate assets reach their maximum when $s = 0.80$ and $a = 5.741849144$.

As we might have expected, the steady state of the bail-in levies rate reaches its minimum at the same point at which aggregate asset reach their maximum ($s = 0.80$ and $\tau = 0.18$). The larger is the bail-in levy rate, the smaller will be the level of aggregate asset. The effect of bail-in levies from enterprises to households is that an increase in bail-in levies has positive effect on consumption that is equal to previous period production but have negative effect on labour costs financing and consequently labour incomes (See Equations **??** and **??**). After taking these two factors together, the steady state effect of an increase in the levy rate on labour income, output and consumption will be negative. For $0.29 \leq s \leq 0.8$, the higher is the financial softness in the economy, the smaller is the steady state of the bail-in levy rate. For higher s this relationship reverses.

It might seem surprising that the bail-in levy rate is decreasing with s (the financial softness of the economy). However, this can well be explained by the fact that s is the proportion of firms that become bailed out once they realise negative profits. The latter depends on how high is the bankruptcy threshold. In this model, one of the main results is that the equilibrium bankruptcy

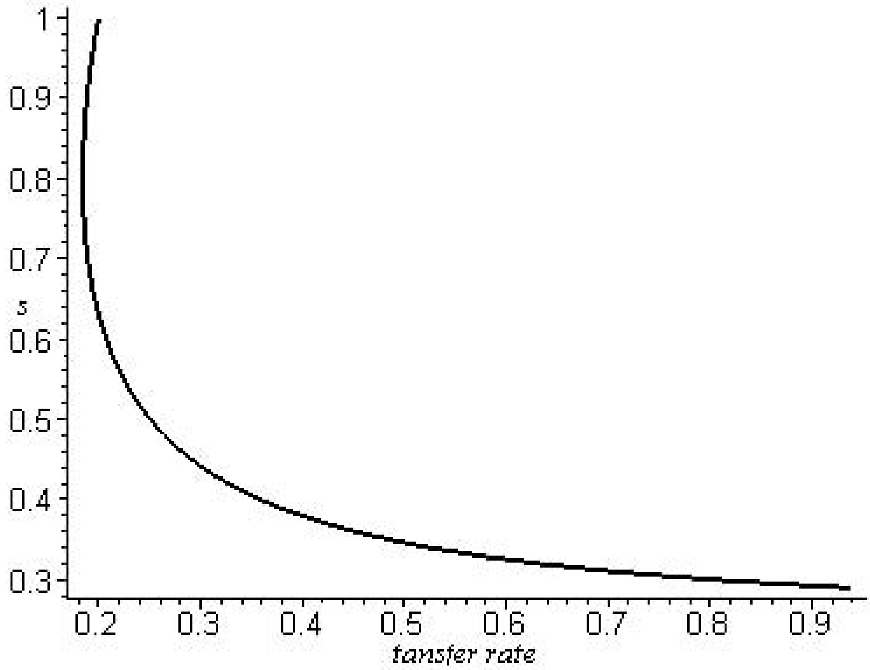


Figure 3: Steady states of the bail-in levy rate for the full range of degrees of financial softness in the economy.

threshold decreases with the financial softness. Therefore, the higher is the financial softness, the lower will be the bankruptcy threshold, the lower will be the realised negative profits and the subsidies (bail-out funds).

Also, Figure 5 plotting the real bail-in levies in absolute terms shows that the amount of bail-in levies varies in a relatively small range.

Real money balances increase as the softness of the economy increases because of the falling inflation rate.

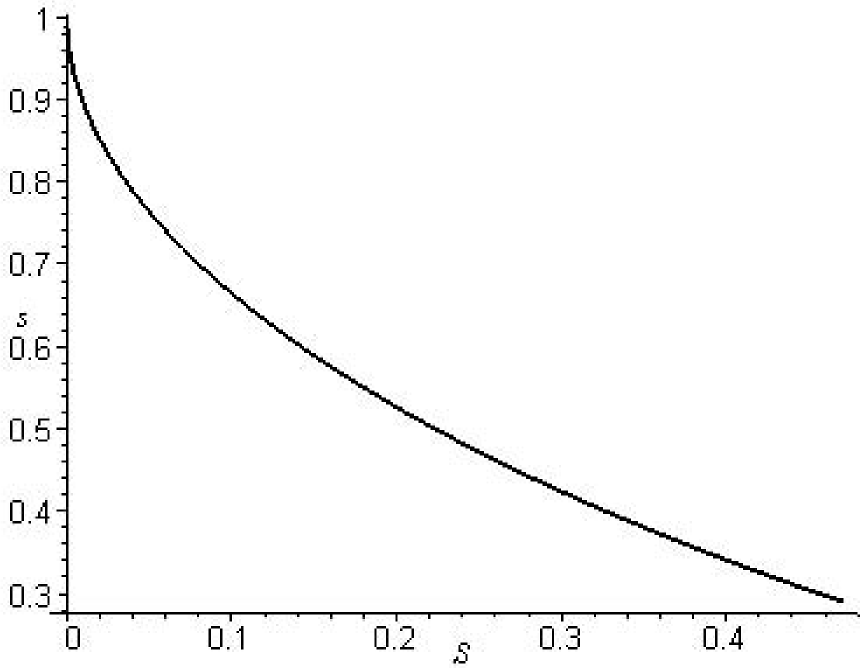
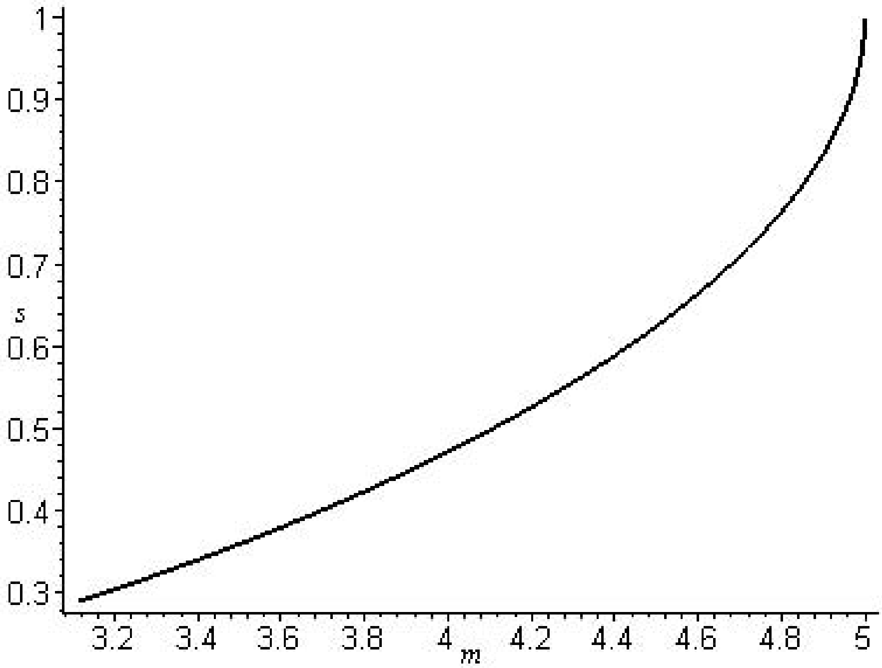


Figure 4: Steady states of the bail-out subsidies for the full range of degrees of financial softness in the economy.



Steady states of the real money balances for the full range of degrees of financial softness in the economy.

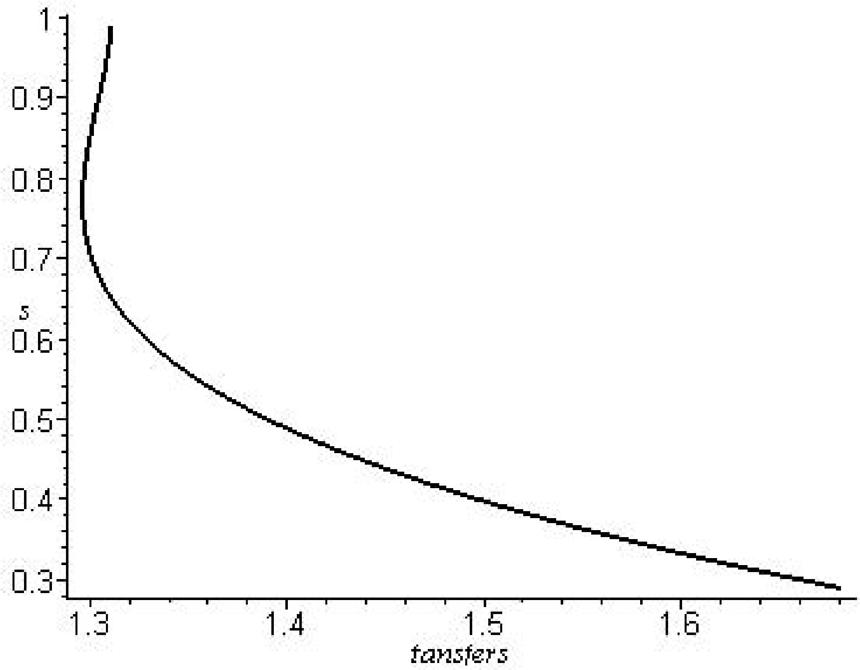


Figure 5: Steady states of the real bail-in levies for the full range of degrees of financial softness in the economy.

3.0.2 Dynamics

First we explore the dynamics of the two key variables $\widehat{\theta}_t$ and φ_t by linearising the reduced system of equations around their steady states:

Equation **??** is represented as

$$Q(\widehat{\theta}_{t+1}, \widehat{\theta}_t, \varphi_{t+1}\varphi_t) = 0$$

and Eq.**??** is represented as

$$N(\widehat{\theta}_{t+1}, \widehat{\theta}_t, \varphi_{t+1}\varphi_t) = 0$$

We apply a first order Taylor approximation to the two equations:

$$\frac{\partial}{\partial\widehat{\theta}_{t+1}} Q\left(\widehat{\theta}_{t+1} - \widehat{\theta}\right) + \frac{\partial}{\partial\widehat{\theta}_t} Q\left(\widehat{\theta}_t - \widehat{\theta}\right) + \frac{\partial}{\partial\varphi_{t+1}} Q\left(\varphi_{t+1} - \varphi\right) + \frac{\partial}{\partial\varphi_t} Q\left(\varphi_t - \varphi\right) = 0$$

$$\frac{\partial}{\partial\widehat{\theta}_{t+1}} N\left(\widehat{\theta}_{t+1} - \widehat{\theta}\right) + \frac{\partial}{\partial\widehat{\theta}_t} N\left(\widehat{\theta}_t - \widehat{\theta}\right) + \frac{\partial}{\partial\varphi_{t+1}} N\left(\varphi_{t+1} - \varphi\right) + \frac{\partial}{\partial\varphi_t} N\left(\varphi_t - \varphi\right) = 0$$

This allows us to find the absolute deviations of the bankruptcy threshold and inflation factor from their steady states. However, to make comparison easier, we would like to find their deviations as a rate of change. We modify the liberalization of the two equations to achieve this.

$$\widehat{\theta}\left[\frac{\partial}{\partial\widehat{\theta}_{t+1}}Q\left(\frac{\widehat{\theta}_{t+1}-\widehat{\theta}}{\widehat{\theta}}\right)\right]+\widehat{\theta}\left[\frac{\partial}{\partial\widehat{\theta}_{t}}Q\left(\frac{\widehat{\theta}_{t}-\widehat{\theta}}{\widehat{\theta}}\right)\right]+\varphi\left[\frac{\partial}{\partial\varphi_{t+1}}Q\left(\frac{\varphi_{t+1}-\varphi}{\varphi}\right)\right]$$
$$+\varphi\left[\frac{\partial}{\partial\varphi_{t}}Q\left(\frac{\varphi_{t}-\varphi}{\varphi}\right)\right]=0$$
$$\widehat{\theta}\left[\frac{\partial}{\partial\widehat{\theta}_{t+1}}N\left(\frac{\widehat{\theta}_{t+1}-\widehat{\theta}}{\partial\widehat{\theta}}\right)\right]+\widehat{\theta}\left[\frac{\partial}{\partial\widehat{\theta}_{t}}N\left(\frac{\widehat{\theta}_{t}-\widehat{\theta}}{\partial\widehat{\theta}}\right)\right]+\varphi\left[\frac{\partial}{\partial\varphi_{t+1}}N\left(\frac{\varphi_{t+1}-\varphi}{\varphi}\right)\right]$$
$$+\varphi\left[\frac{\partial}{\partial\varphi_{t}}N\left(\frac{\varphi_{t}-\varphi}{\varphi}\right)\right]=0$$

We approximate the dynamics of the variables, close to their steady state as: $\left(\frac{\widehat{\theta}_{t+1}-\widehat{\theta}}{\widehat{\theta}}\right)=\widetilde{\theta}_{t+1}$, $\left(\frac{\widehat{\theta}_{t}-\widehat{\theta}}{\widehat{\theta}}\right)=\widetilde{\theta}_{t}$, $\left(\frac{\varphi_{t+1}-\varphi}{\varphi}\right)=\widetilde{\varphi}_{t+1}$, $\left(\frac{\varphi_{t}-\varphi}{\varphi}\right)=\widetilde{\varphi}_{t}$. This way we can measure the path of the variables in terms of rate of change from their steady states. After applying the liberalization and rearranging, the system obtains the form:

$$\widetilde{\theta}_{t+1}=a_{11}\widetilde{\theta}_{t}+a_{12}\widetilde{\varphi}_{t}$$
$$\widetilde{\varphi}_{t+1}=a_{21}\widetilde{\theta}_{t}+a_{22}\widetilde{\varphi}_{t}$$

We estimate the eigenvalues of this reduced system and find that the system is stable for all feasible values of s which are $(0.29,1)$. One of the eigenvalues is smaller than unity and the other one is larger than unity. Once out of equilibrium due to a shock, the economy converges to its equilibrium, following a unique path.

The stability of the system is estimated for different values of financial softness in the economy s. Explaining the notations in the table - γ_1 is the unstable eigenvalue ($|\gamma_1|>1$) and γ_2 is the stable eigenvalue ($|\gamma_2|<1$) are the eigenvalues of the system and c_{12} and c_{22} are the two values of the eigenvector associated with the stable eigenvalue (γ_2).

Since both the bankruptcy threshold and the inflation factor are affected by expectations, they are both free variables. In this case both of the eigenvalues should be larger than unity in order to guarantee a unique solution. This suggests the existence of potentially indeterminate sunspot equilibria. However, we only analyse the minimum state variable solution since this is one which is frequently analysed by applied economists. There is a unique path on which the economy can converge towards the steady state.

To find the stable path, we eliminate the impact of the unstable eigenvalue on the dynamics of the system. The path of φ and $\widehat{\theta}$ measured in percentage deviations from their steady states is:

$$\widetilde{\varphi}_{t}=\widetilde{\varphi}_{0}\gamma_{2}^{t}$$

$$\widetilde{\theta}_{t}=\widetilde{\varphi}_{0}\frac{c_{21}}{c_{22}}\gamma_{2}^{t}$$

, where $\widetilde{\varphi}_0$ is the initial percentage shock on the inverse rate of inflation due to change in expectations.

For example, consider a case when the financial softness of the economy increase, and the public becomes aware that at period 0, s will be 25% higher rising from 0.4 to 0.41.

Higher softness in the economy, means that the steady state of the bankruptcy threshold will decrease and the inflation factor will increase. As portrayed on the phase diagram - Figure 7, the steady state equilibria lie on the line AC. The old equilibrium is point A which is characterised by $\widehat{\theta}$, that is 2.9% away from the new equilibrium, and φ, that is -4% away from the new equilibrium. The economy cannot go directly from point A to point E. Instead φ will jump so that the economy arrives at point B, that is characterised by φ being 10% from equilibrium and $\widehat{\theta}$ being 2.9% from equilibrium. The φ and $\widehat{\theta}$ will subsequently converge to equilibrium following the path BE. Because variables inflation and inflation factor are inversely related, changes in inflation will be with the opposite sigh compared to changes in the inflation factor. Therefore, when the softness of the economy increases, inflation will have to fall. Initially, inflation overshoots: instead of falling straight to its equilibrium value, it initially rises and with time converges to its lower equilibrium level.

The speed of convergence, the time it takes for the economy to converge to its steady state, depends on the stable eigenvalue. We can observe that the higher is the softness of the economy, the closer is the stable eigenvalue to 1, and the more time it takes for the economy to converge to its equilibrium. Therefore, when the financial softness is high, even small shocks will have persistent effect on the economy.

Knowing the paths of the bankruptcy threshold and the inflation factor, we can find the dynamic paths of all the other variables considered in our model. There are two factors that determine their paths: firstly, these variables' responsiveness towards changes of the bankruptcy threshold and the inflation factor, and secondly, the path of the bankruptcy threshold and inflation factor determined by the reduced dynamic system. We find the linearized approximation of the variables in question, and analyse their dynamics.

$$\frac{i_t - i}{i} = \frac{\partial i_t}{\partial \widehat{\theta}_{t+1}} \left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}} \right) \frac{\widehat{\theta}}{i}$$

We find that the coefficient $\frac{\partial}{\partial \widehat{\theta}_{t+1}} i_t$ is negative. This means that positive changes in the level of the bankruptcy threshold lead to positive changes in the interest rate. If no bail-out existed, higher expected bankruptcy threshold would lead to higher number of bankruptcies and more bad loans for the banks. To cover those, the bank would charge higher interest rates.

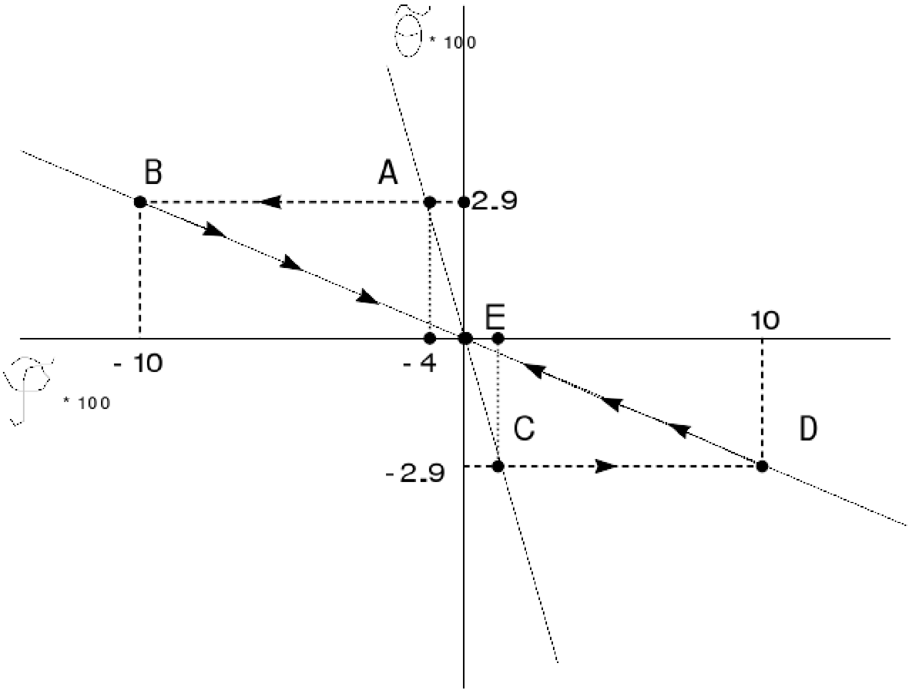


Figure 6: Phase diagram

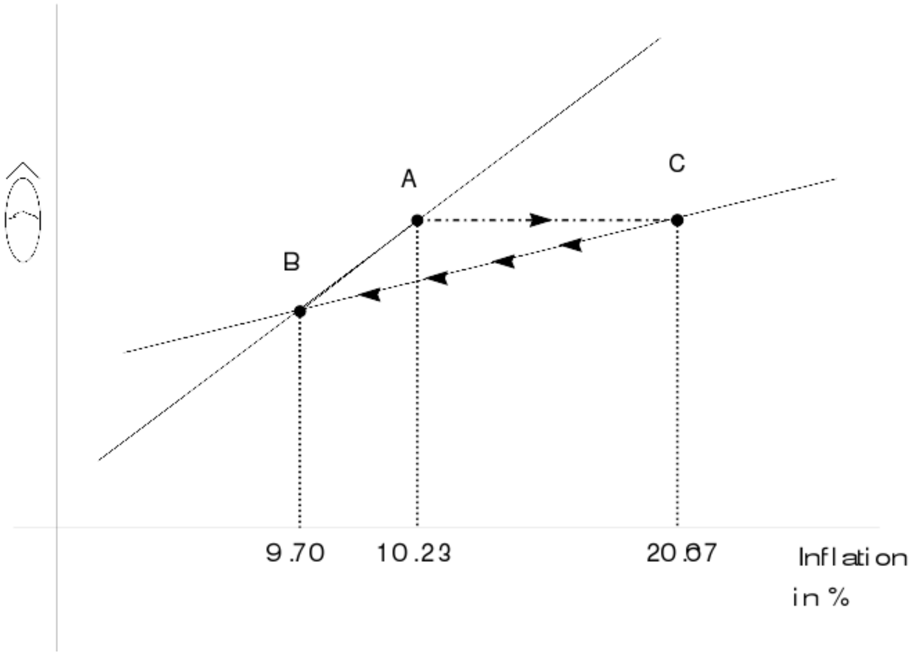


Figure 7: Phase diagram

The coefficient $\frac{\partial}{\partial \widehat{\theta}_{t+1}} i_t$ declines in absolute terms with the financial softness. This means that bail-out acts towards diminishing the effect of the bankruptcy threshold on the interest rate. At the same time, the speed of convergence decreases with the financial softness.

From the fact that $y = x^{\alpha}$

$$\frac{y_t - y}{y} = \frac{1}{y}\left[i \frac{\partial y_t}{\partial i_t} \left(\frac{i_t - i}{i} \right) + \varphi \frac{\partial y_t}{\partial \varphi_{t+1}} \left(\frac{\varphi_{t+1} - \varphi}{\varphi} \right) + \widehat{\theta} \frac{\partial y_t}{\partial \widehat{\theta}_{t+1}} \left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}} \right) \right]$$

The results of the liberalization show that positive changes in the inflation factor will cause negative changes in output. This is because positive changes in the inflation factor are connected with a decline in the future price level and discourages production. Also, positive changes in the interest rate will cause negative changes in output since this increases the cost of production. The bankruptcy threshold influences output through two channels: it stimulates output through the bail-out term $k(.)$ and decreases output through its influence on the interest rate. When the financial softness (s) is below the medium, the first effect of the bankruptcy threshold on output prevails. When the financial softness is above the medium, the second effect prevails. The initial deviations after a shock increase with the financial softness. The speed of convergence to the steady state decrease with the financial softness.

$$\frac{\tau_t - \tau}{\tau} = \frac{1}{\tau}\left[\widehat{\theta} \frac{\partial \tau_t}{\partial \widehat{\theta}_t} \left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}} \right) + \widehat{\theta} \frac{\partial \tau_t}{\partial \widehat{\theta}_{t+1}} \left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}} \right) \right]$$

Changes in the bail-in levy rate are affected negatively by changes in the current bankruptcy threshold and positively by changes in the next period's expected bankruptcy threshold. The higher is the current bankruptcy threshold, the less funds will be possible to transfer to the households in the current period. The higher is the expected bankruptcy threshold in the next period, the more attractive will be the idea to transfer the funds in the current period. We linearize and evaluate the path of the bail-in levy rate in terms of percentage deviations from the steady state. We find that for medium and higher financial softness the deviations from the steady state are very small. For low degrees of financial softness the initial deviations are larger but the path converges to equilibrium relatively fast.

$$\frac{T_t - T}{T} = \frac{1}{T}\left[\tau \frac{\partial T_t}{\partial \tau_t} \left(\frac{\tau_t - \tau}{\tau} \right) + y \frac{\partial T_t}{\partial y_{t-1}} \left(\frac{y_{t-1} - y}{y} \right) + \widehat{\theta} \frac{\partial T_t}{\partial \widehat{\theta}_t} \left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}} \right) \right]$$

The changes in the amount of the bail-in levies from the firms to the households are positively influenced by changes in the bail-in levy rate and changes in the net revenues of the firm. The latter depend on changes in output and the bankruptcy threshold. After evaluating the joint effect of these factors, we find that changes of the amount of the bail-in levies are positively related to the next period's bankruptcy threshold and negatively related to the current bankruptcy threshold and inflation factor. The initial deviations after a shock increase with the financial softness. The speed of convergence to the steady state decrease with the financial softness.

$$\frac{(S/P)_t - S/P}{S/P} = \frac{1}{S/P}\left[y\frac{\partial\,(S/P)_t}{\partial y_{t-1}}\left(\frac{y_{t-1}-y}{y}\right) + \widehat{\theta}\frac{\partial\,(S/P)_t}{\partial\widehat{\theta}_t}\left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}}\right)\right]$$

Changes in the amount of the bail-out in real terms is proportionate to changes in output and the bankruptcy threshold. Given the financial softness, the higher is the output and the bankruptcy threshold, the more funds will be needed to bail-out a proportion of the troubled enterprises. After estimating these joint effects, we find that changes in the bail-out are affected positively by changes in the bankruptcy threshold and the inflation rate. The initial percentage deviations decrease with the financial softness. The speed of convergence decreases with the financial softness.

The path of the inflation rate is determined by:

$$\chi_t = \varphi\left[\frac{\partial\chi_t}{\partial\varphi_t}\left(\frac{\varphi_t-\varphi}{\varphi}\right)\right]\chi$$

Negative deviations of the inflation factor from the steady state lead to positive deviations of the inflation rate from the steady state. The speed convergence decrease with the financial softness.

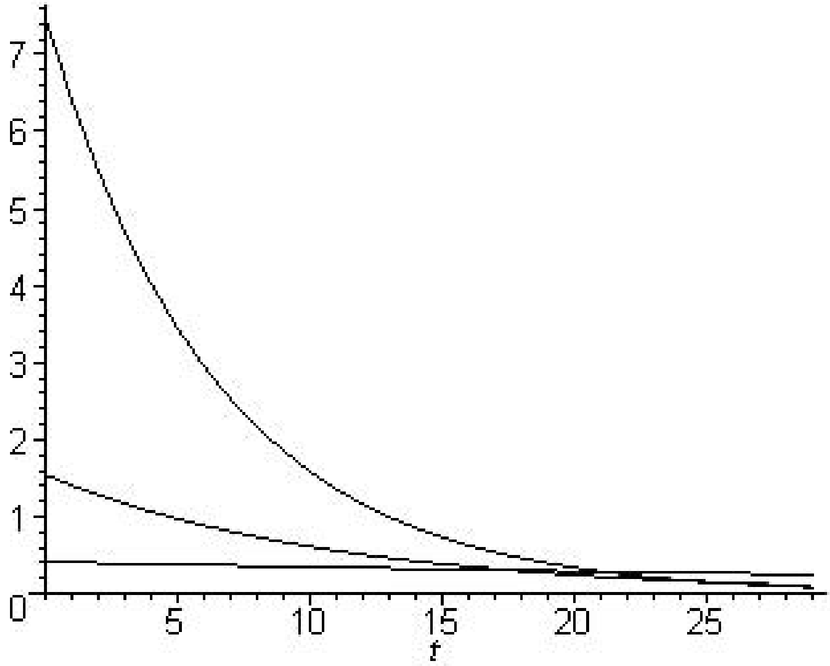
$$\frac{m_t - m}{m} = \frac{\varphi}{m}\left[\frac{\partial m_t}{\partial\varphi_{t+1}}\left(\frac{\varphi_{t+1}-\varphi}{\varphi}\right)\right]$$

The results of the liberalization show that positive deviations of the inflation factor from the steady state lead to positive deviations of the real money balances from the steady state.

$$\frac{\mu_t-\mu}{\mu} = \frac{1}{\mu}\left[y\frac{\partial\mu_t}{\partial y_{t-1}}\left(\frac{y_{t-1}-y}{y}\right) + m\frac{\partial\mu_t}{\partial m_{t-1}}\left(\frac{m_{t-1}-m}{m}\right) + \widehat{\theta}\frac{\partial\mu_t}{\partial\widehat{\theta}_t}\left(\frac{\widehat{\theta}_t-\widehat{\theta}}{\widehat{\theta}}\right)\right]$$

The results of the liberalization show that positive deviations of the output and previous period's real money balances lead to negative deviations of the money growth rate. Positive deviations of the current bankruptcy threshold lead to positive deviations in the money growth rate. The initial deviation decreases with the financial softness. The speed of convergence to the steady state decreases with the financial softness.

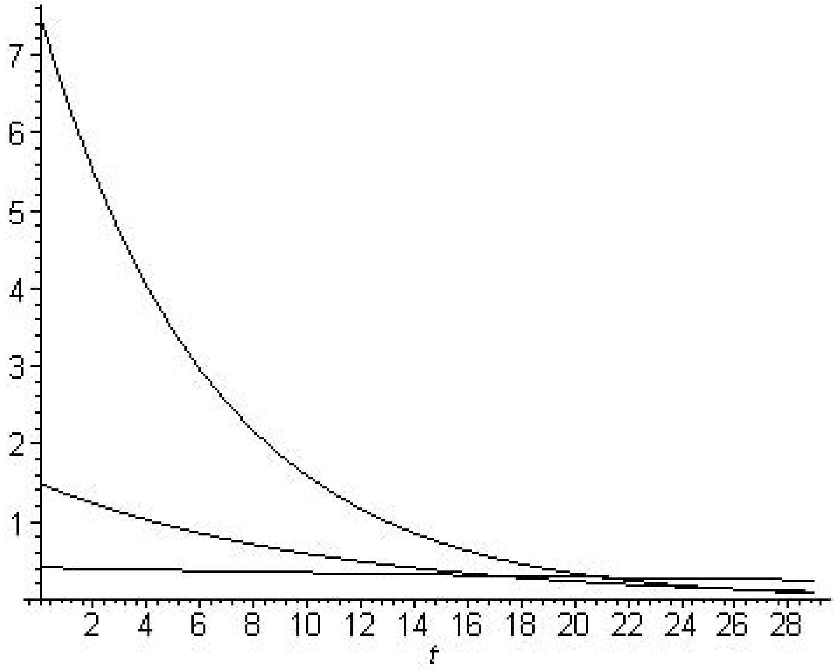
Figure 8: **Time path of the bankruptcy threshold, in terms of percentage deviations from its steady state, when s=0.3, s=0.5 and s=0.7**



Higher softness in the economy, means that the steady state of the bankruptcy threshold will decrease and the inflation factor will increase. As portrayed on the phase diagram - Figure 7, the steady state equilibria lie on the line AC. The old equilibrium is point A which is characterised by $\widehat{\theta}$, that is 2.9% away from the new equilibrium, and φ, that is -4% away from the new equilibrium. The economy cannot go directly from point A to point E. Instead φ will jump so that the economy arrives at point B, that is characterised by φ being 10% from equilibrium and $\widehat{\theta}$ being 2.9% from equilibrium. The φ and $\widehat{\theta}$ will subsequently converge to equilibrium following the path BE. Because variables inflation and inflation factor are inversely related, changes in inflation will be with the opposite sigh compared to changes in the inflation factor. Therefore, when the softness of the economy increases, inflation will have to fall. Initially, inflation overshoots: instead of falling straight to its equilibrium value, it initially rises and with time converges to its lower equilibrium level.

The speed of convergence, the time it takes for the economy to converge to its steady state, depends on the stable eigenvalue. We can see that the higher is the softness of the economy, the closer is the stable eigenvalue to 1, and the more time it takes for the economy to converge to its equilibrium. Therefore, when the financial softness is high, even small shocks will have persistent effect on the economy.

Figure 9: **Time path of the interest rate, in terms of percentage deviations from its steady state, when s=0.3, s=0.5 and s=0.7.**

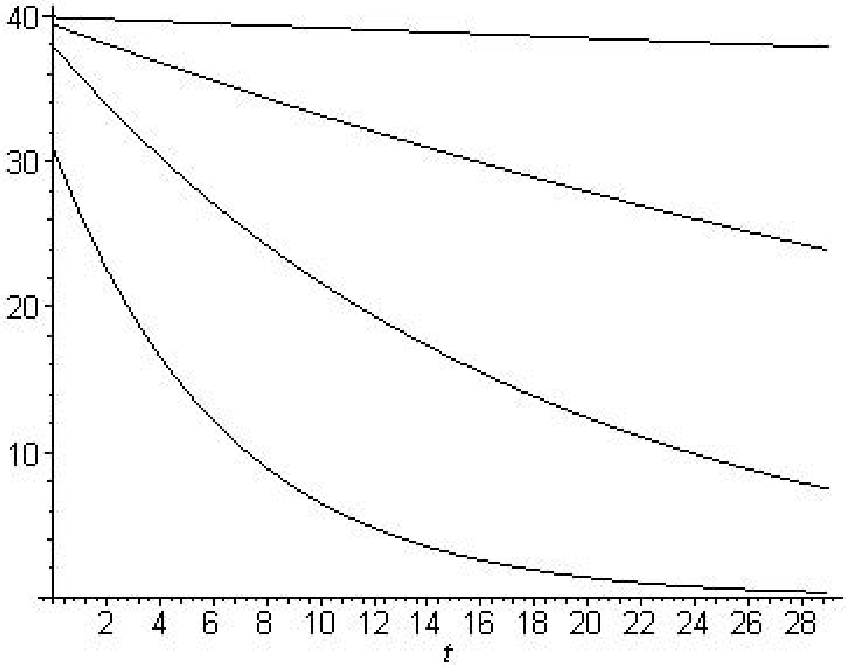


Knowing the paths of the bankruptcy threshold and the inflation factor, we can find the dynamic paths of all the other variables considered in our model. There are two factors that determine their paths: firstly, these variables' responsiveness towards changes of the bankruptcy threshold and the inflation factor, and secondly, the path of the bankruptcy threshold and inflation factor determined by the reduced dynamic system. We find the linearised approximation of the variables in question, and analyse their dynamics.

$$\frac{i_t - i}{i} = \frac{\partial i_t}{\partial \widehat{\theta}_{t+1}} \left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}} \right) \frac{\widehat{\theta}}{i}$$

We find that the coefficient $\frac{\partial}{\partial \widehat{\theta}_{t+1}} i_t$ is negative. This means that positive changes in the level of the bankruptcy threshold lead to positive changes in the interest rate. If no bail-out existed, higher expected bankruptcy threshold would lead to higher number of bankruptcies and more bad loans for the banks. To cover those, the bank would charge higher interest rates. The coefficient $\frac{\partial}{\partial \widehat{\theta}_{t+1}} i_t$ declines in absolute terms with the financial softness. This means that bail-out act towards diminishing the effect of the bankruptcy threshold on the interest rate. At the same time, the speed of convergence decreases with the financial softness.

Figure 10: **Time path of the output, in terms of percentage deviations from its steady state, when s=0.3, s=0.5, s=0.7 and s=0.09**



From the fact that $y = x^{\alpha}$

$$\frac{y_t - y}{y} = \frac{1}{y}\left[i\frac{\partial y_t}{\partial i_t}\left(\frac{i_t - i}{i}\right) + \varphi\frac{\partial y_t}{\partial \varphi_{t+1}}\left(\frac{\varphi_{t+1} - \varphi}{\varphi}\right) + \widehat{\theta}\frac{\partial y_t}{\partial \widehat{\theta}_{t+1}}\left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}}\right)\right]$$

The results of the linearisation show that positive changes in the inflation factor will cause negative changes in output. This is because positive changes in the inflation factor are connected with a decline in the future price level and discourages production. Also, positive changes in the interest rate will cause negative changes in output since this increases the cost of production. The bankruptcy threshold influences output through two channels: it stimulates output through the bail-out term $k(.)$ and decreases output through its influence on the interest rate. When the financial softness (s) is below the medium, the first effect of the bankruptcy threshold on output prevails. When the financial softness is above the medium, the second effect prevails. Figure 10 graphs the paths of output in terms of percentage deviations from the steady state when $s = 0.3$, $s = 0.5$, $s = 0.7$ and $s = 0.9$. The initial deviations after a shock increase with the financial softness. The speed of convergence to the steady state decrease with the financial softness.

$$\frac{\tau_t - \tau}{\tau} = \frac{1}{\tau}\left[\widehat{\theta}\frac{\partial \tau_t}{\partial \widehat{\theta}_t}\left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}}\right) + \widehat{\theta}\frac{\partial \tau_t}{\partial \widehat{\theta}_{t+1}}\left(\frac{\widehat{\theta}_{t+1} - \widehat{\theta}}{\widehat{\theta}}\right)\right]$$

Changes in the bail-in levy rate are affected negatively by changes in the current bankruptcy threshold and positively by changes in the next period's expected bankruptcy threshold. The higher is the current bankruptcy threshold, the less funds will be possible to transfer to the households in the current period. The higher is the expected bankruptcy threshold in the next period, the more attractive will be the idea to transfer the funds in the current period. We linearize and evaluate the path of the bail-in levy rate in terms of percentage deviations from the steady state. We find that for medium and higher financial softness the deviations from the steady state are very small. For low degrees of financial softness the initial deviations are larger but the path converges to equilibrium relatively fast.

$$\frac{T_t - T}{T} = \frac{1}{T}\left[\tau \frac{\partial T_t}{\partial \tau_t}\left(\frac{\tau_t - \tau}{\tau}\right) + y\frac{\partial T_t}{\partial y_{t-1}}\left(\frac{y_{t-1} - y}{y}\right) + \widehat{\theta}\frac{\partial T_t}{\partial \widehat{\theta}_t}\left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}}\right)\right]$$

The changes in the amount of the bail-in levies from the firms to the households are positively influenced by changes in the bail-in levy rate and changes in the net revenues of the firm. The latter depend on changes in output and the bankruptcy threshold. After evaluating the joint effect of these factors, we find that changes of the amount of the levies are positively related to the next period's bankruptcy threshold and negatively related to the current bankruptcy threshold and inflation factor. Figure 11 graphs the path of the real bail-in levies in terms of percentage deviations from the steady state when $s = 0.3$, $s = 0.5$, $s = 0.7$ and $s = 0.9$. The initial deviations after a shock increase with the financial softness. The speed of convergence to the steady state decrease with the financial softness.

$$\frac{(S/P)_t - S/P}{S/P} = \frac{1}{S/P}\left[y\frac{\partial\left(S/P\right)_t}{\partial y_{t-1}}\left(\frac{y_{t-1} - y}{y}\right) + \widehat{\theta}\frac{\partial\left(S/P\right)_t}{\partial \widehat{\theta}_t}\left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}}\right)\right]$$

Changes in the amount of the bail-out in real terms is proportionate to changes in output and the bankruptcy threshold. Given the financial softness, the higher is the output and the bankruptcy threshold, the more funds will be needed to bail-out a proportion of the troubled enterprises. After estimating these joint effects, we find that changes in the bail-out are affected positively by changes in the bankruptcy threshold and the inflation rate. Figure 12 graphs the path of the real bail-out funds in terms of percentage deviations from the steady state when $s = 0.3$, $s = 0.5$, $s = 0.7$ and $s = 0.09$. The initial percentage deviations decrease with the financial softness. The speed of convergence decreases with the financial softness.

The path of the inflation rate is determined by:

Figure 11: **Time path of the real bail-in levies, in terms of percentage deviations from its steady state, when s=0.3, s=0.5, s=0.7 and s=0.09.**

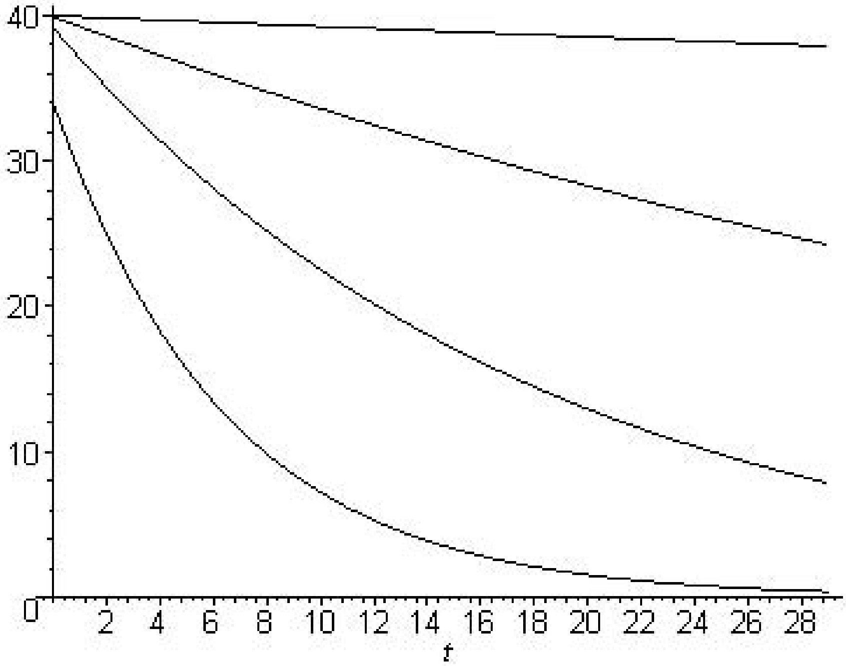


Figure 12: **Time path of the real bail-out funds, in terms of percentage deviations from its steady state, when s=0.3, s=0.5, s=0.7 and s=0.09.**

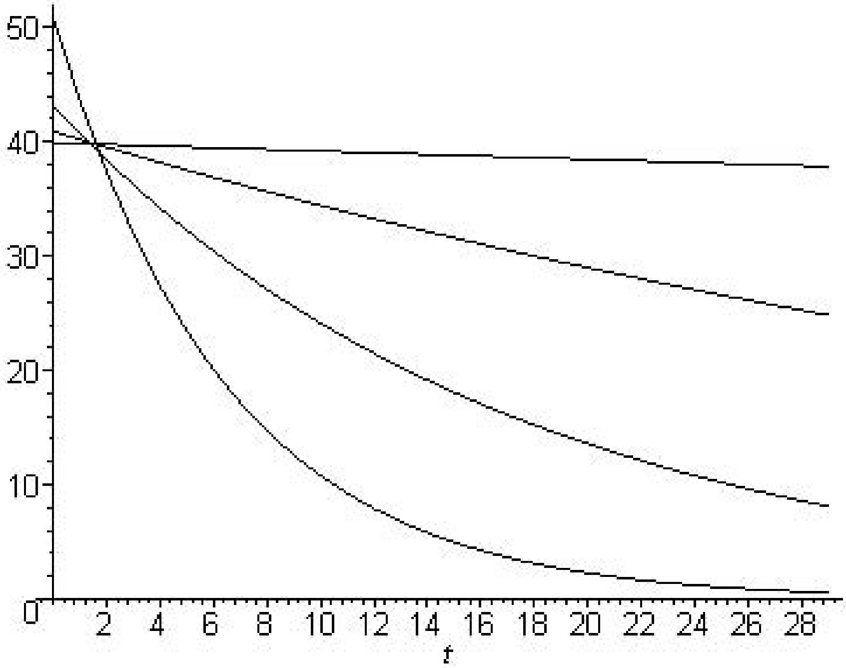
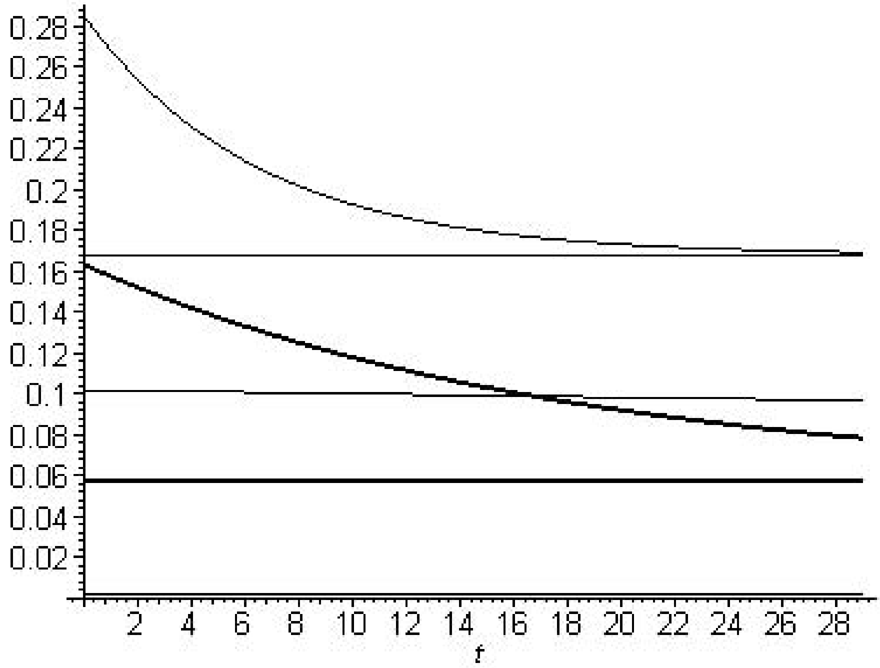


Figure 13: **Time path of the inflation rate when s=0.3, s=0.5, s=0.7 and s=0.09.**



$$\chi_t = \varphi \left[\frac{\partial \chi_t}{\partial \varphi_t} \left(\frac{\varphi_t - \varphi}{\varphi} \right) \right] \chi$$

Negative deviations of the inflation factor from the steady state lead to positive deviations of the inflation rate from the steady state. The speed convergence decrease with the financial softness (see Figure 13).

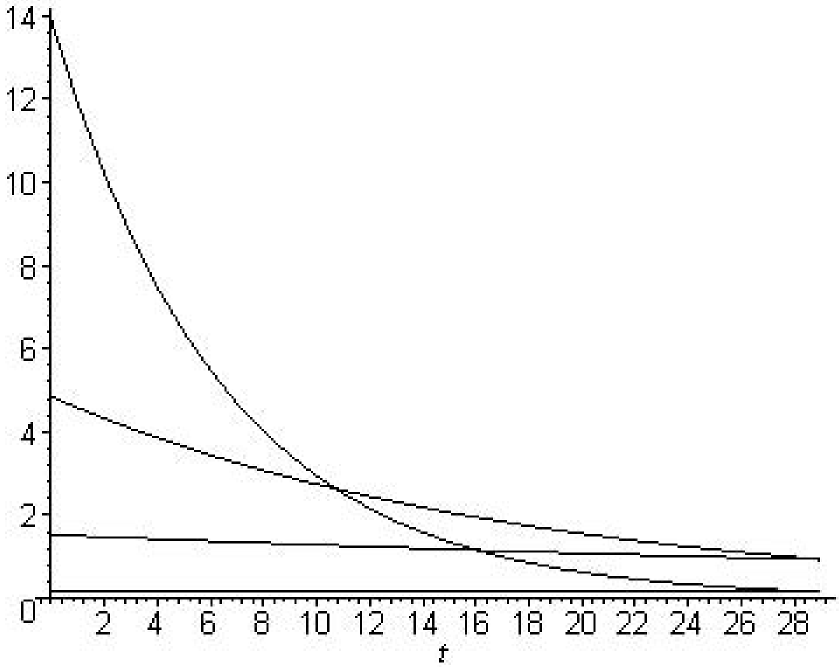
$$\frac{m_t - m}{m} = \frac{\varphi}{m} \left[\frac{\partial m_t}{\partial \varphi_{t+1}} \left(\frac{\varphi_{t+1} - \varphi}{\varphi} \right) \right]$$

The results of the linearisation show that positive deviations of the inflation factor from the steady state lead to positive deviations of the real money balances from the steady state.

$$\frac{\mu_t - \mu}{\mu} = \frac{1}{\mu} \left[y \frac{\partial \mu_t}{\partial y_{t-1}} \left(\frac{y_{t-1} - y}{y} \right) + m \frac{\partial \mu_t}{\partial m_{t-1}} \left(\frac{m_{t-1} - m}{m} \right) + \widehat{\theta} \frac{\partial \mu_t}{\partial \widehat{\theta}_t} \left(\frac{\widehat{\theta}_t - \widehat{\theta}}{\widehat{\theta}} \right) \right]$$

The results of the linearisation show that positive deviations of the output and previous period's real money balances lead to negative deviations of the money growth rate. Positive deviations of the current bankruptcy threshold lead to positive deviations in the money growth rate. Figure 14 graphs the path of the money growth rate in terms of percentage deviations from its steady state when $s = 0.3$, $s = 0.5$, $s = 0.7$ and $s = 0.09$. The initial deviation decreases with

Figure 14: **Time path of the money growth rate, in terms of percentage deviations from its steady state, when s=0.3, s=0.5, s=0.7 and s=0.09.**



the financial softness. The speed of convergence to the steady state decreases with the financial softness.

4 Concluding remarks

This paper presents a dynamic general equilibrium approach to analysing recent policy proposals on a resolution fund for direct recapitalisation of banks financed by banks. The model incorporates endogenous risk inherent in any economic activity due to imperfect information and incomplete financial markets giving rise to dependence on banking finance and the possibility of default. Financial intermediaries could assume this risk ex-ante and support economic activity in expectation of ex-post monetary refinancing and recapitalisation by the state. The downside of such policies and practices would be the resulting financial repression, i.e. erosion of household savings, and the installment of moral hazard incentives in the economic decisions of banks and enterprises.

Under perfect competition and banking financial intermediation, a banking levy and monetary refinancing would jointly share the burden of stimulating the economic activity between the households and enterprises. However, long-term equilibrium is incompatible with negligible monetary refinancing, which indicates the desirability of their co-existence. Medium degree of monetary refinancing encourages entrepreneurs to assume more risk and reach maximum long-term output. This is compatible with relatively low banking levy and inflation rate placing relatively lower burden on both households and producers. However, caution is justified, since at relatively higher degrees of monetary refinancing the positive real effect wears off and reverses. The economy becomes more vulnerable to short-term shocks possessing comparatively low speed of convergence towards long-term equilibrium.

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