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REWORKING THE STANDARD MODEL OF COMPETITIVE MARKETS: THE ROLE OF FUZZY LOGIC AND GENETIC ALGORITHMS IN MODELLING COMPLEX NON-LINEAR ECONOMIC SYSTEMS

Peter Smith, Visiting Fellow
IDPM, University of Manchester

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University of Manchester
External Affairs Office
Harold Hankins Building, Precinct Centre,
Oxford Road, Manchester M13 9QH, UK
Tel: +44-161 275 2814
Email: idpm@man.ac.uk Web: http://idpm.man.ac.uk

REWORKING THE STANDARD MODEL OF COMPETITIVE MARKETS: THE ROLE OF FUZZY LOGIC AND GENETIC ALGORITHMS IN MODELLING COMPLEX NON-LINEAR ECONOMIC SYSTEMS

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Abstract

Some aspects of economic systems (eg, nonlinearity, qualitative variables) are intractable when incorporated into models. The widespread practice of excluding them (or greatly limiting their role) produces deviations of unknown size and form between the resulting models and the reality they purport to represent. To explore this issue, and the extent to which a change in methodology can improve tractability, a combination of two techniques, fuzzy logic and genetic algorithms, was applied to the problem of how the sellers in a freely competitive market, if initially trading at different prices, can find their way to supply/demand equilibrium. A multi-agent model was used to simulate the evolution of autonomously- learnt rule-governed behaviour, (i), under perfect competition, and (ii), in a more commercially realistic environment. During the learning process, markets may lack a true equilibrium price, and therefore sellers in such a model cannot be price-takers in the conventional sense; instead, it was stipulated that they would set an asking price, buyers would shop around for cheap supply, and the sellers would revise their pricing policy according to its profitability. Each firm's pricing policy was embedded in a fuzzy ruleset; the rulesets were improved over time by successive passes of the genetic algorithm, using profit level as a measure of Darwinian fitness. The simulated evolution was repeated over a random sample of 10 markets.

Under perfect competition, sellers' asking prices converged onto the theoretical equilibrium price. This performance was maintained when either uncertainty in demand or a more commercially realistic set of dynamics was introduced. However, when both these features were introduced simultaneously, different, substantially lower equilibrium prices were reached. In both cases, autonomous learning by the sellers suppressed the instability that might have been expected to result from the introduction of a number of nonlinearities. Other possible applications of the methodology are discussed, along with some of its implications.

Keywords: competition, markets, Walrasian Crier, equilibrium, fuzzy logic, genetic algorithms, evolutionary algorithms

INTRODUCTION

It is widely accepted that many aspects of economic (and socio- economic) systems are difficult to incorporate into models because of the mathematical intractability that results. These aspects include nonlinearity, the presence of inherently qualitative variables, and inconvenient statistical properties that make analysis and forecasting difficult. They contribute to a number of economic problems in which a static (but not necessarily stable) equilibrium can be shown to exist, but is either uncomputable, or there is no known mechanism by which it can be reached (Dosi *et al*, 1999; see also Stemp and Herbert, 2003).

Other disciplines have encountered similar problems, and a consensus has begun to build up across a number of fields – including control engineering, robotics, and computational biology and neurobiology – about how such problems should be addressed (Luger, 2001). This alternative consensus (on the investigation of complex systems) favours a switch from the axiomatic/ deductive approach, to one that relies on simulation and experimentation. Over the last few years, it has begun to infuse the more flexible fringes of economics, as shown by the special issue of *Journal of Economic Dynamics and Control* (Vol 24, see Dechert and Hommes, 2000) on complex nonlinear dynamics and computational methods. Many of its practitioners make extensive use of two techniques, fuzzy logic (FL), and various types of evolutionary algorithm (EAs), of which genetic algorithms (GAs) are most commonly used.

FL can work with vaguely defined models, process qualitative propositions, and handle approximations; the latter ability is the basis of its usefulness in situations with nonlinearity and complex dynamics. FL is commonly applied in the form of a fuzzy system (FS), a ruleset that might, for example, determine the response of agents in a model to market conditions. Smithson (1987) discusses the validity of FL as a model of agents' decision processes in complex situations; Ross (1995) gives a thorough review of the theory and methods; and Terano et al (1994) discuss a wide range of industrial and management applications. GAs can be used to improve rulesets of the kind just mentioned on the basis of their relative fitnesses; and they can be used to endow the agents in multi-agent simulations with the power to learn their own rules for survival and prosperity, independent of the experimenter's theories. The classic account of GA is in Goldberg (1989); Reeves and Rowe (2002) present a more complete and up-to-date account.

The main hypothesis of the current work is that a combination of FL and GA may render more realistic models of economic systems tractable; that, for example, it could give us the ability to handle models with non-linear responses to market conditions; actions whose effects are lagged over several periods; liquidity constraints (which imply the possibility of bankruptcy, and a varying aggregate supply function); and complex relations between buyers and sellers.

The problem of the Walrasian Crier has been chosen as the entry-point for this study. This problem concerns process by which the firms in a market, if initially trading at disparate prices, can find their way to the perfect competition[2] equilibrium point. (It is not a trivial problem, even in a dynamically simple model such as the standard microeconomic one.) It was originally identified by Walras, in his pioneering work on general equilibrium at the end of the 19th century (see Hunt, 1992: 340); Walras acknowledged that he was unable to resolve the need for such an agent, and the issue has remained with us.

There is also a subsidiary hypothesis: that models of complex competitive markets may not share all the properties of the standard one (for example, they might not have stable attractors, or they might have stable attractors which do not allocate resources in an optimal, efficient way). However, investigation of the usefulness of the methodology remains the study's primary aim.

Fuzzy Logic (FL)

Often, it is possible to find a qualitative description of a complex system and base a management regime on that description, even when it is not possible to express that knowledge in a set of simultaneous partial differential equations that can be solved to yield a generally-applicable principle (see, for example, Terano et al, 1994). In such situations, FL can provide an alternative. In the current context, a firm in such a setting might manage its affairs through a set of conditional rules of the following kind:

> If the current profitability is *low*, and
> If inventory levels are *low*,
> Then set asking price *high*[3].

The convention is to write the last line of such a structure in the form "Then prices are high". The resemblance to a conventional syllogism is obvious; and, in a context in which the rules are always operated, there is no real difference between a rule and a syllogism. FL is a valid method for processing such syllogisms, provided certain conditions are met:

> i) The fuzzy terms (italicised in the example) must have a meaning that is generally understood within the relevant community, even if its members may disagree about the classification of borderline cases between, say, "low" and "moderate" levels of inventory. (We have no difficulty in interpreting such terms – for example, "growth in the third quarter is likely to be only *moderate*" – when we encounter them in an economic commentary, and we know the context. However, without FL, we have no way of incorporating this ability into our theoretical analyses.)
>
> ii) The conclusions have to be based on valid theory or empirical evidence.
>
> iii) The syllogisms have to be part of a fuzzy system or ruleset, providing a response to every combination of levels of the input variables of interest (profit and inventory in the example).
>
> iv) It must be feasible to collect and analyse the data and generate responses in real time: FL copes with systems with difficult dynamics through repeated cycles of observation and reaction.
>
> v) The problem must be one for which FL's hybridization approach to ambiguous or borderline cases is meaningful. When fuzzy logic outperforms classical methods in complex dynamical systems[4], its advantage appears to derive from its ability to create hybrid solutions for time T, which are influenced by any elements of ambiguity in the data from time T-1; the method restrains over-commitment to a single interpretation of what was going on. This may be neither possible nor desirable in certain types of problem.

FL uses the same theorems, rules, and inference mechanisms as conventional logic[5], with only two exceptions. The exceptions are *The Law of the Excluded Middle* (which stipulates that things which are capable of being either X or not-X must be one or the other, with no intermediate state allowed); and *The Law of Contradiction* (which forbids anything from being both X and not-X)[6]. The reason for these exclusions is that fuzzy categories have to be allowed to overlap (see below). Logicians and philosophers have shown deep antipathy towards this change; even the philosopher Willard Quine, known for his work on self-referential paradoxes, is quoted as saying "it is hard to face up to the rejection of anything so basic [as the above two laws]" (McNeill and Freiburger, 1994: 60). However, FL underpins the normal human ability to deal with fuzzy, vague, and ambiguous knowledge; it is an equally rigorous but *different* calculus of propositions from conventional logic.

The fundamental difference between FL and conventional logic is that FL uses membership functions, which say *how good a member* of a particular fuzzy set the case at hand is, rather than the truth values of conventional logic, which say *whether or not* it is a member of a crisp set. Figure 1 shows a set of membership functions, A, B, and C, each defining a fuzzy set, "low", "moderate", and "high", respectively, covering the range 0.0 – 80.0 units of some hypothetical inventory. Each function gives the membership (in itself) of any level of inventory. Each function has a domain, ie, the range spanned by its base; inputs outside of this domain do not activate it, and it is irrelevant to them. Each function also has a core (in this case, a single value), with a membership value of 1.0; inputs that lie within (or at) the core value(s) are perfect, or typical members. The nature of individual rules in a fuzzy system are commonly determined by theoretical knowledge about the behaviour of these typical cases[7]. As drawn, the triangular function A, for example, implies that the type case of "low" is in fact zero inventory, but that somewhat higher values should be recognized as "low-ish", and allowed a (declining) influence on decisions, all the way up to 40 units. (At the same time, the membership of the particular instance in B, "moderate inventory", is increasing, and acquiring a growing influence.) Membership functions do not have to be triangular, or even segmentally linear; however, the triangular version is computationally convenient, and is widely used.

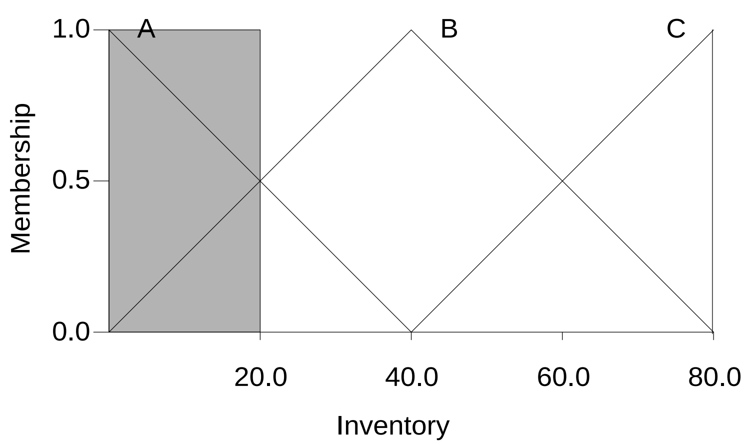

Figure 1: Three Overlapping Numerically-based Membership Functions, A, B, & C.
A and C are right-angled triangles; B is an isosceles triangle.
The shaded area represents a crisp alternative to A.

The contrasting term to fuzzy is *crisp*. Conventional logic uses crisp sets, which have precise boundaries, and allow no ambiguity about which set a particular case belongs to; for example, "low inventory" might be defined crisply as greater than zero, and less than or equal to 20 units – the area shown in grey in the Figure.

In the models described in this study, each rule defines a response to a different combination of conditions (inputs). The membership functions for the two inputs used are both numerically-based, as in the above example. The commonest inference method for numerically-based fuzzy problems is illustrated in Figure 2. This is part of a 2-input, 1-output fuzzy system, which is composed of rules such as:

```
RULE A …

RULE B
If <input 1 is moderate>
and <input 2 is low>
then <output is moderate>

RULE C
If <input 1 is moderately high>
and <input 2 is moderately low>
then <output is high>

RULE D … etc.
```

A given pair of inputs can have any combination of full-, partial-, or zero- membership in either rule. As in conventional logic, the truth of an AND operation is the lesser of the truth values of its inputs. Figure 2(a) shows how this principle works. Because the values of the inputs lie within the domains of both B and C, both rules are activated (other, non-activated rules are not shown). In each case, AND selects the lower of the two membership values; this value, $x_{m,}$ measures how good a member of the rule (ie, of both its clauses, jointly) that particular combination of inputs is. If the inputs have less than perfect membership in the rule, this has to be reflected in that rule's contribution to the overall response of the fuzzy system to those inputs; and this is achieved by truncating the profile of the output membership function at the level of x_m. For a rigorous justification of this procedure, see Ross (1995: 239*ff*)[8]; note that truncation tends to affect the weight given to the rule's contribution much more than its value. The aggregate effect of the two rules is obtained by summing them algebraically (Fig 2(b)); and the summed profile is converted to a single crisp value (*defuzzified*, Fig 2(c)), by taking the location of its centroid as the representative value. (These are the options chosen for this study; Ross [*op cit*: 135 *ff*] discusses alternatives.)

Applications of FL to the problem of rendering complex economic systems more tractable are few. Tay and Linn (2001) applied it to the Santa Fe artificial stock market (Arthur *et al*, 1997), because they felt that the original formulation (described briefly below) had created a somewhat implausible model of the learning process in which agents (the traders on the market) had to apply a very large library of library of conditional rules. FL not only resolved this problem, but improved the match between the behaviour of the artificial market and its real-world counterparts.

Evolutionary Algorithms

EAs provide tools for studying the evolution of complex systems; the technique has been widely used for optimizing control variables in complex systems (see, for example, Terano *et al*, 1994), although in the current study the main focus is on the evolution and mutual adaptation of a population of firms. These techniques all rely on the fact that solutions to a wide range of problems can be coded as strings of digits. These strings may represent a single quantity (crisp or fuzzy), a vector of quantities, or a vector of variables specifying a choice among rules or strategies. For example, in this study, there were fifteen rules, arising from a combination of five levels of one input variable crossed with three levels of the other. Each rule might be specified by three sets of numbers, one for each of the two input membership functions, and one for the output membership function; and each set of numbers might consist of three numbers, specifying the ends of the domain, and the position of the peak (core). This would enable the ruleset of one firm to be encoded in a string of 135 real numbers. (A much simpler arrangement than this was eventually adopted, see below.)

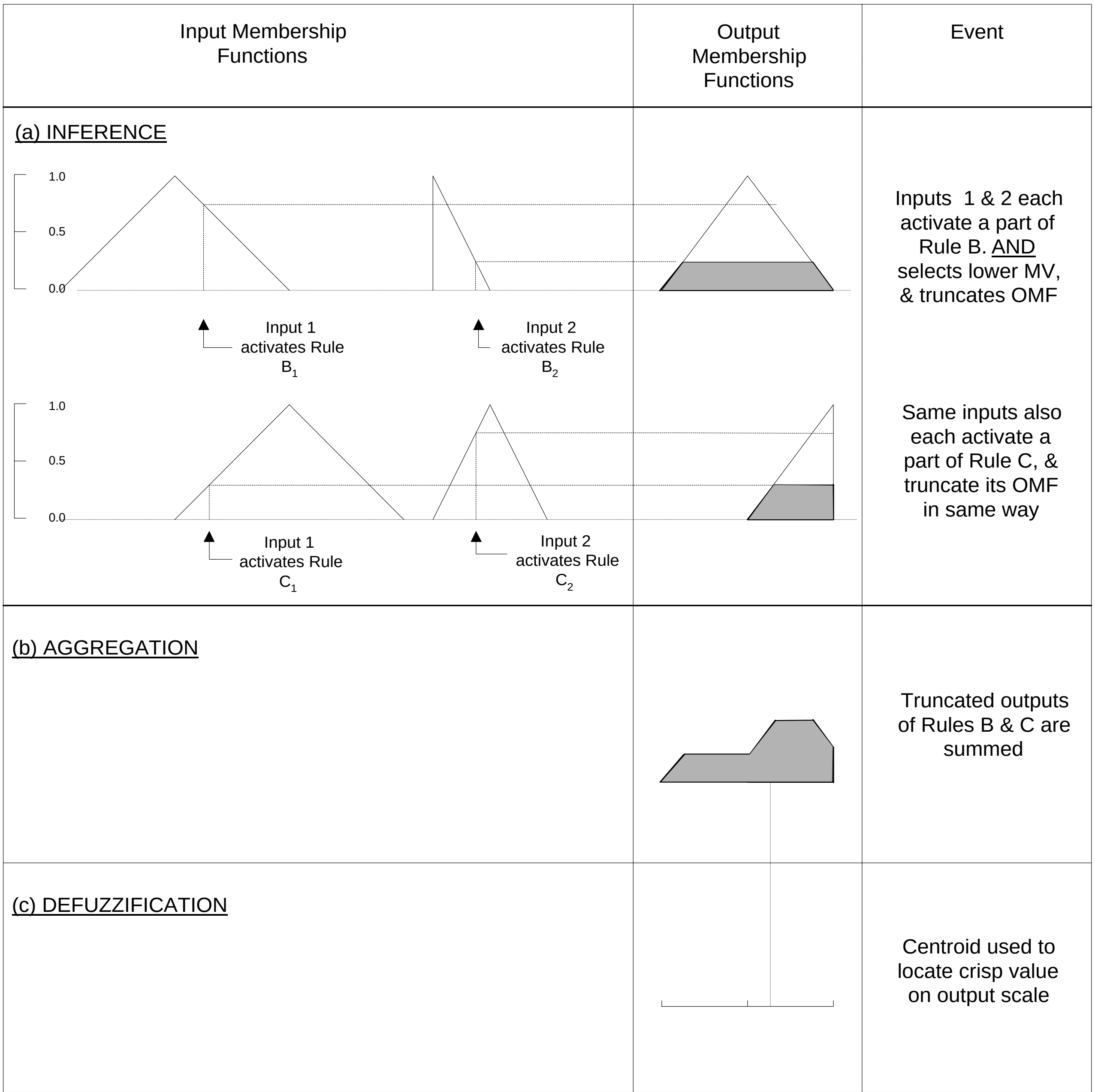

Figure 2: Inference in Fuzzy Systems

MV = Membership value
OMF = Output membership function

Such strings are analogous to the string of genetic information on a chromosome. This suggests that a population of such strings might be made to evolve in a manner closely similar to biological evolution, provided that the investigator can (i) define a suitable encoding scheme, (ii) identify a suitable fitness measure, and (iii) ensure that this influences the frequency of matings (better strings getting the opportunity to produce more "offspring", of course) in an appropriate way. Once rulesets have been identified as parents of the next generation, their strings are paired up, material is exchanged between them in a process based on the crossover that happens in biological ("wet") genetics, and mutations are inserted. The offspring are used to replace either the whole or part of the existing population.

The chief variants on the basic genetic mechanism for stochastic search are:

- *Evolutionary Strategies,* in which the problem is coded as a string of real numbers, and mutation is the primary method of search;
- *Genetic Algorithms*, which are often (but not necessarily) binary-coded, and which use both crossover and mutation;
- *Classifiers*, which breed conditional rules governing the responses of agents; and
- *Evolutionary Programming*, where the genetic process produces segments of executable program governing their responses.

In the specific form of GA, the technique became widely known after the publication of Goldberg's (1989) book, although various members of the wider class of evolutionary strategies had already been in use for more than two decades (Back, 1996). Goldberg advocated using codings based on short alphabets (specifically, the binary one, [0,1]); and he attributed the success of this version of the algorithm to a feature called "implicit parallel processing", and to the "building blocks" hypothesis. (The first is the alleged increased rate of progress resulting when the evaluation of one sub-string of a solution provides some information about related strings; for example, knowing the fitness of a substring 10001 conveys *some* information about all strings with the format 1***1, where * is a wild-card character, representing either 1 or 0. The second is the ability of high-fitness substrings to persist through the generations, and contribute to the emergence of elite overall solutions. Reeves and Rowe (2002) discuss these points at length.)

After Goldberg's book, there was a huge growth in interest in the subject. The potency of the method was undoubtedly greatly exaggerated, to the point where – as Reeves and

Rowe (2002) point out – it was claimed to be a general-purpose method for solving complex problems, including even NP-hard ones[9]. This led to the pattern of activity described by Goldberg (2003), in which individual investigators and teams experimented with the method on simple problems; obtained encouraging results; but were then disappointed by the failure of the technique to work on scaled-up problems.

There have been various responses to this state of affairs. One is to conclude that the whole approach is misguided; its advocates base their case on the *No Free Lunch* (NFL) Theorem, which asserts that (subject to some fairly restrictive conditions), the performance of any one evolutionary method is no better than random search, when averaged across all relevant problems (Reeves and Rowe, 2002). Another has been to re-analyse the core theory, often with particular emphasis on matching specific versions of the methodology with classes of problems; Back (1996) and Reeves and Rowe (*op cit*) are both examples of this school. A third response has been to examine the factors that make these larger problems less responsive to GA. Goldberg (2003) is a good example of this school; he and his co-workers have gone on to develop techniques such as Messy Genetic Algorithms (MGA), in which the length of the coded string and sequence of its sub-units are themselves genetically determined. A respected alternative view is that GA's prime role should be exploring the evolution of systems, rather than optimising functions (Holland, 1992).

The FL / GA Combination

These two techniques work together quite well, and there is an extensive literature on the combination (see Cordon *et al,* 2001), although the combination has not been used much in economics applications. One of its main advantages it that it can be used to optimize – or study the evolution of – a set of natural-language rules, which can then be compared to alternative prescriptions for action in the particular arena (in this case, economics). This is not true of all approaches to machine learning; learning in neural networks, for example, results in a set of changes distributed across the network. Interpretability has been a significant issue in economic applications of the methodology: Tay and Linn's (2001) fuzzy version of the Santa Fe Artificial Stockmarket is considerably easier to interpret than the mass of rules produced by the classifier algorithm which was used in the original study; and Dosi *et al* (1999) experienced some difficulty in interpreting the output of their evolutionary programming.

Applications in the Economic and Social Sciences

There has been a wide range of applications of genetic approaches to analysing complex social and economic phenomena. The most significant of these focus on systems of heterogeneous agents, with substantial nonlinearities. Published work includes the dynamics of attaining (or at least seeking) exchange rate equilibria in a nonlinear system of heterogeneous agents (Arifovic and Gencay, 2000); cobweb-type adjustments in populations of individually-motivated heterogeneous agents (Arifovic, 1994; Dawid and Koppel, 1998); pricing under oligopoly (Dosi *et al*, 1999); the problems for the rational expectations hypothesis of the coordination of expectations among agents in dynamically complex settings, in which expectations and system behaviour influence each other (Chiarella and He, 2003; Negroni, 2003); and investigations of the sources of deviations of the real behaviour of markets for financial assets from the predictions of the Efficient Market Hypothesis (Arthur *et al* 1997, Tay and Linn, 2001). In addition to these strictly economic investigations, there have been a number of important studies of emergent institutions and behaviours, including collaboration (Casti, 1992); and the establishment of transaction networks within a range of markets, which indicate that it is rational – in complex systems – for buyers and sellers to form long-term relationships (these are outlined by Tesfatsion, 2001).

In those cases in which researchers have tested the genetic learning approach on a standard, simplified and linearised version of a problem, the agents have learned the standard solution (see, for example, Vriend, 2000; Dosi *et al*, 1999; Dawid and Koppel, 1998; and Lettau, 1997). Performing such a test is not always easy, because of the incomplete nature of the standard model of perfectly competitive markets. In each of these cases, once a more complex set of conditions was imposed, the agents learned a different response; for example, the agents in the oligopolistic situation modeled by Dosi *et al* opted for a cost-plus pricing policy. These alternative, genetically-learned solutions are commonly stable; the major exception is provided by the work of Arifovic and Gencay on foreign exchange rate determination. This is a rather special case, because the model of the exchange rate process they used is indeterminate, providing a continuum of monetary equilibria, each with its corresponding rate; unsurprisingly, the rate fluctuates chaotically (*sensu stricto*). Dawid (1999) modeled learning (of asking and offer prices) in a sealed double-auction, with two separate populations of learners, the suppliers and the buyers; this also stabilized.

With very few exceptions (e.g., the Sandia model of the USA economy, Basu *et al*, 1996), the transformation of genetic information about the constitution of agents is converted into fitness via a very small set of rules, representing all the dynamics of the technical and economic domain – Negroni's (2001) study is based on a one-dimensional economy, for example. In some cases (such as the study by Arifovic and Gencay exchange rate determination), this has allowed continuity of analysis with older non-genetic, analytic studies of the same model; however, in others it may represent a missed opportunity to represent real systems more fully, and the cost may be a very substantial loss of generality, since it will usually be impossible to say much about the range of applicability of the results of ultra-simple models. It is difficult to believe that, in all such cases, the simplification has had no consequences for the behaviour of the systems being modeled; it is also unclear why this strategy has been so widely adopted, given the level of awareness that agent-based models with autonomous learning implemented through evolutionary algorithms offer a route away from excessive simplification, and towards enhanced realism. The additional computation – of a very straightforward kind – is scarcely a plausible reason.

In addition, the models have mostly been genetically very simple, often searching for a value of a single parameter. The majority of studies have relied on GA (generally with binary coding and overlapping generations). Exceptions to this statement include Dosi *et al* (1999), who used evolutionary programming; the Santa Fe artificial stock market, whose agents operated through a system of classifiers (crisp in the case of the original work by Arthur *et al*, 1997; fuzzy in the case of the later work by Tay and Linn, 2001); and Beltratti and Margarita (1996), who used neural networks. Vriend (2000) used both GA and classifiers in his comparison of the outcome of different learning processes.

Initially, in the mid- to late- 1990s, there was considerable enthusiasm for evolutionary approaches in general (and genetic algorithms in particular) as potential means of handling models of complex heterogeneous and nonlinear systems (see, for example, Tesfatsion, 2001, LeBaron, 2000, or Bullard and Duffy, 1998). From this, and the ambitious choice of topics in many of the earlier studies, one might have expected a growth of a wide range of increasingly realistic studies of complex economic systems which would pose a challenge to the capacity of the underlying genetic computing methodology.

This has not happened. Although there is a small number of studies in which agents in complex settings directly adapt their strategies in the light of experience, without a formal

GA, such as Westerhoff's (2003) work on the effects of a Tobin tax in a population of heterogeneous traders, the general tendency has been to focus on theoretical analyses (eg, Reichmann, 1999, 2001), and more philosophical studies with a social-darwinist or Schumpeterian background. The latter include Loasby (2000) on the evolution of institutions; Guth and Peleg (2001) on the linkage between increasing complexity and the validity of the standard (economic) assumption that maximizing the short-run payoff to self always pays (and, indeed, defines "economic rationality"); Knudsen (2002) on the semantic problems associated with the use of the "survival of the fittest" metaphor in economics; and the study by Berninghaus *et al* (2003) of the relationship between rational choice and evolutionary models.

It is not clear how far this represents a temporary hiatus in the flow of publications, and how far it reflects Goldberg's crisis of expectations. However, it may be significant that there has been little or no discussion of the problems of epistasis and genetically deceptive problems in the economics literature – this would have been expected as a prelude to any significant increase in the complexity of the problems being addressed.

The Standard Model of Perfectly Competitive Markets[10]

It is useful at this point to review the case which underpins the influence of the standard model of perfect competition on the imaginations of both economists and policymakers, to the exclusion of more complex representations of socio- economic systems; at the same time, this will give us the opportunity of identifying some of the deficiencies of that model, which are critical to the experimental details of the current work. The standard model is usually introduced graphically, by way of a diagram in which quantity is on the x-axis, price is on the y-axis, and in which the supply function increases monotonically, and the demand function decreases in the same fashion (see, for example, Samuelson and Nordhaus, 1992, Chapter 4). The demand function represents an aggregation of the demand functions of individual consumers. The supply function is an aggregation (summed parallel to the quantity axis) of firms' individual supply functions, which reflect the law of diminishing returns; individual supply curves also express each firm's marginal cost function. An equilibrium point exists where the graphs of the supply and demand functions intersect; this defines the true price of the commodity, at which demand and supply are in balance. The supply/ demand structure of the model used in the current work adheres to this intuitively appealing structure, avoiding "peculiar" features (such as backward-bending supply curves, and extreme elasticities[11]).

Within this structure, the diminishing-returns effect ensures that profit-maximizing firms will set their production level so that the cost of the marginal (ie, the last incremental) unit of production is equal to the price, since a lower level of production sacrifices some potential profit, and a higher level will incur losses. Firms are price-takers: they are sufficiently numerous to be unable (individually, and without collusion) to influence the market price by their production decisions, and can only accept the established price as it is manifested in the market. The model is silent on what happens before that price is established; it is also generally silent on all issues of how firms and markets adjust to change, and other dynamic issues, including decisions on plant renewal and replacement. The cobweb theorem, of course, addresses the issue of how markets home in on the equilibrium price, but assumes that " … the market price is always set at a level which clears the market" (Chiang, 1984: 562), ie, that the market functions as a single-minded entity, motivated by a primary need to ensure that demand and supply balance.

The most powerful form of this model is an axiomatic system (see, for example Eckert and Leftwich, 1988: 44*ff*), which can be used to show that a general equilibrium exists, at which all markets in an economy are at equilibrium; and to show that that equilibrium (although driven entirely by self-interest) has a number of optimal, socially attractive features. (In this context, optimality is judged by the Pareto criterion: according to this, an economic arrangement is optimal if no one can be made better off without making someone else worse off.) If we accept the claim that the Pareto criterion is valid and value-free, and some implicit assumptions about system dynamics, then it can be shown that freely competitive markets will ensure that everyone is as well off as possible, given the available resources and technology; and that the system of prices generated will ensure that resources are allocated in the unique way that makes this possible. Under these conditions, it is reasonable to maintain that the prices of goods and services are valid measures of their value to society. It can also be shown that there is no possibility of exploitation within such a system, since competition will ensure that the prices of all goods and services are made up of *only* the prices of the inputs used in producing them – and those prices are themselves both just and optimal. In such a system, there is no surplus of the kind envisaged by Marx (Hunt, 1992: 270) for any of the parties to appropriate.

Like all models, this has limitations. The optimality of the end-results of free competition – as envisaged in the standard model – has been disputed on various grounds. These include the model's weak treatment of dynamic processes (in a change-prone environment, optimality should refer to the quality of the adjustment process, as well as to equilibrium states that may only exist for short periods), and some well-known problems concerned with the impact of time and risk on the nature and attainability of equilibrium. (These are summarized by Ormerod, 1994 and 2000.) Others have challenged the Pareto criterion itself; Cullis and Jones (1998: 1) discuss whether it is value-free, in the context of alternative ideas on economic justice.

METHODS

General

The experimental study consisted of a series of simulations, in which the evolution of firms' rules of behaviour – driven by the profit motive – was followed under a variety of treatments. In the main study, these consisted of three methods of bargaining – each giving rise to a distinct set of dynamics – between buyers and sellers, crossed with three levels of statistical noise (the latter was applied to the level of demand, affecting the reliability of the signal firms receive about their price levels, relative to the state of the market). The resulting 3x3 factorial design was repeated in each of 10 blocks, each block being a different market, with its own supply and demand functions. The levels for the first factor (bargaining) represented (i), the standard micro-economic model of perfect competition ("standard"); (ii), a more commercially realistic one ("fully dynamic"), and (iii) an intermediate treatment. The latter two are described below, but the standard model presented a difficulty mentioned earlier: it does not specify what firms should do before there is a well-established market price – at which stage, they cannot be price-takers, of course. It was decided to build the model around the real situation in which firms set an asking price. In the fully dynamic and intermediate treatments, consumers then shop around for the best bargain, on a best-of-three-quotes basis. For reasons that are explained below, this provides sellers with a very uninformative signal about their relative price level; this is so far from what the standard model envisages for non-equilibrium situations that an alternative mechanism for demand allocation was sought. In the standard treatment only, each firm was allocated what would have been its share of the aggregate demand, if all firms had been trading at its asking price. This at least preserves the idea that firms get consistent and informative signals from the market. In the other two treatments, the shopping process allocates demand among firms.

The fundamental entities in the models used are the sets of rules that determine its asking price in the light of its current state; these take the form of fuzzy systems, each of which is embedded in a firm. The capacity for learning and improving these rulesets – is provided through a GA. The firms have no permanent existence outside of the major period in which they were created – it is the rulesets which continue and grow through the successive major periods; each has a numbered slot, into which the firms which will embody it are placed. Initially, each market has 50 firms, and 1000 consumers; the latter number is fixed, but, under the fully dynamic treatment, the number of firms in can vary over time, because of the possibility of bankruptcy and incomers. The fitness of a ruleset is measured by the cumulative profit that it generates for the firm in which it is embedded. Rulesets interact, because the firms compete in the market. The primary focus of the investigation is the evolution of the system of rulesets as a whole, rather than the finding of some optimal set of rules; it is assumed that firms would continue to adjust their pricing policies as long as some of their neighbours make greater profits than themselves.

Evolution occurs across two sorts of time periods: major periods (effectively, generations, although only one member is replaced at each, see below), and, nested within them, minor (trading) periods. Nine minor periods were used per generation. During minor periods, firms carry out normal commercial activities, including production and re-investment in plant; at the end of each sequence of minor periods, one pass of the genetic algorithm is executed, bringing that major period to an end. Events happen in real time. All events (including, for example, consumers soliciting quotes from firms) happen in some sequence, even if that sequence is randomly determined. Minor periods are equated with a year for interest and depreciation purposes, but otherwise the *duration* of the periods has no particular significance. Firms have to learn on-line, taking in data as it arises, and producing responses at the appropriate time.

Some parameters were held constant for the whole series of studies: the return on capital and the main interest rates (deposit, ordinary loans, and emergency finance); the blockiness factor for capital investment; the level of overheads, as a fraction of total gross margin; and the proportion of profit retained as a reserve to finance operations and plant replacement. Two other parameters are used to drive the noise generator which creates a stationary autoregressive moving-average (ARMA) sequence; this is used to impose the three levels of uncertainty on demand: nil; sufficient to give the demand a coefficient of variation (CoV) of 5%; and sufficient to give it a CoV of 10%.

The sequence[12] of events as the behaviour of firms evolves, in one market, is as follows:

i) Randomly-chosen supply and demand functions are generated. These define the particular market, and are used to calculate a number of quantities for the "typical" firm under the standard conditions, starting with the equilibrium price and output level which would be expected under perfect competition. (The former are referred to below as the nominal price, nominal quantity, etc.) The two functions are disaggregated to give the corresponding functions for individual firms and consumers. (Consumers are identical, but firms are not – see below.) It is now possible to combine nominal price, the individual firm's revenue, and (through the relationship between the individual supply curve and variable costs) to estimate the typical firm's total variable costs and gross margin at equilibrium. The total and per-period cost of capital equipment is set at such a level that a pre-determined figure for return on capital is achieved; this calculation assumes a fixed depreciation life (5 minor periods). Capital equipment is blocky: each firm has enough to produce its share of the nominal aggregate output, and does this with 10 units of plant. Nominal profit is calculated next, by deducting capital costs, depreciation, and overheads.

ii) The software loops in sequence through the 9 treatments (3 levels of noise, crossed with 3 methods of bargaining).

iii) Within each treatment, the first major cycle is started, with the generation of a sufficient number of random but well-formed rulesets for each of the initial population of firms.

iv) The first minor cycle is started. A population of firms is generated, in such a way that each firm is a stochastic variant of the "typical" firm, with all the data it needs to function: the coefficients for its individual supply function, its commercial history (eg, levels of debt and inventory), and a defined stock of productive plant. (The variation in each data item is controlled by a single parameter, set at a value which ensured a 5% CoV in the prices of the firms generated.). The firms set their asking prices using their fuzzy rulesets, and then interact with consumers and – indirectly, through the relevant competitive process – with other firms, according to the trading procedures defined for the current treatment. As part of this, the incomers procedure may allow the actual population to expand above the core number; these additional firms are assigned rulesets from the current core population at random, to minimize distortion of the dynamics of the system; however, only data from the core population itself are included in the reported averages.

v) The required number of minor cycles is completed; the fitness of each ruleset is measured by the cumulative profit it has generated.

vi) At the end of the major period, one pass of the genetic algorithm is executed; this leads to the replacement of one of the existing rulesets; with this amendment, the process loops back to step (iii), and steps (iii), (iv), and (v) are repeated until the defined number of major periods is completed (1000).
vii) At the completion of step (vi) the process returns to step (ii), until all the treatment combinations have been dealt with.
viii) The process is then repeated from step (i), until the required number of sample markets has been processed.

Sampling

The sample markets (described in Table 1) are based on a random sample of all possible pairs of supply and demand functions which:

- have an equilibrium point in the box defined by an aggregate demand greater than zero and less than 100,000 arbitrary units; and a price greater than zero and less than 100 arbitrary units;
- in the case of the demand function, are linear, with quantity monotonically decreasing with increasing price, and, in the case of the supply function, are quadratic, monotonically increasing with price;
- have (absolute) price elasticities of demand and supply that, at equilibrium, lie in the range 0.5 to 1.5.

Table 1. Summary Description of Sample Markets

Market	*Equilibrium Price*	*Equilibrium Quantity*	*Price Elasticity of Demand*	*Price Elasticity of Supply*
A	68.38	33067	-1.29	1.06
B	35.88	55160	-1.37	0.74
C	83.88	67266	-0.52	0.59
D	35.23	57335	-1.35	0.68
E	72.56	45570	-0.70	1.44
F	85.53	65783	-1.23	0.88
G	25.05	35154	-1.48	1.18
H	8.81	51846	-1.25	0.91
I	24.37	69421	-0.98	1.26
J	91.12	46506	-0.94	1.34

Pricing, Production, Investment, and Trading Procedures for the Treatments

Procedures Common to All Treatments

The first action of a minor period is processing the input data from the preceding one; this consists of the firm's previous profit level, and its inventory level. Firms are expected to know enough about the market to appreciate that half the nominal profit is very low, and 150% of it is very high. (In fact neither of these limits is approached after the first few major periods.) They have no knowledge of the position of where the nominal price lies within these limits (and, under some treatments, the stable price that develops is not half-way between them). Similarly, firms recognize a level of inventory equal to one period's nominal production as very high. These ranges are divided into 5 and 3 overlapping fuzzy domains respectively; both are of the form shown in Figure 1. (Values outside these limits are treated as being at the nearest limit.) These inputs fire one or more rules in the firm's fuzzy system, producing an asking price; this sets its production level (by back-calculation from the marginal pricing relationship), and the target level for production capacity used in replacing superannuated plant. From here on, the three models diverge.

The Fully Dynamic Treatment (D)

In the fully dynamic treatment, consumers engage in competitive shopping: each in turn calls individual firms, gets three quotations from suppliers who still have stock available, and buys from the cheapest; the level of this price determines the quantity purchased[13]. (A consumer always contacts its previous cheapest supplier first, but the other contacts are randomly selected.) Each firm's asking price and sales level (plus interest on holdings of working capital, see below) determine its revenue; and its variable, capital, and general overhead costs are derived from the disaggregated, stochastic versions of the corresponding functions/ quantities for the typical firm of the sample market. If the calculated production level exceeds the capacity of the existing plant, it is reduced accordingly. A defined fraction of profit is retained as working capital (large accumulations of which may be released as extraordinary dividends).

Profit is calculated in normal accounting terms, with actual expenditures on plant converted to depreciation; each firm has a working capital account, which is used to monitor its cashflow/liquidity position; retained profits are placed in this account. Losses drain down the working capital account. A firm can survive one trading period of serious losses, but then has to borrow at an increased rate to fund its operations; a second successive period of losses results in bankruptcy. This is lenient in that it implies that bills are, in effect, settled only once in each minor period – in reality, firms can face debilitating cashflow problems within the production cycle, of course.

If the firm survives the bankruptcy test, the tranche of its plant that has reached the end of its depreciation life is retired, and new plant is purchased to bring the complement up to what is needed to match the current price level (but it does not contribute to productive capacity until the next period). Cash-rich firms can buy new plant outright, others must borrow, the amortization costs forming part of the capital costs mentioned earlier. New firms can come in at the end of any minor period: the probability of any vacant slot being filled by a newcomer is (1.0 – fraction of consumers whose demand was completely satisfied in that period).

Standard (S) and Intermediate (I) Treatments

By contrast, the standard treatment imposes a fixed population of firms, each always provided with a suitable complement of plant. To implement the standard treatment, the shopping, cashflow/ bankruptcy, plant replacement, and incomers mechanisms are disabled. Profit accounting is, of course, retained so that the fitness calculation can be completed (for this treatment, profit excludes capital costs, but the fitnesses of all firms/ rulesets are affected to the same extent). The intermediate treatment is identical with the standard one, except that the competitive shopping mechanism remains enabled, and is used to allocate demand between firms.

Treatment Codes.

In what follows, the treatments are identified by the following two-character codes: the first character (N, M, or H) denotes the noise level; the second one (S, I, or D) denotes the bargaining mechanism. An asterisk is used in either position indicates that the code applies across all levels of that treatment, eg, *S refers to all standard treatments, regardless of noise level. In addition, the two treatment combinations of special interest, NS and HD, are referred to as representing "standard" (microeconomic) and "commercial" conditions, respectively.

RESULTS

Preliminary Experimentation

Coding and Genetics

At the beginning of the study, an attempt was made to apply the binary coding approach originally advocated by Goldberg (9189) to convex, trapezoidal input and output membership functions. The resulting strings were very long; this – combined with an attempt to include the optimization of the choice of input variables – prevented convergence. A number of changes were therefore made:

(i) The choice of variables was established by manual experimentation; this also provided some understanding of the broad features of the dynamics of the system[14].

(ii) Binary coding (and digital mutation) was dropped in favour of real number coding and gaussian mutation.

(iii) The coding of the membership functions (MFs) was drastically simplified, to reduce the size and dimensionality of the search space. A fixed grid of input MFs was adopted, and the action of the GA was restricted to the output MFs; with the previous change, this reduced the length of the strings considerably.

- The fixed, orthogonal, input grid consists of three levels of inventory, and five levels of profitability. As a result, the MFs are 3-dimensional, consisting of 15 intersecting tetragons, constructed so that their profile, viewed parallel to the y-axis (inventory), consists of 5 triangular MFs of the form illustrated in Fig. 1; and, viewed parallel to the x-axis (profit), 3 triangular MFs of that same form. (The membership level is on the z-axis.)
- For the output MFs, a range for the output (asking price) was set at 50% to 150% of the nominal price; one-fifth of this range was adopted as the width of the base of a standard, two-dimensional isosceles-triangular MF, and only the position of the central peak of the triangle was coded. With five price levels and three inventory levels (overlapping, in both cases), this means that each firm's ruleset can be encoded in only 15 real numbers. Each of these numbers specifies the response to one of the 15 cells of the input grid, in the form of the location of the peak (core) of the corresponding output MF. (The justification for this apparently crude procedure is that fuzzy systems are, by definition, insensitive to the fine detail of the placing of the ends of the domains of membership functions.)

(iv) Instead of complete replacement of generations, tournament selection was used in the GA. At each pass, 3 pairs of potential "parents" was selected at random, and the fittest of each of these subgroups were paired up for crossover. In the latter process, two random numbers, n_1 and n_2, between 1 and the total number of loci (n_3) were generated. If (n_1 + n_2) < (n_3 - 1), the n_2 loci following the locus numbered n_1 were exchanged between the strings encoding the rulesets of the two parents; otherwise, all the loci after n_1 were exchanged. (This procedure avoids the situations in which crossover either always disrupts a particular combination of first and last elements, or else never disrupts it.) One resultant string from the first subset of parents was then subjected to a small amount of mutation (1 locus per chromosome was selected at random, and altered by adding a random normal variate, generated with zero mean and variance equal to 2 ½ % of the equilibrium price), and substituted for the rule with the lowest fitness. This combination appeared to be effective, and was used throughout the main investigation.

Behaviour of Manually-adjusted Models

In these models, the GA was disabled, and the models tuned manually. The greatest difficulty in constructing stable models in this way arose from the poor quality of the market signal: the cheapest-of-3-quotes shopping method used is surprisingly efficient at channelling demand to the firms offering the cheapest goods: most firms are either sold out, or left with their entire production unsold. As a result, most firms "know" only that they are above the equilibrium, or below it, but not by how much, because of the lack of any graded response by their customers. This creates a potential trap: adjusting rulesets to attempt to find the price that maximizes profits can push firms over the hidden divide – defined by position in the price ranking, rather than absolute price level – between high profits and massive losses resulting from a collapse in sales. Firms are vulnerable in this way not only to their own errors, but to adjustments in pricing policy by other firms. The resulting instability of individual firms' prices was so great (compared to what the perfect competition model leads us to expect) that it seemed unreasonable to use it as the basis for the standard treatment. Alternatives were tried (including one in which firms spied on competitors' prices and inventory levels), but these were no better; this is why the mechanism described above was adopted.

All this suggested that the firm's current level of profit and its inventory level might be useful input variables in the construction of an asking price. With low (or zero) inventory levels, low profit levels suggest prices are much too low; moderate profits suggest there is room for increasing prices somewhat; and high profits suggest that the firm may be approaching the vulnerable position just described, and, at most, should only increase its prices cautiously. (Large losses accompanied by high inventory indicate that the firm's relative price is too high, of course.) A range of fuzzy systems was constructed on this basis, and adjusted manually; while none of these showed long-term stability (after 20-30 trading periods, the number of live firms in any one period tended to become unstable), the results were sufficiently encouraging to use this choice of input variables in the main study. During this phase, it was noted that rules (within firms) could interact to produce consequences that took more than 1 minor period to become apparent (eg, a rule which led to very low profits in one period might invoke another rule in the next period which made an excessive compensatory change, leading to further repercussions).

During these preliminary explorations, it also became apparent that, while firms' prices might converge fairly rapidly, their profit levels much converged more slowly; it was decided to run each evolution for 1000 "generations" (at each of which only one member-ruleset was replaced).

The Main Trial

Average Final Converged Prices

An analysis of variance was carried out for the main factorial design of three bargaining mechanisms and three levels of noise. There were no significant effects of sample market, nor were there any significant interactions between market and any of the treatments; dynamics, noise, and the interaction between them were all significant at the 0.001 level. These treatments and their interaction accounted for 78% of the total variance; there were no obvious irregularities in the residuals.

The pattern of the results can best be seen by examining the effect of the bargaining treatments at each level of noise, see Table 2A. With no noise, all three types of dynamics lead to price convergence at a level very close to the nominal value; the same is true for standard dynamics, at all three levels of noise. However, with the other combinations of dynamics and noise, there is a progressive warping of the response surface: with intermediate dynamics, as the noise level increases, the converged price falls in two approximately equal steps to approximately 95% of nominal (ie, the perfect competition equilibrium price); while, under fully dynamic conditions, the corresponding decreases are to 95% and 92%, respectively. This suggests that the demand-allocation process itself contributes substantially to the price depression.

Table 2A. Average Converged Price Levels by Treatment (as percentage of nominal).			
	Dynamics		
Noise	Standard	Intermediate	Fully Dynamic
Nil	99.44	100.71	100.41
Moderate	99.55	97.53	95.60
High	100.1	95.50	92.28

The bargaining/noise interaction appears to arise from the effect noted during the preliminary investigations: any firm operating at the upper end of the price range is vulnerable to downward price adjustments made by other firms (which can leave it with high production and of variable costs, and no sales), so that the top of the price-ranking is a dangerous location. Under the *D treatments, the number of firms was free to find its own level under the combined influence of the bankruptcy and incomers mechanisms. In the case of HD, the final number averaged approximately 1% more than the fixed population size of 50 used for the *I and *S treatments. An examination of the (static) aggregate supply and demand curves suggested this was too small to explain the price depression as a supply effect. To exclude this possibility in the dynamic situation of the evolving markets, the HD treatment was repeated with the addition of a constraint to prevent the incomers mechanism from raising the number of firms above the core level of 50. This produced the same average depression (to 3 significant figures) as in the original HD series, and the prob-value for a t-test of difference between the two series was over 0.99.

Average Final Converged Profit levels

Profits

Similar effects were found when the analysis was repeated for profit, see Table 2B, although the picture is affected by two factors: the role of fixed overheads in profit exaggerates the effect of price changes; and the profit level in the standard and intermediate treatments is raised, because they do not bear the cost of plant replacement (see 2.3 above), and use the options for financing plant replacement differently.

Table 2B. Average Converged Profit Levels by Treatment (as percentage of nominal).			
	Dynamics		
Noise	Standard	Intermediate	Fully Dynamic
Nil	105.35	115.01	132.54
Moderate	104.32	89.52	85.28
High	100.10	63.55	48.35

Probability Distribution of Firms' Profits

Individual firms' profit levels within any one minor period tend to have a distribution that is far from normal. For example, when the standardized profit levels from each market in the 5th minor of generation 500 were grouped together, the distribution differed significantly from normal ($p<0.001$), and appeared to be generated by 3 distinct processes with different means and variances, the switch between processes being a stochastic function of the individual firm's place in the price-ranking. The three processes were:

- a mid-range process, generating typical profits;
- a low range process, resulting from activation of the trap for high-asking prices, which was responsible for about 1% of events spread across the -3σ to -7σ[15] range; and
- a high-range process was responsible for about 0.4% of events, spread across $+3\sigma$ to $+7\sigma$ range. This arose where, for various reasons (eg, a positive demand disturbance from the noise generator), the trap was not activated, and firms got away (temporarily) with a high asking-price.

As a result, the graph of individual firms' profits against price-ranking is extremely variable in form between periods: it can have a high peak or a deep trough at the highest prices; in either of those cases, it can fall away at the low end of the price range, or it can be either approximately level or very rough across the whole range.

Learning the Rules

The correlation (across markets) between the nominal prices and final average prices under the NS treatment is over 0.99; there is no significant difference between the levels of the learned prices and the corresponding nominal prices as judged by paired-sample t-tests. The correlation between nominal prices and the converged prices is equally high (and the bias equally low) for all the treatments with nil noise, N*, or standard bargaining, *S.

Once both noise and a more complex bargaining regime are in place, the situation changes markedly, with the substantial depressions of price and profits noted. Under conditions in which firms set asking prices, buyers shop for the cheapest supplies, events happen in real time, plant depreciates and has to be renewed, and liquidity constraints bite, the standard model of competitive markets ceases to be a close approximation to reality when the market signal becomes noisy. Then, the system does still achieve equilibrium, but the equilibria are quantitatively different from the theoretical ones.

A sub-sample of three randomly selected sample markets (D,F and I) were re-run five times each, primarily to examine the form of the rules (see below); however, they also supplied some data on the consistency of the results of the GA/FL combination. Closely similar results were obtained in each case; the coefficient of variation of the depression of the equilibrium price under the HD treatment was small in each case, varying between 1 and 3 percentage points. The learning process operated consistently.

The Form of the Rules

By quite early in the simulations, the majority of hits were focused on a small sub-set of rules; in the case of the commercial treatment, HD, Rule 4 (activated by low profit and low inventory), and Rule 7 (moderate profit and low inventory) accounted for 85% of the total hits in the final major period.

To make interpretation of the rules easier, the output value (asking price) for each rule, for each market, was converted to a percentage of the corresponding the normal price. For the commercial treatment, the output values for Rules 4 and 7 were 94% and 89% of the nominal price, respectively. For the core rules, these values were quite stable across markets, the coefficient of variation being approximately three percentage points. These results are consistent with the behaviour observed in the preliminary investigation, and can be expressed in the following form[16]:

if <profit is low> *and if* < inventory is low> *then* < asking price is moderate>
if <profit is moderate> *and if* <inventory is low> *then* <asking price is low-moderate>

The other rules are much more variable, with coefficients of variation in the range of 20-30%; given the consistency of final converged prices across repeats, there are two possible explanations for this. Firstly, it may be that the overall behaviour of the fuzzy system is insensitive to any but the most frequently-fired rules; and secondly, that the two high-impact rules are fixed relatively early in the evolution (which is true), that one or more of the other rules acquires a random bias, and other rules are then adjusted over time to compensate for this. There is some evidence the latter: if the HD treatment is repeated with only one minor period per generation, learning is severely disrupted, and there are very few hits on any rules other than the core ones; this suggests that these other rules do collectively exert an important influence on the quality of the fuzzy system developed.

Rates of Convergence

The convergence rates for the two treatments whose contrast is of greatest interest (NS and HD, the standard and commercial models) are shown in Table 3, using the range between the second-lowest and second-highest observation as a measure of convergence[17]. This trimmed range corresponds to the 2^{nd} - 98^{th} percentile range (exactly in the case of NS, and approximately in the case of HD and MD, where the size of the population of firms can fluctuate from the initial size of 50). The patterns for the intermediate mode of price setting lie between those shown, and the patterns for the other noise levels are similar. Convergence is somewhat slower in the more challenging environment of the HD treatment. The convergence onto a common level of profitability is much less regular, see Table 4; in the more challenging dynamic environment (HD), this range falls from

Figure 3. Price Convergence (difference between 2^{nd} and 98^{th} percentiles, as % of final mean)

Generation	HD	NS
10	50.1	78.5
50	38.0	46.0
100	25.1	15.3
200	18.3	8.9
500	18.5	5.0
1000	7.6	4.3

Table 4. Profit Convergence (difference between 2nd and 98th percentiles, as % of final mean)		
Generation	HD	NS
10	783	487
50	860	256
100	444	111
200	412	95.7
500	461	64.5
1000	108	54.3

about 800 percent of the final mid-range point in the early generations, to 100 percent at the 1000th generation. The corresponding figures for the standard treatment are 500% and 50%, respectively. The difference between these rates of convergence and those for price is due to the complex relationship between price and profit. (Price affects the total variable costs and demand; both relationships are nonlinear, the demand one markedly so.)

SUMMARY AND DISCUSSION

Summary of Findings

A combination of fuzzy logic and genetic algorithms was used to give an autonomous learning capability to each member population of producers in a multi-agent model of a competitive market; the agents were heterogeneous in terms of their supply functions and asset profiles. The rules specified an asking price for each vendor firm, in the light of its profit and inventory levels in the previous period. The initial rulesets were random but well-formed, so that the firms in which they were embedded started off trading at different prices; the genetic algorithm then honed these rulesets over a number of generations, using profit (taken over a fixed period) as the measure of the relative fitness of the different rulesets. During this process, rulesets and the resulting prices converged in the manner described below.

The software with which the evolution of firms' rules was simulated could be set to provide an environment very close to that of the standard model of competitive markets, or to provide a trading environment much more like commercial reality. (There was also an intermediate option, included for exploratory purposes.) In all three cases, firms had to set an asking price, and learn from buyers' responses how to improve their pricing policy. Under the standard treatment, demand was allocated on a basis which ensured that their level of demand reflected the relationship between their asking-price and the theoretical market-clearing one; this basis was somewhat artificial, but this reflects the absence of any corresponding mechanism in the standard model. In the commercial case, demand was allocated by a shopping process, in which buyers sought out the cheapest sources of supply, and had to deal with persistent inventory, a liquidity constraint, and the possibility of bankruptcy. These software settings were used, together with a facility for injecting statistical noise into the level of demand, to impose a 3 x 3 factorial combination of treatments on the evolution of the simulated market systems, consisting of the three types of market dynamics (standard, intermediate, or commercial) crossed with three levels of noise. The simulated evolution was repeated for 10 sample markets, each a random sample from a population of markets with straightforward supply and demand functions.

Under the standard conditions, the evolution of the rulesets caused firms' prices to converge onto the theoretical equilibrium point, ie, the equilibrium that would have been expected under pure competition, given the aggregate supply and demand functions in the particular sample market. However, even when prices had converged quite tightly (the range of individual firms' prices was some 4% of the final value, across all markets), profits remained distinctly more unstable (range some 55% of the final value); this is because of the complex and unstable relationship between price and profit.

Neither the introduction of noise into the level of demand or the imposition of the commercial pattern of dynamics, singly, affected the model's ability to locate the theoretical equilibrium. However, with both noise in the demand signal and commercial dynamics, the position of the equilibrium changed, the price being depressed below the theoretical equilibrium one. In the case of the high-noise, fully dynamic commercial treatment, the depression was approximately 8%; profit was reduced much more sharply, by over 50% for the treatment mentioned, largely because of the effect of overhead costs. Convergence was also poorer, with the final converged ranges for price and profit being approximately 8% and 100%, respectively. The position of this equilibrium is determined by the balance between the upward pull of unsatisfied demand and the downward drag of firms' reluctance to bear the risks associated with higher prices, rather than between demand and the marginal equalization principle.

The price depression appears to be a response to the poor quality of the information obtainable from the market: even the fairly crude method used here to link buyers to the best bargains works well enough to ensure that, in any one period, most firms are either totally sold-out, or make no sales at all. This means that the great majority of firms know only that they are above or below the current market-clearing price (which, in a turbulent situation, may not be the theoretical equilibrium). This will be true of any reasonably efficient bargaining mechanism, of course. As a result, firms which are high in the price rankings, but still selling all of their goods, are in a vulnerable position: the watershed – between those firms that are sold out and those that are not – can shift (eg, as a result of adjustments made by other firms), leaving them with high costs and no revenue. In these circumstances, it pays firms to frame their aspirations about prices cautiously.

This risk-evasion strategy is rather different from risk-discounting in the normal sense: the size of the hazard is not proportional to the amount of some activity undertaken (such as undertaking a particular line of production, or holding a particular type of asset). Also, a market for this sort of risk would face the problem that the relevant probability distribution is fat-tailed, and very far from normally-distributed; such distributions have played a significant part in a number of financial debacles –see, for example, Lowenstein (2002: 71) on the part that they played in the Long Term Capital Management affair.

The results reported here fit into the wider pattern of studies of evolutionary algorithms in the socio- economic arena. Where there is a model embodying with naïve dynamics (such as the standard model of perfect competition) which has a static equilibrium, agents readily learn to find that equilibrium, even in cases where specifying a route or a mechanism to that point has been a substantial theoretical conundrum. Relaxing the simplifying assumptions in favour of a more complex and realistic set of conditions introduces the potential for interaction between agents' individual choices, and the possibility of costly instability. Under these circumstances, agents typically learn some pattern of mutual accommodation, and their choices lead to a different equilibrium from the naive one. It does of course follow that these alternative equilibria do not have the optimising properties ascribed to perfect competition, etc. However, this is unsurprising, given that it is very unlikely that a complex system will have the same attractor as a heavily linearised version of itself; and that concerns about the Pareto optimality of the equilibria associated with dynamically naive models of markets, or of economic reform, have been around for a long time (see, for example, Newbery and Stiglitz (1982), and Lancaster and Lipsey, 1956).

Issues and Implications

Related Problems

A number of questions (all concerning environments with commercially-realistic dynamics) need to be investigated before the general significance of the findings can be properly assessed. The most important of these appear to be:

- What other sorts of bargaining mechanisms produce deviations from the theoretical equilibrium price?
- How would the existence of geographical or other allegiances within the market, between buyers and sellers, affect the results?
- How does the observed depression of the equilibrium price compare with risk premiums derived in other ways?
- Is it possible that a different pattern of behaviour would have emerged if the firms had been given the possibility of evolving niche strategies? (The niches might be different pricing policies, or different production methods.) This would increase the computational burden somewhat, but would add no difficulties of principle (Cordon *et al*, 2001: 63).
- How do the deviations from the equilibrium price propagate through an interlinked set of markets, and how does this impact on the pricing of firms' resources?
- How do they affect other areas of economic theory? (For example, are we safe in assuming that two economies, engaging in international trade, will operate at their respective production possibility frontiers?)
- How well can autonomous learning models of the kind used here adapt to an environment characterized by continual change?

Further Applications of the Methodology.

A number of other problems can be re-framed in terms of how the responses of different groups of rational agents will co-evolve under different regimes. A selection would include extending existing models of the endogenous development of market institutions (see Tesfatsion, 2000), to learn more about their properties; and studying stability and growth in a structure such as the EC (an imperfect market, with geography and politics, and subject to regionally-differential shocks). Like the other studies cited above, this one has certainly not exhausted all the potential of the GA/ FL combination. The problem addressed required only a fairly low-level application of GA principles, without recourse to any of the more advanced techniques available; and it did not make any explicit use of fuzzy logic's ability to deal with hybrid cases, or its ability to address problems with important qualitative aspects.

Many problems concerning the provision of public goods and the management of common property natural resources are hybrid ones, that is, hybrid between some extreme pure cases, for which some theory exists (not necessarily exclusively within economics). In the public goods arena, for example, there is theory relating to the extremes of the rivality versus and non-rivality and excludable versus and non-excludable axes (Cullis and Jones, 1998); and there is also relevant theory on the emergence of collaborative behaviour available from political science (see, eg, Casti, 1992b). The problem is making a synthesis of this knowledge which can be applied to practical cases. This is well within the scope of fuzzy logic (the aggregation phase of the normal inference mechanism – see Section 1.1 above – routinely deals with borderline cases as hybrids).

Qualitative variables arise in situations where we have some theory which is in narrative rather than mathematical form, and which is difficult to apply to practical situations because of the lack of any accepted method of formalising it. This lack lies behind Stiglitz' (2004) appeal for a more sensitive and flexible application of macro economic theory to nations in crisis, where orthodox strategy has led to political instability, or unacceptable impacts on the poorer strata of society. With fuzzy logic, it is possible to specify derogations from strict orthodoxy in a way that is clear and transparent, and expresses some challengeable model of the processes whose effects are claimed to justify the derogation. Without it, the standard objection that such derogations will open the floodgates to particularism and corruption is probably valid. There is also a good deal of practical experience with fuzzy modeling of systems which are structurally or dynamically complex (see, eg, Terano *et al*, 1994, or Ross, 1995), and/or have important qualitative dimensions (such as "market sentiment", or "the level of political tension").

Free-Riding

A key feature of the markets modeled in the main part of the current study is that would-be free-riders can only exploit the stable market created by the mutual restraint of others by moving to the top of the price-rankings – the very situation that is most exposed to the penalties for defection from the implicit charter of behaviour. A similar mechanism appears to operate in the extended tournaments of Prisoners Dilemma discussed by Casti (1992b): attempting to exploit the collaborative behaviour that spontaneously evolves ensures that – for the two leading strategies, TIT-FOR-TAT and PAVLOV – would-be exploiters suffer at least as badly as anyone else from their own mis-behaviour. In both cases, there is nonlinearity in the form of feedback. In this study, it takes the form of relatively impersonal repercussions from the system, whereas in the case of the players in these tournaments, it is direct (but limited) personal retaliation. In strictly linear systems any such repercussions are excluded, by definition; there, rational conduct does not need to take account of repercussions, whether directed and personal, or delivered impersonally by the dynamics of the system. This is not true in the more general case. Smith (2001) examines the implications of system complexity for our concept of rationality in decision-making.

Realism and Stability

Many critics – notably, Ormerod (2000) – have suggested (i), that economics is unrealistic to the extent that it excludes nonlinear phenomena, and (ii), that accommodating them will necessarily introduce a distressing degree of instability. The current findings suggest that the second link in this chain is a weak one: in this case at least, autonomous learning in a reasonably complex nonlinear situation has not given rise to the expected level of instability.

These results presented above may be a special case of a more widely-applicable principle: that, in economic systems where the short-term self-interest of agents can precipitate costly instability, and where free-riding is difficult because the free-rider is among the agents most exposed to the resulting penalties, agents will learn to act with mutual restraint. We cannot (yet) say much about the rate of such learning, or how readily agents can transfer learning between contexts. If the period is long, and transfer is limited, then one would expect the sort of flip-flop instability that Ormerod anticipates. However, what he puts forward as part of the foundations for "*A New General Theory of Social and Economic Behaviour* "[18] does seem very close to parts of Rene Thom's catastrophe theory (Thom, 1975). That, too, was proposed as an alternative model for a very wide range of social and economic phenomena – but its performance never matched it advocates' promises, largely because it proved too difficult to produce plausible models which generated the mathematical surfaces required by the underlying theory. (See Thompson, 1982, on modelling social change, for example; and Casti, 1992: Chapter 2, discusses applications and criticisms of the concept.) With learning whose conclusions are transferable between contexts, it would be more reasonable to expect something of the kind found in this study.

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Notes

[1] Address for correspondence: Springcott, Chittlehamholt, Devon EX37 9PD, UK. Email: manindev@yahoo.com

[2] "Perfect competition" describes markets in homogeneous goods, with enough agents to ensure that no-one can affect the price through production or purchasing decisions, with mobile resources, and with no "artificial" constraints. This differs from "pure" competition, in which all agents have complete, accurate, and instantaneous knowledge of the market.

[3] Because the investigation addresses the actions of firms before there is a market price, its firms cannot, of course, be price-takers. The models use are variants on the situation in which firms set an asking price, and react to the consequences.

[4] Kosko (1992: 379*ff*), for example, discusses the improved performance of fuzzy controllers relative to the classic least-squares Kalman filter in aircraft landing systems.

[5] "Conventional logic" refers to all styles of logic that rely on crisp, unambiguous sets, and use two discrete truth values; this, of course, includes Boolean and Aristotelian logic.

[6]Strategies – such as *reductio ad absurdum* – that rely on these laws are also banned, of course.

[7] They may also be determined empirically, see Ross (*op cit*, pp 371*ff*).

[8] He also defines procedures for cases in which the input scales are qualitative.

[9] Some of the confusion may have resulted from the fact that there are genetic methods of finding good *approximations* for a number of well known NP-hard problems, including the Travelling Salesman Problem.

[10] This is a discussion paper, and some of the colleagues I hope to hear from may not be economists – hence this section.

[11] Percentage change in quantity in response to a (small) percentage change in price.

[12] This was implemented via a purpose-written set of software in FORTRAN.

[13] Where a selected firm has insufficient stocks, demand may be split among firms on a price-weighted basis.

[14] Here, "system" refers to the combination of GA, a sample market, and a repertoire of commercial actions for its firms defined by one or other of the main treatments.

[15] The standard deviation of the unpartitioned total population of profit levels.

[16] If these seem unduly cautious, it has to be remembered that they relate to a range of from 50% to 150% of the nominal price; the realised price is below the midpoint of this range.

[17] The full range is much more unstable, largely as a result of the presence of newly-created rules from the GA.

[18] The full title of his book is *Butterfly Economics: A New General Theory of Social and Economic Behaviour.*