Chasing the Gap:
Speed Limits and Optimal Monetary Policy*

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Abstract

Speed limit monetary policy rules incorporate a response to the change in the output gap. Speed limit rules feature in the influential DSGE model of Smets and Wouters but are not widely used. This may reflect their association with policymaking under commitment. In this paper we derive optimal speed limit monetary policy rules under both discretion and commitment using a simple New-Keynesian DSGE model with habit persistence. A novel feature of our model is the inclusion of the lagged output gap in the Phillips Curve. Empirical tests suggest that the behaviour of US monetary policymakers during the "Great Moderation" can best be characterised by a speed limit policy rule obtained under discretion. Simulations reveal that optimal policy rules under discretion and commitment imply similar impulse responses for the output gap but rather different impulse responses for inflation.

Keywords: optimal monetary policy, speed limit, New Keynesian model

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1 Introduction

Speed limit monetary policy rules, which incorporate a response to the change in the output gap, have been suggested as an alternative to the well-known Taylor Rule (e.g. Walsh, 2003a and McCallum and Nelson, 2004). Speed limit policy rules have occasionally been used, for example in the influential DSGE model of Smets and Wouters (2003, 2007), but are not common. One possible reason for this is that speed limit policy rules are associated in the literature with policymaking under commitment (eg McCallum and Nelson, 2004), a monetary policy framework that is often regarded as infeasible. By contrast, optimal speed limit policy rules have not been derived under discretion (widely seen as more feasible than commitment), with the exception of Walsh (2003a), who argues that this could be done if the speed limit were to replace the output gap in the objective function of the policymaker[1].

In this paper, we argue that optimal monetary policy rules derived under both discretion and commitment contain speed limit effects if there is habit persistence in household utility (Fuhrer, 2000). The essence of our argument is very simple. An optimal monetary policy rule is obtained by combining the aggregate demand relationship with the optimality condition for monetary policy; if either of these contains speed limit terms, so will the resultant rule. The existing literature does not assume habit persistence. In this case, the aggregate demand relationship cannot be written in a form that contains speed limits. Any speed limit terms must therefore come from the optimality condition. This condition contains speed limits terms under commitment but not under discretion. Therefore optimal speed limit policy rules are obtained under commitment but not discretion. In this paper, by contrast, we assume habit persistence. The aggregate demand relationship can be written in a form that contains speed limit terms. As a result, the optimal policy rule contains speed limit effects irrespective of the optimality condition and therefore we obtain optimal speed limit policy

[1] Speed limit policy rules have been proposed as a response to imperfect knowledge of the equilibrium rate of output (Orphanides and Williams, 2002, Orphanides 2003, Walsh, 2003b); we do not consider this aspect of the literature in this paper

rules under both discretion and commitment. Habit persistence is supported by extensive empirical evidence (eg Smets and Wouters, 2003, 2007, Fuhrer and Rudebusch, 2004, Bouakez et al, 2005, Christiano et al, 2005, Ravn et al 2006, 2008). This suggests that speed limits may be a common feature of optimal monetary policy.

We use a simple micro-founded New Keynesian DSGE model with habit persistence. A novel feature of our model is that the Phillips Curve includes the lagged output gap. We argue that this is a consequence of habit persistence. The New Keynesian Phillips Curve relates inflation to real marginal costs. These reflect the marginal rate of substitution between consumption and leisure; with habit persistence, this is affected by consumption in the previous period. As a consequence, real marginal costs reflect both current and lagged levels of output, leading to the inclusion of the lagged output gap in the Phillips Curve. The addition of this term affects the optimality conditions for monetary policy. Under discretion, the optimal choice of the output gap is a function of the current and expected future inflation rates. This is an extension of the familiar "leaning against the wind" condition, which is a contemporaneous relationship between the output gap and the inflation rate. Under commitment, the optimality condition relates the change in the output gap (ie the speed limit) to current and expected future inflation rates; this also extends the usual optimality condition.

We analyse optimal monetary policy using two alternative loss functions, the quadratic and a model-based approximation to social welfare. Using the quadratic enables us to derive optimal speed limit policy rules in a simple and transparent framework and to relate our results to the previous literature. Optimal policy rules derived under discretion and commitment have distinctive features. With discretion, the policy rules contain both speed limits and the level of the output gap (the policy rule assumed by Smets and Wouters, 2003, 2007, has this form) and are thus are an augmentation of the familiar Taylor rule. With commitment, by contrast, policy rules contain speed limit terms but not the output gap (the policy rules estimated by, among others, Stracca, 2006, have this feature); they are an alternative to the Taylor rule.

The distinctive features of speed limit rules under discretion and commitment enable us to discriminate between them empirically. Doing this using US data for 1983-2006, we find that the restrictions implied by optimal speed limit policy rule under discretion are not rejected by the data whereas the restrictions implied by the policy rules under commitment are rejected (as are the restrictions implied by a simple Taylor rule). This suggests that the behaviour of US monetary policymakers during the "Great Moderation" can best be characterised by a speed limit policy rule obtained under discretion.

We calculate theoretical impulse response functions using a calibration of our model, focussing on the policy trade-off caused by a supply shock. Our simulations show that optimal policy under discretion and commitment implies similar impulse responses of the output gap but rather different impulse responses for inflation. They also show that variations in the strength of habit effects in household utility make little difference to impulse responses under optimal discretion and commitment, but lead to very different impulse responses if policy follows a simple Taylor Rule. This illustrates how optimal policy adapts to changes in structural characteristics of the economy, delivering similar outcomes for different configurations of the structural parameters, in contrast to more ad-hoc policies such as the Taylor Rule (Svensson, 2003).

The remainder of the paper is structured as follows. We outline our model in section 2) and then derive and discuss optimal speed limit rules in section 3). In section 4), we discuss how to discriminate between policy rules under discretion and commitment empirically and present tests that suggest the data favour a speed limit policy rule under discretion. We present impulse responses from a calibration of our model in section 5) and highlight characteristics of speed limit policy rules. There are a number of caveats to our model and estimates; we discuss these and conclude in section 6).

2 The Model

We use a simple New Keynesian DSGE model[2] in which households supply homogenous labour inputs and purchase differentiated goods, while firms hire labour and produce goods. The goods market is monopolistically competitive but the labour market is competitive. There is a continuum of households; each has an inter-temporal utility function given by

$$(1)\ E_t \sum_{k=0}^{\infty} (e^{\epsilon_t^D}\beta)^k \left\{ \frac{(C_{t+k}(j)-\mu C_{t+k-1})^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+k}(j)^{1+\eta}}{1+\eta} \right\}$$

where j indexes the household, $C_t(j) = (\int_0^{\infty} C_t(j,q)^{\frac{\theta-1}{\theta}} dq)^{\frac{\theta}{\theta-1}}$ is a consumption index that aggregates individual goods $C_t(j,q)$, $C_t = \int_0^{\infty} C_t(j)dj$ is aggregate consumption, $N_{t+k}(j)$ are hours worked; β is the discount factor, σ is the inverse of the elasticity of inter-temporal substitution, μ measures the strength of habit formation, η is the inverse of the labour supply elasticity and χ denotes the relative weight on hours worked. $e^{\epsilon_t^D}$ represents a preference shock: we assume $\epsilon_t^D = \rho_D \epsilon_{t-1}^D + \varrho_t^D$ where $0 \leqslant \rho_D \leqslant 1$ and ϱ_t^D is distributed as $N(0, \sigma_D^2)$.

We characterise habit persistence in household utility using the external (or "shallow") habit formation approach of Abel (1990) and Campbell and Cochrane (1999) and follow Constantinides (1990) and Smets and Wouters (2003, 2007) in expressing habits in terms of the quasi-difference between current and lagged consumption. Alternatives include expressing habits in terms of the ratios of current and lagged consumption within an external habits framework and using the internal (or "deep") habit approach of Ravn et al (2006). We use the simple external habits formulation as it allows us to obtain analytic solutions for optimal monetary policy rules[3].

The budget constraint of households is

[2] Without habit persistence, the model simplifies to that outlined in chapter 8) of Walsh (2010).

[3] Optimal monetary policy with habit persistence is analysed through simulations in a more complex model of external habit persistence model by Levine et al (2008) and Corrado et al (2012) and in a model with internal habit persistence by Leith et al (2012).

(2) $C_t(j) + \frac{B_t(j)}{P_t} \leqslant \frac{(1+i_{t-1})B_{t-1}(j)}{P_t} + \frac{W_t(j)}{P_t}N_t(j) + \frac{\Pi_t(j)}{P_t} - T_t$

where $B(j)$ are holdings of bonds, P is the aggregate price level given by $P_t = (\int_0^\infty P_t(q)^{1-\theta}dq)^{\frac{1}{1-\theta}}$ where $P_t(q)$ is the price of good q, i is the nominal interest rate, W is the nominal wage, $\Pi(j)$ are nominal profits distributed to household j and T is a lump-sum tax. Optimising with respect to consumption and labour supply implies equality between the real wage and the marginal rate of substitution,

(3) $\frac{W_t(j)}{P_t} = \chi N_t(j)^\eta (C_t(j) - \mu C_{t-1})^\sigma$

Firms have the production function

(4) $Y_t(k) = AN_t(k)$

where $Y_t(k)$ and $N_t(k)$ are output and employment at firm k and A is productivity[4]. Firms are able to reset their price with probability $(1 - \xi)$ and maintain the same price with probability ξ. There is no indexation of prices for firms that are not able to reset their price. Real marginal cost is given by

(5) $(1 - \nu)(\frac{W_t}{P_t})$

where ν is a subsidy paid to ensure an efficient level of output, financed from lump-sum taxation on households.

Monetary policy seeks to minimise the output gap, defined as $x_t = y_t - y_t^e$ where y represents output and y^e represents the socially efficient level of output. Following Woodford (2003), we assume that the efficient level of output differs from the flexible-price (or "natural") level of output, y^n, because of a supply shock, so

(6) $y_t^e = y_t^n + \epsilon_t^S$

[4] We do not include productivity shocks as these make it more difficult to obtain analytic solutions for optimal monetary rules using an approximation to social welfare. As discussed in Walsh (2010, section 8.3.5), productivity shocks would be a component of the shock to aggregate demand in a linearised model expressed in terms of the otuput gap.

We assume $\epsilon_t^S = \rho_S \epsilon_{t-1}^S + \varrho_t^S$ where $0 \leqslant \rho_S \leqslant 1$ and ϱ_t^S is distributed as $N(0, \sigma_S^2)$.

Solving the model and using a first-order linearization around the steady-state yields aggregate demand and supply equations given by

(7) $\hat{x}_t = \frac{\mu}{1+\mu}\hat{x}_{t-1} + \frac{1}{1+\mu}E_t\hat{x}_{t+1} - \frac{1-\mu}{\sigma(1+\mu)}(i_t - E_t\pi_{t+1}) + \zeta_t$

and

(8) $\pi_t = \beta E_t \pi_{t+1} + \kappa\hat{\varphi}_t$

where $\hat{x}$ is the deviation of the output gap around its' steady-state value, $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$is the inflation rate, i is the deviation of the nominal interest rate from its' steady-state value, $\hat{\varphi}$ represents log linear deviations of real marginal cost around the steady-state, $\kappa = \frac{(1-\xi)(1-\beta\xi)}{\xi}$ and $\zeta_t = \frac{(1-\mu)(1-\rho_D)}{\sigma(1+\mu)}\epsilon_t^D$. These relationships are familiar from the literature. Equation (7) is an aggregate demand relationship with habit persistence. Equation (8) is a New Keynesian Phillips Curve expressed in terms of marginal cost.

We next combine the defintion of marginal cost in (5), the equality between the real wage and the marginal rate of substitution in (3) and the definition of the efficient level of output in (6), which gives

(9) $\hat{\varphi}_t = (\eta + \frac{\sigma}{1-\mu})\hat{x}_t - \frac{\sigma\mu}{1-\mu}\hat{x}_{t-1} + (\eta + \frac{\sigma}{1-\mu})\epsilon_t^S - \frac{\sigma\mu}{1-\mu}\epsilon_{t-1}^S$

Combining (8) and (9) gives the Phillips Curve expressed in terms of the output gap

(10) $\pi_t = \beta E_t\pi_{t+1} + \lambda\hat{x}_t - \delta\hat{x}_{t-1} + \lambda\epsilon_t^S - \delta\epsilon_{t-1}^S$

where $\lambda = \kappa(\eta + \frac{\sigma}{1-\mu})$, $\delta = \kappa\frac{\sigma\mu}{1-\mu}$ and $\zeta_t = \frac{(1-\mu)(1-\rho_D)}{\sigma(1+\mu)}\epsilon_t^D$. Compared to the existing literature, this Phillips curve includes two extra terms, the lagged output gap and the lagged supply shock[5]. These reflect the impact of habit persistence in the household utility function on the marginal rate

[5] The implications of this are further analysed by De Tina and Martin (2014).

substitution between consumption and leisure and hence on marginal cost. If $\mu = 0$ then (10) simplifies to the New Keynesian Philips Curve $\pi_t = \beta E_t \pi_{t+1} + (\eta + \sigma)\hat{x}_t + (\eta + \sigma)\epsilon_t^S$.

3 Optimal Monetary Policy

3.1 Optimal Monetary Policy with a Quadratic Loss Function

We first analyse optimal monetary policy when policymakers have a simple quadratic loss function, given by

(11) $\Lambda = \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{2}\pi_{t+j}^2 + \frac{\alpha}{2}\hat{x}_{t+j}^2 \right]$

If policymakers choose the output gap under discretion in order to minimise this, subject to the Phillips curve, the optimality condition is

(12) $\hat{x}_t = -\frac{\lambda}{\alpha}\pi_t + \frac{\beta\delta}{\alpha}E_t\pi_{t+1}$

This generalises the familiar optimality condition under discretion, reflecting the lagged output gap term in the Phillips Curve; if there are no habit effects, (12) simplifies to the standard "leaning against the wind" condition, $\hat{x}_t = -\frac{\lambda}{\alpha}\pi_t$. Re-writing the aggregate demand relationship as

(13) $\hat{x}_t = \frac{1}{1+\mu}E_t\Delta\hat{x}_{t+1} + \frac{1}{1+\mu}E_t\Delta\hat{x}_t + \hat{x}_{t-1} - \frac{1-\mu}{\sigma(1+\mu)}(i_t - E_t\pi_{t+1}) + \zeta_t$

and combining this with (12), we obtain the optimal monetary policy rule under discretion with a quadratic loss function, given by

(14) $i_t = \frac{\sigma(1+\mu)\lambda}{\alpha(1-\mu)}\pi_t + (1 - \frac{\sigma\beta\delta(1+\mu)}{\alpha(1-\mu)})E_t\pi_{t+1} + \frac{\sigma}{(1-\mu)}E_t\Delta\hat{x}_{t+1} + \frac{\sigma}{(1-\mu)}E_t\Delta\hat{x}_t$
$+ \frac{\sigma(1+\mu)}{(1-\mu)}\hat{x}_{t-1} + \frac{\sigma(1+\mu)}{(1-\mu)}\zeta_t$

This policy rule has two equally-weighted forward- and backward-looking speed limit terms, demonstrating that optimal speed limit policy rules can be obtained under discretion. The presence of these speed terms reflects the speed limit terms in the aggregate demand relationship. The policy rule is an augmentation of the Taylor rule, which is obtained if $\mu = 0$.

The optimality condition under commitment (adopting the timeless perspective, Woodford, 2003) is

$$(15)\ \Delta\hat{x}_t = -\tfrac{\lambda}{\alpha}\pi_t + \tfrac{\beta\delta}{\alpha}E_t\pi_{t+1}$$

This again differs from the familiar optimality condition through the addition of a term in the expected future inflation rate, reflecting the lagged output gap in the Phillips Curve. Substituting this into the aggregate demand relationship, we obtain the optimal monetary policy rule under commitment with a quadratic loss function, given by

$$(16)\ i_t = \tfrac{\sigma\mu\lambda}{\alpha(1-\mu)}\pi_t + (1 - \tfrac{\sigma\beta\delta\mu}{\alpha(1-\mu)})E_t\pi_{t+1} + \tfrac{\sigma}{(1-\mu)}E_t\Delta\hat{x}_{t+1} + \tfrac{\sigma(1+\mu)}{(1-\mu)}\zeta_t$$

This rule is simpler than under discretion, containing a single speed limit and no term in the output gap. It is therefore an alternative to the Taylor rule. Under commitment, the speed limit term in the policy rule is not dependent on persistence in aggregate demand[6]. This is consistent with the previous literature, in which speed limit policy rules were derived under commitment, but not discretion, in models without habit persistence in household utility.

3.2 Optimal Monetary Policy with an approximation to social welfare

The quadratic loss function is convenient but essentially arbitrary. Leith et al (2012) derive a second-order approximation to household utility in our case, given by

$$(17)\ \Lambda^{sw} = \sum_{j=0}^{\infty}\beta^j\left[\tfrac{\theta}{2\kappa}\pi_{t+j}^2 + \tfrac{\eta}{2}\hat{x}_{t+j} + \tfrac{\omega}{2}(\hat{x}_{t+j} - \mu\hat{x}_{t+j-1})^2\right]$$

[6] There are technical issues regarding the derivation of the speed limit policy rule under commitment in the existing literature, discussed in Blake (2012).

where $\omega = \frac{\sigma}{(1-\mu)(1-\beta\mu)}$.

With this alternative loss function, the optimality condition under discretion is

$$(18)\ \eta\hat{x}_t + \omega(\hat{x}_t - \mu\hat{x}_{t-1}) - \beta\mu\omega(E_t\hat{x}_{t+1} - \mu\hat{x}_t) = -\frac{\lambda\theta}{\kappa}\pi_t + \frac{\beta\delta\theta}{\kappa}E_t\pi_{t+1}$$

Combining this with the aggregate demand relationship in (13), we obtain the optimal monetary policy rule under discretion, given by

$$(19)\ i_t = \frac{\sigma(1+\mu)\lambda\theta}{\kappa\Theta(1-\mu)}\pi_t + (1 - \frac{\sigma\beta\delta\theta(1+\mu)}{\kappa\Theta(1-\mu)})E_t\pi_{t+1} + \frac{\sigma}{(1-\mu)}(1 - \frac{(1+\mu)\beta\mu\omega}{\Theta})E_t\Delta\hat{x}_{t+1}$$

$$+\frac{\sigma}{(1-\mu)}(1 - \frac{(1+\mu)\beta\mu\omega}{\Theta})E_t\Delta\hat{x}_t + \frac{\sigma(1+\mu)}{(1-\mu)}(1 - \frac{(1+\beta)\mu\omega}{\Theta})\hat{x}_{t-1} + \frac{\sigma(1+\mu)}{(1-\mu)}\zeta_t$$

where $\Theta = (\eta + \omega(1+\beta\mu^2))$. Although more complex than the quadratic case, this policy rule is similar, comprising the same variables and again containing two speed limit terms with the same coefficients.

The optimality condition under commitment is

$$(20)\ \eta\Delta\hat{x}_t + \omega(\Delta\hat{x}_t - \mu\Delta\hat{x}_{t-1}) - \beta\mu\omega(E_t\Delta\hat{x}_{t+1} - \mu\Delta\hat{x}_t) = -\frac{\lambda\theta}{\kappa}\pi_t + \frac{\beta\delta\theta}{\kappa}E_t\pi_{t+1}$$

from which the optimal policy rule can be derived as

$$(21)\ i_t = \frac{\sigma\mu\lambda\theta}{\kappa\Theta(1-\mu)}\pi_t + (1 - \frac{\sigma\beta\delta\theta\mu}{\kappa\Theta(1-\mu)})E_t\pi_{t+1} + \frac{\sigma}{(1-\mu)}(1 - \frac{\beta\omega\mu^2}{\Theta})E_t\Delta\hat{x}_{t+1}$$

$$-\frac{\sigma\mu^2\omega}{\Theta(1-\mu)}E_t\Delta\hat{x}_{t-1} + \frac{\sigma(1+\mu)}{(1-\mu)}\zeta_t$$

Compared to the quadratic case, this policy rule contains an additional term, reflecting the presence of the lagged speed limit in the first order condition.

4 Empirical Evidence

In this section we investigate whether empirical evidence is consistent with our analysis. Our theoretical model is simple and stylised and is unlikely to be able to match key features of the data. We therefore do not estimate the structural model developed in section 2) and 3). Instead, we examine empirical evidence for the distinctive characteristics of optimal speed limit policy rules under discretion and commitment that were derived in the previous section.

We test three empirical hypotheses. First, we test whether there are significant speed limit effects in estimated monetary policy rules. If there are not, the Taylor rule is an adequate description of monetary policy. Second, we test whether the output gap can be excluded from the empirical policy rules. If it can, this suggests that policymakers are acting under commitment. Third, we test whether empirical policy rules contain two speed limit terms with identical coefficients; if they do, this suggests policymakers are acting under discretion.

To facilitate testing, we simplify the optimal policy rules. Since the lagged output gap is the only observable state variable in the theoretical model, we can write $\pi_t = \nu_\pi x_{t-1} + \zeta_t^\pi$ and $\hat{x}_t = \nu_x \hat{x}_{t-1} + \zeta_t^x$; where ν_π and ν_x are functions of the structural parameters and ζ_t^π and ζ_t^x are functions of exogenous shocks, which are not observed by the econometrician. Combining these, we obtain[7]

(22) $E_t \pi_{t+1} = \nu_x \pi_t$

This enables us to combine the two inflation terms in the optimal policy rules into a single term. For example, we can express (19) as

(19') $$i_t = (\nu_x + \frac{\sigma(1+\mu)(\lambda-\nu_x\beta\delta}{\kappa\Theta(1-\mu)})\pi_t + \frac{\sigma}{(1-\mu)} E_t \Delta\hat{x}_{t+1} + \frac{\sigma}{(1-\mu)} E_t \Delta\hat{x}_t + \frac{\sigma(1+\mu)}{(1-\mu)}\hat{x}_{t-1} + \frac{\sigma(1+\mu)}{(1-\mu)}\zeta_t$$

with corresponding adjustments to the other optimal policy rules.

[7] For "reasonable" parameter values, these relationships have roots within the unit circle.

We use two empirical models, given by

(23) $i_t = \phi_1 \pi_t + \phi_2 E_t \Delta \hat{x}_{t+1} + \phi_3 E_t \Delta \hat{x}_t + \phi_4 \hat{x}_{t-1} + \varepsilon_t^{i1}$

and

(24) $i_t = \phi_1 \pi_t + \phi_2 E_t \Delta \hat{x}_{t+1} + \phi_3 E_t \Delta \hat{x}_{t-1} + \phi_4 \hat{x}_{t-1} + \varepsilon_t^{i2}$

where ε_t^{i2} and ε_t^{i2} are error terms. The hypothesis that monetary policy rules do not contain speed limit terms implies $\phi_2 = \phi_3 = 0$ in (23) and (24). The hypothesis that monetary policy rules do not contain output gap terms implies $\phi_4 = 0$ in (23) and (24). The hypothesis that the coefficients on the speed limit terms are identical under discretion implies $\phi_2 = \phi_3$ in (23). Table 1) presents F-tests of these hypotheses. The hypothesis that monetary policy rules do not contain speed limit terms is strongly rejected. The hypothesis that monetary policy rules do not contain output gap terms is also rejected[8]. However, the hypothesis that the coefficients on the speed limit terms are identical is not rejected. This evidence is consistent with policymakers following an optimal monetary policy rule under discretion.

Table 1)
Hypothesis Tests

	equation (23)	equation (24)
no speed limits	0.001	0.001
no output gap	0.005	0.030
speed limits identical	0.538	

Notes: table presents p-values from F-tests of hypotheses described in text

Estimates of monetary policy rules are presented in Table 2). Column (i) is the Taylor rule, while columns (ii)-(iii) contain estimates of the optimal

[8] We also note that the estimate of ϕ_4 is negative, contrary to the predictions of the policy rule under commitment.

policy rules in (23) and (24). The estimated parameters in columns (ii) and (iii) are larger than those in column (i). This suggests, consistent with the simulations in the following section, that policy with a speed-limit monetary policy rule is more active than with a Taylor rule.

Table 2)

Parameter Estimates

	Taylor Rule	Equation (23)	Equation (24)
π_t	2.517 (0.995)	3.295 (0.737)	2.549 (0.615)
$\hat{x}_{t-1}$	0.905 (0.387)	2.158 (0.603)	1.118 (0.507)
$E_t\Delta\hat{x}_{t+1}$		6.762 (2.282)	3.229 (2.538)
$\Delta\hat{x}_t$		5.147 (1.608)	
$\Delta\hat{x}_{t-1}$			4.297 (1.438)
s.e	0.516	0.537	0.452
exog	0.507	0.226	0.190

Notes: standard errors in parentheses; exog is the p-value of the test for exogeneity of the instruments

5 Simulations

We have developed a New Keynesian DSGE model with habit effects in household utility, arguing that optimal monetary policy rules in this case contain speed limit effects. In this section we present simulations to illustrate the implications of this model. We focus on two issues: what is the impact of optimal policy under discretion, as opposed to optimal policy under commitment? And how does the response of the economy to shocks differ when there are habit effects in household utility compared to the case where there are not?

The difference in responses when policymakers act under discretion and commitment are explored in figure 1). We present impulse responses from models that combine the structural relationships in (7) and (10) with the optimal policy rule in (19) (discretion), or in (21) (commitment). We also present impulse responses for a model in which policymakers follow a Taylor Rule. We calibrate the structural parameters using values from Smets and Wouters (2007), summarised in Table 3). We set $\mu = 0.71$, $\sigma = 1.39$, $\eta = 1.92$, $\xi = 0.65$, $\theta = 10$ and $\beta = 0.99$. For the Taylor Rule, we assume a response to inflation of 1.5 and a response to the output gap of 0.125 . We focus on the impact of supply shocks as these generate interesting policy trade-offs (preference shocks are exactly offset by optimal policy rules). The persistence of the supply shocks is assumed to be $\rho_S = 0.9$.

Table 3)

Calibrated Parameters

μ	σ	η	ξ	θ	β	ρ_S	σ_S
0.71	1.39	1.92	0.65	10	0.99	0.9	0.09

Figure 1a) shows the impulse responses of the output gap following a supply shock. The responses under discretion and commitment are similar; both give a monotonic response of the output gap to the shock. By contrast, the response of output with the Taylor Rule displays a marked hump shape. This suggests that optimal policy smooths out the hump-shaped response that is otherwise implied by the lagged output gap term in the aggregate demand relationship. Movements in the output gap are smaller with the Taylor Rule compared to the optimal policy rules. Figure 1b) shows the impulse responses of the inflation rate following the same supply shock. Here, discretion and commitment produce rather different responses. Under discretion, the impulse response is again monotonic. With commitment, we observe the rapid return of inflation to steady-state that is characteristic of the behaviour of inflation in this case. With the Taylor rule, movements in inflation are markedly larger than with the optimal policy rules. Table 4) reports the values of the loss function in (17) implied by these simulations.

Optimal policy under discretion and commitment leads to quite similar levels of loss; the loss implied by the Taylor Rule is substantially larger.

Figure 1c) depicts the responses of the nominal interest rate. The policy response to the supply shock is strongest under discretion as, lacking the ability to affect expected inflation, policymakers sharply increase the policy rate in order to counteract the impact of the supply shock on inflation. Under commitment, policymakers are able to exploit their control over expected inflation, allowing them to raise the policy rate by somewhat less than under discretion. The policy rate under the Taylor Rule initially rises by less than with the optimal policy rules, but the rate returns to steady-state slowly, implying that the policy rate is higher with the Taylor Rule over most of the response period.

We illustrate the effects of habit persistence by simulating the same model but where we set $\mu = 0$. Figure 2a) plots the impulse responses of the output gap. The responses under discretion and commitment are very similar to those obtained with with $\mu = 0.71$, whereas the response with the Taylor Rule is rather different, with a sharper reduction in the output gap. Figure 2b) plots the impulse responses of the inflation rate. We again observe that the responses under discretion and commitment are very similar to those obtained with $\mu = 0.71$, whereas the response with the Taylor Rule is again different, with a smaller increase in the inflation rate. The impact of changing the value of μ on the impulse responses is reflected in the values of the loss function, shown in table 4).

Table 4)

Values of Loss Implied by Simulations

	$\mu = 0.71$	$\mu = 0$
Discretion	0.1157	0.0757
Commitment	0.1002	0.0656
Taylor Rule	0.4329	0.2367

These findings illustrate the point, stressed by Svennson (2003), that optimal policy adapts to changes in the structural parameters characterising the economy ; specifically, the parameters of the optimal policy rules in (19) and (21) are functions of μ and so changes in this parameter lead to changes in the parameters of the policy rule. This feature enables optimal policy rules to deliver similar outcomes for different configurations of the structural parameters. By contrast, the Taylor Rule does not adapt to changes in structural parameters and so delivers differing outcomes as these parameters change. This is illustrated in Figure 2c), which plots the impulse responses of the interest rate in this case. Under discretion, the response of interest rates to the supply shock is smaller than in the case where $\mu = 0.71$; with less persistence in the economy, policymakers are able to deliver the same outcome with a smaller policy response. Under commitment, there is also a smaller response of the interest rate to the supply shock; on impact the policy rate falls, before rebounding in the subsequent period. The policy rate implied by the Taylor Rule is less responsive to changes in the degree of persistence; the change in the interest rate implied by the Taylor Rule is larger than the changes under discretion or commitment. This is in contrast to the responses obtained when $\mu = 0.71$; in that case, movements in the interest rate on impact are smaller with the Taylor Rule.

6 Conclusions

In this paper we have derived, estimated and simulated optimal speed limit monetary policy rules. We have derived optimal speed limit policy rules under discretion and commitment, with a quadratic loss function and using a loss function that is an approximation to the household utility function. In each case, the optimal policy rule contained speed limit terms. We have presented econometric evidence that suggests that the optimal speed limit policy rule under discretion is consistent with the behaviour of US monetary policymakers during the Great Moderation. Given all this, we would argue that optimal speed limit rules under discretion merit further investigation.

However our model is not perfect. Our theoretical model was extremely simplified and stylized. As a result, it cannot match the main features of macroeconomic data. The model can be extended in two main ways. The first is to allow for a more sophisticated representation of habit effects, for example adopting the formulations of Ravn et al (2006) or Corrado et al (2012). We do not expect this extension to alter the argument that optimal monetary policy with habit effects implies speed limit policy rules. This is because speed limits derive from dynamics in the aggregate demand relationship; these extensions will not eliminate these dynamics. The second extension is to allow for persistence in the Phillips Curve as well as the aggregate demand relationship. Again, we do not expect this extension to affect our basic argument as it will not affect aggregate demand dynamics. It will, however, add another layer of complexity to the model.

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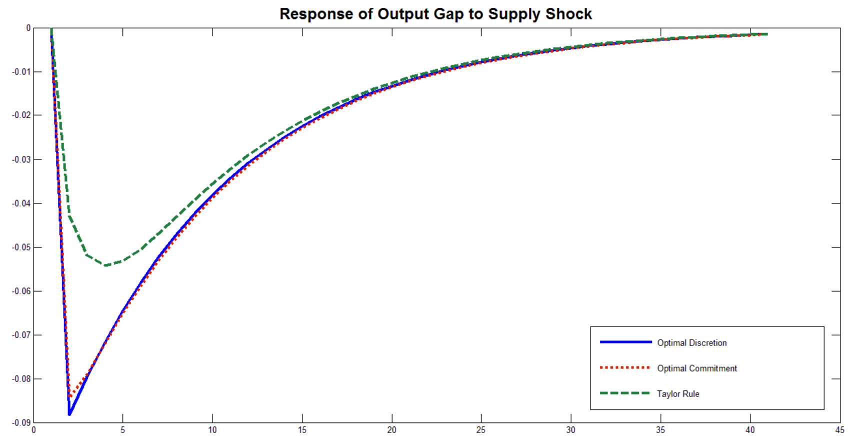
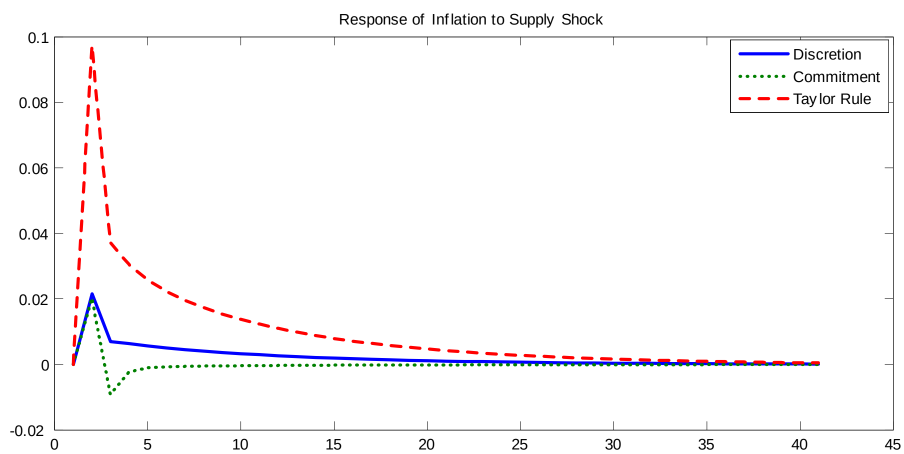
Figure 1a)
Response of Output Gap to Supply Shock Under Alternative Policy Rules ($\mu = 0.71$)
1a



$_1$.pdf

Figure 1b)
Response of Inflation to Supply Shock Under Alternative Policy Rules ($\mu = 0.71$)
1b



$_2$.pdf

Figure 1c)

Response of Interest Rate to Supply Shock Under Alternative Policy Rules ($\mu = 0.71$)

1c

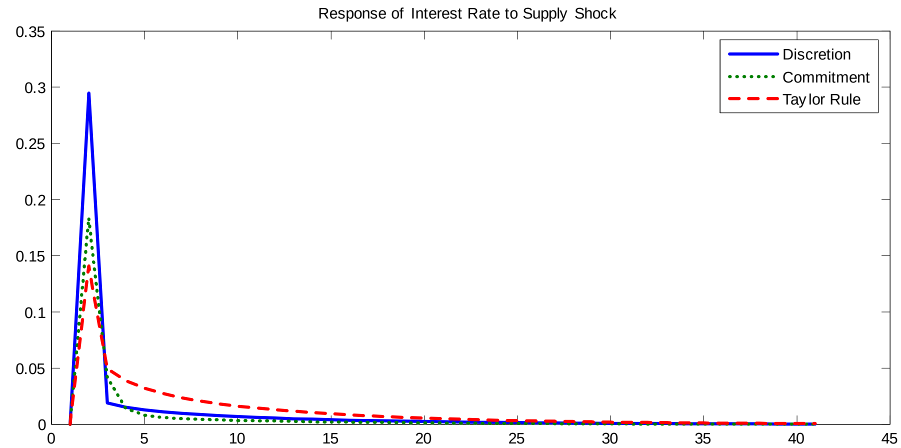


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Figure 2a)

Response of Output Gap to Supply Shock Under Alternative Policy Rules ($\mu = 0$)

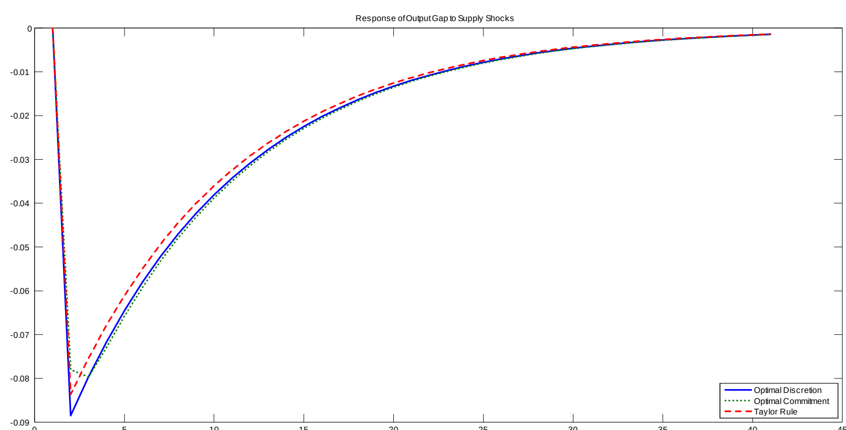


Figure 2b)

Response of Inflation to Supply Shock Under Alternative Policy Rules ($\mu = 0$)

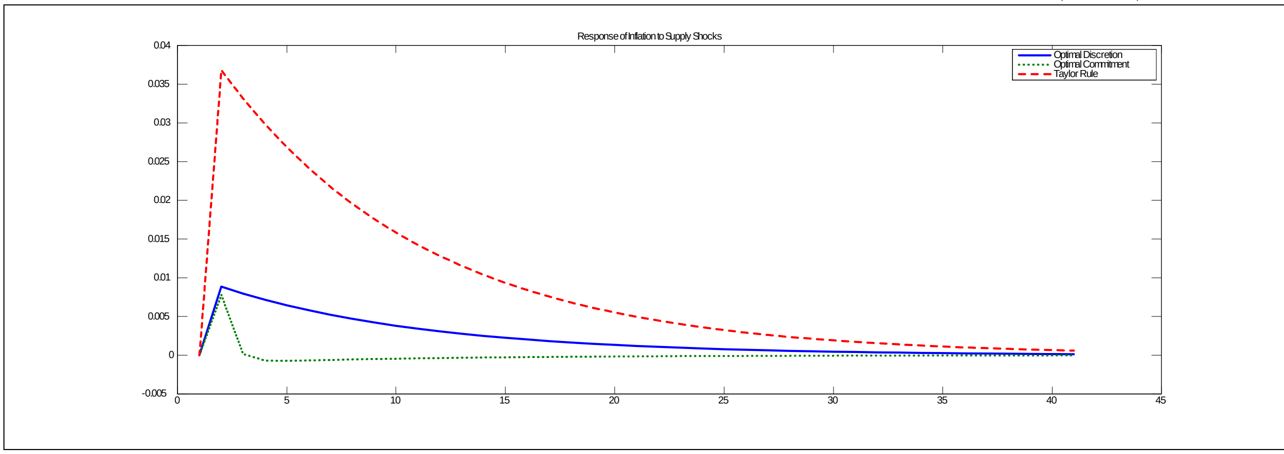
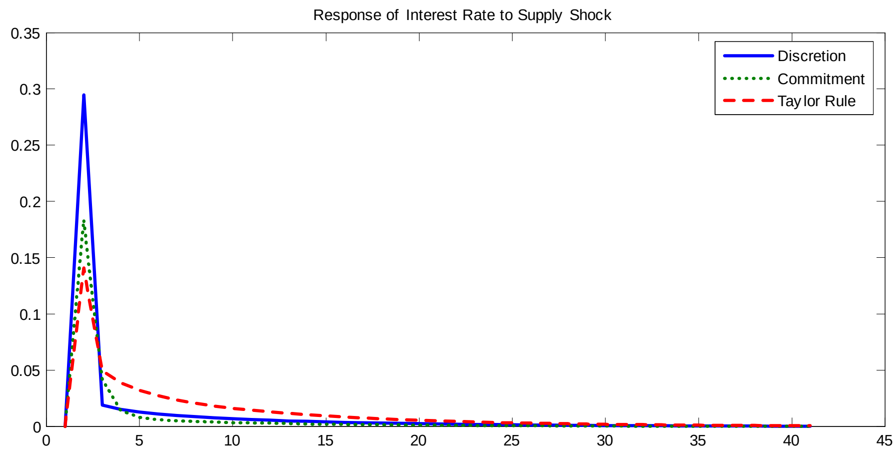


Figure 2c)

Response of Interest Rate to Supply Shock Under Alternative Policy Rules ($\mu = 0$)

1c



$_6.pdf$