Analysis of Monetary Policy Responses After Financial Market Crises in a Continuous Time New Keynesian Model

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Abstract

We develop a dynamic stochastic full equilibrium New Keynesian model of two open economies based on stochastic differential equations to analyse the interdependence between monetary policy and financial markets in the context of the recent financial crisis. The effect of bubbles on stock and housing markets and their transmission to the domestic real economy and the contagious effects on foreign markets are studied. We simulate adjustment paths for the economies under two monetary policy rules: an open-economy Taylor rule and a modified Taylor rule, which takes into account stabilisation of financial markets as a monetary policy objective. We find that for the price of a strong hike in inflation a severe economic recession can be avoided under the modified rule. Using Bayesian estimation techniques, we calibrate the model to the case of the United States and Canada and find that the resulting economic adjustment paths are similar to those of the theoretical model.

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1 Introduction

Many OECD countries are still recovering from the worst financial and economic crisis since the Great Depression. One important lesson to be learned from this experience is that stable financial markets are a precondition for macroeconomic stability. Indeed, the crisis was a forceful reminder that there are important linkages not only between different domestic financial markets but also between international financial markets, meaning that shocks originating in one financial market in one country can generate huge spillovers to other financial markets in that same country as well as to financial markets in other countries. Particularly in North America and Europe, there has been an unprecedented use of monetary policy to stabilise financial markets and the real economy. However, this use of stabilisation policy generates spillovers to other countries too, a fact that is often ignored.

Continuing to act as though there is such a thing as purely national policy making in a globalised world could facilitate the spread of a crisis to other countries, with foreseeably unfortunate results, if history is any guide. We believe a better understanding of the influence of financial market spillovers, as well as of foreign monetary policy, is crucial for the appropriate conduct of national monetary policy. Thus, it is important to learn more about the consequences of policymakers' reactions to financial turmoil and how these vary based on the degree of importance they attach to domestic and foreign financial markets.

Although the recent financial crisis spurred the development of macroeconomic models that help us understand what happened, how to clean up the mess, and what we can do to avoid another crisis, most of these studies either use techniques from finance or macroeconomics (Aït-Sahalia, Cacho-Diaz, and Laeven 2010; Bekaert, Hoerova, and Lo Duca 2013; El-Khatib, Hajji, and Al-Refai 2013; Gertler and Karadi 2011; Gertler, Kiyotaki, and Queralto 2012).In this paper, we build a bridge between both approaches to achieve a better understanding of the effects of financial markets on real markets. We study spillover effects between financial markets as well as from financial markets to the real economy, both within one economy and across economies, and analyse the economic consequences of different monetary policy responses. For this purpose, we develop a dynamic stochastic full equilibrium New Keynesian model of two open economies based on stochastic differential equations. In our simulation analysis, we compare a standard open-economy Taylor rule that focuses on stabilising output, inflation, and the exchange rate to a modified Taylor rule that additionally takes into account financial market stabilisation.

We argue that academic research can aid central banks by analysing to what extent financial markets should be taken into consideration when formulating monetary policy. We thus explore the consequences of treating seriously the interaction between financial markets, monetary policy, and the real economy in a globalised world by developing a fully dynamic theoretical modelling framework. We are particularly interested in shedding light on the relationship between financial markets, financial crises, and monetary policy, which can be characterised by a substantial degree of simultaneity.

Our paper makes several contributions to the literature. First, we follow ball and derive the Taylor rule within the model by employing the nominal interest rate and the exchange rate as monetary policy targets. Moreover, we follow Bekaert, Cho, and Moreno (2010) and model a financial market sector, which allows consistent inclusion of financial markets in the policy rule. Faia and Monacelli (2007) provides empirical evidence that the inclusion of financial market variables in the Taylor rule has significant impact on actual decision-making processes. In a similar vein, Belke and Klose (2010) estimate Taylor rules for the European Central Bank (ECB) and the Federal Reserve (Fed) and include asset prices as additional monetary policy targets. Moreover, we account for simultaneity between monetary policy and financial markets by incorporating several financial markets (i.e., markets for foreign exchange, bonds, and stocks). The issue of simultaneity is empirically analysed by Bjornland and Leitemo (2009), Rigobon (2003), and Rigobon and Sack (2003). A theoretical discussion is given in hildebrand.

Second, we combine finance research with macroeconomic theory by switching to a continuous-time framework. This allows us to employ advanced techniques from the finance literature, such as jump-diffusion processes, in modelling financial markets. Technically, we transform the New Keynesian model into stochastic differential equations and compute solutions by using advanced numerical algorithms. We believe that forging this link between methods from economics, finance, and mathematics is a unique and valuable contribution.

Third, to discover whether our theoretical analysis has real-world value, we study the interaction between the United States and Canada. In this part of the analysis, we estimate model parameters using Bayesian estimation techniques and compare the simulated adjustment paths to those from our model based on *a priori* calibration.

The remainder of the paper is structured as follows. Section 2 derives the theoretical model. In Section 3, we study the effects of financial market turmoil using dynamic simulations based on a calibrated version of the theoretical model, whereas we employ empirically estimated parameters in Section 4. Section 5 concludes.

2 Derivation of the Theoretical Model

2.1 Placing the Model in the Literature

Our open-economy model starts with a typical New Keynesian (NK) approach, Blinder (1997), Clarida, Gali, and Gertler (1999), Romer (2000), and Woodford (1999) and, in line with Clarida, Gali, and Gertler (2002), incorporates the exchange rate. Extensions by Gali and Monacelli (2005) and Engels (2009) introduce Calvo pricing, which we adopt, too. Following Leitemo and Soderstrom (2005), we include exchange rate uncertainty in our NK model and analyse different monetary policy rules.

Bekaert, Cho, and Moreno (2010) discusses including the financial market as a sector in an extended NK model. Another attempt to include an advanced financial market is made by Brunnermeier and Sannikov (2012), albeit not in an NK model. Other related research concentrates on monetary policy transmission channels. For example, Cï¿½rdia and Woodford (2010), Curdia and Woodford (2008), and Woodford (2010), include the credit channel in the NK model, while Christiano, Motto, and Rostagno (2010) and Gertler and Kiyotaki (2010) model a banking sector as a financial intermediary.

The switch from discrete to continuous time is in line with papers by Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b), but has not to date been used in an NK approach. As argued in Hayo and Niehof (2013), using a continuous-time framework makes it possible to consistently include modern approaches in finance in our open-economy NK macroeconomic framework. To the best of our knowledge, this is a unique modelling approach.

2.2 Households

We apply the model in Hayo and Niehof (ibid.) but extend it with a more advanced IS curve and financial markets with the aim of analysing monetary policy transmission under various circumstances. Our core model is based on the New Keynesian three-equation model proposed by Ball (1998) (respectively Allsopp and Vines (2000), Blanchard (2007, 2008), Clarida, Gali, and Gertler (1999), and Gali and Monacelli (2005)) and inclusion of financial markets follows Bekaert, Cho, and Moreno (2010). Furthermore, in line with Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b), we switch to a continuous-time framework. Following rotemberg98, we consider a standard analytical framework with a representative household that is endowed with a continuum of goods-specific skill to be supplied to a differentiated product industry. Our derivation is based on Razin and Yuen (2002) in terms of procedure and notation.

The household operates as a consumer with access to domestic and foreign goods. Aggregate supply follows Ball (1998) and Blanchard (2007, 2008) and evolves from a Calvo pricing equation. In line with Ball (1998) and Benigno and Woodford (2005) a fraction of firms act on a 'rule of thumb' rather than engaging rational behaviour. Furthermore, we apply AR(1) processes as disturbances as in Jensen (2002) and Ramon and Vazquez (2006).

We assume that the economy is inhabited by a continuum of consumers/ producers $j \in [0,1]$. First, we consider a consumption index, such as that of Dixit and Stiglitz (1977), C_t (P_t) which consists of domestic goods $c_t(j)$ produced by firm j and foreign goods $c_t(j)^*$ produced by a foreign firm j, n a fraction of goods which are produced domestically, and θ, which is the price elasticity of demand for individual goods. Second, we define a production index, P_t, using in addition $p_t(j)$ and $p_t^*(j)$, the prices of the individual domestic and foreign goods, and ϵ, the nominal exchange rate.

$$C_t = \left[\int_0^n c_t(j)^{\frac{\theta-1}{\theta}} dj + \int_n^1 c_t^*(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

$$P_t = \left[\int_0^n p_t(j)^{1-\theta} dj + \int_n^1 (\epsilon p_t^*(j))^{1-\theta} dj \right]^{\frac{1}{1-\theta}} .$$

Households try to find the least expensive combination of individual goods $c_t(j)$. This minimisation problem can be written as

$$\min_{c_t(j), c_t^*(j)} \int_0^n p_t(j) c_t(j) dj + \epsilon \int_n^1 p_t^*(j) c_t^*(j) dj$$

Solving this equation by forming a Langrangian and deriving the first order conditions (FOC) reveals the typical characteristic of a Dixit-Stiglitz consumption index, namely

$$c(j) = C_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} ,$$

which describes the demand for good j. As $\theta \to \infty$, individual goods become closer substitutes and individual firms have less market power.

Households seek to maximise the discounted sum of expected utilities subject to a period-by-period budget constraint. Using a constant relative risk aversion utility function (CRRA), the representative household's lifetime utility can be written as

$$U = E \sum_{t=0}^{\infty} \beta^t \left(u \left(C_t, \frac{M_t}{P_t} \right) - \int_0^n v(h_t(j))dj \right)$$

where β is a subjective discount factor, and M denotes the money supply. $h(j)$ is the supply of type j-labour to the production of the good of variety j

$$P_t C_t = \int_0^n p_t(j) c_t(j) dj + \epsilon_t \int_n^1 p_t^*(j) c_t^*(j) dj$$

$$P_t \Pi_t = \int_0^n w_t(j) h_t(j) dj + \int_0^n \Pi_t(j) dj$$

$$P_t C_t + \left(\frac{i_t}{1+i_t} \right) M_t + B_t + \epsilon_t B_t^* = M_{t-1} + (1 - i_{t-1}) B_t + \epsilon_{t-1,t} (1 + i_{t-1}^*) B_t^* + P_t \Pi_t$$

where B (B^*) is the domestic (foreign) currency value of domestic (foreign) borrowing, $\epsilon_{t,t-1}$ the forward exchange rate and i (i^*) domestic (foreign) interest rates. $w(j)$ is the wage per unit labour of type j and $\Pi(j)$ profit income from firms of type j. Under perfect capital mobility, covered interest parity prevails:

$$1 + i_t = (1 + i_t^*) \left(\frac{\epsilon_{t,t-1}}{\epsilon_t} \right).$$

For simplicity, consumer utility is assumed to be separable with respect to consumption and real money balances. Maximising the expected utility under the budget constraint gives the Euler equation for the optimal temporal allocation of consumption. Thus, we solve

$$aE \sum_{t=0}^{\infty} \beta^t \left(u \left(C_t, \frac{M_t}{P_t} \right) - \int_0^n v(h_t(j))dj \right)$$
$$- \sum_{t=0}^{\infty} \lambda_t \left(P_t C_t + \left(\frac{i_t}{1+i_t} \right) M_t + B_t + \epsilon_t B_t^* - (M_{t-1} + (1 - i_{t-1}) B_t + \epsilon_{t-1,t} (1 + i_{t-1}^*) B_t^* + P_t \Pi_t \right)$$

and obtain the following FOCs

$$
\begin{aligned}
C_t &: u_c\left(C_t, \frac{M_t}{P_t}\right) - \lambda_t P_t = 0 \\
C_{t+1} &: E\left(\beta u_c\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)\right) - \lambda_{t+1} P_{t+1} = 0 \\
C_t^* &: u_c\left(C_t^*, \frac{M_t}{P_t}\right) - \lambda_t P_t^* = 0 \\
C_{t+1}^* &: E\left(\beta u_c\left(C_{t+1}^*, \frac{M_{t+1}}{P_{t+1}}\right)\right) - \lambda_{t+1} P_{t+1}^* = 0 \\
B_t &: -\lambda_t + E\left(\lambda_{t+1}(1+i_t)\right) = 0 \\
B_t^* &:= -\lambda_t \epsilon_t + E\left(\lambda_{t+1}\epsilon_{t,t+1}(1-i_t^*)\right)
\end{aligned}
$$

Using the FOCs, we can derive

$$
\beta(1+i_t)P_t E\left(\frac{u_c\left(C_t, \frac{M_t}{P_t}\right)}{P_{t+1}}\right) = \beta(1+i_t^*)\frac{\epsilon_{t,t+1}}{\epsilon_t}P_t^* E\left(\frac{u_c\left(C_t^*, \frac{M_t}{P_t}\right)}{P_{t+1}^*}\right).
$$

This yields the optimal conditions for the choice of labour supply and the consumption-saving decision

$$
\begin{aligned}
\frac{v_h(h_t(j))}{u_c(C_t)} &= \frac{w_t(j)}{P_t} \\
\frac{u_c(C_t)}{u_c(C_{t+1})} &= \beta(1+r^*)
\end{aligned}
$$

where r^* is the world real interest rate.

2.3 Firms

Firms minimise their costs by choosing the lowest possible amount of labour subject to producing the firm specific good $c_t(j)$

$$
\min_{h_t(j)} \int_0^1 \frac{w_t(j)}{p_t(j)} h_t(j) df
$$

and given the production function for the firm specific good

$$
y_t(j) = A_t f(h_t(j)),
$$

where A_t is a random productivity shock. Applying the Lagrangian and deriving FOCs yields real marginal cost s

$$s_t(j) = \frac{w_t(j)}{P_t A_t f'\left(f^{-1}\left(\frac{y_t(j)}{A_t}\right)\right)}$$

Using the optimal labour supply condition, this can be rewritten as

$$s(y, C, A) = \frac{v_h\left(f^{-1}\left(\frac{y}{A}\right)\right)}{u_c(C) A f'\left(f^{-1}\left(\frac{y}{A}\right)\right)}.$$

Trade-wise, price-setting firms face world demand for their products so that

$$c(j) = C_t \left(\frac{p_t(j)}{P_t}\right)^{-\theta} \text{ and}$$
$$y_t(j) = Y_t^w \left(\frac{p_t(j)}{P_t}\right)^{-\theta},$$

where $y(j)$ is the quantity of good j supplied by the firm to meet world demand, P_t is the aggregate price index, and $Y^w = Y^h + Y^f$ is the index for all goods produced around the world, with production indices for domestic and foreign goods

$$Y^h = \int_0^n \left(\frac{p_t(j) y_t(j)}{P_t}\right) dj \text{ and}$$
$$Y^f = \int_0^n \left(\epsilon \frac{p_t^*(j) y_t(j)}{P_t}\right) dj.$$

In a second step, firms maximise profits, i.e., the difference of selling the individual good, $c_t(j)$ minus the costs of producing this good $\pi_t c_t(j)$. Profit maximisation is achieved by setting prices $p_t(j)$ for the individual goods, subject to the relevant demand curve and the assumption that prices are sticky. Following Calvo (1983), in each period, a fraction γ of firms is not able to change its price and must continue with the price chosen in the previous period. This can be solved by maximising profit under optimal labour supply conditions. Flexible price firms set the price as a mark up $\frac{\theta}{\theta-1}$ over the marginal cost.

$$\frac{p_{1t}}{P_t} - \frac{\theta}{\theta - 1} s(y_{1t}, C_t, A_t) = 0$$

Inflexible price firms choose the price so as to maximise expected discounted profit.

$$E\left(\left(\frac{1}{1+i_{t-1}}\right)Y^w P_t^{\theta-1}\left(\frac{p_{2t}}{P_t}-\frac{\theta}{\theta-1}s(y_{2t},c_t,A_t)\right)\right)=0$$

This reveals the optimal price setting rule. In the extreme case of fixed prices $\gamma = 1$ (with P_t price index) this can be expressed as

$$P_t = \left(n\left(\gamma p_{1t}^{1-\theta}+(1-\gamma)p_{2t}^{1-\theta}\right)+(1-n)(\epsilon_t p_t^*)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$
$$\frac{p_t}{P_t} = \frac{\theta}{1-\theta}s(Y_t^n, C_t^n, A_t).$$

2.4 The New Keynesian Phillips Curve

To obtain a tractable solution, we log-linearise the equilibrium conditions around the steady state. For the derivation of the New Keynesian Phillips curve (NKPC) in a general equilibrium framework, we use firms' optimal price setting rule. In a steady state we find $\beta(1+r^*) = 1$. Moreover, in a *deterministic* steady state there is $A_t = A$, $\epsilon_t = \epsilon$, $p_t^* = p^*$, $C_t = C$. Every variable with a 'hat' indicates the proportional deviation from its deterministic state ($\hat{x_t} = \log(x_t/x) \approx \frac{x_t - x}{x}$).

$$s(y,C,A) = \frac{v_h\left(f^{-1}\left(\frac{y}{A}\right)\right)}{u_c(C)Af'\left(f^{-1}\left(\frac{y}{A}\right)\right)}$$
$$\hat{s}_t - \hat{s}_t^{\,n} = \omega(\hat{y}_t - \hat{Y}_t^{\,n}) + \sigma^{-1}(\hat{C}_t - \hat{C}_t^{\,n})$$
$$\omega = \left(\frac{v_{hh}(y/A)}{v_h f'} - \frac{f''(f^{-1}(.))(y/A)}{f'(f^{-1}(.))f'(.)}\right)$$
$$\sigma = -\left(\frac{u_{cc}c}{u_c}\right)$$

Applying this equation to price setting yields

$$\frac{p_{1t}}{P_t} - \frac{\theta}{\theta-1}s(y_{1t},C_t,A_t) = 0 \text{ and}$$
$$E\left(\left(\frac{1}{1+i_{t-1}}\right)Y^w P_t^{\theta-1}\left(\frac{p_{2t}}{P_t}-\frac{\theta}{\theta-1}s(y_{2t},c_t,A_t)\right)\right)=0.$$

Thus, we find that the following equations hold:

$$\begin{aligned}
\log(p_{1t}) &= \log(P_t) + \omega(\hat{y_{1t}} - \hat{Y_t}^n) + \sigma^{-1}(\hat{C}_t - \hat{C_t}^n) \\
\log(p_{2t} &= E(\log(P_t) + \omega(\hat{y_{2t}} - \hat{Y_t}^n) + \sigma^{-1}(\hat{C}_t - \hat{C_t}^n) \\
\log(P_t) &= n(\gamma \log(p_{1t} + (1-\gamma)\log(p_{2t})) + (1-n)\log(\epsilon_t p_t^*)
\end{aligned}$$

We define the inflation rate as $\pi_t = \log(P_t/P_{t-1})$ and the real exchange rate as $e_t = \epsilon_t \frac{P_t^*}{P_t}$. Log-linearising the demand function

$$\begin{aligned}
c(j) &= C_t \left(\frac{p_t(j)}{P_t}\right)^{-\theta} \\
y_t(j) &= Y_t^w \left(\frac{p_t(j)}{P_t}\right)^{-\theta}
\end{aligned}$$

yields

$$\hat{y_{jt}} = \hat{Y_t}^W - \theta(\log(p_{jt} - \log(P_t))$$

Substituting the equations and rearranging terms leads to

$$\begin{aligned}
\log(p_{1t} &= \log(P_t) + \left(\frac{\omega}{1+\theta\omega}\right)(\hat{Y_t}^W - \hat{Y_t}^n) + \sigma^{-1}\left(\frac{1}{1+\theta\omega}\right)(\hat{C}_t - \hat{C_t}^n) \\
\log(p_{2t}) &= E\left(\log(P_t) + \left(\frac{\omega}{1+\theta\omega}\right)(\hat{Y_t}^W - \hat{Y_t}^n) + \sigma^{-1}\left(\frac{1}{1+\theta\omega}\right)(\hat{C}_t - \hat{C_t}^n)\right) \\
&= E(\log(p_{1t})
\end{aligned}$$

Taking into account the price index and the unanticipated rate of inflation, this implies

$$\begin{aligned}
\log(P_t) - E(\log(P_t)) = &\left(\frac{\gamma}{1-\gamma}\right)(\log(p_{1t} - \log(P_t)) \\
&+ \left(\frac{1-n}{n}\right)\left(\left(\frac{1}{1-\gamma}\right)\log(e_t) - E(\log(e_t))\right)
\end{aligned}$$

Replacing $\log(p_{1t})$ yields in the open-economy NKPC

$$\pi_t - E(\pi_t) = \left(\frac{\gamma}{1-\gamma}\right)\left(\left(\frac{n\omega}{1+\theta\omega}\right)(\hat{Y}_t^h - \hat{Y}_t^n) + \left(\frac{(1-n)\omega}{1+\theta\omega}\right)(\hat{Y}_t^f - \hat{Y}_t^n)\right.$$
$$\left. + \left(\frac{1-n}{n}\right)\left(\left(\frac{1}{1-\gamma}\right)\log(e_t) - E(\log(e_t)\right)\right)$$

To simplify the expression, we write the NKPC in Equation 1 as

$$\pi_t = \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e(e_{t-1} - e_{t-2}) + \eta_t, \tag{1}$$

where π is the rate of inflation, y the output gap, e the exchange rate, α_is are weighing parameters, and η is a standard AR(1) process.

2.5 The Investment and Savings Curve

The aggregate demand curve follows ball and starts with a simple open economy. Furthermore, following Allsopp and Vines (2000) and Clarida, Gali, and Gertler (1999), we account for foreign output. As we are interested in financial market and monetary policy spillover effects in times of crises, we extend Hayo and Niehof (2013) by including asset prices in the investments and savings (IS) curve (Stracca 2010).

Technically, the IS curve is constructed in Equation 2 by deriving the Euler equation and log-linearising around the steady state.

$$\beta(1+i_t)P_t E\left(\frac{u_c\left(C_t, \frac{M_t}{P_t}\right)}{P_{t+1}}\right) = \beta(1+i_t^*)\frac{\epsilon_{t,t+1}}{\epsilon_t}P_t^* E\left(\frac{u_c\left(C_t^*, \frac{M_t}{P_t}\right)}{P_{t+1}^*}\right)$$

Rearranging the equations yields the IS curve

$$y_t = \lambda_y y_{t-1} - \lambda_i(i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y_{t-1}^* - \lambda_e e_{t-1} + \lambda_p p_{t-1} + \epsilon_t, \tag{2}$$

where y denotes the output gap, i the nominal interest rate, π the inflation rate, e the exchange rate, and p a linear system of financial market equations. ϵ is assumed to follow a standard AR(1) process and λ's are weighing parameters.

2.6 The Exchange Rate

Since our analysis is focussed on the short run, the long-run oriented purchasing power parity may not hold. However, we allow exchange rate adjustment to incorporate the uncovered interest parity condition and, in line with **mccallum94 and batini00**; Ball (1998), we model the exchange rate as a function of the nominal interest rate and inflation. To allow for an explicit analysis of exchange rate bubbles, we follow the more comprehensive approach in Batini and Nelson (2000) and add another state variable to reflect the potential burst of a bubble (see Equation 3.). This implies an explosive time series parameter such that a closed solution cannot be computed. A detailed description of the computation, length and value of the additional variable is provided by Batini and Nelson (ibid.).

$$e_t = \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i_t^* - \pi_t^*) + \psi_t + \varphi_t \quad (3)$$

In line with previous equations, we define the error term ψ_t as an AR(1) process.

2.7 The Financial Markets

We follow Bekaert, Cho, and Moreno (2010) and model assets in discrete time at first and switch to continuous time afterward. The pricing kernel is based on the IS equation (thus on consumption).

Duffie and Kan (1996) shows that state variable dynamics and pricing kernel processes in affine term structure models must be linear. Furthermore, shocks must be conditionally normal, which is a condition fulfilled by AR(1) processes. Moreover, all equations are linear in their dynamics and thereby satisfy the requirements for affine term structure models as well. We assume

$$E(M_{t+1}, R_{t+1}) = 1$$

for the pricing kernel process M and a n-asset R. If $M > 1$ the no-arbitrage condition is fulfilled (Bekaert, Cho, and Moreno 2010). Logarithmising the pricing kernel yields

$$ln(M_{t+1}) = -i_t - \frac{1}{2}\Lambda_t' D_t \Lambda_t - \Lambda_t' \epsilon_{t+1}$$

Λ_t is a vector (of the number of equations in the model), which mainly contains the dependent variables (inflation, output, etc.), the inverse of the elasticity of intertemporal substitution (derived in the maximising problem of the households), and a household's habit of shifting consumption from one period to the next. Assuming that D does not change over time reduces the equation to a normally distributed price of risk model. Taking $\Lambda_t = \Lambda_0$ we attain a homoscedastic model cox, which belongs to the model class of the IS curve.

Because our derivation of the IS curve assumed a particular preference structure, the pricing kernel is given by the intertemporal consumption marginal rate of substitution of the model

$$m_{t+1} = ln(\psi_t) - \sigma y_{t+1} + (\sigma + \eta) y_t - \eta y_{t+1} + (g_{t+1} - g_t) - \pi_{t+1}. \tag{4}$$

where σ is the inverse of the intertemporal rate of substitution, η is the habit of shifting consumption to another period, and ψ is the demand shifting factor, which are all elasticities in the preferences of the households, that is part of $u\left(C_t, \frac{M_t}{P_t}\right)$. g_t is a logarithmic demand shock. Logarithmic asset prices are modelled as affine functions of the state variables

$$p_t = \beta + \beta_p p_{t-1} + \beta_i (i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t. \tag{5}$$

Note that p is a vector including different financial market instruments. Bekaert, Cho, and Moreno (2010) show that the macroeconomic variables and the term spreads follow a first-order VAR with complex cross-equation restrictions.

Our specification of the financial sector p eflects our assumption of simultaneously interacting stock and bond markets. Following Heston (1993) we model the stock market as a stochastic volatility model. This is an extension of Black and Scholes (1973) and takes into account a non-lognormal distribution of the asset returns, leverage effect, and mean-reverting volatility, while continuing to be analytically tractable. To make the assumption of highly interacting markets, we include the foreign stock market, as well as house prices, exchange rate, output, and interest rate in the drift term. For example, including the output gap in the stock market is in line with Cooper and Priestley (2009) and Vivian and Wohar (2013); a general approach of incorporating macroeconomic factors in stock returns is developed by Pesaran and Timmermann (1995). Moreover, Fama and Farber (1979) provides evidence for common factors in bond and stock markets.

In line with Bayer et al. (2010), house prices are modelled as stochastic differential equations taking into account local risk, national risk, and idiosyncratic risk. This allows modelling house prices in an asset pricing environment. As before, we account for macroeconomic variables in the drift term. Consistent with empirical findings by Adams and Füss (2010), Agnello and Schuknecht (2011), Capozza et al. (2002), and Hirata et al. (2012) , we include the real interest rate, the output gap, and the derived asset from the stock market in the drift term to account for the interconnectedness. To analyse call prices, we apply the extended Black-Scholes formula as in Kou (2002).

In total, we have a system of seven financial market equations: two stock market equations, two bond market equations, two calls, and the exchange rate[1]

$$
\begin{aligned}
dS &= (S_t((r-\lambda\mu) + \rho_b b_t + \rho_b^* b_t^* + \rho_s^* S_t^* + \rho_i i_t + \rho_y y_t + \rho_e e_t + \rho_\pi \pi_t)dt \\
&\quad + \sqrt{V_t} dW_S(t) + \sum_{i=1}^{dN_t} J(Q_i) && (6) \\
dV_t &= \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_V(t) \\
dh_t &= (\gamma_h h_t + \gamma_S S_t + \gamma_s^* S_t^* + \gamma_h^* h_t^* + \gamma_y y_t - \gamma_i(i_t - \pi_t))dt \\
&\quad + \sigma_1 dW_h^1(t) + \sigma_2 dW_h^2(t) + \sigma_3 dW_h^3(t) && (7) \\
e_t &= \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i_t^* - \pi_t^*) + \psi_t + \varphi_t, && (8)
\end{aligned}
$$

where $J(Q)$ is the Poisson jump-amplitude, Q is an underlying Poisson amplitude mark process ($Q = \ln(J(Q)+1)$), and $N(t)$ is the standard Poisson jump counting process with jump density λ and $E(dN(t)) = \lambda dt = Var(dN(t))$. dW_s and dW_v denote Brownian motions. β is the long-term mean level, α the speed of reversion, and σ the instantaneous volatility. e_t is the exchange rate equation derived above.

2.8 The Monetary Policy Rule

In line with Ball (1998), Leitemo and Soderstrom (2005), Lubik and Schorfheide (2007), Lubik and Smets (2005), Svensson (2000), and Taylor (1993), we apply an open-economy Taylor rule, which also takes into account financial markets. Given exchange rate e and interest rate i an optimal policy rule minimises the loss function

$$Var(y) + \mu_1 Var(\pi) + \mu_3 Var(y^*) + \mu_3 Var(p)$$

[1] We keep the simple exchange rate specification from ball to stick to the common New Keynesian framework.

The variation in the parameters μ_i defines the range of efficient policies. To implement our rule, we follow ball and place the exchange rate equation into the IS curve, the NKPC and the financial market equation.

$$
\begin{aligned}
y_{t+1} &= \lambda_y y_t + \frac{\lambda_i}{\theta_i}(e_t - \theta_e e_{t-1} + \theta_i^*(i_t^* - \pi_t^*)) - \lambda_y^* y_t^* - \lambda_e e_t \\
\pi_{t+1} &= \pi_t - \alpha_y y_t - \alpha_e(e_t - e_{t-1}) \\
p_{t+1} &= \beta + \beta_p p_t + \frac{\beta_i}{\theta_i}(e_t - \theta_e e_{t-1} + \theta_i^*(i_t^* - \pi_t^*)) + \beta_y y_t + \beta_e e_t
\end{aligned}
$$

As we take into account i and e, we can define the state variables of the model by two expressions corresponding to terms on the right-hand sides of equations

$$
\begin{aligned}
\lambda_y y + \frac{\lambda_i}{\theta_i}\theta_i^*(i_t^* - \pi_t^*) - \lambda_y^* y_t^* - \frac{\lambda_i}{\theta_i}\theta_e e_{t-1} \\
\pi_t - \alpha_y y + \alpha_e e_{t-1} \\
\beta + \beta_p p + \frac{\beta_i}{\theta_i}\theta_e e_{t-1} + \frac{\beta_i}{\theta_i}\theta_i^*(i_t^* - \pi_t^*) + \beta_y y
\end{aligned}
$$

Combining these parts and rearranging the parameters yields to the Taylor rule in Equation 9

$$
(1 - \omega_e)\, e_t + \omega_i i_t = \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y_t^* \tag{9}
$$

Thus, our Taylor rule accounts for domestic and foreign output, the exchange rate, inflation and the financial market[2]. The rule facilitates analysis of spillover effects between financial markets and monetary policy as well between foreign and domestic policy. Moreover, by including the financial sector consisting of various markets, we account for a direct relationship between monetary policy and financial markets. Rigobon (2003) and Rigobon and Sack (2003) show empirically that including this aspect is useful.

[2] Note that foreign output enters the equation by spillover effects on domestic output

2.9 The Complete Model

Our model in discrete time, as derived above, is summarised in Equation 10.

$$\begin{aligned}
y_t &= \lambda_y y_{t-1} - \lambda_i(i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y^*_{t-1} - \lambda_e e_{t-1} + \epsilon_t \\
\pi_t &= \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e(e_{t-1} - e_{t-2}) + \eta_t \\
e_t &= \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i^*_t - \pi^*_t) + \psi_t(+\varphi_t) \\
p_t &= \beta + \beta_p p_{t-1} + \beta_i(i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t \\
i_t &= \frac{1}{\omega_i}\left((1-\omega_e)e_t + \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y^*_t\right)
\end{aligned} \tag{10}$$

Reflecting the approach by (Hayo and Niehof 2013) and work by Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b), we switch to a continuous time framework by taking first differences of the equations and using stochastic differential equations to model the shocks as Brownian motions. To simplify interpretation, he parameters continue to have the same names as previously. Prior parameters can be recovered by solving the system of linear equations given in Equation (11).

$$\begin{aligned}
dy &= (\lambda_y y - \lambda_i i + \lambda_i \pi - \lambda_{y^*} y^* - \lambda_e e)dt + \sigma_y dW^y_t \\
d\pi &= (\alpha_\pi \pi + \alpha_y y - \alpha_e e)dt + \sigma_\pi dW^\pi_t \\
de &= (\theta_e e + \theta_i i - \theta_\pi \pi - \theta_{i^*} i^* + \theta_{\pi^*}\pi^*)dt + \sigma_e dW^e_t \\
dp &= (\omega_p p + \omega_e e + \omega_y y + \omega_i i - \omega_\pi \pi + \beta)dt + \sigma_p p dW^p_t \\
di &= (\gamma_i i + \gamma_\pi \pi + \gamma_y y - \gamma_{y^*} y^* + \gamma_e e + \gamma_p p)dt
\end{aligned} \tag{11}$$

Note that p can be specified as a vector including various assets, each of which can be included linearly and also priced differently.

Regarding stability, we rely on Lyapunov techniques (Khasminskii 2012). Our continuous time model can be rewritten as:

$$X(t) = X(t_0) + \int_0^1 AX(t)dt + \int_0^1 \sigma(t)X(t)dW(t), \tag{12}$$

where A is a matrix (therefore, a linear function) and b is a vector. X represents our dependent variables (output gap, inflation, exchange rate etc.). A Lyapunov function is a scalar function $V(t,x)$ defined on $\mathbb{R}^n$ that is continuous, positive definite, $V(0) > 0$ for all $x \neq 0$, and has continuous first-order partial derivatives at every point of D. The differential generator is

$$LV(t,x) = \frac{\partial V(t,x)}{\partial t} + \sum_{i=1}^{l}(Ax)_i \frac{V(t,x)}{\partial t} + \frac{1}{2}tr(b'Xb) \tag{13}$$

The trivial solution of a stochastic differential equation is called stable, if the differential generator of the Lyapunov function is negative definite $LV < 0$ for all values in a neighbourhood $D\backslash\{0\}$. We apply the Euklidean norm

$$V(x) = \|x\|_2^2 = \left(\sqrt{\sum |x_i|^2}\right)^2$$

as a Lyapunov function and apply the differential operator. Given the parameters introduced in below we derive

$$\frac{1}{(e^2 + p^2 + (p^*)^2 + \pi^2 + \pi^*)^2 + i^2 + (i^*)^2 + y^2 + (y^*)^2)^{\frac{3}{2}})} Terms$$

$Terms$ depends both on the dependent variables as well as on the parameters. Note that the numerator converges more slowly toward zero than does the denominator. Thus, stability of the dynamic system depends on the appropriate choice of parameters using sufficiently small values in the neighbourhood of 0. This also shows that the trivial solution is weakly stable. In our case, stability of the trivial solution is guaranteed. In economic terms, this means that if an economy drifts away from its steady state, it will return to it or, alternatively, it will not move very far away.

Note that we concentrate our dynamic analysis on short-run adjustments. Within this time frame, there is no guarantee that the variables will actually return to their starting values. If we analyse scenarios that are too far away from the neighbourhood of the trivial solution, stability is no longer guaranteed and trajectories may show no tendency for returning to the long-term equilibrium.

3 Studying Effects of Financial Market Turmoil

To shed light on the contagious effects of financial crises for highly connected domestic and foreign real economies, we study a crash in the stock and housing markets. Our empirical examples in the later part of the study are Canada and the United States.

To study how monetary policy reacts to financial distress, we first apply an open-economy Taylor rule that takes into account the domestic and foreign output gap, inflation, and the exchange rate. In a second simulation, we extend the Taylor rule to include stabilisation of financial markets as an additional objective, with the aim of comparing advantages and disadvantages of both policy rules. In Figures 1-3, blue lines represent central bankers who incorporate financial markets; black lines represent traditional central bankers. Solid lines represent the domestic economy; dashed lines the foreign economy.

We commence the analysis by simulating a minor financial market shock, which, taking into account neutrality of money, should not affect the real economy much. Afterward, we simulate the development of a bubble and its subsequent collapse on (domestic) stock and housing markets and analyse the two different types of monetary policy reaction, the first ignoring and the second incorporating financial markets in the policy rule. Technically, we take the mean of 100,000 simulations with 0,01 time steps. We use a normalised Euler-Maruyama scheme to simulate the trajectories of the stochastic differential equations.

In our first experiment, we study a domestic stock market slump for the case that monetary policy does not directly react to financial markets. We find that the maximum drop in the output gap is 0.25 . As financial markets are positively connected, the drop in the domestic stock market causes a slump in the domestic housing market as well as a gradual decline of foreign financial markets.

To stabilise the economy, monetary policymakers slowly reduce the nominal interest rate by 0.50 percentage points (pp). Following recovery of the output gap and the inflation rate - about half-way through the observation period - interest rates begin to rise. The interest rate drop causes minor inflationary pressure. The exchange rate appreciates only exiguously, so that the negative spillover effects on foreign output and inflation are small. The combination of a slight decrease in output, a slight increase inflation, and a slight appreciation of the foreign currency, induces foreign monetary policymakers to cut the nominal interest rate by about 0.10 pp. This keeps the foreign real economy more or less unaffected by the spillovers from the domestic stock market slump.

Second, we study the case that monetary policy reacts directly to financial market movements. In this case, a slump in the stock market causes monetary policymakers to react more drastically and drop the nominal interest rate by 0.25 pp within one quarter of the simulated observation period. As a consequence, the output gap reacts strongly to the improved borrowing conditions, and the expected recession becomes, instead, a boom, which leads to a notably higher inflation rate. As a reaction to the booming

Figure 1: **Stock Market Slumps**

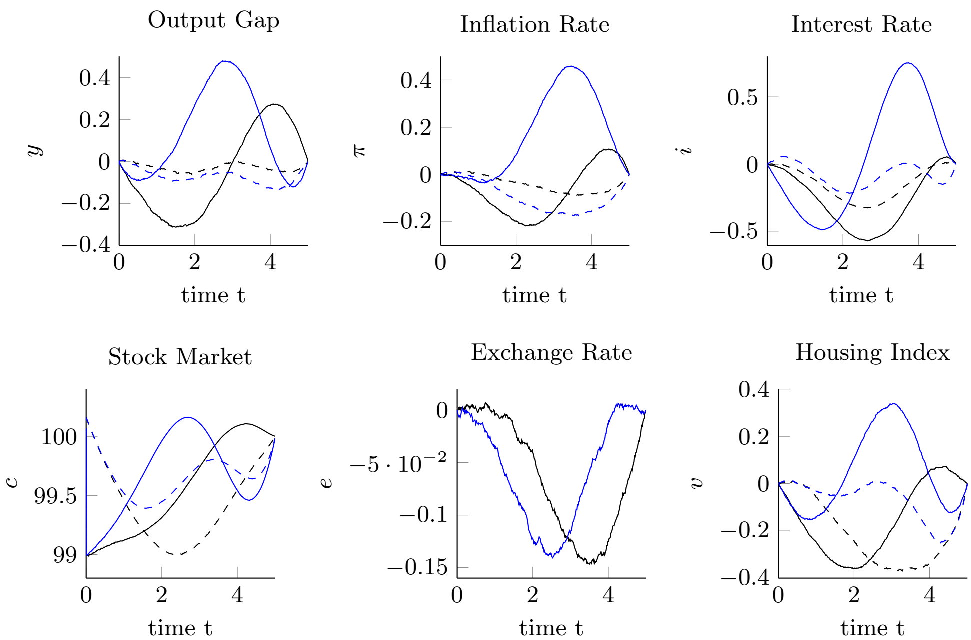
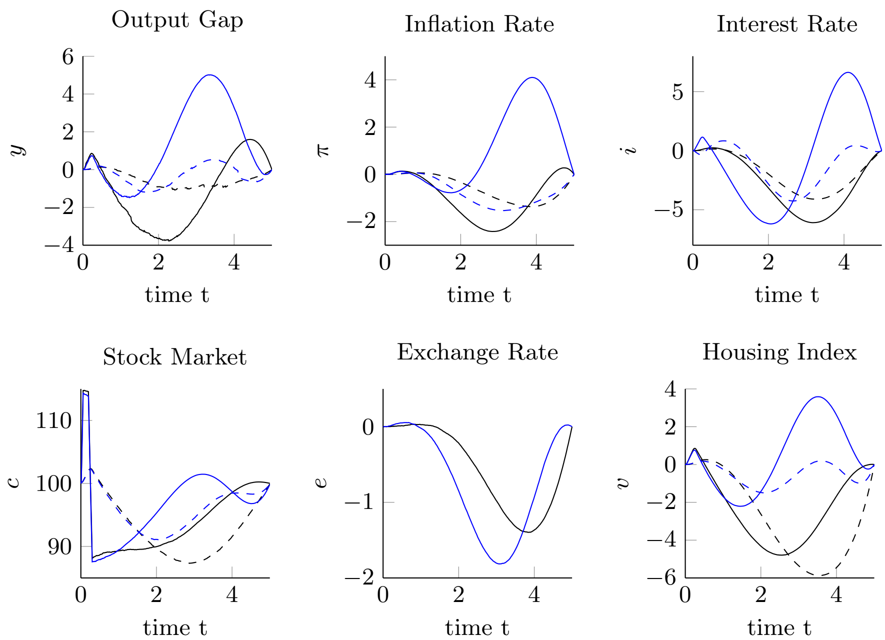


Figure 2: **Stock Market Crisis**



real economy and the inflation hike, monetary policy is tightened and the economy is brought back to equilibrium. Domestic financial markets show pronounced volatility, reflecting this stop-and-go policy.

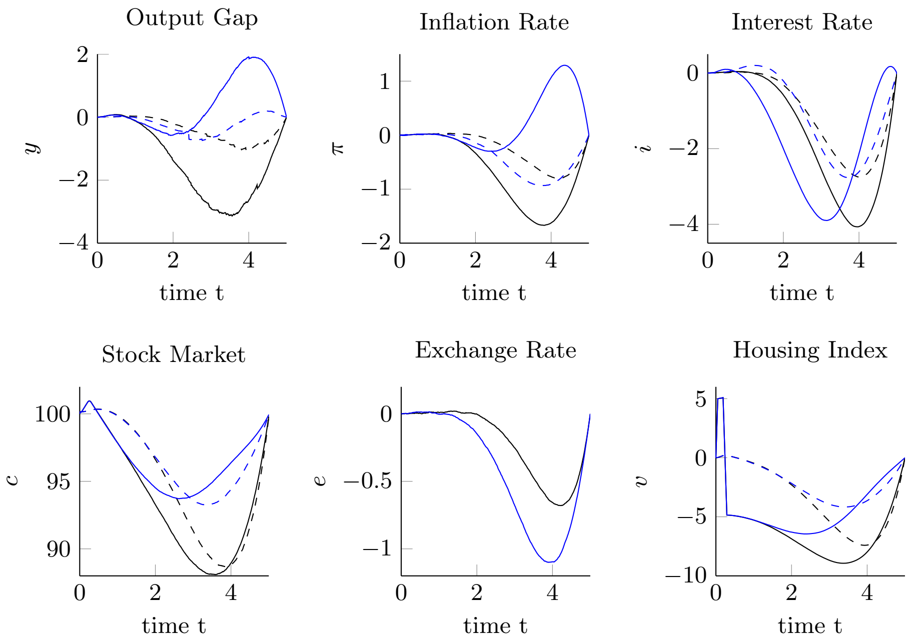
Regarding international spillovers, we find that the domestic output boom causes positive spillovers to the foreign country. Specifically, the positive business cycle effect cannot quite offset the negative spillover from the stock market. As a consequence of increased output, the (domestic) exchange rate first depreciates, causing slight foreign deflation, and then appreciates in tandem with the decreasing output gap. Domestic financial market volatility spills over to foreign financial markets, which then also experience an up-and-down movement, albeit of a smaller amplitude. Finally, foreign monetary policy is also characterised by this cyclicality.

In our second experiment, we analyse the build-up and collapse of a stock market bubble. After the bubble appeared on the stock market, there is a short period of stability before the stock market crash.

First, we study the case where the Taylor rule does not directly take financial markets into account. As stock market assets also reflect firms' value, the burst of a bubble on the stock market causes actual output to drop below potential output. This financial market effect leads to a notable drop in economic activity and a corresponding drop on the housing market. The minimal turning point of 4 per cent of the output gap occurs after half of the period. The negative output gap causes a falling inflation rate. Given these strong downward movements of output and inflation, central bankers begin lowering the interest rate. However, the reaction is relatively slow to begin with but gains momentum as the recession gains steam. The minimal turning point of the interest rate occurs at -5 pp in the third quarter of the observation period. As a consequence of this massive decrease in interest rates, the output gap slightly overshoots before reaching zero, whereas the inflation rate goes directly back to the baseline. After the crash, the domestic stock market's recovery is virtually linear, whereas the housing market has a u-shaped reaction.

As markets are strongly connected, the stock market crash spills over to foreign stock markets and the foreign housing market suffers from contagion, too. Consequently, foreign real markets are hit by recessionary pressure. Foreign output moves below potential, about -1 pp, and causes deflationary pressure. Afterward, monetary policy reactions are generally similar to those in the domestic economy. Monetary policy reacts rather tardily and tends to let the economic situation deteriorate before a massive interest rate decrease brings variables back to equilibrium.

Figure 3: **House Market Crisis**



Second, we study the case that central bankers incorporate financial markets in their decision rule. In this case, we find that the crisis proceeds at a relatively moderate pace. A severe drop in the nominal interest rate of 5 pp relatively early within the simulated period stabilises the output gap at a minimum of -0.3 pp. However, the quick and severe cut in the nominal interest rate causes substantial overshooting in real activity, with the output gap increasing to 5 pp. As a consequence, inflation rises to 4 per cent. Given that the economy has moved from a moderate recession into a fully-fledged boom, monetary policy reacts again and increases the interest rate to 5 pp in the last third of the analysed period. With respect to international transmission of the stock market crash, we find that the strong linkages between the economies lead to severe spillovers to the foreign country. Although they occur with a notable time lag, the reactions in the foreign country are generally similar to those of the domestic economy, but of smaller magnitude. One exception is the foreign inflation rate. Foreign inflation does not experience a hike as did domestic inflation following the expansionary monetary policy, as the appreciation of the foreign currency helps keep rising prices under control. Our third experiment focuses on the origin of the recent financial and economic crisis - the housing market. We construct a house market bubble and observe what happens after it bursts. There are obvious spillover effects: first to the financial markets in general and, second, to the real economy. First, we assume that the monetary policy rule does not directly account for the stock market. We discover that there are contagion effects from the housing market to the stock market, which experiences a downturn. In a secondary reaction, this spills over to the real economy. The resulting drop in output and inflation leads to an expansionary monetary policy reaction, which helps restore equilibrium on all markets at the end of our simulation window. There are spillovers to the foreign economy that lead to similar, but more muted, movements of the economic variables. Generally, this scenario is qualitatively identical to the stock market crash.

We next assume that monetary policymakers react directly to the stock market crisis, which hits the real economy with a short time lag after the house price bubble bursts. To stabilise the economy, monetary policymakers drop the nominal interest rate by four per cent. As in the case of the stock market cash, the central bank reacts very quickly to the crisis and avoids a notable drop of the output gap. However, we observe higher inflation and more volatility in the adjustment process back to equilibrium.

4 Employing Empirically Estimated Parameters

To see how the findings from our model match up with what happens in the real world, we estimate parameters for the United States and Canada. As we are using continuous-time equations, we rely on stochastic estimation (approximate Bayesian computation; see Beaumont, Zhang, and Balding (2002)). To account for the occurrence of the recent financial crisis, we split the sample in two parts, before and after the outburst.

Wright (2008) explains the fundamental idea of Bayesian model averaging (which was first set out Leamer (1985)). Consider a set of n models $M_1, ..., M_n$, which have the same functional form but differ in terms of parameter values. The researcher knows that one of these models is the true model, but does not know which one. She has prior beliefs about the probability that the i-th model is the true model, which we write as $P(M_i)$. Then she observes data D, and updates her beliefs to compute the posterior probability that the i-th model is the true model,

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{\sum_{j=1}^{n} P(D|M_j)P(M_j)}$$

For each model, we can compute an in-sample forecast. In the presence of model uncertainty, the overall forecast weighs each of these forecasts by the posterior for that model. This gives the minimum mean square error forecast. Thus, the researcher has to specify the set of models, the model priors, $P(M_i)$, and the parameter priors. Our discrete model has the form

$$y = X_i \beta_i + \epsilon$$

To obtain a closed-form solution, we assume that the regressors are exogenous. Two inputs are crucial for obtaining plausible results via Marcov Chain Monte Carlo (MCMC) estimations: the choice of priors and the choice of initial values. Our choice of prior distributions for NK models is similar to that of Smets and Wouters (2007), Negro et al. (2007) or Lindé (2005).

We follow Kimmel (2007), Wright (2008) or Jones (2003) and choose normal distributions for financial instruments. The financial parameters take the natural conjugate g-prior specification, so that each prior for a financial parameter is $N(0, \sigma^2(X_i' X_i)^{-1})$, conditional on σ^2. Zellner and Palm (2004) shows one can then calculate the required likelihood of the model analytically, and the priors for the parameters can be considered adequate .

In Bayesian analysis, one cannot use improper priors for model-specific parameters because they are unique only up to an arbitrary multiplicative constant and thus their use would lead to an indeterminacy of the model posterior probabilities (Kass and Raftery 1995). Thus, in our estimations, we use informative priors for each parameter[3].

An overview of the priors is given in the first four columns of Table (1). We run 50,000 simulations to obtain our results, with an average acceptance ratio of about 40-50 per cent.

Data are obtained from the Bureau of Economic Analysis, the Federal Reserve St. Louis, the US Bureau of Labor Statistics, Statistics Canada, Datastream, and the OECD database. We employ quarterly data from 1971:Q1 to 2013:Q1. The output gap is obtained as the transitory component after applying the HP filter to logged quarterly GDP. The monetary policy interest rate is the short-term money market rate. The inflation series is constructed as $400(CPI_t/CPI_{t-1} - 1)$. Regarding financial variables, we employ the major stock index in the United States, S&P, and the major Canadian index, TSX. We also include the housing market, represented by changes in house prices.

Table 1: Priors and Posteriors for the Extended NK model (USA & Canada)

Model	D	M	V	Posterior	
				No Financial Markets	Incl Financial Markets

[3] Note that in some cases New Keynesian literature applied improper priors to compare them to their results. In this case they applied the uniform distribution with zero mean and standard deviation of one

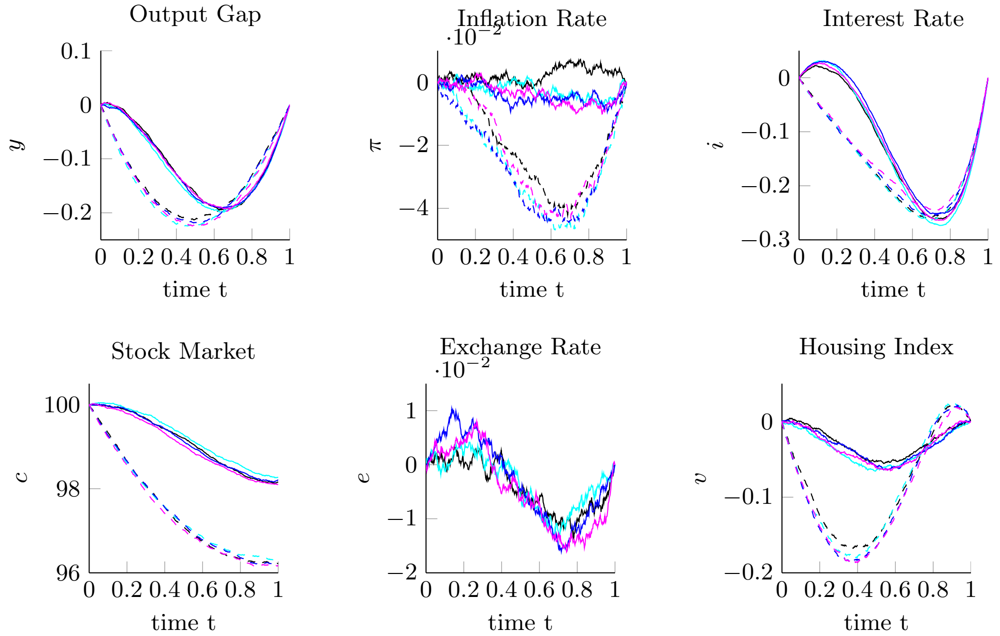
					Pre-Crisis		After Crisis		Pre-Crisis		After Crisis	
					USA	CA	USA	CA	USA	CA	USA	CA
λ_y	0.95	B	1.0	0.1	0.88	0.99	0.92	1.00	0.99	0.22	1.00	1.00
λ_i	0.99	B	1.0	0.3	0.63	0.86	1.00	0.40	0.97	0.18	0.60	0.72
λ_y^*	0.70	B	1.0	0.3	0.99	0.69	0.49	0.38	0.99	1.00	0.98	0.70
λ_s	0.60	B	1.0	0.5	0.96	0.41	0.83	0.75	1.00	0.65	0.97	0.58
λ_e	0.60	B	1.0	0.3	0.99	0.99	1.00	0.64	0.95	0.11	0.51	0.99
α_π	1.00	1.0	-	-	2.02	1.58	0.71	1.49	1.93	1.33	1.28	0.27
α_y	0.40	IG	0.5	0.5	-1.23	0.70	0.81	0.53	2.04	2.02	0.09	0.69
α_e	0.80	IG	0.5	0.5	1.68	1.55	1.09	0.74	0.97	2.11	0.69	1.67
θ_e	0.40	B	1.0	0.3	0.42	-	-0.35	-	0.98	-	0.99	-
θ_i	0.20	B	1.0	0.3	0.75	-0.72	0.25	-0.99	0.45	0.63	-0.16	-0.38
ω_h	0.80	N	0.0	5.0	1.35	1.43	1.09	1.67	0.97	0.31	0.92	0.54
ω_h^*	0.50	N	0.0	5.0	0.23	1.03	0.25	1.69	0.62	0.53	1.07	-0.62
ω_s	0.30	N	0.0	5.0	-0.15	0.63	-0.29	0.51	0.25	0.67	0.10	0.51
ω_i	0.60	N	0.0	5.0	0.69	0.63	1.09	-0.29	0.04	0.04	0.32	0.32
ω_i^*	0.40	N	0.0	5.0	0.64	0.10	1.55	-0.65	1.75	0.30	0.09	1.82
ω_y	0.50	N	0.0	5.0	-0.29	-0.28	0.58	0.88	-0.04	0.19	1.37	0.34
ω_e	0.50	N	0.0	5.0	1.74	-0.32	1.09	0.28	0.90	-0.28	1.03	0.83
ω_h	0.30	N	0.0	5.0	-0.35	-0.21	0.19	0.78	1.35	0.73	-0.18	0.75
ω_h^*	0.20	N	0.0	5.0	0.92	-0.24	-0.20	1.66	0.99	-0.96	0.41	0.26
ω_s	0.50	N	0.0	5.0	1.92	0.62	0.99	0.42	-0.64	0.53	0.20	1.21
ω_s^*	0.30	N	0.0	5.0	-0.11	0.26	-0.71	-0.45	-0.23	1.14	1.32	-0.90
ω_e	0.60	N	0.0	5.0	1.21	1.92	0.39	-0.20	0.46	-0.02	0.93	2.14
ω_y	0.50	N	0.0	5.0	-0.01	0.59	0.92	0.57	0.72	-0.45	0.92	0.47
ω_i	0.50	N	0.0	5.0	0.22	0.45	0.11	1.64	1.44	0.04	-0.34	-0.26
ω_π	0.40	N	0.0	5.0	0.26	1.15	1.59	1.03	0.89	0.98	0.19	0.91
γ_y	0.90	IG	0.5	0.5	1.17	0.62	1.31	0.97	0.83	1.84	0.81	1.68
γ_y^*	0.50	IG	0.5	0.5	0.76	1.18	0.70	0.54	1.55	1.92	1.94	1.13
γ_π	0.30	IG	0.5	0.5	-0.49	0.89	0.46	0.55	0.12	0.41	0.74	1.70
γ_e	0.90	IG	0.5	0.5	-	-	-	-	1.18	-0.86	1.54	-0.28
γ_h	0.40	IG	0.5	0.5	-	-	-	-	2.37	1.34	2.01	-0.22
γ_h^*	0.20	IG	0.5	0.5	-	-	-	-	-1.07	2.61	-0.58	1.68
γ_s	0.40	IG	0.5	0.5	-	-	-	-	1.04	-0.37	0.61	0.31
γ_s^*	0.20	IG	0.5	0.5	-	-	-	-	1.53	0.28	0.26	-0.91

γ_i	0.60	B	1.0	0.1	1.00	0.96	0.70	0.37	0.99	-0.74	0.79	0.08
γ_i^*	0.20	B	1.0	0.1	2.44	0.99	-0.43	-0.70	0.69	0.85	1.02	0.99
K	0.30	-	-	-	-	-	-	-	-	-	-	-
κ	1.50	-	-	-	-	-	-	-	-	-	-	-
λ	4.00	-	-	-	-	-	-	-	-	-	-	-
μ	-0.05	-	-	-	-	-	-	-	-	-	-	-
r	0.40	-	-	-	-	-	-	-	-	-	-	-
ρ	0.40	-	-	-	-	-	-	-	-	-	-	-
σ	0.40	-	-	-	-	-	-	-	-	-	-	-
σ_v	0.20	-	-	-	-	-	-	-	-	-	-	-
θ	0.005	-	-	-	-	-	-	-	-	-	-	-
θ_2	0.01	-	-	-	-	-	-	-	-	-	-	-
σ_y	0.10	B	1.0	0.1	0.96	-0.86	-0.81	0.95	-0.98	0.62	-0.93	-0.45
σ_π	0.10	B	1.0	0.1	0.66	0.65	0.98	0.95	-0.42	-0.39	-0.97	0.61
σ_e	0.10	B	1.0	0.1	0.38	-	0.20	-	-0.66	-	-0.17	-
σ_h	0.10	B	1.0	0.1	-0.11	-0.79	0.16	0.65	0.35	0.70	-0.30	0.48
σ_s	0.10	B	1.0	0.1	-0.38	0.38	0.31	0.39	-0.99	-0.35	0.86	-0.55[4]

Since our main purpose is to study the impact of monetary policy, we concentrate on the reaction of central banks. In general, our estimations are consistent with the parameters values used in the theoretical analysis above. However, there are huge differences in some parameters before and after the crisis, especially those involving the financial markets. First, output gap and inflation appear to be more important before the crisis than afterward, whereas the exchange rate and house markets become more important after the crisis. Moving from the parameter estimates to trajectories in Figur (4) shows that, in general, there is no big difference before and after the financial crisis in terms of adjustment dynamics. Our estimations reveal the inability of monetary policy to prevent a recession after the financial market crisis. Overall, the magnitude of movements in this scenario is quite similar to what we find in the theoretical simulations in (2). Although

[4] Note that in some cases New Keynesian literature applies improper priors to compare them to their results. In this case, they apply a uniform distribution with zero mean and standard deviation of one

Figure 4: **Estimation Results (after a minor stock market slump in the US)**



some parameters in the policy rule changed, the resulting effects on the real economy are small. Thus, we show that financial market crises cause domestic and international spillovers to the real economy, regardless of whether monetary policy includes financial markets in its reaction function. The advantage of including financial variables is that a major crisis can be avoided, as the monetary policy response is quick and strong. The disadvantage, however, is that the economy faces a higher inflation rate and more volatility.

5 Conclusions

In this paper, we extend the well-known, open-economy New Keynesian model of Clarida, Gali, and Gertler (1999) and Lubik and Schorfheide (2007) in two important ways. First, we include a well-developed financial sector and, second, we apply stochastic differential equations and conduct the analysis in a continuous-time framework. This allows us to employ classic research from the field of finance and model the financial sector by including the market for foreign exchange, the housing market, and the stock market, both in the domestic as well as in the foreign economy, thereby taking into account international economic interdependence. Applying stochastic differential equations allows us to rely on established research, as provided by Merton (1973). In particular, we specify the financial markets as jump-diffusion processes and the Black-Scholes equations Black and Scholes (1973) for call prices. Furthermore, we employ Lyapunov techniques Khasminskii (2012) to analyse the stability of the solutions and steady-state properties. Thus, in our analysis, we combine New Keynesian macroeconomic analysis, classic finance research, and standard mathematical procedures.

Our main research question concerns the effects of financial crises on real markets, which we investigate by comparing theoretical and empirical results. We find theoretical and empirical evidence for spillover effects of monetary policy. Furthermore, we observe that including financial markets in the Taylor rule mitigates the effect of a financial crisis at the expense of an overshooting in the inflation rate and greater volatility of all economic variables.

Our continuous-time estimation compares our model findings with real-world data from the United States and Canada. Employing quarterly data over the period 1971:Q1: to 2013:Q1, we use estimates based on Bayesian estimation techniques to derive the model's parameters. To analyse changes in monetary policy during the financial crisis, we split the dataset and run the simulations for each half separately. The simulation results support our findings from the theoretically parameterized model. We find spillover effects from monetary policy if conducted in the United States, but only very small effects if the policy is initiated by the Bank of Canada. Moreover, US monetary policy appears to have a larger effect on Canada than Canadian monetary policy itself. This finding is consistent with evidence reported by Hayo and Neuenkirch (2012) on how monetary policy communication impacts financial markets in the United States and Canada. We find no evidence that the Taylor rules of either central bank incorporated stock prices. In line with our theoretical analysis, this might have amplified the effects of the crisis but avoided increasing inflation and higher volatility.

Our study has some interesting policy implications. Taking financial markets directly into account in the Taylor rule mitigates the severity of economic recessions in the aftermath of financial crises. However, the price could be a higher inflation rate and more volatility of other variables. While this may be a small price to pay in the case of a severe crisis, during normal times, the typical up and down movements of financial markets will be translated to and magnified in other economic variables. Given the rarity of major crises in advanced economies, this suggests that perhaps monetary policy should not include financial variables in its reaction function, but should have a discretionary emergency plan so that the rule-based monetary policy can be quickly replaced by discretionary policy after a major financial crisis.

Regarding the international dimensions, we find evidence that not only financial and real shocks, but also monetary policy actions spill over to other countries. Thus, monetary policy in one country can substantially affect financial markets in other countries, even to the extent of triggering booms and busts. The impact and size of the effect depends on, first, the linkage between the markets and, second, the structure of the markets. Policymakers, particularly those of very open and well-integrated countries, should take into account that spillovers from larger countries could have effects that (depending on the degree of interaction) might even be larger than the effects of domestic policies. In contrast, we find evidence in support of the conventional wisdom that big countries and relatively closed economies can design and engage in policy primarily based on domestic factors.

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