

What is Formal Logic?

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Philosophy

What is logic about?

- Sometimes said logic not concerned with meaning - it is ***form*** not ***content***. But this is misleading.
- Logic studies meanings of ***logical constants***.
- Formal systems are syntactic objects
- Semantics teaches us about ***logical truth*** and ***logical consequence***.

In the beginning

- Aristotle: *sylogistic logic*
- **Categorical propositions** have 4 LFs:
 - A: universal affirmative All S are P
 - E: universal negative No S is P
 - I: particular affirmative Some S are P
 - O: particular negative Some S are not P
- LFs property of **quantity** and **quality**, not **content** (from subject and predicate).

Problems for Aristotelian Logic

- ‘Socrates is mortal’ and ‘all men are mortal’ have same (S-P) LF.
- No real formalization of logic/patterns of inference.
- No extension to ***modal operators***.
- No analysis of ***sentence connectives***.

Modern Logic

- Revolution in logic: Frege, Russell, Peano, Boole, Schroeder, etc.
- Birth of Mathematical Logic
- Logicism and axiomatics: PL, QL, PM
- Modal Logic
- Metalogic
- Non-Classical Logic

Formal Logic

- Formal Systems - axiomatics and natural deduction
- Proofs
- Specification of logical form - e.g. theory of descriptions
- Interpreting formal systems, model theory and possible world semantics

*14.1. $\vdash \cdot [(ix)(\phi x)] \cdot \psi(ix)(\phi x) \equiv (\exists b) : \phi x \equiv_x \cdot x = b : \psi b$
 [*4.2. (*14.01)]

In virtue of our conventions as to the scope intended when no scope is explicitly indicated, the above proposition is the same as the following:

*14.101. $\vdash \cdot \psi(ix)(\phi x) \equiv (\exists b) : \phi x \equiv_x \cdot x = b : \psi b$ [*14.1]

*14.11. $\vdash \cdot E!(ix)(\phi x) \equiv (\exists b) : \phi x \equiv_x \cdot x = b$ [*4.2. (*14.02)]

*14.111. $\vdash \cdot [(ix)(\psi x)] \cdot f\{(ix)(\phi x), (ix)(\psi x)\} \equiv :$
 $(\exists b, c) : \phi x \equiv_x \cdot x = b : \psi x \equiv_x \cdot x = c : f(b, c)$

Dem.

$\vdash \cdot *4.2. (*14.04.03) \cdot \supset$

$\vdash \cdot [(ix)(\psi x)] \cdot f\{(ix)(\phi x), (ix)(\psi x)\} \equiv :$

$[(ix)(\psi x)] : [(ix)(\phi x)] \cdot f\{(ix)(\phi x), (ix)(\psi x)\} :$

[*14.1] $\equiv \cdot [(ix)(\psi x)] \cdot (\exists b) : \phi x \equiv_x \cdot x = b : f\{b, (ix)(\psi x)\} \cdot :$

[*14.1] $\equiv \cdot (\exists c) : \psi x \equiv_x \cdot x = c : (\exists b) : \phi x \equiv_x \cdot x = b : f(b, c) \cdot :$

[*11.55] $\equiv \cdot (\exists b, c) : \phi x \equiv_x \cdot x = c : \psi x \equiv_x \cdot x = b : f(b, c) \cdot \supset \vdash \cdot \text{Prop}$

*14.112. $\vdash \cdot f\{(ix)(\phi x), (ix)(\psi x)\} \equiv :$

$(\exists b, c) : \phi x \equiv_x \cdot x = b : \psi x \equiv_x \cdot x = c : f(b, c)$

[Proof as in *14.111]

In the above proposition, we assume the convention explained on p. 174 after the statement of *14.03.

*14.113. $\vdash \cdot [(ix)(\psi x)] \cdot f\{(ix)(\phi x), (ix)(\psi x)\} \equiv \cdot f\{(ix)(\phi x), (ix)(\psi x)\}$
 [*14.111.112]

This proposition shows that when two descriptions occur in the same proposition, the truth-value of the proposition is unaffected by the question which has the larger scope.

*14.12. $\vdash \cdot E!(ix)(\phi x) \cdot \supset : \phi x \cdot \phi y \cdot \supset_{x,y} \cdot x = y$

Dem.

$\vdash \cdot *14.11 \quad \supset \vdash \cdot \text{Hp} \cdot \supset : (\exists b) : \phi x \equiv_x \cdot x = b$

$\vdash \cdot *4.38 \cdot *10.1 \cdot *11.11.3 \cdot \supset$

$\vdash \cdot \phi x \equiv_x \cdot x = b : \supset : \phi x \cdot \phi y \cdot \equiv_{x,y} \cdot x = b \cdot y = b \cdot$

[*13.172] $\quad \supset_{x,y} \cdot x = y$

$\vdash \cdot (2) \cdot *10.11.23 \cdot \quad \supset \vdash \cdot (\exists b) : \phi x \equiv_x \cdot x = b : \supset : \phi x \cdot \phi y \cdot \exists_{x,y} \cdot x = y$

$\vdash \cdot (1) \cdot (3) \cdot \quad \supset \vdash \cdot \text{Prop}$

*14.121. $\vdash \cdot \phi x \equiv_x \cdot x = b : \phi x \equiv_x \cdot x = c : \supset \cdot b = c$

SECTION B]

Dem.

$\vdash \cdot *10.22 \cdot \quad \supset \vdash \cdot$

[*13.191]

$\vdash \cdot *4.71 \cdot \quad \supset \vdash \cdot$

[*10.11.27] $\quad \supset \vdash \cdot$

[*10.281]

[*13.195]

$\vdash \cdot (2) \cdot *5.32 \cdot \supset \vdash \cdot \phi$

$\vdash \cdot (1) \cdot (3) \cdot \quad \supset \vdash \cdot \text{Pr}$

The two following are the analogy with *14.1 of couples (*55 and *5

*14.113. $\vdash \cdot \phi(x, w) \equiv$

Dem.

$\vdash \cdot *11.01 \cdot \quad \supset \vdash \cdot \phi(x$

$= \phi(x$

[*13.01] $\quad = \phi(x,$

$\vdash \cdot *4.71 \cdot \quad \supset \vdash \cdot \phi(x,$

$\supset :$

[*11.11.02] $\quad \supset \vdash \cdot \phi(x,$

$\supset :$

[*11.041] $\quad \supset :$

[*11.041] $\quad \supset :$

[*11.041] $\quad \supset \vdash \cdot \phi(x, y$

[*11.041] $\quad \supset \vdash \cdot \text{Prop}$

[*11.041] $\vdash \cdot (\exists x, y) : \phi(x, y)$

$\equiv \cdot (\exists x, y)$

[*11.041] $\supset \vdash \cdot \supset$

[*11.041] $\supset \vdash \cdot \supset$

Logic and Philosophy

- ‘I hold that logic is what is fundamental in philosophy, and that schools should be characterized rather by their logic than by their metaphysic’ (Bertrand Russell).
- Logic central to ***Analytic Philosophy***.
- As a method, the method of logical analysis prizes clarity, rigour, precision.

An example

- Ontological Argument (roughly!):
 - God has all perfections
 - Existence is a perfectionTherefore:
 - God exists
- Kant: existence is 'not a predicate'.
- Frege: existence is a higher-order function (quantifier).

Relevance outside Philosophy

- Clearly precision, clarity, rigour, etc. are virtues in all academic work.
- For students, logic introduced in a more informal way (Critical Thinking) to provide foundation for how to argue and think validly, clearly, and rationally.
- Exposes rhetorical ploys and fallacies.

Examples

- Formal fallacies: begging question, affirming the consequent, etc.
- Informal fallacies: Straw Man argument, fallacy of equivocation, Ad Hominem, Appeal to ignorance, authority, etc.
- Recognizing these fallacies is a crucial intellectual skill we should provide to all students.

Philosophy of Logic

- Some research topics:
- Universalism vs Pluralism: one logic or many?
- Logic and Language:
 - Is language logical?
 - Is logical form linguistic?
- Logic and Metaphysics: what are possible worlds, propositions, etc?

Logic @ Manchester

- PHIL10041: Critical Thinking
- PHIL10052: Introduction to Logic
- PHIL20252: Philosophical Logic
- PHIL30011: Special Author: Russell

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