Confirmatory factor analysis

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Historical Origins

- Factor analysis has been the prime statistical technique for the development of structural theories in social science, such as the hierarchical factor model of human cognitive abilities, or the Five Factor Model of personality.

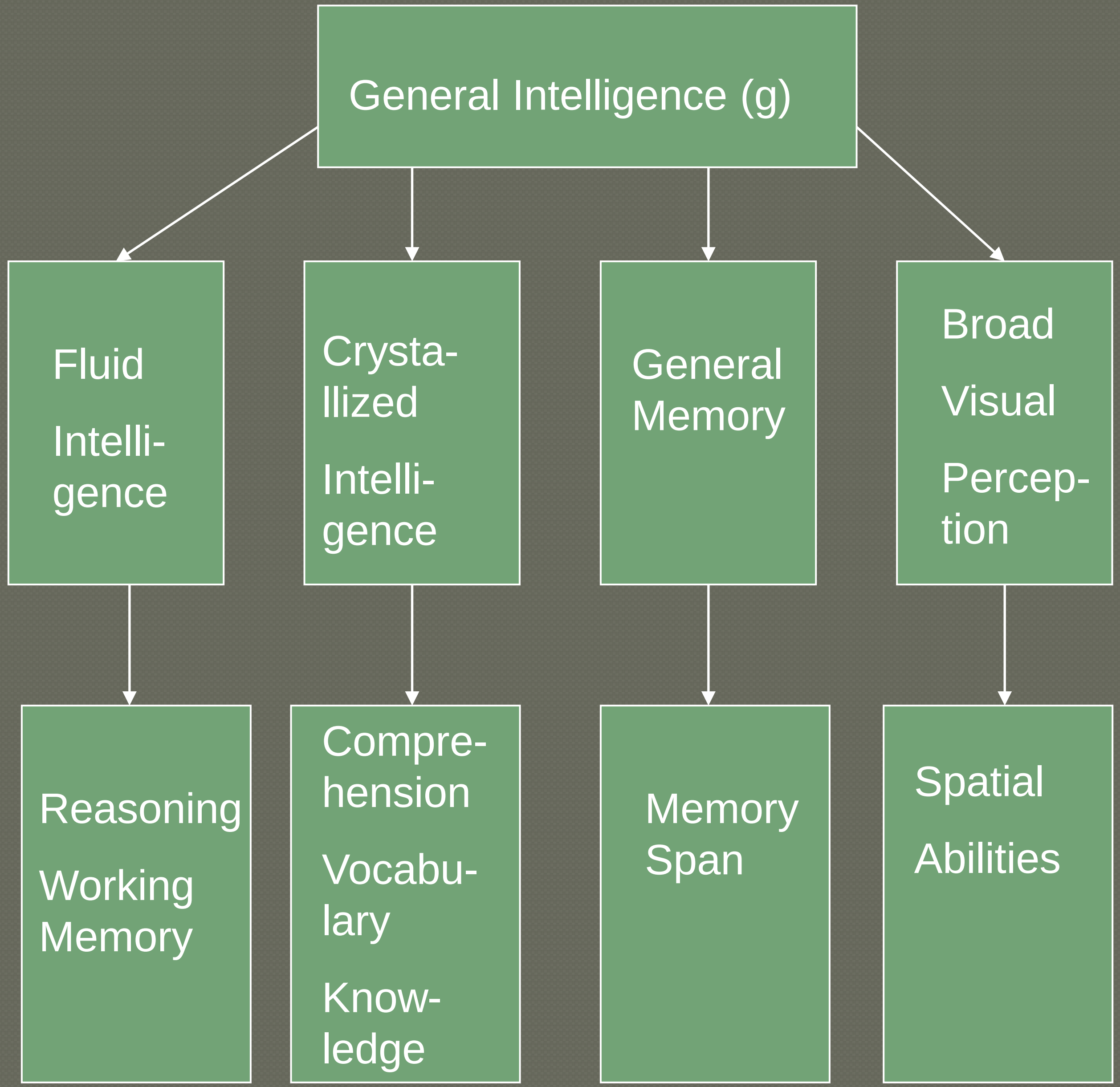

Figure 1. Part of Carroll's hierarchical factor model of human cognitive abilities

Essence of Confirmatory Factor Analysis

- A structural model is hypothesized in advance
- This model specifies:
  - The number of factors
  - The relationship between observed variables and factors
  - Error terms which comprise unique factors plus measurement error

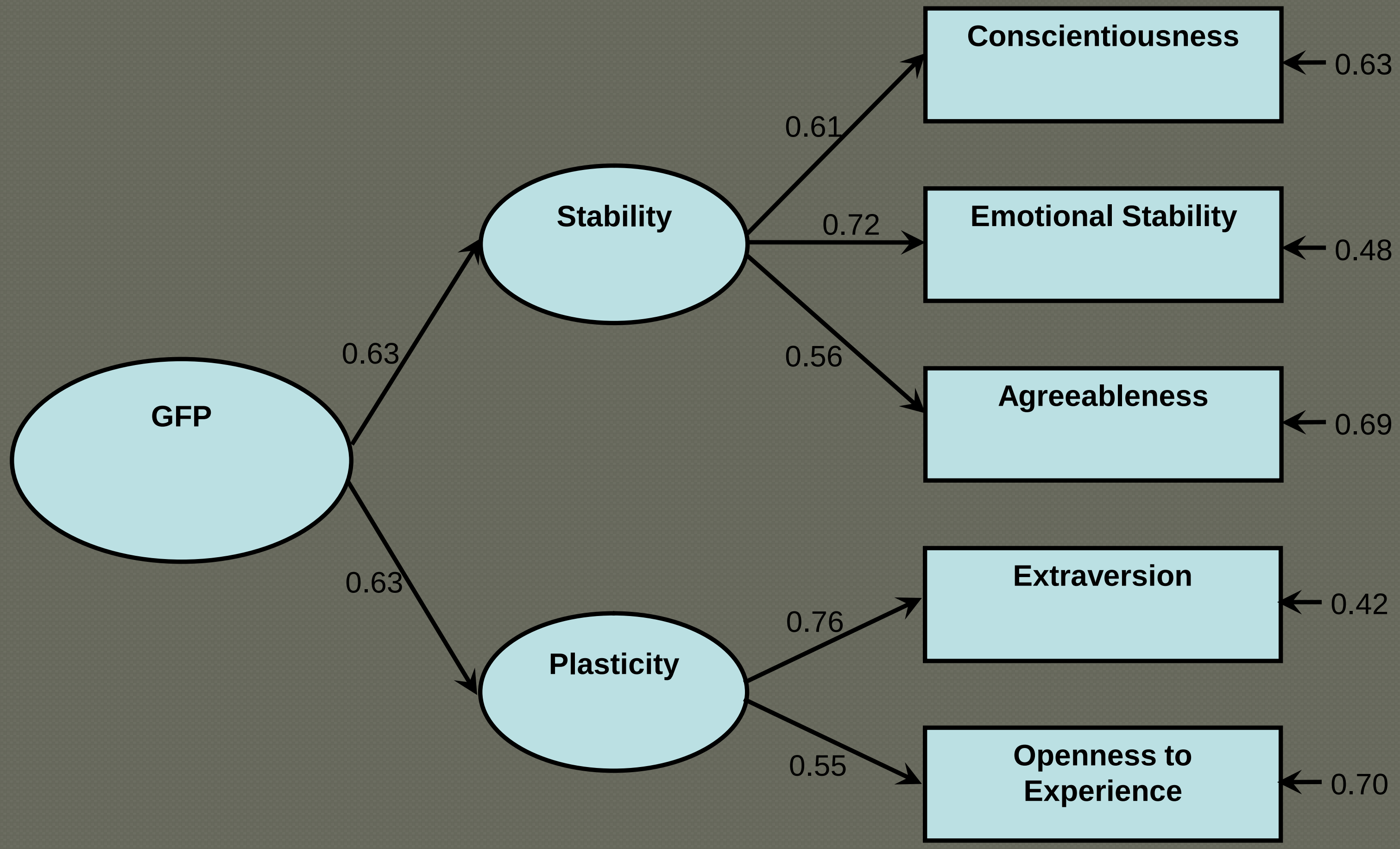

Figure 2. The General Factor of Personality (GFP) in the Big Five.
A hierarchical model of personality from the Big Five to the Big Two to the GFP.
Estimates in the figure are based on medians derived from Digman's (1997)
14 samples. From Rushton and Irwing (2008).

Estimation of a Factor Model

- Data is a sample correlation matrix
- The model is specified a priori:
  - The number of factors
  - The relationship between observed variables and factors
  - Error terms which comprise unique factors plus measurement error
- Model parameters (factor loadings and error variances) are estimated so that the factor model generates a correlation matrix which is as close an approximation to the sample correlation matrix as possible. This is called the model implied correlation matrix.

Table 1.

Inter-scale correlations from Digman's (1997) 14 Studies of the Big Five factors (N = 4,496).

	Openness	Conscientiousness	Extraversion	Agreeableness	Emotional Stability
		Sample			
Openness	1.00				
Conscientiousness	0.19	1.00			
Extraversion	0.42	0.17	1.00		
Agreeableness	0.07	0.35	0.09	1.00	
Emotional Stability	0.12	0.42	0.23	0.41	1.00
		Model Implied			
Openness	1.00				
Conscientiousness	0.13	1.03			
Extraversion	0.42	0.18	1.00		
Agreeableness	0.12	0.36	0.17	1.02	
Emotional Stability	0.16	0.47	0.22	0.43	1.04
		Residuals			
Openness	0.00				
Conscientiousness	0.06	-0.03			
Extraversion	0.00	-0.01	0.00		
Agreeableness	-0.05	-0.01	-0.08	-0.02	
Emotional Stability	-0.04	-0.04	0.01	-0.02	-0.04

Note: From Rushton and Irwing (2008).

Estimation

- The preferred method of estimating the parameters of a factor model given data which is continuous and multivariate normal is maximum likelihood.
- Maximum likelihood solutions have a number of desirable properties with two of the most important being that:
  - Parameter estimates are a close approximation to population parameters (consistency);
  - The fit function times $(N - 1)$ approximates a chi-square distribution, which provides a significance test of overall model fit.

Maximum likelihood estimation

- The basic model equation for the common factor model is:
  - $Y = \Lambda X + \Psi E$
- It can be shown that the model implied covariance matrix is:
  - $\Sigma = \Lambda\Phi\Lambda' + \Theta$
- Maximum likelihood begins with an approximate solution, e.g. least squares
- It then minimizes the fit function:
  - $F = \ln|\Sigma(\Theta)| + tr(S\,\Sigma(\Theta)^{-1}) - \ln|S| - \rho$

Fit statistics

- The likelihood ratio test:
  - F (N – 1) follows a chi-square distribution
- Problem is that the likelihood ratio test is dependent on N, and is invariably significant in large samples, hence many approximate fit statistics have been suggested:

Recommended approximate fit statistics

- Root mean square error of approximation (RMSEA): cut value of < 0.06
- Tucker-Lewis Index (TLI): cut value of ≥ 0.95
- Standardized Root Mean Square Residual (SRMSR): cut value of ≤ 0.05

Identification

- No more model parameters than are contained in the sub-diagonal of the sample covariance matrix can be estimated. However, it is desirable that there are many fewer model parameters than this, resulting in what is called an overidentified model.
- In order to ensure overidentification, it is usually recommended that there are at least 3 to 4 indicators per factor.
- In order to establish the scale of the factor either the variance is set at 1 or one of the factor loadings is set at 1.
- It is possible to set up other restrictions in order to identify models.

Simple structure

- To facilitate interpretability, Thurstone suggested that factors should conform to simple structure. Commonly, this is interpreted as meaning that each indicator should load on one and only one factor, and that all other factor loadings of each indicator should be equal to 0.

Unidimensionality

- The related idea of unidimensionality specifies that each factor should measure one and only one thing.
- A necessary condition for simple structure and unidimensionality to be achieved is that the model contains the correct number of factors

Eigenvalue

- The amount of variance explained by a factor is related to something called the **eigenvalue.**
- **The eigenvalue** is the sum of the squares of the factor loadings.
- The total eigenvalue of all the factors is equal to the sum of the number of items.

No. of factors

- In exploratory factor analysis the eigenvalue forms the basis of a number of heuristics in order to determine the number of factors
- The two commonest rules are:
  - The scree test which is based on a plot of the eigenvalues against the factor number.

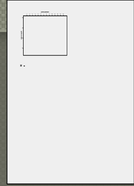

Figure 3. Scree plot of 14 scales from the US standardization sample of the WAIS-III

Kaiser or Eigenvalue criterion

- Since each item has an average eigenvalue of 1, a factor with an eigenvalue less than 1 would explain less variance than an individual item. It is, therefore, proposed that the number of factors is equal to all those factors with eigenvalues ≥ 1.
- Neither of these rules is correct

Practical way of determining the number of factors

- Base initial estimates on theory or one of the better heuristics (e.g. parallel analysis or Velicer's Map test) used in exploratory factor analysis. (A new and probably better alternative is exploratory structural equation modelling.)
- Test this solution for fit using confirmatory factor analysis
- If the model fits then accept it. If the model does not fit then use the exploratory solution which contains one more factor
- Continue until the CFA model fits

The Five Factor Model: A moral tale

- Gordon Allport: Comprehensive search of words describing personality – found around 18,000
- Allport & Odbert (1936), 171 unique trait names
- Cattell used the 171 unique trait names of Allport & Odbert (1936), to obtain ratings of 100 men by colleagues. This was repeat for 208 men with a shortened list

Five Factor Model

- Based on reduced list of 35 adjectives identified by Cattell (Tupes & Christal, 1961; Norman, 1963)
- MaCrae and Costa (1985, 1987, 1990) extended the above work to produce the five-factor model
  - Neuroticism
  - Extroversion
  - Openness
  - Agreeableness
  - Conscientiousness

Cautions

- The Five Factor Model is based on a very restricted list of adjectives
- The predominant method of analysis was Principal Components, not common factor analysis
  - The principal components model is an observed variable model
  - It assumes no errors of measurement
- The Five Factor Model does not fit when tested using CFA
- Conclusion: CFA is not appropriate for analysing personality data.

Recommended computer package

- Muthén, L. K. & Muthén, B. O. (2010). Mplus Version 6. Los Angeles, CA: Muthén & Muthén

Recommended sources

- Mplus website: http://www.statmodel.com/
- Kline, R. B. (2005). *Principles and Practice of Structural Equation Modelling*. London: The Guilford Press.
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