Factor Income Taxation in a Horizontal Innovation Model*

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Abstract

We consider the optimal factor income taxation in a standard R&D model with technical change represented by an increase in the variety of intermediate goods. We show that the model has no transitional dynamics. Redistributing the tax burden from labor to capital will in most cases increase the employment rate in equilibrium. This has opposite effects on two distortions in the model, one due to monopoly power, the second to the incomplete appropriability of the benefits of inventions. Their relative momentum determines the sign of the welfare effect of the redistribution. We show that, for parameter values consistent with available estimates, the optimal tax rate on capital will be sizable.

Keywords: *Capital Income Taxes, R&D, Growth Effect, Welfare Effect.*

JEL classification: *E62, H21, O41*

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1 Introduction

This paper examines how the tax burden should be distributed between capital and labor income in a basic R&D model of endogenous growth.[1] The main message of the extensive literature on the optimal taxation of factor incomes, as summarised by Atkeson et al. (1999) is the following: taxing capital is a bad idea in the long-run.[2] This conclusion was first exposited in Chamley (1985) in the context of simple single sector models of exogenous growth, but has proved robust in a variety of settings, including models where capital-holders are distinct from workers (Judd 1985), overlapping generations models (Erosa and Gervais 2002) and models with human capital accumulation (Jones et al. 1997). We add that most quantitative investigations suggest that capital taxes should be zero or very small even in the short run (see Atkeson et al. 1999). The literature on endogenous growth tends to reinforce the message that capital income should not be taxed, as taxing it would have adverse effects on the rate of growth which would compound over time (see the survey in Jones and Manuelli 2005).

We check if the "Chamley-Judd result" message also holds in a standard model of horizontal innovation, with an infinitely lived representative agent, originally proposed by Rivera-Batiz and Romer (1991) and known as the "lab-equipment model". Given its flexibility and simplicity this model has provided a tractable framework for analyzing a wide array of issues in economic growth, sinthetized in Gancia and Zilibotti (2005). Entrepreneurs spend a fixed cost in order to develop new intermediate goods, over the production of which they then enjoy eternal monopoly power. Output in the final goods production sector is linear in the number of intermediate goods used so unbounded growth is possible. There are two inefficiencies in the model, a static one stemming from market power in the intermediate goods sector, and a dynamic one stemming from the uncomplete appropriability of the social surplus from innovating.

We extend this benchmark model by explicitly analysing the decision to supply labor as well as by introducing government spending. We assume that the only tax instruments available are linear income taxes, that the government determines the amount of revenue it wants to generate as a constant fraction of income, that a constant fraction of this revenue is transferred back to consumers and, finally, that the government budget is balanced at all times. The tax rates (i.e. the labor income tax rate and the interest income tax rate) must adjust endogenously.

[1] The taxation of capital involves many different kind of taxes, some on stocks (eg wealth tax, tax on bequests, property tax of capital), some on the income from savings (from the corporate income tax, the tax on interest and dividends, the taxation of capital gains). By a tax on capital income in our model we mean a tax on income from savings. In the model there is no capital in the physical sense, but wealth accumulates in the form of of patents.

[2] More precisely the Ramsey tax system advocates a high tax on initial capital stock (or on capital income in the initial period) and a zero tax on capital income in future times. However, the Ramsey results hinge on the assumption that the government can commit to zero tax in the future, as there is a problem of dynamic inconsistency (see Martin 2010).

This gives what has become known as a "Ramsey Problem": maximize social welfare through the choice of taxes subject to the constraints that final allocations must be consistent with a competitive equilibrium with distortionary taxes so that a pre-specified amount of fiscal revenue is raised.

In a model with endogenous growth, the common trend between output and government expenditure cannot be ignored, so what we pre-specify here is the ratio between these variables and not the absolute amount of tax revenue. Furthermore, to isolate the effects of taxation, rather than of more complex public interventions, we assume government revenues do not directly affect the marginal utility of private consumption and leisure or the marginal productivity of factors of production.

In this setting we derive an expression for the optimal tax rate on capital income as a function of tastes and technology, which shows that the tax rate will not in general be zero. We then move to the analysis of calibrated versions of the model, and find that the tax rate on capital is sizable thus bridging the gap between economic theory prescriptions and the fact that in developed economies capital taxes are far from zero.[3].

Studies based on R&D models similar to ours have generally found that taxing savings is detrimental to growth and welfare (e.g. Lin and Russo 1999 and 2002 and Zeng and Zhang 2002). In particular our work complements Zeng and Zhang (2007), who study fiscal issues adopting our same specification of the horizontal innovation model. However while we focus on the optimal distribution of the tax burden between capital and labor, they compare the effects on growth of subsidizing R&D investment to the effects of subsidizing final output or subsidizing the purchase of intermediate goods. They consider distortionary taxation (i.e. taxes on labor income) but rule out taxes on interest income.

To understand intuitively our findings, consider that if the shift in the tax burden from capital to labor increases employment and as a consequence the productivity of intermediates, the demand for each of them is increased. The production of each intermediate will then be more profitable, and the distortion due to monopoly power lower. Also, the invention activity, financed by household savings, is more rewarding the greater the prospective demand, and therefore the profits from a new product are. So a higher employment increases coeteris paribus the return to saving and linearly increases growth. However the increase in the tax on capital which is the counterpart to the reduction in the tax on labor directly discourages savings and growth. This could worsen the dynamic inefficiency. A third distortion in the model is created by government expenditure itself as agents do not internalize the fact that higher income will lead to more unproductive public expenditure. Taxing both labor and capital

[3]See McDaniel (2007) for recent estimates of effective tax rates on capital.

income reduces this distortion (see Marrero and Novales 2007). For reasonable parameters' values the interplay between the various channels through which the tax program has effects means that the optimal tax on capital is not only positive but very sizable–given the levels of public spending observed in advanced economies.

Reasons why the Chamley result may not hold have been proposed in the literature. In general when there are distortions in the economy the Chamley result does not hold: imperfect competition can lead to the recommendation of a negative rather than zero capital income tax (Judd 2002, Aghion et al 2013).

A way in which taxing capital can be good is when government spending increases the marginal productivity of capital, as in Baier and Glomm (2001), Barro and Sala-i-Martin (1992), Guo and Lansing (1999), Turnovsky (1996, 2000), Chen (2007) and Zhang et al. (2008).[4] More counter examples to the optimality of a zero tax on physical capital can be found in human capital models (see Ben-Gad 2003, de Hek 2006 and Chen and Lu 2013). The presence of an informal sector the income from which cannot be taxed or of other restrictions on the taxation of factors are also grounds for the positive taxation of capital income (see Correia 1996, Penalosa and Turnovsky 2005 and Reis 2011). Chamley (2001), Ho and Wang (2007) and Imrohoroglu (1998), among others have emphasized that if households face borrowing constraints and/or are subject to uninsurable idiosyncratic income risk, so that excessive savings arise, then the optimal tax system will in general include a positive capital income tax. Asea and Turnovsky (1998) and Kenc (2004) find that increasing the tax rate on capital income may increase growth in a stochastic environment. Many papers (e.g. Cremer et al. 2003, Hendricks 2003, 2004, Erosa and Gervais 2002, Song 2002, Uhlig and Yanagawa 1996 and Yakita 2003) show that in life cycle/OLG models the optimal capital income tax in general is different from zero, as such tax can facilitate the intergenerational trasmission of wealth. Conesa et al. (2009) quantitatively characterize the optimal capital income tax in an overlapping generations model with idiosyncratic, uninsurable income shocks and find it to be significantly positive at 36 percent.Piketty Saez (2012) find an optimal 50-60 percent tax rate on capitalized inheritances.

The arguments developed in these models as grounds for a positive rate of capital taxation are unrelated to ours as we model a perfect foresight closed economy with infinite lived agents, no effect of government expenditures on the rate of return of private factors of production, no human capital accumulation, no subsidies to investment. The mechanism works through increased employment ratio as in Pelloni and Waldmann (2000) based on Romer (1986). Aghion et al (2013) show a similar effect in a creative destruction model.

[4] In Zhang et al. (2008) the government should tax net capital income more heavily than labor income, however this is because investment is subsidized at the same rate at which net capital income is taxed.

However, our paper like most in this literature, can be seen as an example of the argument that it is the presence of market failures that makes capital income taxation desirable. In other words, ours are second-best results.

Often in the papers on taxation and growth, only the growth, not the welfare effect of the tax experiments are calculated, if in the market equilibrium growth is lower than optimal, because there is an implicit presumption that higher growth means more welfare as, through compounding, growth effects always prevail over level effects. However while, as we show, in our model growth is inefficiently low in the absence of taxes, even when the introduction of the tax lowers growth there might be a positive welfare effect. In our calibrated examples, this counterintuitive effect arises with parameters' values well within the range of selections adopted in other settings in public finance, quantitative growth theory and business cycle analysis.

A complete assessment of the welfare effects of the tax program we consider has to include an analysis of its effect on the dynamic properties of the model. In fact it has recently been shown that factor taxes can affect the stability of the dynamic equilibrium of a market economy. In particular, Ben-Gad (2003), Chen and Lee (2007), Guo (2004), Mino (2001), Park (2009), Palivos et al. (2003), Pelloni and Waldmann (1998) and (2000), Raurich (2001), Schmitt-Grohe and Uribe (1997) and Wong and Yip (2010) among others have shown that the introduction of taxes and government spending and /or of non separability of preferences over consumption and leisure (which we assume) may make the equilibrium exhibit local indeterminacy. However, this is not the case in this model, which, as we show, features a unique unstable balanced growth path.

The rest of the paper is organized as follows: in section 2 the model is presented; in section 3 the general equilibrium conditions of the model are described; section 4 analyzes the labor supply effect, the growth effect and the welfare effect of shifting the tax burden from labor to capital; section 5 presents some calibrated examples and derives the optimal tax rates for various sets of parameters; section 6 presents the social planner's solution and section 7 concludes. Most proofs are relegated to the Appendices.

2 The Model

2.1 Households

We assume that in the economy there is a continuum of length one of identical households.[5] Each has utility U given by:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \left(\frac{1}{1-\sigma} C^{1-\sigma} h(H) \right) dt \tag{1}$$

[5] As Zeng and Zhang (2007) note, normalizing the population to unity removes from the analysis of taxes the "scale effect" discussed by Jones (1995).

where C is consumption, H labor, $\rho > 0$ is the rate of time discount and $1/\sigma > 0$ is the intertemporal elasticity of substitution. As Zeng and Zhang (2007) note, normalizing the population to unity removes from the analysis of taxes the so called "scale effect". The following conditions ensure non satiation of consumption and leisure:

$$h(H) > 0 \tag{2}$$

and

$$(1-\sigma)h'(H) < 0. \tag{3}$$

Strict concavity of instantaneous felicity imposes:

$$(1-\sigma)h''(H) < 0 \tag{4}$$

$$\sigma\frac{h''h}{(\sigma-1)} - h'^2 > 0. \tag{5}$$

The instantaneous budget constraint consumers face is given by:

$$\dot{F} = r(1-\tau_r)F + \pi_n(1-\tau_r)N + w(1-\tau_w)H - C + tY. \tag{6}$$

Households derive their income by loaning entrepreneurs their financial wealth F (of which all have the same initial endowment), by profits π_n (net of the interest payments) of the N firms and by supplying labor H to firms, taking the interest rate r and the wage rate w as given. Capital income is taxed at the rate τ_r while labor income is taxed at the rate τ_w. They also receive a lump-sum transfer tY, where t is a constant and Y is aggregate production of the final good. Optimization at an interior point implies that the marginal rate of substitution between leisure and consumption equals their relative price:

$$\frac{h'}{h} = \frac{w(1-\tau_w)(\sigma-1)}{C}. \tag{7}$$

Optimal consumption and leisure must also obey the intertemporal condition:

$$-\sigma\frac{\dot{C}}{C} + \frac{h'}{h}\dot{H} = \frac{\dot{\lambda}}{\lambda} = \rho - r(1-\tau_r) \tag{8}$$

where $\lambda = c^{-\sigma}h$ is the shadow value of wealth. Given a no Ponzi game condition the transversality condition imposes:

$$\lim_{t\to\infty} \lambda F \exp(-\rho t) = 0. \tag{9}$$

2.2 Firms

In this economy there are a final goods sector and an intermediate goods sector. The former is perfectly competitive, whereas the latter is monopolistic. R&D activity leads to an expanding variety of intermediate goods. All patents have an infinitely economic life, that is, we assume no obsolescence of any type of intermediate goods.

The production function of firm i in the final goods sector is given by:

$$Y(i) = AL(i)^{1-\alpha} \int_0^N x(i,j)^\alpha di \tag{10}$$

where $Y(i)$ is the amount of final goods produced and $L(i)$ is labor used by firm i and $x(i,j)$ is the quantity this firm uses of the intermediate goods indexed by j. For tractability both i and j are treated as continuous variables. We assume $0 < \alpha < 1$. The final goods sector is competitive and we assume a continuum of length one of identical firms. We can then suppress the index i to avoid notational clutter. Firms maximize profits given by

$$Y - wL - \int_0^N P(j)x(j)dj \tag{11}$$

where w is the wage rate and $P(j)$ is the price of the intermediate good j. By profit maximization, the demand for good j is given by:

$$x(j) = L\left(\frac{A\alpha}{P(j)}\right)^{\frac{1}{1-\alpha}} \tag{12}$$

and labor demand by:

$$w = (1-\alpha)\frac{Y}{L}. \tag{13}$$

Since the firms in the final goods sector are competitive and there are constant returns to scale their profits are zero in equilibrium. In contrast the firms which produce intermediate goods with patent which they invent then earn monopoly profits for ever. The cost of production of the intermediate good j, once it has been invented, is given by one unit of the final good.

The present discounted value at time t of monopoly profits for firm j, or in other words the value of the patent for the j^{th} intermediate good $V(j,t)$ at time t is:

$$V(j,t) = \int_t^\infty (P(j)-1)x(j)e^{-\overline{r}(s,t)(s-t)}ds \tag{14}$$

where $\overline{r}(s,t)$ is the average interest rate during the period of time from t to s. The inventor of the j^{th} intermediate good chooses $P(j)$ to maximize $(P(j)-1)x(j)$ where $x(j)$ is given by (12), so for each j, the equilibrium price and quantity are:

$$P(j) = P = \frac{1}{\alpha} \tag{15}$$

and

$$x(j) = x = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}. \tag{16}$$

The price is higher than the marginal cost of producing good j, and the quantity produced, $x(j)$, is therefore lower than the socially optimal level. This is in fact the first inefficiency in the model, a straitforward consequence of market power in the intermediate sector.

Plugging equation (16) in equation (10) gives us equation

$$Y = NLA^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} \tag{17}$$

while plugging (17) in (13) we have:

$$w = N(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{18}$$

Profits are given, as a consequence of (16) and (15), by:

$$\pi = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}(\frac{1}{\alpha} - 1). \tag{19}$$

A higher labor supply implies a higher quantity of each intermediate goods and thus higher profits in equilibrium. This means there is an externality to labor in the model, because when deciding labor supply workers will not take into account this positive effect on profits. So a tax program leading to increasing L can increase welfare by reducing the inefficiency due to monopolistic conditions. In section 6 we show formally that in this market economy employment is always lower than its efficient level.

The cost of development of new products is η and there is free entry in the market for inventions. Intermediate goods firms will push the price of a patent to equate its cost. Here a second inefficiency in the model appears, which is due to an appropriability problem: only the discounted value of profits, as opposed to all of social surplus originating from an invention, is taken into account when deciding whether to pay for research leading to innovation, so that its pace will be too low.

If we drop the j index in V, (14) can be written as the Hamilton-Jacobi-Bellman equation:

$$r = \frac{\pi}{V} + \frac{\dot{V}}{V} \tag{20}$$

which allows us to interpret it from an asset pricing perspective. The return on holding a blueprint, rV, is given by dividends π, plus the capital gains, i.e. the change in its value V. In the appendix, we show that, in a growing economy, we must have $V = \eta$ in equilibrium at all times, while $\pi_n = 0$.[6] But if $V = \eta$ at all times, (20), given (19), implies that in equilibrium we will have:

$$r = C_1 L \tag{21}$$

[6] Our proof is an extension to the case of a variable L, to the one offered in Acemoglu (2009) for the case of a fixed L.

with

$$C_1 \equiv \frac{1}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha).$$

The higher is labor supply the higher is the interest rate. As the sales of each intermediate good and therefore profits are increasing in labor supply, for their present discounted value to be equal to the given cost of an invention, the interest rate will have to increase.

2.3 Government

We assume government consumption G equals a fixed fraction, g, of gross aggregate output: $G = gY$. We rule out a market for government bonds and assume that the government runs a balanced budget. As part of the revenue from income taxes is transferred back to in equilibrium we have:

$$r\tau_r F + \tau_w wL = (t + g)\, Y \tag{22}$$

where on the left-hand side we have inflows and on the right-hand side we have outflows. Our assumption of a given g is made mainly for convenience but the public expenditure components that might be seen as exogenous in actual economies (from public wages, the payments of interest on public debt etc.) are far from zero and have remained fairly stable, as a percentage of output, over the last decades. Marrero and Novales (2007) document this and show that factor income taxes may be preferable to lump-sum taxes under the assumption of a given g, as the former allow an internalization of the fact that higher income will lead to extra public spending. This simple effect is also at work in our model.

2.4 Market Equilibrium

In calculating the equilibrium in the final goods market, intermediate goods used in production, xN, are subtracted from final production Y to obtain total value added. All investment in the model is investment in research and development of new intermediate goods $\eta\dot{N}$. The economy-wide resource constraint is therefore given by:

$$Y - xN = C + \eta\dot{N} + gY. \tag{23}$$

We are now ready for the following:

Definition 1 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households (6), (7), (8) and (9), the profit maximization conditions for firms in the final goods sector, (12) and (13) (or 18), and for firms in the intermediate goods sector, (15) (or 16) and (21), with the government budget constraint (22) and with the market clearing conditions for labor ($H = L$), for wealth ($F = VN$), and for the final good, (23).*

The following relationship between before-tax labor income and before-tax capital income holds in equilibrium:

$$wL/rF = \frac{1}{\alpha}. \tag{24}$$

From (22) and (24) we can then infer that:

$$\tau_w = \frac{t+g}{1-\alpha} - \alpha\tau_r. \tag{25}$$

From the definition of equilibrium we can now arrive at the following:

Proposition 2 *The competitive equilibrium conditions in the model give rise to the following differential equation for labor:*

$$\dot{L} = \frac{B(L)}{A(L)} \tag{26}$$

where

$$A(L) \equiv \left(\frac{\sigma h''}{h'} + \frac{h'}{h}(1-\sigma)\right) \tag{27}$$

and

$$\begin{aligned} B(L) \quad &\equiv \quad \frac{\sigma}{\alpha}C_1\frac{h(1-\sigma)}{h'}\left(1+\alpha\tau_r - \frac{g+t}{(1-\alpha)}\right) + \\ &\rho + C_1L\left(\tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right)\right). \end{aligned} \tag{28}$$

Proof. See appendix. ■

Given the definition of B in (28) we can write:

$$B(L) \equiv \sigma\left(g_N(L) - g_C(L)\right)$$

with

$$g_N(L) \equiv \frac{1}{\alpha}C_1\frac{h(1-\sigma)}{h'}\left(1+\alpha\tau_r - \frac{t+g}{(1-\alpha)}\right) + C_1L\left(1+\frac{1}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)\right)$$

and

$$g_C(L) \equiv \frac{C_1L\left(1-\tau_r\right)-\rho}{\sigma}.$$

The two curves g_N and g_C,represented in figure 1, admit an economic interpretation which will be useful when considering the effect of taxes. The g_N curve represents the equilibrium rate of increase in N for a given L, while g_C represents the equilibrium rate of increase in C, for a given L. That g_C slopes up is obvious, that the same is true for g_N as well is shown in Appendix A. g_C always has a constant slope, while the curvature of g_N depends on $(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)$, which is not possible to sign in general. In the figure the slope of g_N is taken to be a constant as well. In a steady state g_C and g_N intersect. More formally:

Proposition 3 *The condition for the existence of a BGP equilibrium in this model in which all variables grow at the same rate is that (26) has a fixed point $\widetilde{L}$ between 0 and 1, implicitly defined by $B(\widetilde{L}) = 0$, consistent with the TVC and with a positive growth rate γ for capital and consumption given by:*

$$\gamma = \frac{C_1 \widetilde{L}(1-\tau_r) - \rho}{\sigma}. \tag{29}$$

Proof. From (7) and (18), in a BGP, i.e. when $\dot{L} = 0$, C and N will grow at the same rate. From (8) this is seen to be given by (29). ■

Restrictions on parameters ensuring existence of a BGP equilibrium will be considered after introducing a specific form for the function h. However for the general case we can establish some interesting results on the uniqueness and stability of the BGP, assuming existence. First of all , given the definition of B in (28) we have:

$$B'(L) \equiv \frac{\sigma}{\alpha} C_1 \left(1 + \alpha\tau_r - \frac{g+t}{(1-\alpha)}\right)(1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right) \tag{30}$$
$$+ C_1 \left(\tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right)\right).$$

We can now state the following:

Proposition 4 *If $\tilde{L}$ defined by $B(\tilde{L}) = 0$ exists, while $\sigma > 1$ or $\sigma < 1$ and $\tau_w < \frac{1-\alpha}{1-\sigma} + \frac{\alpha\tau_r}{(1-\sigma)} + \frac{\sigma}{1-\sigma}\left(1 - \frac{g}{(1-\alpha)}\right)$ then $B'(\tilde{L}) > 0$. This implies the BGP equilibrium is unique and locally determinate, and there is no transitional dynamics to it.*

Proof. See Appendix A. ■

In our calibrations α goes from 0.40 to 0.50, while σ is generally taken to be higher than 0.5. As the necessary conditions for $B'(\tilde{L})$ negative require somewhat unrealistic parameters' values (in particular very high τ_w, α and g and very low σ), from now on we concentrate mainly on the case of a determinate and unique BGP equilibrium. In figure 1 we therefore represent the determinacy case, ie the case in which the g_C curve is flatter than the g_N curve when the two intersect.

3 Effects of Taxes

3.1 Effect on labor

It is relatively simple to calculate the effect of taxes on employment in this model because the wage rate does not vary with employment. As said above equilibrium labor supply $\tilde{L}$ can be expressed as the solution to $B(\tilde{L}) = 0$. The effect of shifting the tax burden from labor to capital can be deduced by using the total derivative of $B(\tilde{L}) = 0$ with respect to labor and the tax (τ_r). This gives us:

$$\frac{\tau_r dL}{\tilde{L} d\tau_r}|_{L=\tilde{L}} = \frac{\frac{\tau_r C_1 \sigma}{\tilde{L}}\left(\frac{(\sigma-1)h}{h'} - \frac{\tilde{L}}{\sigma}\right)}{B'(\tilde{L})}. \tag{31}$$

With $B'(\tilde{L}) > 0$, the case on which we focus, this derivative signs as the numerator of the fraction, while in the appendix we show that the TVC can be rewritten as:

$$\frac{(\sigma-1)h}{h'} > \tilde{L}. \tag{32}$$

This is, in light of (7), the well known condition that consumption must be higher than labor income for dynamic efficiency. For $\sigma > 1$, we can easily see that we will always have $\frac{d\tilde{L}}{d\tau_r} > 0$. We are therefore ready to state the following:

Proposition 5 *An increase in the tax rate on capital income whose proceeds are used to reduce the tax on labor income will increase employment, given determinacy, if and only if* $\frac{(\sigma-1)h}{h'} > \frac{\tilde{L}}{\sigma}$.

To interpret the condition we can use figure 2. Given determinacy we know that the g_C curve is flatter than the g_N curve when the two intersect. When τ_r goes up, g_{C_1} goes down by $\frac{C_1\widetilde{L}}{\sigma}$ (becomes g_{C_2} in the figure). The g_{N_1} goes down by $C_1\frac{h(1-\sigma)}{h'}$and becomes g_{N_2}. For the tax program to increase labour we want the shift in the first curve to be smaller than the shift in the second curve. If $\sigma > 1$ this will always be the case. Also the bigger, coeteris paribus, is σ, the bigger the difference in shift will be. In fact a big σ makes the shift in g_C smaller, for given $\widetilde{L}$: the less people are willing to substitute consumption intertemporally, the less will they respond to a change in the decrease in the return to their savings.

Moreover we notice that the compensated (Frisch) elasticity of labor supply with respect to the wage, ε_F, is given by:

$$\varepsilon_F = \frac{1}{L(\frac{h"}{h'} + \left(\frac{1}{\sigma} - 1\right) h^{-1}h')}. \tag{33}$$

The partial derivative $\partial\varepsilon_F/\partial\sigma = \frac{L\varepsilon_F^2 h^{-1}h'}{\sigma^2}$ is positive if $h' > 0$, i.e. if $\sigma > 1$, so an increase in the net wage will produce a stronger effect on employment the higher is σ. Intuitively if $\sigma > 1$, then $U_{CL} > 0$, i.e. leisure and consumption are substitutes, so not only lowering the tax on labor (with no income effect on impact given the redistributive nature of the program) will make for an increase in labor due to the substitution effect, but also increasing the tax on capital ie lowering the relative price of consumption today in terms of consumption tomorrow, will make consumption more attractive and leisure less attractive, further pushing labor up.

For $\sigma < 1$, leisure and consumption are complements, so the increase in the tax on capital could in theory lead to more, rather than less, leisure.

It may be interesting to note however that the data for developed countries tend to show that the higher the tax on labor income is the lower the yearly hours worked per adult are (see Ohanian et al. 2008 and references therein).

3.2 Effect on Growth

The growth effect of an increase of τ_r (and a correponding decrease in t_w), given (29) is:

$$\frac{d\gamma}{d\tau_r} = \frac{r}{\sigma}\left(\frac{(1-\tau_r)}{\tau_r}\frac{\tau_r d\tilde{L}}{\tilde{L} d\tau_r} - 1\right). \tag{34}$$

Not surprisingly the condition for the tax change to be growth increasing is stricter than the condition for it to be employment increasing, because for growth to increase we need the net interest rate to increase not just the gross interest rate, which is a linear function of the employment rate.

Focusing on the case of a determinate equilibrium the condition for the tax to be improving growth (just by combining (31) and (34) is:

$$(1-\tau_r)\frac{C_1\sigma}{\tilde{L}}\left(\frac{(\sigma-1)h}{h'} - \frac{\tilde{L}}{\sigma}\right) - B'(\tilde{L}) \geq 0. \tag{35}$$

We will see that this is different from the condition for the tax to improve welfare.

3.3 Effect on Welfare

Given γ, the BGP rate of growth, and $\widetilde{L}$ the BGP labor supply, it is possible to calculate maximum lifetime utility V along a balanced growth path:

$$V = \int_{t=0}^{\infty} e^{-[\rho-\gamma(1-\sigma)]t}\left(\frac{1}{1-\sigma}C(0)^{1-\sigma}h(\tilde{L})\right)dt. \tag{36}$$

In the appendix B it is shown how to express V as the following differentiable function of the tax rate τ_r and of equilibrium employment $\tilde{L}$ (itself a function of τ_r):

$$V = \frac{(\eta N(0))^{1-\sigma}}{1-\sigma}\frac{\left(\frac{\sigma-1}{h'(\tilde{L})}\frac{C_1(1+\alpha\tau_r - \frac{t+g}{1-\alpha})}{\alpha}\right)^{1-\sigma} h^{2-\sigma}}{\frac{C_1\tilde{L}(1-\tau_r)(\sigma-1)+\rho}{\sigma}}. \tag{37}$$

The effect on welfare of an increase in τ_r is then positive if $\frac{dV}{d\tau_r}$ is positive. To simplify calculations, we consider the following monotonically increasing transformation of V: $\frac{\log[(1-\sigma)V]}{1-\sigma}$. $\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r}$ signs as $\frac{dV}{d\tau_r}$ but is easier to manipulate algebraically so we will use it. We have:

$$\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r} = \frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tilde{L}}\frac{d\tilde{L}}{d\tau_r} + \frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r}. \tag{38}$$

In Appendix B we derive the following:

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tilde{L}} = -\frac{h''}{h'} + \frac{(2-\sigma)h'}{(1-\sigma)h} + \frac{C_1(1-\tau_r)}{C_1\tilde{L}(1-\tau_r)(\sigma-1)+\rho} \tag{39}$$

and

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r} = \frac{\alpha}{1-\tau_w} - \frac{r}{r(1-\tau_r)(\sigma-1)+\rho}. \tag{40}$$

Substituting (39), (31) and (40) in (38), we arrive at the following :

Proposition 6 *The sufficient and necessary condition for an increase in the tax rate on capital income whose revenue is used to reduce the tax on labor income to improve welfare is:*

$$\frac{C_1}{B'(\tilde{L})}\left(-\frac{h''}{h'} + \frac{(2-\sigma)h'}{(1-\sigma)h} + \frac{C_1(1-\tau_r)}{C_1\tilde{L}(1-\tau_r)(\sigma-1)+\rho}\right)\left(\frac{\sigma(\sigma-1)h}{h'} - \tilde{L}\right) + \tag{41}$$

$$\left(\frac{\alpha}{1-\tau_w} - \frac{r}{r(1-\tau_r)(\sigma-1)+\rho}\right) \geq 0.$$

If a value for τ_r exist such that for this τ_r (41), which signs as the derivative of welfare with respect to τ_r, is zero while it is strictly positive for lower tax rates, the expression in (41) gives us an implicit expression for the optimal tax rate, given the tax program.[7]

The condition for the tax program to increase growth is different from the condition for it to increase welfare, so the possibility is open that increasing the tax on capital, thus lowering growth may be a Pareto improvement. This result goes against the widely held belief that when growth is suboptimal, as we prove below is the case in this model even with no taxes on capital, further decreasing it cannot possibly increase welfare, no matter what static gains would go with the reduction, as the adverse growth effects always prevail by compounding over time. However, in the next section we will show that our surprising finding is more than a theoretical possibility and that for specifications of tastes and technology parameters commonly used in calibration exercises it is possible for the tax program to induce Pareto improvements but reduce growth. In fact while raising a tax on savings will often reduce growth in our simulations, not to tax them is generally inefficient.

[7] Solving the Ramsey problem by chosing the instrumental variables (here the tax rates) that maximize the indirect utility functions derived by the private agents reaction in a decentralized economy is known as the dual formulation.

3.4 Model Specification and Calibration

We consider here the following class of functions for the disutility of labor:

$$h(L) = (1-L)^{1-\chi} \tag{42}$$

where either $\chi > 1$ and $\sigma > 1$ or $\chi < 1 < \chi + \sigma$ and $0 < \sigma < 1$. First we notice that when h is specified as in (42), from (28), $B(\tilde{L}) = 0$ can be written as:

$$\tilde{L} = \frac{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w) - \frac{\rho}{C_1}}{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w) + \tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right)}. \tag{43}$$

$\tilde{L}$ as in (43), will be equal to employment in a BGP equilibrium if it is positive, less than 1, consistent with positive growth and with the TVC. Using (42), from (30) the condition for determinacy is seen to be:

$$B'(\tilde{L}) \equiv \frac{\sigma}{\alpha}C_1(1-t_w)\frac{(\sigma-1)}{(\chi-1)} + C_1\left(\tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right)\right) \geq 0. \tag{44}$$

We also have:

Proposition 7 *Necessary and sufficient conditions for the existence of a determinate equilibrium with positive growth are that* $\frac{\sigma}{\alpha}\frac{(1-t_w)(\sigma-1)}{(\chi-1)} + \tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right) > 0$ *and that* $\frac{\rho}{C_1}$ *belongs to the open interval* (a,b) *with*

$$a = \max\left\{1 - \sigma - \tau_r - \frac{\sigma}{\alpha}\frac{1-\alpha-g}{(1-\alpha)}, \frac{\frac{\sigma}{\alpha}\frac{(1-t_w)(1-\tau_r)(\sigma-1)^2}{(1-\chi)}}{\frac{\sigma}{\alpha}\frac{(\sigma-1)(1-t_w)}{(\chi-1)} + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right) + \tau_r\sigma}\right\}$$

and

$$b = \min\left\{\frac{\sigma}{\alpha}\frac{(\sigma-1)(1-t_w)}{(\chi-1)}, \frac{\frac{(\sigma-1)}{(\chi-1)}(1-t_w)(1-\tau_r)}{\frac{(\sigma-1)}{(\chi-1)}(1-t_w) + \alpha + 1 - \frac{g}{(1-\alpha)}}\right\}.$$

Proof. See Appendix B ■

For the sake of completeness we also consider the indeterminacy case, only possible when $\sigma < 1$. We have:

Proposition 8 *Necessary and sufficient conditions for the existence of an indeterminate equilibrium with positive growth are that* $\frac{\sigma}{\alpha}\frac{(1-t_w)(\sigma-1)}{(\chi-1)} + \tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right) < 0$ *and that* $\frac{\rho}{C_1}$ *belongs to the open interval* (a', b') *with*

$$a' = \max\left\{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w), \frac{\frac{(\sigma-1)}{(\chi-1)}(1-t_w)(1-\tau_r)}{\frac{(\sigma-1)}{(\chi-1)}(1-t_w) + \alpha + 1 - \frac{g}{(1-\alpha)}}\right\}$$

and

$$b' = \min\left\{1-\tau_r-\sigma-\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right), \frac{\frac{\sigma}{\alpha}\frac{(\sigma-1)^2}{(1-\chi)}(1-t_w)(1-\tau_r)}{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)+\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)+\tau_r\sigma}\right\}.$$

Proof. See Appendix B. ■

From (37), given (42), we have:

$$V = \frac{(\eta N(0))^{1-\sigma}}{1-\sigma}\frac{\left(\frac{(\sigma-1)C_1(1+\alpha\tau_r-\frac{t+g}{1-\alpha})}{\alpha}\right)^{1-\sigma}\left(1-\tilde{L}\right)^{2-\chi-\sigma}}{\frac{C_1\tilde{L}(1-\tau_r)(\sigma-1)+\rho}{\sigma}} \tag{45}$$

By Proposition 6, a positive welfare effect of a redistribution of the tax burden from labor to capital, given (42), and considering (44), requires:

$$\frac{\left(\frac{2-\sigma-\chi}{(1-\sigma)(1-\tilde{L})}-\frac{C_1(1-\tau_r)}{C_1\tilde{L}(1-\tau_r)(\sigma-1)+\rho}\right)\left(\frac{\sigma(\sigma-1)(1-\tilde{L})}{(1-\chi)}+\tilde{L}\right)}{\frac{\sigma}{\alpha}\frac{(1-\sigma)}{(1-\chi)}\left(1+\alpha\tau_r-\frac{g+t}{(1-\alpha)}\right)-1+\tau_r+\sigma+\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)} \tag{46}$$
$$+\frac{\alpha}{1+\alpha\tau_r-\frac{g+t}{1-\alpha}}-\frac{r}{r(1-\tau_r)(\sigma-1)+\rho}\geq 0.$$

(37) To calculate the optimal asset income tax we plug in (46) the expression for $\tilde{L}$ given by (43) and we equate the result to zero. The root of the non linear equation in τ_r we obtain gives us the optimal value of the tax, for each six-tuple of parameters $\{\sigma,\alpha,g,\rho,\chi,C_1\}$. We then check that the inequality in (46) holds strictly for values of τ_r lower than the root so found. For all the parameterizations we consider, the expression on the LHS of the inequality in (46) is always decreasing in τ_r for $0\leq\tau_r\leq 1$, so the stationary point of the welfare function we find by equating the expression to zero does indeed correspond to a maximum.

We now use (46) to calculate the optimal tax rates for reasonable values of the parameters. [8]We are completely aware that this model is not rich enough in number of variables to fit the data well. So the aim of our exercise cannot be the finding of precise quantitative results, but rather the understanding of possible mechanisms of action of policy not noticed before in the literature.

Several objects needed for the calculations have closed real-world counterparts so their calibration is relatively straightforward, while our other choices in feeding numbers to the model follow related studies (in particular Comin and Gertler 2006, Jones and Williams 2000, Strulik 2007 and Zeng and Zhang 2007).

[8]The matlab code for generating results is available at www.alessandrapelloni.it.

First, we set values for the the 7-tuple $\left\{\gamma, \rho, \tilde{L}, \sigma, \alpha, g, \tau_r, t\right\}$. These values imply values for r and C_1 (through 29), for τ_w (through 25), and for χ (through 43). We then solve (46), given the values $\{\rho, \sigma, \alpha, g, \chi, C_1\}$.

For the intertemporal elasticity of substitution and time preference parameter we follow Zeng and Zhang (2007) and set $\sigma = 1.5$, $\rho = 0.04$ in our baseline economy. The former is closer to the value used in DSGE models of OECD economies than to the estimates of the parameter, which tend to be lower than unity (see Alan and Browning 2010 for a recent study).[9] As a robustness check we consider $\sigma = 2$ and $\sigma = 0.9$. As in most studies we set our central value for the rate of time discount ρ equal to 0.04 and alternatively consider 0.03 and 0.05. Coming to labor supply, in 2005 the average US worker used 21 percent (24 percent) of her (his) time endowment to work, while the German one 13 percent.[10] So we choose 0.17 as our benchmark value and use $\{0.13, 0.21\}$ for our sensitivity analysis.

Coming to the the value of $1/\alpha$, which is the monopoly markup on intermediates, we infer it from the ratio of intermediate consumption to gross output, which is α^2 in our model. The US intermediate consumption takes up around 0.45 of gross output, hence the mark-up $1/\alpha$ is set at 1.49. This value exceeds the range [1.05,1.37] used by Jones and Williams (2000) but is lower than the 1.6 used by Comin and Gertler (2006). The latter note that while direct evidence is missing, given the specialized nature of these products an appropriate number for $1/\alpha$ would be at the high range of the estimates of markups for other types of goods in the US.[11] The alternative values we consider for the ratio of intermediate consumption to gross output are $\{0.40, 0.50\}$.

For the initial growth rate, we use 2 percent, as the values used in related researches include 1.25 percent (Jones and Williams 2000), and 3 percent (Zeng and Zhang 2007).

The unweighted average of total tax revenue over GDP in OECD countries in 2005 was 35 percent. In the US the percentage was 27.1 and in France 44.1.[12] We define this ratio as the variable $T_N \equiv (g + t)(1 - \alpha^2)^{-1}$, and take 35 percent as our benchmark for it, using 44 and 27 percent in our alternative parameterizations. Government final consumption as a percentage of GDP $G_N = g(1 - \alpha^2)^{-1}$ represents in OECD countries around 40 percent of total

[9] A logarithmic specification is often adopted for the period utility function, so as to match the observed variability of output, working hours, and investment observed in the US economy, while a value of 1.6 is estimated by Smets and Wouters (2003) for European economies.

[10] Source: US Bureau of Labor Statistics, current Population Survey, March 2005. For further discussion see chapter 2 of Borjas (2010).

[11] Zeng and Zhang (2007) assume a benchmark value of α at 0.3, leading to a mark-up as big as 3.33. Cross-country comparisons show that in some other OECD countries the estimated markup value is higher than in the US. Neiss (2001) estimates for 24 OECD countries the mean of the markup to be 2.03 with standard deviation 0.78.

[12] Source: OECD Tax Database.

Government expenditure so we take G_N to be equal to 10 percent and let it vary between 5 and 15 percent.

For our baseline case we consider an initial capital income tax rate of 25 percent, close to the average tax rate on capital income estimated by McDaniel for the US in the period 1995-2007.

Our choices and results as regards the baseline economy are summarized in Table 1:

Table 1: Baseline Economy: Parameterization and Results

Parameters and Steady State Variables Set	Value
rate of time discount: ρ	0.04
initial labor: $\tilde{L}$	0.17
intermediate consumption to gross output ratio: α^2	0.45
intertemporal elasticity of substitution (inverse): σ	1.5
tax revenue to GDP ratio: T_N	0.35
government consumption to GDP ratio: G_N	0.10
initial capital income tax rate: τ_r	0.25
initial GDP per capita growth: γ	0.02
Steady State Variables under Optimal Taxation	
optimal capital income tax rate:$\widehat{\tau}_r$	0.29
optimal labor income tax rate: $\widehat{\tau}_w$	0.39
optimal labor: $\hat{L}$	0.175
optimal growth: $\hat{\gamma}$	0.019
change in welfare using $\widehat{\tau}_w$	0.082

A first comment is that the capital income tax rate associated with maximum utility $\widehat{\tau}_r$, at 28.81 percent is even higher than the initial rate of 25 percent. So one can say that the prescription arising from our simple model is more or less in line with the levels of capital income taxation observed in the real world. Under our scheme, an increase in welfare is consistent with a negative growth effect. This is especially interesting because in this model the market equilibrium generates an inefficiently low growth rate (as shown in next section), while there is a generally shared view that growth effects always tend to prevail over level effects, as regards their impact on welfare. This is definitely not the case here.

We now move to some sensitivity analysis, so as to clarify the role of the various parameters . Our alternative parameterizations and results are reported in Table 2.

Table 2: Sensitivity Analysis

	$\widehat{\tau}_r$	$\widehat{\tau}_w$	$\widehat{L}$	$\widehat{\gamma}$	$\Delta W/\lvert W\rvert$
α^2=0.40	0.26	0.40	0.17	0.019	0.06
α^2=0.50	0.32	0.37	0.18	0.018	0.10
σ=0.9	0.15	0.48	0.16	0.023	0.01
σ=2	0.36	0.34	0.18	0.017	0.18
L=0.13	0.31	0.37	0.14	0.018	0.09
L=0.3	0.23	0.43	0.30	0.020	0.05
ρ=0.03	0.28	0.40	0.17	0.019	0.08
ρ=0.05	0.31	0.38	0.18	0.018	0.08
T_N=0.27, G_N=0.1	0.17	0.34	0.16	0.022	0.02
T_N=0.44, G_N=0.1	0.43	0.45	0.20	0.015	0.22
T_N=0.35, G_N=0.05	0.30	0.39	0.18	0.018	0.08
T_N=0.35, G_N=0.20	0.28	0.40	0.17	0.019	0.07
γ=0.01	0.33	0.36	0.18	0.008	0.09
γ=0.03	0.27	0.40	0.17	0.029	0.07

We can now draw a detailed map of the effects at work in delivering our results. We can see that the optimal τ_r is decreasing in the markup $1/\alpha$ and initial $\tilde{L}$ and increasing in σ, ρ and T_n.

First of all let us recapitulate the chain of reactions triggered by shifting the tax burden from labor to capital. On impact, lowering the tax on the wage while increasing the tax on interest income will cause labor supply to increase, because of the positive substitution effect, in the absence of an income effect, and assuming, as is empirically plausible, that the effect of the complementarity between consumption and leisure is not too strong. The increased labor supply induces a higher demand for the intermediate goods. Since the price of intermediate goods is greater than their marginal cost, increased demand for an intermediate good has a first order benefit for its inventor. This spillover from labor to profits is increasing in α (in fact the income share of capital is $\frac{\alpha}{1+\alpha}$). The increase in profits induces a higher demand for investment in R&D so the interest rate will rise. But the after-tax interest rate will in most cases be smaller than the interest rate with a zero tax on capital income. The BGP growth rate, as a monotonically increasing function of the after-tax interest rate, also decreases. As in the model a positive externality is associated with the invention activity driving growth, this decrease lowers welfare.

The parameter α also has an effect on the second externality in the model. The effect of an invention on the present discounted value of income is given by the cost of inventing divided by the income share of capital, that is $\eta\,(1+\alpha)\,\alpha^{-1}$, while the inventor only considers the part of the contribution to production that goes to capital income, that is η. The spillover here is represented by $\eta\alpha^{-1}$. Clearly this is decreasing in α: the higher the share of profits the lower the dynamic externality.

The tax shift from labor to capital helps to internalize the static spillover (positively related to α), but worsens the dynamic spillover (negatively related to α), a higher α makes for a higher optimal tax on capital income, through this double action.

To explain the role of σ in determining the optimal tax rates, again we must bear in mind that the advantage of pushing up the tax on capital and down the tax on labor is contingent on the policy increasing labor. A bigger increase in labor will make for a bigger reduction in the monopoly distortion and a relatively less important worsening of the appropriability failure. The increase depends on the Frisch elasticity of labor supply, equal to $\frac{\sigma(1-L)}{L(\sigma+\chi-1)}$given (42). This value is increasing in σ (when $\sigma > 1$) and decreasing in $\not L$ and χ. Since a lower labor means a higher Frisch elasticity, lowering the labor income tax will have a stronger effect on labor. Most of the values for ε_F implied by our calibrations are between 2 and 3. In particular, in the benchmark parametric space, the Frisch elasticity is 2.87. No consensus exists on a single number for the Frisch elasticity, as values used in macroeconomic calibrations to be consistent with observed fluctuations in employment over the business cycle are much larger than microeconometric studies would suggest. The values arising in our examples are within the range of the values used in the macro studies.[13]

Moreover, for a given effect of the tax program on the net interest rate, the higher is σ the lower will be the effect on the growth rate and therefore the less important the worsening of the dynamic inefficiency: a lower intertemporal substitution elasticity of consumption means consumers care relatively more about the current increase in consumption (which is lower than future consumption in a growing economy) than about the decrease in future consumption(which is higher). So, when the instantaneous consumption is increased along with employment this increment is given more weight than the future loss.

With higher subjective discount rate ρ, although consumption will grow at a lower rate with a higher tax on capital, this dynamic loss is discounted more heavily thus making for a higher optimal tax on capital income.

As to T_N, the ratio of government tax revenue to GDP, the higher it is, the higher the tax on wages (and the lower labor) will have to be for a given tax rate on capital.

[13] In King and Rebelo (1999) the needed elasticity is 4. This is also the value used by Prescott (2006) to explain differences in hours worked across OECD due to taxes. One explanation for this divergence between micro and macroestimates is that indivisible labor generates extensive margin responses that are not captured in micro studies of hours choices (e.g. Rogerson and Wallenius 2009). Imai and Keane (2004) find that the Frisch elasticity of labor supply may be as high as four, when taking into account that measured wages are less than the shadow wage because the second also reflects the value of on-the-job human capital accumulation. Finally Domeij and Floden (2006) point out that ignoring borrowing constraints will induce a (50%) downward bias in elasticity estimates.

Coming to the composition of the government budget we see that the lower is G_N, for given T_N, the lower the tax rate on capital will have to be. The presence of g (linearly related G_N) introduces a third externality in the model as agents when choosing how much to work and save do not take into account that a higher income will push up government consumption. Taxes correct this spillover, which, as g decreases, becomes relatively stronger for working than for saving. As for the effect of starting with a low γ that is easy to explain: low γ implies, coeteris paribus, a low interest rate, ie a low C_1. On its turn C_1 low, for given L, by (43) implies a low χ. This pushes up the Frisch elasticity and down the optimal tax on labor.

3.5 Comparison between the market economy and the social planner's economy

In this subsection we compare the social planner's equilibrium with the market equilibrium. Our main aim is to rule out that our result on the possibility that welfare is improved while the growth rate is reduced is due to the fact that the BGP growth rate in the market economy is higher than the social optimum.

Variables keep the same meaning as in the market economy, but the index s is used to show they characterize the social optimum. Let $X_s \equiv \int_0^{N_s} X_s(i)di$, where $X_s(i)$ is the amount of each type of the intermediate goods in the social planner's economy and X_s is the total amount produced of such goods. Then the final output in equilibrium can be expressed as

$$Y = AL_s^{1-\alpha} \int_0^{N_s} X_s(i)^\alpha di. \tag{47}$$

The Hamiltonian for the social planner's problem is:

$$J = \frac{C_s^{1-\sigma} h(L_s) e^{-\rho t}}{1-\sigma} + \frac{\mu}{\eta} \left(A(1-g) L_s^{1-\alpha} \int_0^{N_s} X_s(i)^\alpha di - C_s - \int_0^{N_s} X_s(i) di \right) \tag{48}$$

where μ is the Lagrangian multiplier attached to the social budget constraint. The social planner decides on the optimal path of the control variables L_s, C_s, and $X_s(i)$, and that of the state variable N_s. The key optimality conditions are:

$$X_s(i) = (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L_s; \tag{49}$$

$$C_s = \frac{(\sigma-1)h(L_s)}{h'(L_s)} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) N_s; \tag{50}$$

and

$$-\sigma \frac{\dot{C}_s}{C_s} + \frac{h'(L_s)}{h(L_s)} \dot{L}_s - \rho = \frac{\dot{\mu}}{\mu} = -\frac{1-\alpha}{\eta} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s. \tag{51}$$

In the balanced growth path, L_s is constant so $\dot{L}_s = 0$. From (51) we get:

$$\frac{\dot{C}_s}{C_s} = \frac{\frac{1-\alpha}{\eta}\left(A(1-g)\right)^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s - \rho}{\sigma}. \tag{52}$$

In equilibrium, the rate of return used by the social planner r_s is then:

$$r_s = \frac{1-\alpha}{\eta}\left(A(1-g)\right)^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s. \tag{53}$$

Substituting (49) into (47) we get

$$Y_s = A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s N_s. \tag{54}$$

The resource constraint can be expressed as:

$$\frac{\dot{N}_s}{N_s} = \frac{Y_s(1-g) - C_s - X_s}{\eta N_s} = \frac{(1-\alpha)\left(A(1-g)\right)^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s\left(1 - \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right)}{\eta} \tag{55}$$

where the second equality uses equations (49), (50) and (54). We use γ_s to denote the BGP growth rate in the centralized economy. In the BGP,

$$\frac{\dot{C}_s}{C_s} = \frac{\dot{N}_s}{N_s} = \gamma_s.$$

The transversality condition requires $0 < \gamma_s < r_s$, which, from (53) and (55) is equivalent to:

$$0 < \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1. \tag{56}$$

This is different from the analogous condition (32) in the market equilibrium. We exploit this difference to compare the steady state labor supply in the social planner's economy and that in the decentralized economy. Given our specification of the utility function in (42), $\frac{(\sigma-1)h(L)}{h'(L)L}$ equals $\frac{\sigma-1}{\chi-1}\frac{1-L}{L}$, which is a strictly decreasing function of L. But then $\frac{\sigma-1}{\chi-1}\frac{1-L_s}{L_s} < 1 < \frac{\sigma-1}{\chi-1}\frac{1-\tilde{L}}{\tilde{L}}$ (by 32 and 56), where $\tilde{L}$ is equilibrium employment in the decentralized economy. We deduce that the steady state labor supply in the social planner's economy is larger than in the market economy.

For optimal growth to be lower than growth in a market economy we would need $C_1\tilde{L}(1-\tau_r) > r_s$, and a fortiori, since $L_s > \tilde{L}$, $C_1 L_s(1-\tau_r) > r_s$, or using the definition of C_1 and (53) $\tau_r < 1 - \left(\frac{1-g}{\alpha}\right)^{\frac{1}{1-\alpha}}$. For realistic α and g this would require a negative τ_r.

4 Conclusions

This study analyses how the tax burden should be distributed betwen factor incomes in the "lab equipment" model of endogenous technological progress, thus complementing the study of fiscal policy in this same model by Zeng and Zhang (2007). Plausible calibrations of our model imply that the optimal tax rate on capital will not in general be zero. In fact,in many of the cases we consider the optimal tax rate is higher than 25 percent. Our analysis helps to make sense of the fact that in advanced economy tax rates on capital are generally well above the zero level generally recommended by the literature. In the model there are two inefficiencies, one related to the market power of firms, the second related to the appropriability problem related to the invention of new products. Shifting the tax burden from labor to capital has opposite effects on these two distortions. The increase in the interest income tax and the corresponding decrease in the labor income tax changes the opportunity cost of leisure without any change in disposable income, so labor supply will increase due to the substitution effect. Raising labor supply increases the quantity of goods produced by monopolistic firms so that the welfare cost of monopoly is reduced. For plausible calibrations of the model, the after-tax interest rate is decreasing in the tax rate on capital and so the growth rate goes down, ie the second distortion (which consists in an inefficiently low rate of growth even with a zeo capital income tax) is worsened. We have shown that the optimal tax on capital income is higher the higher the elasticity of labor supply, the lower the elasticity of intertemporal substitution in consumption, the lower the income share of labor, the higher the rate of time discount and the higher the ratio between goverment spending and income.

We find that the sign of the growth effect of a tax program is not necessarily the same as that of the welfare effect and that the two effects should be analysed separatedly, even in models when growth is sub-optimal.

In future research we plan to explore the generality of the result along two main directions: i.e. considering a richer tax structure that includes consumption taxes, and considering a model of vertical as well as of horizontal innovation. Further developments would be considering home production and the dependence of the marginal utility of leisure on its economy-wide average level.

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A Proofs for Section 2

A.1 Proof that $V = \eta$ in a growing economy.

$V > \eta$ is never possible because of the free entry assumption in the research market. On the other hand if $V < \eta$, no research would be done so that $\dot{N} = 0$, and from the economy-wide resource constraint we would have $Y - xN = C$, or, using (16) and (17),

$$C = \left(1 - \alpha^2\right) NLA^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}. \tag{57}$$

Plugging this, together with (13), in (7), the equilibrium level of employment would be implicitly given by:

$$\frac{h}{h'} = \frac{L\left(1 - \alpha^2 - g\right)}{(1-\alpha)(1-\tau_w)(\sigma - 1)}. \tag{58}$$

So if this equation had a solution for L between 0 and 1, this solution would define the equilibrium level of employment in a growthless economy, L_{ng}. Plugging L_{ng} in (57) and (19), the consumption level and the profit level in this growthless economy would also be given. With labor and consumption fixed over time, the Euler equation (8) implies an interest rate equal to $\frac{\rho}{1-\tau_r}$. Now suppose that $V = V_0 < \eta$. If $\frac{\rho}{1-\tau_r} - \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha} - 1)}{V_o} > 0$, or if, in other words $r - \frac{\pi}{V_0} > 0$, then, by (20), $\frac{\dot{V}}{V} > 0$. So V will increase and, since π and r will stay the same, $r - \frac{\pi}{V}$ will increase as well, ie $\frac{\dot{V}}{V}$ will be increasing. This implies that in finite time V will get to η, but then $\frac{\dot{V}}{V} > 0$ will be no longer possible. It would then become profitable to invest in inventions and growth would start. However this would require a jump in C and L (no longer dictated by 57 and 58) which would violate the equilibrium conditions of agents. In analogous fashion, if $\frac{\rho}{1-\tau_r} - \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha}-1)}{V_o} < 0$ that is if $r - \frac{\pi}{V} < 0$, V would be decreasing at an increasing rate, reaching the value 0 in finite time. If that happened (20) could not hold any longer. So again we would have a contradiction. Finally if $\frac{\rho}{1-\tau_r} = \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha}-1)}{V_o}$, then $V_o < \eta$ would be the equilibrium price of existing patents and the economy would never grow.

Summing up we can say that in a growing economy we must have $V = \eta$ at all times.

A.2 Proof of Proposition 2

Using the factor exhaustion condition that the wage bill plus total interest payments is equal to GDP, and the fact just established that growth requires $V = \eta$, we have $Y - xN - gY = wL + r\eta N$, $Y - xN - gY = C + \eta\dot{N}$, while substituting for C using equation (7), given the relationships between factor incomes (24) and between the tax rates (25), while expressing final income in terms of the interest rate by combining (21) and (17), we can write (23) as:

$$\frac{\dot{N}}{N} = \left(\frac{1}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right) + 1\right) r + \frac{h(1-\sigma)}{h'}\left(1 + \alpha\tau_r - \frac{t+g}{(1-\alpha)}\right)\frac{r}{\alpha L}. \quad (59)$$

Differentiating (7) with respect to time we obtain:

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}. \quad (60)$$

Plugging this expression for $\frac{\dot{C}}{C}$ in (8) we obtain:

$$\frac{\frac{h'}{h}\dot{L} - \rho + r(1-\tau_r)}{\sigma} - (h'/h - h''/h')\dot{L} = \frac{\dot{N}}{N}. \quad (61)$$

Finally if we substitute in (61) the expression for $\frac{\dot{N}}{N}$ given by (59) we obtain:

$$\dot{L} = \frac{\rho - r(1-\tau_r) + \sigma\left(\frac{1}{\alpha}\left(1 - \frac{g}{(1-\alpha)}\right) + 1\right) r + \frac{\sigma h(1-\sigma)}{h'}\left(1 + \alpha\tau_r - \frac{t+g}{(1-\alpha)}\right)\frac{r}{\alpha L}}{\frac{h'}{h} - \sigma(h'/h - h''/h')}$$

and using (21) we get (26) in the text.

A.3 Proof that g_N is upward sloping:

We have:

$$g\prime_N(L) = \frac{\sigma}{\alpha}C_1\left(1+\alpha\tau_r - \frac{g+t}{(1-\alpha)}\right)(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right) \tag{62}$$
$$+C_1\left(\sigma+\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)\right).$$

This is positive for all values of σ. When $\sigma < 1$, this is obvious as $h'' < 0$, so $(1-\sigma)\left(1-\frac{h''h}{h'^2}\right) > 0$.

When $\sigma > 1$ the sign of $1-\frac{h''h}{h'^2}$ and therefore of the first term on the right hand side of 62 is ambiguous. However, rearranging we have:

$$g'_N(L) = \frac{\sigma}{\alpha}C_1\left(1+\alpha\tau_r - \frac{g+t}{(1-\alpha)}\right)\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)$$
$$+C_1\sigma(1-\tau_r)+\frac{\sigma}{\alpha}C_1\frac{t}{(1-\alpha)}.$$

If $\left(1-\frac{h''h}{h'^2}\right) < 0$, obviously $1+(1-\sigma)\left(1-\frac{h''h}{h'^2}\right) > 1 > 0$, but even if $1-\frac{h''h}{h'^2} > 0$, by 5, $1-\frac{h''h}{h'^2} < 1-\frac{\sigma-1}{\sigma} = \frac{1}{\sigma}$, so $1+(1-\sigma)\left(1-\frac{h''h}{h'^2}\right) > 1+(1-\sigma)\frac{1}{\sigma} = \frac{1}{\sigma} > 0$. This completes the proof.

We have so established the following result which we will use in the next proof:

$$1+(1-\sigma)\left(1-\frac{h''h}{h'^2}\right) \geq 0, \sigma \lesseqgtr 1. \tag{63}$$

A.4 Proof of Proposition 4

The proof is divided into two parts. In the first part we prove that $B'(\tilde{L}) > 0$ implies uniquess and determinacy of the BGP, with no transitional dynamics to it. In the second part we prove that if $\sigma > 1$ or if $\sigma < 1$ and $\tau_w < 1 - \alpha(1-\tau_r)(1-\sigma)$, then $B'(L) > 0$ for all L, hence $B'(\tilde{L}) > 0$.

First Part

Any point of intersection, assuming it exists, between the two curves g_N and g_C, both continuous and differentiable, defines a vector (whose lenghth may be one) of BGP equilibria $\tilde{L}$. If at a point of intersection $B\prime > 0$ the g_N curve crosses the g_C curve from below. But a continuous function cannot cross another continuous function from below twice in a row. This establishes uniqueness of equilibrium given its existence if $B\prime(\tilde{L}) > 0$.

To study the dynamic nature of a fixed point of (26), i.e. of BGP labor supply, we have to sign $d\dot{L}(\widetilde{L})/d\widetilde{L}$. If this derivative is positive the fixed point $\widetilde{L}$ is a repeller and the BGP is locally determinate. If $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is negative then $\widetilde{L}$ is an attractor, i.e. there is local indeterminacy. $A(L)$ as defined in (27), is always strictly positive for all values of L, by the negative definiteness condition of the hessian of the utility function (4), so the differential equation (26) is defined for all values of L beween 0 and 1. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). So $B'(\tilde{L}) > 0$ implies $d\dot{L}(\widetilde{L})/d\widetilde{L} > 0$.

We have therefore established that if $B\prime(L) > 0$, the equilibrium value of L, $\widetilde{L}$, will be unique and unstable. This implies that no other value of L is consistent with the general equilibrium conditions. Since for a given L the ratio between C and N is given, from (7) and (18), this means that in this model the economy will always be on a BGP.

Second Part Given the definition of B in (28), taking the derivative we have:

$$\frac{B'(L)}{C_1} \equiv \frac{\sigma(1-\sigma)\left(1+\alpha\tau_r - \frac{g+t}{(1-\alpha)}\right)}{\alpha}\left(1-\frac{hh''}{(h')^2}\right) \tag{64}$$

$$+\tau_r - 1 + \sigma + \frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right).$$

When $\sigma > 1$ the sign of $1-\frac{h''h}{h'^2}$ and therefore of the first term on the right hand side of 65 is ambiguous. However, rearranging we have:

$$\frac{B'(L)}{C_1} = \frac{\sigma\left(1+\alpha\tau_r - \frac{g+t}{(1-\alpha)}\right)}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) \tag{65}$$

$$+(\sigma-1)(1-\tau_r) + \frac{\sigma}{\alpha}\frac{t}{(1-\alpha)}.$$

In light of 63, the first term on the right hand side of 65 will be positive. So with $\sigma > 1$, determinacy is established.

When $\sigma < 1$, $h'' < 0$, so $(1-\sigma)\left(1-\frac{h''h}{h'^2}\right) > 0$. However the second term on the right hand side of 64 could be negative, making indeterminacy theoretically possible. However considering that by 5 $1-\frac{h''h}{h'^2} > \frac{1}{\sigma}$, we see that:
$\frac{B'(L)}{\sigma} > \frac{1}{\alpha}C_1\left(1+\alpha\tau_r-\frac{g+t}{(1-\alpha)}\right)\frac{(1-\sigma)}{\sigma} + C_1\left(\frac{\tau_r-1}{\sigma}+1+\frac{1}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)\right).$

So indeterminacy would require (using 25): $\tau_w > \frac{1-\alpha}{1-\sigma} + \frac{\alpha\tau_r}{(1-\sigma)} + \frac{\sigma}{1-\sigma}\left(1-\frac{g}{(1-\alpha)}\right)$.

A.5 Proof that the TVC can be written as $\left(1+\frac{(1-\sigma)h}{h'\tilde{L}}\right)<0$.

The condition (9) implies that the BGP rate of growth, γ, is lower than $r(1-\tau_r)$. Using (59) to express γ the condition becomes:

$$0>\gamma-r(1-\tau_r)=r\left(\frac{1}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)+\tau_r\right)+\frac{h(1-\sigma)}{h'}\left(1+\alpha\tau_r-\frac{t+g}{(1-\alpha)}\right)\frac{r}{\alpha L}.$$

Notice that: $\left(\frac{1}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)+\tau_r\right)>\frac{1}{\alpha}\left(1+\alpha\tau_r-\frac{t+g}{(1-\alpha)}\right)=\frac{1-\tau_w}{\alpha}>0$. So for the inequality to hold $\frac{(\sigma-1)h}{h'\tilde{L}}>1$ is needed.

B Proofs for Section 3

B.1 Proof of equations 38 and 39

By solving the integral in (36) we obtain:

$$V=\frac{1}{1-\sigma}\frac{C(0)^{1-\sigma}h(\tilde{L})}{\rho-\gamma(1-\sigma)}.$$

By using (7), (21) and (25) we can write:

$$C(0)=\eta N(0)\frac{(\sigma-1)h}{h'}\frac{C_1\left(1+\alpha\tau_r-\frac{t}{1-\alpha}\right)}{\alpha}.$$

Using (29) we have:

$$\rho-\gamma(1-\sigma)=\frac{r(1-\tau_r)(\sigma-1)+\rho}{\sigma}.$$

Combining we get (37)in the text. Taking the log of V in (37) we get:

$$\frac{\log[(1-\sigma)V]}{1-\sigma}=\log(\eta N(0))+\log\left(\frac{\sigma-1}{h'}\right)+\log\left(\frac{C_1(1+\alpha\tau_r-\frac{t+g}{1-\alpha})}{\alpha}\right)+$$
$$\frac{2-\sigma}{1-\sigma}\log(h)-\frac{1}{1-\sigma}\log\left(\frac{r(1-\tau_r)(\sigma-1)+\rho}{\sigma}\right),$$

hence

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial L}=-\frac{h''}{h'}+\frac{(2-\sigma)h'}{(1-\sigma)h}+\frac{C_1(1-\tau_r)}{C_1L(1-\tau_r)(\sigma-1)+\rho}$$

which is (39) in the text. We also have:

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r}=\frac{\alpha}{1+\alpha\tau_r-\frac{t+g}{1-\alpha}}-\frac{r}{r(1-\tau_r)(\sigma-1)+\rho},$$

which is (38) in the text.

B.2 Proof of Proposition 7

For $\tilde{L}$ to be positive we need the numerator and denominator of the ratio on the RHS of (43) to be either both positive or both negative. Since, in a determinate equilibrium, the denominator is positive we need the numerator to be positive as well or $\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)-\frac{\rho}{C_1}\geq 0$. Positive growth, by (29) requires:

$$\frac{(1-\tau_r)\frac{(\sigma-1)}{(\chi-1)}(1-t_w)}{\frac{(\sigma-1)}{(\chi-1)}(1-t_w)+\alpha+1-\frac{g}{(1-\alpha)}}>\frac{\rho}{C_1}. \tag{66}$$

For $\sigma>1$ just from visual inspection of (43) $\tilde{L}$ is always less than 1 while the TVC is guaranteed to hold, so there are no related parametric restrictions to be imposed.

For $\sigma<1$, from (43), $\tilde{L}$ less than 1 requires $1-\sigma-\tau_r-\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)<\frac{\rho}{C_1}$ while the TVC (which dictates that the rate of growth of consumption is lower than the interest rate net of tax) requires $\frac{\left(\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)\right)(1-\tau_r)(1-\sigma)}{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)+\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)+\tau_r\sigma}<\frac{\rho}{C_1}$.

B.3 Proof of Proposition 8

For $\tilde{L}$ positive we need $\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)<\frac{\rho}{C_1}$, for growth positive by (29): $\frac{\frac{(\sigma-1)}{(\chi-1)}(1-t_w)(1-\tau_r)}{\frac{(\sigma-1)}{(\chi-1)}(1-t_w)+\alpha+\left(1-\frac{g}{(1-\alpha)}\right)}<\frac{\rho}{C_1}$.

For $\tilde{L}$ less than one: $\frac{\rho}{C_1}<1-\tau_r-\sigma-\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)$. For the TVC, we need $C_1(1-\tau_r)\tilde{L}(\sigma-1)>-\rho$, or given (43): $\frac{\rho}{C_1}<\frac{\frac{\sigma}{\alpha}\frac{(\sigma-1)^2}{(1-\chi)}(1-t_w)(1-\tau_r)}{\frac{\sigma}{\alpha}\frac{(\sigma-1)}{(\chi-1)}(1-t_w)+\frac{\sigma}{\alpha}\left(1-\frac{g}{(1-\alpha)}\right)+\tau_r\sigma}$.

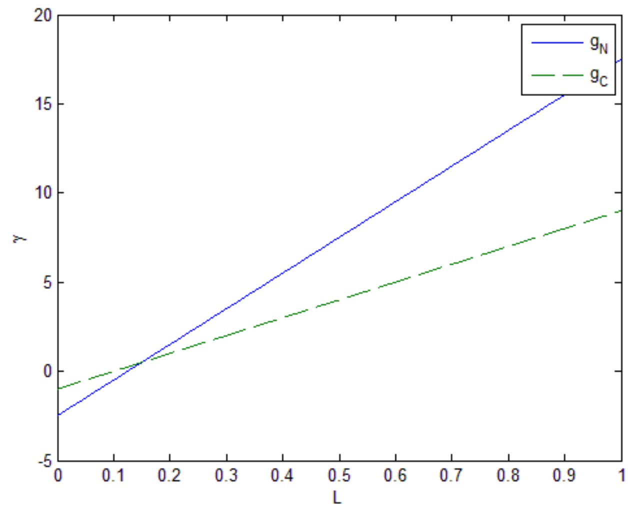

Figure 1: BGP equilibrium.

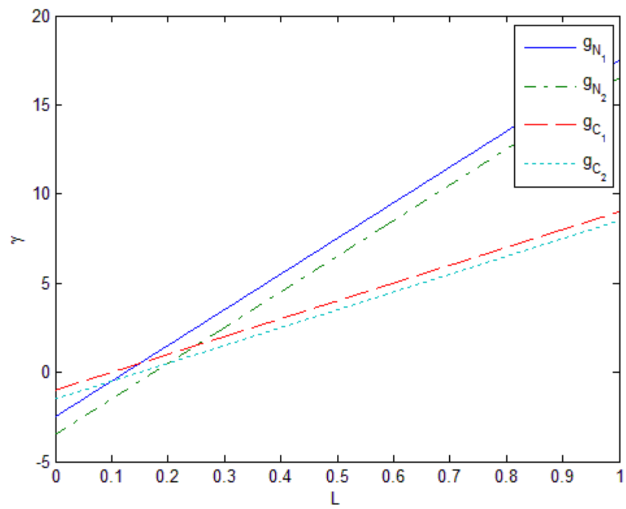

Figure 2: Effects of raising τ_k.