Adaptive Learning, Incomplete Knowledge and Unemployment Volatility

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Abstract

We show that relaxing the assumption of Rational Expectations (RE) in a standard search and matching model has the potential to generate amplification in labour market variables as indicated by US data. We present a model in which agents update their expectations as new information becomes available using a Recursive Least Square algorithm. Firms choose vacancies by making infinite horizon forecasts over (un)employment rates, wages and discount rates at each point in time. We find that, in a model where agents make forecasts based on incomplete knowledge, firms have a much greater incentive for vacancy posting because they tend to overestimate the present discounted value of future profits per hire at the time the TFP shocks hits the economy. We show that small forecast errors tend to accumulate as the time horizon of the forecast increases to infinity by comparing our results with Euler Equation learning. We find that the model with adaptive learning and incomplete knowledge matches reasonably well the first and second moments of the forecast errors of unemployment in the Survey of Professional Forecasters.

Keywords: adaptive learning, bounded-rationality, search and matching frictions

JEL codes: E24, E25, E32, J64

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1 Introduction

Shimer (2005) shows that the standard search and matching model driven by Total Factor Productivity (TFP) innovations has a hard time replicating the amplification of labour market variables, such as unemployment, vacancies and the measure of labour market tightness, as shown in the US data. One of the main assumptions of the standard search and matching model is that agents are fully endowed with rational expectations (RE) and, hence, firms make no systematic errors when forecasting future profits per hire. As most inter-temporal decisions, hiring choices depend crucially upon what agents' perceive the future to hold. The aim of this study is thus to investigate the role of expectations and expectation formation for studying hiring decisions.

The idea that *expectations and expectational errors* play a relevant role in explaining business cycle fluctuations goes back to Pigou (1927) and Keynes (1936). The recent financial crisis has instilled a renewed interest in studying the role of expectations for explaining business cycles - see for example Jess and A. (1994), Beaudry and Portier (2006), Jaimovich and Rebelo (2009) and Lorenzoni (2009) amongst others. In this study we take the view that expectations, expectation formation and expectational errors are also important for explaining labour market dynamics in an otherwise standard search and matching model. Systematic errors due to *incomplete knowledge* in our setting can be perceived as waves of optimism and pessimism because small errors tend to propagate as the time horizon of the forecast increases to infinity and, once compounded, have the potential to explain the large variation in labour market variables as observed in the US data. In this sense, our findings are related to Howitt and McAfee (1992) but our focus is on the accumulation of systematic mistakes in agents' forecasts due to incomplete knowledge in a model with fully optimising agents rather than on the assumption of rationality.

Following Eusepi and Preston (2011), we make the assumption that agents have a misspecified set of beliefs because they are unable to correctly understand the mapping from TFP innovations to the aggregate state of the economy and market prices. The economy is self-referential in that changes to the beliefs about about prices, (un)employment and profits affect current vacancy posting decisions, which in turn reinforces beliefs. We believe that incomplete knowledge is consistent with the fact that agents make forecasts as if they were econometricians to make forecasts by running simple regressions with (un)employment as the only dependant variable and treat TFP innovations simply as residuals of their regressions. Following Preston (2005) and Mitra et al. (2013), we assume that agents update their beliefs as new information becomes available to carry out the Infinite Horizon forecasts that are necessary to make current decisions. Since the belief system in our model is misspecified, we follow Sargent (1999), Cho et al. (2002), Evans and Honkapohja (2001) and Branch and Evans (2006a) to characterise the resulting *self-confirming/restricted perception/misspecification equilibrium* from which we compute the main statistical properties of the simulated series. Agents in our model are unable to accurately forecast wages due to incomplete knowledge, giving rise to the expectational errors that are at the source of labour market amplification. We find support for this hypothesis by comparing the performance of the forecast errors generated in the model against the equivalent data drawn from the Survey of Professional Forecasters.

By comparing the performance of the model with Infinite Horizon (IH) learning and the model featuring Euler Equation (EE) learning, we find that only small systematic errors that compound into large errors, as the time horizon of the forecast goes to infinity, can generate sufficient amplification as shown in the data. We find that a positive TFP innovation gives a greater incentive for vacancy creation relative to the RE model because the present discounted value of profits per hire increases sharply when agents beliefs are misspecified. Since future wage forecasts are subject to expectational errors, whilst the returns per hire are not, firms tend to overestimate the present discounted value of future profits per hire and to post increasingly more vacancies with the arrival of a TFP innovation. The forecast errors die out much faster under EE learning and incomplete knowledge simply because of the nature of EE learning, which involves making only one-step ahead forecasts rather than infinite horizon forecasts, with the resulting forecast errors being not sufficiently large to match the amplification in the data. Moreover, the possibility of consumption smoothing can explain why the volatility of output is lower relative to the volatilities of vacancies and unemployment. The assumption of symmetric beliefs across households and firms implies that consumption choices are not affected directly by TFP innovations but indirectly through updating beliefs as new information becomes available. Hence, consumption decisions in our model exhibit less variability relative to vacancy posting decisions.

A large number of studies has attempted to provide solutions to the *unemployment volatility puzzle*. Two of the most prominent solutions up to date consist of making relatively simple modifications to the standard search and matching model with RE beliefs. The first approach proposed by Hall (2005) and Shimer (2005) suggests the introduction of real wage rigidities in order to generate larger variation in profits per hire. This means that, as wages - efficiently negotiated under Nash bargaining - cannot fully absorb the productivity shifts, there is a further incentive for vacancy creation. The rationale behind such result is simple: if quantities display little amplification after TFP innovations, then fixing prices has the potential to generate the expected results. Menzio (2005), Gertler et al. (2008), Christoffel and Kuester (2008), Gertler and Trigari (2009), Blanchard and Gali (2010) and Hertweck (2013) amongst others embed this idea into a general equilibrium setting to show that the standard model augmented for real wage rigidities performs reasonably well in terms of amplification. The second popular approach postulated by Hagedorn and Manovskii (2008) carries out a simple calibration exercise that sets the value of non-market activity close to the value of search to the worker. This calibration results in a higher outside option for the worker that translates into in a higher steady state value of wages and a smaller value of steady state profits per hire. And lower profits in turn imply higher labour market volatility.

The list of solutions to the unemployment volatility puzzle is not exhaustive. Menzio and Shi (2011), Colciago and Rossi (2011), Di Pace and Faccini (2012) and Alves (2012) propose an alternative mechanism that requires mark-up fluctuations either through the process of entry and exit, deep habits or trend inflation. Quadrini and Trigari (2008) and Gomes (2011) emphasise the interactions between private and public employment, Reiter (2007) develops a model with embodied technological change, Guerrieri (2008) and Robin (2011) assess the role of heterogeneity and disequilibrium unemployment and, finally, Petrosky-Nadeau (2013) and Petrosky-Nadeau and Wasmer (2013) study the role of financial frictions for vacancy posting decisions.

This alternative characterisations of the labour market have been criticised on the following grounds: a) microeconometric evidence by Pissarides (2009) and Haefke et al. (2013) is suggestive that wages for newly hired workers are highly cyclical and b) Costain and Reiter (2008) suggests the implied elasticity of unemployment benefits relative to unemployment arising from this calibration is implausibly larger than suggested by the data and c) much debate is centered around the choice of an appropriate measure of mark-ups, with the added criticism, pioneered in the work of Rotemberg (2008), that the volatility of mark-up fluctuations needed to generate amplification is excessively large and d) some solutions are rather complex and more difficult to take to the data.

The common feature of the solutions up to date is the assumption that agents are endowed with RE beliefs, so agents' forecasts carry no systematic mistakes. As pointed out by Mortensen and Nagypal (2007), the performance of the standard model featuring RE beliefs depends on the variability of next period profits per hire. Our solution differs from theirs in that, as we relax the RE assumption in a setting where agents make infinite horizon forecasts, the behaviour of the *present discounted value of profits per hire* rather than the behaviour of flow profits per hire is key for amplification. This means that productivity shocks affect on impact the present discounted value of profits because agents make systematic forecast errors about the path of future wages. Since firms carry out infinite horizon forecasts to choose how many vacancies to posts, firms' *discounting* of revenues and costs per hire turns out to be central for our results. Moreover, our solution relies on fairly standard assumptions about the general structure of the model: a) wages for newly hired workers are highly cyclical, b) our calibration ensures that our results are not driven by the HM calibration, c) we assume both perfect competition at level of the goods market and symmetric beliefs across agents and households and d) our solution is conceptually very simple.

The remainder paper is organised as follows. Section 2 presents the model environment. Section 3 discusses the main mechanism of amplification. Section 4 describes our calibration strategy and evaluates the quantitative performance of the model both from the point of view of labour market variables and the forecast errors of unemployment. Section 5 concludes.

2 Model

We propose a simple model featuring labour market search and matching frictions as in Mortensen and Pissarides (1994) and a form of adaptive learning following Preston (2005) and Mitra et al. (2013). Our model economy is inhabited by two types of agents: households and firms. Each households consists of a continuum of workers that search for jobs if unemployed and work for firms if employed. Following Andolfatto (1996), we make the assumption of perfect risk sharing at the household level so that both employed and unemployed members of the household consume equal amounts. Firms post job vacancies and employ workers with a lag so as to produce final goods using labour as the only input of production. Households consume the final goods supplied by firms. We assume that agents form their expectations by updating their beliefs as new information becomes available. Agents make Infinite Horizon (IH) forecasts about the future path of factor prices, unemployment and profits by running simple regressions that include the (un)employment rate as the only regressor in order to make current decisions about consumption and vacancy posting.

2.1 Labour Market

The labour market is frictional in that, from the perspective of the firm, it is costly to post vacancies and, from the standpoint of workers, searching for jobs is a time consuming process. Every period firms create new vacancies, sought by unemployed workers who are continuously looking for new job opportunities. Following Shimer (2010), we assume that workers that are matched at time t become productive at the beginning of next period, $t+1$. Worker-firm matches break up at the exogenous rate, $\rho \in (0,1)$. The aggregate number of matches, m_t, depends positively on both the unemployment rate, u_t, and aggregate vacancies, v_t and we assume that the matching process is guided by a matching function that exhibits constant returns to scale

$$m(v_t, u_t) = \bar{m} v_t^{\sigma} u_t^{1-\sigma}, \tag{1}$$

where $\bar{m}$ denotes the efficiency of the matching process, σ the elasticity of the matching function with respect to aggregate vacancies and the unemployment rate is given by $u_t = 1 - n_t$. We define the measure of labour market tightness as

$$\theta_t = v_t / u_t. \tag{2}$$

Due to the assumption of constant returns in the matching technology, we define the job finding rate as $m(v_t, u_t)/u_t = m(v_t/u_t, 1) = f(\theta_t)$ and the job filling rate as

$$m(v_t, u_t)/v_t = m(1, u_t/v_t) = q_t. \tag{3}$$

2.2 Households

In our model economy there is a continuum of large households of measure $h \in [0,1]$. Each household h maximises his or her life-time utility

$$\mathcal{H}\left(s_{t-1}^h, n_t^h\right) = \tilde{E}_t^h \sum_{s=t}^{\infty} \beta^{s-t} \mathcal{U}\left(c_s^h, n_s^h\right), \tag{4}$$

where $\tilde{E}_t^h$ denotes the subjective expectation of household h at time t. The period utility of household h depends positively on consumption, c_t^h, and negatively on employment, n_t^h and is defined by

$$\mathcal{U}\left(c_t^h, n_t^h\right) = \log c_t^h - \chi n_t^h,$$

where $\chi > 0$ is a parameter that captures the disutility of employment at the level of the household. The flow budget constraint reads

$$s_t^h = w_t^h n_t^h + \pi_t + s_{t-1}^h r_{t-1} - c_t^h, \tag{5}$$

where s_t^h denotes household savings at the end of period t, r_t the real interest rate, w_t^h the pooled wage rate at the household level and π_t^h firm profits, which are rebated to the household at the end of each period.[1] In addition, the employment rate at the household level evolves according to

$$n_{t+1}^h = (1-\rho) n_t^h + \left(1 - n_t^h\right) f(\theta_t). \tag{6}$$

Moreover, the unemployment rate at the household level is defined by

$$u_t^h = 1 - n_t^h. \tag{7}$$

Household h chooses the level of consumption to maximise his or her life-time utility, equation (4), subject to the inter-temporal budget constraint, equation (5), and law of motion of employment, equation (6). The household's problem can be written in terms of the following Bellman equation

$$\max_{c_t^h, s_t^h} \quad \mathcal{H}(s_{t-1}^h, n_t^h) = \mathcal{U}(c_t^h, n_t^h) + \beta \tilde{E}_t \mathcal{H}(s_t^h, n_{t+1}^h),$$

and the constraints (6) and (5). $\mathcal{H}\left(s_{t-1}^h, n_t^h\right)$ is the value function of the household, which depends on last period savings and the period employment rate. By combining the first order conditions of this problem with respect to c_t^h and s_t^h, we obtain the standard Euler equation

$$\mathcal{U}_c\left(c_t^h, n_t^h\right) = \beta r_t^{-1} \tilde{E}_t \mathcal{U}_c\left(c_{t+1}^h, n_{t+1}^h\right). \tag{8}$$

This condition states that household's marginal utility derived from consumption and labour at time t must equal to the marginal utility of consumption derived at time $t+1$ expressed in terms of time t. The envelop condition with respect to n_t^h gives

$$\mathcal{W}_t^h = w_t^h + \frac{\mathcal{U}_n\left(c_t^h, n_t^h\right)}{\mathcal{U}_c\left(c_t^h, n_t^h\right)} + \beta[1-\rho-p(\theta_t)] \tilde{E}_t^h \mathcal{W}_{t+1}^h, \tag{9}$$

where $\mathcal{W}_t^h = \frac{\mathcal{H}_n\left(s_{t-1}^h, n_t^h\right)}{\mathcal{U}_c(c_t^h, n_t^h)}$ denotes the net marginal value to the household - expressed in utility terms - of having an additional household member employed at time t. This condition states that the net marginal value of employment to the household is equal to the net flow value of employment, the distance between the wage and the utility cost of working, plus the net continuation value of employment.

[1] We make the simplifying assumption that all the firms are equally owned by all the household in the economy.

By iterating forwards the budget constraint (5) and substituting for future values of savings, we can find an expression for consumption of household h as a function of both human and non-human wealth

$$c_t^h + \tilde{E}_t^h \sum_{j=1}^{\infty} D_{t,t+j}^{-1} c_t^h = s_{t-1}^h r_{t-1} + w_t^h n_t^h + \pi_t + \tilde{E}_t^h \sum_{j=1}^{\infty} D_{t,t+j}^{-1} (w_{t+j}^h n_{t+j}^h + \pi_{t+j}^h),$$

where $D_{t,t+j} = \prod_{i=0}^{j-1} r_{t+i}$, $j \geq 1$. We assume that the transversality condition, $\lim_{j\to\infty} D_{t,t+j}^{-1} s_{t+j}^h = 0$ holds. Using the Euler equation (8), we can simplify the inter-temporal budget constraint as follows

$$\frac{c_t^h}{1-\beta} = s_{t-1}^h r_{t-1} + w_t^h n_t^h + \pi_t + \tilde{E}_t^h \sum_{j=1}^{\infty} D_{t,t+j}^{-1} (w_{t+j}^h n_{t+j}^h + \pi_{t+j}). \tag{10}$$

Following Mitra et al. (2013), we assume the households use a consumption rule based on a linearisation of (10) around the steady state values $(\bar{c}, \bar{w}, \bar{n}, \bar{\pi}, \bar{r})$. We use the upper bar over the variable x to denote the steady state value of variable x and the hat over the variable x to mean the deviations of the variable x from its steady state value. After linearising the household's behavior rule reads as

$$\frac{\widehat{c}_t^h}{1-\beta} = \bar{r}\widehat{s}_{t-1}^h + \bar{s}^h \widehat{r}_{t-1} + \bar{n}\widehat{w}_t^h + \bar{w}\widehat{n}_t^h + \widehat{\pi}_t + \sum_{j=1}^{\infty} \beta^j \tilde{E}_t^h \left[\bar{n}\widehat{w}_{t+j}^h + \bar{w}\widehat{n}_{t+j}^h + \widehat{\pi}_{t+j} - (\bar{w}\bar{n} - \bar{\pi})\, \beta \sum_{i=0}^{j-1} \widehat{r}_{t+i} \right]. \tag{11}$$

2.3 Firms

Our model economy features a continuum of large firms of measure $h \in [0,1]$. Each firm f employs labour to produce consumption goods using a decreasing return to scale technology of the form

$$y_t^f = z_t \left(n_t^f\right)^{\alpha}, \tag{12}$$

where y_t^f denotes output at the firm level, n_t^f employment and $\alpha \in (0,1)$ the elasticity of output with respect to labour. The productivity innovation z_t follows an exogenous process given by

$$\ln z_{t+1} = \varrho \ln z_t + \epsilon_{t+1} \quad \text{with} \quad \epsilon_t \sim N(0, \varsigma), \tag{13}$$

where $\varrho \in (0,1)$ denotes the persistence of the technology process and ϵ_t is an i.i.d. innovation with mean zero and dispersion ς.

Since posting vacancies is costly, the profits of firm f at time t can be written as

$$\pi_t^f = z_t \left(n_t^f\right)^{\alpha} - w_t^f n_t^f - \kappa v_t^f, \tag{14}$$

where $\kappa > 0$ denotes the unitary cost of opening a vacancy and v_t^f the number of job openings at the firm level. The problem of firm f is to choose the process v_t^f so as to maximise the present discounted value of expected profits, which can be written as

$$\max_{v_{t+j}^f} \left\{ \pi_t^f + \sum_{j=1}^{\infty} \tilde{E}_t^f \left[D_{t,t+j}^{-1} \pi_{t+j}^f \right] \right\} \quad \text{for} \quad j \geq 0 \tag{15}$$

subject to the law of motion of employment at the firm level

$$n_{t+1}^f = (1-\rho)\, n_t^f + v_t^f q_t. \tag{16}$$

The Bellman equation of firm f of this problem can be written as

$$\max_{v_t} \quad \mathcal{J}(n_t^f) = \pi_t^f + r_t^{-1}\mathcal{J}\left(n_{t+1}^f\right),$$

so the problem of firm f is to maximise this equation subject to (12) and (16). The first order condition with respect to v_t reads

$$\kappa = \mu\left(\theta_t\right) r_t^{-1} \tilde{E}_t^f \mathcal{V}_{t+1}^f, \tag{17}$$

where $\mathcal{V}_t^f = \mathcal{J}'(n_t^f)$ denotes the marginal value of having an additional worker employed at the firm. Equation (17) states that the marginal cost and marginal benefit of posting a vacancy must be equal. The envelop condition with respect to n_t^f is

$$\mathcal{V}_t^f = \alpha z_t \left(n_t^f\right)^{\alpha-1} - w_t^f + (1-\rho)\,\tilde{E}_t^f \mathcal{V}_{t+1}^f. \tag{18}$$

This condition simply states that the value of having additional worker employed at the firm must be equal to the flow value of employing an additional worker - the marginal productivity of the additional worker net of the wage costs - plus the continuation value of employment at the firm.

By leading equation (18) one period forward, multiplying both side of the expression by the stochastic discount factor, $\frac{1}{r_t}$, taking the expectation at time t and combining the resulting expression with equation (17), we obtain the job creation condition

$$\frac{\kappa}{q\left(\theta_t\right)} = \tilde{E}_t^f \frac{1}{r_t}\left[\alpha z_{t+1}\left(n_{t+1}^f\right)^{\alpha-1} - w_{t+1}^f + (1-\rho)\frac{\kappa}{q\left(\theta_{t+1}\right)}\right].$$

This condition is central to our analysis since it determines the optimal number of vacancies that firm f would like to post. The condition simply states that the cost of hiring in the margin must be equal its benefit. By iterating forwards, we can express the job creation condition in terms of the expectations about future profits per hire

$$\frac{\kappa}{q\left(\theta_t\right)} = \sum_{j=1}^{\infty}(1-\rho)^{j-1}\,\tilde{E}_t^f D_{t,t+j}^{-1}\left[\alpha z_{t+j}^f\left(n_{t+1}^f\right)^{\alpha-1} - w_{t+j}^f\right], \tag{19}$$

where $\lim\limits_{j\to\infty}(1-\rho)^{j-1}\,\tilde{E}_t^f D_{t,t+j}^{-1}\left[\frac{\kappa}{q(\theta_{t+j})}\right] = 0$ holds. In line with the household problem, firms use a vacancy posting rule based on the linearisation of equation (19) around steady state values of $\bar{w}$, $\bar{n}$, $\bar{q}$ and $\bar{r}$. The firms' behavioral rule therefore reads

$$\begin{aligned}\bar{\lambda}_1\bar{q}\widehat{v}_t^f \quad = \quad & \beta\bar{\lambda}_1(1-\rho)\,\widehat{n}_t^f + \left[\frac{\kappa}{\bar{q}^2} + \beta\bar{\lambda}_1\bar{v}\right]\widehat{q}_t + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t^f\bar{\lambda}_1\widehat{n}_{t+j}^f + \\ & + \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t^f\left[\bar{\lambda}_2\widehat{z}_{t+j} - \widehat{w}_{t+j}^f - \left(\bar{z}\bar{\lambda}_2 - \bar{w}\right)\beta\sum_{i=0}^{j-1}\widehat{r}_{t+i}\right],\end{aligned} \tag{20}$$

where $\bar{\lambda}_1 = \alpha(\alpha-1)\bar{z}\bar{n}^{\alpha-2}$ and $\bar{\lambda}_2 = \alpha\bar{n}^{\alpha-1}$.[2] Note that these parameters become equal to 1 when the production function exhibits constant returns to scale.

2.4 Wage Negotiation

Nash bargaining is used to characterise how wages for newly hired workers are determined. The negotiated wage $w_t^{h,f}$ is set to maximise the joint surplus of a match between a worker and a firm,

$$w_t^{h,f} = \arg\max\left(\mathcal{W}_t^{h,f}\right)^{\xi}(\mathcal{V}_t^f)^{1-\xi},$$

[2] See appendix for a more detailed derivation of this expression.

where $\xi \in (0,1)$ denotes the workers' bargaining power or the share of the surplus the worker is able to take. The first order condition of this problem then yields the standard sharing rule that tells us how the joint surplus of a match is split between a worker and a firm,

$$\frac{\mathcal{W}_t^{h,f}}{\mathcal{V}_t^f} = \frac{\xi}{1-\xi}. \tag{21}$$

2.5 Aggregation and Market Clearing

We make the assumption that households and firms share the same set of beliefs about the future. This assumption is reasonable because a) firms are equally owned by households, b) we work in an environment with large households, which also ensures that members of the households coordinate of expectations, c) firms' employees are members of the household and d) wage negotiations guarantee that each worker-firm pair coordinates on expectations. All these frequent and economic interactions between households and firms helps us simplify the analysis from the viewpoint of expectation formation. The fact that we assume that expectation are homogeneous across households and firms, $\tilde{E}_t^h = \tilde{E}_t^f = \tilde{E}_t$, results in a symmetric equilibrium. i.e., $n_t^h = n_t^f = n_t$, $c_t^h = c_t$ and $v_t^f = v_t$ for all h, f and t.

The market clearing condition in the goods market requires to sum up the period budget constraints and period profits over both $h$and f reads

$$c_t + \kappa v_t = y_t. \tag{22}$$

Thus, we can integrate (11) and (20) over h and f to get the following two aggregate equilibrium conditions

$$\frac{\widehat{c}_t}{1-\beta} = \bar{n}\widehat{w}_t + \bar{w}\widehat{n}_t + \widehat{\pi}_t + \mathcal{S}_{w,t}^H + \mathcal{S}_{n,t}^H + \mathcal{S}_{\pi,t}^H - \mathcal{S}_{r,t}^H \tag{23}$$

and

$$\bar{\lambda}_1\bar{q}\widehat{v}_t = \beta\bar{\lambda}_1(1-\rho)\widehat{n}_t + \left[\frac{\kappa}{\bar{q}^2} + \beta\bar{\lambda}_1\bar{v}\right]\widehat{q}_t + \mathcal{S}_{z,t}^F + \mathcal{S}_{n,t}^F - \mathcal{S}_{w,t}^F - \mathcal{S}_{r,t}^F, \tag{24}$$

where

$$\begin{aligned}
\mathcal{S}_{w,t}^H &= \sum_{j=1}^{\infty}\beta^j\bar{n}\tilde{E}_t\widehat{w}_{t+j}, & \mathcal{S}_{n,t}^H &= \sum_{j=1}^{\infty}\beta^j\bar{w}\tilde{E}_t\widehat{n}_{t+j},\\
\mathcal{S}_{\pi,t}^H &= \sum_{j=1}^{\infty}\beta^j\tilde{E}_t\widehat{\pi}_{t+j}, & \mathcal{S}_{r,t}^H &= (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\left[\beta^{j+1}\tilde{E}_t\sum_{i=0}^{j-1}\widehat{r}_{t+i}\right],\\
\mathcal{S}_{z,t}^F &= \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_2\tilde{E}_t\widehat{z}_{t+j}, & \mathcal{S}_{n,t}^F &= \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\tilde{E}_t\widehat{n}_{t+j},\\
\mathcal{S}_{w,t}^F &= \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t\widehat{w}_{t+j}, & \mathcal{S}_{r,t}^F &= (\bar{z}\bar{\lambda}_2-\bar{w})\sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^{j+1}\sum_{i=0}^{j-1}\tilde{E}_t\widehat{r}_{t+i}.
\end{aligned}$$

To obtain (23) we make the assumption that initial financial wealth of all households is zero. Since all agents make the symmetric decisions, then aggregate financial wealth is also zero in subsequent periods. The $\mathcal{S}_t$ variables denote the discounted sums of future forecast and are key for understanding the dynamic properties of the model that we shall discuss at a later stage.

To obtain the aggregate wage equation, we replace $\mathcal{W}_t^h$, $\mathcal{V}_t^f$ into (21) and impose symmetric beliefs across firms and household,

$$w_t^{h,f} = \xi\alpha z_t(n_t^f)^{\alpha-1} + \xi\kappa\theta_t + (1-\xi)\chi c_t^h,$$

The bargained wage is thus a weighted average between the marginal productivity of the additional worker plus the cost of replacing the worker and the opportunity cost of working. We then linearise the above equation around steady state and then integrate it over h and f in order to find an expression for the aggregate wage

$$\widehat{w}_t = \xi\bar{\lambda}_2\widehat{z}_t + \xi\bar{\lambda}_1\widehat{n}_t + \xi\kappa\widehat{\theta}_t + (1-\xi)\,\chi\widehat{c}_t. \tag{25}$$

Similarly, we linearise the unemployment rate (7), the production function (12), the profit function (14), the law of motion of employment (16), the good market clearing condition (22) and then integrate over h and f to get the following conditions

$$\widehat{n}_t + \widehat{u}_t = 0, \tag{26}$$

$$\widehat{y}_t = \bar{n}^{\alpha}\widehat{z}_t + \bar{z}\alpha\bar{n}^{\alpha-1}\widehat{n}_t, \tag{27}$$

$$\widehat{\pi}_t = \bar{z}\alpha\bar{n}^{\alpha-1}\widehat{n}_t + \bar{n}^{\alpha}\widehat{z}_t - \bar{n}\widehat{w}_t - \bar{w}\widehat{n}_t - \kappa\widehat{v}_t, \tag{28}$$

$$\widehat{n}_{t+1} = (1-\rho)\,\widehat{n}_t + \bar{v}\widehat{q}_t + \bar{q}\widehat{v}_t,. \tag{29}$$

and

$$\widehat{c}_t + \kappa\widehat{v}_t = \widehat{y}_t. \tag{30}$$

Moreover, we linearise equations (2), (3) and (13) to get

$$\bar{\theta}\widehat{u}_t + \bar{u}\widehat{\theta}_t = \widehat{v}_t, \tag{31}$$

$$\widehat{q}_t = (\sigma-1)\,m\bar{\theta}^{\sigma-2}\widehat{\theta}_t \tag{32}$$

and

$$\widehat{z}_{t+1} = \varrho\widehat{z}_t + \epsilon_{t+1}. \tag{33}$$

Hence, a stationary competitive equilibrium is a set of stationary processes $\widehat{c}_t$, $\widehat{y}_t$, $\widehat{v}_t$, $\widehat{w}_t$, $\widehat{r}_t$, $\widehat{\pi}_t$, $\widehat{n}_t$, $\widehat{u}_t$, $\widehat{\theta}_t$, $\widehat{q}_t$ that, given the exogenous stochastic process $\{z_t\}_{t=0}^{\infty}$ and the initial conditions z_t, n_t and s_{t-1}, satisfy the system of equations consisting of (23)-(33).

2.6 Infinite-Horizon Learning

We deviate from the rational expectations hypothesis and assume instead that agents forecast the values of the variables of interest based on their own belief systems by running simple regressions behaving as if they were econometricians. We follow Eusepi and Preston (2011) in that the productivity process, equation (13), is both observable and known to all agents. We further assume that agents have complete knowledge about the steady state as indicated in (11) and (20) but incomplete knowledge about the dynamics because a) do not fully understand how the productivity shocks influence the future wage bargaining processes and b) believe that productivity shocks can potentially generate long-term drifts because the observed productivity process is near unit root.[3] Agents observe their own objectives, their constraints as well as the realisations of aggregate variables and prices but have no knowledge about the other agent's preferences and beliefs. They simply do not know that their decisions are identical to those of the other agents.

Agents' beliefs over the behaviour of n_t, r_t, w_t, and π_t are expressed in levels rather than as deviations from their respective steady state values. [4] In particular, we assume that agents have the following Perceived Law of Motions (PLMs)

$$n_{t+1} = b_n + a_{nn}n_t + \mu_{nt},$$

$$x_t = b_w + a_{wn}n_t + \mu_{xt}, \quad \text{for} \quad x_t = \{\pi_t, w_t, r_t\}.$$

[3]One may also follow Mitra et al. (2013) to let agents learn about steady state and to study policy changes.
[4]This assumption is innocuous to the results.

where μ_t is a noise. Note that this belief system depends on the employment rate, n_t, defined as $n_t = 1 - u_t$, so agents beliefs therefore depend on the unemployment rate. In terms of the specification of the belief system, the key difference with the rational expectation solution is that the variable $\widehat{z}_t$ is ommitted. This is a reasonable assumption because TFP innovations are usually retrieved as residuals from the view point of the econometrician. This assumption is central to our results because it generates systematic and persistent mistakes in agents' forecasts.[5]

Agents update their beliefs over time by revising the value of the a parameters through a Recursive Least Square algorithm as new information becomes available to them. At the beginning of each period, agents inherit the parameters of their belief system from the previous period, make forecasts and compute the present discounted sums that allows them to make consumption and vacancy posting decisions at every point in time. The interest rate is determined simultaneously with all other agents' economic decisions to clear both goods and labour markets. The wage rate is determined through the process of Nash bargaining between each firm and worker pair. At the end of each period, agents are informed about factor prices, (un)employment and profits and they then update their beliefs accordingly in the following period.

Let $A_t = \begin{pmatrix} A_{n,t} & A_{w,t} & A_{r,t} & A_{\pi,t} \end{pmatrix}$, where $A_{n,t} = \begin{pmatrix} b_n & a_{nn} \end{pmatrix}'$, $A_{w,t} = \begin{pmatrix} b_w & a_{wn} \end{pmatrix}'$, $A_{r,t} = \begin{pmatrix} b_r & a_{rn} \end{pmatrix}'$, $A_{\pi,t} = \begin{pmatrix} b_\pi & a_{\pi n} \end{pmatrix}'$, $C_t = \begin{pmatrix} n_t & w_{t-1} & r_{t-1} & \pi_{t-1} \end{pmatrix}'$ and $B_t = (1, n_t)'$. As in Evans and Honkapohja (2001), the we use that agents use a Recursive Least Square (RLS) algorithm to update their beliefs

$$A_t = A_{t-1} + \gamma R_t^{-1} B_{t-1} \left(C_t - A_{t-1}' B_{t-1} \right)',$$

where the precision matrix reads as

$$R_t = R_{t-1} + \gamma \left(B_{t-1} B_{t-1}' - R_{t-1} \right)$$

and $\gamma \in (0,1)$ denotes the constant gain learning parameter. A higher value of γ implies that agents put more weight on current information relative to past information.

3 Learning, Incomplete Knowledge and Amplification

In this section we study the transmission mechanism of TFP innovations to disentangle the source of the amplification in the model. We first describe the transmission mechanism of shocks in the standard search and matching model with RE beliefs in order to understand the source of amplification in the model with IH learning and incomplete knowledge.

A TFP shock has well-understood implications in the search and matching model. Positive TFP innovations shift the production frontier and the labour demand schedule, by increasing labour productivity and by raising marginal profits per hire. Since wages in the standard model are negotiated efficiently through the process of Nash bargaining, the shift in technology increases both the marginal revenues as well as current and future wages per hire. Firms' profits are in turn positively related to the expected returns of a match and negatively related to future costs. Firms would like to hire in the margin so long as the expected profits per hire increase. In the RE model however wages absorb most of the shift in productivity and firms' profits tend to increase only marginally, leading to little incentive for vacancy creation.

[5] The fully-specified beliefs consist of simply adding the TFP innovation as an additional explanatory variable to the above regressions. Hence, the fully-specified model consist of the following set of equations

$$n_{t+1} = b_n + a_{nn} n_t + a_{nz} \widehat{z}_t + \mu_t,$$

$$x_t = b_x + a_{xn} n_t + a_{xz} \widehat{z}_t + \mu_t, \quad \text{for} \quad x_t = \{\pi_t, w_t, r_t\}.$$

To gain on intuition we reproduce the job creation condition in linearised form[6]. Without loss of generality, we set the value of α equal to 1 and make the simplifying assumption that the stochastic discount factor plays only a minor role for vacancy posting

$$-\frac{\widehat{q}_t}{\bar{q}} \approx \frac{\bar{q}}{\kappa}\tilde{E}_t \sum_{j=1}^{\infty} (1-\rho)^{j-1} \beta^j \left[\widehat{z}_{t+j} - \widehat{w}_{t+j}\right]. \tag{34}$$

This expression states that the cost of hiring in the margin must be equal to the marginal benefit of hiring, which consists of the distance between marginal returns and costs in present discounted terms. Given the parameters of the model, we then conjecture that the RE solution of the linearised model is given by

$$\begin{aligned}
\widehat{n}_{t+1} &= \bar{a}_{nn}\widehat{n}_t + \bar{a}_{nz}\widehat{z}_t, \\
\widehat{w}_t &= \bar{a}_{wn}\widehat{n}_t + \bar{a}_{wz}\widehat{z}_t, \\
\widehat{z}_{t+1} &= \varrho\widehat{z}_t + \epsilon_{t+1},
\end{aligned}$$

where $0 < \bar{a}_{xn} < 1$ and $0 < \bar{a}_{xz} < 1$ denote the elasticities of the variable x with respect to employment and productivity respectively. Substituting the RE solution into the job creation condition gives the following expression

$$-\frac{\widehat{q}_t}{\bar{q}} \approx \frac{\bar{\beta} q}{\kappa}\left\{\frac{\varrho}{1-\beta(1-\rho)\varrho}\widehat{z}_t - \frac{\bar{a}_{wn}\bar{a}_{nn}}{1-\beta(1-\rho)\bar{a}_{nn}}\widehat{n}_t - \frac{\bar{a}_{wn}\bar{a}_{nz} + \varrho\bar{a}_{wz}(1-\beta(1-\rho)\bar{a}_{nn})}{[1-\beta(1-\rho)\bar{a}_{nn}][1-\beta(1-\rho)\varrho]}\widehat{z}_t\right\}.$$

Two important remarks follow from a first inspection of this expression. First, the vacancy posting decision responds to shifts in productivity, $\widehat{z}_t$, but not necessarily in employment, $\widehat{n}_t$, because the latter only affects the vacancy posting with a lag. Second, the absolute values of the two elasticities with respect to $\widehat{z}_t$ in this expression are indeed very close, which indeed reflects the fact that productivity innovations have very little impact on the present discounted value of profits per hire. When firms correctly forecast future revenues and wages per hire, the variability of profits per hire is dampened because wages, due to the assumption of efficient Nash bargaining, tend to absorb large part of the productivity innovation. As a result, the RE model is unable to generate sufficient amplification in vacancies.

By building on this result, we argue that the present discounted value of profits per hire are largely responsive to shifts in productivity in the IH learning model with incomplete knowledge. Relative to the RE model, firms tend to overestimate the future path of profits per hire by underestimating wage costs. This is due to the fact that under IH learning small forecasting errors compound into large errors when the time horizon of the forecasts is increased. An alternative learning mechanism such as Euler Equation (EE) learning, which only requires agents to make one-step ahead forecasts as in model with RE beliefs, cannot match the amplification generated under IH learning. EE learning combined with incomplete knowledge generates one-step ahead forecast errors are indeed too small to generate sufficient volatility[7]. This feature of the model can also be interpreted as hiring firms becoming overoptimistic about the future profitability of a match, which is driven purely by the misspecification in their belief system. This measure of the misspecification in the job creation condition is given by

$$\frac{\bar{a}_{wn}\bar{a}_{nz} + \varrho\bar{a}_{wz}(1-\beta(1-\rho)\bar{a}_{nn})}{[1-\beta(1-\rho)\bar{a}_{nn}][1-\beta(1-\rho)\varrho]}\widehat{z}_t.$$

[6]The vacancy creation condition in the standard model is written in terms of one-step ahead forecasts about profits per hire and labour market conditions. As shown earlier, this condition is equivalent to making an infinite-horizon forecast of future profits per hire.

[7]Unlike the IH learning approach, EE learning assumes that agents make inter-temporal choices decisions based on the first order difference equations (the Euler equation and vacancy posting condition in our model), characterised in the RE solution. This means that under EE learning agents only need to make only one-step ahead forecasts of the relevant variables. For prominent studies with EE learning, please refer to Bullard and Mitra (2002) and Evans and Honkapohja (2003). For a detailed derivation of the EE learning model, refer to the Appendix.

Since period wages are negotiated efficiently, wages do however increase directly after the shock hits the economy and indirectly through changes in labour market conditions. One feature of our model is that it exhibits a dichotomy between period wages and future wages in that we assume that agents are unable to understand the complexity involved in future wage negotiations and run simple regression models to forecast future wages. The misperception that profits increase largely with the arrival of a TFP innovation leads firms to post an increasingly large number of vacancies. Analytically, the job creation condition under IH learning with incomplete knowledge reads as follows

$$-\frac{\widehat{q}_t}{\bar{q}} \approx \frac{\beta\bar{q}}{\kappa}\left[\frac{\varrho}{1-\beta(1-\rho)\varrho}\widehat{z}_t - \frac{a_{wn}a_{nn}}{1-\beta(1-\rho)a_{nn}}\widehat{n}_t\right].$$

A simple comparison of the elasticities of the TFP innovations between the baseline model and the RE model can give a sense of the relative amplification. It is straight forward to show that the impact of a TFP innovation is greater in the IH model with incomplete knowledge relative to the RE model. More precisely, the job creation multiplier with respect to z_t is

$$\frac{\varrho}{\varrho - \bar{a}_{wn}\bar{a}_{nz} - \varrho\bar{a}_{wz}\left[1-\beta(1-\rho)\bar{a}_{nn}\right]} \quad \text{with} \quad \varrho \neq \bar{a}_{wn}\bar{a}_{nz} + \varrho\bar{a}_{wz}\left[1-\beta(1-\rho)\bar{a}_{nn}\right],$$

times larger than its RE counterpart. This means that the impact multiplier is greater than 1 so long as $0 < \bar{a}_{wn}\bar{a}_{nz} + \varrho\bar{a}_{wz}\left[1-\beta(1-\rho)\bar{a}_{nn}\right] < \rho$.

There is a trade-off between current and expected wages. Period wages tend to increase because workers are more productive, the labour market is tighter and households consume higher amounts. Note however that only the forecast of future wage negotiations enters the job creation condition and affects job creation. This means that, even though period profits per hire fall after a TFP innovation, the present discounted value of future returns per hire increase relative to future wage costs.

The system is however self-referential in the sense that beliefs about the future path of wages tend to have an impact on future vacancy creation and employment. Shifts in the current economic conditions in turn affect the belief system and so on. From the job creation condition, it follows that the updated values of a_{wn} and a_{nn} have an impact on job creation through shifts in employment in subsequent periods. Learning takes care of the persistence in the productivity process that is omitted in agents' beliefs. A positive productivity innovation leads to an increase in employment, which in turn raises the costs of future employment relations. In addition, a productivity innovation tends to increase the values of a_{nn} and a_{wn} in subsequent periods, increasing the expected wage costs. The updated forecasts of future wages exhibit large variation from period to period due to the fact that a_{nn} and a_{wn} are updated as new information becomes available. Larger variation in the forecast of wages in turn generates larger amplification in profits per hire.

4 Numerical Results

4.1 Calibration

We set the values of the parameters of the model following a standard calibration exercise. First, we choose some parameter values using *a priori* information. Second, the choice of the remaining parameters ensures that the stationary equilibrium of the model matches a number of stylised facts as observed in the post-WWII US economy. One period of time in our model corresponds to one month in the data.

The parameters that are set using *a priori* information are the subjective discount factor, β, the exogenous separation rate, ρ, the worker's bargaining power, ξ, and the elasticity of the matching function with respect to vacancies, σ. The value of β is set to 0.996, which implies an annual real interest rate of about 4%. The value of ρ is calibrated to 0.033 in order to match the evidence that jobs last on average for two years and a half as estimated in Davis et al. (1996).

Table 1: Calibrated Parameter Values - Monthly

Description	Parameter	Value
Discount Factor	β	0.996
Parameter in the Utility Function	χ	0.528
Efficiency of the Matching Technology	$\bar{m}$	0.566
Elasticity of the Matching Function	ξ	0.5
Bargaining Power	σ	0.5
Unitary Vacancy Posting Cost	κ	0.192
Separation Rate	ρ	0.033
Elasticity of Output w.r.t. Employment	α	1
Productivity Level	$\bar{z}$	1
Persistence of Productivity Shocks	ϱ	0.98
St. Dev. of Productivity Shocks	ς	0.01
Gain Parameter	γ	0.002

We set the value of σ in line with the literature. This value lies within the plausible interval of $[0.5, 0.7]$ as surveyed by Petrongolo and Pissarides (2001). In order to facilitate comparability with the existing literature, ξ is chosen to be equal to 0.5. The choice of the values of σ and ξ ensures that Hosios (1990) condition is met. The elasticity of output with respect to employment, α, is set to 1, which corresponds to a constant returns to scale technology in the process of production. We calibrate the persistence of the technology shock, ϱ_a, to 0.98 and the standard deviation of the productivity shocks to be 0.01 in line with Shimer (2010).

The remaining three labour market parameters, namely the unitary vacancy cost, κ, the efficiency of the matching function, $\bar{m}$, and the disutility parameter, χ, are set to match: *i*) a vacancy cost to output ratio of 0.01 in line with Andolfatto (1996), Gertler and Trigari (2009) and Blanchard and Gali (2010); *ii*) a vacancy filling rate of 27.8% as estimated by Shimer (2005), which is consistent with a quarterly rate of 70% as in Trigari (2006) and den Haan et al. (2000); *iii*) an unemployment rate of 6%, which corresponds with the standard ILO definition of unemployment for the post-WWII US average.

The resulting replacement ratio – computed as the disutility of work over the wage (χ/w) – is equal to 0.89%, which is above the value of 70% suggested by Mortensen and Nagypal (2007). According to a study by Hagedorn and Manovskii (2008), a total replacement ratio of around 95% can generate labour market fluctuations that are in line with the empirical evidence. Their study argues that, if the outside option of the worker is high (this happens when both ξ is low and the replacement ratio high), then the size of firm's steady state profits is small and can generate greater amplification in labour market variables. The replacement ratio in their study is equal to the value of non-market activity that includes both unemployment subsidies and the value of leisure. Costain and Reiter (2008) have criticised this calibration because it gives rise to excessive sensitivity of unemployment to unemployment benefits. Although unemployment benefits are absent in our model, our prior belief is that, if such subsidies were to be introduced, a greater deal of amplification could potentially be generated. The resulting replacement ratio ensures that our results are not driven by the Hagedorn and Manovskii effect. This will become evident once we examine the amplification properties of the model under RE beliefs. Table 1 provides a summary of the parameters used in the baseline calibration of our hypothetical model economy.

We choose the gain parameter in the learning algorithm to be $\gamma = 0.002$ (equivalent to a value of 0.006 in the corresponding quarterly model), which implies that agents use past data to update their beliefs for roughly 42 years ($1/0.002 = 500$ month). There is lack of consensus in the learning literature concerning the constant gain parameter, which ranges from 0.002 to 0.035 at quarterly frequencies. See for example Eusepi and Preston (2011), Branch and Evans (2006b), Milani (2007) and Orphanides and Williams (2007). The value chosen for this parameter is

Table 2: Summary Statistics, quarterly US data.

		y	n	v	u	θ	c
σ_x/σ_y		1	0.55	9.25	8.65	17.51	0.79
Autocorrelation		0.81	0.89	0.90	0.88	0.90	0.80
Correlation matrix	y	1	0.81	0.85	-0.79	0.84	0.86
	n	-	1	0.91	-0.97	0.96	0.66
	v	-	-	1	-0.91	0.98	0.71
	u	-	-	-	1	-0.98	-0.62
	θ	-	-	-	-	1	0.68
	c	-	-	-	-	-	1

Notes. Standard Deviations and correlations in this table correspond to quarterly series, detrended using a Hodrick-Prescott filter with smoothing parameter of 1600.

relatively small value because we exclude from policy considerations but lies within the range of parameters suggested in the literature. In a robustness exercise, we show that the choice of the gain parameter is not central for our results.

4.2 US Data

In this section we compare the main statistical properties of the simulated labour market series, particularly focusing on second moments, against the US data. It is standard practice, in one-worker-one-firm models, to compute the standard deviations of labour market data relative to productivity, but, in large-firm models, the standard measures that capture the volatilities of the data are typically divided by the corresponding measure of output. Since our model is a large-firm model of the labour market, we take the latter approach.

The seasonally adjusted series on (un)employment is taken from the Bureau of Labour Statistics (BLS). As a proxy for vacancies we merge the seasonally adjusted help-wanted advertising index released by the Conference Board with vacancy series calculated by Barnichon (2010). Aggregate output is measured as seasonally adjusted real GDP, which is drawn from the National Income and Product Account (NIPA) tables 1.1.6 and 1.1.5. All data series are quarterly and cover the period ranging from 1951Q1 to 2011Q4. Table 2 summarises the main cyclical properties of the logged detrended series.[8]

One of the most salient features in the data is the high cyclicality of unemployment, vacancies and labour market tightness as reported in Table 2. In particular, both vacancies and unemployment are about 9.25 and 8.65 times more volatile than the aggregate output respectively. Moreover, the measure of labour market tightness is around 17.51 times more volatile than output. As pointed out by Shimer (2005), the standard search and matching model with RE beliefs fails to match the data along this dimension. Another well known stylised fact in labour market dynamics is the negative relationship between vacancies and unemployment, also known as the Beverigde curve.

4.3 Simulation results

We initiate the simulation of our baseline model from the RE solution and then generate series for 3932 periods using the learning algorithm previously stated. We employ a projection facility – rarely used in our simulations – that ensures that the model converges in the limit to a stable and stationary solution also known in the learning literature as the *self-confirming equilibrium/misspecification* equilibrium. We then discard the first 3200 periods and keep the remaining 732 observations, which correspond to 61 years of data, so as to guarantee that the simulated

[8]We use the Hodrick-Prescott filter with a smoothing parameter of 1600 to detrend the series.

Table 3: Summary Statistics, IH with Incomplete Knowledge

		y	n	v	u	θ	c
σ_x/σ_y		1	0.50	10.31	7.83	17.19	0.93
Autocorrelation		0.95	0.93	0.57	0.93	0.85	0.94
Correlation matrix	y	1	0.94	0.76	-0.94	0.88	1.00
	n	-	1	0.79	-1.00	0.93	0.93
	v	-	-	1	-0.79	0.96	0.71
	u	-	-	-	1	-0.93	-0.93
	θ	-	-	-	-	1	0.85
	c	-	-	-	-	-	1

Notes. Relative standard deviations, autocorrelations and correlation coefficients in this table correspond to the quarterly simulated series expressed in percentage deviations from the steady state values.

Table 4: Learning, Knowledge and Amplification

	σ_n/σ_y	σ_v/σ_y	σ_u/σ_y	σ_θ/σ_y	σ_c/σ_y	σ_w/σ_y
Complete Knowledge (CK)						
RE	0.03	0.50	0.43	0.91	1.01	0.98
IH Learning	0.03	0.50	0.43	0.91	1.01	0.98
EE Learning	0.03	0.50	0.43	0.91	1.01	0.98
Incomplete Knowledge (IK)						
EE Learning	0.17	3.08	2.60	5.58	0.98	1.26
IH Learning	0.50	10.31	7.97	17.91	0.93	1.94

Notes. Relative standard deviations in this table correspond to the quarterly simulated series expressed in percentage deviations from the steady state values.

series are free from any transitional dynamic considerations. We then repeat this procedure 2500 times to ensure that our results are not contaminated by single random draws. Since our model is calibrated for monthly frequencies and the US data is reported only in quarterly frequencies, we then convert the simulated series from monthly frequencies into quarterly frequencies. Finally, we transform the simulated series from deviations into percentage deviations by simply dividing by the corresponding steady state values.

Table 3 reports the statistical properties of the simulated series of interest under Infinite Horizon (IH) learning and Incomplete Knowledge (IK). The table shows that the search and matching model with IH learning can replicate the second moments of the US labour market remarkably well. Vacancies and unemployment are about 10.31 and 7.83 times more volatile than output respectively. We find that, although the model underestimates the relative volatility of unemployment and overestimates the relative volatility of vacancies only slightly, the general performance of the model in terms the amplification is highly satisfactory. This can be seen from comparing the relative standard deviation of labour market tightness, which is 17.89 in the model with the 17.51 in the data. Hence, we argue that replacing the conventional RE assumption with a form of adaptive learning in which agents have incomplete knowledge has the potential to generate substantial amplification in vacancies, unemployment and the measure of labour market tightness. It follows from table 3 that vacancies display a slightly lower persistence relative to the data (and to the RE model).

In order to check how our simulations compare with the standard results, we simulate a structural search and matching model but instead endow the economic agents first with RE beliefs and then with adaptive learning and complete knowledge.[9] Table 4 shows that the RE model generates very little amplification in labour market variables. Moreover, the table also shows the relative standard derivations of the variables of interest under the different expectational and knowledge assumptions. We find that endowing agents with complete information under both IH and EE learning can only perform as well as the RE model. The table also shows that the assumption of incomplete knowledge per se can generate more amplification relative to the RE model but falls short of matching the US labour data. The reason behind this finding has to do with the fact that, while in the IH model with Incomplete Knowledge (IK) small mistakes compound into large mistakes, the EE specification of the model produces forecast errors that last only for one period. The IH model with IK generates large compounded errors that are at the source of amplification. This can be interpreted as large waves of pessimism and optimism that affect vacancy posting decisions. Although not reported in the table below, all models do a reasonable good job at predicting the negative correlation between vacancies and unemployment.

4.4 Expectational Errors

The forecast of future variables plays a key role in our model since both households and firms must form their expectations of future (un)employment rates, wages, interest rates and profits so as to make consumption and job posting choices. As shown in Section 3, the expectational errors are central in matching the amplification of labour market variables with the data. In this section we would therefore like to analyse how well the statistical properties of the forecast errors generated by our model compared with those in the data as reported by the Survey of Professional Forecasters.

As indicated by the PLMs, agents forecast all future variables based on their expectations of future (un)employment rates. We can therefore compare the performance of the forecast errors of future (un)employment rates generated in the model relative to the data. We take the quarterly forecasts of unemployment rates from the Survey of Professional Forecasters, which are collected by the Federal Reserve Bank of Philadelphia. The individual forecasts are released in the middle month of each quarter using information at the beginning of the quarter, date when the questionnaires are sent to the individual forecasters to carry out their expert forecasts. Forecast data are available from 1968Q4 to 2011Q4 and both mean and median of the forecasts are reported up to four quarters ahead. We compute the forecast errors by removing from the forecast data the actual realisation of the unemployment rates up to four periods ahead using BLS data. We then divide the resulting error by the average unemployment rate over the period, which is equal to 6.26%.

Table 5 reports the statistical properties of the forecast errors, including their relative volatility with respect to the HP series of output, the autocorrelation of the forecast errors and their correlation with output growth and the first difference of unemployment. We find that the forecast errors are much more volatile relative to output over the 4-quarter horizon. The table shows that the one quarter-ahead forecast errors is almost 6 times more volatile than output. In addition, the quarterly forecast errors display high serial correlation, which is stronger when the information sets available at the time of the forecast contains the same amount of information. Furthermore, over the business cycle, the forecast errors on unemployment rate are all pro-cyclical, which means agents tend to over-predict unemployment during expansions and under-predict in recessions.

Table 5 also reports the statistical properties of the simulated forecast errors expressed in percentage deviations from the steady state unemployment rate under both RE and IH learning with Incomplete Knowledge. The simulation methods used here are essentially the same as described in previous subsection, with the exception that the simulation horizon after

[9] The RE model is solved following Sims (2002).

Table 5: Forecast Properties

		FE_t^{Q1}	FE_t^{Q2}	FE_t^{Q3}	FE_t^{Q4}
	σ_x/σ_y	4.86	6.70	8.59	10.44
	$\rho\left(x_t, x_{t-1}\right)$	0.69	0.78	0.84	0.88
	$\rho\left(x_t, x_{t-2}\right)$	0.34	0.40	0.53	0.63
Data	$\rho\left(x_t, x_{t-3}\right)$	0.16	0.14	0.23	0.37
	$\rho\left(x_t, x_{t-4}\right)$	0.02	-0.01	0.04	0.14
	$\rho\left(x_t, \Delta u_t\right)$	-0.75	-0.60	-0.51	-0.43
	$\rho\left(x_t, \Delta y_t\right)$	0.53	0.51	0.48	0.44
	σ_x/σ_y	0.04	0.13	0.19	0.23
	$\rho\left(x_t, x_{t-1}\right)$	0.06	0.24	0.58	0.72
	$\rho\left(x_t, x_{t-2}\right)$	0.04	0.02	0.13	0.34
RE	$\rho\left(x_t, x_{t-3}\right)$	0.03	-0.01	-0.04	0.04
	$\rho\left(x_t, x_{t-4}\right)$	-0.02	-0.05	-0.04	0.03
	$\rho\left(x_t, \Delta u_t\right)$	-0.06	-0.04	-0.02	0.00
	$\rho\left(x_t, \Delta y_t\right)$	0.08	0.10	0.07	0.05
	σ_x/σ_y	1.81	6.55	8.13	8.87
	$\rho\left(x_t, x_{t-1}\right)$	0.39	0.66	0.84	0.87
	$\rho\left(x_t, x_{t-2}\right)$	0.28	0.55	0.66	0.74
IH IK	$\rho\left(x_t, x_{t-3}\right)$	0.27	0.42	0.53	0.59
	$\rho\left(x_t, x_{t-4}\right)$	0.23	0.38	0.45	0.50
	$\rho\left(x_t, \Delta u_t\right)$	-0.48	-0.34	-0.29	-0.26
	$\rho\left(x_t, \Delta y_t\right)$	0.56	0.42	0.34	0.30

Notes. The term $\rho(x_1, x_2)$ stands for the correlation coefficient between variables x_1 and x_2. Data from Survey of Professional Forecasters. Forecast errors are defined as $FE_t^{Qj} = E_t u_{t+j} - u_{t+j}$ for $j = 1, 2, 3, 4$.

convergence is now set to be 512 (months) - matching the time span of the available forecast data - and the unemployment rate set to 6.26%. The simulated monthly forecast errors series are also converted to quarterly frequencies by simple averaging. It follows from the table that the performance of the RE model at matching the statistical properties of the forecast errors in the data is rather poor. The RE model predicts a very low relative volatility of the forecast errors, negligible autocorrelation and no pro-cyclicality. The baseline model performs better relative to the RE model in matching the statistical properties of the forecast errors. The learning model generates a sizeable relative volatility of the forecast errors, strictly positive autocorrelations, positive correlation with the output growth and negative correlation with the first difference of unemployment. The one-quarter ahead forecast errors are more serially correlated and display a greater deal of amplification in the data relative to what the learning model suggests. These results are mainly due to the fact that the TFP innovations are highly persistent in a setting where agents make IH forecasts.

4.5 Impulse Responses

In this section we study how labour market variables respond to a 1% positive TFP shock and also carry out a comparative analysis with alternative models featuring different expectational and informational assumptions. In particular, we compare the performance of the baseline model against the model with RE beliefs and the model featuring EE learning and IK. The reason behind the choice of these has to do with the fact that a) we would like to compare how the model performs against the standard search and matching model and b) we would like to analyse whether the type of learning that matters for amplification.

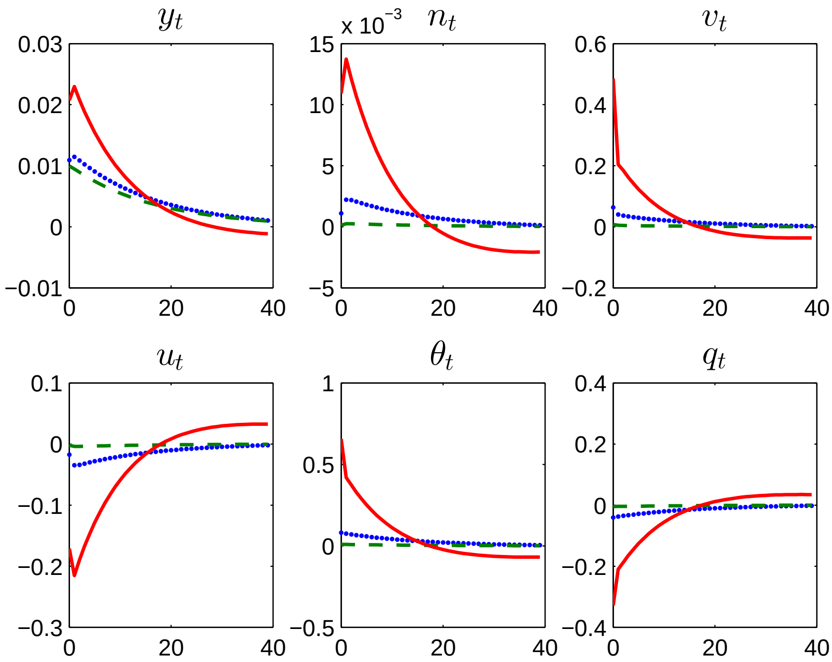
Following Eusepi and Preston (2011), the impulse responses of the two learning models under analysis are computed by simulating the model twice over 3200+120 periods. The first 3200 periods ensure convergence towards the *self-confirming/restricted perception/misspecification equilibrium.* We add to the first simulation a positive 1% productivity shock in period 3201 and compute the impulse responses as the distance between the two resulting set of impulse responses from the period 3201 onwards. This experiment is then repeated 2500 times. We then report the averaged impulse responses of the variable of interest. Furthermore, we convert the simulated series into quarterly frequencies and plot the percentage deviations of the variables from their corresponding steady state values.

Figure 1 shows the impulse responses of aggregate output, (un)employment, vacancies, labour market tightness and the job filling rate to a positive TFP innovation under learning and RE. Although not reported in the figure, we find that the responses under RE are indeed very similar to the responses generated in the models featuring adaptive learning and fully-specified beliefs. The impulse responses under RE display negligible amplification relative to the baseline model and less amplification relative to the EE learning model. Firms endowed with RE beliefs do not make any systematic mistakes when forecasting the future path of wages, which means that expected wages tend to absorb most of the productivity increase by offsetting the increase in revenues per hire brought about by the technology innovation. As a consequence, both unemployment and labour market tightness respond only marginally to a TFP innovation.

Very distinct dynamic responses are observed under learning and incomplete information to a positive TFP innovation. Following a positive technology shock, the incentive for vacancy creation increases sharply, leading to more employment and to a sharp fall in unemployment. As labour market tightens and the job finding probability falls, the marginal cost of posting a vacancy tends to increase, matching the rise in the marginal benefit associated with vacancy posting. The response in unemployment is however sluggish relative to the response in vacancies given that, as opposed to vacancies, unemployment is pre-determined and displays higher persistent. Vacancies fall sharply after the first quarter and its adjustment displays greater relative persistence. Vacancies are more responsive to a TFP innovation relative to consumption because a) households have access to financial instruments that allow them to smooth their consumption over time, b) the vacancy posting decision is akin to an investment decision in human capital and c) firm's make systematic mistakes by overestimating the present discounted value of profits per hire on impact. All variables revert back to their steady state under both RE and adaptive learning. Finally, the model featuring EE learning stands between the RE model and the baseline model. The dotted line in figure 1 shows that a positive TFP innovation increases the incentive for vacancy posting under EE learning because the rise in productivity is not fully reflected in the expectations of wages, generating more amplification in expected profits per hire.

Similar to Eusepi and Preston (2011), figure 1 shows that the combination of adaptive learning and incomplete knowledge increases the internal propagation mechanism of the model. In particular, the response of aggregate consumption and employment to a TFP innovation is greater than one-to-one in our model but around one-to-one under RE (and adaptive learning with fully-specified beliefs). i.e., the response of output to a TFP innovation of the same order of magnitude is much larger under the baseline model. The absence of physical capital, the introduction of employment and the high persistence of the productivity process can help explain the larger internal propagation mechanism of our model relative to Eusepi and Preston (2011). One point of difference with their model is that we consider persistent TFP shocks as opposed to productivity growth innovations, so learning internalises the lack of knowledge about the influence of persistence on agents' beliefs. The higher persistence of the TFP innovation in the job creation condition increases on impact the discount factor of future revenues per hire relative to the discount factor associated with future wages. The latter being simply a function of the way in which agents update their beliefs. This leads firms to overestimate the present discounted value of profits per hire, which then translates into further incentives for vacancy creation. Thus,

Figure 1: Impulse responses: Labour Market Variables



Notes. Impulse responses to a one percent increase in productivity are measured along the vertical axes in percentage deviations from the steady state. The horizontal axes display the number of quarters after the shock. The solid line shows the impulse responses under adaptive learning, the dotted lines denote the impulse responses under EE learning with incomplete knowledge and the dashed lines the impulse responses under RE.

the labour market becomes tighter and the job filling rate falls after a TFP innovation, which then increases the costs of posting a vacancy in equilibrium. More vacancies in equilibrium translate into higher employment and production. The second point of difference is that the absence of capital, together with the assumption of constant returns to scale in labour, implies that output displays relatively less endogenous persistence and it is relatively more responsive.

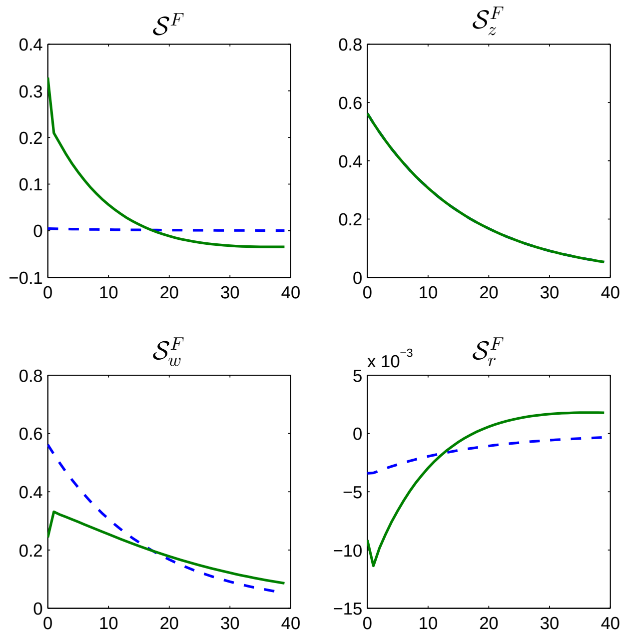
To disentangle the results under IH with IK, we plot the infinite sums of equation (24). As explained in the previous section, the infinite sum S_t^F, which is equal to the present discounted value of profits, responds much more strongly under baseline model relative to its RE. Figure 2 decomposes the infinite sum, S_t^F, into three components, $S_{z,t}^F$, $S_{w,t}^F$ and $S_{r,t}^F$. This figure shows that the order of magnitude of $\mathcal{S}_{r,t}^F$ is smaller relative to the $\mathcal{S}_{z,t}^F$ and $\mathcal{S}_{w,t}^F$. This means that, absent physical capital, the stochastic discount factor displays little variation relative to the other two infinite sums.[10] Moreover, the figure shows that the size of the response of $\mathcal{S}_{z,t}^F$ is much larger than that of $\mathcal{S}_{w,t}^F$ after a TFP innovation. The distance between $\mathcal{S}_{z,t}^F$ and $\mathcal{S}_{w,t}^F$ explains the strong amplification mechanism of our model and the high variation in vacancy posting on impact. The figure also shows that the sizes of the responses of the these sums under RE are of equal magnitudes, which helps explain the negligible amplification generated by standard search and matching model.

4.6 Robustness

In this section we check the sensitivity of our results to alternative parameterisations of our model. We first consider a calibration of the model with decreasing returns to scale in labour and then consider two values of the constant gain parameter.

[10]Note that this result is consistent with assumption made in the previous section.

Figure 2: Impulse responses: Infinite Sums in the Job Creation Condition



Notes. Impulse responses to a one percent increase in productivity are measured along the vertical axes in percentage deviations from the steady state. Horizontal axes display the number of quarters after the shock. The solid line shows the impulse responses with adaptive learning, the dotted lines denote the impulse responses under EE learning with IK and the dashed lines the impulse responses under RE.

The calibration of our baseline model assumes constant returns to scale in labour. We carry out an alternative calibration exercise to check whether our results depend on the choice of the elasticity of output with respect to employment. In line with models featuring physical capital, we set the value of α equal to 0.67 and simulate the model under this new parameterisation. The main conceptual difference with the baseline calibration is that the demand curve under decreasing returns to labour is downward sloping rather than constant. However, this alternative calibration does not change the transmission mechanism of TFP innovations because the effect of employment in the job creation condition, as stated in equation (24), has only a minor impact on the present discounted value of profits per hire. We find that the results from this simulation are in line with the simulations of our baseline model. The main difference is that setting a lower value of α reduces the replacement ratio to 0.79% and therefore reduces the amplification of labour market variables only slightly. Table 6 shows that labour market variables still display a great deal of variation relative to all other specifications.

We then consider two alternative values of the gain parameter so as to check for the sensitivity of our results to the way in which agents discount past information. We choose the following two parameters of γ: 0.001 and 0.004. Table 6 reports the relative standard derivations of labour market variables under the two alternative parameterisations. Our simulations show that the values of the gain parameter have very little influence in terms of the statistical properties of the simulated data because this parameter only affects the amount of time it takes for the model to reach the *self-confirming/restricted perception/misspecification equilibrium* but does not alter the overall dynamic properties of labour market variables. A smaller gain parameter indicates that agents put more weight on past data relative to current data to update beliefs, which means that time it takes for the effects of TFP innovations to vanish from agents information set is longer. The amplification results are therefore independent of the choice of the parameter γ.

Table 6: Robustness

	σ_n/σ_y	σ_v/σ_y	σ_u/σ_y	σ_θ/σ_y	σ_c/σ_y	σ_w/σ_y	$\sigma_{(E^*_{t-1}w_t)}/\sigma_y$
Baseline	0.50	10.31	7.97	17.91	0.93	1.94	1.15
Alternative parameterisation of γ							
CK ($\gamma = 0.004$)	0.03	0.50	0.43	0.91	1.01	0.98	0.96
IK ($\gamma = 0.001$)	0.50	10.73	7.88	17.47	0.93	1.96	1.15
IK ($\gamma = 0.004$)	0.48	10.71	7.59	16.96	0.94	1.89	1.04
Alternative parameterisation of α							
IK ($\alpha = 0.67$)	0.45	10.12	7.09	15.83	0.94	2.42	1.40

*N*otes. Relative standard deviations this table correspond to the quarterly simulated series expressed in percentage deviations from the steady state values under combinations of alternative parameterisations (γ and α) and knowledge (Complete and Incomplete Knowledge).

5 Conclusion

We develop a highly stylised model of the labour market with Infinite Horizon (IH) adaptive learning and we show that the model is able to match US labour data remarkably well in terms of its amplification. In the standard search and matching model the vacancy positing decision depends crucially on what firms expect the present discounted value of profits per hire to be. We relax both the assumptions of Rational Expectations (RE) and complete knowledge to study the role of expectation formation on job creation. We make the assumption that agents run simple regressions behaving as if they were econometricians, using the (un)employment rate as the main regressor, and update their beliefs as new information becomes available in order to make infinite horizon forecasts about future wages and to choose how many vacancies to post in equilibrium.

In particular, we find that Total Factor Productivity (TFP) innovations under IH learning and incomplete knowledge can generate greater incentive for vacancy creation relative to RE - and other adaptive learning specifications of the search and matching model with complete knowledge - and to Euler Equation (EE) learning. A TFP innovation under Infinite Horizon learning and incomplete knowledge increases the firms' present discounted value of profits per hire. Firms become overoptimistic after a positive TFP innovation because they make small systematic forecast errors over wages that compound into large errors as the time horizon of the forecast extends to infinity. We show that the model with Euler Equation learning generates more amplification relative to the RE counterpart, but it falls short of replicating the amplification observed in the US labour data. We find that the role of expectation formation plays a key role in hiring decisions and that incomplete knowledge can provide a solution to the *unemployment volatility puzzle*.

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A IH learning

A.1 Incomplete Knowledge

We define the following expressions

$$\begin{aligned}
\bar{n}_t^e &= \frac{b_n}{1-a_{nn}}, \\
\bar{w}_t^e &= b_w + a_{wn}\bar{n}_t^e, \\
\bar{r}_t^e &= b_r + a_{rn}\bar{n}_t^e, \\
\bar{\pi}_t^e &= b_\pi + a_\pi \bar{n}_t^e,
\end{aligned}$$

where $\bar{n}_t^e$, $\bar{w}_t^e$, $\bar{r}_t^e$ and $\bar{\pi}_t^e$ are the agents' perceived steady state values at period t. We also define as deviations of the variables from the perceived steady state

$$\begin{aligned}
\widehat{n}_t^e &= n_t - \bar{n}_t^e, \\
\widehat{w}_t^e &= w_t - \bar{w}_t^e, \\
\widehat{r}_t^e &= r_t - \bar{r}_t^e, \\
\widehat{\pi}_t^e &= \pi_t - \bar{\pi}_t^e.
\end{aligned}$$

In the model with incomplete knowledge, the Perceived Laws of Motion (PLMs) are

$$\begin{aligned}
\widehat{n}_{t+1}^e &= a_{nn}\widehat{n}_t^e, \\
\widehat{w}_t^e &= a_{wn}\widehat{n}_t^e, \\
\widehat{r}_t^e &= a_{rn}\widehat{n}_t^e, \\
\widehat{\pi}_t^e &= a_{\pi n}\widehat{n}_t^e.
\end{aligned}$$

Thus, future expectations about employment, wages, interest rates and profits can be written as

$$\begin{aligned}
\tilde{E}_t\widehat{n}_{t+j}^e &= a_{nn}^j\widehat{n}_t^e, \\
\tilde{E}_t\widehat{w}_{t+j}^e &= a_{wn}a_{nn}^j\widehat{n}_t^e, \\
\tilde{E}_t\widehat{r}_{t+j}^e &= a_{rn}a_{nn}^j\widehat{n}_t^e, \\
\tilde{E}_t\widehat{\pi}_{t+j}^e &= a_{\pi n}a_{nn}^j\widehat{n}_t^e.
\end{aligned}$$

We are now ready to derive the behavior rules of households and firm including the forms of the sums. Firstly, we derive the firms' vacancy posting rule, equation (20). The linearisation of equations (16) and (2.3) read

$$\widehat{n}_{t+1}^f = (1-\rho)\,\widehat{n}_t^f + \bar{v}\widehat{q}_t + \bar{q}\widehat{v}_t^f$$

and

$$\begin{aligned}
-\frac{\kappa}{\bar{q}^2}\widehat{q}_t &= -\frac{\alpha\bar{z}\bar{n}^{\alpha-1}-\bar{w}}{\bar{r}^2}\widehat{r}_t + \frac{1}{\bar{r}}\alpha\bar{z}(\alpha-1)\bar{n}^{\alpha-2}\widehat{n}_{t+1}^f + \frac{1}{\bar{r}}\alpha\bar{n}^{\alpha-1}\widehat{z}_{t+1} - \frac{1}{\bar{r}}\widehat{w}_{t+1}^f + \\
&\quad + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t^f\left[\alpha\bar{z}(\alpha-1)\bar{n}^{\alpha-2}\widehat{n}_{t+j}^f + \alpha\bar{n}^{\alpha-1}\widehat{z}_{t+j} - \widehat{w}_{t+j}^f\right] - \\
&\quad -(\alpha\bar{z}\bar{n}^{\alpha-1}-\bar{w})\beta^2\sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^{j-1}\sum_{i=1}^{j}\tilde{E}_t^f\widehat{r}_{t+i-1}
\end{aligned}$$

respectively. Combining the above expressions to eliminate $\widehat{n}_{t+1}$, we have equation (20) in the main text

$$\begin{aligned}\bar{\lambda}_1\bar{q}\widehat{v}_t^f &= \beta\bar{\lambda}_1(1-\rho)\widehat{n}_t^f + [\frac{\kappa}{\bar{q}^2} + \beta\bar{\lambda}_1\bar{v}]\widehat{q}_t + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t^f\bar{\lambda}_1\widehat{n}_{t+j}^f + \\ &+\sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t^f\left[\bar{\lambda}_2\widehat{z}_{t+j} - \widehat{w}_{t+j}^f - (\bar{z}\bar{\lambda}_2 - \bar{w})\beta\sum_{i=0}^{j-1}\widehat{r}_{t+i}\right],\end{aligned}$$

where $\bar{\lambda}_1 \equiv \alpha(\alpha-1)\bar{z}\bar{n}^{\alpha-2}$, $\bar{\lambda}_2 \equiv \alpha\bar{n}^{\alpha-1}$. We then integrate expression (20) over f to get expression (24) in the main text

$$\bar{\lambda}_1\bar{q}\widehat{v}_t = \beta\bar{\lambda}_1(1-\rho)\widehat{n}_t + \left[\frac{\kappa}{\bar{q}^2} + \beta\bar{\lambda}_1\bar{v}\right]\widehat{q}_t + \mathcal{S}_{z,t}^F + \mathcal{S}_{n,t}^F - \mathcal{S}_{w,t}^F - \mathcal{S}_{r,t}^F,$$

where

$$\mathcal{S}_{z,t}^F = \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_2\tilde{E}_t\widehat{z}_{t+j} = \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_2\varrho^j\widehat{z}_t = \frac{\beta\varrho\bar{\lambda}_2}{1-(1-\rho)\beta\varrho}\widehat{z}_t,$$

$$\begin{aligned}\mathcal{S}_{w,t}^F &= \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t\widehat{w}_{t+j} = \\ &= \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j(\bar{w}_t^e - \bar{w}) + \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t\widehat{w}_{t+j}^e = \\ &= \frac{\beta(\bar{w}_t^e - \bar{w})}{1-\beta(1-\rho)} + a_{wn}\sum_{j=1}^{\infty}\beta^j(1-\rho)^{j-1}a_{nn}^j\widehat{n}_t^e = \\ &= \frac{\beta(\bar{w}_t^e - \bar{w})}{1-\beta(1-\rho)} + a_{wn}\frac{\beta a_{nn}}{1-\beta(1-\rho)a_{nn}}\widehat{n}_t^e,\end{aligned}$$

$$\begin{aligned}\mathcal{S}_{n,t}^F &= \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\tilde{E}_t\widehat{n}_{t+j} = \\ &= \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1(\bar{n}_t^e - \bar{n}) + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\tilde{E}_t\widehat{n}_{t+j}^e = \\ &= \frac{\bar{\lambda}_1\beta^2(1-\rho)(\bar{n}_t^e - \bar{n})}{1-\beta(1-\rho)} + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1 a_{nn}^j\widehat{n}_t^e = \\ &= \frac{\bar{\lambda}_1\beta^2(1-\rho)(\bar{n}_t^e - \bar{n})}{1-\beta(1-\rho)} + \frac{\beta^2(1-\rho)a_{nn}^2\bar{\lambda}_1}{1-\beta(1-\rho)a_{nn}}\widehat{n}_t^e.\end{aligned}$$

To compute $\mathcal{S}_{r,t}^F$ firstly note that

$$\sum_{j=1}^{\infty}(1-\rho)^j\beta^j j = \frac{\beta(1-\rho)}{[1-\beta(1-\rho)]^2}$$

and that

$$\begin{aligned}\sum_{j=1}^{\infty}(1-\rho)^j\beta^j\frac{a_{nn} - a_{nn}^{j+1}}{1-a_{nn}} &= \frac{\beta(1-\rho)a_{nn}}{(1-a_{nn})[1-\beta(1-\rho)]} - \frac{\beta(1-\rho)a_{nn}^2}{(1-a_{nn})[1-\beta(1-\rho)a_{nn}]} = \\ &= \frac{\beta(1-\rho)a_{nn}}{[1-\beta(1-\rho)a_{nn}][1-\beta(1-\rho)]}.\end{aligned}$$

Thus,

$$
\begin{aligned}
\mathcal{S}^F_{r,t} &= \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^{j-1} \beta^{j+1} \sum_{i=0}^{j-1} \tilde{E}_t \widehat{r}_{t+i} = \\
&= \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^{j-1} \beta^{j+1} \widehat{r}_t + \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^{j} \beta^{j+2} \sum_{i=1}^{j} \tilde{E}_t \widehat{r}_{t+i} = \\
&= \frac{\bar{\lambda}_3 \beta^2}{1-\beta(1-\rho)} \widehat{r}_t + \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^j \beta^{j+2} (\bar{r}^e_t - \bar{r}) j + \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^j \beta^{j+2} \sum_{i=1}^{j} \tilde{E}_t \widehat{r}^e_{t+i} = \\
&= \frac{\bar{\lambda}_3 \beta^2}{1-\beta(1-\rho)} \widehat{r}_t + \frac{\bar{\lambda}_3 \beta^3 (1-\rho)(\bar{r}^e_t - \bar{r})}{[1-\beta(1-\rho)]^2} + \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^j \beta^{j+2} \sum_{i=1}^{j} a_{rn} a^i_{nn} \widehat{n}^e_t = \\
&= \frac{\bar{\lambda}_3 \beta^2}{1-\beta(1-\rho)} \widehat{r}_t + \frac{\bar{\lambda}_3 \beta^3 (1-\rho)(\bar{r}^e_t - \bar{r})}{[1-\beta(1-\rho)]^2} + \bar{\lambda}_3 \sum_{j=1}^{\infty} (1-\rho)^j \beta^{j+2} a_{rn} \frac{a_{nn} - a^{j+1}_{nn}}{1-a_{nn}} \widehat{n}^e_t = \\
&= \frac{\bar{\lambda}_3 \beta^2}{1-\beta(1-\rho)} \widehat{r}_t + \frac{\bar{\lambda}_3 \beta^3 (1-\rho)(\bar{r}^e_t - \bar{r})}{[1-\beta(1-\rho)]^2} + \frac{\bar{\lambda}_3 \beta^3 (1-\rho) a_{rn} a_{nn}}{[1-\beta(1-\rho)a_{nn}][1-\beta(1-\rho)]} \widehat{n}^e_t,
\end{aligned}
$$

where $\bar{\lambda}_3 \equiv \alpha \bar{z} \bar{n}^{\alpha-1} - \bar{w}$.

Now we compute the sums in the household's consumption behavioral rule. Recall the linearised life-time budget constraint, equation (11), in the main text,

$$
\frac{\widehat{c}_t}{1-\beta} = \bar{n}\widehat{w}_t + \bar{w}\widehat{n}_t + \widehat{\pi}_t + \mathcal{S}^H_{w,t} + \mathcal{S}^H_{n,t} + \mathcal{S}^H_{\pi,t} - \mathcal{S}^H_{r,t},
$$

where

$$
\begin{aligned}
\mathcal{S}^H_{\pi,t} &= \sum_{j=1}^{\infty} \beta^j \tilde{E}_t \widehat{\pi}_{t+j} = \sum_{j=1}^{\infty} \beta^j (\bar{\pi}^e_t - \bar{\pi}) + \sum_{j=1}^{\infty} \beta^j \tilde{E}_t \widehat{\pi}^e_{t+j} = \\
&= \frac{\beta(\bar{\pi}^e_t - \bar{\pi})}{1-\beta} + a_{\pi n} \sum_{j=1}^{\infty} \beta^j a^j_{nn} \widehat{n}^e_t = \frac{\beta(\bar{\pi}^e_t - \bar{\pi})}{1-\beta} + a_{\pi n} \frac{\beta a_{nn}}{1-\beta a_{nn}} \widehat{n}^e_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^H_{w,t} &= \sum_{j=1}^{\infty} \beta^j \tilde{E}_t \bar{n} \widehat{w}_{t+j} = \bar{n} \sum_{j=1}^{\infty} \beta^j (\bar{w}^e_t - \bar{w}) + \bar{n} \sum_{j=1}^{\infty} \beta^j \tilde{E}_t \widehat{w}^e_{t+j} = \\
&= \frac{\beta \bar{n} (\bar{w}^e_t - \bar{w})}{1-\beta} + \bar{n} a_{wn} \sum_{j=1}^{\infty} \beta^j a^j_{nn} \widehat{n}^e_t = \frac{\beta \bar{n} (\bar{w}^e_t - \bar{w})}{1-\beta} + \bar{n} a_{wn} \frac{\beta a_{nn}}{1-\beta a_{nn}} \widehat{n}^e_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^H_{n,t} &= \sum_{j=1}^{\infty} \beta^j \bar{w} \tilde{E}_t \widehat{n}_{t+j} = \beta \bar{w} \widehat{n}_{t+1} + \sum_{j=2}^{\infty} \beta^j \tilde{E}_t \bar{w} \widehat{n}_{t+j} = \\
&= \beta \bar{w} \widehat{n}_{t+1} + \bar{w} \sum_{j=2}^{\infty} \beta^j (\bar{n}^e_t - \bar{n}) + \bar{w} \sum_{j=2}^{\infty} \beta^j \tilde{E}_t \widehat{n}^e_{t+j} = \\
&= \beta \bar{w} \widehat{n}_{t+1} + \frac{\beta^2 \bar{w} (\bar{n}^e_t - \bar{n})}{1-\beta} + \bar{w} \sum_{j=2}^{\infty} \beta^j a^j_{nn} \widehat{n}^e_t = \\
&= \beta \bar{w} \widehat{n}_{t+1} + \frac{\beta^2 \bar{w} (\bar{n}^e_t - \bar{n})}{1-\beta} + \bar{w} \frac{\beta^2 a^2_{nn}}{1-\beta a_{nn}} \widehat{n}^e_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^H_{r,t} &= (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\left[\beta^{j+1}\sum_{i=0}^{j-1}\tilde{E}_t\widehat{r}_{t+i}\right] = \\
&= (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\beta^{j+1}\widehat{r}_t + (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}_{t+i}] = \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\beta^{j+2}(\bar{r}^e_t-\bar{r})j + (\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}^e_{t+i}] = \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + (\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}a_{rn}a^j_{nn}\widehat{n}^e_t] = \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + (\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}\beta^{j+2}a_{rn}\frac{a_{nn}-a^{j+1}_{nn}}{1-a_{nn}}\widehat{n}^e_t = \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3 a_{nn}a_{rn}}{(1-\beta a_{nn})(1-\beta)}\widehat{n}^e_t.
\end{aligned}
$$

A.2 Complete Knowledge

Now we consider learning rules that take the same form as the RE solution. The PLMs therefore read as

$$n_{t+1} = b_n + a_{nn}n_t + a_{nz}\hat{z}_t,$$

$$x_t = b_x + a_{xn}n_t + a_{xz}\hat{z}_t, \quad \text{for} \quad x_t = \{\pi_t, w_t, r_t\}.$$

We then redefine, $A_{n,t} = \begin{pmatrix} b_n & a_{nn} & a_{nz}\end{pmatrix}'$, $A_{w,t} = \begin{pmatrix} b_w & a_{wn} & a_{wz}\end{pmatrix}'$, $A_{r,t} = \begin{pmatrix} b_r & a_{rn} & a_{rz}\end{pmatrix}'$, $A_{\pi,t} = \begin{pmatrix} b_\pi & a_{\pi n} & a_{\pi z}\end{pmatrix}'$, $B_t = \begin{pmatrix} 1 & n_t & \widehat{z}_t\end{pmatrix}'$. The constant gain RLS formulas for updating the PLM parameters as depicted in section 2.6 still hold. Under current PLMs, both households' and firms' behavioural rules are of the same general forms as in (23) and (24) respectively, although the infinite sum terms are different. We now derive the infinite sums. First, using the definition of $\bar{n}^e_t$, $\bar{w}^e_t$, $\bar{r}^e_t$ and $\bar{\pi}^e_t$ as well as those of $\widehat{n}^e_t$, $\widehat{w}^e_t$, $\widehat{r}^e_t$, $\widehat{\pi}^e_t$, the PLMs can then be written as

$$
\begin{aligned}
\widehat{n}^e_{t+1} &= a_{nn}\widehat{n}^e_t + a_{nz}\hat{z}_t, \\
\widehat{w}^e_t &= a_{wn}\widehat{n}^e_t + a_{wz}\hat{z}_t, \\
\widehat{r}^e_t &= a_{rn}\widehat{n}^e_t + a_{rz}\hat{z}_t, \\
\widehat{\pi}^e_t &= a_{\pi n}\widehat{n}^e_t + a_{\pi z}\hat{z}_t.
\end{aligned}
$$

Let $k_t = \begin{pmatrix} \widehat{n}^e_t & \widehat{z}_t\end{pmatrix}'$, $\Lambda = \begin{pmatrix} a_{nn} & a_{nz} \\ 0 & \varrho \end{pmatrix}$. Therefore,

$$\tilde{E}_t k_{t+j} = \Lambda^j k_t$$

and

$$
\begin{aligned}
\tilde{E}_t\widehat{n}^e_{t+j} &= \begin{pmatrix} 1 & 0\end{pmatrix}\Lambda^j k_t, \\
\tilde{E}_t\widehat{w}^e_{t+j} &= \begin{pmatrix} a_{wn} & a_{wz}\end{pmatrix}\Lambda^j k_t, \\
\tilde{E}_t\widehat{r}^e_{t+j} &= \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}\Lambda^j k_t, \\
\tilde{E}_t\widehat{\pi}^e_{t+j} &= \begin{pmatrix} a_{\pi n} & a_{\pi z}\end{pmatrix}\Lambda^j k_t.
\end{aligned}
$$

The infinite sums are given by

$$\begin{aligned}
\mathcal{S}^F_{w,t} &= \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t\widehat{w}_{t+j} = \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j(\bar{w}^e_t-\bar{w}) + \sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^j\tilde{E}_t\widehat{w}^e_{t+j} = \\
&= \frac{\beta(\bar{w}^e_t-\bar{w})}{1-\beta(1-\rho)} + \begin{pmatrix} a_{wn} & a_{wz}\end{pmatrix}\sum_{j=1}^{\infty}\beta^j(1-\rho)^{j-1}\Lambda^j k_t = \\
&= \frac{\beta(\bar{w}^e_t-\bar{w})}{1-\beta(1-\rho)} + \begin{pmatrix} a_{wn} & a_{wz}\end{pmatrix}\beta\Lambda[I-(1-\rho)\beta\Lambda]^{-1}k_t,
\end{aligned}$$

$$\begin{aligned}
\mathcal{S}^F_{n,t} &= \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\tilde{E}_t\widehat{n}_{t+j} = \\
&= \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1(\bar{n}^e_t-\bar{n}) + \sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\tilde{E}_t\widehat{n}^e_{t+j} = \\
&= \frac{\bar{\lambda}_1\beta^2(1-\rho)(\bar{n}^e_t-\bar{n})}{1-\beta(1-\rho)} + \begin{pmatrix} 1 & 0\end{pmatrix}\sum_{j=2}^{\infty}(1-\rho)^{j-1}\beta^j\bar{\lambda}_1\Lambda^j k_t = \\
&= \frac{\bar{\lambda}_1\beta^2\left(1-\rho\right)\left(\bar{n}^e_t-\bar{n}\right)}{1-\beta(1-\rho)} + \begin{pmatrix} 1 & 0\end{pmatrix}\bar{\lambda}_1(1-\rho)\beta^2\Lambda^2\left[I-\left(1-\rho\right)\beta\Lambda\right]^{-1}k_t,
\end{aligned}$$

$$\begin{aligned}
\mathcal{S}^F_{r,t} &= \bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^{j+1}\sum_{i=0}^{j-1}\tilde{E}_t\widehat{r}_{t+i} = \\
&= \bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^{j-1}\beta^{j+1}\widehat{r}_t + \bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^{j}\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}_{t+i} = \\
&= \frac{\bar{\lambda}_3\beta^2}{1-\beta(1-\rho)}\widehat{r}_t + \bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^j\beta^{j+2}(\bar{r}^e_t-\bar{r})j + \bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^j\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}^e_{t+i} = \\
&= \frac{\bar{\lambda}_3\beta^2}{1-\beta(1-\rho)}\widehat{r}_t + \frac{\bar{\lambda}_3\beta^3(1-\rho)(\bar{r}^e_t-\bar{r})}{[1-\beta(1-\rho)]^2} + \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}\bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^j\beta^{j+2}\sum_{i=1}^{j}\Lambda^i k_t = \\
&= \frac{\bar{\lambda}_3\beta^2}{1-\beta(1-\rho)}\widehat{r}_t + \frac{\bar{\lambda}_3\beta^3(1-\rho)(\bar{r}^e_t-\bar{r})}{[1-\beta(1-\rho)]^2} + \\
&+ \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}\bar{\lambda}_3\sum_{j=1}^{\infty}(1-\rho)^j\beta^{j+2}(\Lambda-\Lambda^{j+1})(I-\Lambda)^{-1}k_t = \\
&= \frac{\bar{\lambda}_3\beta^2}{1-\beta(1-\rho)}\widehat{r}_t + \frac{\bar{\lambda}_3\beta^3(1-\rho)(\bar{r}^e_t-\bar{r})}{[1-\beta(1-\rho)]^2} + \\
&\quad + \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}\frac{\bar{\lambda}_3\beta^3(1-\rho)\Lambda[I-\beta(1-\rho)\Lambda]^{-1}}{1-\beta(1-\rho)}k_t,
\end{aligned}$$

$$\begin{aligned}
\mathcal{S}^H_{\pi,t} &= \sum_{j=1}^{\infty}\beta^j\tilde{E}_t\widehat{\pi}_{t+j} = \sum_{j=1}^{\infty}\beta^j\left(\bar{\pi}^e_t-\bar{\pi}\right) + \sum_{j=1}^{\infty}\beta^j\tilde{E}_t\widehat{\pi}^e_{t+j} = \\
&= \frac{\beta\left(\bar{\pi}^e_t-\bar{\pi}\right)}{1-\beta} + \begin{pmatrix} a_{\pi n} & a_{\pi z}\end{pmatrix}\sum_{j=1}^{\infty}\beta^j\Lambda^j k_t = \\
&= \frac{\beta\left(\bar{\pi}^e_t-\bar{\pi}\right)}{1-\beta} + \begin{pmatrix} a_{\pi n} & a_{\pi z}\end{pmatrix}\beta\Lambda\left(I-\beta\Lambda\right)^{-1}k_t,
\end{aligned}$$

$$
\begin{aligned}
\mathcal{S}^H_{w,t} &= \sum_{j=1}^{\infty}\beta^j\tilde{E}_t\bar{n}\widehat{w}_{t+j} = \bar{n}\sum_{j=1}^{\infty}\beta^j(\bar{w}^e_t-\bar{w}) + \bar{n}\sum_{j=1}^{\infty}\beta^j\tilde{E}_t\widehat{w}^e_{t+j} \\
&= \frac{\beta\bar{n}(\bar{w}^e_t-\bar{w})}{1-\beta} + \begin{pmatrix} a_{wn} & a_{wz}\end{pmatrix}\bar{n}\sum_{j=1}^{\infty}\beta^j\Lambda^j k_t = \\
&= \frac{\beta\bar{n}(\bar{w}^e_t-\bar{w})}{1-\beta} + \begin{pmatrix} a_{wn} & a_{wz}\end{pmatrix}\bar{n}\beta\Lambda(I-\beta\Lambda)^{-1}k_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^H_{n,t} &= \sum_{j=1}^{\infty}\beta^j\bar{w}\tilde{E}_t\widehat{n}_{t+j} = \beta\bar{w}\widehat{n}_{t+1} + \sum_{j=2}^{\infty}\beta^j\tilde{E}_t\bar{w}\widehat{n}_{t+j} \\
&= \beta\bar{w}\widehat{n}_{t+1} + \bar{w}\sum_{j=2}^{\infty}\beta^j(\bar{n}^e_t-\bar{n}) + \bar{w}\sum_{j=2}^{\infty}\beta^j\tilde{E}_t\widehat{n}^e_{t+j} \\
&= \beta\bar{w}\widehat{n}_{t+1} + \frac{\beta^2\bar{w}(\bar{n}^e_t-\bar{n})}{1-\beta} + \begin{pmatrix} 1 & 0\end{pmatrix}\bar{w}\sum_{j=2}^{\infty}\beta^j\Lambda^j k_t \\
&= \beta\bar{w}\widehat{n}_{t+1} + \frac{\beta^2\bar{w}(\bar{n}^e_t-\bar{n})}{1-\beta} + \begin{pmatrix} 1 & 0\end{pmatrix}\bar{w}(\beta\Lambda)^2(I-\beta\Lambda)^{-1}k_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^H_{r,t} &= (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}[\beta^{j+1}\sum_{i=0}^{j-1}\tilde{E}_t\widehat{r}_{t+i}] = \\
&= (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\beta^{j+1}\widehat{r}_t + (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}_{t+i}] \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + (\bar{w}\bar{n}+\bar{\pi})\sum_{j=1}^{\infty}\beta^{j+2}(\bar{r}^e_t-\bar{r})j + (\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}\tilde{E}_t\widehat{r}^e_{t+i}] \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}(\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}[\beta^{j+2}\sum_{i=1}^{j}\Lambda^i k_t] \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + \\
&\quad + \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}(\bar{w}\bar{n}+\bar{\Pi})\sum_{j=1}^{\infty}\beta^{j+2}(\Lambda-\Lambda^{j+1})(I-\Lambda)^{-1}k_t \\
&= \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^2}{1-\beta}\widehat{r}_t + \frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3(\bar{r}^e_t-\bar{r})}{(1-\beta)^2} + \begin{pmatrix} a_{rn} & a_{rz}\end{pmatrix}\frac{(\bar{w}\bar{n}+\bar{\pi})\beta^3\Lambda(I-\beta\Lambda)^{-1}}{1-\beta}k_t.
\end{aligned}
$$

B EE learning

Now consider a learning model in which agents only make one-period ahead forecasts only. Therefore, households and firms make use of the Euler equation (8) and job creation condition (2.3) to make their optimal decisions. The whole economic system under EE learning consists of equations (25)-(33) in addition to those two linearised behaviour rules derived here below.

B.1 Incomplete knowledge

Comparable with the learning rule in our section 2.6, we assume the agents have the following PLMs,

$$n_{t+1} = b_n + a_{nn}n_t$$

$$x_t = b_x + a_{xn} n_t, \quad \text{for} \quad x_t = \{w_t, \theta_t, c_t\}.$$

Let $A_{n,t} = \begin{pmatrix} b_n & a_{nn} \end{pmatrix}'$, $A_{w,t} = \begin{pmatrix} b_w & a_{wn} \end{pmatrix}'$, $A_{\theta,t} = \begin{pmatrix} b_\theta & a_{\theta n} \end{pmatrix}'$, $A_{c,t} = \begin{pmatrix} b_c & a_{cn} \end{pmatrix}'$, $B_t = \begin{pmatrix} 1 & n_t \end{pmatrix}'$, $A_t = \begin{pmatrix} A_{n,t} & A_{w,t} & A_{\theta,t} & A_{c,t} \end{pmatrix}$, $C_t = \begin{pmatrix} n_t & w_{t-1} & \theta_{t-1} & c_{t-1} \end{pmatrix}'$. The RLS formulas for updating beliefs are as follows

$$A_t = A_{t-1} + \gamma R_t^{-1} B_{t-1} \left(C_t - A'_{t-1} B_{t-1} \right)',$$
$$R_t = R_{t-1} + \gamma \left(B_{t-1} B'_{t-1} - R_{t-1} \right)$$

We then turn to the derivation of the optimal behavior rules, the counterpart of (23) and (24) as in the IH learning case. Applying the PLMs, we have

$$\begin{aligned}
\tilde{E}_t n_{t+1} &= A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}', \\
\tilde{E}_t \widehat{w}_{t+1} &= A'_{w,t} \begin{pmatrix} 1 & \tilde{E}_t n_{t+1} \end{pmatrix}' - \bar{w} = A_{w,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{w}, \\
\tilde{E}_t \widehat{\theta}_{t+1} &= A'_{\theta,t} \begin{pmatrix} 1 & \tilde{E}_t n_{t+1} \end{pmatrix}' - \bar{\theta} = A'_{\theta,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{\theta}, \\
\tilde{E}_t \widehat{c}_{t+1} &= A'_{c,t} \begin{pmatrix} 1 & \tilde{E}_t n_{t+1} \end{pmatrix}' - \bar{c} = A'_{c,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{c}.
\end{aligned}$$

The linearised version of Euler equation reads

$$\tilde{E}_t \widehat{c}_{t+1} = \widehat{c}_t + \beta \bar{c} \widehat{r}_t$$

We then have, after substituting for the expectation of $\tilde{E}_t \widehat{c}_{t+1}$, that

$$\widehat{c}_t + \beta \bar{c} \widehat{r}_t = A'_{c,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{c},$$

which is the consumption rule under EE learning.

The linearized version of vacancy posting condition reads

$$-\frac{\kappa}{\beta \bar{q}^2} \widehat{q}_t + \frac{\kappa}{\bar{q}} \widehat{r}_t = \alpha \bar{z} (\alpha - 1) \bar{n}^{\alpha-2} \widehat{n}_{t+1} + \alpha \bar{n}^{\alpha-1} \widehat{z}_{t+1} - \tilde{E}_t \widehat{w}_{t+1} - \frac{\kappa (1-\rho) \bar{m} (\sigma - 1) \bar{\theta}_t^{\sigma-2}}{\bar{q}^2} \tilde{E}_t \widehat{\theta}_{t+1}.$$

After substituting the expectations of $\tilde{E}_t \widehat{w}_{t+1}$ and $\tilde{E}_t \widehat{\theta}_{t+1}$ respectively, we have that

$$\begin{aligned}
\alpha \bar{z} (\alpha - 1) \bar{n}^{\alpha-2} \widehat{n}_{t+1} \quad &+ \quad \alpha \bar{n}^{\alpha-1} \varrho \widehat{z}_t + \frac{\kappa}{\beta \bar{q}^2} \widehat{q}_t - \frac{\kappa}{\bar{q}} \widehat{r}_t = \\
&= \frac{\kappa (1-\rho) \bar{m} (\sigma-1) \bar{\theta}_t^{\sigma-2}}{\bar{q}^2} \left[A'_{\theta,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{\theta} \right] + \\
&\quad + A_{w,t} \begin{pmatrix} 1 & A'_{n,t} \begin{pmatrix} 1 & \bar{n} + \widehat{n}_t \end{pmatrix}' \end{pmatrix}' - \bar{w},
\end{aligned}$$

which is the vacancy posting rule.

B.2 Complete Knowledge

One can also consider the learning rules which take the same form as RE solution. Thus, the PLMs read

$$n_{t+1} = b_n + a_{nn} n_t + a_{nz} \widehat{z}_t$$

and

$$x_t = b_x + a_{xn} n_t + a_{xz} \widehat{z}_t, \quad \text{for} \quad x_t = \{w_t, \theta_t, c_t\}.$$

We then redefine, $A_{n,t} = \begin{pmatrix} b_n & a_{nn} & a_{nz} \end{pmatrix}'$, $A_{w,t} = \begin{pmatrix} b_w & a_{wn} & a_{wz} \end{pmatrix}'$, $A_{\theta,t} = \begin{pmatrix} b_\theta & a_{\theta n} & a_{\theta z} \end{pmatrix}'$, $A_{c,t} = \begin{pmatrix} b_c & a_{cn} & a_{cz} \end{pmatrix}'$, $B_t = \begin{pmatrix} 1 & n_t & \widehat{z}_t \end{pmatrix}'$. The RLS formulas for updating beliefs in the last section still hold. Applying the PLMs, we obtain

$$\begin{aligned}
\tilde{E}_t n_{t+1} &= A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}', \\
\tilde{E}_t \widehat{w}_{t+1} &= A'_{w,t}\begin{pmatrix} 1 & \tilde{E}_t n_{t+1} & \widehat{z}_t \end{pmatrix}' - \bar{w} = A'_{w,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{w}, \\
\tilde{E}_t \widehat{\theta}_{t+1} &= A'_{\theta,t}\begin{pmatrix} 1 & \tilde{E}_t n_{t+1} & \widehat{z}_t \end{pmatrix}' - \bar{\theta} = A'_{\theta,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{\theta}, \\
\tilde{E}_t \widehat{c}_{t+1} &= A'_{c,t}\begin{pmatrix} 1 & \tilde{E}_t n_{t+1} & \widehat{z}_t \end{pmatrix}' - \bar{c} = A'_{c,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{c}.
\end{aligned}$$

The consumption rule and vacancy posting rule following the same logic read as follows

$$\widehat{c}_t + \beta\bar{c}\widehat{r}_t = A'_{c,t}\begin{pmatrix} 1 & \tilde{E}_t n_{t+1} & \widehat{z}_t \end{pmatrix}' - \bar{c} = A'_{c,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{c}$$

and

$$\begin{aligned}
\alpha\bar{z}(\alpha-1)\bar{n}^{\alpha-2}\widehat{n}_{t+1} &+ \alpha\bar{n}^{\alpha-1}\varrho\widehat{z}_t + \frac{\kappa}{\beta\bar{q}^2}\widehat{q}_t - \frac{\kappa}{\bar{q}}\widehat{r}_t = \\
&= \frac{\kappa(1-\rho)\bar{m}(\sigma-1)\bar{\theta}_t^{\sigma-2}}{\bar{q}^2}\left[A'_{\theta,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{\theta}\right] + \\
&+ A'_{w,t}\begin{pmatrix} 1 & A'_{n,t}\begin{pmatrix} 1 & \bar{n}+\widehat{n}_t & \widehat{z}_t \end{pmatrix}' & \widehat{z}_t \end{pmatrix}' - \bar{w}
\end{aligned}$$

respectively.