

Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty*

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Abstract

This paper studies the interaction of government debt and banking markets. Both markets are known to be fragile: excessively responsive to fundamentals and prone to strategic uncertainty. Our analysis highlights the spillover from fragility in debt markets on banks and, through government bailout of troubled banks, spillovers from banks to the debt market. This interaction, termed a ‘diabolic loop’, is driven by government willingness to bail out banks and the resulting incentives for banks not to self-insure through equity buffers. We provide conditions such that the ‘diabolic loop’ is a Nash Equilibrium of the interaction between banks and the government.

1 Introduction

We must break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks. This works both ways. Making banks stronger, including by restoring adequate capital levels, stops banks from hurting sovereigns through higher debt or contingent liabilities. And restoring confidence in sovereign debt helps banks, which are important holders of such debt and typically benefit from explicit or implicit guarantees from sovereigns. (Christine Lagarde, April 17, 2012)

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Following the Greek sovereign debt write-down in 2011, the four largest Greek banks made losses of more than 28 billion euros (or 13% of GDP).¹ This was enough to wipe out almost all of their combined equity capital. In 2010, the Irish government ran an unprecedented peace-time deficit, reaching 32% of GDP as it bailed out its banking system. Under the weight of nationalized banks' losses, Ireland was forced to seek financial support from the IMF and the EU in November 2010.

These are two recent examples of a 'diabolic loop' between banks and sovereigns. In the case of Greece, banks that were otherwise solvent, were made insolvent by the default of their sovereign whose debt they were holding.² In the case of Ireland, a government which had previously had one of the lowest levels of debt to GDP in Europe, suffered a withdrawal of funding as markets became concerned about the contingent liabilities involved in bailing out its large, insolvent banking system. Throughout the rest of southern Europe, this 'diabolic loop' has operated in a less dramatic fashion but has nevertheless contributed to ongoing strains in sovereign and bank debt markets.

In this paper we build a model of the channels that transmit fragility between debt and banking markets and evaluate policy measures for the stabilization of these markets. The framework combines the canonical model of sovereign debt fragility (Calvo (1988)) with the canonical model of banking instability (Diamond and Dybvig (1983)). Put differently, the framework studies the interaction of strategic complementarities in debt and financial markets.

Sovereign debt fragility arises due to a strategic complementarity between the buyers of government bonds as in Calvo (1988). Since the government's ability to repay debt depends inversely on the real interest rate it has to pay, this opens up the possibility of self-fulfilling pessimistic equilibria in which the high interest rate needed to compensate bond holders for high expected default risk weakens the government's solvency and validates the pessimistic default expectations. Banks face liquidity and solvency risks as they provide liquidity insurance to their depositors while holding risky assets such as government debt. The collapse of intermediation (either because of 'runs' or fundamental shocks to solvency) then leads to large output and welfare costs to the real economy.

The key contribution of our paper is to examine the interactions between these two sources of financial fragility. Motivated from the European experience, we consider two channels whose interactions complete the 'diabolic loop'.

The first is the strong tendency by banks to hold (their own) government debt both as a long-term investment and as a source of liquidity. Table 1 shows data on European banks' government debt holdings which was released as part of the EBA stress test conducted in 2011.³ The table focuses on the so-called 'peripheral Eurozone countries' whose debt had come under pressure during the

¹National Bank of Greece, Alpha Bank, Pireus and Eurobank.

²The term 'diabolic loop' was evidently coined by Markus Brunnermeier in a presentation on the Euro Crisis at the July 2012 NBER Summer Institute.

³European Banking Authority.

sovereign debt crisis which began in 2010.

Two things are immediately apparent from the table. First of all, the exposure of southern European banks to EEA sovereign debt is very high.⁴ The average GIIPS, (Greece, Ireland, Italy, Portugal and Spain), bank holds 15.8% of risk-weighted assets in EEA government securities. The second fact highlighted in the table is that banks are heavily invested in the debt of their own government.

Table 1: European Holding of Sovereign Debt

	All GIIPS	Greece	Spain	Ireland	Italy	Portugal
EEA30 government debt	15.8%	36.2%	11.8%	10.6%	17.5%	11.7%
of which domestic government debt	14.5%	25.9%	11.2%	6.8%	15.5%	8.9%

European banks' holdings of EEA and domestic government debt as a percentage of total risk weighted assets. Source: 2011 EBA Stress Test.

The second channel arises due to the explicit (via deposit insurance) or implicit guarantees that governments provide to their banking systems. One of the contributions of the paper is to provide conditions for governments to provide guarantees.

The interactions of these two elements create an economic environment where pessimism in debt markets is transmitted to the banking system and amplified due to powerful feedback effects on the financial health of the sovereign. When the government debt market switches to a pessimistic (high interest rate, high default risk) equilibrium, government bond prices fall and the banks holding the bonds suffer losses. At this point (due to the high output costs of bank defaults), governments are forced to intervene and bail their banks out, further increasing government debt at precisely the point when high interest rates are making repayment difficult. The result is a further decline in government debt prices, leaving a deeper hole in bank balance sheets and requiring a larger bailout. This is the 'diabolic loop' between government debt and the banking system.

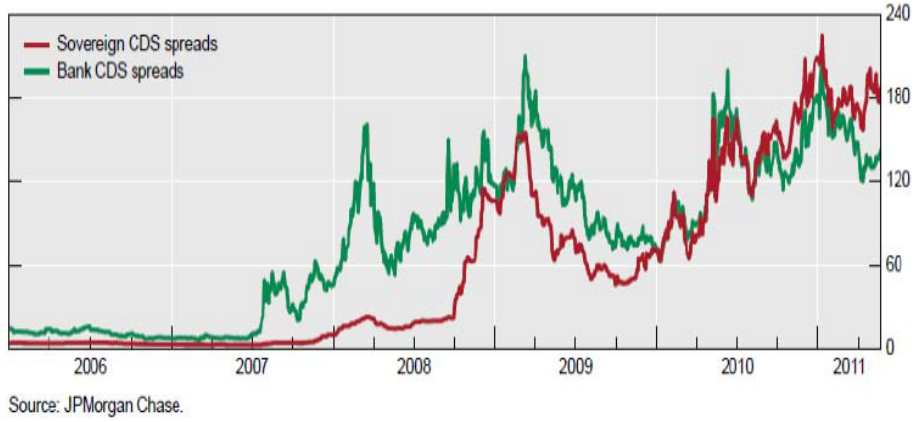
There is ample evidence for the interaction we study in the paper. Figure 1 (reprinted from Hannoun (2011)) shows that European sovereign and bank CDS prices have co-moved very closely since the crisis began. The figure clearly demonstrates that when government and bank balance sheets become closely intertwined, their default probabilities becomes highly correlated too.

In this paper we study the policy options of the crisis-prone country in isolation of other members of a currency or economic union.⁵ We consider two ways policymakers and private agents (in

⁴European Economic Area. This includes the 27 EU countries as well as Norway and Switzerland.

⁵This is the subject of independent related work by Uhlig (2013) who also appeals to moral hazard in order to

Figure 1: Credit Default Swaps



Christine Lagarde’s words) ‘break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks’.

On the banking side, equity cushions can break the adverse feedbacks between banks and sovereigns.⁶ Banks that hold adequate capital against potential sovereign risks become completely insulated from developments in debt markets, severing a key channel of crisis transmission from governments to the banking system. However, when banks expect deposit insurance to be provided *ex post*, the incentive for them to self-insure by building up equity buffers against losses disappears.

On the sovereign side, we examine a key policy which affects the ‘diabolic loop’ - the *ex post* choice of whether to provide bailout assistance to the banking system during a crisis. We argue that in the likely case when the collapse of the financial system is very costly for the real economy, governments always provide DI *ex post*, thus removing the need for banks to self-insure *ex ante* by issuing equity.

The paper is structured as follows. Section 2 outlines the baseline model, section 3 describes the optimistic equilibrium while section 4 describes the pessimistic equilibrium and the sovereign-banking feedbacks that it triggers. Section 5 examines the government’s decision on whether to provide deposit insurance *ex post*. Section 6 examines the way equity buffers can achieve the first best and discusses banks’ incentives for equity issuance. Section 7 synthesizes the results

explain banks’ tendency to hold large quantities of government debt. The main reason for moral hazard in Uhlig (2013) is inadequate collateral haircuts imposed by the central bank which allow weak country banks to profitably default on the central bank in bad states of the world.

⁶Acharya, Drechsler, and Schnabl (2011) also models the balance sheet linkages between banks and sovereigns but do not consider how anticipated bailouts affect banks’ incentives to hold government bonds and/or issue equity to guard against sovereign exposures

by characterizing the Nash equilibria of the interactions between the banks and the government. Section 8 concludes.

2 Framework

Time lasts for three periods: 0, 1 and 2. The model has two principal components. The first is a banking relationship between intermediaries and depositors, following Diamond and Dybvig (1983). The second component is the pricing of government debt, following Calvo (1988) and others.⁷ The debt is partially held by banks.

The intermediation process and pricing of government debt are linked in a couple of ways. First, the value of the government debt held by the banks affects their solvency. Second, the potential and realized needs to bailout the financial sector influences the value of government debt. Third, banking problems affect the real economy and impact on the size of the tax base, thus adding a further valuation effect on government debt. These interactions can be activated by either fundamental shocks or self-fulfilling expectations influencing the value of government debt.⁸

There are four types of agents: households, banks, investors and the government. We discuss the choices and objectives of these agents and then characterize the equilibria.

2.1 Households

Households are of size 1. They have an endowment of goods d at $t = 0$ with preferences

$$V_0^H = \pi u(c_1 + \beta c_2) + (1 - \pi) u(\beta c_1 + c_2).$$

Here β is close to 0. With probability π they are *early* consumers who prefer consuming at $t = 1$ and with probability $1 - \pi$ they are *late* consumers who prefer consuming at $t = 2$. The shares of early consumers at the aggregate level is fixed at π . We assume $u(\cdot)$ is strictly increasing, strictly concave and $u(0)$ is finite.

⁷There are now a number of papers building on Calvo (1988), including Cole and Kehoe (2000) and, more recently, Corsetti and Dedola (2012), Roch and Uhlig (2012), Cooper (2012) and Farhi and Tirole (2014).

⁸In a recent paper, Farhi and Tirole (2014) use a similar framework to look at the effects of fundamental shocks. In contrast to our model, all agents are risk neutral so that the insurance through bank contracts is not central to their analysis.

In addition, Farhi and Tirole (2014) do not consider banks' equity issuance decisions on the 'diabolic loop'. This is very important for our paper because, as we will show subsequently, significant investments in government bonds are not a problem *per se* as long as banks hold significant capital buffers against sovereign exposures.

2.2 Banks

Following Diamond and Dybvig (1983), consumers can share liquidity risk through the banking system. Banks construct a portfolio for households which provides the needed liquidity while still taking advantage of longer term investment opportunities. As is well understood, it is this interaction of liquidity needs and illiquid investment that can lead to fragility in the banking system.

Banks are competitive. They raise deposits d from households in period 0. Banks invest in two types of assets in period 0. They can buy government bonds b_0 at price q_0 . These bonds do not pay a coupon at the middle date but can be traded in the secondary market. Second, banks can make long term investments i_0 that return $R > 1$. These investments have a liquidation value at the middle date of $0 \leq \varepsilon \leq 1$.

Ex ante banks offer contracts to consumers. The contract specifies the level of early, denoted c^E , and late consumption, denoted c^L . Decisions on holding of government securities (b_0) and long term investment (i_0) are also made *ex ante*. Banks can adjust their portfolios in the middle period. The optimal contract solves:

$$\max_{c^E, c^L, b_0, i_0, b_1, l_1, L_1} \pi u(c^E) + (1 - \pi) u(c^L) \quad (1)$$

such that

$$i_0 + q_0 b_0 \leq d \quad (2)$$

$$\pi c^E \leq q_1 (b_0 - b_1) + \varepsilon l_1 - L_1 \quad (3)$$

$$(1 - \pi) c^L \leq b_1 + R(i_0 - l_1) + r^b L_1. \quad (4)$$

From (3), the funding for the payment to the early households comes from three sources. First, the bank can sell some of the government debt it acquired in period 0 to the investors to obtain goods for early consumers. Second, the bank could liquidate some of the illiquid investment, denoted l_1 in (3). The liquidation of the illiquid technology is equivalent to having access to a storage technology with a return of ε between period 0 and 1. Finally, the bank could extend loans to investors or other banks, denoted L_1 in (3), at a rate r^b . We refer to this as a loan in the interbank market.

From (4), the consumption of late households is financed by the bonds held until the last period as well as the return on the illiquid investment that was not liquidated in the middle period. Further, the bank has the returns to investor loans made at the middle date.

2.3 Investors

Investors are risk neutral agents (of size 1) with endowments in periods t of A_t for $t = 0, 1, 2$. They consume in periods 1 and 2 with preferences given by $c_1 + \frac{c_2}{R}$. The assumption that investors

discount at $\frac{1}{R}$ will determine the asset returns in equilibrium. These agents invest their endowments in government debt and lend to/borrow from banks (L_1^I).⁹ They can also directly invest in the illiquid technology.

In the first period, investors allocate their endowment to the purchase of government debt, and illiquid investments, respectively:

$$A_0 = q_0 b_0^I + i_0^I. \quad (5)$$

The budget constraint in period 1 is:

$$c_1^I = A_1 + q_1(b_0^I - b_1^I) + L_1^I \quad (6)$$

as the investor can purchase government debt of b_1^I and borrow from banks. The budget constraint in period 2 is:

$$c_2^I = (1 - \tau) A_2 + b_1^I + R i_0^I - r^b L_1^I \quad (7)$$

where τ is the tax rate on investor's endowment. In period 2, the endowment of the investor is augmented by the returns to bond holdings and the long term investments minus the repayments on bank loans.

The investors' endowment at the final date, A_2 , serves as the tax base for debt service. Its value depends on the operations of the intermediation process as well as the default choice of the government.

The dependence of A_2 on the intermediation process captures the disruptive effects of a breakdown in the financial system. For one specific model of this, see Gennaioli, Martin, and Rossi (2013). In our model, the disruption has the effect of reducing the endowment of the investors and hence the tax base. The output loss is parameterized by ψ in (8) where $\mathbb{1}\{B\} = 1$ if the intermediation process breaks down.

In addition, following Eaton and Gersowitz (1981) government default leads to output costs. This is reflected in the reduction in the $(1 - \gamma \mathbb{1}\{G\})$ term in the investors' endowment where $\mathbb{1}\{G\} = 1$ if the government defaults.

Specifically, the investor's endowment in the last period is given by:

$$A_2 = \bar{A}(1 - \psi \mathbb{1}\{B\})(1 - \gamma \mathbb{1}\{G\}). \quad (8)$$

The parameters (ψ, γ) will be important for determining the default choice as well as the government's decisions on protecting the intermediation process.

⁹We introduce equity later in the analysis.

2.4 The Government

The government issues debt B_0 at price q_0 in period 0 to fund government expenditure G_0 . This is two-period debt with repayment due in period 2. At the middle date, it may issue additional debt to support the intermediaries in period 1 and to finance period 1 government expenditure G_1 . As the analysis proceeds, the debt issued in period 1 will be tied to the price of debt and will create the basis for multiple equilibria. At the end of period 1, the debt outstanding is B_1 .

The government taxes investors' endowments A_2 at the final date. The tax rate required to meet the total obligations of the government is equal to

$$\tau = \frac{B_1}{A_2}.$$

By taxing investors' endowments, the government taxation does not directly impact the intermediation process. Any frictions that impinge on the deposit contract, such as sequential service, are irrelevant for the government's ability to collect taxes. However, the tax base does depend on the functioning of the intermediation process, as in (8).

To introduce the possibility of default into the analysis, assume the government's capacity to tax the endowment of the investors is random and drawn from a known probability distribution $F(\bar{\tau})$ with associated density $f(\bar{\tau})$.¹⁰ The uncertainty about tax capacity, denoted $\bar{\tau}$, is realized at the final date. This naturally leads to the possibility of default due to bad fundamentals (as opposed to strategic default): a low realization of $\bar{\tau}$ could trigger government insolvency despite a large tax base (A_2).

If $\bar{\tau} < \frac{B_1}{A_2}$, the government must default on its obligations where $A_2 = \bar{A}(1 - \psi \mathbb{1}\{B\})$. The probability of default is therefore equal to $F\left(\frac{B_1}{A_2}\right)$ while the probability of repayment is given by $1 - F\left(\frac{B_1}{A_2}\right)$. Once the government is forced to default, it defaults fully.

The debt is priced by risk neutral investors who discount the future at rate $\frac{1}{R}$. To avoid arbitrage, the price q_1 must satisfy

$$\frac{1 - F\left(\frac{B_1}{A(1 - \psi \mathbb{1}\{B\})}\right)}{R} = q_1. \quad (9)$$

3 Optimistic Equilibrium

A rational expectations equilibrium has two components. One is the optimal contract between a bank and its depositors. The second is the valuation of the government debt. These components

¹⁰Alternatively, A_2 could be random.

are linked, in part, because the contractants in the banking system hold government debt and take asset prices as given.

This section characterizes a particular equilibrium of the model in which there is no default. To do so, we make two provisional assumptions to develop the framework. Both of these assumptions are subsequently relaxed.

First, assume there exists a solution to (9) in which there is no default. That is, if lenders believe that the default probability is zero, the debt burden of the government will always be below its minimal tax capacity. Hence we call this the ‘optimistic equilibrium’. In an optimistic equilibrium, there is no default so that $A_2 = \bar{A}$ and $F(\frac{B_1}{A}) = 0$.

Second, the banking contract and lending behavior assumes that the optimistic equilibrium will occur with certainty in period 1. That is, neither lenders nor those party to the banking contract recognize any underlying strategic uncertainty.

Given these assumptions, there exists an equilibrium in which $q_0 = q_1 = \frac{1}{R}$ and $r^b = R$. Markets clear at these prices, given the solution of the banking problem.

3.1 Optimal Contract

Given debt prices q_0 and q_1 , the optimal contract for the banking system solves (1) subject to the constraints as described in section 2.2. This problem generates a demand for government debt by the banking system. Under the assumption of optimism, neither the banks nor the depositors anticipate variations in the price of government debt.

The banks hold a portfolio of government debt and long-term illiquid investment. They provide for the consumption of early households by selling government debt to investors in period 1. When the liquidation value of the illiquid investment, ε , is less than one, trading government debt strictly dominates liquidating the long-term investment. At $\varepsilon = 1$, the bank is indifferent between liquidation and the selling of government debt and we assume there is no liquidation in this case either.

Proposition 1. *In the optimal banking contract with $q_0 = q_1 = \frac{1}{R}$: (i) $c^L > c^E$ and (ii) $l_1 = 0$.*

Proof. The first order conditions to the contracting problem in (1) are:

$$u'(c^E) - \lambda^E = 0 \tag{10}$$

$$u'(c^L) - \lambda^L = 0 \tag{11}$$

$$q_0\phi = q_1\lambda^E \tag{12}$$

$$\phi = R\lambda^L \tag{13}$$

$$(\varepsilon\lambda^E - R\lambda^L)l_1 = 0. \quad (14)$$

Combining the first four conditions and using $q_0 = q_1 = \frac{1}{R}$:

$$u'(c^E) - Ru'(c^L) = 0. \quad (15)$$

This condition implies property (i): $c^L > c^E$ for all $R > 1$ as $u(\cdot)$ is strictly concave. Using (14), l_1 is zero, and strictly so if $\varepsilon < 1$, as $\lambda^E = R\lambda^L$ from (12) and (13). ■

In the subsequent discussion, let (c^{*E}, c^{*L}) denote the optimal contract characterized in Proposition 1. We will refer to this as the first best contract. The property that $c^{*L} > c^{*E}$ implies that depositors have an incentive to reveal their true taste types.¹¹

From (1), there are other elements of the bank's problem to determine: (b_0, i_0, b_1, L_1) . To implement the optimal contract, it is sufficient that $(b_0 = \frac{\pi c^{*E}}{q_1}, i_0 = \frac{(1-\pi)c^{*L}}{R})$ and $(b_1 = L_1 = 0)$. That is, trades in period 1 are not needed in the case of optimism, though they can be important under pessimism.

3.2 Equilibrium

Given the banking contract, the last step in constructing an equilibrium is to guarantee market clearing. There are three markets to consider: (i) the period 0 market for government debt, (ii) the period 1 market for government debt and (iii) the interbank loan market.

Proposition 2. *There exists an optimistic rational expectations equilibrium with $q_0^* = q_1^* = \frac{1}{R}$, $r^b = R$ and the banking contract given by (c^{*E}, c^{*L}) .*

Proof. The equilibrium conditions are driven by the investors. We assume that the aggregate endowment of the investors is larger than the stock of government debt in period 0. The investors can either put their endowment directly in the illiquid technology and obtain R or purchase two period government debt. They are indifferent between these options if $q_0 = \frac{1}{R}$. If this condition holds, they are willing to purchase any of the government debt not held by the banking system. Since investors have linear utility of $c_1 + \frac{1}{R}c_2$, investors are indifferent between consuming their period 1 endowment and buying one period government debt if $q_1 = \frac{1}{R}$. Assuming that investors period 1 endowment is sufficiently large, if $q_1 = \frac{1}{R}$, the investors will purchase the debt sold by the banks in period 1 and the new debt issued by the government in period 1.

Thus, at these prices, all markets clear. The excess supply of government debt in period 0 is purchased by the investors. The stock of government debt held by bank is sold to the investors

¹¹As is well understood, there may also exist a bank runs equilibrium in this environment. That is not the focus of this analysis and is left aside to focus on crises emanating from uncertainty over government debt repayment.

along with any new debt in period 1. The market for government debt clears in both periods. Given that $q_0^* = q_1^* = \frac{1}{R}$, the probability the government will default is zero.

Further, at $r^b = R$, investors are indifferent both with respect to the timing of their consumption and the composition of their portfolio. This indifference guarantees market clearing in the interbank market at zero trades.

The fact that the first best contract is provided in equilibrium comes from Proposition 1. In equilibrium, banks will hold enough debt to finance their payment to early consumers at the anticipated period 1 price: $b_0 q_1 = \pi c^{*E}$. The debt is sold to the investors for goods and those goods are transferred to the early consumers. There are no liquidations in an optimistic equilibrium. ■

In the discussion that follows, we refer to the allocation characterized by Proposition 2 as the **first best allocation**. At this allocation, a government with the ability to redistribute the endowments of households and investors in period 0 would have no incentive to do so. In this way, this allocation will serve as a benchmark for *ex ante* comparisons of other allocations.

4 Pessimism

Depending on government policies, the optimistic equilibrium may not be the only outcome. This comes from other equilibrium valuations of government debt.

This section outlines the impact of an unexpected drop in confidence in the value of government debt. The discussion highlights the adverse spillover effects from strategic uncertainty regarding government debt on the banking system. To the extent that households and banks ignored the prospect of the pessimism, the effects identified here may be excessive. The next section considers various remedies to the banking contract once the possibility of pessimism is recognized.

4.1 Debt Fragility

To see how other equilibria might arise, suppose that the debt outstanding at the end of period 1 depends on the price of government debt, i.e. $B_1(q_1)$. In that case, (9) is replaced by

$$\frac{1 - F\left(\frac{B_1(q_1)}{A(1-\psi\mathbb{1}\{B\})}\right)}{R} = q_1. \quad (16)$$

Looking at the government sector alone, any value of q_1 that satisfies (16) is part of a rational expectations equilibrium.

If $B_1(q_1)$ is decreasing in q_1 , then the right side is increasing in q_1 . As q_1 increases, the amount of debt outstanding decreases and the probability of repayment increases with q_1 . As we shall see,

in this case, government debt is fragile in the sense that it is subject to multiple equilibria. That is, there may be multiple solutions to (16).

First we look at conditions under which $B_1(q_1)$ is decreasing in q_1 . Then we look at the consequences of this dependence.

4.1.1 Government Bailout

A leading reason for $B_1(q_1)$ to be increasing in q_1 comes from government support of the banking system, either through deposit insurance (DI) or debt repurchases. In this section we assume that these *ex post* interventions do take place and study their implications for the nature of the sovereign-banking interactions. We subsequently derive conditions under which it is optimal for the government to provide DI *ex post*.

Suppose that the government provides deposit insurance so that banks are able to meet the liquidity needs of all households claiming to be early consumers without liquidating any of the illiquid investment. The government finances deposit insurance by selling more debt to the investors.

If the banking contract offers each early consumer c^{*E} , then the maximal amount of deposit insurance the government provides is

$$DI = \pi c^{*E} - q_1 B_0^B. \quad (17)$$

Here $q_1 < q_1^*$ is an arbitrary value of government debt in period 1 and B_0^B is the quantity of debt held by a representative bank.¹²

An alternative way to support the banking system is through debt buybacks. In the event of pessimism, the government purchases its own bonds from banks at the optimistic price q_1^* . This insulates the assets of banks from variations in the value of debt, leaving them with adequate resources to meet the demands of early households.

To do so, the government must sell additional debt equal to $DB_1 = (q_1^* - q_1)B_0^B$. As with deposit insurance, this additional debt burden will have an influence on the period 1 price of debt and thus the amount of debt buybacks required. This is determined, as characterized below, through the equilibrium price of debt.

In our environment, the provision of DI and a debt buy-back program are equivalent ways to support depositors. That is, either depositors are supported by direct transfers supported by new debt through the DI system, or through support to the banks via debt financed purchases of bank held debt, with the proceeds transferred to depositors. In environments where there are other

¹²In equilibrium the number of banks is not determined. For convenience, we normalize the number of banks to unity and therefore B_0^B is also the aggregate quantity of debt held in the banking system.

claimants on the banks, such as equity holders, these programs may not be equivalent. Or, if the provision of DI required costly liquidations, then the policies would also differ. Since our model has neither of these features, as the discussion progresses, we use the two policies interchangeably.

Thus, under these policies, the transfers made to the banks would be given by $T(q_1) = \pi c^{*E} - q_1 B_0^B = (q_1^* - q_1) B_0^B$. Note that $T(q_1)$ is decreasing in q_1 . If this transfer is debt financed, then the government would need to issue $B_1(q_1) = \frac{T(q_1)}{q_1}$ units of new debt to support the banking system.

With government support of the banking system, $B_1(q_1)$ will be decreasing. In this case, there may be multiple solutions to the valuation of government debt given in (16).

4.1.2 Financing of Government Spending

Suppose the government is committed to spending $G_1 > 0$ in period 1. It is willing to sell whatever new debt is necessary to finance this level of real spending. Thus, the new debt issued will equal $\frac{G_1}{q_1}$. Thus the debt outstanding at the end of period 1 would be $B_1(q_1) = B_0 + \frac{G_1}{q_1}$ which is clearly decreasing in q_1 .

4.1.3 Multiple Valuations of Government Debt

Given these arguments for $B_1(q_1)$, to be decreasing, the potential for multiple solutions to the debt valuation equation, (16). Figure 2 illustrates the equilibria.¹³ The function $[1 - F(\frac{B_1(q_1)}{A(1-\psi_1\{B\})})]/R$ is the ‘debt valuation equation’ because it determines the price of government debt (as a function of itself). It is depicted as the black dashed curve. The points of intersection of this curve and the 45-degree line are solutions to (16).

There is a point labeled ‘optimism’ in Figure 2 where the default probability is zero so that $q_1 = \frac{1}{R}$. That is, $F\left(\frac{B_1(q_1)}{A_2}\right) = 0$. This corresponds to the valuation of government debt in the optimistic equilibrium characterized in Proposition 2. In an optimistic equilibrium, $T(q_1) = 0$ implying $B_1 = B_0 + RG_1$.

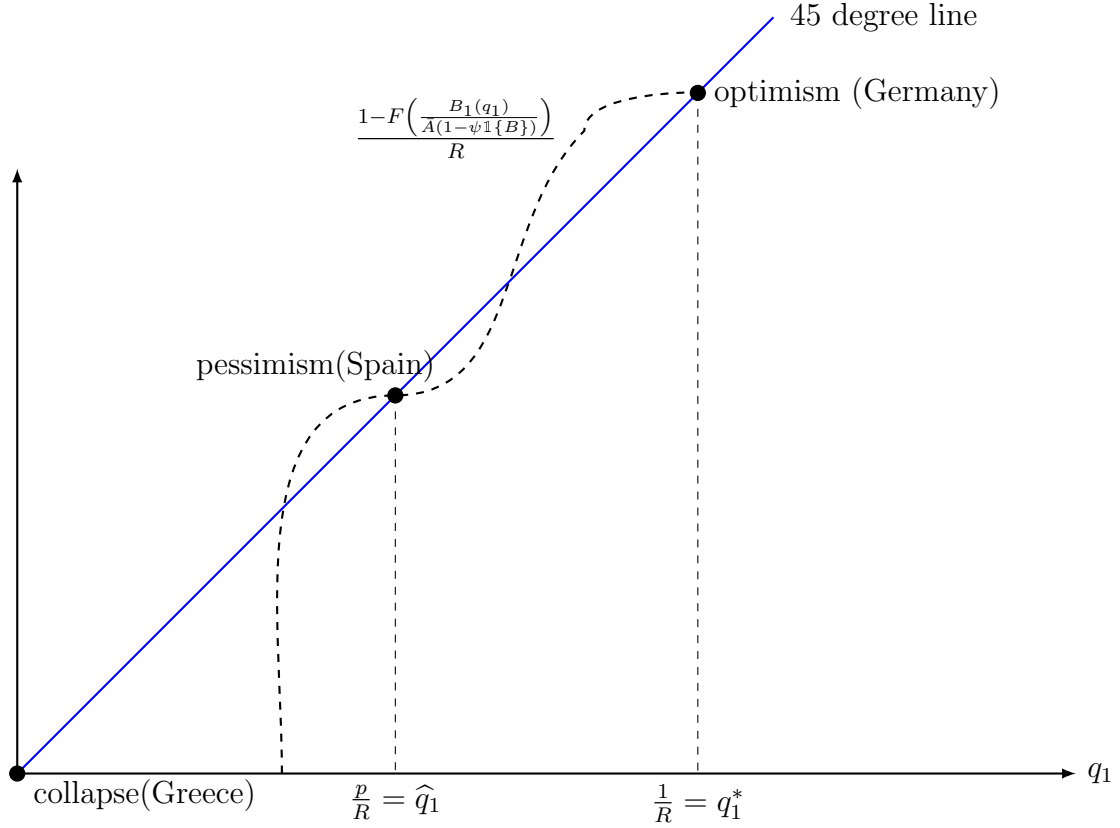
In addition, there are other equilibria in which the value of debt, q_1 , is lower. These are labeled

¹³The slope of the debt valuation equation is given by $-f\left(\frac{B_1}{A_2}\right) \frac{B'_1(q_1)}{A_2}$ where $f(\cdot)$ is the density associated with the distribution function $F(\cdot)$. This expression is zero at high levels of q_1 when government debt is very far from the default point. This is the case when the density of the tax capacity random variable $f\left(\frac{B_1}{A_2}\right) = 0$ when the stock of government debt is at its value in the optimistic equilibrium $B_1 = B_0 + RG_1$.

The curve crosses the x-axis at the point at which the price of government debt becomes so low that the government is insolvent with probability 1. The location of this intersection point depends on the support of the distribution of the tax capacity shock.

At the ‘pessimism’ point, the debt valuation curve is assumed to have a slope less than unity implying that the pessimistic equilibrium would be stable under a dynamic adjustment of private beliefs.

Figure 2: Fiscal Fragility



‘pessimism’ and ‘collapse’ in the figure. The resulting higher debt obligation in period 2 generates a positive probability of default and thus lower values of q_1 .

The multiplicity stems from the financing of a bailout and/or government spending. In what follows, it will be useful to distinguish the case in which $G_1 > 0$ alone is sufficient for multiple solutions to the debt valuation equation.

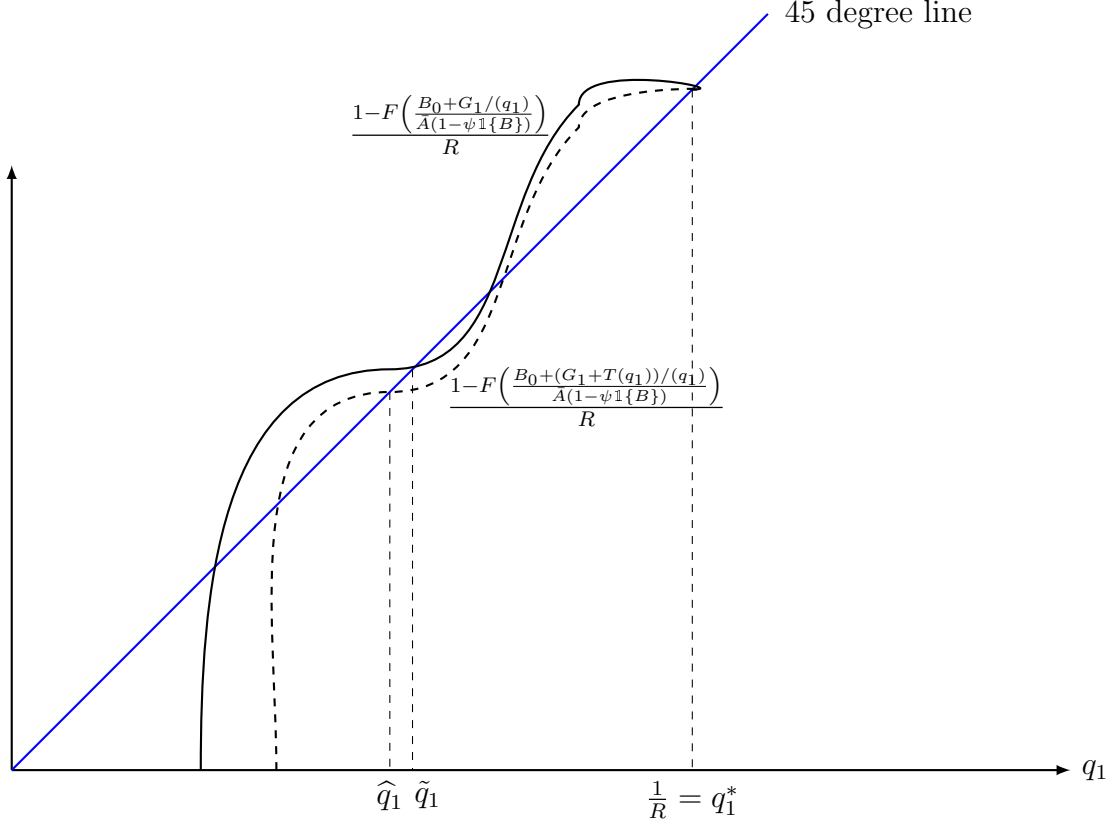
Assumption 1. *There are multiple solutions to (16) when $B_1(q_1) = B_0 + \frac{G_1}{q_1}$.*

Assumption 1 guarantees that strategic uncertainty in the debt market exists even in the absence of bailouts similarly to the Calvo (1988) model. The inclusion of bailouts provides a powerful amplification mechanism of the effects of strategic uncertainty in the government debt market by adding a further negative dependence of bond issuance on the debt price.

This is illustrated in Figure 3. There the solid curve assumes no bailouts while the dashed one allows $T(q_1) > 0$. By Assumption 1, there are multiple crossings of the solid and the 45 degree lines. Debt prices in locally stable pessimistic equilibria are shown as \hat{q}_1 and \tilde{q}_1 . Due to local stability, the pessimistic debt price without a bailout is higher than that with $T(q_1) > 0$.

Once $B_1(q_1)$ is decreasing in q_1 , it is straightforward to construct multiple solutions of (16) since there is flexibility in the choice of the distributions of tax capacities, $F(\cdot)$. The key to the multiplicity, both with and without $T(q_1)$ decreasing in q_1 , is fact that both sides of (16) are increasing in q_1 .

Figure 3: Fiscal Fragility: The Impact of $T(q_1)$



The solid curve displays the case in which $T(q_1) \equiv 0$. The dashed curve allows $T(q_1) > 0$.

4.2 Crisis

To see the impact of pessimism, assume that at $t = 0$ the equilibrium probability of repayment is considered to be unity. That is, the strategic uncertainty, i.e. the potential sunspot, influencing government debt is not considered by private agents in the pricing of government debt and the operation of the intermediaries.

Then, at the middle date, an unanticipated debt sunspot shock realizes, increasing the probability of government default. Referring to Figure 2, imagine the shift in beliefs puts the government

debt market at the point labeled ‘pessimism’. The price of government debt satisfies:

$$\frac{p}{\widehat{q}_1} = R \quad (18)$$

where ‘hat’ variables denote realizations conditional on being in a pessimistic equilibrium and $p < 1$ is the repayment probability.¹⁴ Since $p < 1$ under pessimism, $\widehat{q}_1 < q_1^*$.

While the strategic uncertainty originates in the government debt market, it affects financial markets. The debt fragility leads to a collapse in the banking system: with the reduced value of government debt, banks are unable to meet their obligations to early and late households.

Proposition 3. *If $\varepsilon < 1$, then the pessimistic debt sunspot triggers bank insolvency and hence the collapse of the banking system.*

Proof. Consider a bank at the middle date. In the optimistic equilibrium, it was selling debt b_0 at price q_1^* to meet the needs to early consumers: $b_0 q_1^* = \pi c^{*E}$. The consumption of late consumers was financed by the return on the illiquid technology: $i_0 R = (1 - \pi) c^{*L}$. With $\widehat{q}_1 < q_1^*$, the bank is unable to meet needs of early households out of its debt holdings. Either it must default on its obligations to the early households or it must liquidate enough of the illiquid investment so that $b_0 \widehat{q}_1 + \varepsilon l_1 = \pi c^{*E}$. As $\varepsilon < 1$, the bank will be unable to meet the demands of both early and late households regardless of whether the government defaults or not. Consequently, the bank is insolvent.

Depositors at the bank realize that the fall in debt prices has made the bank insolvent. This leads to a run on the bank. The bank liquidates its illiquid investment and uses the resources to meet a fraction of the withdrawals of c^{*E} . ■

If $\varepsilon = 1$ and *ex post* the government does not default on its debt, then the bank will have enough resources to pay late households. That is, if the bank does not sell any of its debt in period 1, then $l_1 = \pi c^{*E}$ implying that the late households will get $R(i_0 - l_1) + b_0 = R(i_0 - \pi c^{*E}) + b_0$ which does equal Ri_0 and thus $(d - \pi c^{*E})$ if the government does not default.

The collapse of the banking system in the pessimistic equilibrium is in stark contrast to its smooth functioning in the optimistic equilibrium. The unanticipated wealth shock which hits banks due to the switch to the pessimistic equilibrium renders the banking system insolvent.

Note that bank solvency is directly affected by movements in the price of government debt on banks’ balance sheets; actual sovereign default is not needed. In our framework, banks rely on government debt sales to fund the withdrawals of the early consumers and consequently a decline in government debt values at the middle date (q_1) leaves them both illiquid and insolvent. This will

¹⁴At this point, which locally stable pessimistic equilibrium is selected is not important.

be enough to trigger implicit or explicit deposit insurance guarantees. Acharya and Steffen (2013) provide evidence that European banks were affected by fluctuations in government debt prices even in the absence of formal default. The paper shows that banks' equity returns positively correlate with returns on peripheral European sovereign bonds (Greece, Portugal, Italy, Ireland and Spain). The size of the correlation is increasing in an individual bank's exposure to peripheral sovereign debt. This is direct evidence that markets saw the value of sovereign bonds as an important determinant of bank valuations and bank solvency.

4.3 Is Pessimism Endemic?

The analysis supports the presence of a 'diabolic loop' between the financial and fiscal sectors. To be clear, in our model, multiple equilibria arise solely from the government sector, through the financing of government purchases and bailouts of intermediaries. A key feature of the model is that the amount of debt issued in period 1 varies inversely with the value of the debt.

Key elements for the multiplicity are the presence of standard deposit contracts and bank holdings of government debt. These features are clearly consistent with evidence. As documented earlier, banks hold a substantial amount of government debt. Further, government actions are consistent with the assumed *ex post* public guarantee of the banking system.

Using our model, we can explore whether these key features are robust in a number of directions. First, does the government actually have an incentive to provide Deposit Insurance to banks? Section 5 studies this question and argues that this support will generally be forthcoming.

Second, the banking contract is simplistic as it ignores the possibility of equity issuance by the bank to potentially buffer against fluctuations in the value of government debt. This makes the banks particularly susceptible to shocks. Section 6 enhances the bank contract to include these elements. We then find conditions under which the diabolic loop remains.

5 Public Response: Provision of Deposit Insurance

In this section we examine the problem of a government which is faced with a pessimism shock to the value of its debt and decides whether to bail out its banks through the provision of DI. The analysis is undertaken given the contract and portfolio decisions of a bank in an optimistic equilibrium.

The bail-out decision entails elements of redistribution between investors and depositors, as in Cooper and Kempf (2013). Moreover, a bail-out avoids costly liquidations, as in Acharya and Yorulmazer (2008) and thus disruptions of the intermediation process possibly at the cost of an

increased probability of default. Our analysis will focus primarily on the second trade-off.

The analysis starts with a shock to the beliefs of investors. This leads, as in the analysis of pessimism, to a lower price of government debt. This would be a solution to (16) with $q_1 < q_1^*$.¹⁵ The question we address is: will the government have an incentive to provide deposit insurance (buy back debt) to support the banking system?

The Appendix derives the payoffs of deposits and investors with and without the provision of deposit insurance. Here we use that structure to write these payoffs assuming that banks provide the first-best contract to depositors, characterized in Proposition 1.

In the event of a pessimistic sunspot, if deposit insurance is not provided, then, as seen in Proposition 3, the bank is insolvent and will close. Some depositors will be served c^{*E} and the remainder have zero consumption. The bank's holdings of government debt are sold to the investors, implying that they hold the entire stock of government debt.

If deposit insurance is provided, then the bank continues to operate. It finances payments to early consumers in period 1 through the sale of government debt to investors, assisted by the government's support. Again investors hold the entire stock of government debt at the end of period 1. The transfer to support the deposit insurance is equivalent to the banks selling bonds to investors at the optimistic price q_1^* . In contrast, when no DI is provided the banks sell bonds to investors at q_1^{NI} . Hence the net cost to investors of DI provision is equal to $(q_1^* - q_1^{NI}) B_0^B$ where B_0^B is the holding of government debt by a representative bank.¹⁶

In this case, the difference in social welfare function between providing DI and not is:

$$W^{DI} - W^{NI} = \pi (1 - k_1) [u(c^{*E}) - u(0)] + (1 - \pi) [u(c^{*L}) - (k_1 u(\beta c^{*E}) + (1 - k_1) u(0))] \quad (19) \\ - \omega (q_1^* - q_1^{NI}) B_0^B + \frac{\omega}{R} [(p^{DI} - p^{NI}) \gamma + \psi] \bar{A}$$

where ω is a welfare weight.¹⁷

There are three terms in (19). The first is the gain by depositors from DI. This is unambiguously positive because $u(c^{*E}) > u(0)$ and $u(c^{*L}) > (k_1 u(\beta c^{*E}) + (1 - k_1) u(0))$. The second term, $-\omega (q_1^* - q_1^{NI}) B_0^B$, is the loss for investors due to higher taxation under DI. The third term is the gain for investors as there are no bank default costs if DI is provided and a (possible) loss from higher expected default costs.¹⁸

To separate these influences, let ω^* satisfy:

¹⁵Exactly which equilibrium obtains is not important as long as there is movement away from the optimistic equilibrium.

¹⁶This is worked out in detail in the Appendix.

¹⁷This welfare weight pertains to payoffs in periods 1 and 2. It may differ from that in period 0, reflecting the crisis in period 1 and thus various pressures on the government to intervene.

¹⁸The sign of $(p^{DI} - p^{NI})$ is not determined *a priori*, as discussed further below.

$$\pi(1 - k_1) [u(c^{*E}) - u(0)] + (1 - \pi) [u(c^{*L}) - (k_1 u(\beta c^{*E}) + (1 - k_1) u(0))] = \omega^* (q_1^* - q_1^{NI}) B_0^B. \quad (20)$$

In this case, the welfare effects of the insurance gains and tax costs just balance.

So leave aside this part, the focus of Cooper and Kempf (2013), to study the effects of DI on economic activity and thus the tax base, i.e. the last term of (19) which is proportional to:

$$(p^{DI} - p^{NI}) \gamma + \psi. \quad (21)$$

The first term, $(p^{DI} - p^{NI}) \gamma$, is the difference in the expected output costs of default due to the provision of DI relative to the situation where DI is not provided. Note that the sign of this term is not determined. That is, the provision of deposit insurance may increase the probability of sovereign default or it may reduce it.

The probabilities that the government repays the debt were given in (48) and (50). The two opposing effects of DI provision on the probability that the government repays the debt are apparent from these expressions. On the one hand, DI provision protects the intermediation process thus maintaining the tax base which is equal to \bar{A} in (48) ('the tax base effect'). On the other hand, DI provision involves fiscal outlays $T_1(q_1^{DI})/q_1^{DI}$ which add to the debt burden ('the debt burden effect'). Which of the two dominates depends on the size of bank government debt holdings, which determine $T_1(q_1^{DI})/q_1^{DI}$, relative to the output costs of banking collapse ψ .

Proposition 4. *There is a critical value ψ^* such that DI is provided when $\psi \geq \psi^*$ and $\omega \leq \omega^*$.*

Proof. Let $\Delta(\psi) \equiv (p^{DI} - p^{NI}) \gamma + \psi$ with p^{NI} depending on ψ from (50). Set $\omega = \omega^*$ so the incentive to provide DI is determined by the sign of $\Delta(\psi)$. The proof shows that $\Delta(\psi)$ crosses zero exactly once as ψ varies between 0 and 1 using: $\Delta(0) < 0$, $\Delta(1) > 0$ and $\Delta'(\psi) > 0$.

$\Delta(0) < 0$ since at $\psi = 0$, $p^{NI} > p^{DI}$ from (50) and (48). To see this, note that at $\psi = 0$, government debt is more valuable if DI is not provided since then default probabilities are lower: $q_1^{NI} > q_1^{DI}$. $\Delta(1) > 0$, since at $\psi = 1$, $p^{NI} = 0$ so that $p^{NI} < p^{DI}$ from (50) and (48).

To prove that $\Delta(\psi)$ is increasing in ψ , the derivative of (21) with respect to ψ implies:

$$\Delta'(\psi) = 1 - \frac{\partial p^{NI}}{\partial \psi} \gamma.$$

The first effect is direct as ψ enters $\Delta(\psi)$. The second is indirect, through the effect of ψ on p^{NI} . p^{NI} and q^{NI} are jointly determined with the equilibrium value of q^{NI} given by (16) with $T(q_1) \equiv 0$ and $p^{NI} = q^{NI} R$. By assumption, the pessimistic equilibrium is a locally stable solution to (16) and $\partial q^{NI} / \partial \psi < 0$. Hence $\partial p^{NI} / \partial \psi < 0$.

From this structure of $\Delta(\psi)$, there is a critical value of ψ such that the positive effect from protecting intermediation is exactly equal to the increased expected sovereign default costs as the result of DI provision. For $\psi \geq \psi^*$ DI is provided.

If DI is provided for a given ψ with $\omega = \omega^*$, then it will be provided if $\omega \leq \omega^*$. From (19), $\omega < \omega^*$ implies that the gains from insurance outweigh the costs so that the first term is positive. ■

The case of costly disruption of intermediation as a basis for bailouts seems powerful. The Lehman Brothers collapse in September 2008 demonstrated clearly how costly the collapse of the financial system can be. Lehman Brothers was allowed to fail, partly at least due to the strong public outcry following the bailouts of Bear Stearns in March 2008 and of Fannie Mae and Freddie Mac during the summer of 2008. Yet, once the real effects of the crisis became apparent, the bailouts were extended more widely than ever before, covering insurance companies such as AIG and even the entire money market mutual fund industry. For this reason, assume DI is provided *ex post* by the government when the banking system is threatened with insolvency without it.

If bank or government defaults are not costly in terms of output (ψ and γ are close to zero) the DI provision decision is entirely determined by the balance of the insurance benefits (to depositors) versus the tax costs (to investors) of providing DI. This balance depends critically on the relative Pareto weight of investors (ω) as demonstrated in equation (20). Hence, as in Cooper and Kempf (2013), DI will still be provided even if $\psi = \gamma = 0$ as long as $\omega \leq \omega^*$.

6 Private Response: Equity Buffer

This section looks at private remedies for the spillovers between banks and governments. Throughout we focus on the contracting problem of a single bank given a government policy of providing a bailout to other banks and financing $G_1 > 0$. Thus the single bank faces uncertainty in the value of government debt.

Subsection 6.1 studies the contracting problem of a single bank **assuming** it is not supported by a government bailout. In this case, equity issuance, in particular, can, if done in sufficient quantities, support the first best contract **without** the need for a bailout. The equity holders absorb the losses which occur when sunspot shocks hit the debt market.

From Proposition 4 the government will provide DI to banks threatened by insolvency. Subsection 6.2 shows how, when banks anticipate such government bailouts, this destroys their incentive to self-insure through building up an equity buffer against sovereign risk.

6.1 Self Insurance Through Equity Buffers

Suppose the bank can raise x_0 goods from investors at the initial date by selling shares in the bank. The equity entitles the owner to a state contingent dividend δ_2 at the final date, which is equal to the residual value of the bank after all depositors have received what was promised to them. Hence the equity will trade at a price e_0 which equals the expected value of dividends at the final date discounted back to time 0 by investors' required rate of return R . Investors will participate in this share offering if $e_0 \geq x_0$. Put differently, the equity investment in the bank will yield the same return as the outside option of the investors to put resources into the two-period, illiquid technology.

With an appropriate equity investment, the bank can offer households the first best contract, i.e. the allocation to early and late consumers, from Proposition 1 even in the presence of variations in debt prices. The resulting allocation insulates the banking system from debt fragility.

6.1.1 Optimal Contract

To show how equity issuance can deal with the source of strategic uncertainty, we focus on a 'bankruptcy-proof' contract in which the bank issues sufficient equity to cover losses on sovereign debt holdings. The contract is further restricted so that allocations to early and late consumers are state noncontingent despite the presence of strategic uncertainty. As we demonstrate, this 'bankruptcy-proof' contract with state noncontingent consumption allocations implements the first-best contract.

Even though the bank's promises to early and late consumers are noncontingent, its investments as well as the dividend payments $\delta_2(s)$ to shareholders are contingent upon the realization of sunspot shocks. Let s index the sunspot shock in the sovereign debt market: $s = 1$ when pessimism occurs and $s = 0$ when it does not. Let $\nu(s)$ be the probability of sunspot state s .

The value of government debt is state contingent. In the optimistic state, the value of government debt is, $q_1(0) = \frac{1}{R}$, characterized in Proposition 2. In the pessimistic state, the value of government debt, $q_1(1) < q_1(0)$ is given by an interior equilibrium from Figure 2.

As banks compete for depositors, the optimal contract maximizes depositor welfare subject to a participation constraint for the equity investors. The optimal contract solves:

$$\max_{\delta_2(s), l_1(s), b_0, b_1(s), L_1(s), i_0, c^E, c^L} \pi u(c^E) + (1 - \pi) u(c^L) \quad (22)$$

such that

$$i_0 + q_0 b_0 \leq d + x_0 \quad (23)$$

$$\pi c^E \leq q_1(s) (b_0 - b_1(s)) + \varepsilon l_1(s) - L_1(s) \quad (24)$$

$$(1 - \pi) c^L \leq b_1(s) + R(i_0 - l_1(s)) - \delta_2(s) + r^b L_1(s) \quad (25)$$

$$e_0 = \frac{\sum_s \nu(s) \delta_2(s)}{R} \geq x_0. \quad (26)$$

The contract is modified in several ways compared to the contract in section 3. First of all, the sale of equity to investors adds further resources x_0 at the initial date. Secondly, the constraint (24) is modified because the choice of how many bonds to roll over $b_1(s)$ and how much to lend to investors $L_1(s)$ are now contingent upon the realization of the government debt sunspot shock. (26) is the participation constraint of the equity investors. It states that the amount investors inject into the bank must be greater than or equal to the expected value of dividends at the final date $(\sum_s \nu(s) \delta_2(s))$ discounted back to period 0 by the investors' opportunity cost of funds R .¹⁹

What remains unchanged is the fact that the promises to early and late consumers are not contingent on the realization of the sunspot. In what follows we do not solve (22) directly but instead show that it can achieve the first best contract and fully insulates depositors from sunspot shocks in sovereign debt markets.

Proposition 5. *Optimal equity issuance implements the first-best contract.*

Proof. The proof argues that there exists e_0 such that (i) the contract with equity supports the first best allocation despite stochastic government debt prices and (ii) investors receive their required rate of return from the equity investment. We first determine the level of equity investment needed to support the first best contract, (c^{*E}, c^{*L}) . We then argue that the return on this equity equals the outside option of the investors.

Let x_0 denote the period 0 investment of equity holders into the bank. The bank's resource constraint becomes:

$$q_0 b_0 + i_0 = d + x_0. \quad (27)$$

In the first best contract, the expected net present value of promises to depositors equals the amount they deposit at the initial date:

$$\pi c^{*E} + \frac{(1 - \pi) c^{*L}}{R} = d. \quad (28)$$

To support the first best allocation, the bank must have sufficient resources to meet the contractual commitment to early consumers regardless of the realized government debt price:

$$\pi c^{*E} = q_1(1) b_0 \quad (29)$$

¹⁹As we discussed in section 2, investors can place unlimited time deposits with banks and these time deposits pay a return of R . Hence risk-neutral investors will require the same expected return from any other asset they invest in.

where $q_1(1)$ is the period 1 price of government debt under pessimism. In this state, the bank sells its entire bond holding in order to pay off early consumers. Promises to late consumers are met through the illiquid investment:

$$(1 - \pi) c^{*L} = R i_0. \quad (30)$$

The cash flow for dividend payments to shareholders is only generated in the optimistic state. The bank rolls over its bond not needed to fund the early consumers:

$$\pi c^{*E} = q_1(0) (b_0 - b_1(0)). \quad (31)$$

The rolled over bond holding is then used to pay dividends to shareholders at the final date:

$$\delta_2(0) = b_1(0).$$

The value of the equity to the shareholder is the discounted expected value of this dividend:

$$e_0 = \frac{\nu b_1(0)}{R}. \quad (32)$$

For the equity investment to be undertaken in equilibrium it must be the case that this expected value equals the equity put into the bank by the investor, i.e. $x_0 = e_0$. Substituting (28), (29) and (30) into (27) yields $x_0 = q_0 b_0 - \pi c^{*E}$. From this the equity investment needed is:

$$x_0 = \nu \frac{(1 - p)}{p} \pi c^{*E}. \quad (33)$$

Combining (29) with (31) we get:

$$b_1(0) = R \frac{(1 - p)}{p} \pi c^{*E}.$$

Hence

$$e_0 = \nu \frac{(1 - p)}{p} \pi c^{*E} = x_0. \quad (34)$$

■

The effect of the equity issuance is to remove all risks for depositors from the bank's bond purchases. This is very intuitive. The deep pocket risk-neutral equity investors absorb all the risk and leave the bank and its depositors with a safe consumption profile.

It is useful to define

$$\kappa^* \equiv \frac{x_0}{b_0} = \nu \frac{(1 - p)}{R} \quad (35)$$

as the amount of equity issued for given purchases of government debt. This is chosen by the bank in its own prudential capital choice (under the belief that no *ex post* assistance will be provided). The form of the capital charge consists of two key parts. It is increasing in the difference between the value of government debt in the optimistic state and the pessimistic state

$$q_1(0) - q_1(1) = \frac{(1-p)}{R}$$

because this difference determines the volatility of bank asset values and the size of dividends in the good state.

When p is small and the government is expected to default with a high probability in the pessimistic state of the world, the bank needs to buy a lot of bonds at the initial date in order to ensure that their value under pessimism is sufficient to meet depositor withdrawals. When the economy remains in the optimistic equilibrium (this happens with probability ν which is also part of the κ^* expression), the difference between bank assets and bank liabilities is very big and shareholders receive a large dividend. So κ^* has the interpretation of a ‘risk weight’ - it is growing in the volatility of government debt prices between the pessimistic and optimistic states of the world.

6.2 The Limits to Self-Insurance

The previous subsection painted an optimistic picture: banks have the ability to self-insure against variations in the value of government debt. But, as we know, this did not happen in practice and a number of European economies did experience severe positive feedback loops between the financial health of governments and their banks. In this section, we explain the limited equity issuance as a response of banks to anticipated bailouts.

Of course, there are other reasons for the fact that banks were exposed to variations in government debt prices. An immediate explanation is some element of irrationality. Either banks did not anticipate the possibility of pessimism at all or did not appreciate the severity of the debt crisis through the complicated linkages from the equilibrium interactions between debt prices, bailouts and bank balance sheets.

But even if banks correctly anticipate the likelihood and severity of sovereign and banking crises, a moral hazard problem exists from the government’s strong incentives to provide DI (see Proposition 4 in section 5). Though it is feasible, banks fail to shield their balance sheets from the impact of the crises anticipating government debt buybacks at above market prices if banks experience losses.

6.2.1 Optimal Contract

From section 5, *ex post* the government will purchase bonds from banks at a price q_1^* which is chosen such that the bank can meet payments to early as well as late consumers for any realization of the government debt sunspot shock s . This government intervention makes the consumption allocation received by depositors riskless. It also obviates the need for self-insurance through equity buffers.

Note that we do not generate this effect through a high cost of equity. The return on equity is R , exactly the same as on all the other assets held by investors. The unwillingness to issue equity comes purely from the desire by the banks to exploit the financial safety net to the maximum extent possible.²⁰

But the effects go further. The implicit bailout assistance which banks rationally anticipate the government to provide *ex post* creates incentives for excessive risk-taking through holdings of fragile government debt. This implication of the model is consistent with the large exposures to sovereign debt documented in Table 1.

The banking contract with expected bailouts but without equity issuance solves

$$\max_{b_0, i_0, L_1(s), b_1(s), c^E, c^L} \pi u(c^E) + (1 - \pi) u(c^L)$$

such that

$$i_0 + q_0 b_0 \leq d$$

$$\pi c^E = q_1^* (b_0 - b_1(s)) - L_1(s) \quad (36)$$

$$(1 - \pi) c^L = R i_0 + b_1(s) + r^b L_1(s). \quad (37)$$

Here the government buys government bonds from the banks at q_1^* , the optimistic equilibrium price, making the return on government bonds riskless. The consumption allocation provided to depositors continues to be independent of the sunspot although the bank's portfolio choice at the middle date ($b_1(s)$ and $L_1(s)$) may not be.

Proposition 6. *The optimal contract under the debt buyback scheme features maximum bank exposure to strategic uncertainty from the government debt market: $i_0 = 0$ and $d = q_0 b_0$. Equity is not issued voluntarily.*

Proof. The debt price reflects investors' valuation which takes the risk of sunspot shocks into account.

$$q_0 = \frac{\nu + (1 - \nu) p}{R}. \quad (38)$$

²⁰To rule out permanent transfers to the banking sector we assume that the government can commit to limit transfers to zero in the optimistic equilibrium. In other words $q_1^* \leq q_1(0)$ so transfers only occur during a fiscal crisis.

The first order conditions for the optimal contract are:

$$u'(c^E) = \lambda^E = 0,$$

$$u'(c^L) = \lambda^L = 0,$$

$$(q_0\phi + q_1^*\lambda^E) b_0 = 0.$$

Using (38) and $q_1^* = 1/R$, the first order condition with respect to bond purchases becomes

$$\left(-\phi + \frac{\lambda^E}{(\nu + (1-\nu)p)}\right) b_0 = 0. \quad (39)$$

The rest of the first order conditions are:

$$(-\phi + R\lambda^L) i_0 = 0, \quad (40)$$

$$(-\lambda^E + r^b\lambda^L) L_1(s) = 0, \quad (41)$$

$$(-q_1^*\lambda^E + \lambda^L) b_1(s) = 0. \quad (42)$$

It is clear from the above first order conditions that $i_0 = 0$ and b_0 is greater than zero. To see why this is, remember that $r^b = R$ and substitute (41) into (39) to get:

$$-\phi + \frac{R}{\nu + (1-\nu)p} \lambda^L = 0.$$

Since $\nu + (1-\nu)p < 1$, $-\lambda^E + R\lambda^L < 0$. Hence $i_0 = 0$ and $q_0b_0 = d$.

Choosing $i_0 = 0$ and $q_0b_0 = d$ maximises the expected payments from the deposit insurance fund. It implies that the net present value of promises to the two types of consumers is:

$$\pi c^E + \frac{(1-\pi)c^L}{R} = \frac{q_1^*}{q_0} d \geq d. \quad (43)$$

Using the first order conditions for c^E , c^L , and L_1 we can see that the promises to the early and late consumers continue to be governed by

$$u'(c^E) = Ru'(c^L). \quad (44)$$

which is the same marginal condition as in the first best. However, the net present value of consumption promises is higher by a factor of $\frac{q_1^*}{q_0}$ compared to the first best. Thus, the consumption of early and late consumers is each higher in this contract compared to the first best.

Since the consumption of all consumers in the banking contract is higher under no equity issuance and maximum sovereign debt exposure, this implies that a competitive bank optimally issues no equity if it is allowed to choose its capital structure freely. ■

Proposition 6 demonstrated two important consequences of the anticipation by banks of government bailouts.²¹ First, the bank invests heavily in fragile government debt. Secondly, the bank does not issue equity despite the absence of any costs of doing so.

The lack of equity issuance is actually the more serious consequence of deposit insurance. Large holdings of risky sovereign debt are not necessarily a problem for bank solvency as long as they are backed by sufficiently large equity buffers. But we saw in Proposition 6 that this is not the case. The key reason for this is that when the bank issues equity, its losses are borne by investors who demand compensation through a competitive return on the funds they provide. In contrast, the government does not require compensation for the bailout assistance it provides *ex post* when the banking system gets into difficulty. Banks and their depositors rationally prefer to resort to free government insurance rather than relying on costly private alternatives.²²

7 The ‘Diabolic Loop’ as a Nash Equilibrium

Having studied the incentives of the government and the banks independently, it is possible to characterize the (sub-game perfect) Nash Equilibria for this economy. The players are the banks, the households and the government. The banks simultaneously and independently move first, setting contracts with households and deciding on their portfolio, including the amount of equity financing. They do so recognizing the possibility of strategic uncertainty influencing the valuation of government debt in period 1. The strategic uncertainty is modeled as a randomization between the optimistic and a (locally stable) pessimistic solution to (16).

Given the choices of the banks, after the sunspot is realized, the government chooses, in period

²¹The results in Proposition 6 crucially depend on the fact that DI will in fact be provided *ex post*. A key determinant of the government’s DI provision is the output cost of bank default as measured by ψ in equation (8). We assumed for simplicity that ψ is a parameter which is independent of the nature of bank portfolios. Another possible assumption is that bank default costs only arise when bankrupt banks are forced to liquidate long term projects early. In other words, when banks hold only government bonds (as shown in Proposition 6), ψ should be very low and possibly zero. Even when $\psi = 0$ we know from section 5 that DI will still be provided if the government cares more about depositors than about investors ($\omega \leq \omega^*$).

Another possibility is that banks behave strategically and hold just enough long term projects to make sure that they are important enough to get bailed out by the government. Banks then place the rest of their deposits in government debt in order to benefit from bailouts.

Such strategic considerations behind banks’ portfolio choices are not crucial for the main message of the paper and we do not pursue them here for the sake of brevity. Instead we assume that ψ is sufficiently large and ω is sufficiently small so that DI is provided even when banks hold only government bonds.

²²Note that we are implicitly assuming here that investors must hold government debt directly. The option of holding bonds via the banking system in order to take advantage of the deposit guarantee is ruled out.

1, whether or not to support the banks. Along the equilibrium path, the expectations underlying the choices of the investors, depositors and the banking contract are fulfilled.

Definition 1. A ***Sub-game Perfect Nash Equilibrium*** (SPNE) is a set of bank equity issuance and debt purchase strategies, a set of government DI provision strategies and a set of realizations of government debt prices as a function of the debt sunspot realizations such that: (i) Individual banks choose the deposit contract, equity issuance and debt purchase strategies to maximize depositor utility conditional upon other banks' equity issuance and debt purchase strategies, the government's DI provision strategy, the exogenous probabilities of government debt sunspot shock realizations and the prices of government debt at these sunspot realizations. (ii) The government chooses whether or not to provide DI in order to maximize social welfare taking bank government debt exposures as given. (iii) the government debt markets clear at each sunspot realization.

The SPNE will depend on the types of regulations available to the government and its ability to commit. One extreme is full commitment. Alternatively, the government may have some limited commitment, for example by being able to condition the provision of deposit insurance on the contract offered to depositors and/or the portfolio of bank assets.

Our approach is to study three cases. At one extreme, a **committed regulator** chooses *ex ante* whether to bailout the banks.²³ At the other extreme, a **weak regulator** is incapable of any kind of commitment and decides whether or not to bailout a financial institutions in order to maximize *ex post* social welfare. In the intermediate case, we have a **strong regulator** who can deny deposit insurance to any financial institution that deviates from the first-best contract and portfolio that supported the optimistic equilibrium characterized in Proposition 2. We establish whether the diabolic loop exists in all these cases.

Proposition 7. A ***committed regulator*** will choose not to bailout the banks. As a consequence, the unique SPNE features banks offering the first best contract and self-insuring through equity issuance.

Proof. Anticipating no bailout, Proposition 5 applies: a bank will have an incentive to use equity financing to self-insure. Under Assumption 1, sunspot equilibria exist even when the government does not bail out the banks and sovereign debt remains risky. Then there is a strict incentive for banks to issue equity to insure depositors against the strategic uncertainty created by $G_1 > 0$. As Proposition 5 shows, this implements the first best contract.

²³Here the regulator is limited to choosing between DI or no DI, including the imposition of a tax on investors to finance DI. We do not consider other *ex ante* tools for redistribution.

If the regulator commits to no bailout, the outcome is given by Proposition 5. If the regulator commits to a bailout, then the outcome is given by Proposition 6. In both cases, the intermediation process is never interrupted so that the welfare loss from $\psi > 0$ is avoided.

With a bailout, the probability of debt default is higher; $p^{NI} > p^{DI}$ from (48) and (50). Hence the expected default cost, due to $\gamma > 0$, is higher under a bailout. Further, with a bailout, the contract offered to households provides higher consumption to both early and late consumers compared to (c^{*E}, c^{*L}) from the first-best contract. Since the consumption allocation in Proposition 5 corresponds with the first-best allocation, the redistribution towards households is not welfare improving. Hence, the allocation without bailout is preferred. ■

Proposition 7 shows that government discretion is a vital ingredient for the existence of the ‘diabolic loop’.²⁴ A committed government that withholds insurance *ex post* can force banks to self-insure in a way that obviates the need for a bailout. In this case, when Assumption 1 holds, strategic uncertainty remains in the government debt market but it does not spill over to the banking system.

But as the subsequent analysis will show, the incentive for banks to self-insure against sovereign exposures is fragile. The less commitment the government has to deny bailouts *ex post*, the more exposed banks will become.

Proposition 8. *When the regulator is **strong** and the conditions stipulated in Proposition 4 hold, there will exist a SPNE in which the banks offer the first best contract to depositors and issue no equity. The regulator bails out the banking system in the event of pessimism.*

Proof. Consider the choice of banks when faced with the strong regulator. If they do not issue equity and if Proposition 4 holds, they will be bailed out *ex post* when they offer the first best contract (but not if they offer another contract).²⁵ From Proposition 5, when the banks issue equity, this can also implement the first best allocation. Hence since government DI provision guarantees the first best allocation to the depositors, banks will have no incentive to issue equity when they provide the optimistic contract to depositors.²⁶

²⁴An important issue, brought out in Proposition 7, is the redistribution from the provision of deposit insurance. As is clear from Proposition 6, competitive banks redistribute the gains from bailout to households. Thus, in our model, the commitment to deposit insurance becomes a tool for redistribution. This redistribution is not welfare improving, given the first-best allocation characterized in Proposition 2.

²⁵When pessimism is anticipated *ex ante*, the period 0 price of government debt is lower than q_0^* , reflecting default risk. As the bank is offering the first-best contract to depositors, it will generate profits. We assume that the strong regulator can tax these profits from the bank and rebate them to the investors. As a result, the social welfare function is identical to that used in Proposition 4.

²⁶If there is an infinitely small equity issuance cost, banks will strictly prefer not to issue equity when the government provides DI.

The equilibrium values for government debt are the solutions to (16). The sunspot process randomizes across optimistic and pessimistic valuations in this set. ■

The inability of the government to fully commit activates the ‘diabolic loop’ because banks anticipate a government bailout and do not self-insure through equity issuance. However, in this example, the ability of the government to deny deposit insurance to banks who offer overly generous terms to depositors and hold excessive amounts of government bonds (as in Proposition 6) helps to reduce the size of government bailouts and contain the amplification of pessimism shocks. However, since the banks do not issue equity, they remain exposed to strategic uncertainty in the government debt market and the ‘diabolic loop’ is alive and well.

Proposition 8 makes clear the outcome for a deposit insurance authority which is able to limit its bailout to banks offering a particular contract to depositors and holding only a limited quantity of government bonds.²⁷ Next we show that when the regulatory authority lacks even this commitment power, the ‘diabolic loop’ becomes even more severe as long as the government has an incentive to bailout the banks *ex post*. This is because, in addition to not issuing equity, the banks choose to increase their holdings of government debt beyond what they hold when pessimism is not anticipated.

Proposition 9. *When the regulator is **weak**, ω is sufficiently low and ψ is sufficiently large, there will exist a SPNE in which banks offer the contract characterized in Proposition 6 and issue no equity. In the event of pessimism the government bails out the banking system. The probability of government default in the pessimistic equilibrium is higher than in the case of the **strong** regulator.*

Proof. The contract characterized in Proposition 6 implies a (c^E, c^L) that satisfy (44), a portfolio of government debt only, and no equity issuance. This contract is optimal when the bank expects a government bailout.

Using the framework in the Appendix, the net gain to deposit insurance

$$\begin{aligned}\Delta &\equiv W^{DI} - W^{NI} \\ &= \pi (1 - k_1) [u(c^E) - u(0)] + (1 - \pi) [u(c^L) - (k_1 u(\beta c^E) + (1 - k_1) u(0))] \\ &\quad - \omega (q_1^* - q_1^{NI}) B_0^B + \frac{\omega}{R} [(p^{DI} - p^{NI}) \gamma + \psi] \bar{A}\end{aligned}$$

can be written for this particular contract and portfolio. As in the proof of Proposition 4, there is a critical value of ω such that the gains to insurance will equal the consumption loss of the investors. At this value, if ψ is large enough, $\Delta > 0$. For ω less than this value, there will be insurance gains

²⁷Namely, the quantity of government bonds which are sufficient to meet withdrawals by the early consumers in the optimistic contract.

to government intervention as well. Consequently, in the event of pessimism, the government will bail out the banks who provide such a contract.

When banks expand their government debt holdings, this increases the required government transfers $T(q_1^{DI}) = (q_1^* - q_1^{DI}) B_0^B$ other things equal. Since we have assumed that the pessimistic equilibrium is a locally stable solution to (16) this also implies that higher B_0^B implies a lower value of q_1^{DI} . ■

Propositions 7, 8 and 9 demonstrate how bank risk-taking in the sovereign debt market grows and the joint fragility of banks and sovereigns worsens with diminished regulator commitment. If governments can commit not to bail out, bank and sovereign balance sheets become disconnected and there is no sovereign-banking loop. If governments can enforce the contract offered but cannot force banks to issue equity against sovereign exposures, then banks rely on bailouts and issue no equity, activating the loop. When no commitment is possible, we get full moral hazard: banks overinvest in government debt and issue no equity. As government commitment declines, the sovereign-banking loop grows in severity.

Finally, it is useful to see if an equilibrium with equity and no bailout could exist even if the government had an incentive to support the banks *ex post*. Suppose the conditions for Proposition 4 hold. Consider a candidate Nash Equilibrium with equity financing and no bailouts. Under Assumption 1, there is strategic uncertainty in the pricing of government debt. In this case, a single bank could deviate and set equity financing to zero, exposing depositors to the strategic uncertainty. With the weight ω low enough, the government will have an incentive to bailout this single bank. Thus the combination of equity financing and no bailouts would not be an equilibrium in this economy. Formally,

Proposition 10. *If Assumption 1 holds and the welfare weight on investors, ω , is sufficiently low, there will not exist a Nash Equilibrium in which the banks self-insure through equity financing and the government does not bailout the banking system.*

Proof. By Assumption 1, at a candidate equilibrium with no bailout and equity financing, there remains strategic uncertainty. A single bank that believes the government will provide a bailout will, following Proposition 6, profitably deviate and set equity financing to zero. In the event of pessimism, for a sufficiently low value of ω the government will indeed bailout the bank. Thus there is no equilibrium with equity financing and no bailout. ■

8 Conclusions

This paper builds a model of the feedback loops between banks and sovereigns in Europe. From Diamond and Dybvig (1983) and Calvo (1988), banks as well as sovereign debt markets are individually subject to powerful sources of strategic uncertainty, which can lead to multiple Pareto-ranked equilibria. Our paper characterizes a ‘diabolic loop’ that links these markets and thus propagates and amplifies the impact of strategic uncertainty.

The interactions work from sovereigns to banks as well as in the opposite direction. Bank solvency is affected by sovereign bond market turmoil because the financial system holds a large amount of (largely domestic) government debt. In turn, government solvency is affected due to the implicit or explicit guarantees extended by governments to their banking systems. These interactions amplify the impact of pessimism in the government debt market. The initial decline in government debt prices reduces bank solvency and causes the implicit government promises to its banks to turn into explicit debt issuance at precisely the time when the government is least able to issue debt on favorable conditions. The higher debt issuance then pushes government debt prices even lower, completing the diabolic loop which has been rocking a number of European economies since 2010. The impact of these feedbacks is to make sovereign-banking crises much more severe than they otherwise would have been.

While we study the effects of strategic uncertainty in debt markets as the initial shock, the model is general enough to accommodate other sources and types of uncertainty. As is well understood, the Diamond and Dybvig (1983) model often has a bank run equilibrium which itself could influence debt valuation through the cost of a government fulfilling obligations to banks. In fact, even if the bank run could be avoided through government intervention, the costs of those actions through debt valuation would be an important part of the calculus concerning the *ex post* provision of deposit insurance. Moreover, fundamental shocks to either the banking system or the government fiscal situation would be magnified and propagated through the mechanisms identified in our model.

Having built a model of the crisis, we study a number of simple remedies for cutting the diabolic loop. One often suggested policy is just to let the banking system fail, imposing losses on depositors. Such a policy, if credible, would have multiple benefits. First, it would reduce the need for bailout assistance to add to government debt during a sovereign crisis. This, taken in isolation, would diminish the crisis amplification mechanisms we study in our paper. Second, when banks know they will not be bailed out, they will issue equity which will absorb losses from sovereign bond holdings without needing government assistance. Hence, bank solvency becomes completely decoupled from government solvency, severing a key linkage that has amplified the financial crisis in a number of EU countries.

The problem with such a commitment to let the banking system fail is that it is not credible. Governments, acting with discretion after a crisis, prefer (under some plausible conditions) to bailout the banking system rather than incur the output losses associated with a breakdown of the intermediation process.

In turn, banks, anticipating that government assistance will be provided, have little incentive to issue equity. This is because they would like to take advantage of the ‘heads-I-win, tails-you-lose’ nature of the financial safety net. If the economy finds itself in an optimistic equilibrium, banks profit from high *ex post* bond returns. When the economy finds itself in the pessimistic equilibrium, the bank expects the government to bail it out in order to protect household deposits. This strategy extracts a transfer from taxpayers to bank depositors which makes the latter better off. As a result, banks rationally prefer to remain exposed to a sovereign debt crisis.

This moral hazard by banks might be corrected by regulatory interventions which impose capital requirements on banks’ sovereign debt holdings until they become insulated from shocks in the debt market. In the light of this finding, it is puzzling that the new Basel III regime continues to favor domestic government debt over other assets by assigning it a zero capital weight. Moreover, domestic government debt continues to be exempt from large exposure limits creating exactly the kinds of incentives for banks to become overexposed to it described in our paper. While beyond the scope of this paper, adding an explicit analysis of government capital regulation seems to us like an interesting avenue for future research.

Future work will analyze the international dimension of the European twin crisis. Financial stability policy in Europe is undergoing major reform with the establishment of the Single Supervisory Mechanism (SSM) and with the use of Outright Monetary Transactions (OMT) by the European Central Bank. We intend to embed our single country model into a multi-country setting and consider the union-wide policies which can limit the economic damage done by the ‘diabolic loop’.

9 Appendix

This appendix provides the framework for assessing the provision of deposit insurance. The starting point is period 1, given a contract to depositor, (c^E, c^L) , the bank’s period 0 portfolio, (B_0^B, i_0^B) and the investors’ period 0 portfolio (B_0^I, i_0^I) . The details of these choices are provided in the analysis that uses this framework.

Assume that the government debt market is in a state of pessimism. Given this situation, the analysis characterizes the payoffs in periods 1 and 2 to households and investors with and without the provision of deposit insurance. Throughout, let $(\mathbb{1}\{G\} = 0)$ denote the states in which the government repays its debt and let $(\mathbb{1}\{G\} = 1)$ denote default states.

9.1 Welfare when DI is provided

First, suppose that the government provides DI to the banks. This transfer is financed through the issue of new debt.

9.1.1 Investors

If DI is provided, at the middle date investors consume the difference between their endowment, the bonds they buy from banks B_0^B and the amount they borrow from banks L_1 :²⁸

$$c_1^{I,DI} = A_1 - q_1^{DI} (B_0^B + (G_1 + T(q_1^{DI})) / q_1^{DI}) + L_1.$$

At the final date, investors consume all their net worth. Their consumption depends on the default decision of the government.

If the government repays its debt, consumption is given by:

$$\begin{aligned} c_2^{I,DI}(\mathbb{1}\{G = 0\}) &= \bar{A} + B_1 + Ri_0^I - RL_1 - \tau\bar{A}. \\ &= \bar{A} + Ri_0^I - RL_1. \end{aligned} \tag{45}$$

where the second equality follows from the fact that the banking system sells its entire holding of government debt in the pessimistic equilibrium. Hence investors own all debt and pay the taxes to pay the debt. The two cancel out ($B_1 = \tau\bar{A}$).

If the government defaults, investors' final period consumption is given by:

$$c_2^{I,DI}(\mathbb{1}\{G = 1\}) = \bar{A}(1 - \gamma) + Ri_0^I - RL_1. \tag{46}$$

Investors' welfare is affected by the default decision due to the output costs of default assumed in our specification of the final date endowment. Investors' welfare when the government repays ($\mathbb{1}\{G\} = 0$) and defaults ($\mathbb{1}\{G\} = 1$) is:

$$W^{I,DI}(\mathbb{1}\{G\}) = c_1^{I,DI} + \frac{1}{R}c_2^{I,DI}(\mathbb{1}\{G\}).$$

9.1.2 Depositors

The utility of depositors when the government provides DI is independent of its decision to default and is given by the utility delivered by the standard banking contract

$$W^{H,DI} = \pi u(c^E) + (1 - \pi)u(c^L).$$

²⁸In the pessimistic equilibrium, banks sell their entire holdings of government debt (B_0^B) to investors in the middle period.

9.1.3 Welfare if DI is provided

Social welfare when DI is provided is:

$$W^{DI} = \pi u(c^E) + (1 - \pi) u(c^L) \quad (47)$$

$$+ \omega [A_1 - q_1^{DI} (B_0^B + (G_1 + T(q_1^{DI})) / q_1^{DI})] + \frac{\omega}{R} (\bar{A} + Ri_0^I) - \frac{\omega}{R} (1 - p^{DI}) \gamma \bar{A}.$$

Here p^{DI} is the probability that the government repays at the final date conditional upon DI being provided:

$$p^{DI} = 1 - F\left(\frac{B_0 + (G_1 + T(q_1^{DI}))/q_1^{DI}}{\bar{A}}\right). \quad (48)$$

In (47), q_1^{DI} denotes the price of government debt in the middle period if there is pessimism and deposit insurance is provided. This is determined from the investor's arbitrage condition of $\frac{p^{DI}}{q_1^{DI}} = R$ and (48). Finding the (p^{DI}, q_1^{DI}) that solves these two conditions is the same as finding a solution to (16), other than the optimistic equilibrium.

9.2 Welfare when DI is not provided

Now assume that the government does not bailout the banks. As a consequence, the banks close and deposits are served sequentially using the available resources of the bank.

9.2.1 Investors

In period 1, if DI is not provided, the consumption of investors is given by:

$$c_1^{I,NI} = A_1 - q_1^{NI} (B_0^B + G_1/q_1^{NI}).$$

The period 2 consumption of investors under repayment and default respectively are given by:

$$c_2^{I,NI} (\mathbb{1}\{G = 0\}) = \bar{A} (1 - \psi) + Ri_0^I$$

and

$$c_2^{I,NI} (\mathbb{1}\{G = 1\}) = \bar{A} (1 - \psi) (1 - \gamma) + Ri_0^I.$$

As the government does not provide a financial safety net, the sovereign crisis leads to a banking crisis that reduces investors' endowments by a factor of $(1 - \psi)$ due to the disruption of the intermediation process. If, in addition, the government ends up defaulting at the final date ($\mathbb{1}\{G\} = 1$) and investors' endowments are reduced further, by the factor $(1 - \gamma)$ which represents the output costs of default in our model.

Investors' welfare when the government does (does not) repay is:

$$W^{I,NI}(\mathbb{1}\{G\}) = c_1^{I,NI} + \frac{1}{R} c_2^{I,NI}(\mathbb{1}\{G\}).$$

9.2.2 Depositors

Consumers run on the bank when no deposit insurance is provided. Hence, their welfare depends on their place in the banking queue and on whether they are early or late consumers. A fraction

$$k_1 = \frac{q_1^{NI} B_0^B + \varepsilon i_0^B}{c^E}$$

get served in equilibrium and they obtain the consumption allocation of the early consumers. Of those who get served, π fraction are early consumers who actually like consuming at the middle date. The rest, $1 - \pi$, are late consumers who get reduced utility from consuming at the early date. Those who do not get served, a fraction $(1 - k_1)$ of the total, receive zero consumption which delivers a low but finite utility $u(0)$. The price of government debt in the middle period if there is pessimism and deposit insurance is not provided is q_1^{NI} .

The total utility of depositors in the regime without DI is given by:

$$W^{H,NI} = k_1 (\pi u(c^E) + (1 - \pi) u(\beta c^E)) + (1 - k_1) u(0).$$

9.2.3 Social Welfare without Deposit Insurance

If there is no bailout, social welfare is given by:

$$\begin{aligned} W^{NI} &= k_1 (\pi u(c^E) + (1 - \pi) u(\beta c^E)) + (1 - k_1) u(0) \\ &+ \omega [A_1 - q_1^{NI} (B_0^B + G_1/q_1^{NI})] + \frac{\omega}{R} (\bar{A} + R i_0^I) - \frac{\omega}{R} (1 - p^{NI}) \gamma \bar{A} - \frac{\omega}{R} \psi \bar{A}. \end{aligned} \quad (49)$$

Here p^{NI} is the probability that the government repays conditional upon no DI being provided, given by:

$$p^{NI} = 1 - F\left(\frac{B_0 + G_1/q_1^{NI}}{\bar{A}(1 - \psi)}\right). \quad (50)$$

Note the presence of ψ in the denominator of $F(\cdot)$ since there are bank defaults when deposit insurance is not provided.

This is determined from the investor's arbitrage condition of $\frac{p^{NI}}{q_1^{NI}} = R$ where p^{NI} is given in (50). Finding the (p^{NI}, q_1^{NI}) that solves these two conditions is the same as finding a solution to (16) with $T(q_1) \equiv 0$, other than the optimistic equilibrium.

9.3 The DI Provision Decision

The difference in the value of the social welfare function between providing DI and not is:

$$\begin{aligned}\Delta &\equiv W^{DI} - W^{NI} \\ &= \pi(1 - k_1) [u(c^E) - u(0)] + (1 - \pi) [u(c^L) - (k_1 u(\beta c^E) + (1 - k_1) u(0))] \\ &\quad - \omega(q_1^* - q_1^{NI}) B_0^B + \frac{\omega}{R} [(p^{DI} - p^{NI}) \gamma + \psi] \bar{A}.\end{aligned}$$

where p^{NI} is given by (50) and p^{DI} is given by (48).

References

- ACHARYA, V., AND S. STEFFEN (2013): “The Greatest Carry Trade Ever? Understanding Eurozone Bank Risks,” NBER Working Paper #19039.
- ACHARYA, V., AND T. YORULMAZER (2008): “Cash-in-the-Market Pricing and the Optimal Resolution of Bank Failures,” *Review of Financial Studies*, 21(6), 2705–42.
- ACHARYA, V. V., I. DRECHSLER, AND P. SCHNABL (2011): “A pyrrhic victory?-bank bailouts and sovereign credit risk,” Discussion paper, National Bureau of Economic Research.
- CALVO, G. (1988): “Servicing the public debt: The role of expectations,” *The American Economic Review*, pp. 647–661.
- COLE, H., AND T. KEHOE (2000): “Self-Fulfilling Debt Crises,” *Review of Economic Studies*, 67(1), 91–116.
- COOPER, R. (2012): “Fragile Debt and the Credible Sharing of Strategic Uncertainty,” NBER Working Paper # 18377.
- COOPER, R., AND H. KEMPF (2013): “Deposit Insurance and Orderly Liquidation without Commitment: Can we Sleep Well?,” NBER Working Paper #19132.
- CORSETTI, G., AND L. DEDOLA (2012): “Fiscal Crises, Confidence and Default A Bare-bones Model with Lessons for the Euro Area,” mimeo.
- DIAMOND, D., AND P. DYBVIG (1983): “Bank Runs, Deposit Insurance and Liquidity,” *Journal of Political Economy*, 91, 401–19.

- EATON, J., AND M. GERSOWITZ (1981): “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, 42(2), 289–309.
- FARHI, E., AND J. TIROLE (2014): “Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops,” mimeo.
- GENNAIOLI, N., A. MARTIN, AND S. ROSSI (2013): “Sovereign Default, Domestic Banks and Financial Institutions,” *Journal of Finance*.
- HANNOUN, H. (2011): “Financial crises: the role of deposit insurance,” Presentation, Bank for International Settlements.
- ROCH, F., AND H. UHLIG (2012): “The Dynamics of Sovereign Debt Crises and Bailouts,” http://www.ecb.europa.eu/events/conferences/shared/pdf/researchforum7/Uhlig_Harald.pdf?090aa93a65cfbc331785bf7f09c79737, mimeo.
- UHLIG, H. (2013): “Sovereign Default and Banks in a Monetary Union,” NBER Working Paper #19343.