

# Balanced budget stimulus with tax cuts in a liquidity constrained economy\*

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March 9, 2014

**Abstract:** This paper examines the macroeconomic effects of unexpected, exogenous, simultaneous, temporary cuts to income tax rates in an economy when the government follows a balanced budget fiscal rule and keeps money supply constant, and private agents face constraints on the ability to finance investments. The main results are that the tax cuts increase output, private consumption, and investment; the increases in output and consumption are significant and long-lasting; and the liquidity constraints play a major role in the shock's long-term persistence. Results are obtained from calibrating a modified version of the DSGE model of liquidity and business cycles by [Kiyotaki and Moore \(2012\)](#). The modifications are twofold: (i) distortionary taxes to labour and dividend incomes are added, and (ii) the government follows a balanced budget fiscal rule and keeps money supply constant. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to implausibly significant variations in the persistence of tax shocks.

**JEL Classification:** E10, E20, E30, E44, E50, E62, H30

**Keywords:** Fiscal policy, taxation, balanced budget, liquidity constraints

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\*I am grateful to John Driffill and Ron Smith for their guidance throughout this paper, and to Ivan Petrella, Stephen Wright, and seminar participants at Birkbeck for useful comments. All errors and omissions are my own.

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# 1 Introduction

This paper examines the macroeconomic effects of cuts to income tax rates in an economy when the government follows a balanced budget fiscal rule and keeps money supply constant, and private agents face constraints on the ability to finance investments. The tax rate cuts are unexpected, exogenous, simultaneous, and temporary. The main results are that the tax cuts increase output, private consumption, and investment; the increases in output and consumption are significant and long-lasting; and the liquidity constraints play a major role in the shock's long-term persistence. Liquidity constraints create demands for two assets of varying liquidity; tax cuts increase the demand for both assets; and while the tax cuts also lead to an increase in supply of the less liquid asset, the liquidity constraints restrict this increase to be small; accordingly, both asset prices increase, and amplify the internal propagation of the shock. Results are obtained from calibrating a modified version of the DSGE model of liquidity and business cycles by [Kiyotaki and Moore \(2012\)](#) (henceforth KM). The modifications are twofold: (i) distortionary taxes to labour and dividend incomes are added, and (ii) the government follows a balanced budget fiscal rule and keeps money supply constant. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to implausibly significant variations in the persistence of tax shocks. The paper contributes to an extensive literature on the effectiveness of fiscal policy for economic stimulation. It belongs to a narrow strand of this literature which explores balanced budget expansion. Results are consistent with those achieved by [Mountford and Uhlig \(2009\)](#) (henceforth MU) , a member of this balanced budget research.

Tax cuts are shown to be expansionary in early works by [Andersen and Jordan \(1968\)](#), [Giavazzi and Pagano \(1990\)](#), [Baxter and King \(1993\)](#), [Braun \(1994\)](#), [McGrattan \(1994\)](#), [Alesina and Perotti \(1997\)](#), and [Perotti \(1999\)](#), and more recently by [Romer and Romer \(2010\)](#), [Mertens and Ravn \(2011a,b, 2012\)](#), and [Monacelli et al. \(2012\)](#). Support for tax cuts is also expressed in blogs by [Hall and Woodford \(2008\)](#), [Mankiw \(2008\)](#), and [Barro \(2009\)](#). And counterfactual experiments by [Blanchard and Perotti \(2002\)](#), [Romer and Romer \(2010\)](#), MU, and [Alesina and Ardagna \(2010\)](#) show that tax cuts produce larger responses than increases in government spending.

Tax cuts with a balanced budget are shown to be expansionary in [Eggertsson \(2010\)](#) and MU. [Eggertsson \(2010\)](#) obtains his results by cutting consumption taxes and simultaneously raising income and wealth taxes to perfectly compensate. MU show that completely financing an unexpected, exogenous increase in government spending with an increase in taxation causes reductions in private consumption and investment on impact, as well as in output from the second period.<sup>1</sup> The converse of this result suggests a recipe for debt-free economic expansion. This paper complements MU by showing that the converse of their result is also true. The novelty of this paper is that while MU obtain their results from an empirical study with vector autoregressions, this paper is a theoretical investigation using a mostly neoclassical DSGE model.

The KM model is chosen for its pair of financial frictions, which resemble an essential

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<sup>1</sup>There is, however, a small increase in output on impact, with a multiplier of 1.3.

feature of the 2007/8 financial crisis. This makes the paper relevant for policy discussions on the crisis. KM is an otherwise neoclassical model, but with constraints on firms' ability to internally and externally finance investments. Commentators argue that the cause of the crisis was the sudden and unexpected deterioration in the value of partially liquid private financial assets, which thus ruined their resaleability and suitability for use as collateral (Brunnermeier (2009), Del Negro et al. (2011), Bigio (2012) and Jermann and Quadrini (2012)). Assets' resaleability and suitability for collateral were thus adversely affected. This event bears a striking resemblance to KM's own negative liquidity shock.

The KM model is theoretically adjusted and/or extended in a series of recent papers. These papers can be classified into two groups. The first group uses the KM model to evaluate the unconventional policies seen in the crisis; Del Negro et al. (2011) and Driffill and Miller (2013) are members of this research, and both show that recessions would have been exacerbated had it not been for government interventions. The second KM-related group returns to the original questions posed by KM on the importance of (i) liquidity shocks for explaining business cycles, and (ii) liquidity constraints for the propagation of productivity shocks. Papers in this group include Salas-Landeau (2010), Bigio (2010, 2012), Ajello (2011), Nezafat and Slavík (2012), Shi (2012), and Jermann and Quadrini (2012).

The inclusion of distortionary taxes and a balanced budget rule is not unique in KM-related literature. Ajello (2011), Shi (2012), and Driffill and Miller (2013) have a balanced budget rule for government. Ajello (2011) also includes distortionary taxes, but he modifies the KM model more extensively than in this paper. The uniqueness of this contribution is that it is the first to examine fiscal shocks in the KM model. What it shares with these papers, the second KM-related group in particular, is showing the macroeconomic significance of KM's liquidity constraints in propagating exogenous shocks. In this case, however, the shocks are to tax rates.

In a wider literature, the significance of this paper is that it shows how a neoclassical model can be modified to produce large responses to fiscal shocks. The New Keynesian model is the workhorse for fiscal policy research. This perhaps follows from papers like Burnside et al. (2004), which shows that the magnitude of observed responses to fiscal shocks are not matched by a standard neoclassical models, but they are matched by models that include habit formation and adjustment costs. Beyond the liquidity constraints, this model is otherwise neoclassical. This paper therefore shows that a host of New Keynesian frictions are not always needed to study fiscal policy. The KM model can be a workhorse for that purpose.

The rest of the paper is organized as follows: Section 2 describes the model in full, and derives conditions which characterize the dynamic equilibrium; Section 3 presents the main results of the tax shock; Section 4 quantifies and comments on the magnitude of shock responses using tax multipliers; Section 5 briefly gives the conclusions of sensitivity analysis on structural parameters and the persistence of tax shocks; Section 6 examines the significance of the results by relating them to similar work in the literature; and Section 7 concludes the paper and outlines avenues for future research. The technical appendix contains the model's calibration, sensitivity analysis of the shock, algebra of proofs and derivations, and the data used in the paper.

## 2 The model

This section defines agents' behaviour and derives conditions that characterise the dynamic equilibrium. The model is an adaption of the [Kiyotaki and Moore \(2012\)](#) framework with government and without storage (henceforth KM). In particular, KM is modified in two ways: (i) distortionary taxes on wage and dividend income are added, and (ii) different policy rules to the ones in KM are used.<sup>2</sup> In particular, the government holds no equity, keeps money supply constant, and adheres to a balanced fiscal budget rule. To make the paper self-contained, a full description of the model is given. [Appendix C](#) contains detailed algebra associated with derivations, simplifications, and proofs.

### 2.1 The environment

The economy exists over an infinite horizon of discrete time periods. It is populated by a unit-mass continuum of *ex ante* identical entrepreneurs, a unit-mass continuum of identical workers, and a government. The population does not grow or decline, and the economy is closed to the rest of the world. There are no financial intermediaries or formal credit markets. All agents consume a perishable general output, which is produced exclusively by entrepreneurs and is the economy's numeraire. All agents own and exchange two assets: money and equity. Money is perfectly liquid and exclusively issued by the government; equity is not perfectly liquid, not perfectly re-saleable, depreciates each period, earns dividends, and is exclusively issued by entrepreneurs. Money and equity are traded in competitive markets at perfectly flexible prices  $p_t$  and  $q_t$ , respectively, both expressed in terms of general output.<sup>3</sup>

### 2.2 Entrepreneurs

#### 2.2.1 Production

Entrepreneurs are the exclusive owners of capital and a homogeneous Cobb-Douglas technology that produces general output with guaranteed success. At the beginning of period  $t$  the representative entrepreneur owns  $k_t$  units of capital. The entrepreneur employs  $l_t$  hours of labour and produces  $y_t$  units of general output at the end of the period according to

$$y_t = A_t k_t^\gamma l_t^{1-\gamma} \tag{2.1}$$

where  $A_t$  is a common level of total factor productivity and  $\gamma \in (0, 1)$  is the capital elasticity of output. The market for general output is perfectly competitive; then, according to [Cobb and Douglas \(1928\)](#), the production function (2.1) exhibits constant returns to scale and  $\gamma$  is also

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<sup>2</sup>KM have two policy rules. One limits the government's spending to the deviation of its own asset holdings from steady state; this rule prevents government spending from exploding. The other, for open market operations, limits the ratio of current to steady state government equity holdings to a weighted sum of productivity and liquidity impulse responses.

<sup>3</sup>In other words,  $p_t$  and  $q_t$  units of general output are exchanged for 1 unit of money and equity, respectively. These are "real" prices. "Nominal" prices are the prices of a unit of general output and equity expressed in terms of money, i.e.  $1/p_t$  and  $p_t/q_t$ , respectively. Inflation in the conventional sense is an increase in the nominal price of general output, or equivalently, a fall in the real price of money.

the share of output accruing to capital.  $A_t$  evolves according to a stationary AR(1) process,

$$A_t = (1 - \rho_A)A + \rho_A A_{t-1} + u_t^A \quad (2.2)$$

where  $A$  is the steady state level of productivity,  $\rho_A \in (0, 1)$  parameterises the degree of persistency of a stochastic productivity shock, and  $u_t^A$  is an exogenously determined, independently, identically, and Normally distributed innovation with zero mean and constant variance  $\sigma_{u_A}^2$ .

The entrepreneur pays workers  $w_t$  units of general output for each labour-hour employed. The rest of the period's general output is the gross profit to the capital used in production,

$$r_t k_t = y_t - w_t l_t \quad (2.3)$$

### 2.2.2 Investment

Capital depreciates during the production process, and a fraction,  $\delta$ , of its stock survives to the end of the period when production is complete. Some time soon after the start of the period, a fraction,  $\pi$ , of entrepreneurs gain access to a homogenous investment technology that converts a unit of general output into a unit of capital with guaranteed success. The production and installation of new capital takes an entire period; so an entrepreneur who invests  $i_t$  units of general output in period  $t$  has

$$k_{t+1} = \delta k_t + i_t$$

units of capital at the end of period  $t$ .

Entrepreneurs are identical *ex ante* to when investment opportunities are revealed.  $\pi$  is independently and identically distributed across time and entrepreneurs. Who gets an opportunity to invest is exogenously determined. Those without investment opportunities carry on with what they have been doing since the start of the period, i.e. producing, consuming, and saving (by purchasing assets); they are the period's "savers". Those with investment opportunities change their behaviour when such opportunities are received; they are the period's "investors". The investment technology is unrestricted in capacity, but the opportunity to use it expires at the end of the period. At the end of the period, investors and savers revert to being *ex ante* identical until the next period's investment opportunities are revealed.

In an attempt to acquire each unit of general output for the investment technology, an investor publicly issues a new unit of equity. However, the investor simultaneously faces two constraints that make such external financing incomplete.

Once an investment project is underway, the entrepreneur's human resource is needed for the entire period to ensure the full amount of new capital is produced. The investor acquires knowledge and skills that are specific to his investment project and cannot be costlessly replicated or replaced. The entrepreneur, however, cannot pre-commit to being involved with the project to its end. Instead, he can guarantee that he will remain with the project for no more than an exogenously determined fraction,  $\theta$ , of its duration. This implies that he can guarantee a maximum of  $\theta$  of an investment's new capital will be produced, which further implies that he can guarantee a maximum of  $\theta$  of new output in the next period when the new capital enters production technologies. Consequently, the investing entrepreneur can credibly raise no more

than  $\theta$  of his investment cost from equity financing. This limitation is called the “borrowing” constraint, and has its origins in [Hart and Moore \(1994\)](#) and [Kiyotaki and Moore \(1997\)](#).<sup>4</sup>

The representative entrepreneur holds  $n_t$  units of equity at the beginning of the period. With an investment opportunity, and after the borrowing limit has been reached, the entrepreneur sells his equity. However, the investor cannot sell all of his equity before the investment opportunity expires. Instead, he can liquidate up to a fraction,  $\phi_t$ , of his holdings in period  $t$ . This limitation is called the “re-saleability” constraint, and is an exogenously determined, intrinsic feature of equity.  $\phi_t$  evolves according to a stationary AR(1) process,

$$\phi_t = (1 - \rho_\phi)\phi + \rho_\phi\phi_{t-1} + u_t^\phi \quad (2.4)$$

where  $\phi$  is the steady state value of  $\phi_t$ ,  $\rho_\phi \in (0, 1)$  parameterises the degree of persistency of a stochastic shock to  $\phi_t$ , and  $u_t^\phi$  is an exogenously determined, independently, identically, and Normally distributed innovation with zero mean and constant variance  $\sigma_{u^\phi}^2$ .

Borrowing and re-saleability constraints are together called “liquidity” constraints. Beyond these limits, the investor completes his investment financing by exchanging money for general output at the market price,  $p_t$ . The representative entrepreneur holds  $m_t$  units of money at the beginning of the period. The entrepreneur cannot lend money or use it as collateral, and therefore holds

$$m_{t+1} \geq 0 \quad (2.5)$$

units at the end of the period. An entrepreneur’s demand for money is motivated by a precaution against falling short of liquidity when financing investment opportunities.<sup>5</sup>

For an investment of  $i_t$  units of general output, the investor raises as much as  $\theta i_t q_t$  units of general output from issuing new equity; this is the investment’s external finance. The remainder of the investment cost,  $(i_t - \theta i_t q_t)$ , is internally financed from liquid funds, i.e. re-saleable equity and money.  $(i_t - \theta i_t q_t)$  therefore represents the total “payment” the investor makes to himself to acquire the non-re-saleable part of his own new equity issue. Put differently, for every unit of investment, the entrepreneur pays himself  $(1 - \theta q_t)$  units of general output to acquire  $(1 - \theta)$  units of his own new equity issue. Or equivalently, for every unit of his own new equity that he retains, the investor effectively pays himself  $q_t^R$  units of general output, where

$$q_t^R = \frac{1 - \theta q_t}{1 - \theta} \quad (2.6)$$

Notice from Equation (2.6) that the higher the market price of equity,  $q_t$ , the more funds the investor externally obtains, and the less he spends on buying his own new equity issue, i.e. the

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<sup>4</sup>An alternative interpretation of  $\theta$  is proposed by [Lorenzoni and Walentin \(2007\)](#): the entrepreneur can “run away” with a fraction,  $(1 - \theta)$ , of the value of his capital at any time, simply because capital is always under his complete control. In models with formal credit markets, unlike this one,  $\theta$  is featured as a credit market friction: due to a limited ability by lenders to enforce loan contracts, lenders request collateral, and lend at most a fraction,  $\theta$ , of the value of collateralised assets. The credit market friction is its more common representation, owing to [Kiyotaki and Moore \(1997\)](#) who show its macroeconomic significance, and to [Carlstrom and Fuerst \(1997\)](#) and [Bernanke et al. \(1999\)](#) who introduce it into dynamic macroeconomic models.

<sup>5</sup>The demand for money is inversely related to the tightness of liquidity constraints, i.e. the values of  $\theta$  and  $\phi_t$ . The tighter the liquidity constraints bind, i.e. the smaller the values, then the greater the need to internally finance investments, and the greater the desire to hold money balances. The converse is true.

smaller his effective payment,  $q_t^R$ ; and conversely.

That non-re-saleable part of his own new equity issue the investor retains is called his “inside” equity. An entrepreneur’s stock of equity that is issued by other entrepreneurs is called his “outside” equity. Inside and outside equity are assumed to be perfect substitutes, i.e. they have the same re-saleability constraint and provide the same rate of return. Inside and outside equity are therefore collectively referred to as “equity”.

For an investment of  $i_t$  units of general output, at the end of the period, the investor buys at least  $(1 - \theta)i_t$  new units of inside equity and remains with at least  $(1 - \phi_t)\delta n_t$  units of non-re-saleable equity. The investor therefore holds

$$n_{t+1}^i \geq (1 - \theta)i_t + (1 - \phi_t)\delta n_t \quad (2.7)$$

units of equity at the end of the period.

**Assumption 1.** An entrepreneur with an investment opportunity borrows and liquidates the maximum quantities of equity that liquidity constraints allow.

With assumption 1, (2.7) becomes binding with equality,

$$n_{t+1}^i = (1 - \theta)i_t + (1 - \phi_t)\delta n_t \quad (2.8)$$

which, if re-structured, gives the entrepreneur’s investment for the period,

$$i_t = \left( \frac{1}{1 - \theta} \right) n_{t+1}^i - \left( \frac{1 - \phi_t}{1 - \theta} \right) \delta n_t \quad (2.9)$$

### 2.2.3 Consumption and saving

Since capital is created only from investment, then each unit of equity in the economy is backed by a unit of capital. Equity therefore depreciates in tandem with capital. Furthermore, the gross profits that accrue to capital represent dividends to the holders of equity. Then each unit of equity earns  $r_t$  in dividends after period  $t$ ’s production is complete. The entrepreneur pays the government a tax on dividend income at a rate of  $\tau_t^{rn}$ . His net dividend income is allocated to consumption and saving, and to investment if the opportunity exists.

An investor in period  $t$  consumes  $c_t^i$  units of general output, invests  $i_t$  units, and saves by acquiring  $(n_{t+1}^i - i_t - \delta n_t)$  and  $(m_{t+1}^i - m_t)$  units of equity and money, respectively, at their market prices. The investor thus faces a budget constraint for period  $t$ ,

$$c_t^i + i_t + q_t(n_{t+1}^i - i_t - \delta n_t) + p_t(m_{t+1}^i - m_t) = (1 - \tau_t^{rn})r_t n_t \quad (2.10)$$

The investor’s budget constraint (2.10) is simplified by substituting Equation (2.9) to obtain (see Appendix C.1)

$$c_t^i + q_t^R n_{t+1}^i = (1 - \tau_t^{rn})r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \delta n_t + p_t(m_t - m_{t+1}^i) \quad (2.11)$$

The RHS of Equation (2.11) is the investor’s net worth: his net dividends from equity holdings, the value of depreciated equity (where a re-saleable fraction,  $\phi_t$ , is valued at the market price

and the non-re-saleable fraction is valued at  $q_t^R$ ), and net sales of money. The LHS expresses what he does with his net worth.

Alternatively, substituting Equation (2.8) into Equation (2.10) gives the investor's resource constraint (see Appendix C.1),

$$c_t^i + (1 - \theta q_t) i_t = (1 - \tau_t^{rn}) r_t n_t + \phi_t q_t \delta n_t + p_t (m_t - m_{t+1}^i) \quad (2.12)$$

The RHS of Equation (2.12) is the total liquid resources available to the investor in period  $t$ : net dividends from equity, a re-saleable portion of equity holdings, and net sales of money. The LHS says how he uses these resources: for consumption and financing that portion of his investment for which he cannot borrow.

A saver in period  $t$  consumes  $c_t^s$  units of general output, and saves the rest of his net income by purchasing  $(n_{t+1}^s - \delta n_t)$  and  $(m_{t+1}^s - m_t)$  units of equity and money, respectively, at their market prices. The saver's budget constraint for period  $t$  is

$$c_t^s + q_t (n_{t+1}^s - \delta n_t) + p_t (m_{t+1}^s - m_t) = (1 - \tau_t^{rn}) r_t n_t \quad (2.13)$$

#### 2.2.4 Optimising behaviour

When investment opportunities are revealed in period  $t$ , the representative investor and saver make optimal choices on  $\{c_t^i, i_t, n_{t+1}^i, m_{t+1}^i\}$  and  $\{c_t^s, n_{t+1}^s, m_{t+1}^s\}$ , respectively. These choices maximise an expected lifetime discounted utility,

$$E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_e(c_j) \right] = U_e(c_t) + E_t[\beta U_e(c_{t+1}) + \beta^2 U_e(c_{t+2}) + \dots] \quad (2.14)$$

subject to the respective budget constraints, (2.11) and (2.13), where  $E_t[\cdot]$  is the expected value conditional on information available in period  $t$ , and  $\beta \in (0, 1)$  is the subjective discount factor, or the inverse of the rate of time preference. The representative entrepreneur's current utility is a natural logarithm of current consumption,

$$U_e(c_t) \equiv \ln c_t$$

Optimal choices are made with uncertainty about investment opportunities in the future, i.e. period  $t+1$ . The entrepreneur's first order conditions yield an Euler equation (see Appendix C.2),

$$\begin{aligned} \pi E_t \left[ \frac{1}{q_t} ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R) U_e'(c_{t+1}^i) \right] \\ + (1 - \pi) E_t \left[ \frac{1}{q_t} ([1 - \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}) U_e'(c_{t+1}^s) \right] \\ = E_t \left[ \frac{p_{t+1}}{p_t} \left( \pi U_e'(c_{t+1}^i) + (1 - \pi) U_e'(c_{t+1}^s) \right) \right] \end{aligned} \quad (2.15)$$

Once an entrepreneur identifies his optimal path for period  $t$ , the Euler equation (2.15) describes the expectation that in period  $t+1$ ,  $1/q_t$  additional units of equity and  $1/p_t$  additional units



of money provide the same marginal utility from consumption. The expression on the RHS of (2.15) is the expected marginal benefit of holding  $1/p_t$  additional units of money in period  $t + 1$ . The expression on the LHS of (2.15) is the expected marginal benefit of holding  $1/q_t$  additional units of equity in period  $t + 1$ . Each unit of equity is expected to earn a net dividend of  $(1 - \tau_{t+1}^{rn})r_{t+1}$  and depreciate in value. If there is an investment opportunity, a re-saleable fraction of depreciated equity,  $\delta\phi_{t+1}$ , will be valued at the market price,  $q_{t+1}$ , while the non-re-saleable fraction,  $\delta(1 - \phi_{t+1})$ , will be valued at its replacement cost,  $q_{t+1}^R$ . If there is no investment opportunity, the depreciated value of a unit of equity will be  $\delta q_{t+1}$ .

**Claim 1.**  $q_t \neq 1 \iff m_{t+1}^i = 0$

*Proof of Claim 1.* See Appendix C.4.

The market price of equity is critical for economic activity. An investor needs at least 1 unit of general output for every unit of equity issued. If  $q_t < 1$  then the investor does not raise enough funds in the market to fulfil his ambition of investing  $i_t$ ; the investor abandons his opportunities, and then all entrepreneurs become savers. If  $q_t > 1$  then the investor materialises his opportunity and sells as much equity as he can within budget and liquidity constraints. The following assumption is therefore made to restrict attention to the case where investment takes place in the economy:

**Assumption 2.**  $q_t > 1$

By Claim 1 and Assumption 2, an investor will not have any money left at the end of a period of investment, i.e.  $m_{t+1}^i = 0$ . He exhausts all of his money in the pursuit of an investment opportunity. In the next period, up to when new investment opportunities are revealed, the current period's investors will be able to replenish their money stocks.

The entrepreneur's logarithmic utility function provides a standard feature that his consumption in each period is a stable fraction,  $(1 - \beta)$ , of his net worth in that period. From Equations (2.11) and (2.13), Claim 1 and Assumption 2, a representative investor and saver therefore consume, respectively,

$$c_t^i = (1 - \beta)([1 - \tau_t^{rn}]r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \delta n_t + p_t m_t) \quad (2.16)$$

$$c_t^s = (1 - \beta)([1 - \tau_t^{rn}]r_t n_t + q_t \delta n_t + p_t m_t) \quad (2.17)$$

The difference in consumption between the two types of entrepreneurs is given by

$$\begin{aligned} c_t^s - c_t^i &= (q_t - q_t^R)(1 - \phi_t)\delta n_t \\ &= (q_t - 1) \left( \frac{1 - \phi_t}{1 - \theta} \right) \delta n_t \end{aligned} \quad (2.18)$$

Assumption 2 therefore implies  $c_t^s > c_t^i$ . As entrepreneurs utilise equity and money towards investment financing, they inter-temporally substitute consumption away from an investing period and towards a saving period. During a period of saving they accumulate equity and money, and do so in an optimal fashion according to the Euler equation (2.15).

Assumption 2 also implies (see Appendix D)

$$\frac{[1 - \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + (1 - \phi_{t+1})\delta q_{t+1}^R}{q_t} < \frac{[1 - \tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t} \quad (2.19)$$

i.e. an investor's equity portfolio generates a lower rate of return than a saver's equity portfolio. This is because of the limited re-saleability of equity for an investor, which forces him to own inside equity that is valued negatively to the market price of equity. Hence, the return on equity is correlated with consumption. This correlation, along with the re-saleability constraint, is what makes equity risky. Money, on the other hand, is free from these risks. Its return does not depend on having an investment opportunity and it is perfectly liquid; these are two reasons why entrepreneurs hold money. Additionally, savers accumulate money in preparation for when they receive investment opportunities and expect to face financing constraints.

The Euler equation (2.15) simplifies to a portfolio balance equation (see Appendix C.3),

$$\begin{aligned} \pi E_t & \left[ \frac{\left( \frac{p_{t+1}}{p_t} \right) - \left( \frac{[1 - \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1 - \phi_{t+1}]\delta q_{t+1}^R}{q_t} \right)}{[1 - \tau_{t+1}^{rn}]r_{t+1}n_{t+1} + [\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R]\delta n_{t+1} + p_{t+1}m_{t+1}} \right] \\ & = (1 - \pi)E_t \left[ \frac{\left( \frac{[1 - \tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t} \right) - \left( \frac{p_{t+1}}{p_t} \right)}{[1 - \tau_{t+1}^{rn}]r_{t+1}n_{t+1} + q_{t+1}\delta n_{t+1} + p_{t+1}m_{t+1}} \right] \end{aligned} \quad (2.20)$$

Equation (2.20) reflects the portfolio balance theory of Tobin (1958, 1969) and demonstrates substitution between assets when their relative price changes. If  $q_t$  rises, for example, then equity's expected return falls, and the entrepreneur substitutes towards money. The substitution represents an increase in demand for money, which in aggregate, *ceteris paribus*, raises  $p_t$ . Substitution moves back and forth until expected portfolio returns between having and not having an investment opportunity are equal. The LHS of Equation (2.20) expresses an expected excess return on money over equity if the entrepreneur becomes an investor. The RHS expresses an expected excess return on equity over money if he becomes a saver. The portfolio balance equation says that the *ex ante* identical entrepreneur equates the expected marginal benefits of receiving and not receiving an investment opportunity. He does this by varying how many units of equity and money he holds.

### 2.3 Workers

Workers are the exclusive owners of labour. They do not own capital or have investment opportunities. In period  $t$  the representative worker supplies  $l_t^w$  hours of labour to entrepreneurs in exchange for a perfectly flexible gross hourly wage of  $w_t$  units of general output. The worker pays the government a tax on wage income at a rate of  $\tau_t^{wl}$ .

The worker holds  $n_t^w$  units of entrepreneur-issued equity and  $m_t^w$  units of government-issued fiat money at the beginning of period  $t$ . Each unit of equity earns gross dividends of  $r_t$  units of general output per period, depreciates at a constant rate of  $(1 - \delta)$  per period, and faces the re-saleability constraint,  $\phi_t$ . Money does not have any of these features. The worker's human resource is non-transferable across time, so he cannot borrow or have negative net worth. His

equity and money holdings are therefore always non-negative, i.e. for all  $t$ ,

$$n_t^w \geq 0 \quad \text{and} \quad m_t^w \geq 0 \quad (2.21)$$

The worker pays the government a tax on dividend income at a rate of  $\tau_t^{rn}$ .

The worker consumes  $c_t^w$  units of general output. The rest of his net income is saved by purchasing  $(n_{t+1}^w - \delta n_t^w)$  and  $(m_{t+1}^w - m_t^w)$  units of equity and money, respectively, at their prevailing market prices. His budget constraint for period  $t$  is given by

$$c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) = (1 - \tau_t^{wl})w_t l_t^w + (1 - \tau_t^{rn})r_t n_t^w \quad (2.22)$$

Subject to Equations (2.21) and (2.22), the worker maximises an expected lifetime discounted utility,

$$E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_w(c_j^w, l_j^w) \right] = U_w(c_t^w, l_t^w) + E_t[\beta U_w(c_{t+1}^w, l_{t+1}^w) + \beta^2 U_w(c_{t+2}^w, l_{t+2}^w) + \dots] \quad (2.23)$$

The worker's utility is additively separable in consumption and leisure,

$$U_w(c^w, l^w) = c^w - \frac{\omega}{1 + \nu} (l^w)^{1+\nu}$$

where  $\omega$  is the relative weight of labour in utility and  $\nu$  is the inverse Frisch elasticity of labour supply.<sup>6</sup>

### 2.3.1 Optimising behaviour

At the start of period  $t$  the representative worker chooses  $\{c_t^w, l_t^w, n_{t+1}^w, m_{t+1}^w\}$  to maximise his expected discounted utility, subject to his budget constraint (2.22). The worker optimally supplies labour until the marginal disutility of work (or equivalently, the marginal utility of leisure) is equal to the real wage rate. First order conditions yield his supply of labour (see Appendix D.1),

$$l_t^w = \left[ \frac{(1 - \tau_t^{wl})w_t}{\omega} \right]^{\frac{1}{\nu}} \quad (2.24)$$

**Claim 2.**  $n_{t+j}^w = 0$  and  $m_{t+j}^w = 0$ , for  $j = 0, 1, 2, \dots$ , i.e. the worker will always choose not to hold equity and money.

*Proof of Claim 2.* From Equations (D.1), (D.3) and (D.4) in Appendix D.1, if the worker decides to hold equity and money, i.e. if  $n_{t+1}^w \neq 0$  and  $m_{t+1}^w \neq 0$  then

$$\frac{\delta q_{t+1} + (1 - \tau_{t+1}^{rn})r_{t+1}}{q_t} = \frac{p_{t+1}}{p_t} = \frac{1}{\beta} \quad (2.25)$$

Equation (2.25) says that holding equity and money will not provide any superior (expected) gains above the discounted marginal utility from consumption,  $1/\beta$ . If the worker has one more

<sup>6</sup>The specification for  $U_w(c^w, l^w)$  implies the disutility from work does not directly affect the utility from consumption. Nezafat and Slavík (2012) point out that this utility specification is unusual in the Real Business Cycle literature, but shows that quantitative results are not sensitive to the choice of functional form.

unit of general output, he gains as much by consuming it as he expects to gain by saving it. Then there is no reason for the worker to save. The worker saves only if there is a marginal benefit from doing so.  $\square$

By Claim 2, the worker’s budget constraint (2.22) simplifies to

$$c_t^w = (1 - \tau_t^{wl})w_t l_t^w \quad (2.26)$$

i.e. in each period the worker’s consumes his entire net wage earnings, thus making him non-Ricardian.<sup>7</sup>

## 2.4 Government

The government is both the fiscal and monetary authority. As the fiscal authority, the government collects  $T_t$  in taxes from entrepreneurs and workers according to

$$T_t = \tau_t^{rn} r_t N_t + \tau_t^{wl} w_t L_t \quad (2.27)$$

where  $N_t$  is the total private sector’s equity holdings at the beginning of period  $t$ , and  $L_t$  is the aggregate number of labour hours employed in the period’s production, or the period’s “employment”. The government consumes  $G_t$  units of general output; this activity does not directly affect the utility of workers and entrepreneurs or create any production externalities.<sup>8</sup>  $G_t - T_t$  is the government’s fiscal balance.

As the monetary authority, the government exclusively and costlessly issues or withdraws fiat money in exchange for general output at the market price, and thereby earns  $p_t(M_{t+1} - M_t)$  units of general output as seignorage, where  $M_t$  and  $M_{t+1}$  are the stocks of money in circulation at the start and end of period  $t$ , respectively.

The government (unconventionally) owns a non-negative stock of entrepreneur-issued equity. At the start of period  $t$  it holds  $N_t^g$  units of equity, which earn dividends at a rate of  $r_t$  units of general output over the period, depreciate at a rate of  $(1 - \delta)$  per period, and faces the re-saleability constraint,  $\phi_t$ . Over the period, the government buys/sells  $|N_{t+1}^g - \delta N_t^g|$  from/to private agents in exchange for general output at the market price. These purchases and sales represent changes in the market supply of equity and are called “open market operations”. Section 2.5.1 elaborates more on this activity in the context of the equity market.

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<sup>7</sup>The non-Ricardian feature is how this model departs from the standard Real Business Cycle (RBC) model and starts to resemble Keynesian IS-LM. [Driffill and Miller \(2013\)](#) algebraically show that the KM model is fundamentally IS-LM by simplifying it to two equations that resemble IS and LM functions. If workers are Ricardian, as in the standard RBC model, then a cut in the income tax rate increases the present value of disposable income, and thus creates a positive wealth effect that induces a rise in saving and a drop in consumption. But here, a cut in the income tax rate increases workers’ consumption; this is shown in Section 3. The non-Ricardian feature of this model arises endogenously. Elsewhere in the literature, where such behaviour is exogenously assumed to hold (in [Galí et al. \(2007\)](#), for example), it is justified by such things as lack of access to financial markets, myopia, or fear of saving. [Campbell and Mankiw \(1989\)](#) provide empirical support for the existence of non-Ricardian behaviour, while [Mankiw \(2000\)](#) reviews microeconomic evidence that supports such behaviour.

<sup>8</sup>[Canova and Pappa \(2011\)](#) note that if a change in government spending affects private agents’ utility (as in [Bouakez and Rebei \(2007\)](#)) and/or creates production externalities (as in [Baxter and King \(1993\)](#)) then the output response is amplified.

The government balances its overall budget in every period. Its consumption and open market operations are financed from taxes, dividends, and seignorage, according to

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \quad (2.28)$$

The economy is subject to exogenous stochastic shocks to policy variables  $N_{t+1}^g$ ,  $M_{t+1}$ ,  $\tau_t^{rn}$ , and  $\tau_t^{wl}$ . These policy variables evolve according to the same stationary AR(1) process,

$$N_{t+1}^g = (1 - \rho_{Ng})N_t^g + \rho_{Ng}N_t^g + u_t^{Ng} \quad (2.29)$$

$$M_{t+1} = (1 - \rho_M)M_t + \rho_M M_t + u_t^M \quad (2.30)$$

$$\tau_t^{rn} = (1 - \rho_{\tau rn})\tau_t^{rn} + \rho_{\tau rn}\tau_{t-1}^{rn} + u_t^{\tau rn} \quad (2.31)$$

$$\tau_t^{wl} = (1 - \rho_{\tau wl})\tau_t^{wl} + \rho_{\tau wl}\tau_{t-1}^{wl} + u_t^{\tau wl} \quad (2.32)$$

where, for  $X_t \in \{N_{t+1}^g, M_{t+1}, \tau_t^{rn}, \tau_t^{wl}\}$ ,  $X$  denotes the steady state value,  $\rho_X$  parameterises the degree of persistency of a shock to  $X$ ,  $u_t^X$  is an exogenously determined, independently, identically, and Normally distributed innovation with zero mean and constant variance  $\sigma_{u^X}^2$ , and  $E[u_t^Y u_t^X] = 0$  for  $X_t \neq Y_t \in \{N_{t+1}^g, M_{t+1}, \tau_t^{rn}, \tau_t^{wl}\}$ . It is assumed that  $|\rho_X| < 1$  so that exogenous shocks are temporary.<sup>9</sup>

## 2.5 The aggregate economy

### 2.5.1 The equity market

At the beginning of period  $t$ , *ex ante* identical entrepreneurs hold a total stock of  $N_t$  units of equity. When investment opportunities are revealed, the period's savers account for a total of  $(1 - \pi)N_t$  units of equity. Savers are the only agents who buy equity; investors sell their equity to finance projects. Savers buy equity from three sources: (i) investors selling  $\phi_t$  of their depreciated equity holdings,  $\pi\delta N_t$ ; (ii) investors issuing  $\theta I_t$  new units of (outside) equity; and (iii) government open market operations,  $(N_{t+1}^g - \delta N_t^g)$ . By the end of the period, the stock of equity held by savers is

$$N_{t+1}^s = (1 - \pi)\delta N_t + \phi_t \pi \delta N_t + \theta I_t - (N_{t+1}^g - \delta N_t^g) \quad (2.33)$$

which is re-expressed as an equity market clearing condition,

$$N_{t+1}^s - \delta N_t^s = \phi_t \pi \delta N_t + \theta I_t - (N_{t+1}^g - \delta N_t^g) \quad (2.34)$$

where the RHS is the aggregate supply of equity. Savers' accumulation of equity, or aggregate saving, satisfies an aggregate portfolio balance equation, from Equation (2.20),

$$\pi E_t \left[ \frac{\left( \frac{p_{t+1}}{p_t} \right) - \left( \frac{[1 - \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1 - \phi_{t+1}]\delta q_{t+1}^R}{q_t} \right)}{[1 - \tau_{t+1}^{rn}]r_{t+1}N_{t+1}^s + [\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R]\delta N_{t+1}^s + p_{t+1}M_{t+1}} \right]$$

<sup>9</sup>With an estimated DSGE model, [Mertens and Ravn \(2011a\)](#) show that responses are different in magnitude, but not in direction, between temporary and permanent fiscal shocks.

$$= (1 - \pi)E_t \left[ \frac{\left( \frac{[1 - \tau_{t+1}^r]r_{t+1} + \delta q_{t+1}}{q_t} \right) - \left( \frac{p_{t+1}}{p_t} \right)}{[1 - \tau_{t+1}^r]r_{t+1}N_{t+1}^s + q_{t+1}\delta N_{t+1}^s + p_{t+1}M_{t+1}} \right] \quad (2.35)$$

If the government sells equity on the open market, then

$$N_{t+1}^g - \delta N_t^g < 0$$

Because of the re-saleability constraint, the government can sell at most  $\phi_t \delta N_t^g$  units, and therefore

$$|N_{t+1}^g - \delta N_t^g| \leq \phi_t \delta N_t^g$$

or equivalently, because the expression inside the absolute value brackets is negative,

$$\begin{aligned} N_{t+1}^g - \delta N_t^g &\geq -\phi_t \delta N_t^g \\ \implies N_{t+1}^g &\geq (1 - \phi_t) \delta N_t^g \end{aligned} \quad (2.36)$$

**Assumption 3.** Within the limits of the re-saleability constraint, the maximum quantity of equity is exchanged in open market operations.

Assumption 3 does not hold in an open market sale when the size of the fiscal shock is smaller than the re-saleable value of government's equity holdings. If Assumption 3 holds then by inequality (2.36), when the government sells equity on the open market, its equity holdings evolve according to

$$N_{t+1}^g = (1 - \phi_t) \delta N_t^g \quad (2.37)$$

Entrepreneurs' total equity holdings depreciate each period, and accumulate from investment financing and government sales. Therefore, when the government sells equity on the open market, aggregate private equity holdings evolve according to<sup>10</sup>

$$N_{t+1} = \delta N_t + \phi_t \delta N_t^g + I_t \quad (2.38)$$

If the government buys equity on the open market, then

$$N_{t+1}^g - \delta N_t^g > 0$$

Aside from its budget constraint (2.28), the government has a limit on how many units of equity it can buy. Because of the re-saleability constraint, private agents can sell at most  $\phi_t \delta N_t$  units to the government, and therefore

$$|N_{t+1}^g - \delta N_t^g| \leq \phi_t \delta N_t$$

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<sup>10</sup>To clarify, aggregate investment adds  $I_t$  to the aggregate equity stock.  $\theta I_t$  is issued as new outside equity and enters equity's supply.  $(1 - \theta)I_t$  does not enter the equity market, but represents new (inside) equity that is retained by investors.

or equivalently, because the expression inside the absolute value brackets is positive,

$$\begin{aligned} N_{t+1}^g - \delta N_t^g &\leq \phi_t \delta N_t \\ \implies N_{t+1}^g &\leq \delta N_t^g + \phi_t \delta N_t \end{aligned} \quad (2.39)$$

Assumption 3 holds in an open market purchase to prevent the government from unnecessarily increasing its equity holdings beyond the need to balance its overall budget (2.28). By Assumption 3 and inequality (2.39), when the government buys equity on the open market, its equity holdings evolve according to

$$N_{t+1}^g = \delta N_t^g + \phi_t \delta N_t \quad (2.40)$$

Private agents retain their non-re-saleable equity stock,  $(1 - \phi_t)\delta N_t$ , and accumulate new units from investment financing. Therefore, when the government buys equity on the open market, aggregate private equity holdings evolve according to

$$N_{t+1} = (1 - \phi_t)\delta N_t + I_t \quad (2.41)$$

From either Equations (2.37) and (2.38) or Equations (2.40) and (2.41), the aggregate equity stock at the end of a period in which there are government open market operations is

$$N_{t+1} + N_{t+1}^g = \delta(N_t + N_t^g) + I_t \quad (2.42)$$

Since every unit of equity produces a unit of capital, then the capital stock,  $K_t$ , is matched by the aggregate equity stock,

$$K_t = N_t + N_t^g \quad (2.43)$$

and Equation (2.42) then becomes

$$K_{t+1} = \delta K_t + I_t \quad (2.44)$$

which is the law of motion for the aggregate capital stock.

If Assumption 3 holds then, from Equations (2.37), (2.38), (2.40) and (2.41), the laws of motion for government and private equity holdings are compactly given by

$$N_{t+1}^g = (1 - \mathbb{1}_t) [(1 - \phi_t)\delta N_t^g] + \mathbb{1}_t [\delta N_t^g + \phi_t \delta N_t] \quad (2.45)$$

$$N_{t+1} = (1 - \mathbb{1}_t) [\delta N_t + \phi_t \delta N_t^g + I_t] + \mathbb{1}_t [(1 - \phi_t)\delta N_t + I_t] \quad (2.46)$$

where

$$\mathbb{1}_t = \begin{cases} 1 & \text{if the government buys equity in period } t \\ 0 & \text{if the government sells equity in period } t \end{cases} \quad (2.47)$$

### 2.5.2 The labour market

From the production function (2.1), the marginal product of labour is

$$\frac{\partial y_t}{\partial l_t} = (1 - \gamma) A_t \left( \frac{k_t}{l_t} \right)^\gamma$$

From Equation (2.3), the first order condition for gross profit maximisation with respect to labour is

$$\begin{aligned} \frac{\partial y_t}{\partial l_t} - w_t &= 0 \\ \implies (1 - \gamma)A_t \left(\frac{k_t}{l_t}\right)^\gamma - w_t &= 0 \\ \implies l_t = k_t \left[\frac{(1 - \gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}} \end{aligned}$$

which is a typical entrepreneur's demand for labour, given his capital stock.

With an aggregate capital stock,  $K_t$ , owned entirely by entrepreneurs, it follows that the aggregate demand for labour is

$$L_t^D = K_t \left[\frac{(1 - \gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}} \quad (2.48)$$

Given the homogeneity and unit mass of the worker population, from Equation (2.24), the aggregate labour supply labour is

$$L_t^S = \left[\frac{(1 - \tau_t^{wl})w_t}{\omega}\right]^{\frac{1}{\nu}} \quad (2.49)$$

The inverse labour demand and supply functions are, respectively,

$$w_t^D = \frac{(1 - \gamma)A_t K_t^\gamma}{(L_t)^\gamma} \quad (2.50)$$

$$w_t^S = \left[\frac{\omega}{1 - \tau_t^{wl}}\right] (L_t)^\nu \quad (2.51)$$

The labour market clears when  $L_t^S = L_t^D$  and the equilibrium real wage and employment are, respectively (see Appendix E),

$$w_t = K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} (1 - \tau_t^{wl})^{-\frac{\gamma}{\gamma+\nu}} [(1 - \gamma)A_t]^{\frac{\nu}{\gamma+\nu}} \quad (2.52)$$

$$L_t = K_t^{\frac{\gamma}{\gamma+\nu}} \omega^{-\frac{1}{\gamma+\nu}} [(1 - \tau_t^{wl})(1 - \gamma)A_t]^{\frac{1}{\gamma+\nu}} \quad (2.53)$$

### 2.5.3 The general output market

Aggregate private consumption is

$$C_t = C_t^i + C_t^s + C_t^w \quad (2.54)$$

where, from Equations (2.16), (2.17) and (2.20), consumption by investors, savers, and workers are, respectively,

$$C_t^i = \pi(1 - \beta)([1 - \tau_t^{rn}]r_t N_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \delta N_t + p_t M_t) \quad (2.55)$$



$$C_t^s = (1 - \pi)(1 - \beta)([1 - \tau_t^{rn}]r_t N_t + q_t \delta N_t + p_t M_t) \quad (2.56)$$

$$C_t^w = (1 - \tau_t^{wl})w_t L_t \quad (2.57)$$

From Equation (2.12), aggregate investment is given by

$$(1 - \theta q_t)I_t = ([1 - \tau_t^{rn}]r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C_t^i \quad (2.58)$$

From Equation (2.1), aggregate production of general output is

$$Y_t = A_t K_t^\gamma L_t^{1-\gamma} \quad (2.59)$$

From Equation (2.3), aggregate gross profits are

$$r_t K_t = Y_t - w_t L_t \quad (2.60)$$

The general output market clears when

$$Y_t = C_t + I_t + G_t \quad (2.61)$$

## 2.6 Steady state

Since  $N_{t+1}^g$  and  $M_{t+1}$  exogenously determined, then (from Equation (2.28)) the government balances its fiscal budget in steady state and in any period in which there are no shocks to these variables. The government can hold any non-negative quantity of equity in steady state; its holdings will earn dividends that exactly match its depreciation. Computing a unique, stable equilibrium for the model requires the following assumption.

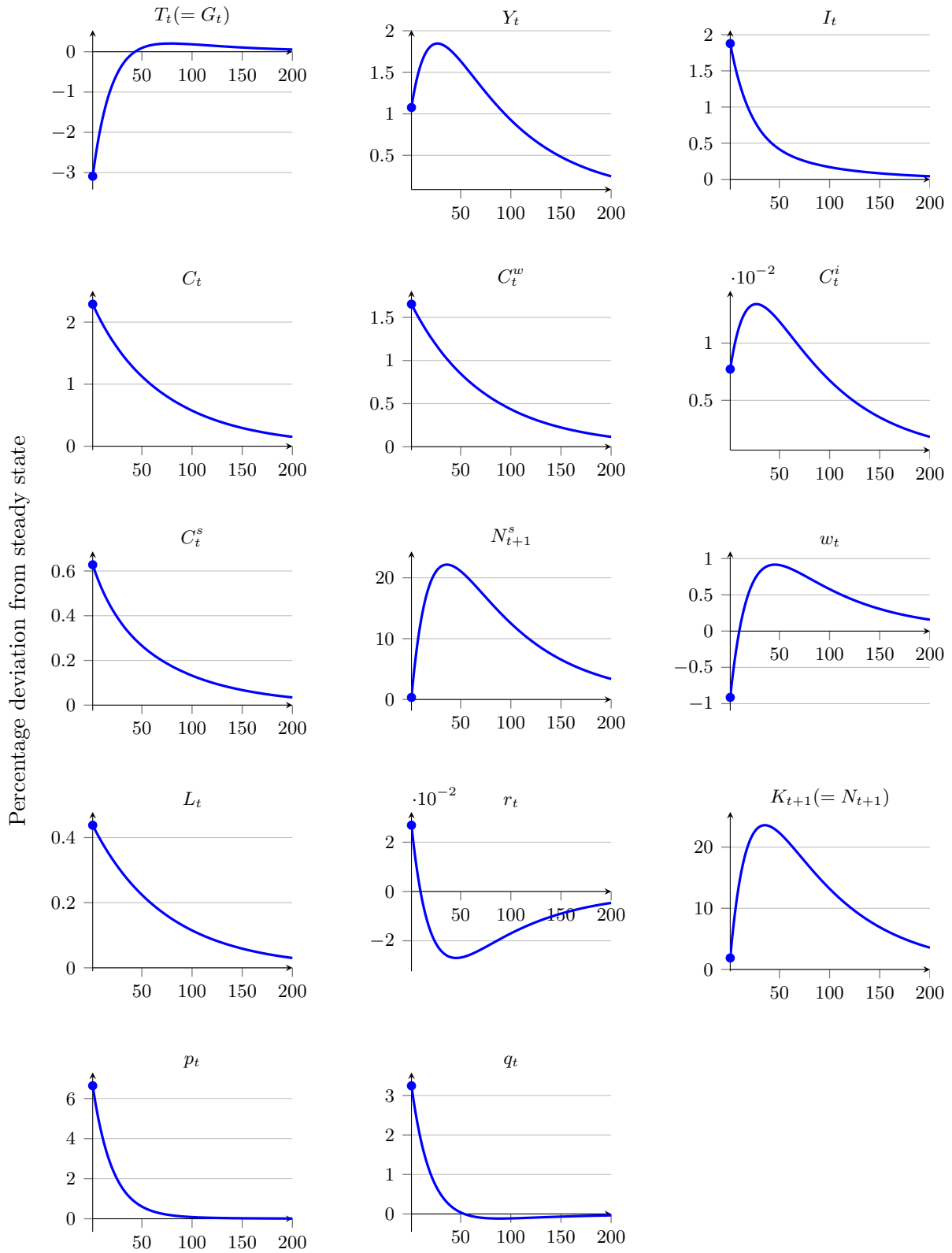
**Assumption 4.** In steady state, the government holds no equity, i.e.  $N^g = 0$ .

Holding privately-issued equity is an unconventional measure by the government. Government has no motive to save, and so it is reasonable to make this assumption.

## 3 Impulse response analysis

This section simulates unexpected, exogenous, temporary cuts to both tax rates (henceforth, a “tax shock”). The model is solved by a second-order approximation around steady state, and the shock is simulated by the quantitative technique of calibration. Appendix A describes the calibration process in detail. Structural parameters are set to the “baseline” values in Table 4 (in Appendix A); these values are taken from similar models in the related literature. Steady state levels and autoregressive (shock) parameters for exogenous variables are set according to values listed in Table 5 (in Appendix A). All parameter values are based on quarterly data. Results are presented as impulse responses, which are graphically illustrated in Figure 1 and summarised in Table 1.

FIGURE 1: Impulse responses of a tax shock: baseline scenario



NOTES: Horizontal axes measure quarters after the shock, starting from quarter 1. Blue dots indicate immediate responses; see the “quarter 1” column of Table 1 for their values.

TABLE 1: Impulse responses of a tax shock: baseline scenario

	Impulse responses (% deviation from steady state)							Quarters to	
	Quarter: 1	2	4	8	20	200	largest	largest	10% of Qu. 1
$T_t$	-3.09	-2.91	-2.57	-1.99	-0.86	0.05	-3.09	1	31
$Y_t$	1.08	1.15	1.29	1.50	1.81	0.25	1.85	27	200
$I_t$	1.88	1.81	1.67	1.44	0.95	0.04	1.88	1	94
$C_t$	2.29	2.25	2.19	2.05	1.72	0.15	2.29	1	167
$C_t^w$	1.65	1.63	1.59	1.50	1.27	0.11	1.65	1	173
$C_t^i$	0.01	0.01	0.01	0.01	0.01	0.00	0.01	27	200
$C_t^s$	0.63	0.61	0.59	0.54	0.43	0.03	0.63	1	156
$N_{t+1}^s$	0.35	2.09	5.25	10.42	19.25	3.38	22.16	36	200
$w_t$	-0.91	-0.79	-0.55	-0.16	0.55	0.16	0.92	45	200
$L_t$	0.44	0.43	0.42	0.40	0.34	0.03	0.44	1	173
$r_t$	0.03	0.02	0.02	0.00	-0.02	0.00	-0.03	46	200
$K_{t+1}$	1.88	3.64	6.82	12.03	20.83	3.56	23.58	35	200
$p_t$	6.63	6.31	5.71	4.67	2.57	0.01	6.63	1	49
$q_t$	3.25	3.07	2.73	2.16	1.02	-0.04	3.25	1	36

NOTES: This table gives impulse responses at certain periods after the tax shock, and the period of time impulse responses take to reach their peak and to converge within 10% of their quarter 1 magnitudes. “200” quarters in the last column means convergence happens some time after 200 quarters, and not in the 200<sup>th</sup> quarter.

### 3.1 Immediate responses

The tax shock is stochastic and lasts just one quarter. Time is counted from when the shock hits. In the first quarter, the cuts to wage and dividend income tax rates are equivalent to 1% and 2.8% of national output, respectively. Tax collections fall by 3.3% *ceteris paribus* (as measured by the variable  $T_t^*$ ) and by 3.1% with endogenous changes in tax bases (as measured by  $T_t$ ). The government continues to hold no stocks of money and equity, and balances its fiscal (and overall) budget by reducing its spending in tandem with taxes. For brevity, the size of impulse responses are not quoted in the text; they are listed in Table 1.

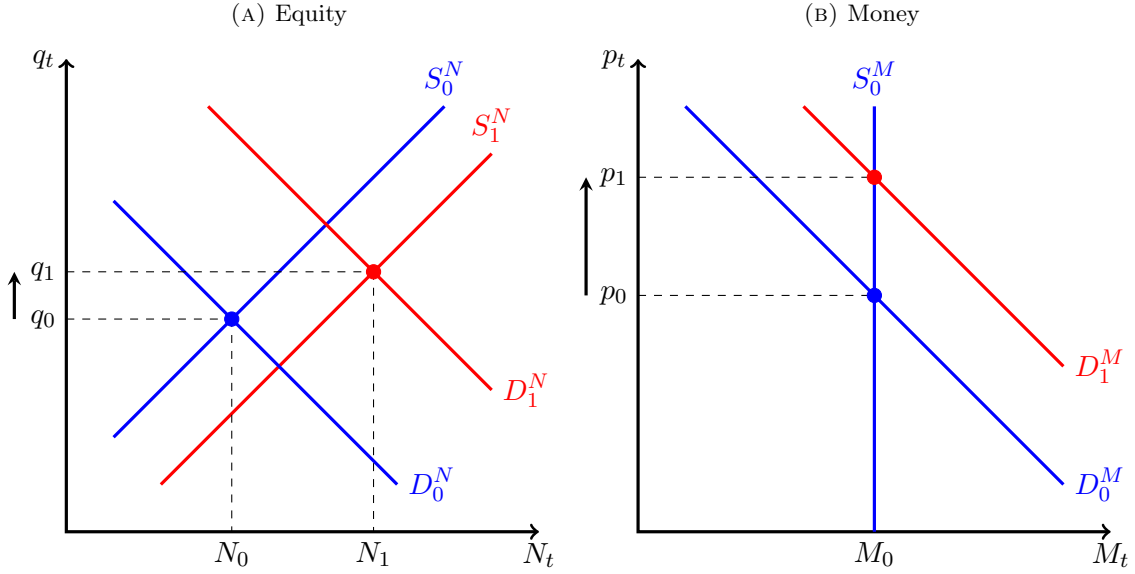
In response to the cut in the rate of tax on wage income, workers increase their labour supply at each and every wage (from Equation (2.49)).<sup>11</sup> In reply, entrepreneurs expand their labour demand and accept a lower wage, and thereby offset any marginal labour productivity losses (from Equation (2.48)). With assumptions of flexible wages and full employment, the labour market therefore adjusts to a lower real wage (by Equation (2.52)) and higher employment (by Equation (2.53)).

Output increases because of more employment (from Equation (2.59)). The other determinants of output are unchanged by the shock. Total factor productivity is exogenously determined (by Equation (2.2)), and the period’s capital stock is determined before the shock hits.

The cut in the dividend income tax improves the net worth of all entrepreneurs. As a result,

<sup>11</sup>More precisely, the baseline setting for  $\nu$  implies that the inverse aggregate labour supply function (2.51) is linear through the origin, as illustrated by Figure 12 in Section B. The shock therefore *pivots* the supply curve outwards.

FIGURE 2: Asset markets and immediate effects of the tax shock



NOTES:  $D$  denotes demand and  $S$  denotes supply. Superscripts  $N$  and  $M$  denote equity and money, respectively. Subscripts 0, 0.5, and 1 denote steady state, intermediate, and at the end of quarter 1, respectively.

these agents all increase their consumption (by Equations (2.55) and (2.56)). Those who receive investment opportunities early in the quarter spend more on investment (from Equation (2.58)), while savers increase their demand for both assets.

Figure 2B illustrates the immediate effects of the shock on the money market. Demand comes from entrepreneurs and supply is fully controlled by the government.  $D_0^M$  and  $S_0^M$  are the initial (i.e. steady state) demand and supply curves, respectively. Savers increase their demand for money after net worth improvements; this is illustrated by a rightward shift of  $D_0^M$  to  $D_1^M$ . With exogenously fixed money supply, the result is a higher price.

Figure 2A illustrates immediate effects of the shock on the equity market.  $D_0^N$  and  $S_0^N$  are the initial (steady state) demand and supply curves, respectively. Demand comes from savers; supply comes from investors (and the government, if  $N_{t+1}^g \neq 0$ ). Savers increase their demand for equity following net worth improvements; this is illustrated by a rightward shift of  $D_0^N$  to  $D_1^N$ . Equity's supply increases as investors issue new issues to finance their investment; this is represented by a rightward shift of  $S_0^N$  to  $S_1^N$ . The increase in supply is small; because of the borrowing constraint, investors issue a small amount of equity relative to the additional investment cost. As a consequence, equity's market price increases.

A demand for money exists because of both liquidity constraints; the borrowing constraint, in particular, restricts the shock-induced increase in equity supply to be small, and significantly contributes to the asset's higher price. Increases in both asset prices amplify the net worth improvements of entrepreneurs, which in turn increases asset demand and investment. This "internal amplification" mechanism originates from the liquidity constraints and is the cause of the long-term persistence of the shock.

Consider an alternative scenario. If the borrowing constraint is calibrated sufficiently loose then the shock encourages investors to issue a considerable amount of new equity, and (as if  $S_1^N$

is further to the right than drawn in Figure 2A) the shock then reduces the price of the asset. This result implies a higher expected return on equity, which encourages agents to substitute away from money. Depending on the relative sizes of this portfolio balance effect and the positive net worth effect, money’s price either increases by a smaller degree than illustrated in Figure 2 by  $p_1$ , or decreases altogether. If the latter result holds, i.e. both asset prices fall, then the *ceteris paribus* effects on entrepreneurs from the cut in the dividend tax rate are (partially or completely) offset. In particular, a first quarter increase in investment is not achieved. Without an increase in investment, aggregate demand decreases. And, as explained in Section 3.2, investment is key to the internal propagation of the shock, particularly for output.

Aggregate private consumption’s largest contributor comes from workers. The fall in the real wage is larger than the rise in employment (see Table 1), and therefore the aggregate gross wages decrease. But because of the drop in the tax rate, workers enjoy a higher aggregate net wage and, being non-Ricardian, consume more general output. Savers and investors consume more because of improvements in the net worth. Accordingly, total consumption increases.

### 3.2 Long-term responses

The steady state level of investment creates a quantity of new capital that exactly replaces depreciated units. Since the shock increases investment above its steady state level, the capital stock increases by the end of the first quarter. By the end of the first quarter, and for each subsequent quarter *ad infinitum*, the tax shock deteriorates at an assumed rate of 5% per quarter and both tax rates asymptotically increase towards their steady state levels.<sup>12</sup> As tax rates start increasing, workers reduce their labour supply, and the real wage increases and employment falls. Long-term economic growth is thus achieved by increases in the capital stock, and investment “internally propagates” the shock.

Furthermore, with more output and a lower aggregate wage bill, gross aggregate dividends are higher. This in part helps entrepreneurs enjoy higher-than-normal net worth. Investment is therefore sustained above steady state, and the capital stock continues to increase. Capital peaks after 35 quarters at 23.6% above steady state. By then, the level of depreciation starts to exceed investment, and the capital stock starts to return to its pre-shock level. This is why capital exhibits a hump-shaped trajectory.

Employment persistently declines since its initial response to the shock. Workers continue to reduce their labour supply as the rate of tax rises towards steady state. The real wage increases, and even overshoots its steady state level in the 10<sup>th</sup> quarter; it peaks just after 45 quarters at 0.9% above steady state (after falling below by 0.9% in the first quarter). The eventual decline in the wage rate occurs when entrepreneurs reduce their labour demand when the capital stock starts decreasing.

Output remains heavily influenced by capital, and even traces the same hump-shaped trajectory. Over its adjustment path, output is kept elevated above its steady state level while it converges. It increases continuously for 27 quarters before returning towards its pre-shock level. But the return is slow, and even after 200 quarters it is still approximately 0.3% above steady state, after being 1.9% above at its peak.

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<sup>12</sup>Sensitivity of shock responses to this 5% assumption is the subject of Section B.3.

The speed of adjustment is indicated by the time a variable takes to get “close” to steady state. A variable is considered “close” to steady state when its percentage deviation is within 10% of the immediate shock response. Figure 1 and the last column of Table 1 show that shock responses last a very long time. Tax revenue gets close to its pre-shock level after 31 quarters; it rises to overshoot steady state around the 50<sup>th</sup> quarter, and exhibits a gentle hump-shaped trajectory as it slowly returns to steady state. Asset prices are the next fastest variables to return to steady state, and they do so asymptotically. Equity and money prices get close to steady state after 36 and 49 quarters, respectively. Investment, private consumption, and employment also have the asymptotic trajectories, and are the next fastest aggregate variables to get close to steady state, after 94, 169, and 173 quarters, respectively. Output, capital, savers’ equity, the real wage, and the dividend rate all have hump-shaped trajectories, and are much slower to return to pre-shock levels, doing so some time after 200 quarters.

## 4 Tax multipliers

The objective of this section is to describe the shock’s responses as either “large” or “small” using multipliers. Tax multipliers measure the response of a variable, over a given period of time, to a drop in government tax revenue by 1 unit of general output in the first quarter due to discretionary cuts in both tax rates, *ceteris paribus*. A negative (positive, respectively) multiplier indicates that the variable increases (decreases, respectively) after the shock. If cumulative multipliers increase (decrease, respectively) with the measured time horizon, then the measured variable converges slower (faster, respectively) towards steady state than a specially-constructed variable  $T_t^*$  (defined below). A variable’s response is described as “large” if the absolute value of its tax multiplier is in excess of unity; otherwise the response is “small”.

Multipliers are more suitable than impulse responses for describing the magnitude of the effects of the shock, for two reasons. Firstly, because of its duality, the shock is not normalised (see the opening paragraph of Section 3.1), and impulse responses are therefore difficult to interpret on their own. Multipliers, on the other hand, measure normalised responses. Secondly, these multipliers disentangle the discretionary change in taxes (i.e. the change in tax rates) from the endogenous component (i.e. the changes in tax bases,  $w_t L_t$  and  $r_t K_t$ ).<sup>13</sup> This section omits any further multiplier analysis of the tax shock, because it would say the same things as the impulse responses analysis in Sections 3.1 and 3.2.

### 4.1 Methodology

Impact and cumulative tax multipliers are calculated for real aggregate variables only. Changes in both tax rates are captured by a single variable,  $T_t^*$ , which represents government tax collections with tax bases held constant to their steady state levels, i.e. from Equation (2.27),

$$T_t^* = \tau_t^{rn} rN + \tau_t^{wl} wL \quad (4.1)$$

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<sup>13</sup>Perotti (2012) highlights the importance of separating discretionary from endogenous changes in taxes by showing they have different effects on output.

TABLE 2: Tax multipliers: baseline scenario

	Immediate impact	Cumulative				
		Quarters: 2	4	8	12	16
$Y_t$	-2.9	-3.1	-3.4	-4.2	-4.9	-5.7
$I_t$	-1.0	-1.1	-1.1	-1.1	-1.1	-1.1
$C_t$	-3.5	-3.6	-3.7	-4.0	-4.2	-4.5
$C_t^w$	-2.1	-2.1	-2.2	-2.3	-2.5	-2.7
$C_t^i$	0.0	0.0	0.0	0.0	0.0	0.0
$C_t^s$	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2
$w_t$	3.1	3.0	2.7	2.1	1.5	0.9
$r_t$	0.0	0.0	0.0	0.0	0.0	0.0
$p_t$	-11.8	-11.8	-11.8	-11.9	-11.9	-11.9
$q_t$	-5.3	-5.2	-5.2	-5.2	-5.1	-5.0

where notations without time subscripts represent steady state (i.e.  $t = 0$ ) values. Changes in  $T_t^*$  therefore represent *ceteris paribus* changes in taxes. The immediate impact multiplier is a ratio of the response of a real aggregate variable in quarter 1 to the change in  $T_t^*$  in quarter 1,

$$\frac{X_1 - X}{T_1^* - T} \quad (4.2)$$

Cumulative multipliers capture a variable's accumulated changes over a period of time. They are measured over 2, 4, 8, 12, and 16 quarters according to

$$\frac{\sum_{t=1}^n (X_t - X)}{\sum_{t=1}^n (T_t^* - T)} \quad (4.3)$$

## 4.2 Results

Output and private consumption have very large, negative responses to the shock, both contemporaneously and cumulatively. On impact, output and consumption increase by 2.9 and 3.5 units of general output, respectively, for every unit the government gives up because of tax rate cuts, *ceteris paribus*. Both these variables' cumulative multipliers increase with the time horizon. This suggests that while the shock itself wears off, its effects on output and consumption continue to propagate. It also confirms the slow convergence that is suggested graphically by Figure 1.

Investment has a moderate increase. A unitary multiplier is observed on impact of the shock, and as the shock wears off, investment converges at a uniformly proportional and slightly slower rate than  $T_t^*$ .

By far, the largest responses are by asset prices. They both increase substantially, with impact multipliers of -11.8 for money and -5.3 for equity, and with similar values for cumulative multipliers over different time horizons.

Both contemporaneously and cumulatively, savers' consumption has a small increase (with multipliers of -0.2), and investors' consumption and the rate of return on capital have insignif-

icant responses (with multipliers close to 0).

## 5 Sensitivity analysis

The robustness of responses to the shock is examined with respect to the calibration of structural parameters and the persistence of tax shocks. For brevity, the details and results of these exercises are provided in Appendix B. The overall conclusion is that shock responses are (i) qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values; (ii) qualitatively and quantitatively robust to small (and plausible) variations in the persistence of tax shocks; and (iii) qualitatively and quantitatively sensitive to significant (and sometimes implausible) variations in the persistence of tax shocks.

### 5.1 Sensitivity to structural parameters

Structural parameter sensitivity analysis is performed systematically by three local methods, all involving repeated simulations of the shock with changes in structural parameter values to their “sensitivity settings”. The first method changes one structural parameter at a time; the second and third methods change combinations of two or more structural parameters. Responses to the shock are quantitatively sensitive to one-at-a-time variation of three structural parameters:  $\beta$ ,  $\gamma$ , and  $\delta$ . The model is also sensitive to combinations of alternative parameter settings, more so when these settings go beyond those of  $\beta$ ,  $\gamma$ , and  $\delta$ . Nevertheless, from changing parameter values either one-at-a-time or in combinations, tax shock responses vary only in magnitude, and not in direction or adjustment trajectories. Finally, comparing baseline responses to alternatives from all possible combinations of parameter values shows that, with the exception of investors’ consumption,  $C_t^i$ , baseline responses are not extreme.

### 5.2 Sensitivity to the persistence of tax shocks

Sensitivity to the persistence of tax shocks,  $\rho_{\tau wl}$  and  $\rho_{\tau rn}$ , is examined by repeatedly simulating the shock with simultaneous use of values above and below the calibrated (and fairly standard) setting. Responses to the shock are quantitatively and qualitatively sensitive to the calibration of the persistence parameters for tax shocks,  $\rho_{\tau rn}$  and  $\rho_{\tau wl}$ . Very small (and plausible) changes in the persistence parameters do very little to alter responses; but with larger (and sometimes implausible) parameter variations there are significant changes in trajectory and convergence. Lowering the level of persistence reveals that investment, savers’ equity, capital, investors’ consumption, and output are still slow to converge to steady state. This suggests that features in the model – in particular, the liquidity constraints – are responsible for their long-term responses to the shock. The mechanism, called the “internal amplification” mechanism, is described in the earlier analysis of the shock.



## 6 Discussion

### 6.1 The KM-related literature: the significance of liquidity constraints

The 2007/8 financial turmoil brought a wave of recent attention to [Kiyotaki and Moore \(2012\)](#) model, for two reasons. First, commentators argue that the cause of the crisis was the sudden and unexpected deterioration in the value of partially liquid private financial assets ([Brunnermeier \(2009\)](#), [Del Negro et al. \(2011\)](#), [Bigio \(2012\)](#) and [Jermann and Quadrini \(2012\)](#)). Assets' re-saleability and collateral suitability were thus adversely affected. This event bears a striking resemblance to KM's negative liquidity shock (i.e. a shock to  $\phi_t$ ). Secondly, the government holding risky, privately-issued assets with limited re-saleability was the central component of the unconventional policy responses to the crisis, whereby these assets were exchanged for safe, liquid, government-issued securities and cash.<sup>14</sup> This is, in fact, KM's main policy implication, that government can inject liquidity to counter-cyclically dampen business cycle fluctuations.

The KM model is theoretically adjusted and/or extended in a series of recent papers. These papers can be classified into two groups. The first group uses the KM model to evaluate the unconventional policies seen in the crisis; [Del Negro et al. \(2011\)](#) and [Driffill and Miller \(2013\)](#) are members of this research, and both show that crisis-induced recessions would have been exacerbated had it not been for government interventions. The second KM-related group returns to the original questions posed by KM on the importance of (i) liquidity shocks for explaining business cycles, and (ii) liquidity constraints for the propagation of productivity shocks. Papers in this group include [Salas-Landeau \(2010\)](#), [Bigio \(2010, 2012\)](#), [Ajello \(2011\)](#), [Nezafat and Slavík \(2012\)](#), [Shi \(2012\)](#), and [Jermann and Quadrini \(2012\)](#).

The inclusions of distortionary taxes and a balanced fiscal budget rule are not unique in the KM-related literature. [Ajello \(2011\)](#), [Shi \(2012\)](#), and [Driffill and Miller \(2013\)](#) have a balanced budget rule for government. [Ajello \(2011\)](#) also includes distortionary taxes, but he modifies the KM model more extensively than in this paper. The uniqueness of this paper's contribution is that it is the first to examine fiscal shocks in the KM model. What this paper shares with the KM-related literature, the second group in particular, is that it shows the macroeconomic significance of KM's liquidity constraints in propagating exogenous shocks. In this case, the shocks are to tax rates.

### 6.2 Fiscal shocks in DSGE models

The New Keynesian model is the workhorse for fiscal policy research. This perhaps follows from papers (such as [Burnside et al. \(2004\)](#)) which show that the magnitude of observed responses to fiscal shocks are not matched by a standard neoclassical models, but they are matched by models that include habit formation and adjustment costs. Beyond the liquidity constraints, this model is otherwise neoclassical, and it is able to produce large responses to fiscal shocks, albeit in a theoretical exercise. This work therefore suggests that New Keynesian frictions are not always needed to study fiscal policy. Instead, a neoclassical model can be modified to produce large responses to fiscal shocks. The essential ingredient of such a modification is the

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<sup>14</sup>The various facilities through which the US government implemented these exchanges are described in [Armantier et al. \(2008\)](#), [Fleming et al. \(2009\)](#), [Adrian et al. \(2009\)](#), and [Adrian et al. \(2011\)](#).

TABLE 3: Multipliers: a survey of the literature

	Output	Consumption	Investment	Employment
Romer and Romer (2010)	-2.9 (10 quarters)			
Mertens and Ravn (2011a)	-2.0 (10 quarters)	-2.0 (10 quarters)	-10.0 (10 quarters)	-1.0 (10 quarters)
Mertens and Ravn (2012)	-1.8 (3 quarters)			
Monacelli et al. (2012)	-2.7 (1 year)		-9.7 (1 year)	-0.5 (2 quarters)

NOTES: This table gives the peak cumulative multipliers from a 1% cut in taxation, and (in brackets) the time after the shock these multipliers are observed. A negative multiplier therefore represents an increase in the variable.

inclusion of KM’s liquidity constraints. The KM model can therefore become a workhorse for fiscal policy research.

### 6.3 The macroeconomics of tax cuts

This paper contributes to the literature on the macroeconomic effects of unexpected, discretionary changes in taxation. Tax cuts are shown to be expansionary in early works by Andersen and Jordan (1968), Giavazzi and Pagano (1990), Baxter and King (1993), Braun (1994), McGrattan (1994), Alesina and Perotti (1997), and Perotti (1999). The same result is shown by more recent contributions from Romer and Romer (2010), Mertens and Ravn (2011a,b, 2012), and Monacelli et al. (2012). These recent papers obtain quantitative results from estimating vector autoregressions and using, as the basis of datasets, the narrative record of exogenous US fiscal shocks developed by Romer and Romer (2010). Their peak cumulative multipliers are given in Table 3.<sup>15</sup> Despite the methodological differences, this paper’s results are consistent with this recent VAR literature. In particular, the literature’s estimated multipliers for output, consumption, and investment are large and negative, and responses exhibit long-term persistence. The difference is that their results suggest weaker output and consumption responses and a much stronger investment response.

This paper’s work is closely related to a small strand of the literature which shows tax cuts are expansionary with a balanced fiscal budget. Eggertsson (2010) uses a New Keynesian model with sticky prices and monopolistic competition, and compares the effects of cutting different tax rates. His recipe for economic stimulus is to cut consumption taxes and raise wage income and wealth taxes. However, he also suggests that liquidity constraints may reverse the intended responses. Results are different between this paper and Eggertsson (2010) because this model does not feature consumption taxes or New Keynesian frictions. Mountford and Uhlig (2009) (henceforth MU) come closest to this paper’s results. MU show that an unexpected, exogenous

<sup>15</sup>Mertens and Ravn (2011a) distinguish between expected and unexpected tax shocks. Table 3 gives their multipliers from *unexpected* shocks.

increase in government spending that is completely financed by an increase in taxation causes reductions in private consumption and investment on impact, as well as in output from the second period.<sup>16</sup> The converse of this result suggests a recipe for debt-free economic expansion. This paper complements MU by showing that the converse of their result is also true. The novelty here is that while MU obtain their results from an empirical study with vector autoregressions, this paper is a theoretical investigation using a mostly neoclassical DSGE model.

## 7 Conclusion

This paper shows that cuts to income tax rates in a liquidity constrained economy increases output, investment, and private consumption. The model is a modification of the mostly neoclassical, DSGE model of liquidity and business cycles by [Kiyotaki and Moore \(2012\)](#). In particular, distortionary taxes and a balanced budget fiscal rule are added to KM. The model is calibrated to be consistent with the KM-related literature. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to implausibly significant variations in the persistence of tax shocks.

This paper is unique in three ways. Firstly, these results are consistent with those obtained by [Mountford and Uhlig \(2009\)](#); but while they use an estimated VAR, this paper complements and supports with a theoretical finding from a calibrated neoclassical model. Secondly, this paper distinguishes itself from the rest of the KM-related literature by being the first to apply the KM model to fiscal shocks; this related literature remains focused on showing the significance of liquidity shocks in explaining business cycles, and the importance of liquidity constraints for propagating productivity shocks. Thirdly, the paper shows how a neoclassical model can be modified to produce large responses to fiscal shocks.

Some opportunities for future research are suggested by this work. One extension is an examination of a cut in taxes without a balanced budget in this environment. Another useful experiment is to cut one tax rate at a time, and determine their relative merits in the economic expansion seen in this paper. It would be interesting to determine the effects of an increase in government spending, with and without a balanced budget. The model can be adjusted by adding New Keynesian type frictions, and then determine the extent to which such frictions produce different results. Finally, taking this study to the data will facilitate the calculation of multipliers that are reliable for quantitatively comparing results with the related literature.

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<sup>16</sup>MU, however, has a small increase in output on impact, with a multiplier of 1.3.

# Technical Appendix

## Appendix A Model calibration

This appendix establishes values for the model’s parameters, with which the tax shock is simulated by the quantitative technique of calibration. All parameter values are based on quarterly data, mostly on the US economy; parameters concerning tax rates are obtained from UK data. Section A.1 describes the choice of structural parameter values, which are summarised in Table 4. The “baseline setting” is used in Section 3 to obtain the main results of the tax shock. All baseline values besides that of  $\omega$  are taken from Del Negro et al. (2011), and are also featured in KM.<sup>17</sup> These values are also consistent with the calibrations of similar models that are derived from KM. The “sensitivity settings” are used in Appendix B to re-simulate the tax shock and assess the sensitivity of results to the model’s calibration. Section A.2 assumes and computes steady state levels and autoregressive (shock) parameters of exogenous variables; these values are summarised in Table 5; Appendix F provides the data used in those exercises.

### A.1 Structural parameters

TABLE 4: Structural parameters: baseline and sensitivity settings

Structural parameters	Symbol	Baseline setting	Sensitivity settings	
			Lower	Higher
Fraction of investment financed by equity	$\theta$	0.185	0.1665	0.2035
Subjective discount factor	$\beta$	0.99	0.98	0.999
Capital’s share in output	$\gamma$	0.4	0.36	<i>n.a.</i>
Survival rate of capital after depreciation	$\delta$	0.975	<i>n.a.</i>	0.98
Probability of investment opportunity	$\pi$	0.05	0.037	0.069
Inverse Frisch elasticity of labour supply	$\nu$	1	0.5	2
Relative utility weight on labour	$\omega$	4.01	3.409	8.15

NOTES: All parameter values are based on quarterly data. All baseline values except that of  $\omega$  are taken from Del Negro et al. (2011). The sources of sensitivity settings are given in this chapter’s text.

#### A.1.1 Liquidity constraint parameters, $\theta$ and $\phi_t$

One challenging aspect of the calibration exercise is finding suitable values for  $\theta$  and  $\phi_t$ . These parameters are not directly observable, and are instead fixed to empirical proxies. Other members of the KM-related literature handle this problem in different ways. One consistent theme in these papers has been a simplification that follows from (earlier versions of) KM:  $\theta$  and  $\phi_t$  are assumed to be equal in steady state, while outside of steady state  $\phi_t$  varies stochastically. The calibration task then comes down to finding an empirical estimate of either parameter.

<sup>17</sup>Del Negro et al. (2011) is based on an earlier, 2008 working paper of KM.

Del Negro et al. (2011) targets  $\phi_t$ . They propose that  $\phi_t$  is a linear function of the steady state value of a “liquidity share” variable, a ratio of liquid assets (empirically, US government liabilities) to total assets (empirically, net claims of private assets). From US data over the period 1952:1 – 2008:4, the authors obtain an average liquidity share of 12.64%. Then, according to the hypothesized linear relationship, they find that a value of 0.185 for  $\phi_t$  is related to a liquidity share of 13%. Del Negro et al. (2011), and therefore KM and this paper, calibrate with  $\phi = 0.185$ ; Driffill and Miller (2013) do the same.

Sensitivity analysis in Section B uses higher and lower settings of 0.2035 and 0.1665, respectively, which represent relaxing and tightening of liquidity constraints by 10% relative to baseline. The higher setting is the highest (common) value for  $\theta$  and  $\phi$  that allows the model to converge to a stable, unique equilibrium.<sup>18</sup>

### A.1.2 Subjective discount factor, $\beta$

Frederick et al. (2002) provide an extensive review of the literature on empirical and experimental studies of  $\beta$  and observe that most arrive at values close to 1, or equivalently, quarterly rates of time preference close to zero, which implies that agents have almost equal preferences for the present and future. More recently, Theodoridis et al. (2012) estimate a VAR based on the Smets and Wouters (2007) DSGE model, but with time-varying parameters, and find that  $\beta$  does not vary over time and is close to, but less than 1. These results support the standard practice in the DSGE literature to fix  $\beta$  very close to 1. The most popular setting is a quarterly discount factor of 0.99, which means a 1% quarterly rate of time preference. This value is selected here. Amongst the KM-related literature, Nezafat and Slavík (2012) shares this setting.

Values above and below, but not far away from, the baseline setting for  $\beta$  are chosen for sensitivity analysis in Section B. A higher  $\beta$  of 0.999 equates agents’ preferences for the present and future. This setting appears in, for example, Fernández-Villaverde (2010), who also investigate fiscal shocks in a calibrated DSGE model with financial frictions. A lower  $\beta$  of 0.98 implies agents are more impatient and prefer the present, and therefore discount future utility by a 2% quarterly rate of time preference.

### A.1.3 Capital’s share in output, $\gamma$

Christensen et al. (1980) estimate an average value of 0.40 for  $\gamma$  in the US between 1947 and 1973. Acemoglu and Guerrieri (2008) obtain a measure over an updated period 1948 – 2005, and not only confirm that this value still holds, but support the Kaldor (1961) fact that it remains constant over time.

Sensitivity from  $\gamma$  relies exclusively on a lower value of 0.36. This setting appears in Shi (2012), Nezafat and Slavík (2012), and Jermann and Quadrini (2012). Lower values of 0.33 and

<sup>18</sup>Some members of the KM-related literature successfully calibrate with higher settings in their own unique models: Shi (2012) set  $\phi = 0.273$  and Bigio (2012) set  $\theta = 0.4$ . Shi (2012) associates  $\phi_t$  with the return on liquid assets; he uses the range that Del Negro et al. (2011) find for annual net returns on US government liabilities, i.e. 1.72% for 1-year maturities to 2.57% for 10-year maturities, and sets an intermediate return of 2% as the target to which  $\phi_t$  is calibrated. Bigio (2012) follows Lorenzoni and Walentin (2007) and sets  $\theta$  to match the aggregate moments of coefficients in a regression by Gilchrist and Himmelberg (1998) of the “great ratio”  $I/K$  against the return on capital and Tobin’s  $q$ . Salas-Landeano (2010) warns against using high parameter values, after finding that the constraints need to be tight for shocks to have significant effects.

0.22 are used by [Bigio \(2012\)](#) and [Fernández-Villaverde \(2010\)](#), respectively. Values above the baseline setting are uncommon in the literature, and are therefore omitted in the analysis.

#### A.1.4 Survival rate of capital after depreciation, $\delta$

A quarterly depreciation rate of 2.5%, or equivalently, an annual rate of  $1 - (1 - 0.025)^4 \approx 10\%$ , is standard in RBC studies on the US economy. Since [King et al. \(1988\)](#), who describe 10% as a “more realistic depreciation rate” (p. 218), this value has been widely used in DSGE calibrations.

Like  $\gamma$ , sensitivity analysis with  $\delta$  relies on just one alternative setting, a higher value of 0.98. Rates above the baseline are not unusual in the literature. [Nezafat and Slavík \(2012\)](#), [Shi \(2012\)](#), and [Bigio \(2012\)](#), for example, use 0.9774, 0.981, and 0.9873, respectively, and at the extreme end, [Fernández-Villaverde \(2010\)](#) uses 0.99.

#### A.1.5 Probability of an investment opportunity, $\pi$

$\pi$  can be related empirically to the fraction of firms that significantly adjust their capital in a given period. From samples of US manufacturing firms, [Doms and Dunne \(1998\)](#) estimate this fraction at 20% in any given year, from which [Del Negro et al. \(2011\)](#) set  $\pi$  to a quarterly rate of  $1 - (1 - 0.2)^{0.25} \approx 5\%$ .

[Cooper et al. \(1999\)](#) and [Cooper and Haltiwanger \(2006\)](#) perform empirical studies similar to [Doms and Dunne \(1998\)](#) and estimate that 14% to 25% of firms significantly adjust their capital in any given year. The difference in estimates between the two sets of studies are down to what the authors consider to be a “significant adjustment” in capital stock. To [Doms and Dunne \(1998\)](#), a “significant adjustment” means more than 10% of a firm’s capital is repaired or replaced, whereas [Cooper et al. \(1999\)](#) and [Cooper and Haltiwanger \(2006\)](#) define it as more than 20%.<sup>19</sup> The interval estimate for the fraction of firms that invest in a year provide upper and lower alternative settings for  $\pi$ . If 14% of firms are assumed to significantly replace or repair their capital in a year then the implied value of  $\pi$  is  $1 - (1 - 0.14)^{0.25} \approx 3.7\%$ . If 25% of firms invest then  $\pi = 1 - (1 - 0.25)^{0.25} \approx 6.9\%$ .

#### A.1.6 Frisch elasticity of labour supply, $1/\nu$

The value of  $1/\nu$  in applied economics is the subject of unresolved debate. On the one hand, empirical microeconomic studies usually find small estimates, i.e. values below 1; a review of the literature by [Contreras and Sinclair \(2008\)](#) shows this. Early work by [MaCurdy \(1981\)](#) and [Altonji \(1986\)](#) find estimates within US data ranging from 0 to 0.5. Since then, most empirical studies, at least those whose samples are selected from males, fall within this range; for example, in [Pencavel \(1986\)](#) and [Domeij and Flodén \(2006\)](#). On the other hand, macroeconomics needs much larger elasticities for calibrating models to match observed business cycle fluctuations in aggregate variables, as [Prescott \(2006\)](#) insists. For example, [Peterman \(2012\)](#) explains that values between 2 and 4 are required to replicate empirical volatility in aggregate labour hours.

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<sup>19</sup>Alternatively, [Gourio and Kashyap \(2007\)](#) consider a “significant adjustment” as investment which amounts to 35% or more of beginning-of-period capital.

The wide micro-macro disparity on the value of  $1/\nu$  is mainly due to sample selection: macroeconomic studies aggregate all individuals, whereas microeconomic studies rely on narrower samples (Peterman (2012) and Chetty et al. (2012)). The results of MaCurdy (1981), for instance, are drawn from prime-aged males. The DSGE literature is fairly consistent in using elastic values. However, there is a subset that applies unitary elasticity in macroeconomic models. This is done by Christiano et al. (2005), following elasticity estimates in Rotemberg and Woodford (1999), and also by Christiano et al. (2013) and Cesa-Bianchi and Fernandez-Corugedo (2013) in their DSGE models with financial frictions. Nezafat and Slavík (2012) also calibrate with Frisch elasticity.

Sensitivity from  $1/\nu$  is assessed from both elastic and inelastic settings. A higher value of 2 is used, following the recommendations of macroeconomists; this value is also used in calibrations by Shi (2012) and Bigio (2012). The upper bound of 0.5 from MaCurdy (1981) and Altonji (1986) is used as the lower sensitivity setting.

### A.1.7 Relative utility weight on labour, $\omega$

This structural parameter is not calibrated by Del Negro et al. (2011). The baseline setting is taken from a similar model by Villa and Yang (2011).  $\omega$  is often calibrated with consideration of  $\nu$ , since together they form labour's coefficient in the worker's utility function. As Hall (1997) points out, researchers have different ways of representing this coefficient.<sup>20</sup>  $\omega$  is usually calibrated to match an average or steady state fraction of time spent in work. According to Villa and Yang (2011), the common assumption in the literature is that individuals spend 8 hours a day in work, or one third of their time. Villa and Yang (2011) assume a utility function similar to the one in this paper, and they set  $\omega = 4.01$ ; the difference between this model and theirs is that they include habit persistence in consumption.

Villa and Yang (2011) is based on Gertler and Karadi (2011), who calibrate  $\omega$  to 3.409 based on estimates by Primiceri et al. (2006). This value is taken as a lower sensitivity setting for  $\omega$ . For a higher sensitivity setting, the value of 8.15 that is set by Nezafat and Slavík (2012) is used. This paper shares modelling similarities with Nezafat and Slavík (2012), including the same utility specification for workers. Their calibration of  $\omega$  is done to match moments in steady state with those found empirically.

## A.2 Steady state and AR(1) parameters of exogenous variables

It can easily be shown (see Appendix E.2) that the variable's own standard deviation,  $\sigma_X$ , allows  $\sigma_{uX}$  to be computed according to

$$\sigma_{uX} = \sqrt{(1 - \rho_X^2)\sigma_X^2} \quad (\text{A.1})$$

$A$  is normalised to 1. King and Rebelo (2000) estimate an AR(1) process for  $A_t$  in natural logarithms and without an intercept, using quarterly US data, and obtain point estimates of 0.979 for the persistence parameter and 0.0072 for the standard deviation of the residuals. The

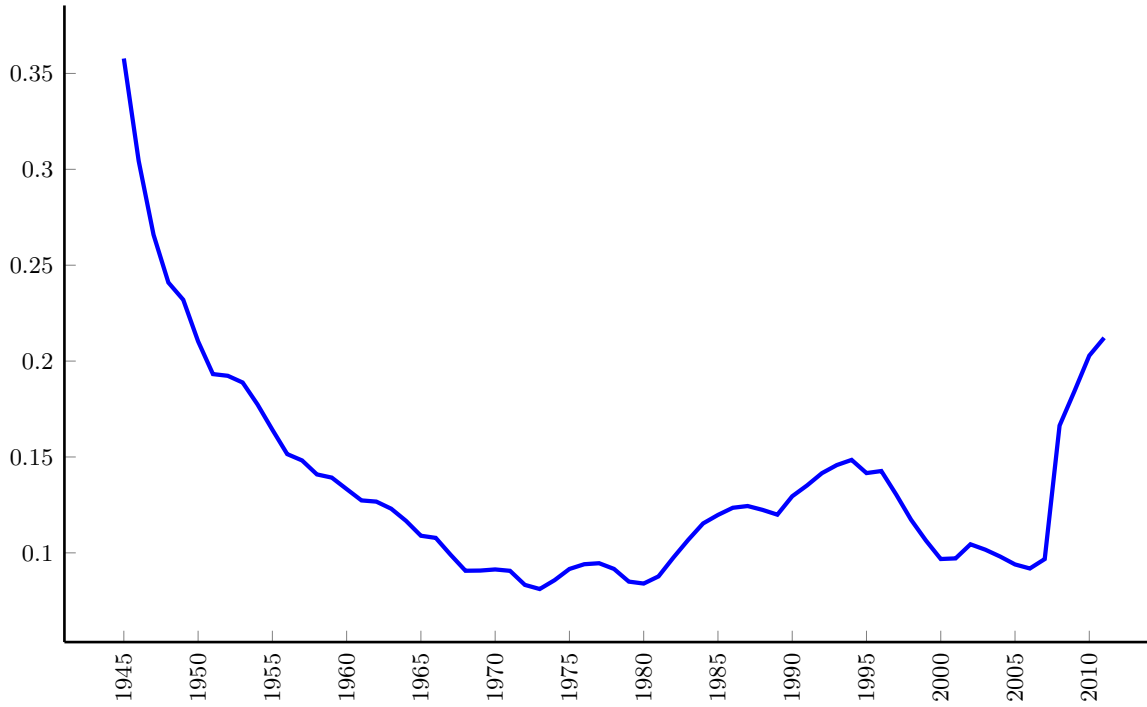
<sup>20</sup>Hall (1997), for instance, normalises  $\omega$  and applies a relative weight to consumption.

TABLE 5: Exogenous variables: steady state levels and autoregressive (shock) parameters

Variable	Steady state level		Persistence parameter		Standard deviation of innovations	
	Symbol	Value	Symbol	Value	Symbol	Value
Aggregate productivity	$A$	1	$\rho_A$	0.979	$\sigma_{uA}$	0.00147
Re-saleable fraction of equity	$\phi$	0.185	$\rho_\phi$	0.95	$\sigma_{u\phi}$	0.00042
Government equity	$N^g$	0	$\rho_{N^g}$	0.95	$\sigma_{uM}$	0.1771
Money supply	$M$	1.95	$\rho_M$	0.952	$\sigma_{uN^g}$	0.00421
Rate of tax on dividends	$\tau^{rn}$	0.207	$\rho_{\tau^{rn}}$	0.95	$\sigma_{u\tau^{rn}}$	0.00349
Rate of tax on wages	$\tau^{wl}$	0.231	$\rho_{\tau^{wl}}$	0.95	$\sigma_{u\tau^{wl}}$	0.00124

NOTES: These values are used to calibrate the stochastic AR(1) processes (2.2), (2.4), (2.29), (2.30), (2.31) and (2.32) which have the general form  $X_t = (1 - \rho_X)X + \rho_X X_{t-1} + u_t^X$  where  $\rho_X$  is the persistence parameter and  $u_t^X$  are innovations. If a variable does not follow a stochastic AR(1) process, then its steady state value is determined endogenously.

FIGURE 3: US liquid assets to total assets



Source: Board of Governors of the Federal Reserve System, US Flow of Funds Statistics, various issues; 2012 data was obtained online from <http://www.federalreserve.gov/releases/z1/current/z1.pdf> and historical data from <http://www.federalreserve.gov/releases/z1/Current/data.htm>.

NOTES: The liquidity share is calculated according to Del Negro et al. (2011). Tables 10 and 11 in Appendix F give the data and metadata, respectively.

value of 0.979 is assumed here for  $\rho_A$ , and by Equation (A.1),

$$\sigma_{uA} = \sqrt{(1 - \rho_A^2)\sigma_A^2} = \sqrt{(1 - 0.979^2)0.0072^2} = 0.00147$$

As mentioned earlier in Section A.1,  $\phi$  is assumed to be equal to  $\theta$  (i.e. 0.185).  $\rho_\phi$  is set



to a standard value of 0.95. An annual time series of the liquidity share variable of [Del Negro et al. \(2011\)](#) is replicated here by following the authors' metadata. The data and metadata are given in Tables 10 and 11, respectively, in Appendix F. The series is illustrated graphically in Figure 3. The liquidity share is relatively stable for half a decade prior to the recent 2007/8 financial crisis. Within this period, the liquidity share has a mean and standard deviation of 0.1110 and 0.0204, respectively. [Del Negro et al. \(2011\)](#) propose that the liquidity share is a linear function of  $\phi_t$ ,

$$LS_t = \phi_0 + 15\phi_t$$

where  $\phi_0$  is a constant.<sup>21</sup> Then

$$\begin{aligned} \sigma_{LS}^2 &= 15^2 \sigma_\phi^2 \\ \implies \sigma_\phi &= \frac{\sigma_{LS}}{15} = \frac{0.0204}{15} = 0.00136 \end{aligned}$$

and by Equation (A.1),

$$\sigma_{u\phi} = \sqrt{(1 - \rho_\phi^2) \sigma_\phi^2} = \sqrt{(1 - 0.95^2) \times 0.00136^2} = 0.00042$$

$\rho_{Ng}$ ,  $\rho_{\tau rn}$ , and  $\rho_{\tau wl}$  are all set to a standard value of 0.95; the setting for  $\rho_M$  is explained below.

The US government started purchasing corporate equities in the third quarter of 2008, as part of the Troubled Asset Relief Program. The natural logarithm of this short time series has a standard deviation of 0.5671 (see Table 12 in Appendix F). Then by Equation (A.1),

$$\sigma_{uNg} = \sqrt{(1 - \rho_{Ng}^2) \sigma_{Ng}^2} = \sqrt{(1 - 0.95^2) \times 0.5671^2} = 0.1771$$

Equation (2.30) is estimated via least squares from quarterly US data over 1987:1 – 2008:1 (i.e. 84 observations); estimation results are summarised in Table 6.  $M_{t+1}$  is taken as the seasonally adjusted, detrended, natural logarithm of the real monetary base. Table 13 in Appendix F provides the data and describes how the series is compiled.  $\rho_M$  is set to the estimated coefficient 0.952 of the lagged dependent variable in the AR(1) regression. The Dickey-Fuller test on

$$\Delta M_{t+1} = (\rho_M - 1)M_t + u_t^M$$

with standard  $t$ -statistic,

$$\frac{\hat{\rho}_M - 1}{SE(\hat{\rho}_M)} = \frac{0.951991 - 1}{0.010284} \approx -5$$

concludes that  $|\rho_M| < 1$  and  $M_{t+1}$  is a trend-stationary series. Figure 4 gives a histogram of the residuals of the regression. The Jarque-Bera test statistic, with p-value of 0.45, does not provide enough statistical evidence to reject a null hypothesis that the regression residuals are normally distributed.  $\sigma_{uM}$  is set to the standard deviation of the regression residuals, 0.004207. The estimated regression coefficients of Equation (2.30) imply a value of 1.95 for  $M$ .

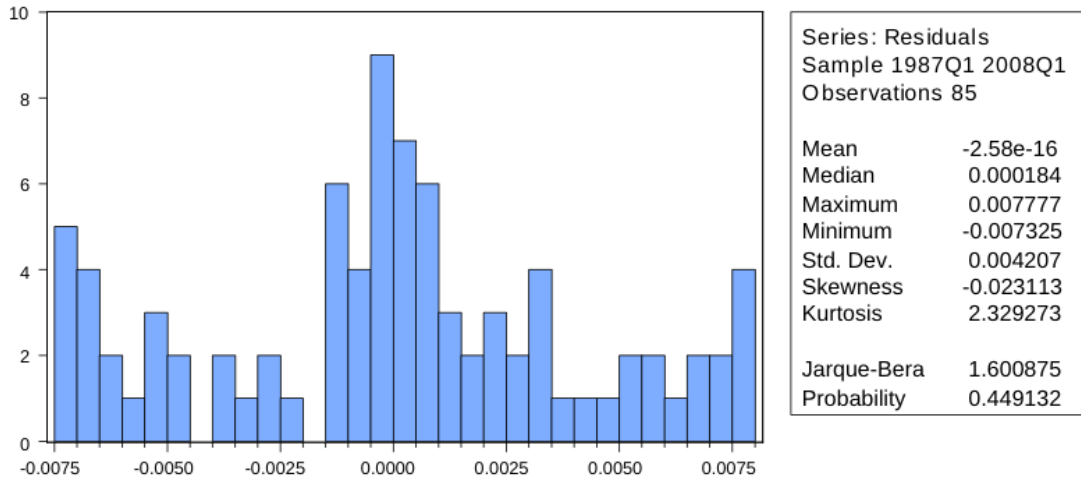
<sup>21</sup>From Figure 3 on page 43 in [Del Negro et al. \(2011\)](#), the straight line appears to travel from 12.5 to 13.25, or 0.75 units along the vertical axis, and from 0.15 to 0.2, or 0.05 units along the horizontal axis, thus giving a slope of  $0.75/0.05 = 15$ .

TABLE 6: Estimation of  $M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M$

Variable	Coefficient	Std. Error	<i>t</i> -Statistic	Prob.
$(1 - \rho_M)M$	0.093483	0.019623	4.763995	0.0000
$\rho_M$	0.951991	0.010284	92.57203	0.0000
R-squared	0.990407	Mean dependent var		1.909519
Adjusted R-squared	0.990292	S.D. dependent var		0.042950
S.E. of regression	0.004232	Akaike info criterion		-8.069100
Sum squared resid	0.001486	Schwarz criterion		-8.011626
Log likelihood	344.9367	Hannan-Quinn criter.		-8.045982
F-statistic	8569.580	Durbin-Watson stat		0.024005
Prob(F-statistic)	0.000000			

NOTES: This table gives the results of estimating equation (2.30) via least squares with a sample of 84 observations from 1987:1 to 2008:1.  $M_{t+1}$  is the seasonally adjusted, detrended, natural log of the real US monetary base. The data is given in Table 13 in Appendix F.

FIGURE 4: Histogram of residuals in the estimation of  $M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M$



Individuals in the US pay tax on income from all sources, not on the type of income earned. Data on dividend and wage taxes is not available from the US. The UK computes taxes by the type of income, including dividend and wage taxes. Parameters related to taxation are therefore drawn from quarterly UK data (which is given in Table 14 in Appendix F). Tax rates are computed as ratios of aggregate taxes to aggregate incomes from wages and dividends. Tax liabilities are used instead of actual tax receipts, to avoid the latter's problems with over/underpayments, late payments, etc. Standard deviations  $\sigma_{\tau wl} = 0.004$  and  $\sigma_{\tau rn} = 0.0112$  for tax rates are observed from the data. The standard deviations of innovations to tax rates are then computed by Equation (A.1):

$$\sigma_{u\tau wl} = \sqrt{(1 - \rho_{\tau wl}^2)\sigma_{\tau wl}^2} = \sqrt{(1 - 0.95^2) \times 0.004^2} = 0.00124$$

$$\sigma_{u\tau rn} = \sqrt{(1 - \rho_{\tau rn}^2)\sigma_{\tau rn}^2} = \sqrt{(1 - 0.95^2) \times 0.0112^2} = 0.00349$$

The UK does not have flat rates of tax on wage and dividend income. In both cases the taxpayer first enjoys a taxable allowance, and any excess amount earned during the tax year is subject to tax. The rate of tax applied on this excess depends on the individual’s income for the fiscal year.  $\tau^{wl}$  and  $\tau^{rn}$  are set to average ratios, 0.231 and 0.207, of aggregate tax liabilities to aggregate incomes from wages and dividends, respectively (see Table 14 in Appendix F).<sup>22</sup>

## Appendix B Sensitivity analysis

This Appendix shows how responses to the tax shock vary with changes to the calibration of structural parameters ( $\theta$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\pi$ ,  $\nu$ , and  $\omega$ ) and the persistence of tax shocks ( $\rho_{\tau rn}$  and  $\rho_{\tau wl}$ ). Structural parameter sensitivity analysis is performed systematically by three local methods, all involving repeated simulations of the shock with changes in structural parameter values to their “sensitivity settings” that are listed in Table 4. The first method changes one structural parameter at a time; the second and third methods change combinations of two or more structural parameters. Sensitivity to the persistence of tax shocks is examined by repeatedly simulating the shock with values of  $\rho_{\tau rn}$  and  $\rho_{\tau wl}$  that are above and below their calibrated (and fairly standard) setting. The conclusions from this Appendix are stated in Section 5.

### B.1 Sensitivity to structural parameters: one-at-a-time parameter variation

The first approach to structural parameter sensitivity is a one-at-a-time (OAT) method: one parameter is changed to one of its sensitivity settings, and all other parameters remain at the baseline; this is done for each and every parameter and for each and every sensitivity setting that is listed in Table 4.<sup>23</sup> This exercise produces 12 sets of results, which are graphically illustrated by impulse responses in Figures 5 to 11. The magnitude of immediate impulse responses from all 12 sensitivity simulations plus the baseline are listed in Table 7.

Since their changes are due to non-uniform changes in parameter values, impulse responses on their own are unsuitable for comparing different scenarios, or for establishing a common criteria to assess sensitivity. An indicator of sensitivity to a particular parameter is constructed for these purposes. The indicator is a ratio of the percentage change in a variable’s first quarter impulse response to the percentage change in a parameter’s value. The indicator is henceforth

<sup>22</sup>These rates are very similar to those computed by [Gomme and Rupert \(2007\)](#) from US data and following a methodology set out by [Mendoza et al. \(1994\)](#) and [Carey and Tchilinguirian \(2000\)](#). [Gomme and Rupert \(2007\)](#) compute income tax rates of 0.22 on wages and 0.2868 on capital. These values, however, are not adopted here for two reasons: (i) [Gomme and Rupert \(2007\)](#) use data on actual government tax collections, which, as indicated earlier in this paper, may be less accurate of the tax burden than tax liabilities data because of errors in tax payments; and (ii)  $\sigma_{u\tau wl}$  and  $\sigma_{u\tau rn}$  are needed here, and from the same dataset these tax rates are obtained. The rates obtained here are also fairly consistent with a dedicated literature that estimates tax rates; [Barro and Sahasakul \(1983, 1986\)](#), [Seater \(1985\)](#), and [Stephenson \(1998\)](#) obtain average wage income tax rates between 0.22 and 0.30 from US data between 1954 and 1994; and [Mendoza et al. \(1994\)](#) obtain average income tax rates between 0.17 and 0.30 for wages and 0.27 and 0.50 for capital.

<sup>23</sup>This method is similar to the “one-factor-at-a-time” method of [Morris \(1991\)](#), but without randomly selecting parameter values.

TABLE 7: Immediate impulse responses of a tax shock: baseline and one-at-a-time parameter changes

	Baseline	$\theta$		$\beta$		$\gamma$	$\delta$		$\pi$		$\nu$		$\omega$	
		L	H	L	H	L	H	L	H	L	H	L	H	
$T_t$	-3.1	-3.1	-3.1	-2.0	-5.1	-1.8	-3.8	-2.9	-3.3	-1.4	-4.6	-3.6	-1.5	
$Y_t$	1.1	1.1	1.1	0.7	1.8	0.7	1.3	1.0	1.1	0.8	0.9	1.3	0.5	
$I_t$	1.9	1.8	1.9	0.9	4.0	1.0	2.2	1.7	2.0	1.0	2.6	2.2	0.9	
$C_t$	2.3	2.3	2.3	1.7	2.9	1.5	3.0	2.2	2.4	1.2	3.0	2.7	1.1	
$C_t^w$	1.7	1.6	1.7	1.0	2.7	1.1	2.1	1.6	1.7	0.9	2.1	1.9	0.8	
$C_t^i$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
$C_t^s$	0.6	0.6	0.6	0.7	0.1	0.4	0.9	0.7	0.6	0.3	0.9	0.7	0.3	
$N_{t+1}^s$	0.3	0.3	0.4	0.2	0.7	0.2	0.4	0.3	0.4	0.2	0.5	0.4	0.2	
$w_t$	-0.9	-0.9	-0.9	-0.7	-1.2	-0.7	-1.0	-0.9	-0.9	-1.4	-0.5	-0.9	-0.9	
$L_t$	0.4	0.4	0.4	0.3	0.6	0.4	0.5	0.4	0.4	0.3	0.4	0.5	0.2	
$r_t$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
$K_{t+1}$	1.9	1.8	1.9	0.9	4.0	1.0	2.2	1.7	2.0	1.0	2.6	2.2	0.9	
$p_t$	6.6	7.8	5.4	2.2	18.2	3.6	5.3	9.6	3.0	3.5	8.9	7.8	3.3	
$q_t$	3.2	3.2	3.3	4.2	2.5	3.3	3.6	3.3	3.4	3.4	3.1	3.2	3.2	

NOTES: This table gives the percentage deviations from steady state in quarter 1 for baseline and one-at-a-time sensitivity scenarios. “L” and “H” refer to the lower and higher sensitivity parameter values, respectively, that are listed in Table 4.

referred to as a “parameter elasticity of impulse response”.<sup>24</sup> A positive elasticity means an increase (or decrease, respectively) in the parameter’s value amplifies (or dampens, respectively) the variable’s immediate impulse response relative to that of the baseline scenario. A negative elasticity means that an increase (or decrease, respectively) in the parameter’s value dampens (or amplifies, respectively) the variable’s immediate impulse response relative to that of the baseline scenario. A variable is considered sensitive to a parameter if the absolute value of the elasticity is greater than 1. The model is considered sensitive to a parameter if the majority of the variables are sensitive to that parameter. Elasticities from all 12 repeated simulations are given in Table 8.

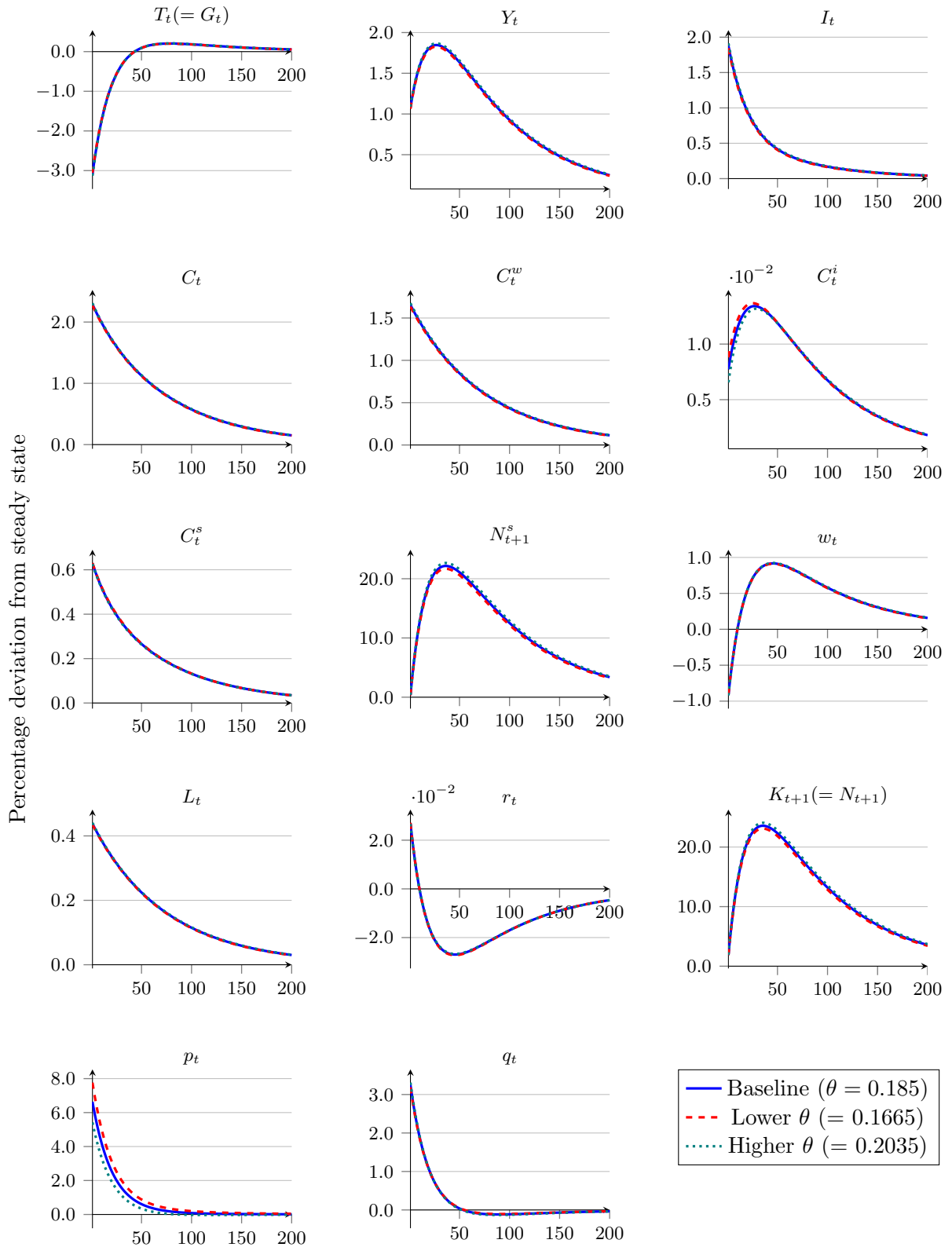
### B.1.1 Liquidity constraints

Figure 5 illustrates the difference in impulse responses among the baseline setting and higher and lower sensitivity settings of  $\theta$  and  $\phi$ . The graphs show little variation in impulse responses. Moreover, parameter elasticities are less than unity in absolute value for all variables except  $C_t^i$ ,  $N_t^s$ , and  $p_t$ . The model can therefore be considered not sensitive to the calibration of liquidity constraint parameters, *ceteris paribus*, once they are tight enough to allow the model to converge to a unique equilibrium.

The replacement cost of equity is inversely influenced by the borrowing constraint (see Appendix E.1). Either directly or indirectly through  $q_t^R$ , the liquidity constraints enter negatively into investors’ consumption (Equation (2.55)) and positively into investment and equity’s sup-

<sup>24</sup>The parameter elasticity resembles the “elementary effects” ratio of Morris (1991).

FIGURE 5: Impulse responses of a tax shock: repeated simulations with lower and higher  $\theta$



NOTES: Horizontal axes measure quarters after the shock, starting from quarter 1. See Table 7 for the values of first quarter impulse responses.

TABLE 8: Parameter elasticities of impulse responses

	$\theta$		$\beta$		$\gamma$	$\delta$		$\pi$		$\nu$		$\omega$	
	L	H	L	H	L	H	L	H	L	H	L	H	
$T_t$	0.1	0.1	36.3	71.9	4.2	47.4	0.2	0.1	1.1	0.5	-1.2	-0.5	
$Y_t$	0.1	0.1	36.3	71.9	3.3	47.4	0.2	0.1	0.5	-0.2	-1.2	-0.5	
$I_t$	0.2	0.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	-1.2	-0.5	
$C_t$	0.1	0.1	24.7	27.5	3.6	58.0	0.1	0.1	0.9	0.3	-1.2	-0.5	
$C_t^{rw}$	0.1	0.1	36.3	71.9	3.3	47.4	0.2	0.1	0.9	0.3	-1.2	-0.5	
$C_t^i$	-1.4	-1.5	24.5	-81.8	4.5	-25.9	0.0	-0.6	1.0	0.4	-1.2	-0.5	
$C_t^s$	0.0	0.0	-5.9	-88.0	4.4	87.1	-0.2	0.0	1.0	0.4	-1.2	-0.5	
$N_{t+1}^s$	1.2	1.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	-1.2	-0.5	
$w_t$	0.1	0.1	20.2	31.4	2.5	22.4	0.1	0.1	-1.1	-0.4	0.0	0.0	
$L_t$	0.1	0.1	20.2	31.4	1.7	22.4	0.1	0.1	0.5	-0.2	-1.2	-0.5	
$r_t$	-0.1	-0.1	-40.5	-34.6	-1.0	-29.4	-0.2	-0.1	-1.1	-0.4	0.0	0.0	
$K_{t+1}$	0.2	0.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	-1.2	-0.5	
$p_t$	-1.7	-1.8	66.7	191.1	4.5	-40.1	-1.7	-1.4	1.0	0.3	-1.2	-0.5	
$q_t$	0.2	0.2	-28.0	-26.6	-0.2	20.2	-0.1	0.1	-0.1	0.0	0.0	0.0	

NOTES: This table gives the percentage change in first quarter impulse responses as a ratio to the percentage change in a single parameter value. “L” and “H” refer to the lower and higher sensitivity parameter values, respectively, that are listed in Table 4.

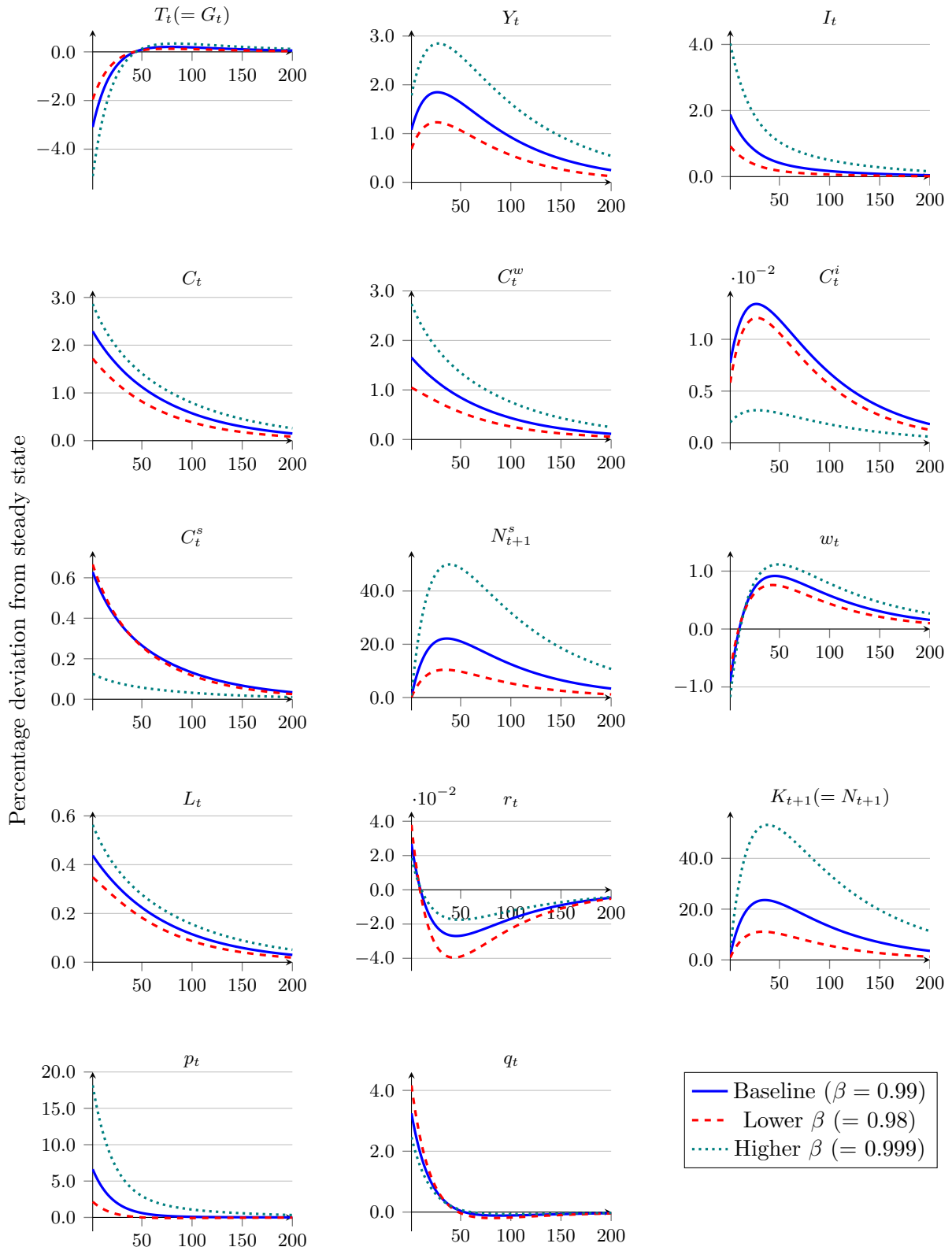
ply (Equations (2.34) and (2.58), respectively). Following a tax shock, the tighter the liquidity constraints (i.e. the lower the values of  $\theta$  and  $\phi$ ), then the higher the increase in investors’ consumption and the lower the increases in equity’s supply and investment; and conversely. This explains why  $I_t$  and  $C_t^i$  have positive and negative parameter elasticities, respectively. It also explains why money’s price is the most sensitive variable to this parameter: from a tax shock, the tighter the liquidity constraints, the smaller the increase in equity’s supply, and the greater the increase in equity’s price (as if  $S_1^N$  is more to the left than it appears in Figure 2A); then by a portfolio balance effect, the greater are the increases in money’s demand and price. KM observe this movement from equity to money when liquidity constraints tighten, in what they called a “flight to liquidity”. *Ceteris paribus*, a tightening of liquidity constraints worsens the appeal of partially liquid assets (i.e. equity) and encourages agents to substitute towards more liquid assets (i.e. money), and thereby increases the price of liquid assets; and conversely.

### B.1.2 Subjective discount factor

Parameter elasticities indicate that the shock responses of all variables are very sensitive to changes in  $\beta$ ; Figure 6 graphically illustrates this. In fact, looking at all the elasticities in Table 8 shows that  $\beta$  produces the greatest amount of sensitivity in all of the OAT simulations. Elasticities are asymmetric, and indicate that the model is more sensitive to raising the parameter’s value above baseline.

$\beta$  enters negatively in entrepreneurs’ consumption (Equations (2.55) and (2.56)). *Ceteris paribus*, increasing  $\beta$  means entrepreneurs are more willing to delay consumption and spend

FIGURE 6: Impulse responses of a tax shock: repeated simulations with lower and higher  $\beta$



NOTES: See the notes in Figure 5.

their net worth more evenly over time.<sup>25</sup> As their patience increase, they consume less in the present. This explains the negative parameter elasticities for  $C_t^i$  and  $C_t^s$  with higher  $\beta$ . The consequences are higher levels of current saving and investment. Combined with a tax shock, increasing  $\beta$  amplifies the shock-induced increases in asset demands. For money, this means a larger price increase compared to the baseline scenario, hence the positive parameter elasticity for  $p_t$ . For equity, there is also a larger supply response; the market adjusts to the shock with a smaller price increase than in the baseline scenario, hence the positive parameter elasticity for  $p_t$ . These variations in asset price impulse responses then propagate throughout the economy.

Conversely, *ceteris paribus*, lowering  $\beta$  means entrepreneurs become more impatient and consume more of their net worth in the present; this implies less saving and investment, and lower asset demands and equity supply. Combined with a tax shock, lowering  $\beta$  produces a smaller increase in  $p_t$  and a larger increase in  $q_t$ . Asset price increases feed back into improvements in entrepreneurs' net worth. Investors therefore consume more. This is why  $C_t^i$  has a positive parameter elasticity with lower  $\beta$ . In other words, investors increase their consumption because of the net worth improvements they enjoy from shock-induced asset price increases; varying  $\beta$  up or down does not interfere with this, hence the difference in the sign of parameter elasticities for  $C_t^i$ . Net worth improvements also increase saving and investment. However, lowering  $\beta$  only partially offsets the increase in savings but completely offsets the increase in investment, hence the negative and positive parameter elasticities for  $C_t^s$  and  $I_t$ , respectively.

### B.1.3 Capital's share in output

This is another parameter for which the model is sensitive to its calibrated value. Figure 7 illustrates this. Elasticities are greater than 1 in absolute value for all variables except  $L_t$  and  $q_t$ . The only variables with negative elasticities are  $q_t$  and  $r_t$ ; impulse responses of other variables are thus smaller when  $\gamma$  is lowered.

$\gamma$  enters the aggregate labour demand and production functions (Equations (2.50) and (2.59), respectively). *Ceteris paribus*, lowering  $\gamma$  positions the inverse aggregate labour demand function leftwards from its baseline calibration; this is illustrated in the first graph of Figure 12. Lowering  $\gamma$  therefore dampens the shock-induced increases in the real wage and labour, and hence output. This explains the positive parameter elasticities of  $w_t$ ,  $L_t$ , and  $Y_t$ . The changes in the goods market then propagate throughout the economy.

### B.1.4 Survival rate of capital after depreciation

Figure 8 shows small differences in tax shock impulse responses between baseline and higher  $\delta$  settings. But the change in  $\delta$ 's setting is very small, and parameter elasticities reveal the change in quarter 1 impulse responses to be relatively much larger. Elasticities are all above 20 in absolute value, making shock responses very sensitive to the parameter's value. In fact, this parameter is the second most sensitive, after  $\beta$ .

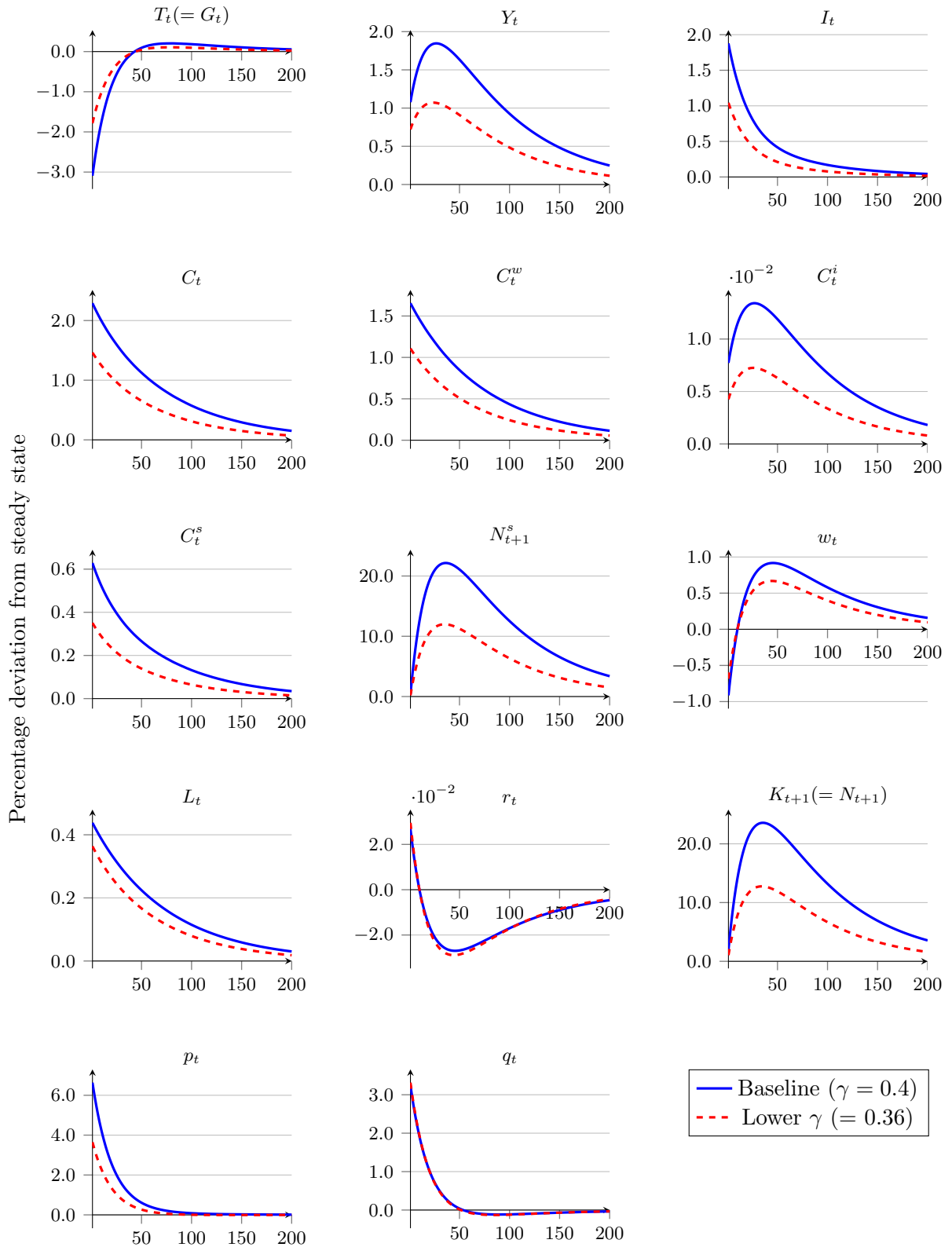
*Ceteris paribus*, a higher  $\delta$  means capital and equity stocks retain more of their value after depreciation each period. This effectively provides net worth improvements to entrepreneurs.

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<sup>25</sup>Given that workers' optimal behaviour involves them not saving for the future (from Equation (2.57)), then such changes are confined to entrepreneurs.

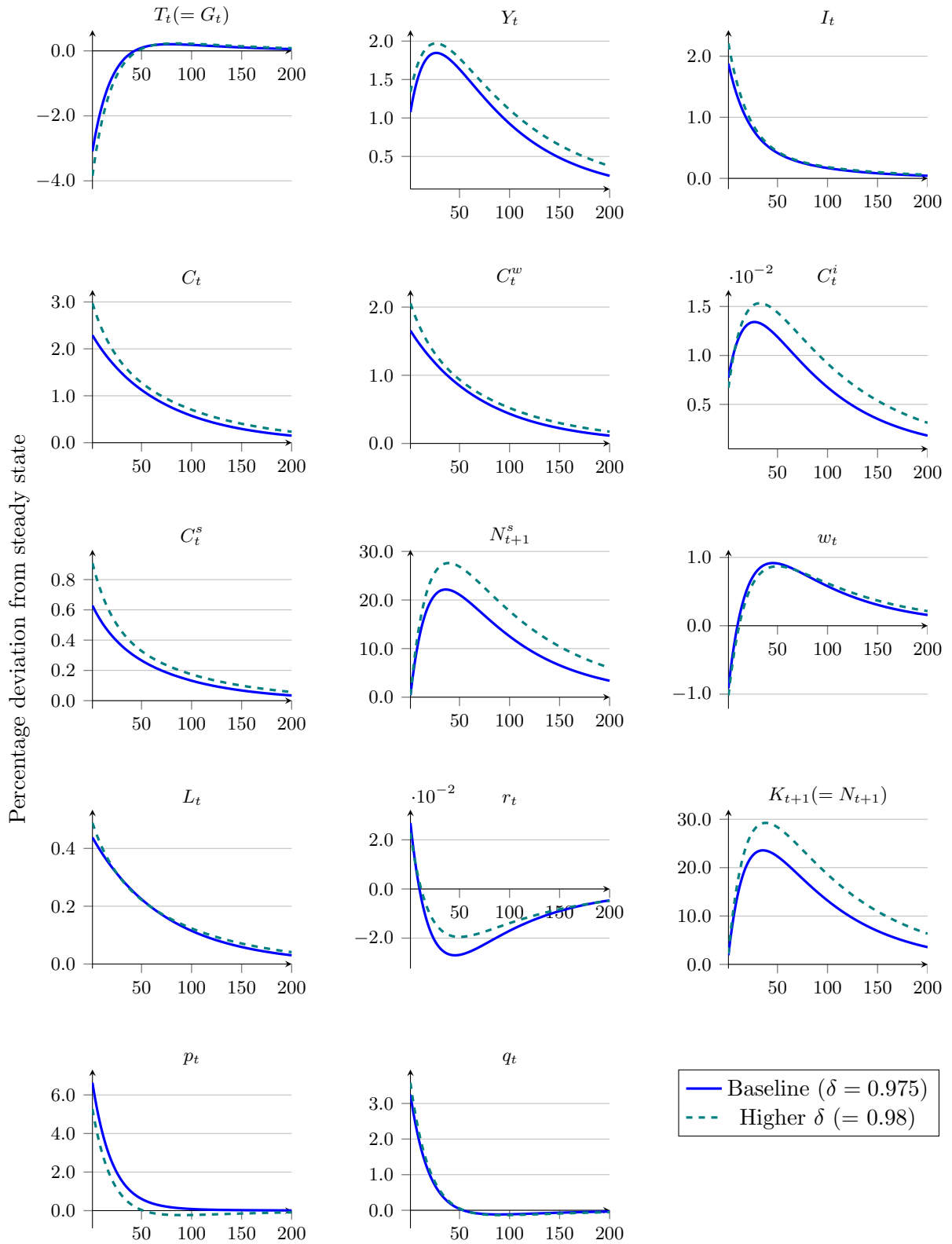


FIGURE 7: Impulse responses of a tax shock: repeated simulation with lower  $\gamma$



NOTES: See the notes in Figure 5.

FIGURE 8: Impulse responses of a tax shock: repeated simulation with higher  $\delta$



NOTES: See the notes in Figure 5.

The consequences are amplified when combined with the shock. However, increasing  $\beta$  improves the appeal of equity. The shock-induced increase in demand for equity is thus amplified, and creates a larger fall in  $q_t$  (as if  $D_1^N$  is further to the right than it appears in Figure 2A). This explains the negative parameter elasticity for  $p_t$ . Moreover, investors are able to invest more, given net worth improvements, and issue more equity, given its greater appeal. They sacrifice consumption for much more investment, hence the negative parameter elasticity for  $C_t^i$ .

### B.1.5 Probability of investment opportunity

Changing the value of  $\pi$  does not significantly alter impulse responses, except for  $p_t$ . This is seen in the deviations of impulse response graphs in Figure 9. Elasticities are fairly similar between lowering and raising the parameter's value relative to its baseline setting.

$\pi$  enters positively into investors' consumption (Equation (2.55)), investment (Equation (2.58)), and the supply of equity (Equation (2.34)), and negatively into savers' consumption (Equation (2.56)). *Ceteris paribus*, raising the value of  $\pi$  increases the population of investors relative to savers, and conversely. Changing the parameter's value therefore shifts activity between investment and saving. The significant changes occur in the asset markets, but these are outweighed by the effects of the tax shock. The economy is therefore hardly affected by variation in the parameter's value, hence the very small elasticities for most variables.

Since a higher  $\pi$  means a smaller population of savers, then the shock-induced increase in money's demand is smaller compared to the baseline scenario (as if  $D_1^M$  in Figure 2B is further to the left than where it is drawn). The increase in  $p_t$  is smaller, hence the negative parameter elasticity. This implies a larger fall in the expected return on money. The portfolio balance effect is stronger, i.e. the substitution-led increase in equity's demand is greater, which produces a greater price increase, and therefore a positive parameter elasticity for  $q_t$ . The converse is true for a lower  $\pi$ .

### B.1.6 Inverse Frisch elasticity of labour supply

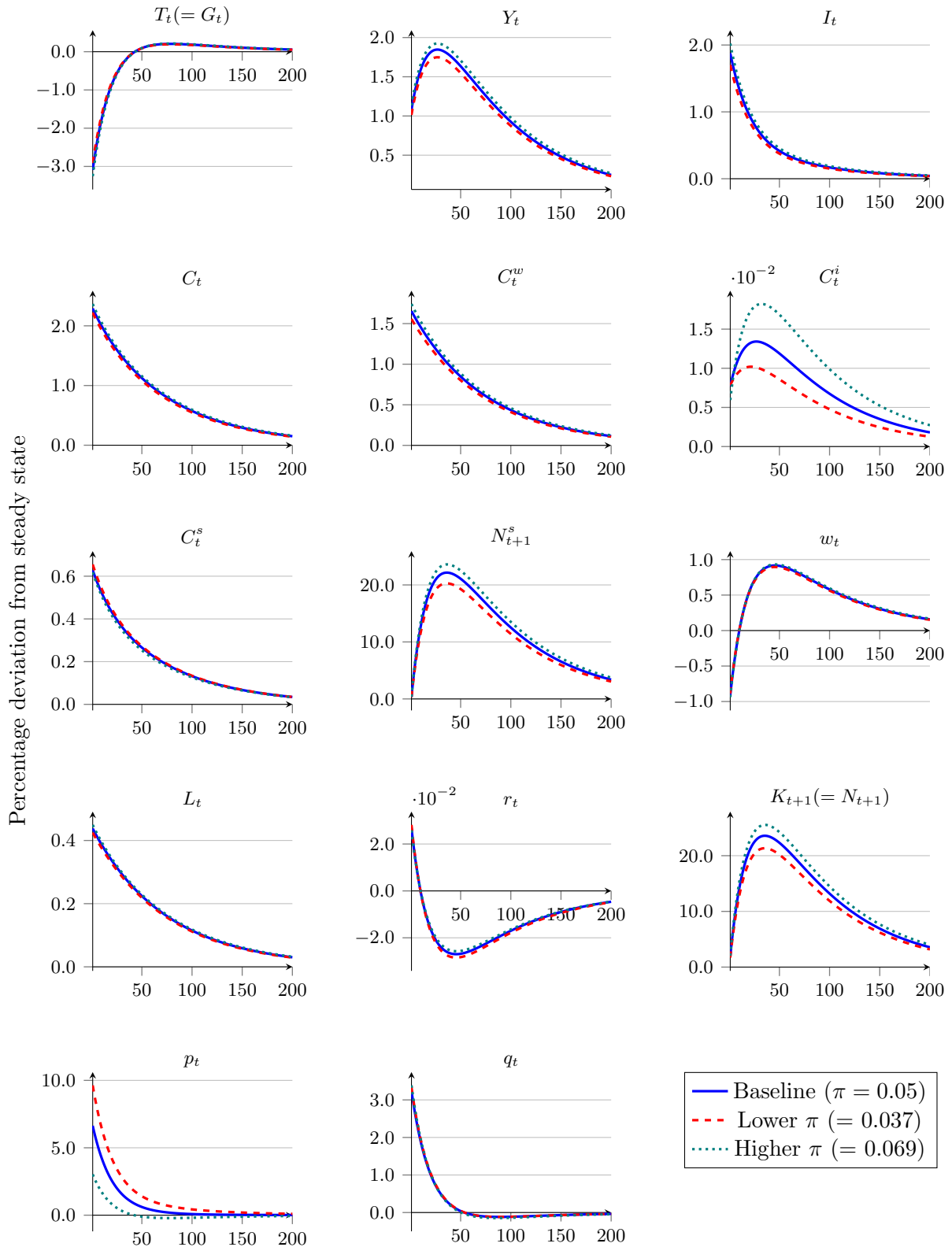
Figure 10 shows some variation in impulse responses from changes in  $\nu$ . Parameter elasticities indicate that a minority of variables are sensitive to the parameter, and even then, the elasticities are marginally above 1. Elasticities indicate that the model is more sensitive to lowering the parameter. Overall, the model cannot be considered sensitive to the calibration of  $\nu$ .

Changing the value of  $\nu$  affects the economy through the aggregate labour supply function. The baseline setting  $\nu = 1$  makes the inverse function (Equation (2.51)) linear in  $w_t$ . If  $\nu < 1$  then the inverse function is convex, and  $\nu > 1$  makes it concave. The second graph of Figure 12 illustrates these variations in shape of labour market curves. These variations are largely responsible for any deviations of impulse responses from the baseline scenario.

### B.1.7 Relative utility weight on labour

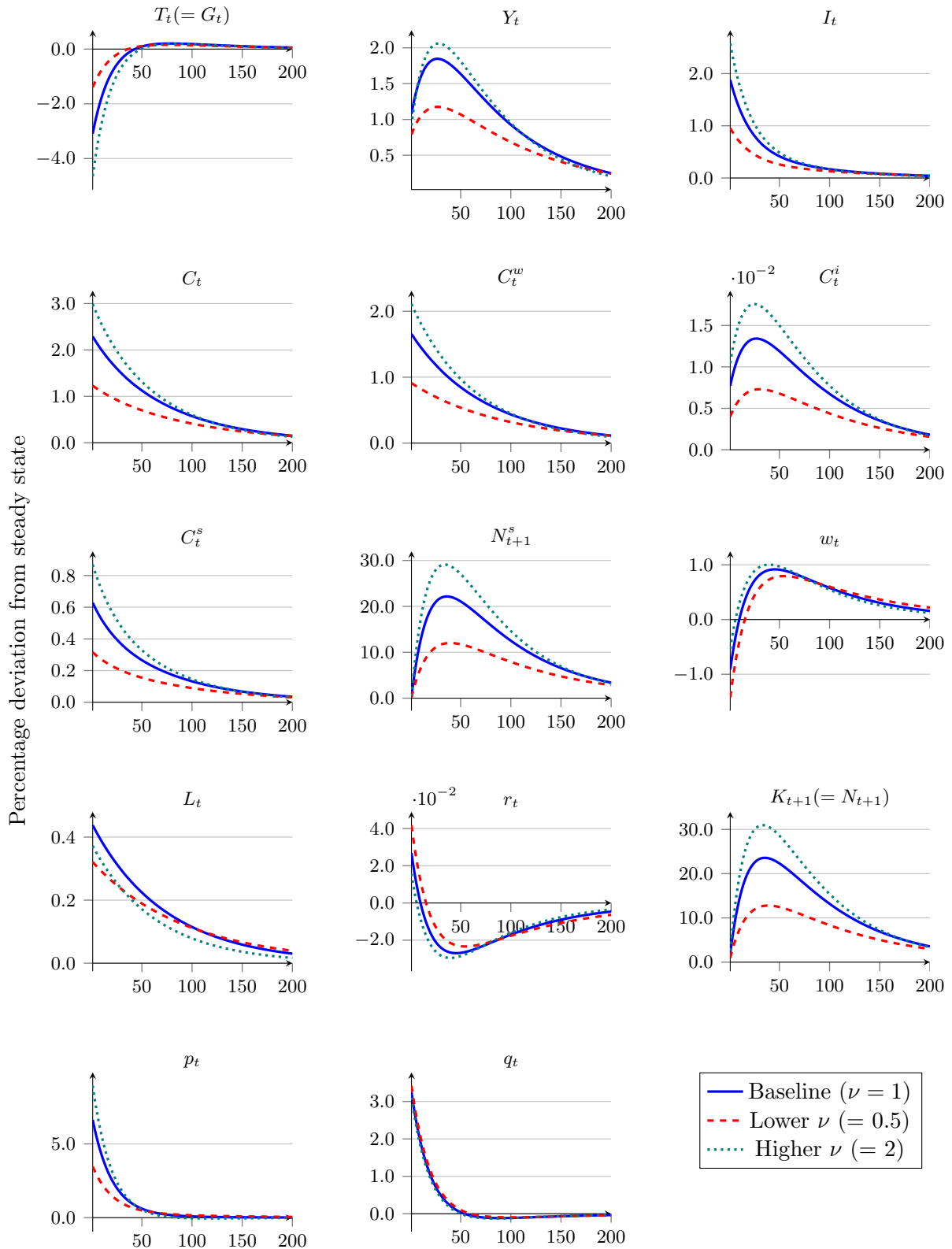
Although large deviations in impulse responses are shown in Figure 11, these are brought on by large changes in the value of  $\omega$ , especially from raising the value above baseline. Parameter elasticities provide a more accurate means of assessing sensitivity. They indicate that the

FIGURE 9: Impulse responses of a tax shock: repeated simulations with lower and higher  $\pi$



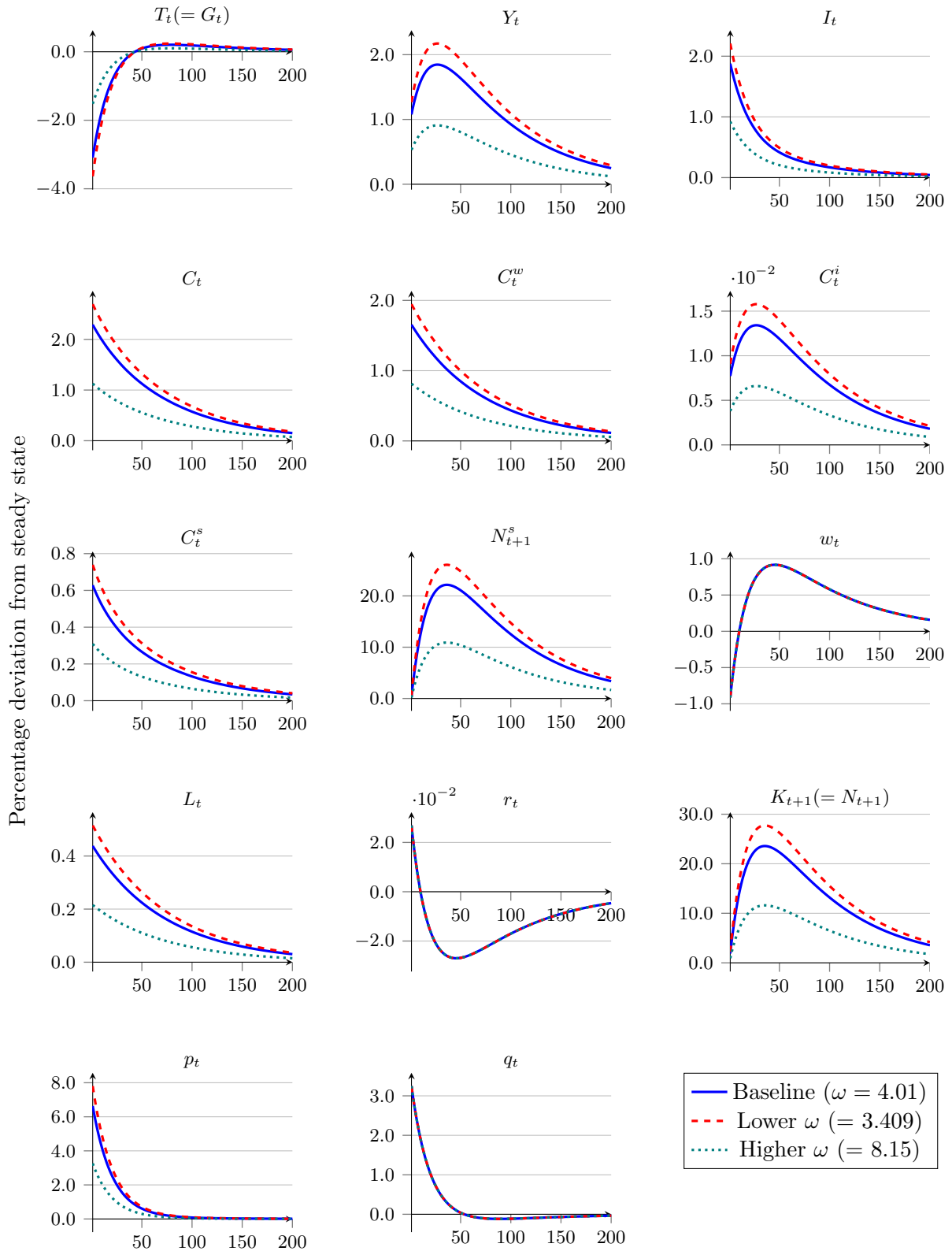
NOTES: See the notes in Figure 5.

FIGURE 10: Impulse responses of a tax shock: repeated simulations with lower and higher  $\nu$



NOTES: See the notes in Figure 5.

FIGURE 11: Impulse responses of a tax shock: repeated simulations with lower and higher  $\omega$



NOTES: See the notes in Figure 5.

model is not sensitive to raising the parameter’s value; no variable has an elasticity above 1 in absolute value. However, lowering the parameter’s value produces large elasticities for most variables. Changes in  $\omega$  in both directions have no effect on  $w_t$ ,  $r_t$ , and  $q_t$ , and produces the same parameter elasticity with the other variables. The overall conclusion is that the model is sensitive to lowering the parameter’s value, but not to raising it.

$\omega$  positively determines the slope of the inverse aggregate labour supply function (Equation (2.51)). The last graph of Figure 12 illustrates how varying  $\omega$ , *ceteris paribus*, affects the labour market. The remarks said above about changes in  $\nu$  can also be said about  $\omega$ .

## B.2 Sensitivity to structural parameters: combinations of sensitivity settings

The second and third approaches both change combinations of two or more structural parameters to their “sensitivity settings” that are listed in Table 4. For compactness, these approaches are called the “Sensitive Combinations” and “All Combinations” methods, respectively.

The Sensitive Combinations method (henceforth SC) uses combinations of sensitivity settings for only those structural parameters which the OAT method determines that the model is sensitive to, i.e.  $\beta$ ,  $\gamma$ , and  $\delta$ . The SC uses 10 combinations of parameter values.<sup>26</sup> The SC builds upon the screening that the OAT method performs, and attempts to capture two or more sensitivity settings from  $\beta$ ,  $\gamma$ , and  $\delta$  that, when combined, produce shock responses that deviate significantly from the baseline. Impulse responses for the SC are presented graphically in Figure 13. Relying on graphical inspection, the SC concludes that responses to the tax shock vary only in magnitude to the calibration of the model, i.e. different assumptions about structural parameters do not change the sign of initial responses or the shape of trajectories.<sup>27</sup>

The All Combinations method (henceforth AC) is more inclusive than the SC, and uses combinations of sensitivity settings for all structural parameters. The AC uses 754 combinations of parameter values, which includes the 10 combinations that are used in the SC.<sup>28</sup> The objective of the AC is to capture any sensitive combination of two or more parameter values outside of those considered in the SC. The AC also avoids any selection bias that the SC may have, despite identification by the OAT method of which parameters are key drivers of sensitivity. Impulse response graphs of the AC closely resemble those in Figure 13 (they are just more densely populated) and are not reported, to avoid repetition. The conclusion of the SC is therefore supported by the AC.

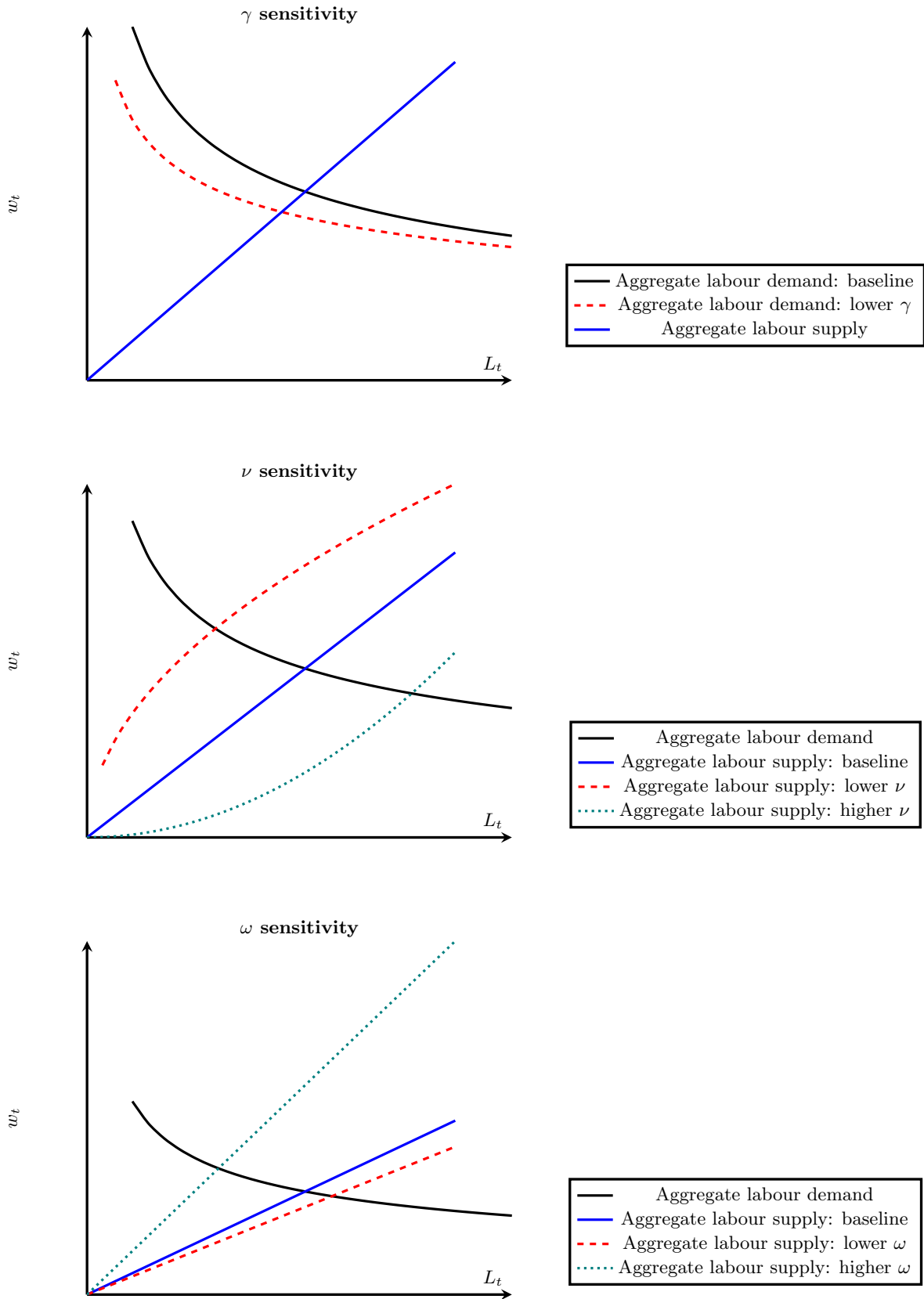
Box plots of immediate impulse responses from both SC and AC are presented in Figure 14. Two conclusions are drawn from inspecting Figure 14. Firstly, with the exception of  $C_t^i$ , baseline responses (marked by a red cross) are not extreme. Secondly, the AC produces more extreme first quarter impulse responses than the SC; since SC combinations are a subset of those used in the AC, then Figure 14 indicates that not only do the parameters identified by the OAT method cause sensitivity, but also certain combinations of any of the parameter values.

<sup>26</sup>An 11<sup>th</sup> combination ( $\beta = 0.999, \gamma = 0.4, \delta = 0.98$ ) does not allow the model to converge to a unique equilibrium.

<sup>27</sup>The literature lacks criteria by which results of such an analysis are to be interpreted; a survey by [Andronis et al. \(2009\)](#) concludes this. Here, the objective is to observe any change in direction or trajectory of impulse responses and to recognize significantly different impulse responses from those of the baseline scenario.

<sup>28</sup>An additional 217 combinations from the AC do not allow the model to converge to a unique equilibrium.

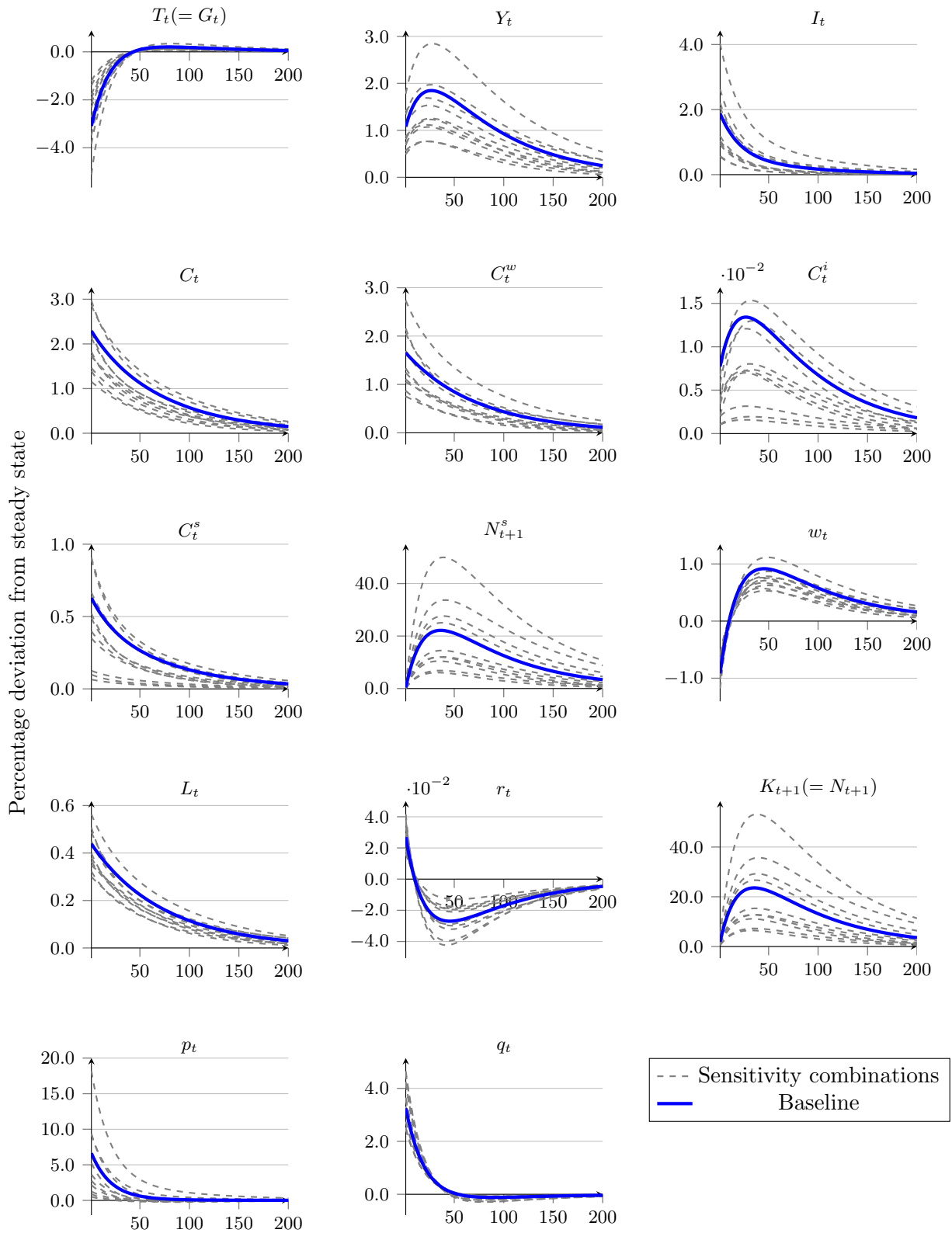
FIGURE 12: Labour market sensitivity to  $\gamma$ ,  $\nu$  and  $\omega$



NOTES: These graphs plot Equations (2.50) and (2.51) using steady state levels of the capital stock and total factor productivity and sensitivity settings for  $\gamma$ ,  $\nu$ , and  $\omega$  that are listed in Table 4.

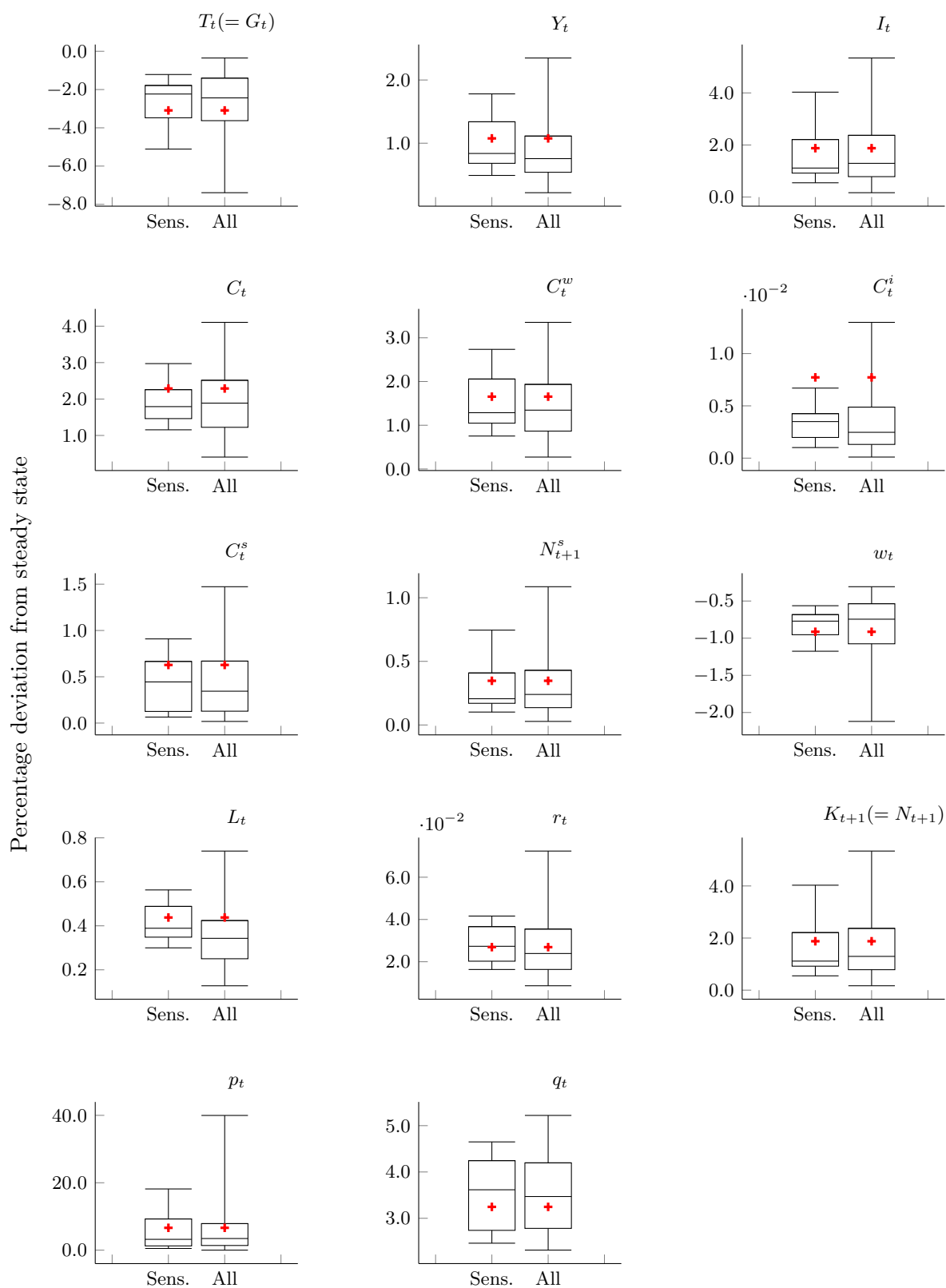


FIGURE 13: Impulse responses of a tax shock: repeated simulations with combinations of sensitivity settings for  $\beta$ ,  $\gamma$  and  $\delta$



NOTES: These graphs plot impulse responses from the “Sensitive Combinations” approach to structural parameter sensitivity. They show impulse responses to the tax shock from 10 repeated simulations with combinations of sensitivity settings for  $\beta$ ,  $\gamma$  and  $\delta$  that are listed in Table 4. An 11<sup>th</sup> combination ( $\beta = 0.999, \gamma = 0.4, \delta = 0.98$ ) does not allow the model to converge to a unique equilibrium.

FIGURE 14: Range of immediate impulse responses of a tax shock: repeated simulations with combinations of sensitivity settings



NOTES: These box plots show the 25<sup>th</sup> and 75<sup>th</sup> percentiles, median, largest and smallest immediate impulse responses from the Sensitive Combinations (labelled “Sens.”) and All Combinations (labelled “All”) approaches to structural parameter sensitivity. Red crosses indicate baseline immediate impulse responses.

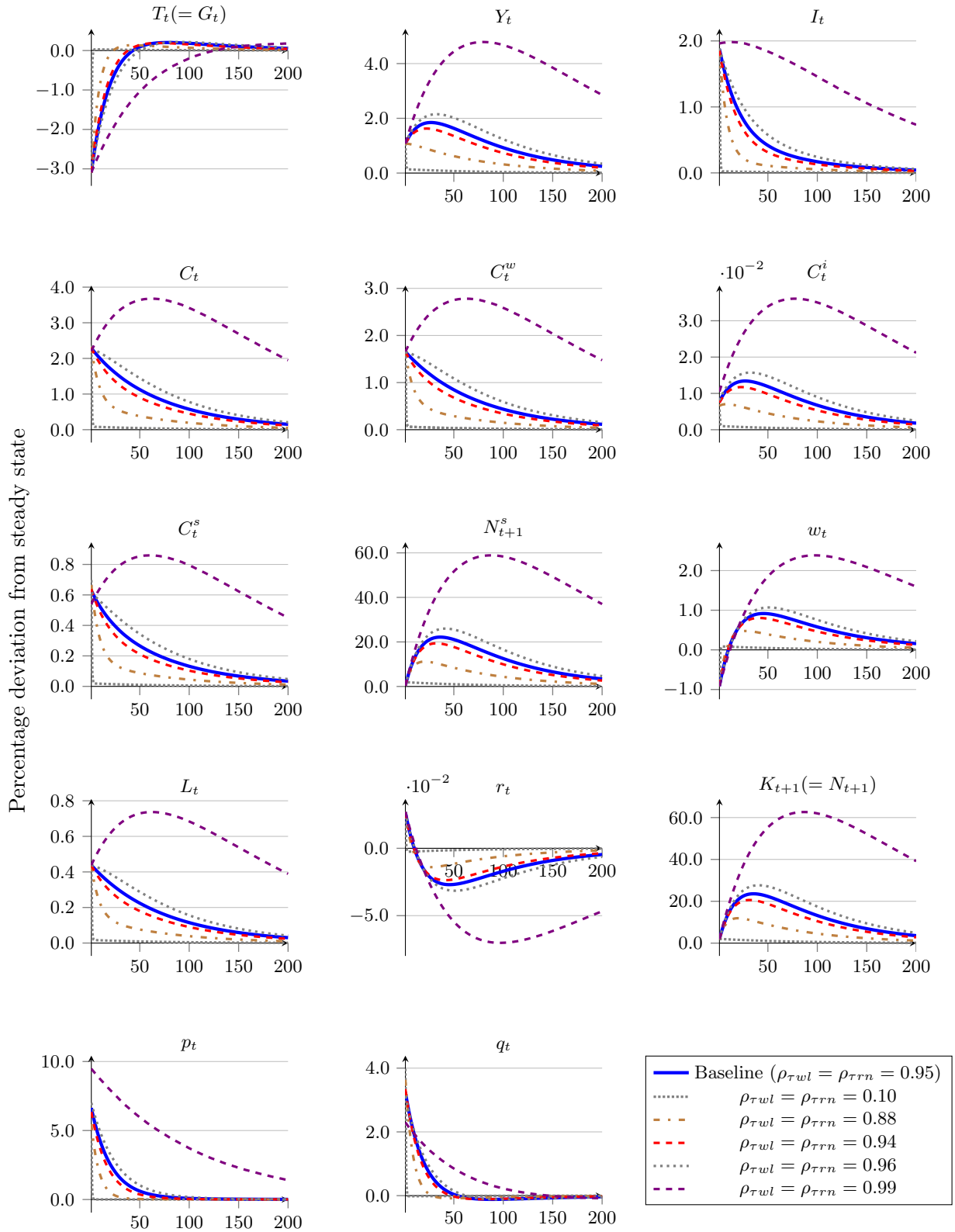
### B.3 Sensitivity to the persistence of tax shocks

Sensitivity to the persistence of tax shocks,  $\rho_{\tau wl}$  and  $\rho_{\tau rn}$ , is examined by repeatedly simulating the tax shock with two values of  $\rho_{\tau rn}$  and  $\rho_{\tau wl}$  that are above (0.99 and 0.96) and three values that are below (0.94, 0.88, and an implausible 0.10) the “baseline” setting (0.95). Baseline values of structural parameters are maintained. Results are illustrated graphically by impulse responses in two ways: Figure 15 gives the usual 200-quarter horizon graphs and shows the variation in adjustment path trajectories, and Figure 16 gives a close-up of the first 20 quarters and shows the divergence of trajectories after the shock’s initial impact. Table 9 gives an indicator of the speed of convergence to steady state: the time it takes for impulse responses to fall within 10% of their immediate impacts.

Very small changes from the “baseline”, by  $\pm 1$  basis point, does not significantly alter the responses of any variable. When  $\rho_{\tau wl}$  and  $\rho_{\tau rn}$  are both increased and decreased to 0.96 and 0.94, respectively, Figure 15 show that the shape and speed of adjustment paths change by very little. Figure 15 also shows that a persistence close to unity, i.e. an increase by 4 basis points, significantly amplifies adjustment paths. All variables except aggregate taxes,  $T_t$ , and asset prices,  $p_t$  and  $q_t$ , now exhibit hump-shaped trajectories and very long shock persistence. Those that had hump-shapes before now have exaggerated humps. For any setting below 0.88, output loses its hump-shaped trajectory. At these levels of persistence, investment falls rapidly towards steady state, and is quickly outpaced by an increasing depreciation.

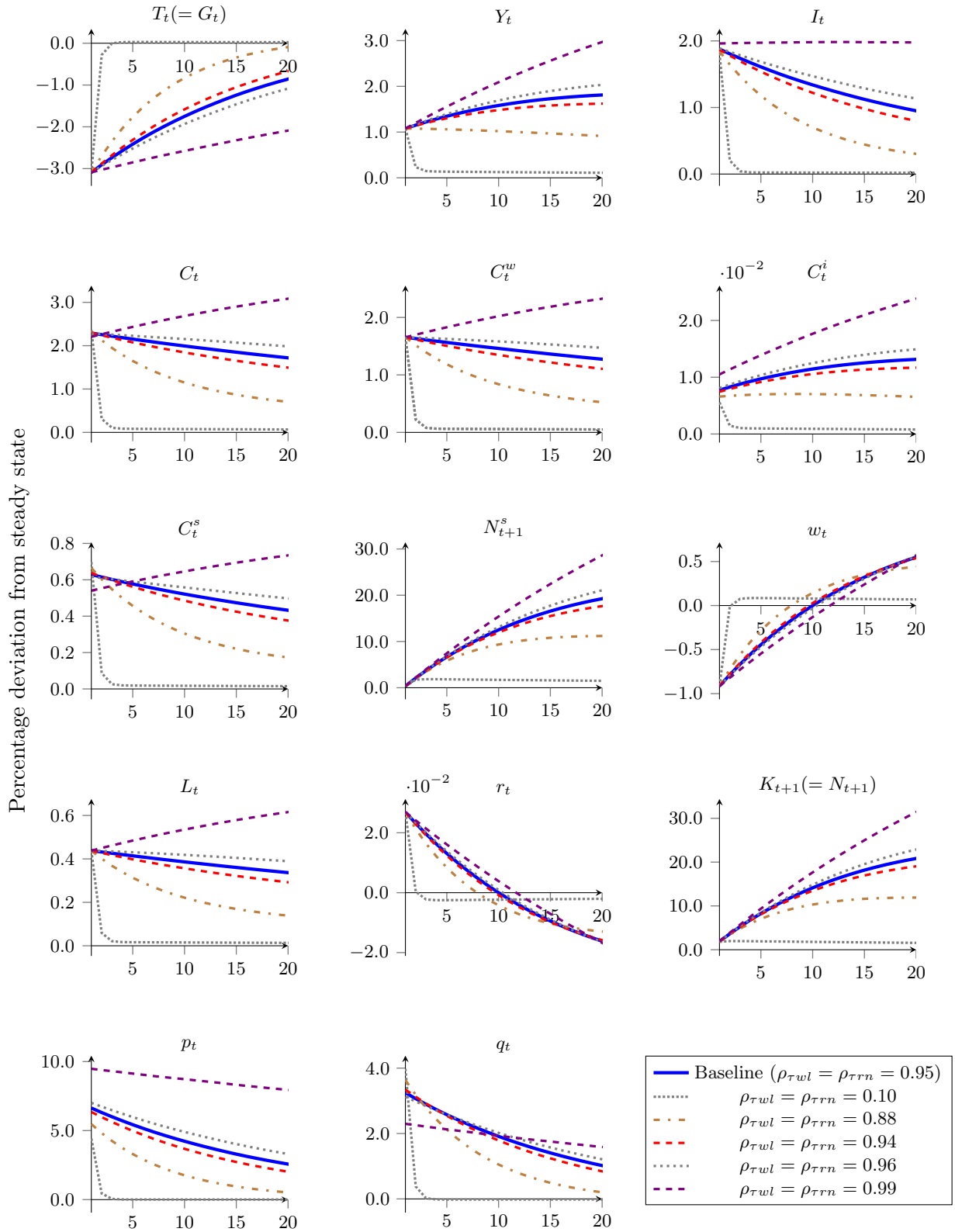
Reducing the persistence parameter down to very low levels reveals those variables whose shock propagations are driven by features of the model. The slowest variables to adjust are (in order) investment, savers’ equity, capital, investors’ consumption, and output. As shown in Table 9, even at the lowest persistence (an implausible 0.10),  $I_t$  and  $N_t^s$  take more than 200 quarters to converge;  $Y_t$  takes more than 5 years, and this is due to the slow convergence of  $K_t$ . The impulse response analysis of the tax shock in Section 3 suggests that investment is supported by asset prices, and consequently net worth, being above steady state levels throughout the process of adjustment; elevated asset prices are, in turn, due to binding liquidity constraints; and once investment is above steady state and below depreciation, the capital stock increases, and so does output. The liquidity constraints are therefore an amplifying feature for the the internal propagation mechanism (i.e. investment-capital-output relationship), or an “internal amplification” mechanism.

FIGURE 15: Impulse responses of a tax shock: repeated simulations with varying persistence of shocks to  $\tau_t^{wl}$  and  $\tau_t^{rn}$



NOTES: These graphs plot impulse responses to the tax shock from repeated simulations with lower-than-baseline settings for persistence parameters,  $\rho_{\tau_{wl}}$  and  $\rho_{\tau_{rn}}$ .

FIGURE 16: Impulse responses of a tax shock: repeated simulations with varying persistence of shocks to  $\tau_t^{wl}$  and  $\tau_t^{rn}$ , 20 quarters



NOTES: These graphs show the first 20 quarters of Figure 15. The same notes apply.

TABLE 9: Quarters after the tax shock when impulse responses converge within 10% of immediate impacts: repeated simulations with varying persistence of shocks to  $\tau_t^{wl}$  and  $\tau_t^{rn}$

	(baseline)									
$\rho_{\tau_{wl}} = \rho_{\tau_{rn}} =$	<b>0.99</b>	<b>0.96</b>	<b>0.95</b>	<b>0.94</b>	<b>0.90</b>	<b>0.89</b>	<b>0.88</b>	<b>0.80</b>	<b>0.50</b>	<b>0.10</b>
$T_t$	90	36	30	27	18	16	15	9	4	1
$Y_t$	200	200	200	200	197	188	181	138	66	21
$I_t$	200	116	93	76	38	33	29	14	4	2
$C_t$	200	193	168	150	102	93	86	43	5	2
$C_t^w$	200	196	172	154	106	98	91	48	5	2
$C_t^i$	200	200	200	200	200	200	194	156	90	46
$C_t^s$	200	180	155	135	86	77	69	27	5	2
$N_{t+1}^s$	200	200	200	200	200	200	200	200	200	200
$w_t$	200	200	200	200	175	167	159	117	44	1
$L_t$	200	196	172	153	106	98	90	48	5	2
$r_t$	200	200	200	200	175	167	159	117	44	1
$K_{t+1}$	200	200	200	200	200	200	200	200	200	181
$p_t$	200	60	48	39	23	21	19	11	4	2
$q_t$	99	41	35	30	19	18	16	10	4	1

NOTES: This table gives the period of time its takes for impulse responses to get within 10% of their quarter 1 magnitudes. Convergence of 200 quarters means some time after 200 quarters, and not in the 200<sup>th</sup> quarter.

## Appendix C Additional algebra

### C.1 The investor's budget and resource constraints

Substituting Equation (2.9) into Equation (2.10) gives

$$\begin{aligned}
& c_t^i + \frac{1}{1-\theta}n_{t+1}^i - \frac{1-\phi_t}{1-\theta}\delta n_t + q_t \left[ n_{t+1}^i - \frac{1}{1-\theta}n_{t+1}^i + \frac{1-\phi_t}{1-\theta}\delta n_t - \delta n_t \right] + p_t(m_{t+1}^i - m_t) \\
& = (1 - \tau_t^{rn})r_t n_t \\
\implies c_t^i + \left[ \frac{1}{1-\theta} + q_t - q_t \frac{1}{1-\theta} \right] n_{t+1}^i & = (1 - \tau_t^{rn})r_t n_t + \left[ \frac{1-\phi_t}{1-\theta} - q_t \frac{1-\phi_t}{1-\theta} + q_t \right] \delta n_t \\
& + p_t(m_t - m_{t+1}^i)
\end{aligned}$$

The coefficients of  $n_{t+1}^i$  and  $\delta n_t$  in the above is simplified as follows:

$$\begin{aligned}
\frac{1}{1-\theta} + q_t - q_t \frac{1}{1-\theta} & = \frac{1}{1-\theta} + q_t \left[ 1 - \frac{1}{1-\theta} \right] \\
& = \frac{1}{1-\theta} + q_t \left[ \frac{1-\theta-1}{1-\theta} \right] \\
& = \frac{1}{1-\theta} - q_t \frac{\theta}{1-\theta} \\
& = \frac{1-\theta q_t}{1-\theta} \equiv q_t^R
\end{aligned}$$

$$\begin{aligned}
\frac{1-\phi_t}{1-\theta} - q_t \frac{1-\phi_t}{1-\theta} + q_t & = \frac{1-\phi_t}{1-\theta} + q_t \left[ 1 - \frac{1-\phi_t}{1-\theta} \right] \\
& = \frac{1-\phi_t}{1-\theta} + q_t \left[ \frac{1-\theta-1+\phi_t}{1-\theta} \right] \\
& = \frac{1-\phi_t}{1-\theta} + q_t \left[ \frac{-\theta+\phi_t}{1-\theta} \right] \\
& = \frac{1-\phi_t-\theta q_t+\phi_t q_t}{1-\theta} \\
& = \frac{1-\phi_t-\theta q_t+\phi_t \theta q_t-\phi_t \theta q_t+\phi_t q_t}{1-\theta} \\
& = \frac{(1-\phi_t)(1-\theta q_t)+\phi_t q_t(1-\theta)}{1-\theta} \\
& = (1-\phi_t) \frac{1-\theta q_t}{1-\theta} + \phi_t q_t \\
& = (1-\phi_t)q_t^R + \phi_t q_t
\end{aligned}$$

thus giving the modified budget constraint (2.11),

$$c_t^i + q_t^R n_{t+1}^i = (1 - \tau_t^{rn})r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \delta n_t + p_t(m_t - m_{t+1}^i)$$

Alternatively, substituting Equation (2.8) into Equation (2.10) gives the resource constraint (2.12),

$$\begin{aligned}
& c_t^i + i_t + q_t[(1 - \theta)i_t + (1 - \phi_t)\delta n_t - i_t - \delta n_t] + p_t(m_{t+1}^i - m_t) = (1 - \tau_t^{rn})r_t n_t \\
\implies & c_t^i + i_t + q_t(1 - \theta)i_t + q_t(1 - \phi_t)\delta n_t - q_t i_t - q_t \delta n_t = (1 - \tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\implies & c_t^i + i_t[1 + q_t(1 - \theta) - q_t] + [(1 - \phi_t) - 1]q_t \delta n_t = (1 - \tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\implies & c_t^i + i_t[1 - \theta q_t] + [-\phi_t]q_t \delta n_t = (1 - \tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\implies & c_t^i + (1 - \theta q_t)i_t = (1 - \tau_t^{rn})r_t n_t + \phi_t q_t \delta n_t + p_t(m_t - m_{t+1}^i)
\end{aligned}$$

## C.2 The entrepreneur's first order conditions

From Equations (2.11), (2.13) and (2.14), the investor's Lagrangian is

$$\begin{aligned}
\mathcal{L}_e^i &= U_e(c_t^i) - \lambda_t^i \left( c_t^i + q_t^R n_{t+1}^i - (1 - \tau_t^{rn})r_t n_t - [\phi_t q_t + (1 - \phi_t)q_t^R] \delta n_t - p_t(m_t - m_{t+1}^i) \right) \\
&+ \pi E_t \left[ \beta \left\{ U_e(c_{t+1}^i) - \lambda_{t+1}^i \left( c_{t+1}^i + q_{t+1}^R n_{t+2}^i - (1 - \tau_{t+1}^{rn})r_{t+1} n_{t+1}^i - [\phi_{t+1} q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R] \delta n_{t+1}^i - p_{t+1}(m_{t+1}^i - m_{t+2}^i) \right) \right\} \right. \\
&\quad \left. + \beta^2 \left\{ U_e(c_{t+2}^i) - \lambda_{t+2}^i \left( c_{t+2}^i + q_{t+2}^R n_{t+3}^i - (1 - \tau_{t+2}^{rn})r_{t+2} n_{t+2}^i - [\phi_{t+2} q_{t+2} + (1 - \phi_{t+2})q_{t+2}^R] \delta n_{t+2}^i - p_{t+2}(m_{t+2}^i - m_{t+3}^i) \right) \right\} + \dots \right] \\
&+ (1 - \pi) E_t \left[ \beta \left\{ U_e(c_{t+1}^s) - \lambda_{t+1}^s \left( c_{t+1}^s + q_{t+1}^s (n_{t+2}^s - \delta n_{t+1}^s) + p_{t+1}(m_{t+2}^s - m_{t+1}^s) - (1 - \tau_{t+1}^{rn})r_{t+1} n_{t+1}^s \right) \right\} \right. \\
&\quad \left. + \beta^2 \left\{ U_e(c_{t+2}^s) - \lambda_{t+2}^s \left( c_{t+2}^s + q_{t+2}^s (n_{t+3}^s - \delta n_{t+2}^s) + p_{t+2}(m_{t+3}^s - m_{t+2}^s) - (1 - \tau_{t+2}^{rn})r_{t+2} n_{t+2}^s \right) \right\} + \dots \right]
\end{aligned}$$

which gives first order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}_e^i}{\partial c_t^i} &= U_e'(c_t^i) - \lambda_t^i = 0 \\
\implies & \lambda_t^i = U_e'(c_t^i)
\end{aligned} \tag{C.1}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_e^i}{\partial c_{t+1}^i} &= \pi E_t[\beta \{U_e'(c_{t+1}^i) - \lambda_{t+1}^i\}] + (1 - \pi) E_t[\beta \{U_e'(c_{t+1}^s) - \lambda_{t+1}^s\}] = 0 \\
\implies & \beta E_t[\pi U_e'(c_{t+1}^i) + (1 - \pi) U_e'(c_{t+1}^s)] = \beta E_t[\pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s] \\
\implies & \pi U_e'(c_{t+1}^i) + (1 - \pi) U_e'(c_{t+1}^s) = \pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_e^i}{\partial n_{t+1}^i} &= -\lambda_t^i q_t^R + \pi E_t[\beta \lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R)] \\
&\quad + (1 - \pi) E_t[\beta \lambda_{t+1}^s (\delta q_{t+1} + [1 - \tau_{t+1}^{rn}]r_{t+1})] = 0
\end{aligned}$$



$$\begin{aligned} \implies \lambda_t^i q_t^R &= \beta \pi E_t [\lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R)] \\ &\quad + \beta (1 - \pi) E_t [\lambda_{t+1}^s (\delta q_{t+1} + [1 - \tau_{t+1}^{rn}] r_{t+1})] \end{aligned} \quad (\text{C.3})$$

$$\begin{aligned} \frac{\partial \mathcal{L}_e^i}{\partial m_{t+1}^i} &= -\lambda_t^i p_t + \pi E_t [\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t [\beta \lambda_{t+1}^s p_{t+1}] \leq 0, \quad m_{t+1}^i \geq 0, \\ &\quad \text{and } \{-\lambda_t^i p_t + \pi E_t [\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t [\beta \lambda_{t+1}^s p_{t+1}]\} m_{t+1}^i = 0 \\ \implies -\lambda_t^i p_t + \pi E_t [\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t [\beta \lambda_{t+1}^s p_{t+1}] &= 0 \quad \text{or } m_{t+1}^i = 0 \\ \implies \lambda_t^i p_t &= \pi E_t [\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t [\beta \lambda_{t+1}^s p_{t+1}] \quad \text{or } m_{t+1}^i = 0 \\ \implies \lambda_t^i &= \beta E_t \left[ \frac{p_{t+1}}{p_t} (\pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s) \right] \quad \text{or } m_{t+1}^i = 0 \end{aligned} \quad (\text{C.4})$$

From Equations (2.11), (2.13) and (2.14), the saver's Lagrangian is

$$\begin{aligned} \mathcal{L}_e^s &= U_e(c_t^s) - \lambda_t^s \left( c_t^s + q_t (n_{t+1}^s - \delta n_t) + p_t (m_{t+1}^s - m_t) - (1 - \tau_t^{rn}) r_t n_t \right) \\ &\quad + \pi E_t \left[ \beta \left\{ U_e(c_{t+1}^i) - \lambda_{t+1}^i \left( c_{t+1}^i + q_{t+1}^R n_{t+2}^i - (1 - \tau_{t+1}^{rn}) r_{t+1} n_{t+1}^i - [\phi_{t+1} q_{t+1} \right. \right. \right. \\ &\quad \left. \left. \left. + (1 - \phi_{t+1}) q_{t+1}^R \right] \delta n_{t+1}^i - p_{t+1} (m_{t+1}^i - m_{t+2}^i) \right) \right\} + \beta^2 \left\{ U_e(c_{t+2}^i) - \lambda_{t+2}^i \left( c_{t+2}^i \right. \right. \\ &\quad \left. \left. + q_{t+2}^R n_{t+3}^i - (1 - \tau_{t+2}^{rn}) r_{t+2} n_{t+2}^i - [\phi_{t+2} q_{t+2} + (1 - \phi_{t+2}) q_{t+2}^R] \delta n_{t+2}^i \right. \right. \\ &\quad \left. \left. - p_{t+2} (m_{t+2}^i - m_{t+3}^i) \right) \right\} + \dots \right] \\ &\quad + (1 - \pi) E_t \left[ \beta \left\{ U_e(c_{t+1}^s) - \lambda_{t+1}^s \left( c_{t+1}^s + q_{t+1} (n_{t+2}^s - \delta n_{t+1}^s) + p_{t+1} (m_{t+2}^s - m_{t+1}^s) \right. \right. \right. \\ &\quad \left. \left. \left. - (1 - \tau_{t+1}^{rn}) r_{t+1} n_{t+1}^s \right) \right\} + \beta^2 \left\{ U_e(c_{t+2}^s) - \lambda_{t+2}^s \left( c_{t+2}^s + q_{t+2} (n_{t+3}^s - \delta n_{t+2}^s) \right. \right. \right. \\ &\quad \left. \left. \left. + p_{t+2} (m_{t+3}^s - m_{t+2}^s) - (1 - \tau_{t+2}^{rn}) r_{t+2} n_{t+2}^s \right) \right\} + \dots \right] \end{aligned}$$

which gives first order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}_e^s}{\partial c_t^s} &= U_e'(c_t^s) - \lambda_t^s = 0 \\ \implies \lambda_t^s &= U_e'(c_t^s) \end{aligned} \quad (\text{C.5})$$

$$\begin{aligned} \frac{\partial \mathcal{L}_e^s}{\partial n_{t+1}^s} &= -\lambda_t^s q_t + \pi E_t [\beta \lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R)] \\ &\quad + (1 - \pi) E_t [\beta \lambda_{t+1}^s (\delta q_{t+1} + [1 - \tau_{t+1}^{rn}] r_{t+1})] = 0 \\ \implies \lambda_t^s q_t &= \beta \pi E_t [\lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R)] \\ &\quad + \beta (1 - \pi) E_t [\lambda_{t+1}^s (\delta q_{t+1} + [1 - \tau_{t+1}^{rn}] r_{t+1})] \end{aligned} \quad (\text{C.6})$$

$$\begin{aligned} \implies \lambda_t^s &= \beta \pi E_t \left[ \frac{1}{q_t} \lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R) \right] \\ &\quad + \beta (1 - \pi) E_t \left[ \frac{1}{q_t} \lambda_{t+1}^s (\delta q_{t+1} + [1 - \tau_{t+1}^{rn}] r_{t+1}) \right] \end{aligned} \quad (\text{C.7})$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_e^s}{\partial m_{t+1}^s} &= -\lambda_t^s p_t + \pi E_t[\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t[\beta \lambda_{t+1}^s p_{t+1}] = 0 \\
\implies \lambda_t^s p_t &= \pi E_t[\beta \lambda_{t+1}^i p_{t+1}] + (1 - \pi) E_t[\beta \lambda_{t+1}^s p_{t+1}] \\
\implies \lambda_t^s &= \beta E_t \left[ \frac{p_{t+1}}{p_t} \left( \pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s \right) \right]
\end{aligned} \tag{C.8}$$

From Equations (C.7) and (C.8),

$$\begin{aligned}
\frac{\lambda_t^s}{\beta} &= \pi E_t \left[ \frac{1}{q_t} \lambda_{t+1}^i ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R) \right] \\
&\quad + (1 - \pi) E_t \left[ \frac{1}{q_t} \lambda_{t+1}^s ([1 - \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}) \right] \\
&= E_t \left[ \frac{p_{t+1}}{p_t} \left( \pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s \right) \right]
\end{aligned}$$

From Equations (C.1) and (C.5), respectively,  $\lambda_{t+1}^i = U_e'(c_{t+1}^i)$  and  $\lambda_{t+1}^s = U_e'(c_{t+1}^s)$ , and substituting Equation (C.2) gives

$$\begin{aligned}
&\pi E_t \left[ \frac{1}{q_t} ([1 - \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R) U_e'(c_{t+1}^i) \right] \\
&\quad + (1 - \pi) E_t \left[ \frac{1}{q_t} ([1 - \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}) U_e'(c_{t+1}^s) \right] \\
&= E_t \left[ \frac{p_{t+1}}{p_t} \left( \pi U_e'(c_{t+1}^i) + (1 - \pi) U_e'(c_{t+1}^s) \right) \right]
\end{aligned}$$

### C.3 The portfolio balance equation

The Euler equation (2.15) simplifies as follows:

$$\begin{aligned}
&\pi E_t \left[ \frac{1}{q_t} ([1 + \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R) U_e'(c_{t+1}^i) \right] \\
&\quad + (1 - \pi) E_t \left[ \frac{1}{q_t} ([1 + \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}) U_e'(c_{t+1}^s) \right] \\
&= E_t \left[ \frac{p_{t+1}}{p_t} \left( \pi U_e'(c_{t+1}^i) + (1 - \pi) U_e'(c_{t+1}^s) \right) \right] \\
\implies \pi E_t \left[ \left( \frac{p_{t+1}}{p_t} \right) U_e'(c_{t+1}^i) \right] &+ (1 - \pi) E_t \left[ \left( \frac{p_{t+1}}{p_t} \right) U_e'(c_{t+1}^s) \right] \\
&= \pi E_t \left[ \left( \frac{[1 + \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R}{q_t} \right) U_e'(c_{t+1}^i) \right] \\
&\quad + (1 - \pi) E_t \left[ \left( \frac{[1 + \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}}{q_t} \right) U_e'(c_{t+1}^s) \right] \\
\implies \pi E_t \left[ \left( \frac{p_{t+1}}{p_t} \right) U_e'(c_{t+1}^i) \right] &- \pi E_t \left[ \left( \frac{[1 + \tau_{t+1}^{rn}] r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R}{q_t} \right) U_e'(c_{t+1}^i) \right] \\
&= (1 - \pi) E_t \left[ \left( \frac{[1 + \tau_{t+1}^{rn}] r_{t+1} + \delta q_{t+1}}{q_t} \right) U_e'(c_{t+1}^s) \right] - (1 - \pi) E_t \left[ \left( \frac{p_{t+1}}{p_t} \right) U_e'(c_{t+1}^s) \right]
\end{aligned}$$

$$\begin{aligned}
&\implies \pi E_t \left[ \left( \frac{p_{t+1}}{p_t} - \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1 - \phi_{t+1}]\delta q_{t+1}^R}{q_t} \right) U'_e(c_{t+1}^i) \right] \\
&= (1 - \pi) E_t \left[ \left( \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t} - \frac{p_{t+1}}{p_t} \right) U'_e(c_{t+1}^s) \right] \\
&\implies \pi E_t \left[ \frac{\left( \frac{p_{t+1}}{p_t} - \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1 - \phi_{t+1}]\delta q_{t+1}^R}{q_t} \right)}{c_{t+1}^i} \right] \\
&= (1 - \pi) E_t \left[ \frac{\left( \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t} - \frac{p_{t+1}}{p_t} \right)}{c_{t+1}^s} \right]
\end{aligned}$$

Then by Equations (2.16) and (2.17), the last line above becomes

$$\begin{aligned}
&\pi E_t \left[ \frac{\left( \frac{p_{t+1}}{p_t} \right) - \left( \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1 - \phi_{t+1}]\delta q_{t+1}^R}{q_t} \right)}{[1 + \tau_{t+1}^{rn}]r_{t+1}n_{t+1} + [\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R]\delta n_{t+1} + p_{t+1}m_{t+1}} \right] \\
&= (1 - \pi) E_t \left[ \frac{\left( \frac{[1 + \tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t} \right) - \left( \frac{p_{t+1}}{p_t} \right)}{[1 + \tau_{t+1}^{rn}]r_{t+1}n_{t+1} + q_{t+1}\delta n_{t+1} + p_{t+1}m_{t+1}} \right]
\end{aligned}$$

#### C.4 Proof of Claim 1

The RHS of Equations (C.3) and (C.6) are identical, thus giving

$$\lambda_t^i q_t^R = \lambda_t^s q_t$$

and from Equations (C.4) and (C.8),

$$\begin{aligned}
m_{t+1}^i \neq 0 &\iff \lambda_t^i = \lambda_t^s \\
&\iff q_t^R = q_t \\
&\iff \frac{1 - \theta q_t}{1 - \theta} = q_t \\
&\iff q_t = 1 \\
\therefore m_{t+1}^i = 0 &\iff q_t \neq 1
\end{aligned}$$

□

## Appendix D Implication of Assumption 2 for expected portfolio returns

$$\begin{aligned}
q_t > 1 &\implies \theta q_t > \theta \\
&\implies 1 - \theta q_t < 1 - \theta \\
&\implies \frac{1 - \theta q_t}{1 - \theta} < 1
\end{aligned}$$

$$\begin{aligned}
& \text{i.e. } q_t^R < 1 \\
& \implies q_t^R < q_t \\
& \implies q_{t+1}^R < q_{t+1} \\
& \implies \frac{(1 + \tau_{t+1}^{rn})r_{t+1} + (1 - \phi_{t+1})\delta q_{t+1}^R}{q_t} < \frac{(1 + \tau_{t+1}^{rn})r_{t+1} + (1 - \phi_{t+1})\delta q_{t+1}}{q_t} \\
& \implies \frac{(1 + \tau_{t+1}^{rn})r_{t+1} + \phi_{t+1}\delta q_{t+1} + (1 - \phi_{t+1})\delta q_{t+1}^R}{q_t} < \frac{(1 + \tau_{t+1}^{rn})r_{t+1} + \delta q_{t+1}}{q_t}
\end{aligned}$$

### D.1 The worker's first order conditions

From Equations (2.22) and (2.23) the worker's Lagrangian is

$$\begin{aligned}
\mathcal{L}_w &= E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_w(c_j^w, l_j^w) - \lambda_t^w \left( c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) - (1 - \tau_t^{wl})w_t l_t^w \right. \right. \\
&\quad \left. \left. - (1 - \tau_t^{rn})r_t n_t^w \right) \right] \\
&= U_w(c_t^w, l_t^w) - \lambda_t^w \left( c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) - (1 - \tau_t^{wl})w_t l_t^w \right. \\
&\quad \left. - (1 - \tau_t^{rn})r_t n_t^w \right) + \beta E_t \left[ U_w(c_{t+1}^w, l_{t+1}^w) - \lambda_{t+1}^w \left( c_{t+1}^w + q_{t+1}(n_{t+2}^w - \delta n_{t+1}^w) \right. \right. \\
&\quad \left. \left. + p_{t+1}(m_{t+2}^w - m_{t+1}^w) - (1 - \tau_{t+1}^{wl})w_{t+1} l_{t+1}^w - (1 - \tau_{t+1}^{rn})r_{t+1} n_{t+1}^w \right) \right] \\
&\quad + \beta^2 E_t \left[ U_w(c_{t+2}^w, l_{t+2}^w) - \lambda_{t+2}^w \left( c_{t+2}^w + q_{t+2}(n_{t+3}^w - \delta n_{t+2}^w) + p_{t+2}(m_{t+3}^w - m_{t+2}^w) \right. \right. \\
&\quad \left. \left. - (1 - \tau_{t+2}^{wl})w_{t+2} l_{t+2}^w - (1 - \tau_{t+2}^{rn})r_{t+2} n_{t+2}^w \right) \right] + \dots
\end{aligned}$$

which gives first order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}_w}{\partial c_t^w} &= \frac{\partial U_w}{\partial c_t^w} - \lambda_t^w = 0 \\
\implies \lambda_t^w &= \frac{\partial U_w}{\partial c_t^w} = 1
\end{aligned} \tag{D.1}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_w}{\partial l_t^w} &= \frac{\partial U_w}{\partial l_t^w} + \lambda_t^w (1 - \tau_t^{wl})w_t = 0 \\
\implies \omega(l_t^w)^\nu &= \lambda_t^w (1 - \tau_t^{wl})w_t
\end{aligned} \tag{D.2}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_w}{\partial n_{t+1}^w} &= -\lambda_t^w q_t + \beta E_t [\lambda_{t+1}^w (\delta q_{t+1} + [1 - \tau_{t+1}^{wl}]r_{t+1})] \leq 0, \quad n_{t+1}^w \geq 0, \\
&\text{and } \{-\lambda_t^w q_t + \beta E_t [\lambda_{t+1}^w (\delta q_{t+1} + [1 - \tau_{t+1}^{wl}]r_{t+1})]\} n_{t+1}^w = 0 \\
\implies \lambda_t^w &= \beta E_t \left[ \frac{\delta q_{t+1} + [1 - \tau_{t+1}^{wl}]r_{t+1}}{q_t} \lambda_{t+1}^w \right] \quad \text{or} \quad n_{t+1}^w = 0
\end{aligned} \tag{D.3}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_w}{\partial m_{t+1}^w} &= -\lambda_t^w p_t + \beta E_t[\lambda_{t+1}^w p_{t+1}] = 0, \quad m_{t+1}^w \geq 0, \quad \text{and } \{-\lambda_t^w p_t + \beta E_t[\lambda_{t+1}^w p_{t+1}]\} m_{t+1}^w = 0 \\ \implies \lambda_t^w &= \beta E_t \left[ \frac{p_{t+1}}{p_t} \lambda_{t+1}^w \right] \quad \text{or} \quad m_{t+1}^w = 0 \end{aligned} \quad (\text{D.4})$$

Substituting Equation (D.1) into Equation (D.2) gives

$$\begin{aligned} \omega (l_t^w)^\nu &= (1 - \tau_t^{wl}) w_t \\ \implies l_t^w &= \left[ \frac{(1 - \tau_t^{wl}) w_t}{\omega} \right]^{\frac{1}{\nu}} \end{aligned}$$

## Appendix E Labour market equilibrium

From Equation (2.48) and Equation (2.48),  $L_t^S = L_t^D$  implies

$$\begin{aligned} \left[ \frac{(1 - \tau_t^{wl}) w_t}{\omega} \right]^{\frac{1}{\nu}} &= K_t \left[ \frac{(1 - \gamma) A_t}{w_t} \right]^{\frac{1}{\gamma}} \\ \implies w_t^{\frac{1}{\nu}} w_t^{\frac{1}{\gamma}} &= \frac{K_t \omega^{\frac{1}{\nu}} [(1 - \gamma) A_t]^{\frac{1}{\gamma}}}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}} \\ \implies w_t^{\frac{\gamma + \nu}{\nu \gamma}} &= \frac{K_t \omega^{\frac{1}{\nu}} [(1 - \gamma) A_t]^{\frac{1}{\gamma}}}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}} \\ \implies w_t &= \left[ \frac{K_t \omega^{\frac{1}{\nu}} [(1 - \gamma) A_t]^{\frac{1}{\gamma}}}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}} \right]^{\frac{\nu \gamma}{\gamma + \nu}} \\ &= \frac{K_t^{\frac{\nu \gamma}{\gamma + \nu}} \omega^{\frac{\gamma}{\gamma + \nu}} [(1 - \gamma) A_t]^{\frac{\nu}{\gamma + \nu}}}{(1 - \tau_t^{wl})^{\frac{\gamma}{\gamma + \nu}}} \end{aligned}$$

Then the quantity of labour is

$$\begin{aligned} L_t &= \left[ \frac{(1 - \tau_t^{wl}) w_t}{\omega} \right]^{\frac{1}{\nu}} \\ &= \left[ \frac{(1 - \tau_t^{wl})}{\omega} \right]^{\frac{1}{\nu}} w_t^{\frac{1}{\nu}} \\ &= \left[ \frac{(1 - \tau_t^{wl})}{\omega} \right]^{\frac{1}{\nu}} \left[ \frac{K_t^{\frac{\nu \gamma}{\gamma + \nu}} \omega^{\frac{\gamma}{\gamma + \nu}} [(1 - \gamma) A_t]^{\frac{\nu}{\gamma + \nu}}}{(1 - \tau_t^{wl})^{\frac{\gamma}{\gamma + \nu}}} \right]^{\frac{1}{\nu}} \\ &= \left[ \frac{(1 - \tau_t^{wl})}{\omega} \times \frac{K_t^{\frac{\nu \gamma}{\gamma + \nu}} \omega^{\frac{\gamma}{\gamma + \nu}} [(1 - \gamma) A_t]^{\frac{\nu}{\gamma + \nu}}}{(1 - \tau_t^{wl})^{\frac{\gamma}{\gamma + \nu}}} \right]^{\frac{1}{\nu}} \end{aligned}$$

$$\begin{aligned}
&= \left[ \frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}-1} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{\gamma}{\gamma+\nu}-1}} \right]^{\frac{1}{\nu}} \\
&= \left[ \frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{-\nu}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{-\nu}{\gamma+\nu}}} \right]^{\frac{1}{\nu}} \\
&= \left[ K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{-\nu}{\gamma+\nu}} [(1-\tau_t^{wl})(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}} \right]^{\frac{1}{\nu}} \\
&= K_t^{\frac{\gamma}{\gamma+\nu}} \omega^{-\frac{1}{\gamma+\nu}} [(1-\tau_t^{wl})(1-\gamma)A_t]^{\frac{1}{\gamma+\nu}}
\end{aligned}$$

### E.1 How $q_t^R$ varies with $\theta$

From Equation (2.6),

$$q_t^R = (1 - \theta q_t)(1 - \theta)^{-1}$$

If  $\theta$  varies, then

$$\begin{aligned}
\frac{\partial q_t^R}{\partial \theta} &= (1 - \theta)^{-1} \frac{\partial}{\partial \theta} (1 - \theta q_t) + (1 - \theta q_t) \frac{\partial}{\partial \theta} (1 - \theta)^{-1} \\
&= (1 - \theta)^{-1} (-q_t) + (1 - \theta q_t)(1 - \theta)^{-2} \\
&= -\frac{q_t}{1 - \theta} + \frac{1 - \theta q_t}{(1 - \theta)^2} \\
&= -\frac{q_t(1 - \theta)}{(1 - \theta)^2} + \frac{1 - \theta q_t}{(1 - \theta)^2} \\
&= \frac{-q_t + \theta q_t + 1 - \theta q_t}{(1 - \theta)^2} \\
&= \frac{1 - q_t}{(1 - \theta)^2}
\end{aligned}$$

By Assumption 2,  $1 - q_t < 0$  and therefore  $\frac{\partial q_t^R}{\partial \theta} < 0$  for  $\theta \in (0, 1)$ .

### E.2 Standard deviation of innovations to exogenous variables: derivation of Equation (A.1)

Consider recursive substitutions of the following AR(1) model for  $X_t$ :

$$\begin{aligned}
X_t &= (1 - \rho_X)X + \rho_X X_{t-1} + u_t^X \\
&= (1 - \rho_X)X + \rho_X \left[ (1 - \rho_X)X + \rho_X X_{t-2} + u_{t-1}^X \right] + u_t^X \\
&= (1 - \rho_X)X + \rho_X(1 - \rho_X)X + \rho_X^2 X_{t-2} + \rho_X u_{t-1}^X + u_t^X \\
&= (1 - \rho_X)X + \rho_X(1 - \rho_X)X + \rho_X^2 \left[ (1 - \rho_X)X + \rho_X X_{t-3} + u_{t-2}^X \right] + \rho_X u_{t-1}^X + u_t^X \\
&= (1 - \rho_X)X + \rho_X(1 - \rho_X)X + \rho_X^2(1 - \rho_X)X + \rho_X^3 X_{t-3} + \rho_X^2 u_{t-2}^X + \rho_X u_{t-1}^X + u_t^X
\end{aligned}$$

$$\begin{aligned}
&= \dots \\
&= (1 - \rho_X)X + \rho_X(1 - \rho_X)X + \rho_X^2(1 - \rho_X)X + \dots + u_t^X + \rho_X u_{t-1}^X + \rho_X^2 u_{t-2}^X + \dots
\end{aligned}$$

where  $X$  is the steady state value of  $X_t$ . Then the variance of  $X_t$  is given by

$$\sigma_X^2 = \sigma_{uX}^2 + \rho_X^2 \sigma_{uX}^2 + \rho_X^4 \sigma_{uX}^2 + \dots = \frac{\sigma_{uX}^2}{1 - \rho_X^2}$$

which implies the standard deviation of innovations to  $X_t$ ,

$$\sigma_{uX} = \sqrt{(1 - \rho_X^2) \sigma_X^2}$$

## Appendix F Data

TABLE 10: US liquid assets, capital and liquidity

Year	Federal government liabilities (\$b)	Capital (\$b)	Liquidity share
1945	262.4	471.0	0.3578
1946	238.4	544.9	0.3044
1947	230.1	635.3	0.2659
1948	222.6	701.2	0.2410
1949	222.3	736.0	0.2320
1950	221.2	830.6	0.2103
1951	222.3	928.2	0.1932
1952	229.1	962.1	0.1923
1953	231.1	992.8	0.1888
1954	232.1	1076.3	0.1774
1955	233.6	1188.9	0.1642
1956	228.8	1281.4	0.1515
1957	228.6	1314.0	0.1482
1958	235.2	1434.0	0.1409
1959	245.3	1516.4	0.1392
1960	242.1	1574.6	0.1333
1961	248.2	1700.8	0.1273
1962	253.7	1748.3	0.1267
1963	258.4	1842.1	0.1230
1964	264.1	1999.5	0.1167
1965	264.8	2166.2	0.1089
1966	268.7	2223.8	0.1078
1967	274.6	2498.9	0.0990
1968	281.4	2822.1	0.0907
1969	285.8	2863.0	0.0908
1970	302.4	3008.0	0.0913
1971	333.3	3344.2	0.0906
1972	349.5	3846.3	0.0833
1973	357.0	4042.8	0.0811
1974	377.8	4028.0	0.0858
1975	476.2	4722.8	0.0916
1976	553.2	5327.2	0.0941
1977	611.9	5856.7	0.0946
1978	673.0	6672.2	0.0916

*Continued on next page*



Table 10: *Continued from previous page*

Year	Federal government liabilities (\$b)	Capital (\$b)	Liquidity share
1979	723.4	7787.0	0.0850
1980	823.6	8982.0	0.0840
1981	923.7	9604.3	0.0877
1982	1101.3	10189.0	0.0975
1983	1290.9	10797.8	0.1068
1984	1501.1	11513.4	0.1153
1985	1744.1	12817.3	0.1198
1986	1971.5	13990.6	0.1235
1987	2109.7	14846.2	0.1244
1988	2242.7	16078.1	0.1224
1989	2404.1	17644.1	0.1199
1990	2671.6	17957.5	0.1295
1991	2985.2	19099.8	0.1352
1992	3276.6	19871.3	0.1416
1993	3543.3	20763.5	0.1458
1994	3710.8	21282.3	0.1485
1995	3868.4	23449.4	0.1416
1996	4048.2	24326.8	0.1427
1997	4097.7	27323.9	0.1304
1998	4089.8	30781.4	0.1173
1999	4232.4	35528.9	0.1064
2000	3808.8	35541.3	0.0968
2001	3855.5	35844.0	0.0971
2002	4118.6	35308.8	0.1045
2003	4549.1	40199.5	0.1017
2004	4930.0	45337.8	0.0981
2005	5273.5	50847.9	0.0940
2006	5475.2	54131.2	0.0919
2007	5797.3	54116.5	0.0968
2008	8980.9	45001.9	0.1664
2009	9990.4	44224.8	0.1843
2010	11707.4	46010.2	0.2028
2011	12444.2	46217.6	0.2121
<b>Average: 1957 - 2007</b>			0.1110
<b>Standard deviation: 1957 - 2007</b>			0.0204

Source: Board of Governors of the Federal Reserve System, US Flow of Funds Statistics, various issues; 2012 data is obtained online from <http://www.federalreserve.gov/releases/z1/current/z1.pdf>, and historical data from <http://www.federalreserve.gov/releases/z1/Current/data.htm>.

NOTES: The liquidity share is calculated according to [Del Negro et al. \(2011\)](#). Table 11 gives the metadata. The liquidity share is illustrated graphically in Figure 3.

TABLE 11: Liquidity share measure: metadata

Item	Reference, Flow of Funds Statistics
<b>Liabilities of the federal government</b>	
T-bills	Table L.105, line 21
Treasury securities	Table L.105, line 22
Less: Holdings by the monetary authority	Table L.108, line 12
Less: Holdings by the budgetary agency	Table L.105, line 23
Reserves	Table L.108, line 32
Vault cash	Table L.108, line 33
Currency	Table L.108, line 34
Currency outside banks	Table L.108, line 41
Less: Remittances to the federal government	Table L.108, line 35
<b>Capital (at market value)</b>	
Capital owned by households:	
Real estate	Table B.100, line 3
Equipment and software of non-profit organisations	Table B.100, line 6
Consumer durables	Table B.100, line 7
Capital owned by the non-corporate sector:	
Real estate	Table B.103, line 3
Equipment and software	Table B.103, line 6
Inventories	Table B.103, line 9
Capital owned by the corporate sector:	
Equity outstanding, market value	Table B.102, line 35
Liabilities	Table B.102, line 21
Less: Financial assets	Table B.102, line 6
Less: Government credit market instruments	Table F.105c, line 33
Less: Trade receivables	Table F.105c, line 43

NOTES: This table follows from the appendix of [Del Negro et al. \(2011\)](#).

TABLE 12: US federal government's corporate equity holdings

<b>Period</b>	<b>Equities (\$m)</b>	<b>ln(Equities)</b>
2008:4	188,676	12.15
2009:1	223,856	12.32
2009:2	157,566	11.97
2009:3	158,847	11.98
2009:4	67,351	11.12
2010:1	50,234	10.82
2010:2	49,613	10.81
2010:3	50,814	10.84
2010:4	49,928	10.82
2011:1	62,137	11.04
2011:2	65,961	11.10
2011:3	59,282	10.99
2011:4	57,813	10.96
2012:1	48,156	10.78
2012:2	43,618	10.68
2012:3	41,134	10.62
<b>Standard deviation</b>		<b>0.5671</b>

*Source:* Board of Governors of the Federal Reserve, US Flow of Funds Accounts, Table L.105, line 11.

NOTES: This table gives the value of equities that were purchased by the US government from financial corporations under the Troubled Asset Relief Program. They are valued at market prices.

TABLE 13: US real monetary base

Period	M1 (\$b)	CPI	CPI, s.a.	Real M1	ln (real M1)	ln (real M1), detrended
1987Q1	730.2	111.200	111.4902	6.549456	1.879382	1.794665
1987Q2	743.9	112.700	112.6327	6.604653	1.887774	1.805112
1987Q3	743.0	113.800	113.6317	6.538671	1.877734	1.814580
1987Q4	756.2	115.300	115.1649	6.566237	1.881941	1.823065
1988Q1	756.2	115.700	116.0019	6.518858	1.874699	1.830602
1988Q2	768.1	117.100	117.0301	6.563269	1.881489	1.837261
1988Q3	781.4	118.500	118.3247	6.603862	1.887655	1.843141
1988Q4	783.3	120.200	120.0592	6.524284	1.875531	1.848370
1989Q1	785.7	121.100	121.4160	6.471140	1.867352	1.853101
1989Q2	779.2	123.100	123.0265	6.333595	1.845868	1.857505
1989Q3	777.8	124.400	124.2160	6.261675	1.834448	1.861763
1989Q4	786.6	125.600	125.4528	6.270086	1.835790	1.866049
1990Q1	795.4	127.400	127.7324	6.227079	1.828907	1.870516
1990Q2	806.1	128.900	128.8230	6.257421	1.833768	1.875304
1990Q3	810.1	130.400	130.2071	6.221627	1.828031	1.880521
1990Q4	819.9	133.500	133.3436	6.148778	1.816253	1.886255
1991Q1	827.2	134.600	134.9512	6.129622	1.813133	1.892556
1991Q2	843.1	135.200	135.1193	6.239672	1.830928	1.899433
1991Q3	861.6	136.200	135.9985	6.335363	1.846147	1.906845
1991Q4	878.0	137.400	137.2390	6.397598	1.855923	1.914708
1992Q1	910.4	138.100	138.4604	6.575167	1.883300	1.922900
1992Q2	943.8	139.500	139.4167	6.769633	1.912447	1.931262
1992Q3	963.3	140.500	140.2922	6.866385	1.926638	1.939612
1992Q4	1003.7	141.800	141.6338	7.086583	1.958203	1.947752
1993Q1	1030.4	142.600	142.9721	7.207000	1.975053	1.955481
1993Q2	1047.7	144.000	143.9140	7.280041	1.985136	1.962601
1993Q3	1084.5	144.400	144.1864	7.521514	2.017768	1.968929
1993Q4	1112.9	145.700	145.5293	7.647259	2.034347	1.974292
1994Q1	1131.6	146.200	146.5815	7.719937	2.043806	1.978553
1994Q2	1141.1	147.400	147.3120	7.746144	2.047195	1.981607
1994Q3	1150.6	148.400	148.1805	7.764856	2.049608	1.983394
1994Q4	1150.1	149.500	149.3248	7.702001	2.041480	1.983893
1995Q1	1151.5	150.300	150.6922	7.641404	2.033581	1.983124
1995Q2	1149.2	151.900	151.8093	7.570023	2.024196	1.981144
1995Q3	1145.4	152.500	152.2744	7.521947	2.017825	1.978043
1995Q4	1137.3	153.700	153.5199	7.408160	2.002582	1.973933

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Table 13: *Continued from previous page*

Period	M1 (\$b)	CPI	CPI, s.a.	Real M1	ln (real M1)	ln (real M1), detrended
1996Q1	1123.5	154.400	154.8029	7.257616	1.982051	1.968957
1996Q2	1124.8	156.300	156.2067	7.200716	1.974180	1.963272
1996Q3	1112.4	157.000	156.7677	7.095847	1.959510	1.957043
1996Q4	1086.3	158.300	158.1145	6.870337	1.927213	1.950445
1997Q1	1081.3	159.100	159.5152	6.778666	1.913780	1.943650
1997Q2	1064.0	160.200	160.1044	6.645666	1.893965	1.936820
1997Q3	1066.3	160.500	160.2626	6.653456	1.895136	1.930096
1997Q4	1065.6	161.600	161.4106	6.601795	1.887342	1.923592
1998Q1	1074.2	161.600	162.0217	6.629976	1.891601	1.917401
1998Q2	1076.7	162.500	162.4030	6.629804	1.891575	1.911592
1998Q3	1075.0	163.200	162.9586	6.596768	1.886580	1.906220
1998Q4	1086.1	164.000	163.8078	6.630330	1.891655	1.901327
1999Q1	1097.4	164.300	164.7287	6.661861	1.896399	1.896941
1999Q2	1102.0	166.200	166.1008	6.634527	1.892287	1.893085
1999Q3	1098.5	166.700	166.4534	6.599445	1.886986	1.889782
1999Q4	1102.2	168.200	168.0029	6.560601	1.881082	1.887054
2000Q1	1121.6	168.800	169.2405	6.627256	1.891191	1.884922
2000Q2	1114.6	171.300	171.1977	6.510600	1.873432	1.883403
2000Q3	1102.8	172.800	172.5444	6.391399	1.854953	1.882516
2000Q4	1098.7	174.000	173.7961	6.321775	1.844000	1.882278
2001Q1	1097.1	175.100	175.5569	6.249255	1.832462	1.882684
2001Q2	1116.1	176.900	176.7944	6.312983	1.842608	1.883708
2001Q3	1139.0	177.500	177.2374	6.426408	1.860416	1.885292
2001Q4	1166.2	177.700	177.4918	6.570445	1.882582	1.887352
2002Q1	1191.3	177.100	177.5621	6.709201	1.903480	1.889789
2002Q2	1187.7	179.800	179.6927	6.609619	1.888526	1.892500
2002Q3	1199.8	180.100	179.8336	6.671724	1.897878	1.895392
2002Q4	1204.7	181.300	181.0876	6.652583	1.895005	1.898369
2003Q1	1227.1	181.700	182.1741	6.735863	1.907446	1.901336
2003Q2	1250.3	183.800	183.6903	6.806566	1.917888	1.904195
2003Q3	1288.3	183.900	183.6280	7.015816	1.948167	1.906854
2003Q4	1297.3	185.000	184.7832	7.020659	1.948857	1.909229
2004Q1	1306.0	185.200	185.6833	7.033482	1.950682	1.911261
2004Q2	1333.2	188.000	187.8878	7.095726	1.959493	1.912916
2004Q3	1340.6	189.400	189.1198	7.088628	1.958492	1.914185
2004Q4	1360.7	190.900	190.6763	7.136177	1.965177	1.915088

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Table 13: *Continued from previous page*

Period	M1 (\$b)	CPI	CPI, s.a.	Real M1	ln (real M1)	ln (real M1), detrended
2005Q1	1366.4	190.700	191.1976	7.146532	1.966627	1.915672
2005Q2	1358.2	194.600	194.4838	6.983614	1.943567	1.916017
2005Q3	1366.9	195.400	195.1109	7.005758	1.946732	1.916233
2005Q4	1375.4	199.200	198.9666	6.912718	1.933363	1.916448
2006Q1	1379.6	198.300	198.8175	6.939028	1.937162	1.916809
2006Q2	1380.9	201.500	201.3797	6.857196	1.925299	1.917474
2006Q3	1369.8	203.500	203.1990	6.741176	1.908234	1.918612
2006Q4	1370.2	201.800	201.5635	6.797856	1.916607	1.920401
2007Q1	1372.6	202.416	202.9442	6.763435	1.911531	1.923007
2007Q2	1378.1	206.686	206.5626	6.671585	1.897857	1.926597
2007Q3	1368.8	208.299	207.9909	6.581058	1.884196	1.931331
2007Q4	1379.7	208.936	208.6912	6.611204	1.888766	1.937350
2008Q1	1379.2	211.080	211.6308	6.517010	1.874416	1.944766

*Sources:* Board of Governors of the Federal Reserve System and Bureau of Labour Statistics.

*NOTES:* Nominal, seasonally adjusted base money, or M1, is obtained from the Board of Governors of the Federal Reserve System; the data represents the stock at the end of the quarter. The all-items, all urban consumers, US city average CPI (1982-84 = 100) is obtained from the Bureau of Labour Statistics. The CPI is seasonally adjusted by the multiplicative moving average method. M1 is deflated by the seasonally adjusted CPI (CPI, s.a.) to obtain real M1. The natural logarithm of real M1 is detrended by the Hodrick-Prescott filter.

TABLE 14: UK taxpayers' earnings and tax liabilities

Tax year	Earnings (£m)		Tax liabilities (£m)		Tax liabilities Earnings	
	Wages	Dividends	Wages	Dividends	Wages	Dividends
1999-00	382,000	18,300	87,650	3,670	0.229	0.201
2000-01	431,500	20,800	98,580	3,900	0.228	0.188
2001-02	444,900	19,400	102,030	4,053	0.229	0.209
2002-03	451,600	20,800	103,900	4,330	0.230	0.208
2003-04	449,000	25,400	103,100	5,290	0.230	0.208
2004-05	496,000	32,600	113,860	6,070	0.230	0.186
2005-06	539,000	37,100	125,640	7,790	0.233	0.210
2006-07	573,000	41,500	135,370	9,000	0.236	0.217
2007-08	611,000	45,700	145,720	9,950	0.238	0.218
2008-09	n.a.	n.a.	142,000	9,380	n.a.	n.a.
2009-10	614,000	50,000	139,100	10,910	0.227	0.218
2010-11	616,000	36,100	138,600	7,700	0.225	0.213
<b>Average</b>					0.231	0.207
<b>Standard deviation</b>					0.0040	0.0112

*Source:* HM Revenue and Customs. UK taxpayers' earnings from employment and UK dividends are obtained from [HM Revenue and Customs \(2012b\)](#), Tables 3.6 and 3.7, respectively. Data for the tax year 2008-09 is not available. Income tax liabilities on earnings and dividends are obtained from [HM Revenue and Customs \(2012a\)](#), Table 2.6. Older issues of these publications are obtained online at [http://webarchive.nationalarchives.gov.uk/20120609144700/http://hmrc.gov.uk/stats/income\\_tax/table2-6a.pdf](http://webarchive.nationalarchives.gov.uk/20120609144700/http://hmrc.gov.uk/stats/income_tax/table2-6a.pdf) and [http://webarchive.nationalarchives.gov.uk/\\*/http://hmrc.gov.uk/stats/income\\_distribution/menu-by-year.htm](http://webarchive.nationalarchives.gov.uk/*/http://hmrc.gov.uk/stats/income_distribution/menu-by-year.htm).



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