Coordination Failure and the Financial Accelerator*

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Abstract

This paper studies the effect of liquidity problems in markets for short-term debt within a DSGE model with leveraged borrowers. Creditors (financial intermediaries) receive imperfect signals regarding the profitability of borrowers (entrepreneurs) and, based on these signals and their beliefs about other intermediaries' actions, choose between rolling over and foreclosing on the debt. Due to the uncoordinated actions of intermediaries, the incidence of rollover is suboptimal, generating endogenous capital scrapping and an illiquidity premium on external finance. As entrepreneurs become more leveraged, the magnitude of the coordination inefficiency increases as do the premiums paid on external finance. The interaction between entrepreneurial leverage and the illiquidity premium generates significant amplification of technology shocks, and predicts that periods of illiquidity in credit markets can generate sharp contractions in output. Two unconventional policy responses are analyzed. Direct lending to entrepreneurs is found to dampen output fluctuations. Equity injections into entrepreneurs' balance sheets, however, are significantly more powerful in dampening the contemporaneous effect of illiquidity shocks, but cause output deviations from potential to persist.

Keywords: Financial accelerator, Business cycles, Global games, Coordination failure, Unconventional policy instruments, Financial crises.

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1 Introduction

Creditors financing a project face a coordination problem. Fear of premature foreclosure by other creditors may lead to preemptive action, undermining the project, and the chances of repayment to those creditors as well. In practice, the existence of coordination problems has an impact on the functioning of many different forms of the credit market, including direct bank loans, lines of credit, commercial paper, corporate bonds and short-term interbank lending. For an example in the bank lending market, Hertzberg et al. (2011) exploit a natural experiment, which compelled banks' to make public negative private assessments about their borrowers. They show that lenders, while learning nothing new about the firm, reduce credit in anticipation of the reaction by other lenders to the same firm. Public information therefore exacerbates lender coordination problems and increases the incidence of financial distress.

Even larger firms with virtually no default risk experience difficulties in rolling over short-term credit. The market for non-financial commercial paper has experienced several liquidity dry ups in recent decades.[1] In the recent financial crisis, the Federal Reserve introduced the Commercial Paper Funding Facility (CPFF) to prevent the market from closing. Notably, none of the issuers who made use of the CPFF defaulted on their debt obligations, suggesting that the liquidity dry up was driven to some extent by coordination problems among creditors, rather than a fundamental increase in the insolvency risk of the issuers.[2]

The most visible and often dramatic manifestation of the coordination failure problem is in the financial sector. At least since Bagehot (1873)'s description of the financial panic of 1866, economists have acknowledged the inherent fragility of the financial sector. The demise of Northern Rock in the UK in 2007 and Bear Stearns and Lehman Brothers in the US in 2008 have been interpretted (see Brunnermeier (2009) and Shin (2009)) as events in which short-term interbank market lenders were unwilling to continue lending to these institutions, for fear that other lenders were doing likewise.[3]

In this paper, the aim is to take the possibility of coordination failure seriously, and

[1] Following the Penn Central bankruptcy in 1970, the Russia/LTCM crisis in 1998 and the Enron/WorldCom episode in 2002.

[2] In fact, the Chapter 11 bankruptcy provision in the U.S. is explicitly designed to address the problem of coordination among creditors; see Jackson (1986). Disorderly liquidation of a company's assets can be an economically inefficient outcome for a business with a fundamentally sound operation but with a debt burden that it cannot service. Chapter 11 affords businesses with excessive debt burdens legal protection to remain a going concern while they are restructured.

[3] It is unclear whether the unwillingness of short-term borrowers to continue lending hastened the bankruptcy of an already insolvent institution or whether the run by creditors scuppered an otherwise sound institution. Disentangling the two effects is incredibly difficult in practice. Morris and Shin (2010) show how the two effects may be isolated in theory.

therefore to model it from first principles and introduce it in the credit market of a standard Dynamic Stochastic General Equilibrium (DSGE) macroeconomic framework. Understanding coordination problems in this framework highlights the role that liquidity and leverage play in the propagation of shocks through an economy. It also offers a useful framework in which unconventional policy responses to shocks can be analyzed. The main findings of the paper are that coordination problems generate a steep equilibrium trade off between the amount that an entrepreneur borrows (i.e. leverages himself) and the illiquidity premium that must be paid on this external finance. This trade off is a result of a suboptimal incidence of debt rollover which results in endogenous capital scrapping. Impulse response analysis shows that a coordination problem among creditors significantly amplifies technology shocks, while illiquidity shocks in credit markets generate a channel for additional volatility in economic activity novel to this paper. The paper analyzes two potential policy instruments to dampen the effect of an illiquidity crisis, namely *direct lending* to and *equity injections* into entrepreneurs. Direct lending is shown to have mildly dampening effects. Equity injections, in contrast, are able to powerfully dampen the contemporaneous effect of falls in liquidity, but tends to cause output to remain away from its steady state for longer. In short, this is because one dollar of additional debt held by the government does not improve the solvency of an entrepreneur materially (which largely determines the extent of the coordination problem), while one dollar of additional equity does. However, government equity injections disincentivize balance sheet adjustment, thus slowing the return of output to its steady state following a shock.

The illiquidity model of creditor coordination failure has two key features. The first is that the borrower has a balance sheet mismatch with longer maturity (or illiquid) assets on one side and shorter maturity (or liquid) liabilities on the other. The second is the existence of multiple creditors who cannot coordinate their actions. When the borrower's debts mature, each creditor must decide whether to rollover or foreclose, taking into account the economic fundamentals of the borrower and the actions of other creditors. In Diamond and Dybvig (1983), this coordination problem is applied to a deposit taking bank.[4] Their model predicts that there are multiple equilibrium outcomes, one of which is a bank run.

Multiple equilibria, however, are problematic for incorporating coordination failure into a general equilibrium framework. In general equilibrium, the pricing of a contract between a borrower and creditor is endogenously determined and therefore requires ex ante knowledge about the probability of different ex post outcomes. The existence of multiple equilibria naturally renders this impossible. The literature on global games offers a solution in this

[4] The literature that emerged from the Diamond and Dybvig (1983) paper is vast. A good overview of this literature, although somewhat out of date, is Gorton and Winton (2003).

regard.[5] The indeterminacy of beliefs in the coordination model with multiple equilibria, as described above, is a consequence of the assumption that creditors' information sets are perfectly symmetric. By introducing idiosyncratic noise into each creditor's signal of a borrower's solvency, the global games literature shows that a unique set of self-fulfilling beliefs will prevail in equilibrium. In this paper the coordination problem in credit markets is therefore modelled in such a way that it can be solved as a sequence of static global games. Specifically, the coordination problem is among financial intermediaries which take deposits from households and provide finance to entrepreneurs.

It is quite reasonable, especially when applying the analysis of this model to the recent financial crisis, to think of the financial intermediaries as depository banks and the set of entrepreneurs as behaving like a shadow banking sector. In effect, entrepreneurs fund themselves with short-term wholesale funding in order to invest in longer term assets. In particular, entrepreneurs manage physical capital. In every period entrepreneurs purchase physical capital from capital producers with the intention of applying their own productivity and then renting the augmented capital to goods producing firms, for use in production. Entrepreneurs fund their physical capital purchases using their own net worth and by issuing one-period debt to a continuum of intermediaries. Only after this debt has been issued do intermediaries observe (with noise) a signal of each entrepreneur's productivity. The contract between the intermediary and entrepreneur allows the creditor to *rollover* (i.e. remain invested) or *foreclose* (i.e. seize its collateral stake from the entrepreneur's physical capital and take over the role of capital management for the remainder of the period).

The payoffs for an intermediary to each action will depend on fundamentals (the realization of the entrepreneur's productivity), the actions of other intermediaries (the proportion of intermediaries that foreclose) and on the (exogenous) intra-period illiquidity of capital. The intra-period illiquidity of capital is an important feature because it ensures, for a sufficiently leveraged entrepreneur, that not all intermediaries can foreclose and still leave the entrepreneur with strictly positive levels of capital with which to continue operating. The result is that entrepreneurs' balance sheets are inherently illiquid, and by extension ensures that beliefs (and higher order beliefs) about other intermediaries' behaviour are important. It is important to note, though, that illiquidity risk is an endogenous outcome in the model. An entrepreneur that is leveraged is not necessarily illiquid. Liquidity risk depends on the relationship between the exogenous illiquidity parameter and the endogenously determined return on foreclosure. In equilibrium, it is optimal for the entrepreneurs to leverage them-

[5] The seminal work on this method of selecting a unique equilibrium is related to Carlsson and Van Damme (1993). For a comprehensive overview of the global games literature, see Morris and Shin (2003). Morris and Shin (1998) and Corsetti et al. (2004) apply the global games methodology in the closely related context of currency crisis.

selves to the extent that they *do* face illiquidity risk.

The global games method solves for a unique switching equilibrium in the coordination game among intermediaries. In this switching equilibrium, all intermediaries will foreclose on an entrepreneur if the entrepreneur's productivity is below a given threshold and rollover otherwise. The central implication is that this (no coordination) threshold is strictly above the perfect coordination threshold. This wedge (between the two thresholds) indicates that in every period there are some entrepreneurs that are liquidated in equilibrium that creditors would not have chosen to liquidate if they had been able to coordinate their actions. It is this wedge, this inefficiency, which captures the market imperfection as a result of coordination failure. As a result, the loan rate charged to entrepreneurs prices in an illiquidity premium over the risk-free rate. And this illiquidity premium depends importantly on the leverage of the entrepreneur.[6] Entrepreneurs who have borrowed a larger proportion of their total financing needs, have a less liquid and therefore a more fragile balance sheet. To account for this heightened illiquidity, contracts with higher capital to net worth ratios also have loan rates with a higher illiquidity premium over the risk-free rate. This market imperfection will generate additional amplification of shocks in the economy.

The key micro-founded coordination game at the heart of this paper has its roots in the models of Morris and Shin (2004), Rochet and Vives (2004) and Goldstein and Pauzner (2005). To understand the macroeconomic consequences of this game, I aggregate and place it within a fully dynamic, general equilibrium setting.

The full model has a continuum of entrepreneurs and intermediaries. Intermediaries interact with households which save via deposits, while entrepreneurs interact with capital producers by buying capital and with firms by renting them capital. The other interactions within the model are relatively standard. In fact, the equilibrium conditions of the full model are the set of first-order conditions and constraints of a canonical real business cycle model, with the addition of two new equations as a result of the existence of the coordination problem among financial intermediaries. The first new equation defines the relationship between the illiquidity premium paid by entrepreneurs on external finance and the leverage ratio. The second equation describes the endogenous evolution of entrepreneurial net worth. It is then the interaction between asset prices (which affect net worth), leverage and the illiquidity premium, which generates an additional mechanism in this model that propagates and amplifies shocks in the economy, via investment decisions to production and consumption outcomes.

[6] Throughout the paper it is useful to use the terms *leverage ratio* and *capital to net worth ratio* interchangeably. Strictly speaking, the leverage ratio is $1 - N/K$ where N/K is the inverse of the capital to net worth ratio. However, since the leverage ratio is a monotonically increasing function of the capital to net worth ratio, it is worth introducing this small inaccuracy for the sake of clarity.

This propagation mechanism is examined under two different crisis scenarios, namely a negative technology shock and an illiquidity shock. An economy-wide technology shock lowers the marginal product of capital and therefore lowers entrepreneurial profitability (net worth), increases leverage, exacerbates the coordination problem for financial intermediaries, and causing an endogenous rise in the illiquidity premium. The higher premium on external finance causes investment demand to fall further than in the frictionless (perfect coordination) real business cycle model. This lowers output, which in turn lowers entrepreneurial net worth further. And so the cycle continues causing an additional multiplier effect in the economy as a result of the coordination problem in the credit market. A similar mechanism operates to exacerbate the effect of an illiquidity shock.

A key insight for policymakers from the illiquidity crisis scenario is that an incremental increase in the persistence of the illiquidity shock has a disproportionate effect on the contraction in output. It suggests that policies aimed at restoring liquidity in credit markets that have seized up are vital for reducing the real economic impact of financial market dislocations.

1.1 Related literature

Theories of coordination problems in credit markets have received little attention in the macroeconomics literature. The vast literature that studies financial frictions in macroeconomic models has broadly followed two paths which find their roots in the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).[7] Kiyotaki and Moore (1997) build on the hold-up model of debt in Hart and Moore (1994). In this model, the inalienability of human capital introduced a binding collateral constraint on lending. Bernanke and Gertler (1989) adopt the costly state verification (CSV) assumption from Townsend (1979). With costly state verification there is an asymmetry in a single borrower-creditor relationship. To ensure that a borrower truthfully reveals the return on a project, the creditor must pay a monitoring cost when the borrower cannot repay his full debt obligation. The model of Bernanke and Gertler (1989), in reduced form, is qualitatively similar to my model.[8] However, there are important differences. First, the microfoundations and the interpretation of the distortion created by coordination problems among creditors, is in itself, a distinct and important fea-

[7] An important contribution to the development of this literature was Carlstrom and Fuerst (1997). More recently Iacoviello and Neri (2010) applied financial frictions to the housing market, Christiano et al. (2008) estimated a medium-scale DSGE model with financial frictions and Faia and Monacelli (2005), Curdia and Woodford (2009), De Fiore and Tristani (2009) and Carlstrom et al. (2010) all analyze optimal monetary policy in the presence of financial frictions. Close in spirit to the model presented here, Angeloni and Faia (2010) introduce a model of bank runs into a DGSE framework to analyze macroprudential policy.

[8] This paper borrows the financial accelerator moniker for use in the title, in recognition of these close ties between the two models.

ture of credit markets. Second, the coordination failure model predicts greater amplification of technology shocks. This is because this model is able to generate a steeper trade off between the risk premium on external finance and the leverage ratio, and because coordination failure in credit markets generates endogenous capital scrapping. Moreover, the model is able to rationalize the recent crisis, which was in part the result of a illiquidity problems in credit markets, and offer a framework in which to analyze the use of various credit policy instruments.

Recently there has been a growing literature, attempting to model the policy responses introduced by the U.S. Federal Reserve and Treasury and other central banks and governments to the financial crisis. The work in this paper is closely related to the work of Gertler and Kiyotaki (2009) and Gertler and Karadi (2010).[9] The contribution of my paper in this regard is to assess these unconventional policy responses in a model which interprets the recent financial crisis as being triggered by an illiquidity shock in credit markets.

The remainder of the paper is structured as follows. Section 2 presents the frictionless DSGE model. Section 3 details the role of the entrepreneurs and financial intermediaries, solves the unique rollover / foreclosure switching equilibrium and the terms of the debt contract. Section 4 performs comparative static analysis of the credit market and compares this model to the costly state verification model. Sections 5 presents impulse response analysis. Section 6 analyzes the effects of direct lending and equity injections following an illiquidity shock. Section 7 concludes.

2 The basic frictionless DSGE model

Before describing the economy with coordination problems in the credit market, the basic DSGE model, in which the financial sector operates without friction, is presented. The basic specification is a purely real, closed economy DSGE model. The lack of a financial friction means it is possible to abstract from the role played by entrepreneurs and financial intermediaries in the economy. The agents of interest in the basic model are the households, capital producers and goods producing firms. Section 3 will explicitly introduce the entrepreneurs and financial intermediaries, while the introduction of my quasi-monetary-fiscal policymaker will wait until Section 6.

[9] See also Sargent and Wallace (1982), Cúrdia and Woodford (2010) and Reis (2009).

2.1 Households

The economy is populated by households of measure one. Households supply labour, L_t, enjoy leisure, $1-L_t$, consume, C_t and save, D_t. The expected lifetime utility of a representative household is:

$$E_0\sum_{t=0}^{\infty}\beta^t\left(\ln\left(C_t-hC_{t-1}\right)-\frac{\chi}{1+\rho}L_t^{1+\rho}\right) \tag{1}$$

where E_t is the expectations operator conditional on date t information, β is the subjective discount factor and $0<h<1$. The model abstracts from many of the frictions that appear in the wider DSGE literature. However, I follow Gertler and Kiyotaki (2009) by including habit formation of consumption and adjustment costs of investment because these features are helpful for reasonable quantitative performance and because they can be kept in the model at minimal additional complexity. The representative household maximizes its expected discounted utility subject to the budget constraint:

$$C_t \leq W_tL_t+R_tD_t-D_{t+1}+\Pi_t-T_t \tag{2}$$

where W_t is the real wage, R_t is the risk-free rate of return on savings, Π_t are profits (from goods producing firms) and T_t are lump sum taxes. Let $U_{C,t}$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor. Then the household's first-order conditions with respect to labour supply and consumption/savings are:

$$\begin{aligned} E_tU_{C,t}W_t &= \chi L_t^{\rho} && (3)\\ E_t\Lambda_{t,t+1}R_{t+1} &= 1 && (4)\end{aligned}$$

where:

$$\begin{aligned} U_{C,t} &\equiv \left(C_t-hC_{t-1}\right)^{-1}-\beta h\left(C_{t+1}-hC_t\right)^{-1}\\ \Lambda_{t,t+1} &\equiv \beta\frac{U_{C,t+1}}{U_{C,t}}\end{aligned}$$

2.2 Capital producers

Each period a representative capital producer buys final (investment) goods, I_t and old depreciated capital, $(1-\delta)\,K_t^*$ and produces new capital. In the basic model, K_t^* is simply K_t. K_t^* and K_t differ only when the coordination problem in the credit market is introduced, where $K_t-K_t^*$ denotes the deadweight cost of coordination failure. K_t^* is used to avoid repeating later in the paper many of the equilibrium equations which are common between the frictionless and fully specified models.

The technology to produce new capital exhibits capital adjustment costs as follows:

$$K_{t+1} = (1-\delta) K_t^* + \Psi\left(\frac{I_t}{K_t}\right) K_t \tag{5}$$

where $\Phi(.)$ is increasing and convex in the ratio of investment to capital. In the steady state, $\Phi(I/K) K = I$, where any variable without a time-subscript denotes its non-stochastic steady state value. The new capital is sold in a perfectly competitive market at price, Q_t:

$$Q_t = \left(\Psi'\left(\frac{I_t}{K_t}\right)\right)^{-1} \tag{6}$$

2.3 Goods producers

Each period the goods producing firms hire labour from households and rent capital, and combine these inputs using a constant returns to scale technology to produce output, Y_t:

$$Y_t = A_t (K_t^*)^{\alpha} L_t^{1-\alpha} \tag{7}$$

where A_t is total factor productivity and follows the exogenous stochastic process, $A_t = A_{t-1}^{\rho_A} \exp\left(\epsilon_t^A\right)$, and $\epsilon_t^A \sim N\left(0, \sigma_A^2\right)$. Output is sold in a perfectly competitive market at a unit price. The relevant first-order conditions of the firms are:

$$R_t^K = \alpha A_t \frac{Y_t}{K_t^*} \tag{8}$$

$$W_t = (1-\alpha) A_t \frac{Y_t}{L_t} \tag{9}$$

where R_t^K is the rental rate of capital. In equilibrium, the risk-free rate of return paid to households on savings is equal to the expected rate of return on capital:

$$E_t\left(\frac{R_{t+1}^K + (1-\delta) Q_{t+1}}{Q_t}\right) = R_{t+1} \tag{10}$$

and:

$$D_{t+1} = Q_t K_{t+1} \tag{11}$$

every period. To close the model, aggregate output is divided between household consumption, investment expenditures and government consumption, G_t:

$$Y_t = C_t + I_t + G_t \tag{12}$$

and the government runs a balanced budget:

$$G_t = T_t$$

Without credit market frictions, the competitive equilibrium matches the social planner's problem which involves choosing aggregate quantities $\{Y_t, C_t, I_t, L_t, K_{t+1}\}$ given the aggregate state $\{C_{t-1}, K_t, A_t\}$ to maximize the representative household's expected discounted utility (subject to resource constraints). The competitive equilibrium of this frictionless economy (a standard real business cycle model) will serve as a benchmark for the fully specified model with coordination problems in the credit markets.

3 Financial frictions via coordination failure

The financial sector is, in general, responsible for transforming household's savings into rentable capital for firms. In a simple version of the world there are two types of agents in the financial sector, namely financial intermediaries and entrepreneurs. The intermediaries accept household savings and use these to fund entrepreneurs. Entrepreneurs (or *capital managers*), in turn, use the borrowed funds to purchase capital (from capital producers), and rent the capital to goods producing firms. If this process operates without friction (i.e. no agency problems or coordination problems), it is possible to abstract from this sector, since the competitive equilibrium is unaltered. It would simply be sufficient to note that, to have no arbitrage, it must hold that the risk-free rate of return paid to households is equal to the expected rate of return on capital, and that the value of total deposits equal the value of the capital stock every period, equations (10) – (11).

In this section, the situation in which intermediation is not frictionless is studied. Specifically, I analyze an environment in which intermediaries face the problem of coordinating their lending decisions to entrepreneurs. Problems arise when entrepreneurs' short-term debt is held by a large number of intermediaries, which then have difficulty coordinating their decisions whether to foreclose or rollover when the short-term debt matures.

Introducing a friction of this kind makes two significant alterations to the set of equilibrium equations that define the economy. The first is that the coordination problem generates an endogenous illiquidity premium on borrowed funds (i.e. a wedge between the left- and right-hand side of equation (10)), which becomes larger, the more leveraged the borrower is (i.e. the higher the capital to net worth ratio). The equilibrium illiquidity premium is solved in two stages. First, by solving for the intermediaries' decision rule over when to rollover and when to foreclose, for a given debt contract. And then to solve for the equilibrium debt contract, given the intermediaries' decision rule. The second alteration to the basic model is that the coordination problem introduces an additional state variable; entrepreneurial net worth, and an additional equilibrium equation which governs the law of motion of this new state variable.[10]

3.1 The environment

Let us begin with the entrepreneurs. Think of them as *capital managers*. They purchase capital from capital producers with the intention of augmenting the capital using their own productivities and renting the augmented capital to goods producing firms.

Formally, there is a continuum of risk-neutral entrepreneurs, indexed by e. At the end of period t, each entrepreneur purchases capital, $K_{t+1}(e)$ at price Q_t from capital producers, using his net worth, $N_{t+1}(e)$ and by issuing debt, $B_{t+1}(e)$, to a continuum of intermediaries:

$$Q_t K_{t+1}(e) = B_{t+1}(e) + N_{t+1}(e) \tag{13}$$

Entrepreneurs, when they make their capital purchase decisions, are homogenous in all respects except for their level of net worth. After the purchase, each entrepreneur *observes* his productivity, which will transform the capital to $\omega_{t+1}(e) K_{t+1}(e)$, where $\omega_{t+1}(e)$ is a random variable with $E(\omega) = 1$, distributed independently over time and across entrepreneurs, with c.d.f. $F(.)$, p.d.f. $f(.)$ and support on $[0, \infty)$. The distribution is assumed to be log-normal and common knowledge.[11] An entrepreneur's expected rate of return per unit of productive capital (i.e. per unit of $\omega_{t+1}(e) K_{t+1}(e)$) is:

$$E_t R^E_{t+1} = E_t \left\{ \frac{R^K_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right\} \tag{14}$$

which is composed of the rental rate (or marginal product of capital), R^K_{t+1} paid by goods producing firms, and any capital gains earned from price movements between the purchase and sale of the capital.

Assume that the debt issued by the entrepreneur, $B_{t+1}(e)$ is short-term. In this

[10] Due to the existence of a friction in the credit market, entrepreneurs are able to earn economic rents. It is these economic rents (profits) that constitute entrepreneurial net worth. Consider the frictionless credit market. In this case, entrepreneurs earn no economic rents and net worth is zero in all periods. Because lending is frictionless, this means that entrepreneurs' are able to finance their capital purchases 100% using debt. In other words, they are 100% leveraged, and pay no risk premium on their debt over the risk-free rate of return.

[11] The idiosyncratic productivity shock is important in the model in order to generate a subset of entrepreneurs that are able to rollover their borrowing each period, and a subset of entrepreneurs that experience foreclosure. We require the distribution of the idiosyncratic productivity shock, ω to have a non-negative support, although the choice of log-normality is not crucial.

Table 1: Sequence of Events in a Given Period

1.	The aggregate return per unit of capital, R_t^E is realized.
2.	Given the realization of R_t^E, the loan rates, R_t^{LR} and R_t^{LF} (and hence $\overline{\omega}_t$ and ω_t^*) from the state-contingent menu are determined.
3.	The idiosyncratic productivity of each entrepreneur, $\omega_t(e)$ is realized.
4.	Intermediaries receive signals $\omega_t(e, f)$ and decide whether to rollover or foreclose using the threshold strategy, $\omega^*(\overline{\omega}_t)$.
5.	Entrepreneurs and intermediaries in possession of capital use their idiosyncratic productivities ($\omega_t(e)$ and γ respectively) to augment their capital.
6.	The augmented capital is rented to goods producing firms at the rental rate, R_t^K.
7.	Once goods production is complete, capital is sold back to the capital producers at price Q_t.
8.	Entrepreneurs repay intermediaries and intermediaries pay households the risk-free rate of return, R_t on deposits.
9.	A proportion of entrepreneurs, $1 - \upsilon$ exit and consume their profits and new entrepreneurs of mass $1 - \upsilon$ enter.
10.	All entrepreneurs receive a small lump sum transfer, T^E from households.
11.	Entrepreneurs use their new net worth, $N_{t+1}(e)$ and establish new borrowing, $B_{t+1}(e)$ subject to state-contingent loan rates, R_{t+1}^{LR} and R_{t+1}^{LF} to purchase new capital $K_{t+1}(e)$ at price Q_t from capital producers.

model there is no beneficial rationale *per se* for the maturity mismatch, as in, for example, the Diamond and Dybvig (1983) model of bank runs in which some depositors might have liquidity needs at the intra-period stage. Instead, I motivate the assumption of short-term debt issuance along the lines of Brunnermeier and Oehmke (2010)'s *maturity rat race*. In their model, it is a negative externality that causes borrowers to use excessively short-term financing. The externality is that shorter maturity claims dilute the value of longer maturity claims. This causes a borrower to successively move towards short-term financing. Complete short-term financing is therefore the only stable equilibrium. This means that the entrepreneur has a maturity mismatch on its balance sheet; its assets (the capital) only earn a return at the end of period $t + 1$ but its liabilities (the debt) must be rolled over at an intra-period stage. Table 1 summarizes the exact timing of events during a given period.

Because entrepreneurs are risk-neutral and households are risk-averse, the debt contract the intermediaries sign has the entrepreneur absorb any aggregate risk.[12] The debt contract

[12] Intermediaries will hold a portfolio of entrepreneurial debt which will perfectly diverisfy the idiosyncratic risk involved. This, plus the feature that the debt contracts will insulate the intermediaries from aggregate risk means that they will hold perfectly safe portfolios which earn the risk-free rate of return in equilibrium.

therefore specifies a *state-contingent* non-default loan rate that is paid at the end of the period, $R^{LR}_{t+1}(e)$. If an intermediary chooses not to rollover at the intra-period stage, the entrepreneur pays the non-default *foreclosure* loan rate, $R^{LF}_{t+1}(e) = R^{LR}_{t+1}(e) / R^{E}_{t+1} Q_t$.[13]

Suppose all intermediaries rollover. It is then possible to define an entrepreneur's solvency condition in terms of a cutoff value of idiosyncratic productivity, denoted as $\overline{\omega}_{t+1}(e)$ such that the entrepreneur has just enough resources to repay the loan:

$$\overline{\omega}_{t+1}(e) R^{E}_{t+1} Q_t K_{t+1}(e) = R^{LR}_{t+1}(e) B_{t+1}(e) \tag{15}$$

Intermediaries however need not rollover. If an intermediary forecloses, the entrepreneur pays $R^{LF}_{t+1}(e) B_{t+1}(e)$ in units of capital (since the entrepreneur has no other assets), which equivalently is $\overline{\omega}_{t+1}(e) K_{t+1}(e)$. Foreclosure, however, incurs a deadweight cost on the entrepreneur. For every unit of capital transferred from the entrepreneur to an intermediary, the entrepreneur pays an additional foreclosure cost of $(1-\lambda_t)$. λ_t, which exists within the $(0,1)$ interval because it measure the proportion of assets that are liquid, is assumed to follow an exogenous stochastic process, $\lambda_t = \lambda (\lambda_{t-1})^{\rho_\lambda} e^{\varepsilon^{\lambda}_t}$, where $\epsilon^{\lambda}_t \sim N(0, \sigma^2_\lambda)$.[14]

The incursion of $(1-\lambda_t)$ is important to generating the financial friction in this model. $(1-\lambda_t)$ is a proxy for the degree of illiquidity in the credit market. When an economic agent is forced to sell an asset quickly in an illiquid market, he must content himself with a price for the asset which is below its fundamental value. It is this feature of credit markets which is captured by the foreclosure cost, $(1-\lambda_t)$. It means that entrepreneurs' balance sheets not only exhibit a maturity mismatch, but also a liquidity mismatch. The intra-period value of an entrepreneur's capital is $\lambda_t K_{t+1}(e)$ while the claims on the entrepreneur if all intermediaries choose to foreclose is $\overline{\omega}_{t+1}(e) K_{t+1}(e)$. When the loan rate is such that $\overline{\omega}_{t+1}(e) > \lambda_t$, the entrepreneur is illiquid and vulnerable to a credit run. It will be shown that $\overline{\omega}_{t+1}(e)$, which is an endogenous variable, is always greater than λ_t in equilibrium as the result of an optimization problem. Thus, the coordination problem exists as an endogenous phenomenon.

3.2 The financial intermediaries' problem

Let us now turn to the financial intermediaries. Intermediaries are perfectly competitive, accept deposits from households (promising to pay a risk-free rate of return) and hold a fully diversified portfolio of entrepreneurial debt. As described above, the debt contracts

[13] As well as reducing the number of choice variables in the determination of the debt contract, this specification of the foreclosure loan rate is quite reasonable as we discuss in the next sub-section.

[14] The steady state value, λ is assumed to be sufficiently within the $(0,1)$ interval and σ^2_λ is assumed to be sufficiently small that the probability of λ_t taking a value outside the $(0,1)$ interval is negligable.

allow intermediaries to foreclose early. By foreclosing, an intermediary seizes an entrepreneur's capital and earns a return directly by taking over the capital management process (i.e. augmenting the capital itself and renting the augmented capital to the goods producing firms).

This setting raises three issues for the modeler. First, it is necessary to solve for the intermediaries' decision rule regarding when to foreclose and when to rollover. Second, in order to insulate themselves from systemic risk (and honour the promise of a risk-free return for depositors), the debt contracts must be state-contingent. This means that the loan rates must be contingent on the realized state of the economy that period. Third, it is necessary to define a break-even condition for intermediaries' which any equilibrium debt contract must satisfy.

Begin with the rollover/foreclosure decision. At this stage, the aggregate state of the economy (including the rate of return on capital, R_{t+1}^{E}) has already been realized and the appropriate loans rates, $R_{t+1}^{LR}(e)$ and $R_{t+1}^{LF}(e)$ have been identified from the state-contingent menu. Next, each intermediary observes a noisy signal of each entrepreneur's idiosyncratic productivity:

$$\omega_{t+1}(e,f) = \omega_{t+1}(e) + \varepsilon_{t+1}(e,f)$$

where f indexes the intermediary. The noise term, ε is independently distributed over time and across entrepreneurs and intermediaries and is drawn from the uniform distribution, $\varepsilon \sim U(-\bar{\varepsilon}, \bar{\varepsilon})$ with mean zero. Following the receipt of the signal, intermediaries must decide individually whether to rollover or foreclose on each of the entrepreneurs' debt it holds.

An intermediary's payoff to either decision (rollover or foreclose) depends on the realization of the entrepreneur's idiosyncratic productivity shock, and on the actions of the other intermediaries. These payoffs are derived in Appendix A and are given in Table 2, where $p_{t+1}(e)$ is the proportion of intermediaries that foreclose, and γ is the capital management productivity of the intermediary (i.e. the intermediaries' equivalent of ω). γ measures the intermediaries' ability to do an entrepreneur's job. When $\gamma = 0$, intermediaries and entrepreneurs are complements in the financial sector - neither agents can do the others' job. When $\gamma > 0$, there is a certain degree of substitutability with intermediaries able to earn a positive return from capital management.[15]

[15] Above a critical value of γ, the intermediaries have no incentive to lend to entrepreneur's since they will be able to earn a higher return by directly buying and managing capital themselves. We therefore assume:

$$\frac{u^{\min} E\left\{R^{E}\right\}}{R} < \frac{1}{\gamma}$$

where $u^{\min}$ defines the worst possible non-negligable probability aggregate shock to R^{E}. See Appendix A

Table 2: Payoffs to Intermediaries

Rollover	Foreclosure		
$\overline{\omega}$	$\gamma\overline{\omega}$	when	$0 \leq p \leq \frac{\lambda}{\overline{\omega}}$ and $\omega \geq \frac{\overline{\omega}(1-p)\lambda}{\lambda - p\overline{\omega}}$
$\frac{\omega}{(1-p)}\left(1 - \frac{p\overline{\omega}}{\lambda}\right)$	$\gamma\overline{\omega}$	when	$0 \leq p \leq \frac{\lambda}{\overline{\omega}}$ and $\omega < \frac{\overline{\omega}(1-p)\lambda}{\lambda - p\overline{\omega}}$
0	$\frac{\gamma\lambda}{p}$	when	$\frac{\lambda}{\overline{\omega}} < p \leq 1$

Note: The payoffs have been normalized by $R^E_{t+1}Q_tK_{t+1}$, and are for a given intermediary's action regarding a given entrepreneur's debt. Since this is a static problem that symmetric intermediaries fact every period, all time and agent indexes have been dropped for clarity.

Consider the top-left entry in Table 2. When the proportion of intermediaries rolling over, $1 - p_{t+1}(e)$ is sufficiently high and the entrepreneur's productivity, $\omega_{t+1}(e)$ is also sufficiently high, an intermediary that rolls over receives the non-default return:

$$\overline{\omega}_{t+1}(e)\, R^E_{t+1}Q_tK_{t+1}(e)$$

given in equation (15). If however, the intermediary rolls over when a sufficiently large number of intermediaries foreclose such that the entrepreneur runs out of capital, the rolled over intermediary gets a zero return (given in the bottom-left entry). There is also an intermediate outcome in which the entrepreneur survives the intra-period stage, but fails to generate a high enough return to fully pay his debt obligations to the rolled over intermediaries. This is given in the middle-left entry.

The right-hand column gives the payoffs from foreclosure. If an intermediary forecloses, but sufficiently few other intermediaries foreclose to cause the entrepreneur to lose all its capital, the intermediary receives $\overline{\omega}_{t+1}(e)\, K_{t+1}(e)$ units of capital, applies his own productivity, γ and earns the gross return R^E_{t+1} on the augmented capital value to give a total return of $\gamma\overline{\omega}_{t+1}(e)\, R^E_{t+1}Q_tK_{t+1}(e)$. This is given in the top- and middle-right entries of Table 2. If the proportion of intermediaries that foreclose is such that the entrepreneur has insufficient capital to pay the foreclosure loan rate, then the entrepreneur's liquid capital, $\lambda_tK_{t+1}(e)$ is divided equally among the foreclosing intermediaries, which will ultimately earn them a return of $\frac{1}{p_{t+1}}\gamma\lambda_tR^E_{t+1}Q_tK_{t+1}(e)$ (given in the bottom-right entry).

Clearly, the payoff structure exhibits strategic complementarities. Up to the critical

for details.

$p_{t+1}(e)$ at which the entrepreneur fails (i.e. runs out of capital), the *net* payoff from rolling over (i.e. the payoff from rollover minus the payoff from foreclosure) decreases as the proportion of intermediaries that foreclose, $p_{t+1}(e)$ becomes larger. So, intermediaries face a strategic environment in which higher-order beliefs regarding the actions of other intermediaries are important.

How do intermediaries behave in this environment? Proposition 1 states that, under certain technical assumptions, there is a unique equilibrium. An intermediary's action is uniquely determined by its signal: It forecloses on entrepreneur e if and only if its signal, $\omega_{t+1}(e,f)$ is below a certain threshold.

Proposition 1 *There is a unique (symmetric) equilibrium in which an intermediary forecloses if it observes a signal, $\omega_{t+1}(e,f)$ below threshold $\omega^*(e)$ and does not foreclose if it observes a signal above.*

Proof. See Appendix A for the proof.[16] ■

In computing the threshold, $\omega^*_{t+1}(e)$, observe that an intermediary with signal:

$$\omega_{t+1}(e,f) = \omega^*_{t+1}(e)$$

must be indifferent between foreclosing and rolling over. The intermediary's posterior distribution of $\omega_{t+1}(e)$ is uniform over the interval $\left[\omega^*_{t+1}(e)-\varepsilon, \omega^*_{t+1}(e)+\varepsilon\right]$. Moreover, the intermediary believes that the proportion of intermediaries which foreclose, as a function of $\omega_{t+1}(e)$ is $p\left(\omega_{t+1}(e), \omega^*_{t+1}(e)\right)$, where:

$$p(\omega(e), \omega^*(e)) = \begin{cases} 1 & \text{if} \quad \omega(e) \leq \omega^*(e) - \varepsilon(e,f) \\ \frac{1}{2} + \frac{\omega^*(e)-\omega(e)}{2\varepsilon(e,f)} & \text{if} \quad \omega^*(e) - \varepsilon(e,f) \leq \omega(e) \leq \omega^*(e) + \varepsilon(e,f) \\ 0 & \text{if} \quad \omega(e) \geq \omega^*(e) + \varepsilon(e,f) \end{cases}$$

Thus, its posterior distribution of $p_{t+1}(e)$ is uniform over $[0,1]$. At the limit (when $\bar{\varepsilon} \to 0$),

[16] Technically, the proof of this proposition requires that there exists an upper and lower dominance region, i.e. a region $\left(\omega^H, \infty\right)$ in which an intermediary would rollover, regardless the actions of other intermediaries, and a region $\left[0, \omega^L\right)$ in which an intermediary would foreclose, regardless the actions of other intermediaries. A formal discussion of this condition is left to Appendix A. However, it is worth noting that when the intermediaries receive a noiseless singal, the region between the lower and upper dominance regions is indeterminate in the sense that, on the interval $\left[\omega^L, \omega^H\right]$ there are multiple equilibria. But, a grain of doubt for intermediaries (i.e. $\bar{\varepsilon}$ arbitrarily close to zero) leads to the starkly different (and very useful!) result given in Proposition 1.

the resulting indifference condition is:[17]

$$\int_{p=\frac{\lambda}{\overline{\omega}}}^{1} -\frac{\gamma\lambda}{p}dp + \int_{p=0}^{\frac{\lambda}{\overline{\omega}}} \left(\frac{\omega^*}{(1-p)} \left(1 - \frac{p\overline{\omega}}{\lambda}\right) - \gamma\overline{\omega} \right) dp = 0 \tag{16}$$

Solving for $\omega_{t+1}^*(e)$ leads to Proposition 2:

Proposition 2 *In equilibrium, with the noise component of intermediaries' signals arbitrarily close to zero, all intermediaries will foreclose on entrepreneur e if $\omega_{t+1}(e) < \omega_{t+1}^*(e)$ and all intermediaries will rollover otherwise, where:*

$$\omega_t^* = \gamma\lambda_t \frac{\frac{\lambda_t}{\overline{\omega}_t}\left(1 - \ln\left(\frac{\lambda_t}{\overline{\omega}_t}\right)\right)}{\frac{\lambda_t}{\overline{\omega}_t} + \left(1 - \frac{\lambda_t}{\overline{\omega}_t}\right)\ln\left(1 - \frac{\lambda_t}{\overline{\omega}_t}\right)} \tag{17}$$

It is a striking result of Proposition 1 that there exists a unique switching equilibrium, even when the noise term in intermediaries' signals is arbitrarily close to 0. It means that in equilibrium entrepreneurs never experience a partial credit run. An entrepreneur experiences a complete credit run with probability $F\left(\omega_{t+1}^*(e)\right)$ and full rollover with probability $1 - F\left(\omega_{t+1}^*(e)\right)$ in period $t+1$.

We are now in a position to get a sense of the inefficiency (or friction) that is generated as a result of the coordination problem. Consider the decision of an intermediary when it is the sole holder of entrepreneur e's debt (or equivalently a scenario in which intermediaries can costlessly and credibly coordinate their actions).[18] If it forecloses it gets the return $\gamma\lambda_t R_{t+1}^E Q_t K_{t+1}(e)$ and if it rolls over it gets the lesser of $R_{t+1}^{LR}(e) B_{t+1}(e)$ and $\omega_{t+1}(e) R_{t+1}^E Q_t K_{t+1}(e)$. The optimal action is to foreclose when $\omega_{t+1}(e) < \gamma\lambda_t$ and rollover otherwise. The efficient, full coordination threshold is $\omega_{eff}^* = \gamma\lambda_t$ and the inefficiency wedge

[17] To be accurate, this indifference condition only correctly specifies the true $\omega^*(e)$ when $\omega^*(e) < \overline{\omega}$. The complete indifference condition is given in Appendix A. However, for reasonable parameterizations of the model, the values of γ and λ are such that this indifference condition is a sufficient characterization of intermediaries' decision rules.

[18] There are two possible benchmarks against which to gauge the inefficiency generated in the credit market. The first, which we apply in the text, is to eliminate the coordination problem by assuming that creditors can perfectly coordinate their actions. The second possibility is to assume that only long-term debt contracts are feasible. This second possibility eliminates the rollover / foreclosure decision and thus eliminates the coordination problem. The basic frictionless RBC model in Section 2 is more akin to this second formulation. Note that under the first formulation, it is efficient to coordinate and foreclose on some entrepreneurs with extremely low realizations of ω (since $\omega_{eff}^* > 0$). Even after taking into account the cost of foreclosure, intermediaries' ability to manage capital means they are able to generate a higher return. It is therefore possible to welfare rank the three scenarios: short-term contracts with perfect coordination is strictly preferred to long-term contracts, but long-term contracts are striclty preferred to short-term contracts without coordination. In Section 5 we benchmark the model of coordination failure against the frictionless RBC model. If we had benchmarked it against an economy with full coordination, the differences in responses would have been even more stark.

as a result of coordination failure is:

$$\frac{\omega^*}{\omega^*_{eff}} = \frac{\omega^*}{\gamma\lambda} = \frac{\frac{\lambda}{\overline{\omega}}\left(1 - \ln\left(\frac{\lambda}{\overline{\omega}}\right)\right)}{\frac{\lambda}{\overline{\omega}} + \left(1 - \frac{\lambda}{\overline{\omega}}\right)\ln\left(1 - \frac{\lambda}{\overline{\omega}}\right)} > 1$$

implying that the probability of foreclosure is higher in the presence of coordination problems than is optimal, $F(\omega^*(e)) > F\left(\omega^*_{eff}\right)$. This leads to Proposition 3:

Proposition 3 *i) The inefficiency wedge,* $\frac{\omega^*}{\omega^*_{eff}}$ *is increasing in the illiquidity,* $\frac{\lambda}{\overline{\omega}}$ *of the entrepreneur:*

$$\frac{\partial\left(\frac{\omega^*}{\omega^*_{eff}}\right)}{\partial\left(\frac{\lambda}{\overline{\omega}}\right)} < 0$$

ii) In the limit, when the entrepreneur is not illiquid, there is no inefficiency:

$$\lim_{\left(\frac{\lambda}{\overline{\omega}}\right)\to 1}\frac{\omega^*}{\omega^*_{eff}} = 1$$

To get a sense of the impulse responses coming in Section 5, suppose $\overline{\omega}_{t+1}(e)$ moves countercyclically with output in the economy (which will be verified later). This implies that in a recession, for a given λ_t, the distortionary effects of coordination problems in the credit market are amplified. In a nut shell, this is the basis of the mechanism by which the financial accelerator operates in the model, amplifying aggregate shocks through the economy.

3.3 The entrepreneurs' problem

Choosing a debt contract involves choosing a state-contingent non-default loan rate and the amount to be borrowed, $\left\{R^{LR}_{t+1}(e), B_{t+1}(e)\right\}$. Since intermediaries must receive an expected rate of return equal to the risk-free rate of return on its lending in order to fulfill its commitments to its depositors and since intermediaries are perfectly competitive and therefore earn zero profits, any debt contract must satisfy the intermediaries' break-even condition.

In equilibrium an intermediary's expected return from holding an entrepreneur's debt is a probability weighted sum of three possible outcomes:

$$\Bigg(\underbrace{\overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\,d\omega}_{\substack{\text{i. Returns on rolled over}\\ \text{bonds that pay in full}}} + \underbrace{\int_{\omega^*}^{\overline{\omega}} \omega f(\omega)\,d\omega}_{\substack{\text{ii. Returns on rolled over}\\ \text{bonds that don't pay in full}}} + \underbrace{\gamma\lambda_t\int_0^{\omega^*} f(\omega)\,d\omega}_{\substack{\text{iii. Returns from}\\ \text{foreclosure}}}\Bigg) R^E_{t+1}Q_tK_{t+1}(e) \qquad (18)$$

There are three outcomes because, as is made clear in Appendix A, for any reasonable parameterization of the model, $\omega^*(\overline{\omega}_{t+1}(e)) < \overline{\omega}_{t+1}(e)$. The entrepreneur either (i), survives the foreclosure stage and is able to repay the intermediaries in full, or (ii), survives the foreclosure stage but is insolvent in which case the rolled over intermediaries share what returns there are, or (iii), the intermediaries foreclose and earn a return by managing the capital themselves. It is convenient to rewrite the expected return as:

$$\left(\Gamma\left(\overline{\omega}_{t+1}\left(e\right)\right)-G_{CF}\left(\omega^{*}\left(\overline{\omega}_{t+1}\left(e\right)\right)\right)\right)R_{t+1}^{E}Q_{t}K_{t+1}\left(e\right)$$

where:

$$\begin{aligned}\Gamma\left(.\right) &= \overline{\omega}\int_{\overline{\omega}}^{\infty}f\left(\omega\right)d\omega+\int_{0}^{\overline{\omega}}\omega f\left(\omega\right)d\omega \\ G_{CF}\left(.\right) &= \int_{0}^{\omega^{*}}\left(\omega-\gamma\lambda_{t}\right)f\left(\omega\right)d\omega \end{aligned} \tag{19}$$

The intermediaries' break-even condition is then:

$$\left(\Gamma\left(\overline{\omega}_{t+1}\left(e\right)\right)-G\left(\omega^{*}\left(\overline{\omega}_{t+1}\left(e\right)\right)\right)\right)R_{t+1}^{E}Q_{t}K_{t+1}\left(e\right)=R_{t+1}\left(Q_{t}K_{t+1}\left(e\right)-N_{t+1}\left(e\right)\right) \tag{20}$$

where R_{t+1} is the risk-free rate of return. The left-hand side of equation (20) is the expected gross return on entrepreneur e's debt and the right-hand side is the intermediary's opportunity cost of holding debt. While this condition only holds in expectation, intermediaries are able to mitigate this risk with a fully diversified portfolio of entrepreneurial debt.

The entrepreneur is the residual claimant on its gross profits and only makes a return when $\omega_{t+1}\left(e\right)>\overline{\omega}_{t+1}\left(e\right)$. The expected net return to an entrepreneur is therefore:

$$\begin{aligned} & \left(\int_{\overline{\omega}}^{\infty}\left(\omega-\overline{\omega}\right)f\left(\omega\right)d\omega\right)R_{t+1}^{E}Q_{t}K_{t+1}\left(e\right) \\ = & \left(1-\Gamma\left(\overline{\omega}_{t+1}\left(e\right)\right)\right)R_{t+1}^{E}Q_{t}K_{t+1}\left(e\right)\end{aligned}$$

By remembering that $\omega_{eff}^{*}=\gamma\lambda_{t}$, it becomes clear that the function $G_{CF}\left(.\right)$ captures the expected share of gross returns lost as a result of the coordination problem. Absent $G_{CF}\left(.\right)$, $\Gamma\left(.\right)$ represents the expected share of gross returns accruing to the intermediaries while the expected share, $1-\Gamma\left(.\right)$ accrues to the entrepreneur.

The entrepreneur's problem is then to choose $K_{t+1}\left(e\right)$ and a menu of $\overline{\omega}_{t+1}\left(e\right)$, one for each realization of the aggregate state, to maximize his expected return, subject satisfying a continuum of break-even conditions, again one for every possible realization of the aggregate state. Let's drop the entrepreneurial index and add an index for the aggregate state, P with conditional probability function $\Pi\left(P_{t+1}|P_{t}\right)$.[19] The Lagrangian that the entrepreneur solves

[19] P_{t} is the vector of the economy-wide exogenous shocks, $\left(A_{t},\lambda_{t}\right)'$.

is:[20]

$$\max_{\{\overline{\omega}_{t+1}\}_P, K_{t+1}} \int_{P_{t+1}} \left(1-\Gamma\left(P_{t+1}\right)\right) R^E_{t+1}\left(P_{t+1}\right) Q_t K_{t+1} d\Pi\left(P_{t+1}|P_t\right) \tag{23}$$

$$+\int_{P_{t+1}} \xi_t\left(P_{t+1}\right)\left(\left(\Gamma\left(P_{t+1}\right)-G_{CF}\left(P_{t+1}\right)\right) R^E_{t+1}\left(P_{t+1}\right) Q_t K_{t+1} - R_{t+1}\left(Q_t K_{t+1} - N_{t+1}\right)\right) dP_{t+1}$$

The solution to this Lagrangian is Proposition 4:

Proposition 4 *The first-order conditions to equation* (23) *yields the following relation between the illiquidity premium and the capital to net worth (leverage) ratio:*

$$\frac{E_t R^E_{t+1}}{R_{t+1}} = \Xi_{CF}\left(\frac{Q_t K_{t+1}(e)}{N_{t+1}(e)}; \lambda_t\right) \tag{24}$$

with $\lim_{QK/N \to 1} \frac{E\left(R^E\right)}{R} = 1$ *and* $\frac{\partial\left(E\left(R^E\right)/R\right)}{\partial(QK/N)} > 0.$

Proof. See Appendix A for full details of the derivation. ■

Proposition 4 describes the critical link between the illiquidity premium and entrepreneurial leverage. Given the value of $K_{t+1}(e)$ that satisfies equation (24), the schedule for $\overline{\omega}_{t+1}(e)$ is pinned down uniquely by the state-contingent constraint on the expected return to debt, defined by equation (20). Equation (24) is the key relationship in the model. It shows that the capital to net worth ratio is increasing in the expected discounted return to capital. Everything else equal, a rise in the expected discounted return to capital reduces the expected default (and foreclosure) probability. As a consequence the entrepreneur can take on more debt and expand its capital expenditure. But the entrepreneur is constrained from increasing his capital purchases indefinitely by the fact that the costs of the coordination problem also rise as the leverage ratio increases.

Aggregating equation (24) across entrepreneurs is quite straightforward. Ex-ante, entrepreneurs are heterogenous only in their net worth. Since $\frac{E_t R^E_{t+1}}{R_{t+1}}$ is common across entrepreneurs, it must be that all entrepreneurs choose $K_{t+1}(e)$ such that $\frac{K_{t+1}(e)}{N_{t+1}(e)} = \frac{K_{t+1}}{N_{t+1}}$

[20] The notation in equation (23) is somewhat cumbersome. For clarity consider the problem when there are two aggregate states of the economy wide technology, $A_t = \{H, L\}$ with probability π and $1-\pi$, repectively. R^E_{t+1} and $\overline{\omega}_{t+1}$ are both dependent on the realization, and by extension, so are ω^*_{t+1}, and the values of $\Gamma(.)$, $G(.)$ and ξ_t. Equation (23) is then:

$$\max_{\overline{\omega}(H), \overline{\omega}(L), K} \left(1-\Gamma(H)\right) R^E(H) QK\pi + \left(1-\Gamma(L)\right) R^E(L) QK(1-\pi) \tag{21}$$

$$+\xi(H)\left(\left(\Gamma(H)-G(H)\right) R^E(H) QK - R(QK-N)\right)$$

$$+\xi(L)\left(\left(\Gamma(L)-G(L)\right) R^E(L) QK - R(QK-N)\right) \tag{22}$$

for all e. This means that while entrepreneurs have heterogenous net worth levels and make heterogenous capital expenditure choices, they all choose their capital expenditure in equilibrium such that they have the same leverage ratio.[21] Thus, the aggregated form of equation (24) is:[22]

$$\frac{E_t R^E_{t+1}}{R_{t+1}} = \Xi_{CF}\left(\frac{Q_t K_{t+1}}{N_{t+1}}; \lambda_t\right) \tag{25}$$

The dynamic behavior of capital demand and the return to capital depend on the evolution of entrepreneurial net worth:

3.4 Evolution of net worth

An entrepreneur's expected net worth at the end of period t is given by:

$$N_{t+1}(e) = \begin{cases} \upsilon\left(\underbrace{\left(\omega_t(e) - \overline{\omega}_t\right) R^E_t Q_{t-1} K_t(e)}_{\text{Entrepreneurial profits}}\right) + T^E & \text{if} \quad \omega_t(e) \geq \overline{\omega}_t \\ T^E & \text{if} \quad \omega_t(e) < \overline{\omega}_t \end{cases}$$

where, with probability $1 - \upsilon$ an entrepreneur is forced to exit the market and consume his current profits from period t.[23] This assumption ensures that entrepreneurs cannot accumulate enough wealth to become fully self-financing. Entrepreneurs (new and incumbent alike) also receive a small transfer from the government every period, T^E.[24] Aggregating over entrepreneurs gives aggregate net worth:

$$N_{t+1} = \upsilon\left(\underbrace{\left(1 - \Gamma\left(\overline{\omega}_t\right)\right) R^E_t Q_{t-1} K_t}_{\text{Aggregate profits}}\right) + T^E$$

and using intermediaries' (aggregate) break-even condition:

$$\left(\Gamma\left(\overline{\omega}_{t+1}\right) - G\left(\omega^*\left(\overline{\omega}_{t+1}\right)\right)\right) R^E_{t+1} Q_t K_{t+1} = R_{t+1}\left(Q_t K_{t+1} - N_{t+1}\right)$$

the evolution of net worth can be written as:

[21] It also follows that $\overline{\omega}_{t+1}(e) = \overline{\omega}_{t+1}$ and $R^{LR}_{t+1}(e) = R^{LR}_{t+1}$ etc.

[22] Equation (25) also includes the exogenous stochastic variable, λ_t and in Section 6 the policymaker's policy instrument governing its direct lending.

[23] New entrepreneurs of mass $1 - \upsilon$ enter the market every period to ensure the mass of entrepreneurs is unaltered.

[24] This ensures that the entrepreneur's problem, given in equation (23) is well defined, which it is not for $N_{t+1}(e) = 0$. For our parameterization, we set T^E arbitrarily close to zero.

$$N_{t+1} = \upsilon \left(\left(1 - G\left(\omega^*\left(\overline{\omega}_t\right)\right)\right) R_t^E Q_{t-1} K_t - R_t \left(Q_{t-1} K_t - N_t\right) \right) + T^E \tag{26}$$

which gives N_{t+1} as a function of N_t. Importantly, N_{t+1} is sensitive to unexpected changes in the aggregate return on capital. To see this, let J_t define aggregate entrepreneurial profits, and let $V_{1,t} \equiv R_t^E - E_{t-1} R_t^E$ and $V_{2,t} \equiv G\left(\overline{\omega}_t\right) R_t^E - E_{t-1} G\left(\overline{\omega}_t\right) R_t^E$. Aggregate entrepreneurial profits can then be rewritten as:

$$J_t = \left(V_{1,t} + V_{2,t}\right) Q_{t-1} K_t + E_{t-1} J_t$$

and the elasticity of aggregate entrepreneurial profits to an unanticipated movement in the return on capital is:

$$\epsilon_t = \frac{\partial \left(J_t / E_{t-1} J_t\right)}{\partial \left(V_{1,t} / E_{t-1} R_t^E\right)} = \frac{E_{t-1} R_t^E Q_{t-1} K_t}{E_{t-1} J_t} \geq 1$$

Next, differentiating the elasticity with respect to the capital to net worth ratio, gives:

$$\frac{\partial \epsilon_t}{\partial \left(Q_{t-1} K_t / N_t\right)} = \frac{E_{t-1} R_t^E R_t}{\left(E_{t-1} J_t\right)^2} > 0$$

Thus, entrepreneurial profits respond elastically to unexpected changes in aggregate returns, and that the elasticity is greater, the more leveraged the entrepreneurs. The key point here, for the propagation and amplification of shocks, is that shocks affect net worth more than proportionally, which in turn impacts on investment expenditure.

The introduction of coordination problems into the financial sector of a basic real business cycle model is almost complete. There are only two more points to clarify. The first is the definition of K_t^* which was used in equations (5), (7) and (8). In this model, the deadweight cost of coordination failure is paid in units of capital. Thus, while the aggregate stock of capital purchased in period t for use in period $t+1$ is K_{t+1}, only $K_{t+1}^* = \left(1 - G_{CF}\left(\overline{\omega}_{t+1}\right)\right) K_{t+1}$ is put to productive use. The deadweight cost of coordination is therefore $G_{CF}\left(\overline{\omega}_{t+1}\right) K_{t+1}$ units of capital. Finally, aggregate entrepreneurial consumption, C_t^E needs to be added to the aggregate resource constraint in equation (12), which becomes:

$$Y_t = C_t + I_t + G_t + C_t^E$$

As a summary, the complete sequence of events that take place in the financial sector in a given period is provided in Table 1.

3.5 Financial frictions via costly state verification

As has been alluded to in several places above, the model with coordination failure generates a similar set of reduced form aggregate equilibrium relationships as the *financial accelerator* model of Bernanke et al. (1999) from very different microfoundations. Specifically, coordination problems among creditors, like Bernanke et al. (1999) generates a risk spread between internal and external finance related to the endogenous evolution of entrepreneurial leverage. However, the two models are quite distinct in terms of the understanding of the key features of credit market dynamics.

While it is beyond the scope of this paper to assess which of the credit market frictions - coordination failure or bankruptcy costs - is empirically more relevant, it is instructive to provide a rigorous comparison of the two models, and highlight how the empirical work could distinguish between these two mechanisms. The Bernanke et al. (1999). model offers a second benchmark (in addition to the frictionless real business cycle benchmark in Section 2) from which to assess the role of coordination failure.

To this end, a stripped down version of Bernanke et al. (1999)'s model appropriate for direct comparison is presented. The financial friction in Bernanke et al. (1999) is the result of a *costly state verification* (CSV) assumption, first introduced by Townsend (1979). In their model, debt also matures every period, but unlike the coordination failure model, there is no possibility of early foreclosure by intermediaries. Instead, the friction is the result of an informational asymmetry between the entrepreneur (the borrower) and the intermediary (the creditor). At the end of a period, an entrepreneur knows his gross profits, $\omega_{t+1}(e) R^E_{t+1} Q_t K_{t+1}(e)$. The intermediary however, does not. As before, the non-default threshold for an entrepreneur is given by $\overline{\omega}_{t+1}$. Suppose the entrepreneur declares that his gross profits were less than the contractual amount owed to the intermediary, and therefore claims to only be able to pay a fraction of his debt obligation. Does the intermediary believe this? How does the intermediary respond? In the costly state verification model, the intermediary is able to pay a monitoring cost in order to observe the entrepreneur's gross profits. The monitoring cost is assumed to be a proportion of gross profits, $\mu\omega_{t+1}(e) R^E_{t+1} Q_t K_{t+1}(e)$, with $0 < \mu < 1$. It turns out that the entrepreneur is incentivized to truthfully reveal his gross profits if the intermediary commits to monitoring the entrepreneur whenever the entrepreneur is insolvent: $\omega_{t+1}(e) < \overline{\omega}_{t+1}$. When monitoring is costless, $\mu = 0$, the model reduces to the frictionless real business cycle benchmark. At the other extreme, when $\mu = 1$, the credit market ceases to function. If however $0 < \mu < 1$, the expected gross return to the intermediary from lending is given by:

$$\left(\underbrace{\overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\, d\omega}_{\substack{\text{i. Returns on debt} \\ \text{that pays in full}}} + \underbrace{(1-\mu)\int_{0}^{\overline{\omega}} \omega f(\omega)\, d\omega}_{\substack{\text{ii. Returns on debt that} \\ \text{doesn't pay in full net of} \\ \text{monitoring costs}}}\right) R_{t+1}^{E} Q_t K_{t+1}(e)$$

Rewrite the expected gross profits accruing to the intermediary as:

$$\left(\Gamma(\overline{\omega}) - G^{CSV}(\overline{\omega})\right) R_{t+1}^{E} Q_t K_{t+1}(e)$$

where $\Gamma(.)$ is as in Section 3.3 and $G_{CSV}(.)$ is:

$$G_{CSV}(\overline{\omega}) = \mu \int_{0}^{\overline{\omega}} \omega f(\omega)\, d\omega \tag{27}$$

The intermediaries' break-even condition under the coordination failure and costly state verification assumptions are clearly similar, except in the interpretation and functional form of the financial friction, $G_{CSV}(\overline{\omega})$ under CSV and $G_{CF}(\overline{\omega}(\omega^*))$ under coordination failure (CF). Under costly state verification, $G_{CSV}(\overline{\omega})$ is the expected monitoring (or agency) cost while in Section 3.3, $G_{CF}(\omega^*(\overline{\omega}))$ was the cost of coordination failure. The differences in the endogenous responses of these two frictions to the same aggregate shocks is what distinguishes the dynamics of the two model economies.

Setting up the entrepreneur's Lagrangian similar to that in Section 3.3 to solve for the menu of state-contingent equilibrium debt contracts under CSV gives:

$$\frac{E_t R_{t+1}^{E}}{R_{t+1}} = \Xi_{CSV}\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right) \tag{28}$$

where $\Xi'_{CSV}(.) > 0$ and $\Xi''_{CSV}(.) > 0$. This equation exhibits the same functional form as equation (25); the external finance premium, $\frac{E_t R_{t+1}^{E}}{R_{t+1}}$ is increasing and convex in the entrepreneurial capital to net worth ratio. The reason for this relationship under costly state verification is that the expected monitoring costs rise as the ratio of borrowing to net worth increases. In the next section, it will be shown that it is the *curvature* of this function that will be the key distinction between the two models, and their dynamics.

As a result of the asymmetry of information, competitive entrepreneurs are able to earn informational rents, leading the evolution of aggregate net worth to follow:

$$N_{t+1} = \upsilon\left(R_t^{E} Q_{t-1} K_t - R_t\left(Q_{t-1} K_t - N_t\right)\right) + T^{E}$$

There is a small discrepancy between the net worth equations under coordination failure (see equation (26)) and costly state verification, above. This discrepancy results from the way in which the costs (whether coordination costs or agency costs) are realized. Under coordination failure, the cost was a loss in units of productive capital. Under costly state verification, the monitoring costs are paid in terms of units of output. Thus, under costly state verification, $K_t^* = K_t$ for all t, but the aggregate resource constraint becomes:

$$Y_t = C_t + G_t + I_t + C_t^E + \underbrace{\mu \left(\int_0^{\overline{\omega}_t} \omega f(\omega)\, d\omega \right) R_t^E Q_{t-1} K_t}_{\text{Deadweight cost of monitoring}}$$

4 Comparative statics and model comparison

This section considers the comparative static properties of the coordination failure model. In particular, it will show graphically the importance of this feature in generating an endogenous illiquidity premium in the model. It also offers an opportunity to understand the important differences between the costly state verification model and the coordination failure model. But, before discussing the models further, it is useful to fully parameterize the models.

4.1 Parameterization

The main goal of the parameterization is to ensure comparability between the coordination failure and costly state verification version of the model. The structural parameters, unrelated to the financial sector, are taken directly from the business cycle literature and are based on quarterly data: the output elasticity with respect to capital is $\alpha = 0.35$; the subjective discount factor is $\beta = 0.99$; the depreciation of capital is $\delta = 0.025$; and price of capital elasticity with respect to the investment to capital ratio is $\varphi = 0.25$. There is also the habit parameter, $h = 0.5$, the utility weight on labour, $\chi = 5.6$, and the inverse of the Frisch elasticity of labour supply, $\rho = 3$.

The financial sector is governed by four exogenous parameters for the coordination failure model. These are the capital management productivity of intermediaries, γ, the steady-state intra-period liquidity of capital, λ, the variance of the distribution of the idiosyncratic shocks, σ_ω^2, and the proportion of entrepreneurs that survive each period, υ. Their values are pinned down by four steady state moments of the model, which approximately match the long-run averages in U.S. data, given in Table 3.[25] The steady state moments are a risk premium of 2 percentage points over the risk-free rate, an annual bankruptcy rate

[25] The costly state verification model has only three exogenous parameters; the monitoring cost parameter, μ, the variance of the idiosyncratic shock distribution and the entrepreneurial survival probability. This means only the first three moments in Table 3 are matched to determine these parameter values.

Table 3: Steady State Moments

	Moment	Description	Value	Source
1.	$R^E - R$	Risk premium†	0.02	Bernanke et al. (1999)
2.	$F(\overline{\omega})$	Bankruptcy rate††	0.03	Bernanke et al. (1999)
3.	K/N	Capital to net worth ratio	2.00	Bernanke et al. (1999)
4.	$\int_0^{\omega^*} \frac{\gamma\lambda}{\omega} f(\omega)\, d\omega$	Average recovery ratio of liquidated assets	0.50	Berger et al. (1996)

† Spread between the prime lending rate and the six month Treasury bill rate. †† Annualized

of 3%, a capital to net worth ratio of 2 (implying a leverage ratio of 50%), and an average recovery ratio of liquidated assets of 50%.

The productivity shock is given autocorrelation, $\rho_A = 0.95$. Estimating the parameters of the exogenous illiquidity shock process using micro-level bond market data is beyond the scope of this paper. Instead, Section 5 shows the sensitivity of impulse responses to varying degrees of persistence of the illiquidity shock processes.

4.2 Comparative static analysis

Section 3 derived the equilibrium relationships between entrepreneurs and intermediaries in the environment in which intermediaries face a coordination problem and Section 3.5 reproduced the key equilibrium equation of Bernanke et al. (1999)'s original financial accelerator model. The basic form of the function is reproduced here:

$$\frac{E_t R^E_{t+1}}{R_{t+1}} = \Xi\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right) \text{ where } \Xi'(.) > 0 \text{ and } \Xi''(.) > 0$$

Both relationships show that the risk premium on external funds is an increasing and convex function of the entrepreneurial capital to net worth ratio. These equilibrium equations can equally be thought of as the supply schedule of loanable funds in the credit market.

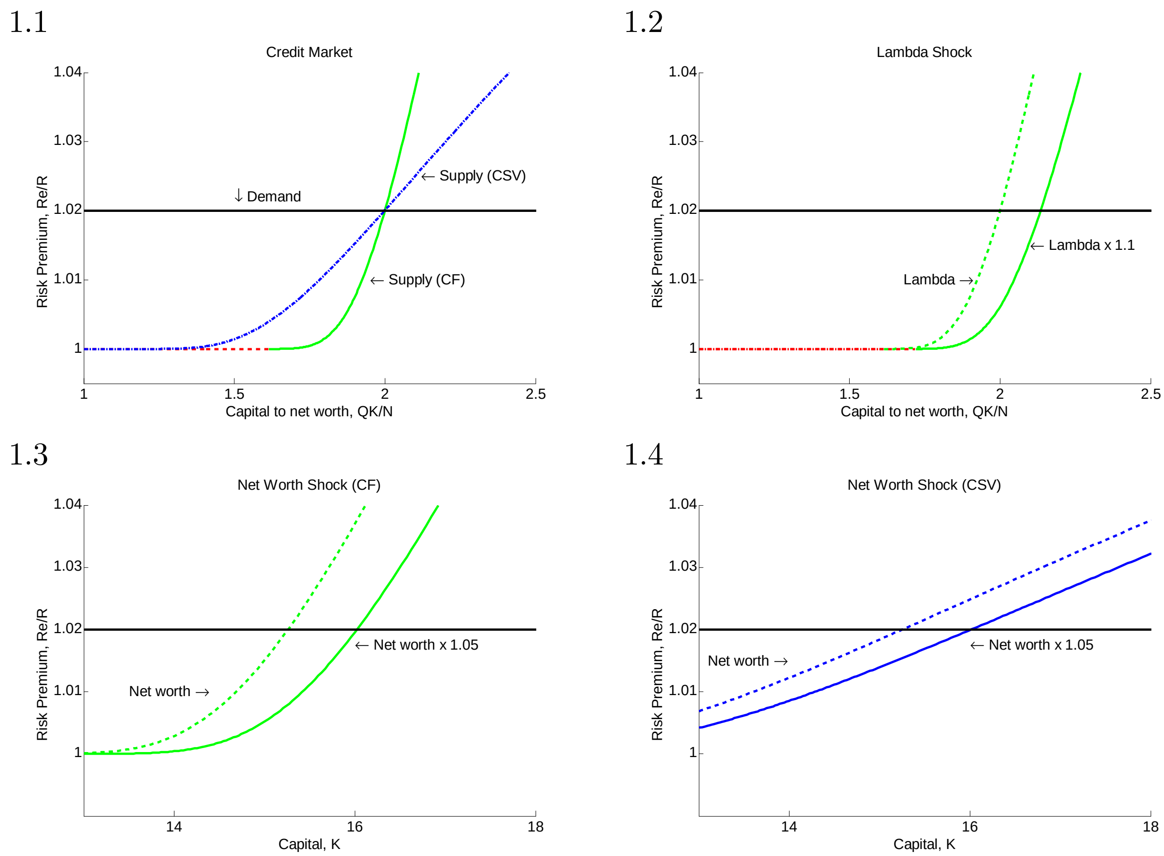
Figure 1.1 shows graphically demand and supply in the credit market for the steady state parameterization of the model (see Section 4.1). The horizontal axis shows the capital

Table 4: Structural Parameters

Parameter	Description	Value
Non-financial sector		
α	Output elasticity w.r.t. capital	0.35
β	Subjective discount factor	0.99
δ	Depreciation of capital	0.025
h	Habit parameter	0.5
χ	Weight on labor in the utility function	5.6
ρ	Inverse Frisch elasticity of labour supply	3
φ	Price of capital elasticity w.r.t. investment to capital ratio	0.25
ρ_A	Technology shock persistence	0.95
Financial sector[†]		
υ	Entrepreneur survival probability	0.954 (0.956)
σ_ω^2	Variance of idiosyncratic shock	0.119 (0.118)
γ	Productivity of financial intermediaries	0.445 (−)
λ	Intra-period tangibility of capital	0.380 (−)
μ	Monitoring cost	− (0.166)
χ^{CF}	Implied elasticity of illiquidity premium w.r.t. capital to net worth ratio.	0.299 (0.095)

[†] Values in parentheses refer to the parameterization of the CSV model

Figure 1: Credit Market Comparative Statics



Note: CF = coordination failure and CSV = costly state verification. Based on the steady state parameterization in Section 5. The red line (horizontal at 1) is when the entrepreneur is liquid, $\lambda > \overline{\omega}$.

to net worth ratio and the vertical axis shows the risk premium on external finance.[26] Owing to constant returns to scale, the demand schedule for capital is horizontal. The supply curve is upward sloping. Notably, for any common steady state, the coordination failure supply curve is always more elastic than the costly state verification supply curve in equilibrium.

Although it is not possible to derive an analytical expression for this result, I offer an heuristic proof of this result. In the coordination failure version, there is no illiquidity premium for low levels of entrepreneurial leverage. This is because, at these levels of leverage, the loan rate is low so that $\lambda_t > \overline{\omega}_{t+1}$. This means that at low levels of leverage, the entrepreneurs are not illiquid, and there is no coordination failure. Additionally, $\overline{\omega}_{t+1}$ is monotonically increasing in the leverage ratio. Only when the leverage ratio rises sufficiently such that $\lambda_t > \overline{\omega}_{t+1}$ do intermediaries face a coordination problem, leading to a positive

[26]Th x-axis can easily be re-labelled as the demand for capital by multiplying through by the steady state level of net worth.

illiquidity premium. Once entrepreneurial leverage is high enough to induce the entrepreneur to become illiquid, an incremental increase in leverage (and thus $\overline{\omega}_{t+1}$) causes the inefficiency cost of coordination failure to rise rapidly. This explains the steepness of the supply schedule. As the leverage of entrepreneurs rise, intermediaries place increasing weight on the beliefs of other intermediaries and less on the fundamentals. Adding leverage near the steady state has a disproportionately large effect on the weight given by intermediaries towards higher order beliefs. Intermediaries know this and know that they respond by foreclosing on increasingly productive entrepreneurs, even though it is inefficient. To compensate themselves for this risk, they demand a sharp increase in the return at which they would be willing to lend.

Costly state verification generates a supply schedule with less curvature. This is in part because expected agency costs begin to bite as soon as entrepreneurs take on external finance. Given that the risk premium is strictly greater in the CSV model for low levels of leverage, and given the convexity of the supply schedule, it follows that the coordination failure model must deliver a steeper supply schedule at the common steady state equilibrium.

Figure 1.2 shows how steady state leverage in the economy would rise as a result of an increase in the intra-period liquidity, λ of the capital stock. This is because a change in λ shifts the position of the critical leverage ratio at which coordination problems between intermediaries begin to appear. Finally, Figures 1.3 and 1.4 show the effect on the steady state capital stock from a 5% increase in entrepreneurial net worth.

In reduced form, the key difference in the two models lies in the specification of the $G(.)$ function as defined by equations (19) and (27) which capture the share of gross returns lost due to coordination failure and costly state verification respectively:

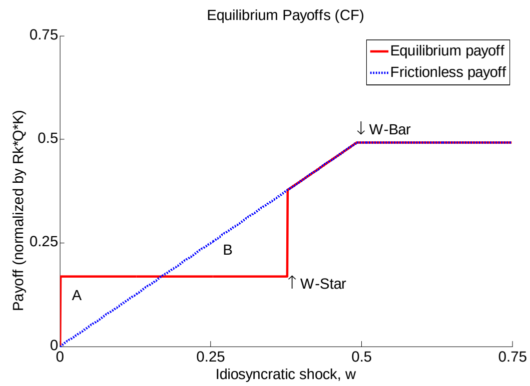
$$\begin{aligned} G^{CF}(.) &= \int_0^{\omega^*(\overline{\omega})} (\omega - \gamma\lambda_t) f(\omega) \, d\omega \\ G^{CSV}(.) &= \mu \int_0^{\overline{\omega}} \omega f(\omega) \, d\omega \end{aligned}$$

In order to visualize the differences, Figure 2 plots the equilibrium payoff function for an intermediary for different realizations of the entrepreneur's idiosyncratic shock realization. The textbook debt payoff function is given by the dotted lines. Figure 2.1 shows the equilibrium payoff structure for the coordination failure model. It is clear to see the three different outcomes bounded by ω^* and $\overline{\omega}$, as expressed in equation (18). The area $B - A > 0$ gives the size of the deadweight loss as a result of coordination failure. Similarly, Figure 2.2 shows the equilibrium payoff structure in the costly state verification model, with the area C denoting the deadweight loss or agency cost associated with this model. Figures 2.3 and 2.4 show the effect on the steady state of changes in λ and μ respectively.

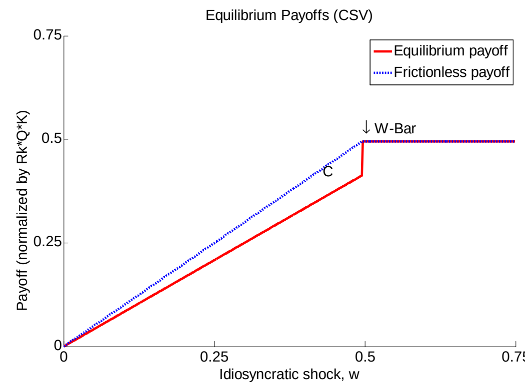
A larger distortion (due to a fall in λ or a rise in μ) leads to a fall in the steady state

Figure 2: Intermediaries' Gross Return in Equilibrium

2.1



2.2



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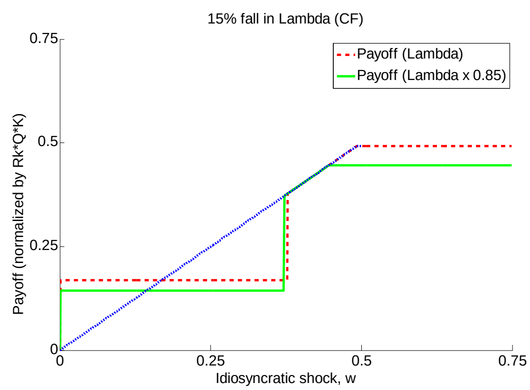
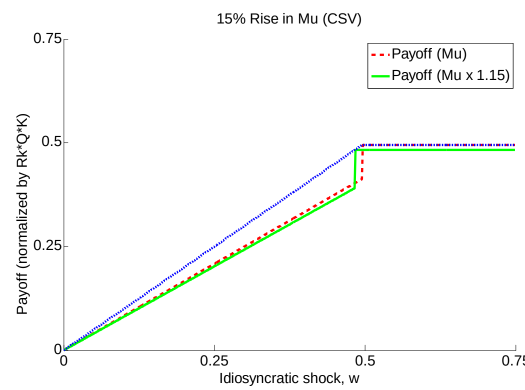
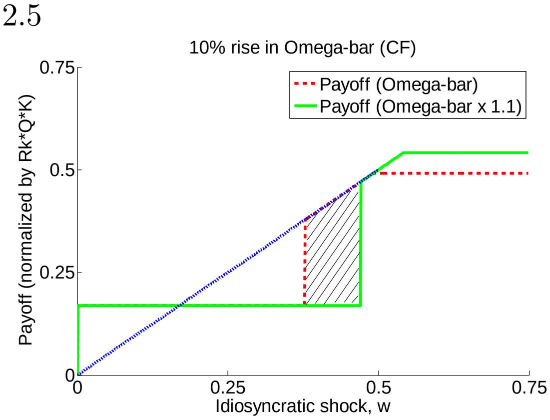


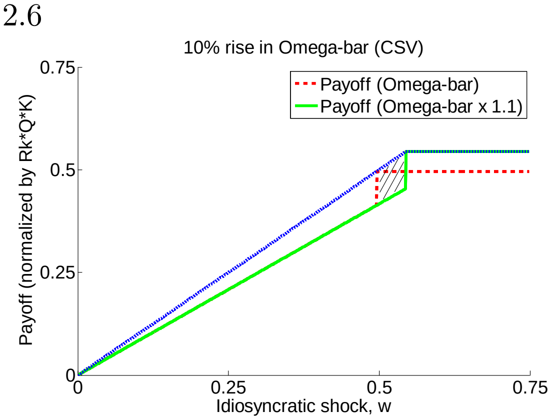
2.4



2.5



2.6



Note: Equilibrium (steady state) payoff functions under the friction of coordination failure (CF) and costly state verification (CSV). The x-axis is the realization of ω and the y-axis is the equilibrium gross return to the intermediaries. The shaded areas denotes the change in the size of the deadweight loss.

leverage of entrepreneurs in the economy. This reduces steady output. Since the distortion in the credit market has a direct effect of investment, it is no surprise that steady state investment is pushed further from its efficient level due to the distortion.

When there is a negative productivity shock, $\overline{\omega}$ rises in both models in order to insulate the intermediaries from the aggregate risk. Figure 2.5 and 2.6 plot the response of the intermediaries expected payoff functions to a 10% rise in $\overline{\omega}$. The shaded areas show the change in the size of the deadweight loss as a result of coordination failure and costly state verification, respectively. It is clear to see that the size of the distortion has a larger effect on the model with illiquid entrepreneurs rather than the model with monitoring costs. Although an analytical proof of this result is absent, these results hold for any parameter configurations in which both models exhibit a common steady state, and the intuition can be traced back to the discussion of the supply schedule for loanable funds in Figure 1. It is useful to keep these comparative static experiments in mind for the comparative dynamics in the next section.

5 Crisis scenario impulse responses

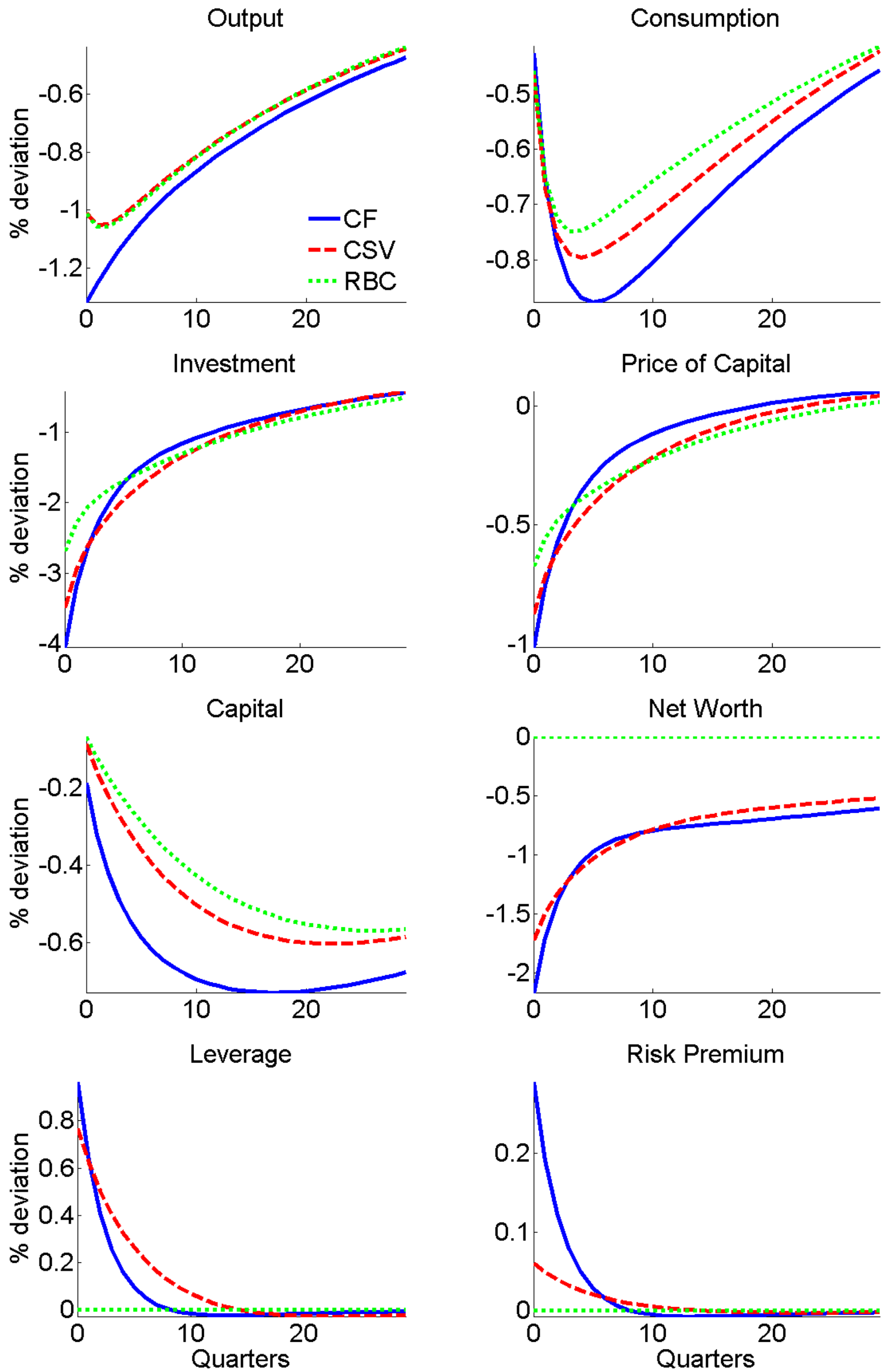
5.1 Productivity shock

Figure 3 shows the reaction to a 1% negative technology shock. The dotted line shows the reaction of the model without any financial frictions while the dashed and solid lines show the responses of the models with costly state verification and coordination failure, respectively. In the basic model without financial frictions, the negative technology shock causes an immediate fall in output and asset prices. Along the transition, output and asset prices return towards their initial steady state levels. It is clear that the inclusion of financial frictions does not alter the qualitative shapes of the responses, but does alter the magnitude of the responses. Notably, the effects on investment, asset prices and the capital stock are larger.

The inclusion of financial frictions introduces several new aggregate variables of interest, specifically entrepreneurial net worth, entrepreneurial leverage and the external finance premia. The negative productivity shock causes a drop in entrepreneurial net worth and an increase in entrepreneurial leverage (entrepreneurs' capital to net worth ratio). The risk-free (deposit) rate falls while the expected return on capital rises, leading to a sharp rise in the external finance premium.[27]

[27]Supplementary material on several alternative shocks (including capital quality and net worth shocks) and the behaviour of the rollover threshold, recovery rates and default and foreclosure rates for both the technology shock and the illiquidity shock is available from the authors website, https://sites.google.com/site/oliverdegroot/.

Figure 3: Technology Shock



Note: 1% negative technology shock

These exaggerated responses are the result of three features of the models with financial frictions. First, the loan rate paid by entrepreneurs is a function of the expected return on capital, which means that it is the entrepreneurs alone who face the aggregate risk. When the negative technology shock hits, the realized return on capital is below its expected return, which drives down the aggregate profits of the entrepreneurs, and hence their net worth. Second, entrepreneurial net worth decreases faster the demand for capital, implicitly causing leverage to rise. Third, higher leverage increases the distortion imposed by the financial friction in the credit market, which causes the premium on external finance to rise.

In the coordination failure model, short-term creditors demand a higher loan rate (i.e. an increase in $\overline{\omega}_{t+1}$) following a negative productivity shock, which increases the illiquidity of entrepreneurs. This causes investment and the price of capital to deviate further from their efficient values in response to a negative technology shock. Investment immediately falls 4.0% following an 1% technology shock in the coordination failure model, relative to a 2.8% fall in the frictionless case. In the costly state verification model, increased leverage increases the agency costs of financial intermediation.

The key distinction between the two financial friction models is that, for the majority of the aggregate variables, the initial response is larger in the coordination failure case, but the responses are less persistent. This offers a dimension along which to empirically test the two models. For example, the initial response of the illiquidity premium is 0.32% relative to the agency risk premium of 0.06%, but after the 6th quarter following the shock, the risk premium in the costly state verification model is further from its steady state level. The reason is that the coordination failure model generates a higher elasticity of the external finance premium relative to the capital to net worth ratio as discussed in Section 4.

Once the model is log-linearized, the difference in curvature of the two supply schedules shown in Figure 3 reduces to the difference in a single parameter, χ, the external finance premium elasticity with respect to the capital to net worth ratio. The log-linearized version of equations (25) and (28) are:

$$\begin{aligned} E_t r_{t+1}^k - r_{t+1} &= \chi_{CF} \left(q_t + k_{t+1} - n_{t+1} \right) + ... \\ E_t r_{t+1}^k - r_{t+1} &= \chi_{CSV} \left(q_t + k_{t+1} - n_{t+1} \right) \end{aligned}$$

with the distinction that $\chi^{CF} = 0.3$ and $\chi^{CSV} = 0.1$. This difference impacts on the impulse responses of the model.

Table 5: Accuary of First Order Model Approximation

Approximation	**Coordination Failure Model**			**Costly State Verification Model**		
Tech. shock	s.d. = 0.1%	s.d. = 1%	Ratio	s.d. = 0.1%	s.d. = 1%	Ratio
First-	−0.0348	−0.348	10	−0.00503	−0.0503	10
Second-	−0.0344	−0.286	8.312	−0.00501	−0.0487	9.719
Third-	−0.0344	−0.324	9.405	−0.00501	−0.0488	9.751

Note: Model with only technology shocks showing the contemporaneous response of the risk premium to a 1 s.d. negative technology shock.

5.2 Checking the accuracy of the solution

As a crude check of the accuracy of the first-order approximation of the model, the following experiment is performed. We solve the model twice, first assuming the standard deviation of the technology shock process is 0.1% and then 1%. Under a first-order approximation, the contemporaneous response of the illiquidity premium to a 1 standard deviation shock will be exactly ten times larger in the second version of the model than in the first. We perform the same exercise for the entire model approximated to second and third order using Dynare[28]. If this scaling of the shock produces a disproportionate change in the impulse responses then it is suggestive that there is some accuracy value to be gained from a higher order approximation.

Table 5 presents the results of this exercise for the contemporaneous impulse response of the illiquidity premium to a 1s.d. fall in productivity. Since the ratio for the third order approximation is further from 10 for the CF model than for the CSV, this implies that the non-linearities play a more important role under coordination failure. However, we take the ratio of 9.4 to be suggestive that a third order approximation to the model does not add significantly to its accuracy to warrant us to disregard the first-order approximation. By simply plotting impulse responses it becomes clear that the differences are not readily visible.[29]

We thus content ourselves with proceeding using a first-order approximation to the equilibrium conditions.[30]

[28] Juillard (1996).

[29] Further details on the higher order appoximations are available from the author on request.

[30] The reason that these higher order approximations may not improve the accuracy of the solution much, is that they are still approximations within the neighbourhood of the deterministic steady state. To be able to characterize the non-linearities generated by the coordination problem, we would really require a global solution to the model. Although this would certainly be a fruitful avenue for future research, it is beyond

5.3 Illiquidity shock

A novel feature of the model in this paper is the ability to model an exogenous liquidity fall in credit markets. This can be thought of as a confidence shock in credit markets. Figure 4 shows the response to a 1% fall in the intra-period illiquidity of capital, using different potential values of the persistence parameter, ρ_λ. The blue-solid, red-dash and green-dot lines refer a ρ_λ of 0.95, 0.85 and 0.75 respectively.

An exogenous fall in the liquidity of capital leads to a rise in the rollover-foreclosure threshold, ω^* which implies a higher incidence of foreclosures. This causes a sharp rise in the illiquidity premium paid by entrepreneurs on external finance. The rise in the premium is the result of both a rise in the expected return on capital as well as a fall in the risk-free deposit rate. The intermediaries want to cut back on the supply of loanable funds. However, the drop in the demand for capital is insufficient to offset the fall in entrepreneurial net worth. The intermediaries can therefore only break even if it sharply lowers the return paid on deposits. Households, for whom income is initially unchanged by the shock, but facing a lower return on savings, generates a temporary consumption boom.[31]

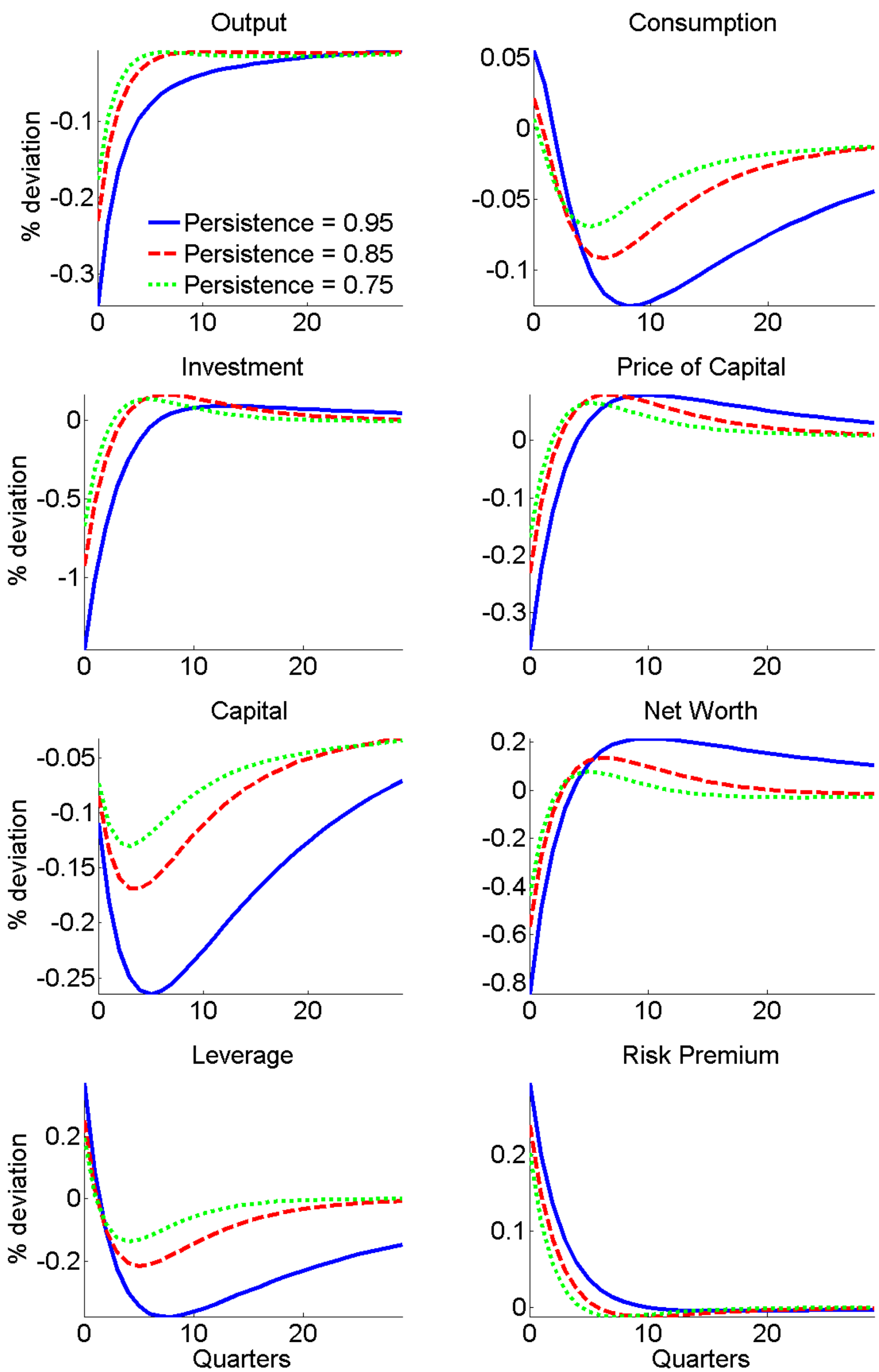
In the transition, the fall in the demand for investment causes a gradual fall in the capital stock. As the impact of the illiquidity shocks recedes, households cut consumption to below its steady state level in order to restore their steady state savings ratio. This requires a long period of deleveraging by entrepreneurs.

Notably, the size and persistence in the response of capital (and hence output) is very sensitive to the persistence of the fall in capital illiquidity. The evolution of capital is relatively gradual. A less persistent shock therefore gives less opportunity for capital to fall and do serious damage to the output potential of the economy. The risk to the length and severity of a recession depends on how long the credit market remains illiquid. There is therefore a rationale for policymakers (monetary or fiscal) to offset the effect of illiquidity in the credit market. And it is to this issue I turn in the next section.

the scope of this paper.

[31]The implication is that when an illiquidity shocks hits, consumption rises temporarily while output and investment falls. This is because of a high intertemporal elasticity of substitution for households which means that the substitution effect dominates the wealth effect. This result is shared by many other papers which incorporate shocks to preferences, investment goods prices or other financial frictions. A possible extension to the model to alleviate this result is to assume that technology shocks and illiquidity shocks are correlated. Indeed, during recessions, markets do seem to experience more illiquidity.

Figure 4: Illiquidity Shock



Note: 1% illiquidity shock

6 Policy responses

The model of coordination failure allows us to analyze two of the unconventional credit market policies adopted by the U.S. Federal Reserve during the recent crisis: Direct lending in credit markets, and equity injections. This section analyzes how these policies work in the context of this model. For related attempts to model credit policy, see Cúrdia and Woodford (2010), Reis (2010) and Gertler and Kiyotaki (2009).

It is important to emphasize that we have in mind that these interventions be used only during crises and not during normal times. In this regard, the net benefits from credit policy should be increasing in the distortion of credit markets, as measured by the illiquidity premiums. Finally, note that these unconventional policies blur the distinction between monetary and fiscal policy. The policymaker might therefore be thought of as some quasi-monetary-fiscal agent.

6.1 Direct lending in credit markets

Direct lending, in the context of the model, refers to the scenario in which the policymaker supplements the private level of lending in the credit markets by providing additional lending directly to entrepreneurs. The policymaker has both advantages and disadvantages relative to the intermediaries. The advantage is that it can obtain funds during crises more easily, and therefore channel them to entrepreneurs with abnormal excess returns. Intermediaries in the model lend only a small fraction of total lending to each entrepreneur. Thus, they have no ability to coordinate actions. The policymaker instead behaves as a single, large market participant. By promising to commit to rollover on its lending, it is able to reduce the liquidity and coordination problem in the market. At the same time, suppose that the policymaker is less efficient at intermediating funds. It faces an efficiency cost, τ per unit for intermediated funds.

To obtain funds, the policymaker issues government debt to households. Government debt and bank deposits are perfect substitutes, both paying the risk-free rate of return, R_{t+1}. An entrepreneur then receives credit from both intermediaries of measure 1 and the policymaker:

$$B_{t+1} = B^p_{t+1} + B^g_{t+1}$$

where p and g indexes the private intermediaries and policymaker (government) respectively. Crucially, the intermediaries have the option to foreclose early on the loan, while the policymaker is assumed *to always rollover*.[32] Let $\mathbb{N}_{t+1}$ be the proportion of total lending that is

[32] This view of direct lending is consistent with the anecdotal evidence. Fiscal-monetary authorities in the crisis implicitely lengthened the maturity structure of borrowers by directly lending at longer maturities

provided by intermediaries, such that:

$$B^p_{t+1} = \mathbb{N}_{t+1} B_{t+1} \text{ and } B^g_{t+1} = (1 - \mathbb{N}_{t+1}) B_{t+1}$$

In other words, the policymaker pledges to lend a fraction of total private lending, where $(1 - \mathbb{N}_{t+1})$ can be thought of as the instrument of credit policy. When $\mathbb{N}_{t+1} = 1$, only intermediaries lend to the entrepreneur. The policymaker lends at the same non-default loan rate as private lenders, $R^{Lg}_{t+1} = R^L_{t+1}$, and does not offer funds at a subsidized rate. However, by expanding the supply of funds available in the market, it will reduce these rates by reducing the illiquidity premium.

The augmented complete-rollover solvency condition for entrepreneurs is:

$$R^L_{t+1} B^p_{t+1} + R^{Lg}_{t+1} B^g = \overline{\omega}_{t+1} R^E_{t+1} Q_t K_{t+1}$$

Rearranging, it is possible to show the debt obligations to the government and intermediaries, respectively:

$$\begin{aligned} R^{Lg}_{t+1} B^g_{t+1} &= (1 - \mathbb{N}_{t+1}) \overline{\omega}_{t+1} R^E_{t+1} Q_t K_{t+1} \\ R^L_{t+1} B^p_{t+1} &= \mathbb{N}_{t+1} \overline{\omega}_{t+1} R^E_{t+1} Q_t K_{t+1} \end{aligned}$$

The analysis of the equilibrium strategies then follow the global games methodology used earlier. The intermediaries decision rule given in equation (17) becomes:

$$\begin{aligned} \omega^* &= \gamma\lambda \frac{\frac{\lambda}{\overline{\omega}} \left(1 - \ln\left(\frac{\lambda}{\mathbb{N}\overline{\omega}}\right)\right)}{\frac{\lambda}{\overline{\omega}} + \left(1 - \frac{\lambda}{\overline{\omega}}\right) \ln\left(1 - \frac{\lambda}{\overline{\omega}}\right)} \\ &= \gamma\lambda \frac{\mathbb{N}x (1 - \ln(x))}{\mathbb{N}x + (1 - \mathbb{N}x) \ln(1 - \mathbb{N}x)} \end{aligned}$$

where $x = \frac{\lambda}{\mathbb{N}\overline{\omega}}$ measures the illiquidity of the entrepreneurs. It is again instructive to consider the rollover threshold relative to the *efficient* rollover threshold. This mean the optimal decision of the intermediaries if they could perfectly coordinate their actions. The efficient threshold in this case becomes:

$$\omega^*_{EFF} = \frac{\gamma\lambda}{\mathbb{N}}$$

than private agents were willing to lend at, or by purchasing commercial paper. See the Federal Reserve's press release here: http://www.federalreserve.gov/newsevents/press/monetary/20081007c.htm and the efficiency wedge, $\frac{\omega^*}{\omega^*_{EFF}}$ is:

$$\frac{\omega^*}{\omega^*_{EFF}} = \mathbb{N}\frac{\omega^*}{\gamma\lambda} = \frac{\mathbb{N}^2 x\,(1-\ln(x))}{\mathbb{N}x + (1-\mathbb{N}x)\ln(1-\mathbb{N}x)}$$

It can again be easily shown that $\frac{\partial\left(\frac{\omega^*}{\omega^*_{EFF}}\right)}{\partial\mathbb{N}} > 0$ for a given $\overline{\omega}$. Thus, for a given $\overline{\omega}$, a fall in $\mathbb{N}$ (i.e. a rise in government intervention) reduces the distortion as a result of coordination problems in the credit market. The augmented break-even condition for households is unchanged except for the $G(.)$ function, capturing the cost of coordination failure, which becomes:

$$G(\overline{\omega}) = \int_0^{\omega^*(\overline{\omega})}\left(\omega - \frac{\gamma\lambda}{\mathbb{N}}\right) f(\omega)\,d\omega$$

The expected return to the entrepreneur is also unchanged. In a partial equilibrium setting therefore, the introduction of direct lending generates an outward shift of the loanable funds supply schedule. This implies that for a given illiquidity premium, entrepreneurs are able to leverage up more, thus increasing demand for investment and capital.

6.2 Equity injections

With equity injections, the policymaker acquires ownership positions in entrepreneurs. As with direct lending, suppose there are efficiency costs associated with government acquisition of equity. Let this cost be τ' per unit of equity acquired. Also, assume that a unit of government equity has the same payout stream as a unit of private equity. Entrepreneurial total net worth is then:

$$N_{t+1} = N^p_{t+1} + N^g_{t+1}$$

Let $\mathbb{C}_{t+1}$ be the proportion of total equity that is privately held, such that:

$$N^p_{t+1} = \mathbb{C}_{t+1}N_{t+1} \text{ and } N^g_{t+1} = (1-\mathbb{C}_{t+1})\,N_{t+1}$$

The intermediaries break-even condition is unchanged. But, it means that the residual profits are split between the entrepreneur and the government. The expected profits of the entrepreneur are therefore:

$$\mathbb{C}_{t+1}\,(1-\Gamma(\overline{\omega}_{t+1}))\,R^E_{t+1}Q_tK_{t+1}$$

Importantly, total net worth rises with the introduction of policymaker into the credit market since entrepreneurial net worth is a state variable. The evolution of total net worth is therefore:

$$
\begin{aligned}
N_{t+1} &= \upsilon\left[\mathbb{C}_t\left(1-G\left(.\right)\right)R_t^E Q_{t-1}K_t^* - R_t\left(Q_{t-1}K_t - N_t^p\right)\right] + N_{t+1}^g \\
&= \upsilon\frac{\mathbb{C}_t}{\mathbb{C}_{t+1}}\left(\phi_t R_t^E Q_{t-1}K_t - R_t\left(Q_{t-1}K_t - N_t\right)\right)
\end{aligned}
$$

Clearly, since the equity injection expands entrepreneurial net worth, this in turn will expand asset demand by a multiple equal to the leverage ratio. One additional important effect of government equity injections is that it reduces the impact of unanticipated changes in asset values on private net worth. Absent government equity, for example, the entrepreneur absorbs entirely the loss from an unanticipated decline in asset values, given that its obligations to outsiders are all in the form of non-contingent debt. With government equity however, the government shares proportionally in the loss.

6.3 Policymaker's budget constraint

For simplicity, assume there is no government spending and that lump sum taxes follow a simple rule:

$$T_t = xD_t \tag{29}$$

to ensure that the policymaker's debt accumulation is non-explosive.[33] When a crisis hits, the initial deficit the policymaker incurs due to the implementation of either direct lending or equity injections, is absorbed via debt issuance. Total receipts and total spending each period is given as follows:

$$
\begin{aligned}
\textit{receipts} \;&:\; \underbrace{\left(1-\mathbb{N}_t\right)\left(\Gamma\left(\overline{\omega}_t\right) - \int_0^{\omega_t^*}\omega f\left(\omega\right)d\omega\right)R_t^E Q_{t-1}K_t}_{\text{Return on direct lending}} + \underbrace{\left(1-\mathbb{C}_t\right)\left(1-\Gamma\left(\overline{\omega}_t\right)\right)R_t^E Q_{t-1}K_t}_{\text{Return on equity injections}} \\
&\quad + \underbrace{D_{t+1}^g}_{\text{Bond issuance}} + \underbrace{T_{t+1}}_{\text{Lump sum taxes}} \\
\textit{spending} \;&:\; \underbrace{\tau\left(1-\mathbb{N}_{t+1}\right)\left(K_{t+1}-N_{t+1}\right)}_{\text{Direct lending}} + \underbrace{\tau'\left(1-\mathbb{C}_{t+1}\right)N_{t+1}}_{\text{Equity injections}} + \underbrace{R_t D_t^g}_{\text{Bond repayment}}
\end{aligned}
$$

Note that the policymaker does not receive the same return on direct lending as the intermediaries. Since the policymaker commits to never foreclose prematurely, they lose the value, $\gamma\lambda_t\omega_t^*$ on lending that is foreclosed early. Importantly though, the presence of the policymaker in the credit market has lowered the threshold, ω_t^*.

[33] This requires $x > \frac{1}{\beta} - 1$

The policymaker therefore has two policy instruments available, $(1 - \mathbb{C}_{t+1})$ and $(1 - \mathbb{N}_{t+1})$. I consider a simple and implementable reaction function that governs the use of these instruments:

$$\begin{array}{lcl} \text{Direct lending} & : & (1 - \mathbb{N}_{t+1}) = a_{DL} \left(\frac{R^E_{t+1}/R_{t+1}}{R^E/R} - 1 \right) \\ \text{Equity injection} & : & (1 - \mathbb{C}_{t+1}) = a_{EQ} \left(\frac{R^E_{t+1}/R_{t+1}}{R^E/R} - 1 \right) \end{array}$$

The policy rule states that the size of the policymakers intervention in the credit market depends positively on the size of the illiquidity premium. This is a reasonable policy rule to consider since the magnitude of the distortion in the credit market, which the policymaker is trying to offset, naturally manifests itself by the size of the illiquidity premium. From a practical policy perspective, the rule is probably also easily implementable since the credit spreads are easily observable (although disentangling illiquidity from credit risk may not be so easy). From anecdotal evidence, it was the sharp rise in spreads during the recent financial crisis that pushed the U.S. Federal Reserve into introducing unconventional credit policies, even before the conventional tool of monetary policy, the nominal interest rate reached the zero lower bound.

6.4 Crisis scenarios with policy responses

In Figure 5 the model economy is again shocked with a 1% fall in the intra-period liquidity of the capital stock (with persistence parameter set at 0.95). The new parameters have been chosen as follows. The parameter on the tax rule in equation (29) is set at $x = 0.05$, and a symmetric inefficiency cost of direct lending and equity injections is assumed at $\tau = \tau' = 1.01$. To policy experiment that is conducted is to ensure that both policies deliver the same contemporaneous increase in government debt (see Figure 6) which results in the parameters of the policy rules being set at $a_{DL} = 3.00$ and $a_{EQ} = 14.02$.[34]

Figure 5 shows that the equity injections are able to mitigate the effects of the initial shock to liquidity better than direct lending. The initial fall in output with no policy intervention was 0.35%. The use of equity injections and direct lending reduced the initial fall in output to -0.11% and -0.27% respectively, a reduction in the former case of more than 65%. The reason is that the equity injection can directly offset the fall in net worth, actually

[34] Since government debt is zero in steady state, government debt is the only variable that has been linearized in levels rather than log-levels. To clarify the magnitude of this policy, we can go through the following back of the envelope calculation. Under standard parameters, annual investment is 10% of the capital stock and investment accounts for 20% of output. The policy paremeter falls to 0.6 indicating that the government owns 0.6% of aggregate net worth (or 0.6% of outstanding entrepreneurial debt). Since, in the steady state, net worth and total borrowing are equal in magnitude, the initial issue of government debt is of the order of 0.6% of annual GDP, or for the US economy, approximately $90bn. This is a relative small number compared to the expansion of the Federal Reserves balance sheet by over $1tn during the crisis.

causing leverage to fall. However, while equity injections reduce the initial impact of the illiquidity shock, they also cause the effects of the illiquidity shock to be persist for longer. The reason should be fairly clear. Remember that without financial frictions, the Modigliani-Miller theorem states that it is irrelevant whether debt or equity financing is used. Once financial frictions are introduced, equity is a cheaper source of funds, exactly because equity avoids the coordination problem - entrepreneurs would like to build up equity so that they don't require debt finance. Direct lending, even though it is with longer term maturities, does not materially improve the solvency of the entrepreneurs. Equity injections therefore are very powerful in mitigating the problem in credit markets (because it lowers the need to access them). However, as the illiquidity premium recedes, the policymaker's withdrawal of equity offsets the recovery in net worth that the entrepreneur would have experienced in the counterfactual scenario without policy intervention.

Direct lending is therefore less powerful because every additional dollar of intervention from the policymaker, although reducing the coordination problem does not mitigate the coordination problem. However, it also means that there are smaller longer term consequences of the policy intervention.

Figure 6 shows how large the effect of the policy responses were in the above scenario. It should be noted though that, by assumption, government debt is funded via lump sum taxes. To get a sense of the true costs of these credit market interventions, it would be necessary to introduce distortionary taxes on labour income. This interesting extension we leave for future research. As a proxy, the policy experiment is calibrated such that the initial debt burden is the same size under both policies.

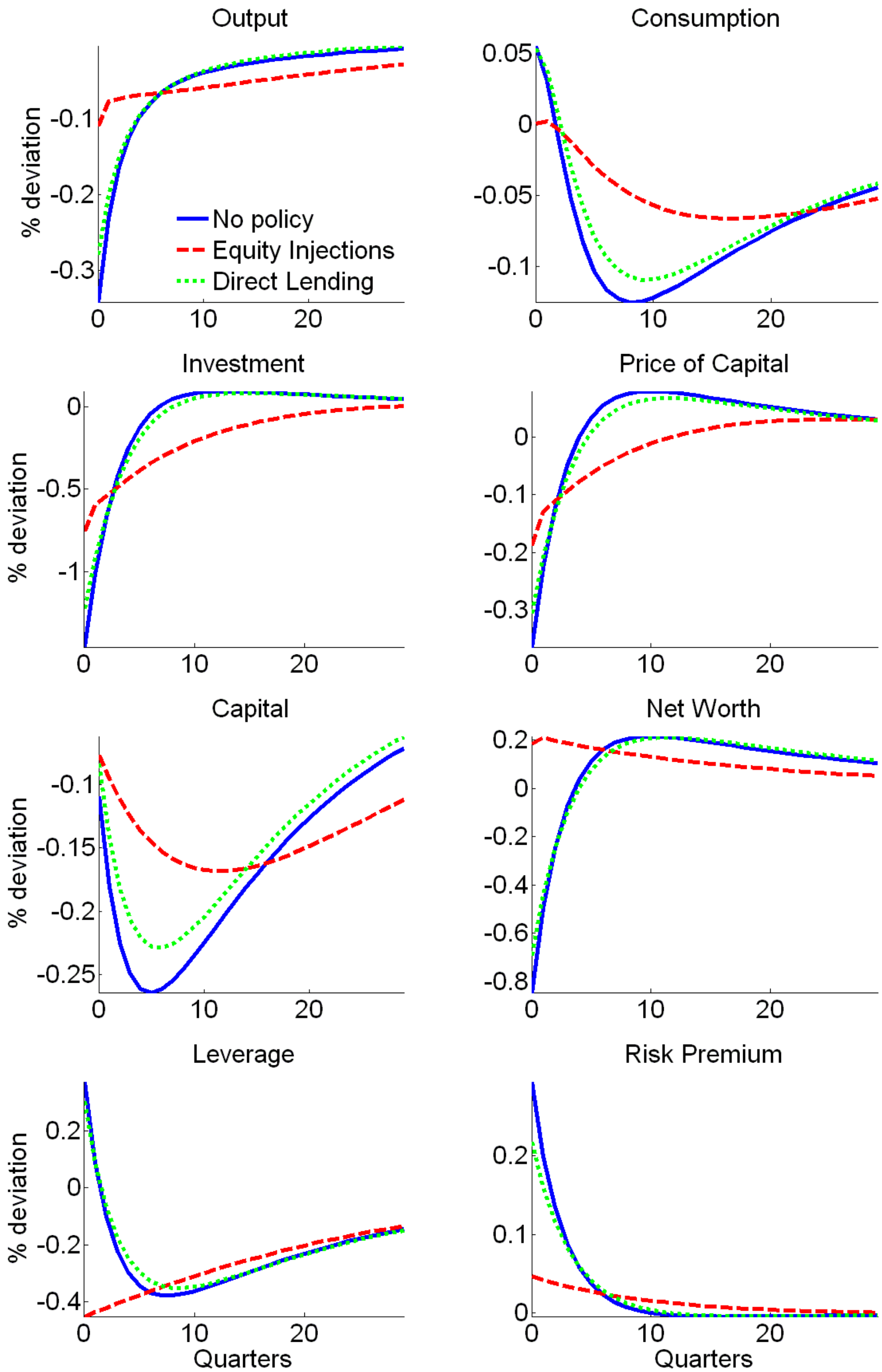
While this policy analysis is not intended to provide strict welfare analysis of different credit market policies, the impulse responses can help qualitatively our understanding of how real economic activity responds to unconventional credit market policies.

7 Discussion and conclusions

This paper incorporates the existence of short-term uncoordinated creditors in credit markets in a DSGE model. The model reveals the relationship between leverage and illiquidity, and the consequence of coordination problems in credit markets for the propagation and amplification of shocks in a dynamic, general equilibrium macroeconomic model. The model generates two implications for policy.

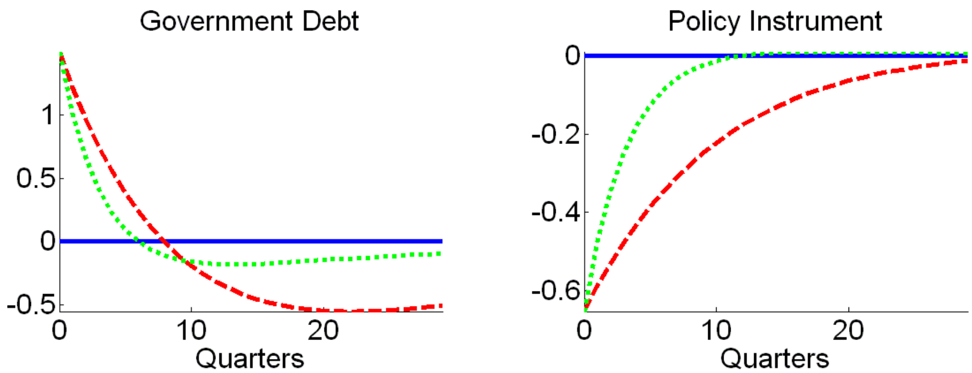
The model reacts to shocks along the lines of the traditional financial accelerator proposed by Bernanke et al. (1999). Replacing the costly state verification assumption in Bernanke et al. (1999). with the assumption of short-term uncoordinated creditors does not

Figure 5: Illiquidity Shock and Policy Response



Note: 1% illiquidity shock with policy intervention

Figure 6: Illiquidity Shock and Policy Response



Note: 1% illiquidity shock with policy intervention.

qualitatively alter the macroeconomics dynamics of the model. This result has implications for the micro-, macro-prudential policy debate. When conducting micro-prudential policy, it is important for the policy maker to understand the frictions and imperfections that exist in the financial markets. However, for macro-prudential policy, the nature of the frictions or imperfections that exist in financial markets can be largely ignored in order to understand the behavior of the macroeconomy to productivity shocks. The reduced form mechanism through which asset prices, leverage and risk premia transmits shocks onto aggregate variables of interest for macroeconomic forecasting and stabilization is very similar for both models of financial frictions.

Having said this, if a policymaker believes that shocks can originate directly as exogenous illiquidity shocks, this paper provides some important insights. It is difficult to dispute that illiquidity in credit markets was not an important component of the recent financial crisis. The results of the impulse response analysis suggest that bouts of illiquidity in asset markets can have painful consequences for the real economy if the bouts of illiquidity persist. In this scenario there is a case for government action to offset the damaging effects of these bouts of illiquidity.

The microfounded coordination problem at the heart of this paper allows us to make a first pass at assessing some of the unconventional credit market policies adopted by the U.S. Federal Reserve during the recent financial crisis, modelled as a large illiquidity shock. In particular, I find that direct lending and equity injections can both offset the initial impact of a fall in credit markets liquidity, and therefore stem the propagation mechanism, causing investment and output to recover more quickly. However, the consequence is that both policies increase the persistent of credit market shocks. Further work in understanding the interaction between policy and liquidity in credit markets, especially with a rigorous welfare criterion, would be an important extension to this line of research.

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A Appendix: The coordination game

This appendix retraces many of the technical aspects of Section 3, in order to ensure completeness by adding additional proofs, details and explanations. Once again, subscripts and indexes have been dropped wherever possible to aid clarity.

A.1 Aggregate return per unit of effective capital

The first thing that requires some explanation is why R_{t+1}^{E} (defined in equation (14) and reproduced here) is the appropriate *gross* return on the *value* of a unit of capital:

$$R_{t+1}^{E}=\frac{R_{t+1}^{K}+(1-\delta)\,Q_{t+1}}{Q_{t}}$$

Consider that an agent (entrepreneur or intermediary) holds X_{t+1} units of effective capital that has a current value of Q_tX_{t+1}. The gross profits earned on this capital are:

$$\Pi_{t+1}=\underbrace{R_{t+1}^{K}X_{t+1}}_{\text{Rent earned}}+\underbrace{(1-\delta)\,Q_{t+1}X_{t+1}}_{\substack{\text{Revenue from selling}\\ \text{the capital}}}$$

where Π_t is the sum of the income from renting the capital plus the income from selling the capital at the end of the period (adjusted for depreciation). It is then clear that the gross return on the value of a unit of capital is:

$$R_{t+1}^{E}=\frac{\Pi_t}{Q_tX_{t+1}}=\frac{R_{t+1}^{K}+(1-\delta)\,Q_{t+1}}{Q_{t}}$$

Importantly, X_{t+1} can be the effective units of capital of an entrepreneur, $\omega_{t+1}(e)\,K_{t+1}(e)$, or the effective units of capital an intermediary might receive from foreclosure, $\overline{\omega}_{t+1}K_{t+1}(e)$ or $\gamma\lambda_tK_{t+1}(e)$. Each unit earns the same gross rate of return, $R_{t+1}^{E}Q_tX_{t+1}$.

A.2 Construction of payoffs in Table 1

This sub-section gives a detailed explanation of the construction of the payoff matrix for intermediaries in Table 2. An entrepreneur owns K units of *raw* capital, of which only λK units are liquid, where $0<\lambda<1$. Suppose a proportion, $0<p<1$ of intermediaries foreclose. The debt contract offers foreclosing intermediaries $\overline{\omega}K$ units, leaving the entrepreneur with:

$$\begin{cases} \left(1 - \frac{p\varpi}{\lambda}\right) K & \text{if} \quad p\varpi K \leq \lambda K \\ \\ \quad 0 & \text{if} \quad p\varpi K > \lambda K \end{cases}$$

units of raw capital. An intermediary that foreclosed is in possession of:

$$\begin{cases} \varpi K & \text{if} \quad p\varpi K \leq \lambda K \\ \\ \frac{\lambda K}{p} & \text{if} \quad p\varpi K > \lambda K \end{cases}$$

units of raw capital. Thus, an entrepreneur *fails* at the intra-period stage (i.e. looses all his capital to foreclosing intermediaries) if the proportion of intermediaries that foreclose, p is in the interval $(\frac{\lambda}{\varpi}, 1]$. When $p \in (\frac{\lambda}{\varpi}, 1]$ the raw liquid capital, λK is divided equally among the foreclosing intermediaries.

The entrepreneur and the intermediaries have productivity ω and γ respectively. They use their productivity to transform the raw capital into *effective* capital (i.e. capital that can be used in the production of final goods). The entrepreneur and the foreclosed intermediaries' therefore hold:

$$\begin{cases} \left(1 - \frac{p\varpi}{\lambda}\right) K \\ \\ \quad 0 \end{cases} \quad \text{and} \quad \begin{cases} \varpi K & \text{if} \quad p\varpi \leq \lambda \\ \\ \frac{\lambda K}{p} & \text{if} \quad p\varpi < \lambda \end{cases}$$

units of effective capital, respectively. Each unit of effective capital, X_{t+1} earns a gross return $R^E_{t+1} Q_t X_{t+1}$. The gross return for a foreclosed intermediary is therefore:

$$\begin{cases} \gamma\varpi R^E QK & \text{if} \quad p\varpi \leq \lambda \\ \\ \frac{\gamma\lambda}{p} R^E QK & \text{if} \quad p\varpi < \lambda \end{cases}$$

If the entrepreneur survives the intra-period stage, $p \in [0, \frac{\lambda}{\varpi}]$, his effective capital will generate a gross return:

$$\omega \left(1 - \frac{p\varpi}{\lambda}\right) R^E QK$$

The debt contract offers rolled over intermediaries a non-default gross return, $\varpi R^E QK$. If the entrepreneur fails at the intra-period stage, a rolled over intermediary receives 0. If the entrepreneur survives the intra-period stage, a rolled over intermediary receives the gross return:

$$\begin{cases} \overline{\omega} R^E QK & \text{if} \quad \omega \left(1 - \frac{p\overline{\omega}}{\lambda}\right) R^E QK \geq (1-p)\, \overline{\omega} R^E QK \\ \\ \frac{\omega}{(1-p)} \left(1 - \frac{p\overline{\omega}}{\lambda}\right) R^E QK & \text{if} \quad \omega \left(1 - \frac{p\overline{\omega}}{\lambda}\right) R^E QK < (1-p)\, \overline{\omega} R^E QK \end{cases}$$

The second line states that if the entrepreneur generates a gross return lower than his debt obligation to the rolled over intermediaries, the gross return will be shared equally among the rolled over intermediaries. In Table 2, the *if* statement is rearranged as follows: $\omega \geq \frac{\lambda(1-p)\overline{\omega}}{(\lambda - p\overline{\omega})}$. Finally the entrepreneur, as the residual claimant on the gross returns, receives:

$$\begin{cases} \left(\omega \left(1 - \frac{p\overline{\omega}}{\lambda}\right) - (1-p)\, \overline{\omega}\right) R^E QK & \text{if} \quad \omega \geq \frac{\lambda(1-p)\overline{\omega}}{(\lambda - p\overline{\omega})} \\ \\ 0 & \text{if} \quad \omega < \frac{\lambda(1-p)\overline{\omega}}{(\lambda - p\overline{\omega})} \end{cases}$$

conditional on him surviving the intra-period stage. If the entrepreneur fails at the intra-period stage he also earns zero gross return. This completes the description of the payoff matrix in Table 2.

A.3 Proof of Proposition 1

When ω is not common knowledge, the game played by intermediaries each period in deciding whether to foreclose or rollover is a *global game*. A general proof that the game described in Section 3 has a unique (symmetric) switching equilibrium (given in Proposition 1) is provided in Morris and Shin (2003). Here, I simply identify the characteristics of the model that fit the conditions for Morris and Shin's proof.

In this paper's game, there are a continuum of intermediaries. Each intermediary receives a private signal, x and has to choose an action, $a \in \{foreclose, rollover\}$. All intermediaries have the same payoff function, u where $u\,(a, p, \omega)$ is an intermediary's payoff if he chooses action a, proportion p of the other intermediaries choose to foreclose and the state is ω. To analyze best responses, it is enough to know the net payoff of rollover rather than foreclosure. The net payoff function is a function, π with:

$$\pi(p, \omega) \equiv u\,(rollover, p, \omega) - u\,(foreclosure, p, \omega)$$

The state, ω is drawn from a continuously differentiable strictly positive density. Importantly, the payoffs in Table 1 satisfy the following six properties:

Condition 5 *State monotonicity:*
$\pi(p, \omega)$ *is nondecreasing in* ω.

Condition 6 *Action single crossing:*

for each $\omega \in \mathbb{R}$, *there exists* p^* *such that* $\pi(p, \omega) < 0$ *if* $p < p^*$ *and* $\pi(p, \omega) > 0$ *if* $p > p^*$.

Condition 7 *Uniform limit dominance:*

There exist $\omega^L \in \mathbb{R}$, $\omega^H \in \mathbb{R}$, *and* $\varepsilon \in \mathbb{R}++$, *such that* 1) $\pi(p, \omega) \leq -\varepsilon$ *for all* $p \in [0, 1]$ *and* $\omega \leq \omega^L$; *and* 2) $\pi(p, \omega) > \varepsilon$ *for all* $p \in [0, 1]$ *and* $\omega \geq \omega^H$.

Condition 8 *Monotone likelihood property:*

If $\overline{x} > \underline{x}$, *then* $h(\overline{x} - \omega) / h(\underline{x} - \omega)$ *is increasing in* ω, *where* $h(.)$ *is the distribution of the noise term.*

Condition 9 *Continuity:*

$\int_{p=0}^{1} g(p)\, \pi(p, \omega) dp$ *is continuous with respect to the signal* x *and density* $g(.)$.

Condition 10 *Strict Laplacian state monotonicity:*

There exists a unique ω^* *solving* $\int_{p=0}^{1} \pi(p, \omega^*) dp = 0$.

Morris and Shin (2003) prove the following result which can be applied to this setting: Let ω^* be defined as in Condition 10. The coordination game played by intermediaries has a unique (symmetric) switching strategy equilibrium, with an intermediary choosing rollover if $x > \omega^*$ and foreclosure if $x < \omega^*$, (see Morris and Shin (2003), page 67-70 and Appendix C).

Let me finish with providing a brief discussion of the conditions that must be satisfied for the argument to go through. Condition 5 states that the incentive to rollover is increasing in ω. Thus, an intermediary's optimal action will be increasing in the state, given the other intermediaries' actions. Condition 2 states that the net payoff should only cross zero once. Thus, the payoff matrix does not need to exhibit strategic complementarities (i.e. exhibit action monotonicity) across the full range of p. It does however have to satisfy this weaker single crossing condition (referred to by Goldstein and Pauzner (2005) as one-sided strategic complementarities). The single crossing condition says that the net payoff function only crosses the zero line once. It is clear from Table 2 that the net payoff, $\pi(p, \omega)$ is decreasing in $p \in \left(0, p^\dagger\right)$ where $p^\dagger = \frac{\lambda}{\omega}$ is the critical mass of foreclosing intermediaries at which the entrepreneur fails at the intra-period stage. Above this the net payoff is increasing in p.

Condition 7 requires that foreclosure is a dominant strategy for sufficiently low states, and rollover is a dominant strategy for sufficiently high states. In other words, there must be ranges of extremely good and extremely bad realizations of ω for an entrepreneur, for which an intermediary's best action is independent of its beliefs concerning other intermediaries' behaviour. Let's start with the lower region. This is when ω is so low that it is better for an intermediary to foreclose, even if all other intermediaries rollover, and is when $\omega < \gamma\overline{\omega}$. More precisely, I define ω^L where the previous statement holds with equality and refer to the interval $[0, \omega^L)$ as the *lower dominance region*. Similarly, I assume an *upper dominance region* $(\omega^H, \infty]$ in which no intermediary would foreclose, independent of its beliefs about other intermediaries' actions. Strictly speaking, the payoff matrix in Table 2 does not exhibit an upper dominance region. To implement the upper dominance region, I assume that there exists an external large economic agent (either private or public) which would be willing to buy the entrepreneur out and pay its liabilities when ω is within the upper dominance region. The two dominance regions are just extreme ranges of the fundamentals at which intermediaries' behaviour is known. This is important because in the choice of an equilibrium action at a given signal, intermediaries must take into account the equilibrium actions at nearby signals. Again, these actions depend on the equilibrium actions taken at further signals, and so on. Eventually, the equilibrium must be consistent with the known behaviour at the dominance regions. Importantly though, the position of the equilibrium threshold point, ω^* does not depend on the exact specifications of the two regions. It is therefore possible to be agnostic about the exact details of the upper dominance region, with ω^H arbitrarily high. Although the payoff matrix in Table 1 does not have an upper dominance region, a number of natural economic stories can justify the assumption that if ω were sufficiently large, all intermediaries would have an dominant strategy to rollover.

Condition 8 is a technical restriction on the noise distribution, which is satisfied by the uniform distribution assumed. Condition 9 is a weak continuity property that is satisfied despite a discontinuity in the payoffs at $p^\dagger = \frac{\lambda}{\overline{\omega}}$. Finally, Condition 10 is used to find the unique threshold equilibrium.

A.4 The rollover/foreclosure decision

Section 3 explains the rollover/foreclosure decision for an equilibrium debt contract under reasonable parameterization of the full model. The description in Section 3, however, is an incomplete characterization of the decision rule for intermediaries for all theoretically feasible values of $\gamma, \lambda \in (0, 1)$ and $\overline{\omega} \in (0, \infty)$. It is possible to separate the decision rule into four regions, depending on the values of γ, λ and $\overline{\omega}$:[35] 1) The *no fragility* case when $\frac{\lambda}{\overline{\omega}} < 1$, 2)

[35] The decision rule is conditional $\overline{\omega}$. When deciding whether to rollover or foreclose, an intermediary takes $\overline{\omega}$ as given. Thus, we consider the full range of $\overline{\omega} \in (0, \infty)$ at this stage. However, $\overline{\omega}$ is an endogenous variable. We will show below that in equilibrium, a large subset of possible $\overline{\omega}$ are never chosen by optimizing agents.

Table 6: Payoffs $\left(\text{x } R^E QK\right)$ when $\lambda \geq \overline{\omega}$,

Rollover	**Foreclosure**	
$\overline{\omega}$	$\gamma\overline{\omega}$	if $\omega \geq \frac{\overline{\omega}(1-p)\lambda}{\lambda - p\overline{\omega}}$
$\frac{\omega}{(1-p)}\left(1-\frac{p\overline{\omega}}{\lambda}\right)$	$\gamma\overline{\omega}$	if $\omega < \frac{\overline{\omega}(1-p)\lambda}{\lambda - p\overline{\omega}}$

the *mild fragility* case when $\frac{\lambda}{\overline{\omega}} < 1$ but $\omega^* < \overline{\omega}$, 3) the *acute fragility* case when $\frac{\lambda}{\overline{\omega}} < 1$ and $\omega^* > \overline{\omega}$ and 4) the *no rollover* case when foreclosure occurs with probability 1.

A.4.1 When $\lambda \geq \overline{\omega}$

Consider first when $\lambda \geq \overline{\omega}$. Under this debt contract, $\overline{\omega}$, the entrepreneur is not illiquid at the intra-period stage which is why this scenario is the *no fragility* case. Even if $p = 1$ (i.e. all intermediaries foreclose) every foreclosing intermediary is guaranteed the contractual $\overline{\omega}K$ units of raw capital, and the entrepreneur always has a positive level of raw capital with which to continue operating after the intra-period stage. The payoff matrix in Table 2 in Section 3 reduces to Table 6.

If $\omega > \gamma\overline{\omega}$ it is optimal for all intermediaries to rollover. If $0 < \omega < \gamma\overline{\omega}$, all intermediaries can guarantee a return $\gamma\overline{\omega}R^E QK$ by foreclosing. However, $p = 1$ is not the equilibrium. If all but one intermediary forecloses the return to the intermediary that rolled over is $\overline{\omega}R^E QK$. Instead, there is a mixed equilibrium. Intermediaries will foreclose up to the point at which the payoff to foreclosure and rollover is equalized. The equilibrium foreclosure rate, $p^\ddagger$ implicitly solves:

$$\begin{aligned} \frac{\omega}{(1-p^\ddagger)}\left(1-\frac{p^\ddagger\overline{\omega}}{\lambda}\right) &= \gamma\overline{\omega} \\ p^\ddagger &= \frac{\lambda}{\overline{\omega}}\frac{(\gamma\overline{\omega}-\omega)}{(\gamma\lambda-\omega)} < 1 \end{aligned}$$

This implies that when the entrepreneur is not fragile to the possibility of a credit run, there is no symmetric foreclosure threshold. In terms of expected payoffs though, it means that all intermediaries are guaranteed the foreclosure gross return $\gamma\overline{\omega}R^E QK$, (although some intermediaries will earn this gross return by rolling over).

A.4.2 When $\lambda < \overline{\omega}$

When $\lambda < \overline{\omega}$, the indifference condition in equation (16) in Section 3 should actually read:

$$\int_{p=\frac{\lambda}{\overline{\omega}}}^{1} -\frac{\gamma\lambda}{p}dp + \int_{p=0}^{\frac{\lambda}{\overline{\omega}}} \left(\left\{ \begin{array}{ccc} \overline{\omega} & \text{if} & \omega^* > \frac{\lambda(1-p)\overline{\omega}}{(\lambda - p\overline{\omega})} \\ & & \\ \frac{\omega^*}{(1-p)}\left(1 - \frac{p\overline{\omega}}{\lambda}\right) & \text{if} & \omega^* \leq \frac{\lambda(1-p)\overline{\omega}}{(\lambda - p\overline{\omega})} \end{array} \right\} - \gamma\overline{\omega} \right) dp = 0 \qquad (30)$$

where the foreclosure threshold, ω^* is the implicit solution to this indifference condition, which reduces to:

$$\begin{cases} \gamma\lambda\frac{\lambda}{\overline{\omega}}\left(\ln\left(\frac{\lambda}{\overline{\omega}}\right) - 1\right) + \omega^*\frac{\lambda}{\overline{\omega}} + \omega^*\left(1 - \frac{\lambda}{\overline{\omega}}\right)\ln\left(1 - \frac{\lambda}{\overline{\omega}}\right) = 0 & \text{if} \quad \omega^* \leq \overline{\omega} \\ & \\ \gamma\lambda\frac{\lambda}{\overline{\omega}}\left(\ln\left(\frac{\lambda}{\overline{\omega}}\right) - 1\right) + \lambda + \omega^*\left(1 - \frac{\lambda}{\overline{\omega}}\right)\ln\left(1 - \frac{\lambda}{\omega^*}\right) = 0 & \text{if} \quad \omega^* > \overline{\omega} \end{cases}$$

A rearranged version of the top line is given in equation (17) in Section 3. Thus, the main text presented the result where $\omega^* \leq \overline{\omega}$. The discussion below will explain why this is the most likely outcome for a reasonable parameterization of the model. $\omega^* \leq \overline{\omega}$ is termed the *mild fragility* case and $\omega^* > \overline{\omega}$ the *acute fragility* case. Under mild fragility, $\omega^* \leq \overline{\omega}$, there is an inefficiency due to the fact that $\omega^* > \omega^*_{EFF}$. However, an entrepreneur that experiences a credit run is technically already insolvent. Under acute fragility, $\omega^* > \overline{\omega}$, even solvent entrepreneurs face illiquidity.

Clearly when $\omega^* = \overline{\omega}$, the two lines in equation (30) coincide. While it is not possible to obtain an analytical solution for ω^* when $\omega^* > \overline{\omega}$, a comparison of the effect of $\overline{\omega}$ on ω^* in the case of mild and acute fragility is possible. First, what determines when the situation changes from mild to acute? By using equation (17), it is possible to rewrite the inequality, $\omega^* < \overline{\omega}$:

$$\frac{1}{\gamma} < \frac{\left(\frac{\lambda}{\overline{\omega}}\right)^2 \left(1 - \ln\left(\frac{\lambda}{\overline{\omega}}\right)\right)}{\left(\frac{\lambda}{\overline{\omega}} + \left(1 - \frac{\lambda}{\overline{\omega}}\right)\ln\left(1 - \frac{\lambda}{\overline{\omega}}\right)\right)}$$

The right-hand-side as a function of $\frac{\lambda}{\overline{\omega}}$ can be shown to be negatively sloped with $\lim_{\frac{\lambda}{\overline{\omega}}\to 0} rhs = +\infty$ and $\lim_{\frac{\lambda}{\overline{\omega}}\to 1} rhs = 1$. Thus, when γ is close to 1 (i.e. the intermediaries are good capital managers and close substitutes for the entrepreneurs) the entrepreneur's position is acutely fragile even when his balance sheet is not very illiquid (i.e. when $\frac{\lambda}{\overline{\omega}}$ is close to 1). The second important point to make is that ω^* is always increasing in $\overline{\omega}$:

$$\frac{\partial \omega^*(\overline{\omega})}{\partial \overline{\omega}} = \begin{cases} \frac{\gamma\left(\frac{\lambda}{\overline{\omega}}\right)^2 - \left(\frac{\lambda}{\overline{\omega}} + \ln\left(1-\frac{\lambda}{\overline{\omega}}\right)\right)\frac{\omega^*}{\overline{\omega}}}{\frac{\lambda}{\overline{\omega}} + \left(1-\frac{\lambda}{\overline{\omega}}\right)\ln\left(1-\frac{\lambda}{\overline{\omega}}\right)} > 0 & \text{if} \quad \omega^* \leq \overline{\omega} \\ \frac{1-\frac{\lambda}{\omega^*}}{1-\frac{\lambda}{\overline{\omega}}} \frac{\frac{\lambda}{\omega^*}\left(\gamma\frac{\lambda}{\overline{\omega}}-1\right) - \ln\left(1-\frac{\lambda}{\omega^*}\right)}{\frac{\lambda}{\omega^*} + \left(1-\frac{\lambda}{\omega^*}\right)\ln\left(1-\frac{\lambda}{\omega^*}\right)} > 0 & \text{if} \quad \omega^* > \overline{\omega} \end{cases}$$

This shows that the foreclosure threshold rises with rollover-solvency threshold, $\overline{\omega}$. In addition it is possible to show that the rate at which ω^* increases with $\overline{\omega}$ is higher when $\omega^* > \overline{\omega}$. Suppose the decision rule in equation (17) is extrapolated beyond to $\omega^* > \overline{\omega}$. Then:

$$\left.\frac{\partial \omega^*(\overline{\omega})}{\partial \overline{\omega}}\right|_{\text{mild}} < \left.\frac{\partial \omega^*(\overline{\omega})}{\partial \overline{\omega}}\right|_{\text{acute}} \qquad \text{when} \quad \omega^* > \overline{\omega}$$

which states that the the foreclosure threshold, and the inefficiency cost of coordination failure, rises more rapidly in $\overline{\omega}$ when there is acute fragility.

The final case to consider is the *no rollover* case. This is when Condition 10 (above) is violated, i.e. $\lim_{\omega^* \to \infty} \int_{p=0}^{1} \pi(p, \omega^*) dp < 0$. To find the boundary between the acute fragility and no rollover case, solve:

$$\lim_{\omega^* \to \infty} \gamma\lambda\frac{\lambda}{\overline{\omega}}\left(\ln\left(\frac{\lambda}{\overline{\omega}}\right) - 1\right) + \lambda + \omega^*\left(1 - \frac{\lambda}{\overline{\omega}}\right)\ln\left(1 - \frac{\lambda}{\omega^*}\right) < 0$$

which reduces to:

$$1 < \gamma\left(1 - \ln\left(\frac{\lambda}{\overline{\omega}}\right)\right) \tag{31}$$

Since the term in brackets is greater than 1 and increasing in the entrepreneur's illiquidity, $\frac{\lambda}{\overline{\omega}}$, it implies that when γ is very high, there is no threshold solution, even for relatively low levels of illiquidity. In this case, intermediaries foreclose with probability 1. Again, it should be stressed that this possibility is never an equilibrium; the endogenous variable $\overline{\omega}$ is never chosen such that equation (31) holds.

In this section I have provided a complete description of intermediaries' decision rules at the intra-period stage, something I did not do in Section 3. This naturally means I need to readdress the expected equilibrium payoff for the entrepreneur and intermediaries respectively, and the equilibrium debt contract.

A.5 Complete characterization of equilibrium payoffs

This section gives a complete characterization of the expected gross returns accruing to both the entrepreneur and the intermediaries. Again, we normalize by $R^E QK$.

Expected payoff to an intermediary =

$$\begin{cases} \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\, d\omega + \int_{\gamma\overline{\omega}}^{\overline{\omega}} \omega f(\omega)\, d\omega + \gamma\overline{\omega}\int_0^{\gamma\overline{\omega}} f(\omega)\, d\omega & \text{if no fragility} \\ \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\, d\omega + \int_{\omega^*}^{\overline{\omega}} \omega f(\omega)\, d\omega + \gamma\lambda\int_0^{\omega^*} f(\omega)\, d\omega & \text{if mild fragility} \\ \overline{\omega}\int_{\omega^*}^{\infty} f(\omega)\, d\omega + \gamma\lambda\int_0^{\omega^*} f(\omega)\, d\omega & \text{if acute fragility} \\ \gamma\lambda & \text{if no rollover} \end{cases}$$

Expected payoff to the entrepreneur =

$$\begin{cases} \int_{\overline{\omega}}^{\infty} (\omega - \overline{\omega})\, f(\omega)\, d\omega & \text{if no fragility} \\ \int_{\overline{\omega}}^{\infty} (\omega - \overline{\omega})\, f(\omega)\, d\omega & \text{if mild fragility} \\ \int_{\omega^*}^{\infty} (\omega - \overline{\omega})\, f(\omega)\, d\omega & \text{if acute fragility} \\ 0 & \text{if no rollover} \end{cases}$$

Using the notation, $\Gamma(.) \equiv \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\, d\omega + \int_0^{\overline{\omega}} \omega f(\omega)\, d\omega$, it is possible to rewrite the expected gross returns for the entrepreneur and the intermediaries as $1 - \Gamma(.) - H(.)$ and $\Gamma(.) - G(.)$ respectively, where $H(.)$ and $G(.)$ are defined as the expected cost of coordination failure for the entrepreneur and the intermediaries, respectively. The $G(.)$ and $H(.)$ functions are given as follows:

$$G(.) = \begin{cases} \int_0^{\gamma\overline{\omega}} (\omega - \gamma\overline{\omega})\, f(\omega)\, d\omega & \text{if no fragility} \\ \int_0^{\omega^*(\overline{\omega})} (\omega - \gamma\lambda)\, f(\omega)\, d\omega & \text{if mild fragility} \\ (\overline{\omega} - \gamma\lambda)\int_{\overline{\omega}}^{\omega^*} f(\omega)\, d\omega + \int_0^{\overline{\omega}} (\omega - \gamma\lambda)\, f(\omega)\, d\omega & \text{if acute fragility} \\ -\overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)\, d\omega - \int_0^{\overline{\omega}} \omega f(\omega)\, d\omega + \gamma\lambda & \text{if no rollover} \end{cases}$$

with derivative:

$$\frac{\partial G(\overline{\omega})}{\partial \overline{\omega}} = \begin{cases} -\gamma F(\gamma\overline{\omega}) & \text{if} \quad \text{no fragility} \\ (\omega^* - \gamma\lambda) f(\omega^*) \frac{\partial \omega^*}{\partial \overline{\omega}} & \text{if} \quad \text{mild fragility} \\ F(\omega^*) - F(\overline{\omega}) + (\overline{\omega} - \gamma\lambda) f(\omega^*) \frac{\partial \omega^*}{\partial \overline{\omega}} & \text{if} \quad \text{acute fragility} \\ -1 + F(\overline{\omega}) & \text{if} \quad \text{no rollover} \end{cases}$$

and:

$$H(.) = \begin{cases} 0 & \text{if} \quad \text{no fragility} \\ 0 & \text{if} \quad \text{mild fragility} \\ \int_{\overline{\omega}}^{\omega^*(\overline{\omega})} (\omega - \overline{\omega}) f(\omega) d\omega & \text{if} \quad \text{acute fragility} \\ \int_{\overline{\omega}}^{\infty} (\omega - \overline{\omega}) f(\omega) d\omega & \text{if} \quad \text{no rollover} \end{cases}$$

with derivative:

$$\frac{\partial H(\overline{\omega})}{\partial \overline{\omega}} = \begin{cases} 0 & \text{if} \quad \text{no fragility} \\ 0 & \text{if} \quad \text{mild fragility} \\ -(F(\omega^*) - F(\overline{\omega})) + (\omega^* - \overline{\omega}) f(\omega^*) \frac{\partial \omega^*}{\partial \overline{\omega}} & \text{if} \quad \text{acute fragility} \\ -1 + F(\overline{\omega}) & \text{if} \quad \text{no rollover} \end{cases}$$

A.6 The contracting problem

The aim of this section is to show that the debt contracting problem outlined in Section 3 produced a monotonic relationship between the illiquidity premium and the leverage ratio. I first develop the theory for the case of no aggregate risk. As discussed in the main text, the details in this subsection follow very closely the contracting problem described in Section A.1. of Bernanke et al. (1999). Notation has been kept relatively similar in order to facilitate easy comparison. I will highlight where the derivations differ importantly from that in Bernanke et al. (1999).

Let the gross rate of return on the value of a unit of effective capital equal R^E. Capital is subject to an idiosyncratic shock, $\omega \in [0, \infty)$ with $E(\omega) = 1$. I assume $F(x) = \Pr(\omega < x)$ is a continuous probability distribution with $F(0) = 0$ and denote by $f(\omega)$ the *pdf* of ω. The equilibrium contract specifies $\overline{\omega}$. In equilibrium the intermediaries earn an expected return equal to:

$$(\Gamma(.) - G(.))\, R^E K = R(K - N)$$

where $\Gamma(.)$ is the gross share of the returns, $R^E K$ going to the intermediaries. The net share of returns going to the intermediaries is $\Gamma(.) - G(.)$, and the share going to the entrepreneur is $1 - \Gamma(.) - H(.)$ where both $G(.)$ and $H(.)$ are expected costs of coordination failure. By definition, $0 < \Gamma(.) < 1$. The assumption made above imply:

$$\Gamma(.) - G(.) > 0 \text{ for all } \overline{\omega} \in (0, \infty)$$

and:

$$\lim_{\overline{\omega} \to 0} \Gamma(.) - G(.) = 0 \ , \ \lim_{\overline{\omega} \to \infty} \Gamma(.) - G(.) = \gamma\lambda$$

Differentiating $\Gamma(.) - G(.)$ there exists an $\overline{\overline{\omega}}$ such that:

$$\Gamma'(.) - G'(.) \lesseqqgtr 0 \text{ for } \overline{\omega} \gtreqqless \overline{\overline{\omega}}$$

implying that the net payoff to the intermediaries reaches a global maximum at $\overline{\overline{\omega}}$. It is also possible to show that:

$$(\Gamma' + H')\, G'' + \Gamma' H'' - (\Gamma'' + H'')\, G' - \Gamma'' H' > 0 \text{ for } \overline{\omega} < \overline{\overline{\omega}}$$

which will guarantee an interior solution. The contracting problem may now be written as:

$$\begin{aligned} &\max_{K, \overline{\omega}} (1 - \Gamma - H)\, R^E K \\ &\text{subject to } (\Gamma - G)\, R^E K = R(K - N) \end{aligned}$$

It is easiest to analyze this problem by first explicitly defining the illiquidity premium, $s = \frac{R^E}{R}$ and then, owing to constant returns to scale, normalizing by net worth and using $k = \frac{K}{N}$, the capital to net worth ratio as the choice variable. Defining V as the Lagrange multiplier on the constraint that intermediaries earn the risk-free rate of return in expectation, the first-order conditions for an interior solution to this problem may be written as:

$$\begin{aligned}
\overline{\omega} &: \Gamma' + H' - V(\Gamma' - G') = 0 \\
k &: (1 - \Gamma + V(\Gamma - G))\, s - V = 0 \\
V &: (\Gamma - G)\, sk - (k - 1) = 0
\end{aligned}$$

Since $\Gamma - G$ is increasing on $\left(0, \overline{\overline{\omega}}\right)$ and decreasing on $\left(\overline{\overline{\omega}}, \infty\right)$, the intermediary never chooses $\overline{\omega} > \overline{\overline{\omega}}$. The first-order condition with respect to $\overline{\omega}$ implies the Lagrange multiplier, V can be written as a function of $\overline{\omega}$:

$$V(\overline{\omega}) = \frac{\Gamma' + H'}{\Gamma' - G'}$$

Taking derivatives obtains:

$$V' = \frac{(\Gamma' + H')\, G'' + \Gamma' H'' - (\Gamma'' + H'')\, G' - \Gamma'' H'}{(\Gamma' - G')^2} > 0 \text{ for } \overline{\omega} < \overline{\overline{\omega}}$$

and taking limits obtains:

$$\lim_{\overline{\omega} \to 0} V(\overline{\omega}) = 1 \ , \ \lim_{\overline{\omega} \to \overline{\overline{\omega}}} V(\overline{\omega}) = +\infty$$

The first-order conditions then imply that $\overline{\omega}$ satisfies:

$$s(\overline{\omega}) = \frac{V}{1 - \Gamma - H + V(\Gamma - G)} \tag{32}$$

where s is the wedge between the rate of return on capital and the risk-free return demanded by intermediaries. Again, computing derivatives:

$$s' = s\frac{V'}{V}\left(\frac{1 - \Gamma - H}{1 - F - H + V(\Gamma - G)}\right) > 0 \text{ for } \overline{\omega} < \overline{\overline{\omega}}$$

and taking limits:

$$\lim_{\overline{\omega} \to 0} s(\overline{\omega}) = 1 \text{ and } \lim_{\overline{\omega} \to \overline{\overline{\omega}}} s(\overline{\omega}) = \frac{1}{\Gamma\left(\overline{\overline{\omega}}\right) - G\left(\overline{\overline{\omega}}(\omega^*)\right)} < \frac{1}{\gamma}$$

Thus, this guarantees a one-to-one mapping between the optimal $\overline{\omega}$ and the illiquidity premium, s. I introduce an assumption that:

$$\frac{1}{\Gamma\left(\overline{\overline{\omega}}\right) - G\left(\overline{\overline{\omega}}(\omega^*)\right)} < \frac{1}{\gamma}$$

If this condition does not hold, then there are occasions on which it is conceivable that the illiquidity premium is so high that intermediaries could earn a higher return by cutting out the intermediaries and buying and managing capital directly.[36] Thus there is a monotonically increasing relationship between foreclosure probabilities and the illiquidity premium on external funds.

The first-order conditions also give:

$$k(\overline{\omega}) = 1 + \frac{V(\Gamma - G)}{1 - F - H} \tag{33}$$

Computing the derivative obtains:

$$k' = \frac{V'}{V}(k-1) + \frac{\Gamma'}{1-F-H}k > 0 \text{ for } \overline{\omega} < \overline{\overline{\omega}}$$

and taking limits:

$$\lim_{\overline{\omega} \to 0} k(\overline{\omega}) = 1 \ , \ \lim_{\overline{\omega} \to \overline{\overline{\omega}}} k(\overline{\omega}) = +\infty$$

Combining equations (32) and (33) expresses the illiquidity premium as an increasing function of the capital to net worth ratio:

$$s = \Xi(k) \text{ with } \Xi'(.) > 0$$

Section A.3 of Bernanke et al. (1999) extend the derivation their proof to show that the relationship between s and k is still monotonically increasing with the introduction of aggregate risk into the problem. This proof places conditions on the differences in the contracting problem under coordination failure and costly state verification. Therefore I do reproduce the proof here.

B Appendix: The DSGE structure

Appendix B provides the details of the full DSGE model including the log-normal distribution of ω, the steady state of the system of equilibrium equations as well as the fully log-linearized

[36] When aggregate risk is reintroduced, the restriction on the risk premium is weakened, since intermediaries still want to insulate households from the aggregate risk. Suppose we decompose R^E_{t+1} into $\widetilde{u}_{t+1} E_t R^E_{t+1}$ where $\widetilde{u}_{t+1}$ is *i.i.d.* over time and has $E_t(\widetilde{u}_{t+1}) = 1$ and *cov* $\left(\widetilde{u}_{t+1}, E_t R^E_{t+1}\right) = 0$. Thus, the realization of R^E_{t+1} has been decomposed into its expected value and its stochastic element. Consider that the support on $\widetilde{u}_{t+1}$ is $\left(u^{\min}, u^{\max}\right)$. Then we need to assume that:

$$\frac{u^{\min} E\left\{R^E\right\}}{R} < \frac{1}{\gamma}$$

which is a weaker condition.

system.

B.1 The distribution of ω

Suppose ω is distributed log-normally. Under the assumption that $\ln(\omega) \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)$, it follows that $E(\omega) = 1$, and:

$$E(\omega \mid \omega > x) = \frac{1 - \Phi\left(\frac{1}{\sigma}\left(\ln x - \frac{\sigma^2}{2}\right)\right)}{1 - \Phi\left(\frac{1}{\sigma}\left(\ln x + \frac{\sigma^2}{2}\right)\right)} \tag{34}$$

where $\Phi(.)$ is the *c.d.f.* of the standard normal. Using this, it is possible to obtain:

$$\Gamma(\overline{\omega}) = \overline{\omega}\left[1 - \Phi(\overline{z})\right] + \Phi(\overline{z} - \sigma) \tag{35}$$

$$G(\overline{\omega}) = \Phi(z^* - \sigma) - \gamma\lambda\Phi(z^*) \tag{36}$$

where $\Gamma(.)$ and $G(.)$ are defined for mild fragility only (see Section A) and where $\overline{z}$ and z^* are related to $\overline{\omega}$ through $\overline{z} \equiv (\ln\overline{\omega} + \sigma^2/2)/\sigma$ and $z^* \equiv (\ln\omega^*(\overline{\omega}) + \sigma^2/2)/\sigma$ respectively. Differentiating with respect to $\overline{\omega}$ gives:

$$\Gamma'(\overline{\omega}) = \left[1 - \Phi(\overline{z})\right] - \overline{\omega}\phi(\overline{z})\,\overline{z}' + \phi(\overline{z} - \sigma)\,\overline{z}' \tag{37}$$

$$G'(\overline{\omega}) = \phi(z^* - \sigma)\, z^{*\prime}\omega^{*\prime} - \gamma\lambda\phi(z^*)\, z^{*\prime}\omega^{*\prime} \tag{38}$$

where $\overline{z}' = 1/(\sigma\overline{\omega})$ and $z^{*'} = 1/(\sigma\omega^*)$. These are used to calculate the first-order approximations of the equilibrium conditions.

B.2 Equilibrium conditions

This section lists the equilibrium conditions for the model. **Consumption savings:**

$$E_t\Lambda_{t,t+1}R_{t+1} = 1 \tag{39}$$

Labour market equilibrium condition:

$$E_tU_{C,t}(1 - \alpha)\frac{Y_t}{L_t} = \chi L_t^{\rho} \tag{40}$$

where:

$$\begin{aligned} U_{C,t} &\equiv (C_t - \mu C_{t-1})^{-1} - \beta\mu\,(C_{t+1} - \mu C_t)^{-1} \\ \Lambda_{t,t+1} &\equiv \beta \frac{U_{C,t+1}}{U_{C,t}} \end{aligned}$$

Expected rate of return on capital:

$$E_t R_{t+1}^E = E_t \left(\frac{\alpha \frac{Y_{t+1}}{\phi_t K_{t+1}} + (1-\delta)\,Q_{t+1}}{Q_t} \right) \tag{41}$$

where:

$$\begin{aligned} \phi_t\left(\omega^*\left(\overline{\omega}_t\right), \lambda_t\right) &\equiv \left\{E\left(\omega \mid \omega > \omega^*\left(\overline{\omega}_t, \lambda_t\right)\right)\Pr\left(\omega > \omega^*\left(\overline{\omega}_t, \lambda_t\right)\right) + \gamma\lambda_t \Pr\left(\omega \leq \omega^*\left(\overline{\omega}_t, \lambda_t\right)\right)\right\} \\ &\equiv 1 - \Phi\left(z^*\left(\overline{\omega}_t, \lambda_t\right) - \sigma\right) + \gamma\lambda_t \Phi\left(z^*\left(\overline{\omega}_t, \lambda_t\right)\right) \end{aligned}$$

where:

$$z^*\left(\overline{\omega}_t, \lambda_t\right) \equiv \left(\ln \omega^*\left(\overline{\omega}_t, \lambda_t\right) + \sigma^2/2\right)/\sigma$$

Aggregate resource constraint:

$$Y_t = C_t + I_t + G_t + C_t^E \tag{42}$$

where:

$$C_t^E = \frac{1-\upsilon}{\upsilon} N_{t+1}$$

Production function:

$$Y_t = \phi_t A_t K_t^{\alpha} L_t^{1-\alpha} \tag{43}$$

Capital accumulation:

$$K_{t+1} = (1-\delta)\,\phi_t K_t + \Psi\left(\frac{I_t}{K_t}\right) K_t \tag{44}$$

where:

$$\Psi\left(\frac{I_t}{K_t}\right) = \frac{1}{1-\varphi}\left(\frac{I}{K}\right)^{\varphi}\left(\frac{I_t}{K_t}\right)^{1-\varphi} - \frac{\varphi}{1-\varphi}\left(\frac{I}{K}\right)$$

External finance premium: The first-order conditions from the entrepreneur's problem:

$$0 = E_t\left(\frac{\left(\Gamma\left(\overline{\omega}_{t+1}\right)G_{\overline{\omega}}\left(\overline{\omega}_{t+1},\lambda_{t+1},\mathbb{N}_{t+1}\right)-\Gamma_{\overline{\omega}}\left(\overline{\omega}_{t+1}\right)G\left(\overline{\omega}_{t+1},\lambda_{t+1},\mathbb{N}_{t+1}\right)\right)}{\Gamma_{\overline{\omega}}\left(\overline{\omega}_{t+1}\right)-G_{\overline{\omega}}\left(\overline{\omega}_{t+1},\lambda_{t+1},\mathbb{N}_{t+1}\right)}\frac{R^E_{t+1}}{R_{t+1}}\right) + E_t\left(\frac{R^E_{t+1}}{R_{t+1}}\right) - E_t\left(\frac{\Gamma_{\overline{\omega}}\left(\overline{\omega}_{t+1}\right)}{\Gamma_{\overline{\omega}}\left(\overline{\omega}_{t+1}\right)-G_{\overline{\omega}}\left(\overline{\omega}_{t+1},\lambda_{t+1},\mathbb{N}_{t+1}\right)}\right) \quad (45)$$

and the intermediaries' break even:

$$\left(\Gamma\left(\overline{\omega}_t\right)-G\left(\overline{\omega}_t,\lambda_t,\mathbb{N}_t\right)\right)\frac{R^E_t}{R_t}\frac{Q_{t-1}K_t}{N_t} = \left(\frac{Q_{t-1}K_t}{N_t}-1\right) \quad (46)$$

Net worth:

$$\mathbb{C}_{t+1}N_{t+1} = \upsilon\mathbb{C}_t\left(\phi_t R^E_t Q_{t-1}K_t - R_t\left(Q_{t-1}K_t - N_t\right)\right) \quad (47)$$

Investment-Q:

$$Q_t = \left(\Psi'\left(\frac{I_t}{K_t}\right)\right)^{-1} \quad (48)$$

where:

$$\Psi'\left(\frac{I_t}{K_t}\right) = \left(\frac{I}{K}\right)^{\varphi}\left(\frac{I_t}{K_t}\right)^{-\varphi}$$

Technology and illiquidity shock processes:

$$A_t = A_{t-1}^{\rho_A}e^{\varepsilon_t^A} \quad \text{and} \quad \lambda_t = \lambda^{1-\rho_\lambda}\lambda_{t-1}^{\rho_\lambda}e^{\varepsilon_t^\lambda} \quad (49)$$

with $\varepsilon_t^A \sim N\left(0,\sigma_A^2\right)$ and $\varepsilon_t^{\aleph} \sim N\left(0,\sigma_{\aleph}^2\right)$ respectively. **Policy rules:**

$$\left(1-\mathbb{N}_{t+1}\right) = a_{DL}E_t\left(\frac{R^E_{t+1}/R_{t+1}}{R^E/R}-1\right) \quad \text{and} \quad \left(1-\mathbb{C}_{t+1}\right) = a_{EQ}E_t\left(\frac{R^E_{t+1}/R_{t+1}}{R^E/R}-1\right)$$

for direct lending and equity injections respectively.

B.3 Non-stochastic steady state

This section lists the conditions for the non-stochastic steady state of the economy. From equation (39):

$$1 = \beta R \quad (50)$$

From equation (40):

$$(1-\alpha)\frac{Y}{L} = \theta C L^{\varepsilon} \quad (51)$$

From equation (41):

$$\frac{Y}{\phi K} = \frac{1}{\alpha}\left(R^E - (1-\delta)\right) \tag{52}$$

where:

$$\phi \equiv 1 - \Phi\left(z^*\left(\overline{\omega}, \lambda\right) - \sigma\right) + \gamma\lambda\Phi\left(z^*\left(\overline{\omega}, \lambda\right)\right)$$

where:

$$z^*\left(\overline{\omega}, \lambda\right) \equiv \left(\ln \omega^*\left(\overline{\omega}, \lambda\right) + \sigma^2/2\right)/\sigma$$

From equation (12):

$$Y = C + I + G + C^E \tag{53}$$

where:

$$C^E = \frac{1-\upsilon}{\upsilon} N$$

From equation (43):

$$Y = \phi K^\alpha L^{1-\alpha} \tag{54}$$

From equation (44):

$$\frac{I}{K} = 1 - \phi\left(1-\delta\right) \tag{55}$$

From equation (45):

$$\frac{R^E}{R} = \frac{\Gamma_{\overline{\omega}}}{\Gamma_{\overline{\omega}}\left(1-G\right) - \left(1-\Gamma\right) G_{\overline{\omega}}}$$

From equation (46):

$$\left(\Gamma - G\right)\frac{R^E}{R}\frac{K}{N} = \frac{K}{N} - 1$$

From equation (47):

$$N = \frac{\upsilon}{1-\upsilon R}\left(\phi R^E - R\right) K \tag{56}$$

From equation (48), $Q = 1$.

B.4 Log-linearized system

This section lists the equilibrium conditions, with variables expressed in terms of log-deviations from their respective non-stochastic steady states. **Consumption savings:**

$$0 = \beta\mu E_t c_{t+2} - \left(1 + \left(1+\mu\right)\beta\mu\right) E_t c_{t+1} + \left(1 + \left(1+\beta\mu\right)\mu\right) c_t - \mu c_{t-1} + \left(1-\beta\mu\right)\left(1-\mu\right) r_{t+1}$$

Labour market:

$$\frac{\beta\mu}{(1-\mu)(1-\beta\mu)}E_t c_{t+1} - \frac{(1+\beta\mu^2)}{(1-\mu)(1-\beta\mu)}c_t + \frac{\mu}{(1-\mu)(1-\beta\mu)}c_{t-1} = (1+\rho)\, l_t - y_t$$

Expected return on capital:

$$\begin{aligned} E_t r^E_{t+1} &= \frac{\alpha\frac{Y}{\phi K}}{\alpha\frac{Y}{\phi K}+(1-\delta)}\left(E_t y_{t+1} - k_{t+1} - b_1 E_t\widetilde{\overline{\omega}}_{t+1} - b_2 E_t\widetilde{\lambda}_{t+1} - b_3 E_t nn_{t+1}\right) \\ &\quad + \frac{1-\delta}{\alpha\frac{Y}{\phi K}+(1-\delta)}E_t q_{t+1} - q_t \end{aligned}$$

where $b_1 \equiv \frac{\phi_{\overline{\omega}}}{\phi}\overline{\omega}$, $b_2 \equiv \frac{\phi_\lambda}{\phi}\lambda$ and $b_3 \equiv \frac{\phi_{\mathbb{N}}}{\phi}$ where ϕ_x for $x = \lambda, \overline{\omega}, \mathbb{N}$ are partial derivatives and where nn_t and cc_t are the log-deviations of $\mathbb{N}_t$ and $\mathbb{C}_t$ respectively. **Aggregate resource constraint:**

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \frac{C^E}{Y}c^E_t$$

where $c^E_t = n_t$. **Production function:**

$$y_t = a_t + \alpha k_t + (1-\alpha)\, l_t + b_1\widetilde{\overline{\omega}}_t + b_2\widetilde{\lambda}_t + b_3 nn_t$$

Capital accumulation:

$$k_{t+1} = \phi(1-\delta)\left(k_t + b_1\widetilde{\overline{\omega}}_t + b_2\widetilde{\lambda}_t + b_3 nn_t\right) + (1-\phi(1-\delta))\, i_t$$

External finance premium:

$$\begin{aligned} 0 &= \Gamma_{\overline{\omega}}\left(E_t r^E_{t+1} - r_{t+1}\right) - \frac{1}{K/N}\frac{\Gamma_{\overline{\omega}}G_{\overline{\omega}\overline{\omega}} - \Gamma_{\overline{\omega}\overline{\omega}}G_{\overline{\omega}}}{(\Gamma_{\overline{\omega}} - G_{\overline{\omega}})}\overline{\omega}.E_t\widetilde{\overline{\omega}}_{t+1} \\ &\quad - \left(\frac{1}{K/N}\frac{\Gamma_{\overline{\omega}}G_{\overline{\omega}\lambda}}{(\Gamma_{\overline{\omega}} - G_{\overline{\omega}})} + \Gamma_{\overline{\omega}}G_\lambda\beta R^E\right)\lambda.E_t\widetilde{\lambda}_{t+1} \\ &\quad - \left(\frac{1}{K/N}\frac{\Gamma_{\overline{\omega}}G_{\overline{\omega}\mathbb{N}}}{(\Gamma_{\overline{\omega}} - G_{\overline{\omega}})} + \Gamma_{\overline{\omega}}G_{\mathbb{N}}\beta R^E\right)\mathbb{N}.nn_{t+1} \end{aligned} \tag{57}$$

and:

$$\begin{aligned} 0 &= \frac{\Gamma_{\overline{\omega}}}{(1-\Gamma)}\overline{\omega}.\widetilde{\overline{\omega}}_t - G_\lambda\beta R^E\frac{K}{N}\lambda.\widetilde{\lambda}_t - G_{\mathbb{N}}\beta R^E\frac{K}{N}\mathbb{N}.nn_t \\ &\quad + \left(\frac{\Gamma - G}{1-\Gamma}\frac{\Gamma_{\overline{\omega}}}{\Gamma_{\overline{\omega}} - G_{\overline{\omega}}}\right)\left(r^E_t - r_t\right) - (q_{t-1} + k_t - n_t) \end{aligned} \tag{58}$$

The elasticity of the external finance premium with respect to the capital to net worth ratio, χ, calculated in Section 5.1 is derived by rolling forward equation (58) by one period and substituting into equation (57) so as to eliminate $\widetilde{\widetilde{\omega}}_t$. **Net worth:**

$$\begin{aligned} n_{t+1} &= \upsilon\left(\frac{\phi R^E K}{N}\left(r_t^E + b_1\widetilde{\widetilde{\omega}}_t + b_2\widetilde{\lambda}_t + b_3 nn_t\right) - \frac{R\left(K-N\right)}{N} r_t + \frac{\left(\phi R^E - R\right)K}{N}\left(q_{t-1}+k_t\right) + Rn_t\right) \\ &\quad - cc_{t+1} + cc_t \end{aligned}$$

Investment-Q:

$$q_t = \psi\left(i_t - k_t\right)$$

Technology and illiquidity shock processes:

$$a_t = \rho_A a_{t-1} + \varepsilon_t^a \text{ and } \widetilde{\lambda}_t = \lambda + \rho_\lambda \widetilde{\lambda}_{t-1} + \varepsilon_t^\lambda$$

Government debt accumulation:

$$\begin{aligned} d_{t+1}^g &= \frac{1}{\beta\left(1+x\right)} d_t^g - \frac{\tau' N}{1+x} cc_{t+1} + \frac{\left(1-\Gamma\right)R^E K}{1+x} cc_t \\ &\quad - \frac{\tau'\left(K-N\right)}{1+x} nn_{t+1} + \frac{\left(\Gamma - pol\right)R^E K}{1+x} nn_t \end{aligned}$$

Policy rules:

$$-nn_t = a_{DL}\left(E_t r_{t+1}^E - r_{t+1}\right) \quad \text{and} \quad -cc_t = a_{EQ}\left(E_t r_{t+1}^E - r_{t+1}\right)$$