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Integrating estimates and targets within a population projection

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Abstract

This paper recognises that the 'base year' of a population projection rarely reflects all the latest population statistics. It reviews mechanisms which can incorporate into a cohort component population projection incomplete and estimated data since the base year, as well as targets for population, housing and employment. The inclusion of estimated data in a projection allows increased accuracy in the early years of a projection, and estimation within the model of back-projected rates. This training phase allows better judgement of likely future rates. Existing methods of including estimates in a population projection model are reviewed, and illustrated by four UK applications using generic projection software, POGPROUP (http://www.ccsr.ac.uk/popgroup/).

Keywords: population projection; consistency; constrain; software; forecast

1. Introduction

Population projection software has become an essential tool of government planning and policy formulation. It provides likely demographic future scenarios which are the basis for planned provision for pensions and the health service, and from which are derived housing and employment needs. Official projections of population generally implement a cohort component model, and use age-specific estimates of fertility, mortality and migration (Shorter et al., 1995; OPCS, 1990). For sub-national areas, projections assist in assessing a wide number of service provision needs, traditionally education and social services, as well as review of electoral district boundaries and strategic understanding of changing economic and social trends.

In cohort component models, a specific time point is fixed as the base year for the projection, such as 1 January 2000 or 30 June 2005. It is usually the latest point for which a reliable population estimate is available with full detail of age, sex and geography. Detail of age is essential for cohort models, which mirror the ageing of the population over time. In many cases this is the most recent year in which a national census has been conducted, but in other cases the census may have been updated from administrative registers, or systems of vital and migrant registration, or estimation. The components of population change are then applied as age-specific fertility, mortality and migration rates to the base population, which determine the projected population at the end of each period after the base year.

This paper is focused on the improved use of cohort component models for strategic planning. It challenges the notion of a fixed base year before which all is estimated accurately and after which all is assumption, and sees estimates and projections as a single integrated continuum. To the extent that projections do not integrate independent information they miss an opportunity for validation and therefore for improvement.

Frequently, extra information on the flows and stocks of population since the base year is available, is relevant to the projection, and should constrain it. Three such cases are distinguished throughout this paper.

- Components of change. Counts of births, deaths or migration are available for years since the base year, but with insufficient detail to create an updated base year population.
- Population. Population estimates become available with insufficient detail or too late to substitute as the base year. This is particularly common for sub-national areas, where population may be estimated from administrative records only for broad age-sex groups or for the total population only.
- Derivatives of population. Derivatives such as the number of households or the size of the labour force may be available, as future targets or expectations based on independent consideration of housing and labour markets; in planning contexts it is desirable to adjust the population projection to be consistent with that extra information. These housing-led, employment-led and population-led future scenarios have direct implications for education, social care and other services.

In each case, the extra information invalidates a population projection that ignores it. At the very least the projection, or rather those who prepared it, are left open to the criticism of not using the latest available information. That information can be generally thought of as an estimate independent of the projection, and thus the concern is to integrate estimates within projections.

It is sometimes necessary to make one projection consistent with another, which has less detail but is accepted as technically or politically robust. Thus projections for sub-regions or social groups are ‘controlled’ to an accepted projection for the whole region or whole population. This is an important subset of the more general case of integrating independent estimates and targets which is considered here.

The regular production of independent data for births, deaths, migration or population directly contradicts the early years of a projection and is a strong reason for the regular review and update of population projections if they are intended to show likely future population change based on past evidence. The third type of additional information, population derivatives such as housing and the labour force, is less accepted as a driver of population change. In the UK the Government Actuaries Department (for national projections) and the three statistical agencies ONS, GRO(S) and NISRA (for sub-national projections) limit their projections to the continuation of existing *demographic* trends. Local government tends to accept demographic projections for their long-term indications, but gives weight to local housing and employment conditions to set short-term trends.

During the 1990s government population estimates based on recorded births, deaths and estimated migration implied a number of households increasing faster than independent information about the changing housing stock, particularly in London (Hollis and Holmans, 2002). The inconsistency could be resolved in three ways: by challenging the estimates of housing stock, by revising upwards the assumed occupation of the housing stock, or by challenging the population estimate itself. Attempts to reconcile the information were not successful, partly because no methodology was available to adjust the population estimates until results were released from the Census of 2001. The Census has in fact revealed a lower population than had been estimated, as well as a changed relationship between population and housing. A rigorous examination with the ability to integrate population and housing estimates in a single model would have highlighted the over-estimation of population at an earlier stage.

Several practical difficulties can arise that are either addressed in the paper or raised again in discussion: the estimates available to integrate into a projection may be inconsistent with other assumptions of the projection; the extra information may be suitable to treat as a soft rather than a hard target; the constraint may be met over one or over several time periods.

Sections 2 and 3 review the published literature and the diverse ways in which existing software for population projections copes with or ignores data for events after the base year. This diversity emerges from the specific origins of each piece of software. A generic approach is suggested in Section 4, and made operational within the POPGROUP software as described in Section 5. Section 6 illustrates the purpose and power of integrating estimates within projections, with applications related to local area population modelling, projections of ethnic group populations, and household projections. A final section reviews the implications of integrating estimates within projections and discusses future developments.

The paper, and the software which it reviews, is rooted in a policy context concerned with evidence and assumptions about the most *likely* future population change. The inclusion of estimates in a population projection usually aims to bring the projection closer to the unknown future population, by including the impact of external or recent events on past trends. For this reason the term forecast is often used rather than projection, and no strong distinction is intended by the use of either term in this paper.

2. Literature

There is little published methodological literature addressing this common practical concern to integrate estimates and targets in a population projection, beyond a concern to make consistent sub-regional and regional projections (Smith et al., 2001; King, 1990). Keilman (1985: 1482) usefully describes a three stage strategy that is used here for the general case: "(1) formulate initial values of model parameters; (2) check and adjust for consistency; (3) translate consistent model variables into adjusted parameter values". He distinguishes between internal and external constraints.

Internal constraints are needed to ensure consistency because of the incomplete specification of a model. For example where marital status is projected, the number of men and women leaving marriage must be equal in each time interval. Similarly, where migration between regions is modelled in a multi-regional projection, the sum of inter-regional out-migrants must equal the sum of inter-regional in-migrants.

External constraints, involving consistency with data outside the model, are the focus of this paper. Keilman discusses only the case where a multi-regional projection is made consistent with an independent aggregate all-region projection, using the example of the Netherlands' regional population projections. This *projected* population constraint is formally the same as the *estimated* population constraint treated in this paper, as both provide direct information about the population after the base year. Keilman proposes two solutions. The first applies the same proportional adjustment to all regions, an adjustment that may be age and sex specific and may be applied directly to components of change. His second solution minimises the deviation between initial and post-constraint age-specific patterns of fertility, mortality and migration rates, given national sums of births, deaths and net migration. We shall consider the situation where the birth and death components are considered to have been measured with insignificant error and therefore are not changed in the adjustment to reach consistency.

Evert Van Imhoff (1992) provides a more general mathematical framework for consistency in multi-dimensional models. A system of demographic parameters results in numbers of events; these are to be adjusted to meet a constraint expressed as a linear combination of some of those events. A mathematical solution achieves Keilman's three stages by iterating between initial values of events from a projection that does not yet meet the constraint, and adjusted values. While the constrained events may be few in number (the total population, or the equality of moves between marital statuses for women and men), all types of events are adjusted to meet the constraint (fertility, mortality, migration and so on). Implementation in van Imhoff's LIPRO software allows for some events to be specified as 'dominant' and therefore not affected by the constraint, and others to be 'passive' and thus adjusted to meet the constraint. Like Keilman, van Imhoff also focuses on internal constraints for inter-regional migration and marital status.

Interest in the impact of demographic constraints has not been limited to this paper's focus on sub-national demographic projections. The 'replacement migration' necessary to fulfil a constraint of maintaining the current population characteristics is a concern of the most developed countries and has been subject of several investigations (for example: United Nations, 2000; Kippen and McDonald, 1998). These do not however discuss the means of achieving population targets within projections.

The approach in this paper adopts Keilman's three stage strategy. It applies his first solution, proportional adjustment extending the constraints to one or more regions, broad age groups, and population derivatives, and specifies algorithms that can be incorporated in common software.

3. Software for population projection

Population projections are undertaken by numerous public and private agencies for specific purpose, usually with in-house software. Reviews of population projection software packages (Willekens and Hakkert, 1996; Bongaarts and Bulatao, 2000: Appendix A) are quickly superseded by developments of existing packages and by new appearances.

Comparative discussion of projections software is also hampered by the wide variation in maintenance and documentation. The cost of redeveloping existing packages limits regular upgrades – there are costs not only in technical development but for users in training and systems development. There tends to be a leap-frogging of functionality as the older packages are held back by the technical platform that was most relevant at the time they were new. Institutional support and user demand for packages help to ensure their survival and development, while those initiated by individual researchers are prone to stagnation as their authors move on to other interests or to retirement. The web pages of some models had not been updated for several years at the time this article was written.

Here we consider only packages which incorporate external constraints in the projection, which are available for general use, and are described on the Internet: FivFiv (2003), RUP (2003), LIPRO (2003), POPGROUP (2003), and Chelmer (2003). The last two have been developed in the UK in response to planning responsibilities of local government, though they each have wider application. Readers considering acquisition of software should check the availability of these and other software. Most packages allow demographic *rates* to vary during a projection; this allows estimates after the base year to be accommodated, if the user has computed updated rates outside the software. However, the computation of rates demands a population denominator beyond the base year and thus is achieved more accurately when enabled within the software when a projection is computed.

RUP and *POPGROUP* both allow counts of births and deaths to be specified after the base year, from which vital rates are computed and output. Specification of migration as a net count is common since the denominator for rates of in-migration from the rest of the world is rarely available. *FivFiv*, *RUP* and *Chelmer* specify migration as net counts during a period; in this way they integrate estimates of migration since the base year. *POPGROUP* allows specification of gross migration flows as an alternative to rates, for any part of the projection period. As with deaths, *POPGROUP* allows migrant flows after the base year to be disaggregated by age and sex and by an extra user-defined state, or entered as totals.

The *Chelmer Model* and *POPGROUP* allow population estimates after the base year to be specified as a constraint, adjusting migration flows to meet them. *LIPRO* has a general consistency algorithm by which any constraint on components of population change or population estimates may be imposed for specified periods of a projection, such that the re-calculated rates may be used as initial rates for subsequent years (Imhoff, 1992). The software authors acknowledge the difficulty of using these constraints in practice. The difficulty is partly due to the generality of specification, and partly due to the lack of convergence of the algorithm on occasions.

FivFiv caters for back or inverse projections where the current population and previous births and deaths are used as constraints in an estimation of earlier demographic rates and earlier populations. This approach is particularly useful when past data is incomplete, as in historical studies and in some developing countries.

Finally, the *Chelmer Model* and *POPGROUP* accommodate target future changes in population derivatives, as required in the context of planning the release of land for housing in the UK. The *Chelmer Model* as described is under development; its distinguishing feature is an accompanying database for the regions, counties, districts and wards of Britain, which allow a forecast without further preparation of input data. *POPGROUP* on the other hand is designed such that the user optionally specifies counts of demographic events, population estimates and population derivatives with a variety of completeness. This distinguishes it in degree from other packages which allow a more limited integration of estimates within projections. This flexibility and its use of Excel as a platform make it suitable for exploring the issues raised in this paper.

4. Methodology

This section presents solutions to combining estimates in projections in a general form. In every case an initial projection of the components of change from population at time t to population at time t+1 are adjusted to meet estimates provided externally.

The three types of external constraints are discussed – components, population, and population derivatives – followed by comments on practical situations where more than one constraint must be met, where constraints are set for a longer time period t to $t+n$, and where smoothed age schedules of vital rates may be preferred. The following section briefly reviews issues arising when implementing the solutions within working software, *POPGROUP*.

Notation:

a	A single element of the classification of population used in a projection. This may be sex by 5-year or single age bands but may also include further social or geographical disaggregation.
A	The constraint category, an aggregate of one or more different elements of a. This may be a broad age group or the total of all males, all persons, or the aggregate of social groups or geographical areas.
P_{at}	Population in category a at time t.
c_{at}	A component of population change from time t to $t+1$.
b,d,m	Births, deaths and migration, when referred to specifically.
r_{at}	Component rate, equal to c_{at} / P_{at}.*

Suffix s=0,1,2 The step of the constraining process. The initial projection before constraint is represented by s=0. The initial components of change c^0_{at} move from population P^0_{at} to population P^0_{at+1}. The initial component rates of change are r^0_{at}.

Underline The collection of a variable across all population categories is represented with underline, thus $\underline{m}$ is a vector containing all migration flows.

• The summation over all population categories. Thus $m_{\bullet}$ is the total migration.

Demographic components as constraints

The constraint c^1_{At} may be an estimate of the number of deaths, available only for broad age-sex groups A. The adjusted projection for time $t+1$ can be computed simply by scaling the initial components and rates by the ratio of initial events to the constraint, for all a within the constraint A.

$$c^1_{at} = c^0_{at} * c^1_{At} / c^0_{At} \text{ for all } a \in A \tag{1}$$

$$r^1_{at} = r^0_{at} * c^1_{At} / c^0_{At} = r^0_{at} * c^1_{at} / c^0_{at} = c^1_{at} / P^0_{at} \tag{2}$$

* The rate is defined in this way for the convenience of computing a projected population at the end of the period from the population at the beginning of the period. It is different from the demographic rate whose denominator is the exposure time of the population, which is often approximated by ½ $(P_{at}+P_{at+1})$ under the assumption that population change is linear during each time period. The definitional difference is slight and the impact on projections negligible, introducing small errors only when population change is fluctuating greatly from one period to another; in these circumstances the linear assumption is also likely to be transgressed.

The population after constraint, P^1_{at+1}, is then as in all cohort component projections, the sum of the constrained components of change added to the population P^0_{at+1}.

Population estimate as constraint

The constraint P^2_{At+1} is an estimate of the population derived independently from the projection. It is denoted with suffix ^{2} to denote that the initial estimate may have already been made consistent with demographic components as constraints. In general, the population constraint could be met exactly by adjusting any of the components of change, or by adjusting the population at time *t*, P^0_{at}.

In many studies births and deaths are recorded with little error, and both base population and later population estimates are taken as fixed constraints (although the adoption of soft targets and uncertain vital rates are discussed later in this section). Then estimates will be adjusted through re-estimation of migration flows only. This is the approach taken in both the Chelmer model and POPGROUP in the UK, and by Smith, Tayman and Swanson in their review of local projection methodology for the USA (2001: 253).

The aim is to express the difference between the target and the initial projected population as the sum of differences between final and initial gross migration flows. For M migration flows,

$$P^2_{At+1} - P^1_{At+1} = \sum_{i=1}^{M} (m^{2i}_{At} - m^{1i}_{At}) . \qquad (3)$$

There may be just one in-flow and one out-flow with M=2, or streams of migration to and from several external areas, with M>2. The M final flows (m^{2i}_{At}) are to be estimated (as below) so that the projection is made consistent with the target population. To solve for these M unknowns given only one population difference, a weight w_i may be given to represent the contribution of each migration flow to the required adjustment, summing to 1 over the M flows:

$$m^{2i}_{At} - m^{1i}_{At} = w_i (P^2_{At+1} - P^1_{At+1}) . \qquad (4)$$

The weights may be set equal. For example with a single in-flow and out-flow, and $w_1 = w_2 = ½$, the difference between the initial population projection and the target population for time t+1 is halved; this adjustment is then added to the in-flow and deducted from the out-flow. Alternatively some flows can be given higher weights to reflect the projectionist's lesser confidence in the initial estimates of those flows. For example, overseas emigration is often particularly difficult to estimate. If flows other than emigration were thought to be very robustly estimated, then the weights would be zero except for the emigration flow which would have a weight of one.

The adjustment calculated in (4) resolves a population constraint into adjustments for the aggregate population group A. This may be a population total, or broad age-sex-social sub-groups. It is distributed to the finest population categories *a* in proportion to the initial distribution of migrants.

$$m^{2i}_{At} - m^{1i}_{At} = w_i (P^2_{At} - P^1_{At})(m^{1i}_{at} / m^{1i}_{At}) \text{, for all } a \in A \qquad (5)$$

For example, with a total population constraint and an initial projection that migrants age 0 in the ith flow are one hundredth of all migrants in that flow, then the adjustment to the age 0 migrants is set to one hundredth of the total required adjustment to that flow.

Finally, the adjustment is added to initial inflows, and deducted from initial outflows, in order to meet the population constraint. Two numerical problems can occur. First, where the aggregate initial migration flow before constraint, $m^{1i}{}_{At}$, is zero, the adjustment cannot be made proportionally as suggested here; instead it can be divided equally to each category a within *A*. This situation is most likely to occur when *A* refers to age bands within small populations which may have zero population for some elderly categories at the base year of the projection. Second, negative gross flows can result from an adjustment greater than the existing flow. These are avoided by adding the negative result to the opposite flow. Thus for example, an inflow of –100 is set to zero while 100 is added to the outflow from the same population category.

Population derivative as constraint

Population derivatives are figures derived from the population projection and its components. Most usually, they are derived as a linear combination of age-sex specific population counts, such as the number of households derived by deducting the population not in households and multiplying by age-sex household headship rates (now known in UK official demography as household representative rates). In general, derivative $D_t = f(\underline{P}_t)$, where $\underline{P}_t$ represents the collection of population results projected for time *t*.

It is possible to alter the function f in order to meet a target population derivative, taking the target as evidence of an error in the assumed headship rates. The headship rates would then be scaled to meet the estimated housing without any change to the population projection. On the other hand, if the estimated housing is considered as evidence of errors in the components of change $c^1{}_{at}$, then these should be adjusted to achieve the target $D^2{}_{t+1}$. This latter option is considered here; a population derivative is considered as an alternative to a population constraint. Where the target is to be met precisely and births and deaths can be assumed accurate as earlier, and *f* is a linear function of the population output, then the required adjustment to the population derivative can be expressed as a function of adjustments to the migration flows.

$$D^2_{t+1} - D^1_{t+1} = f(\underline{P}^2_{t+1}) - f(\underline{P}^1_{t+1}) = f(\underline{P}^2_{t+1} - \underline{P}^1_{t+1}) = f(\sum_{i=1}^{M}(\underline{m}^{2i}_t - \underline{m}^{2i}_t)) \qquad (6)$$

where $\underline{m}$ represents the collection of *a*-specific migration flows. Similarly to equation (3), the aim is to express the adjustment to the population derivative as the impact of adjustments to M migration flows.

A solution is again derived by weighting each migrant flow by w_i. These weights can sometimes be derived from knowledge of the housing and labour markets. For example, a housing target or constraint has usually been considered as an attraction or deterrent to short distance migration, while employment is usually taken to affect longer distance migration. If short and long distance migration flows are specified separately in the projection model then their weights for the constraint are set accordingly.

To solve equation (6), the impact of migration on the derivative is required. This is achieved by considering the impact of a single migrant shared between all the population categories and migrant flows. The 'composition' of this single migrant is based on the initially projected flows, by dividing each $m^{1i}{}_{at}$ by the total number of migrants in the flow across all population categories, $m^{1i}{}_{\bullet t}$. The flows are weighted as above, with weights summing to one over the M flows, so that:

$$\sum_{i=1}^{M} w_i \underline{m}_t^{1i} / m_{\bullet t}^{1i} = \sum_{i=1}^{M} \sum_{a} w_i m_{at}^{1i} / m_{\bullet t}^{1i} = 1 \qquad (7)$$

The impact on the population derivative of this single representative migrant, by comparison with the required adjustment to the population derivative, provides an adjustment to the total number of migrants, summed over M flows and all population categories:

$$\sum_{i=1}^{M} (m_{\bullet t}^{2i} - m_{\bullet t}^{1i}) = (D_{t=1}^{2} - D_{t=1}^{1}) / f(\sum_{i=1}^{M} w_i \underline{m}_t^{1i} / m_{\bullet t}^{1i}) \qquad (8)$$

The right hand side of equation (8) provides the *total* adjustment to migrants required to meet the target population derivative. It is a single value; for example a value of -1000 would indicate that a deduction of 1000 migrants with the composition of the weighted initial flows will produce the target population derivative.

The separation of this quantity into M flows and each population category is achieved using the representative migrant again:

$$\underline{m}_t^{2i} - \underline{m}_t^{1i} = (\sum_{i=1}^{M} (m_{\bullet t}^{2i} - m_{\bullet t}^{1i})) * (w_i \underline{m}_t^{1i} / m_{\bullet t}^{1i}) \qquad (9)$$

The first term of the right hand side of equation (9) is the single value calculated from equation (8). For each flow *i*, this value multiplies the weighted composition of the representative migrant

As described earlier for population constraints, the adjustments are added to initial inflows and subtracted from initial outflows, avoiding negative counts of migrants by adjusting the opposite flow.

To summarise: housing, employment or other similar derivative constraints are achieved by scaling each age-sex-social group migrant flow by the single factor that will reproduce the constraint, but allowing for different weights for each flow. This is a development to sub-populations of the 'plus-minus method' of Shryock and Siegel (1973) for adjusting gross flows to implement a constraint of a single net migration total.

Multiple constraints, long-term constraints, and soft targets

The solutions presented above are not by any means the last words on constraints. A brief comment on three aspects indicates where development is in order.

First, there may be multiple constraints available for the same projection year. In the framework described above, constraints of different types are handled by applying component constraints prior to constraints on population or population derivatives.

Multiple constraints of the same type are also common; for example the number of migrants is usually estimated not only for one age-sex category but for every age-sex category. They are each applied as above with an independent impact on the projected population. A problem arises when two constraints are not independent. This commonly occurs when geographical sub areas or social divisions such as ethnic group are projected. A more finely grained age-sex constraint is available without the subdivision than is available with it. For example the total population has been estimated for small areas while more detailed estimates are available for the district containing those areas.

In the notation used above, constraints are available for two sets of population categories $\underline{A_1}$ and $\underline{A_2}$ that represent marginal totals of a more detailed classification $\underline{A_3}$ for which a constraint is not available. The implementation of each constraint impacts on the other: they are not independent. Using the example of small areas, applying the constraint for the age-sex composition of the larger area gives an estimated total population in each small area inconsistent with the information in the second constraint. A solution is to estimate a more detailed constraint for classification $\underline{A}_3$ consistent with the constraints for both $\underline{A_1}$ and $\underline{A_2}$. A suitable estimation method is provided by Iterative Proportional Fitting (Bishop et. al., 1975). It is sensible to take initial values of the constraint for $\underline{A_3}$ from the initial projection, iteratively scaling them alternately to be consistent with the constraints for $\underline{A_1}$ and $\underline{A_2}$ until no value of $\underline{A_3}$ differs between consecutive cycles more than a pre-determined small convergence criterion. The practical implementation of Iterative Proportional Fitting in Excel is described by Paul Norman (2002) and in SPSS by Simpson and Tranmer (2004).

A further important case of multiple constraints involves population constraints and derivative constraints, typically where a prior population for a larger area is to constrain projections for sub areas that have observed or planned housing constraints. To achieve consistency the headship rates for each smaller area must also be adjusted.

Second, constraints have been accommodated above for a single time period. Population and population derivatives are often presented as estimates available at the end of longer intervals of say 2 or 5 years, or as a target 10-20 years ahead. In this case the question may be: which *constant* adjustment applied over *n* time periods, will produce a projected population consistent with the constraint? The solution is only possible within an iterative framework, where the solution is amended after each interim projection with successive solutions providing closer consistency with the constraint. The iterations would continue until a pre-determined small convergence criterion is attained. The interaction of migration, births and deaths that occurs *within* one time period has been ignored in this paper, as is usual within projection models (Keilman, 1985).

An alternative solution is to impose extra constraints, one for each time period between the current population and the given constraint by interpolation, to which the single-period solutions can be applied. While this solution will ensure neither constant numbers of migrants nor constant migration rates, the aggregate migration over the longer period is of practical interpretation and use.

Third and finally, the results of constraining population projections to estimates in the ways described can imply uneven, unrealistic age-schedules of component rates. When constraining component rates to age-band and sex specific counts of events by scaling the single-age rates within each as described above, the results are likely to cause unrealistic discontinuity of rates at the boundaries of the age-bands. While methods could be developed to smooth the adjusted rates for single ages while maintaining consistency with the constraining counts in age-bands, the smoothed pattern is likely to remain 'lumpy' while the impact on the projection is likely to be insignificant.

Unusual schedules of migration rates re-calculated after integrating age-specific components of change or population estimates as constraints, will also be due to the inaccuracies of the constraints themselves, or indeed of the base population. The constraint to a detailed population estimate may imply implausible age or sex composition of migration for the prior period, which in turn throws doubt on the constraint itself. In some cases it may be worth imposing model age schedules of components of population change at the cost of treating the constraint as a soft target, to be met only approximately. For historical settings, Jim Oeppen has suggested Generalised Inverse Projection solutions that adjust demographic components to minimise deviations from the constraining population estimate while maintaining a standard schedule of mortality and a smooth development of migration over time (Oeppen, 1993). In any case, it is important to examine the implied component rates following constraint to estimated population and population derivatives, as a check on the plausibility of those constraints.

5. Constraints implemented in POPGROUP

The POPGROUP forecasting software is used widely in the UK, mainly by local government organisations that have funded it but also in academic teaching and research. Its construction and functionality and its implementation of constraints are described in this section, to illustrate the practical application of constraints within an operational forecasting model. POPGROUP (POPGROUP, 2003; Andelin and Simpson, 2002) consists of Microsoft VBA routines that create and work with Excel workbooks according to user input. Single year of age and sex are supplemented by one extra dimension, 'Group'. Groups are labelled by the user, often representing geographical areas, social categories such as ethnic group, or population categories such as urban and rural. The software, with example workbooks, is further described at http://www.ccsr.ac.uk/popgroup/, which holds example workbooks.

Separate workbooks refer to the base population and each component: fertility, mortality and pairs of migration flows (in and out flows between each Group and two external regions). Different files of the same component refer to alternative assumptions developed by the user. The workbooks containing assumptions for a particular forecast are specified in a scenario file, from which the forecast is initiated. Detailed output files contain the components and the final population at single year of age, derived summary rates and the impact of any constraints. A reporter workbook provides charts and tabulations, which can be further enhanced by the user's own Excel skills.

The forecast for each Group is independent except as affected by the constraints as discussed below. Transition between Groups is allowed at birth to reflect fertility when

Groups are ethnic or racial categories; this facility is not as yet extended to all ages to provide a fully multi-dimensional projection model. Households are optionally derived by applying age-sex-Group headship rates for each of nine household types, after subtraction of persons not in households. The labour force is optionally derived by applying age-sex specific economic activity rates; students may be specified separately. The age-sex headship and activity rates are for 5-year age groups or amalgamations of these, as chosen by the user. Users are frequently concerned with the demographic impact on dwellings and jobs, which in POPGROUP are linked to households and the labour force, either through overall conversion rates or through separate rates of vacancy, sharing, second homes, commuting and employment.

Mandatory input is minimised in order to allow a variety of data suitable to a variety of users. A single schedule of age-specific mortality and fertility with a sex ratio at birth may be applied to each Group's base population in every year of the forecast. Rates may then be overridden or multiplied by factors specific to age-sex categories or Groups, in any chosen years.

Constraints as discussed in this paper are specified by the user. An estimated component of change may be specified for any period on the relevant workbook, as a number of events for each Group or as a single constraint on the sum of all Groups. In either case the constraint may be a total number of events, or contain age-sex detail (generally 5-year age-sex groups for migrants and deaths, males and females for births). Constraints may be continued forward from a specified year, or linearly interpolated between two years in which they are explicitly provided. They are implemented as in equations (1) and (2) above.

Population estimates are similarly specified as constraints for any year subsequent to the base year, for each Group or as one constraint on the population sum of all Groups. The constraint may be of the total population, for five-year age groups, or for single years of age. Where constraints for the Group and the sum of all Groups are both specified, the all-Groups information must be more detailed than the Group constraint (else it could be derived from the latter and would be redundant); the extra detail for each Group is derived by Iterative Proportional Fitting as described earlier. Constraints separated by more than one year are linearly interpolated for intervening years. The constraints are implemented as in equations (3) to (5) above.

Population derivatives are computed for each year of the population projection, but a constraint is specified as a *change* in number of households, dwellings, labour force or jobs, as it is the change that is the lever that can be influenced by policy. Again the constraint may be specified for any year subsequent to the base year, for each Group or as one constraint on the population sum of all Groups. No age or sex subdivision of the constraint is permitted when the solution is implemented as in equations (6) to (9) above. However, a further option allows an age-sex specific constraint to be met by automatic adjustment of the headship rates or economic activity rates while maintaining the population projection unaltered.

6. Applications

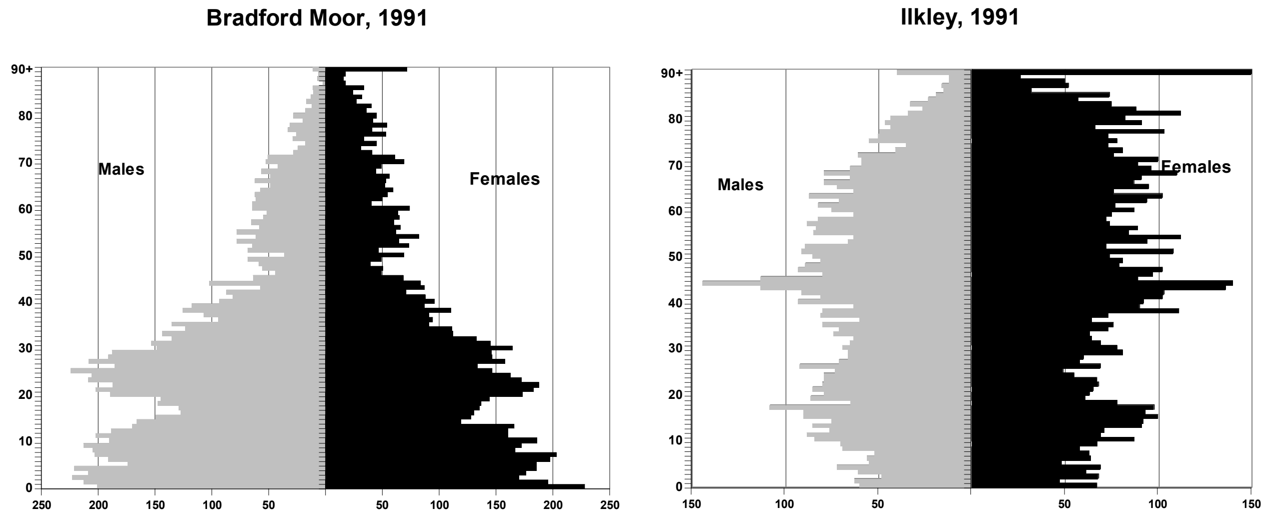
To illustrate the strength that estimates can add to a projection, this section describes four practical applications. They use extra information on the three types of constraints described above: components of population change (in the first two applications), population stock and population derivatives (in the third and fourth applications). The extra information makes population projections more responsive to the variety of evidence on population change that often exists beyond the base year of a projection, and helps to inform the assumptions made for the future. In each case the projection was implemented in POPGROUP, using the arithmetical procedures described earlier.

Integrating local vital statistics with projected national fertility and mortality trends

National statistical agencies make population projections for national areas and sometimes for regions. In Britain such projections are made by the Office for National Statistics for all local government Districts (Wood et al., 1999). Local government organisations need projections for smaller areas to inform electoral boundary reviews, land-use plans and services that are delivered via local offices (Simpson, 1998).

For a cohort component projection of local area population, the base population is provided from the decennial national census. Figure 1 reveals very different age structures for two of the thirty electoral ward of Bradford District. As in the ONS projection of Districts, the national projected trend of fertility and mortality may be used for each local area (migration is discussed in the third example below). Estimates of births and deaths allow the national trend be anchored in local experience.

Figure 1. The contrasting population composition of Bradford wards. Bradford Moor and Ilkley, 1991



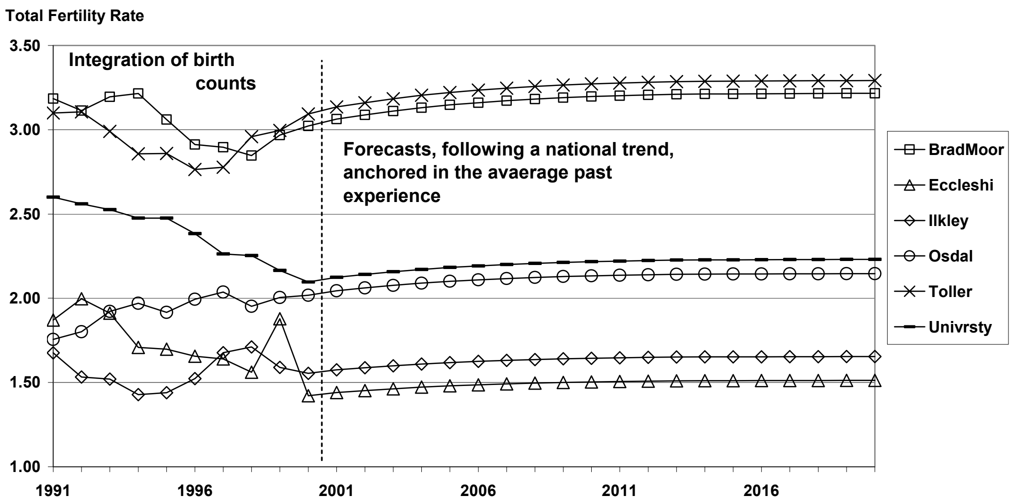
Source: Bradford Council, based on 1991 Census.

Vital statistics are published regularly by ONS for electoral wards (areas of 1,000-20,000 population, recording the numbers of registered births and deaths, broken down by sex and broad age at death. These data can be used as constraints after the base year as described above. This in effect scales the national schedule of age-specific fertility and mortality rates to be consistent with the vital statistics. Figure 2 shows the resulting Total (Period) Fertility Rate as estimated for 1991-2000. It shows fertility projected forward using a differential from the national projection equal to the average differential observed during the period 1991-2000.

In the projection as described each ward has age-specific fertility rates that differ from the national rate by a ward-specific factor which is the same for all ages. The schedule of rates has the same curve for each ward, but this curve is raised or lowered to be consistent with the numbers of births and deaths entered as constraints. Without the integration of birth counts, each area would be projected with the same birth rate, making a difference of over 50% to the child population projected for some wards. Where data are available to estimate age-specific rates separately for each ward, these may be additionally specified. However, in spite of considerable differences in the local age patterns of fertility, their use in a projection makes negligible difference to the projection (Williamson and Simpson, 2002). The integration of recently observed numbers of births and deaths as constraints captures most local differences, at least in as much as they affect a population projection.

The assumption that a future trend in fertility copied from national studies applies equally to each ward may not be the best that can be made. Vital rates are affected by factors that may themselves change over time, as may be the cause of University ward's steady fertility decline in Figure 2. An example of these factors is now discussed.

Figure 2. Total Period Fertility Rate, estimated and projected for Bradford wards



Source: Williamson and Simpson, 2002.

Table 1: Data for a population projection by ethnic group

Input to projection	**Age detail**	**Ethnic group detail**	**Time period(s)**	**Data source**
Base population	Single year of age	Each ethnic group	1991	Census
Fertility rates	Five-year age groups of mother	Each ethnic group	1991	Maternity records
Births	Not applicable	Each ethnic group	Each year 1991-1999	Maternity records
Mortality rates	Single year of age	Total of all ethnic groups	1991	National projection
Deaths	Five year age groups	Total of all ethnic groups	Each year 1991-1997	Government vital statistics
Migration rates within Britain	Five year age groups	Total of all groups, with one all-ages differential between South Asian groups and Other	1991	National census, electoral register
Migration rates to/from overseas	Five year age groups	Total of all groups	1991	National census
Migrants within Britain	Five year age groups	Total of all ethnic groups	Each year 1991-1998	National Health Service Central Register
Migrants to overseas		No estimates		
Migrants from overseas	Five year age groups	Each South Asian group	Each year 1991-1999	Port Health Notifications

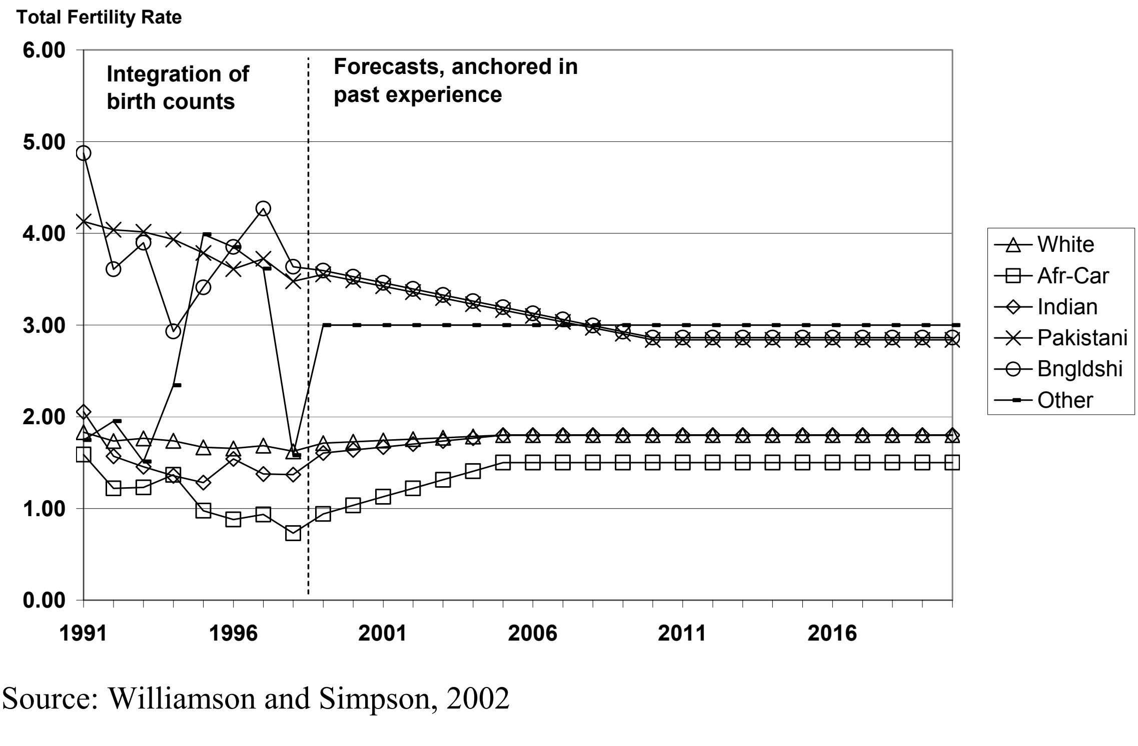
Note: in each case, males and females are recorded separately

Integrating estimated components of change having diverse detail: ethnic groups

This second example also integrates within a forecast selected components of population change after the base year. In Britain, ethnic group has become a dimension of public policy, such that ethnic groups' demographic change helps to determine appropriate services and to monitor equal opportunities. As an extra dimension of demographic projections ethnic group also adds accuracy by disaggregating population change to sub-groups that have widely different demographic rates. To some extent these differences are explained by recent immigration of family members, particularly for the Pakistani and Bangladeshi groups whose major period of migration is relatively recent and continues. There are also cultural and other factors associated with health, employment and social exclusion that influence each demographic rate.

The demographic data available for projections by ethnic group is incomplete and varies within Britain (Haskey, 2002). The projection reported here is more fully described in a report from Bradford Council (2000). In Bradford District, the estimates that were integrated within the projection are described in Table 1. Different components – births, deaths and migration flows – were available with different detail since the projection base year. For example, births were available separately for each ethnic group, but deaths and some migrant flows only for the total without an ethnic group disaggregation. In each case, POGROUP integrated the estimated component of change as described earlier.

Figure 3. Fertility estimated and forecast for six ethnic groups



Source: Williamson and Simpson, 2002

The output from 1991-1999, the period in which estimates were available to constrain the projection, was analysed before finalising assumptions for the future from 1999 for which there were no available estimates. Figure 3 shows the estimated and forecast fertility for each ethnic group. The Pakistani and Bangladeshi groups' fertility has been high but falling steadily. The fall is not expected to result in complete convergence with other groups, partly because immigration continues, and partly because the very low average economic activity of Muslim women is associated with higher fertility. Fertility of the African-Caribbean population (the three 1991 Census categories of 'Black') indicates the ratio of African-Caribbean children to African-Caribbean women. It is depressed because of the high number of unions between African-Caribbean women and men of other groups, whose children are not recorded as African-Caribbean. As the complement of the same phenomenon, the measured fertility of the 'Other' group which includes many children of multiple origin is expected to continue at a high level; the small number in this group makes volatile the rates measured in the1990s.

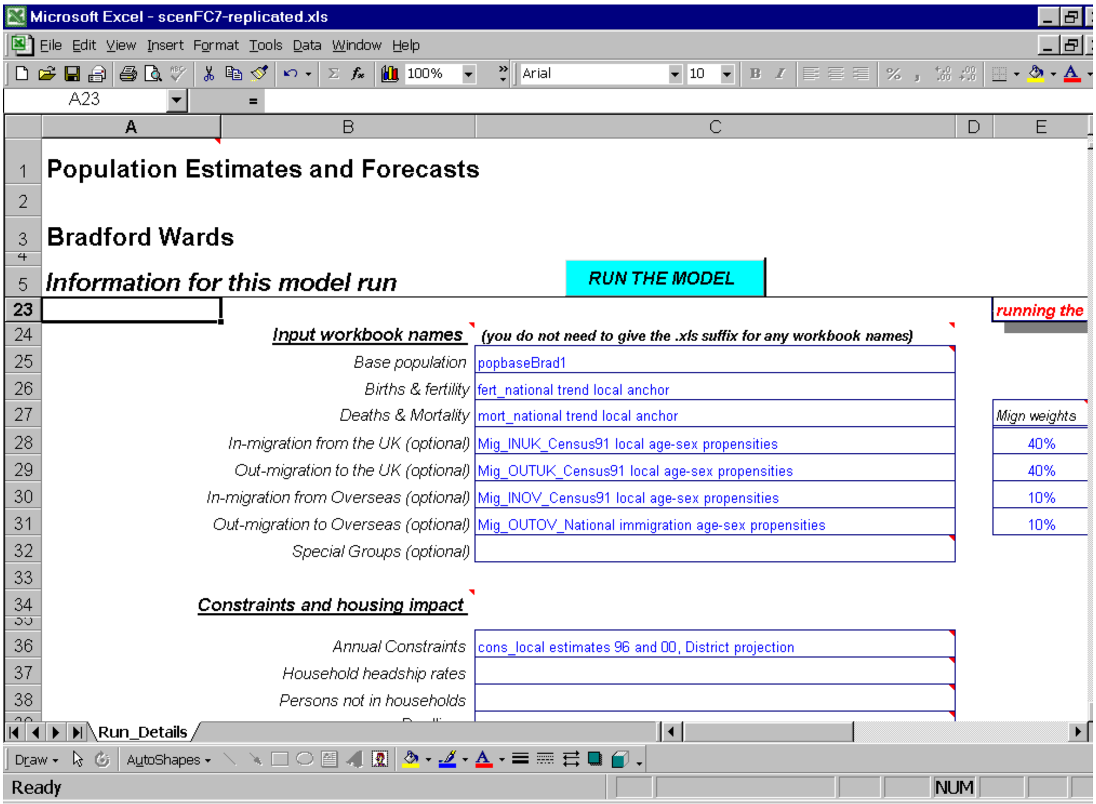
Integrating population estimates to improve migration assumptions

This third example shows the value of integrating population estimates within a projection in order to measure the impact of migration. It extends the first example above where vital statistics were incorporated. Migration rates for local areas can be derived from the UK census but are not ideal. They relate only to the year immediately prior to the census, they do not include international emigration flows (these people having by definition left the national census area and population), the small numbers involved for any one local area limit the age-breakdown available, and they frequently suffer from incomplete response (Rees and Duke-Williams, 1997). In Britain, changes of registration with general practitioners (the provider of primary health care) are monitored and provide detailed estimates of migration between regional and District areas, but are not available for local areas as they omit substantial numbers of short-distance moves (Scott and Kilbey, 1999).

A solution is available when a population estimate is available for at least one time point beyond the base year. In this example population estimates were available for each ward for 1996 and 2000 with 5-year age-sex composition (Simpson, 1998, part III.6). In other areas where estimates are not made between censuses, there will nonetheless be population estimates at two successive censuses. Using the later years as a population constraint, POPGROUP performs a back-projection, deducting the estimated natural change in order to infer the adjustment to migration required to meet the constraints at each age.

In this case a further constraint was imposed, to meet the District population (the total across all wards), which had been estimated independently for each year from 1992 to 2000, by sex and single year of age. As described earlier, the procedure first invoked Iterative Proportional Fitting to create constraints for each single year of age for each ward, consistent with the given ward and District constraints, before applying the back projection to estimate migration. Initial migration rates were based on data from the Census. These are adjusted by POPGROUP as described earlier, to meet the population constraints in each year, and then taken forward for use in the forecasts.

Figure 4. POPGROUP scenario workbook integrating population constraints

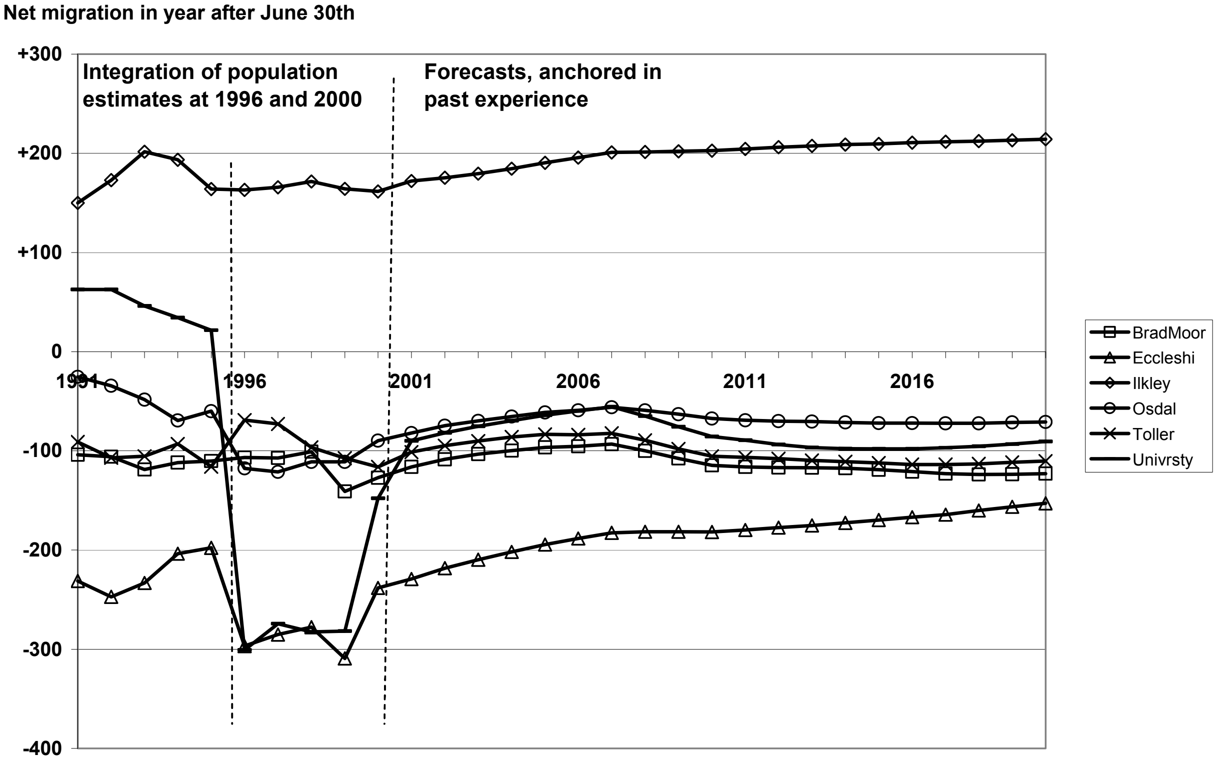


The constraint provides only the net impact of migration. To divide it between gross national and international migration flows, weights were attached approximating the weight of each flow measured in the census. Figure 4 shows the specification of this forecast in a POPGROUP scenario workbook.

Figure 5 shows the resulting net migration in each year of the projection from 1991 to the latest population estimate 2000. The projection beyond 2000 was based on the understanding gained from the period 1991 to 2000; it could be supplemented with variations expected from analysis of future housing developments. From comparison with Figure 2 the natural growth in wards with highest fertility tends to be compensated by a net loss due to migration to other parts of the UK; conversely, those wards with lowest fertility tend to gain through migration. The exception is Eccleshill which lost to both natural change and migration, due to a demolition programme of social housing stock during the period. A compensating relationship between youthful central city and more elderly surrounding areas is a common future of Britain in the late 20th and early 21st centuries.

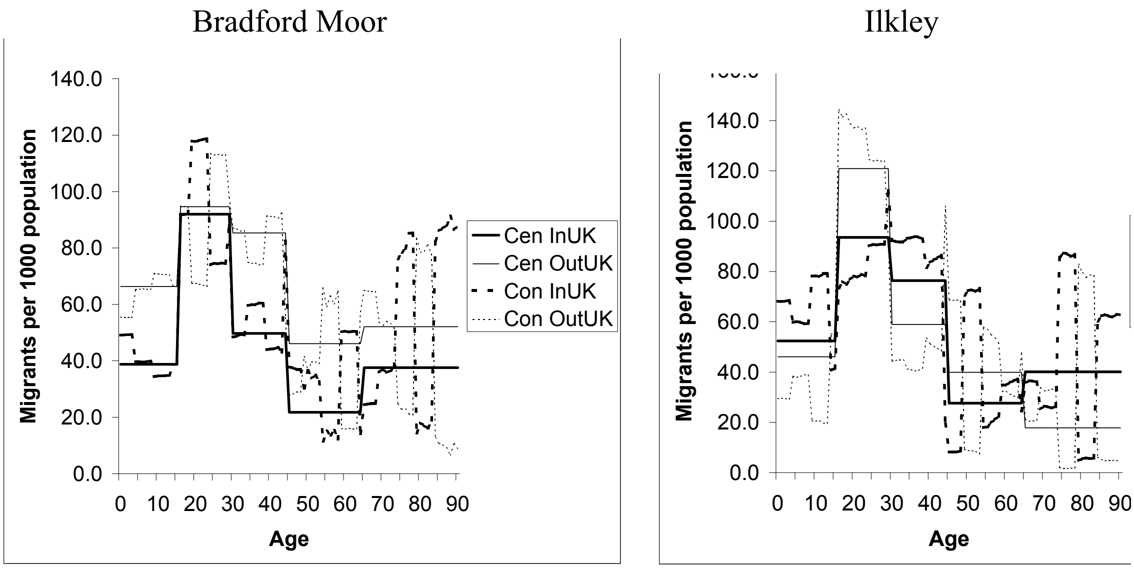
Figure 6 shows age-specific migration rates for two wards before and after the constraint. All rates are expressed as a ratio of migrants per thousand population in the area at the beginning of the annual period. The initial schedules of age-specific rates, taken from the 1991 Census, distinguish only five broad age groups. The adjustment to the initial rates after integrating population estimates has clear characteristics. First, because the weights given to in- and out-migration flows in the adjustment were equal,

Figure 5: Net migration constrained to population estimates



Source: Williamson and Simpson, 2002

Figure 6. Male migration rates before and after constraint of census-based migration rates to population estimates



Note: Cen: 1991 Census, migration rates; Con: migration rate calculated for 1991-2000 after constraint to 1996 and 2000 population estimates.
Source: Williamson and Simpson, 2002.

the distance of the in- and out- constrained migration flows are of a similar distance from the unconstrained rates. They deviate in opposite directions, as an increased in-flow has the same impact on the population as a decreased out-flow. Second, for the Ilkley area, the Census net in-migration of children, and net out-migration of young adults are both deepened by the evidence of the 1990s, with additional age detail provided by the estimates' 5-year age groups. Third, at some older ages adjacent age-groups appear to be adjusted in opposite directions. This may suggest errors in the age-distribution of the population estimates.

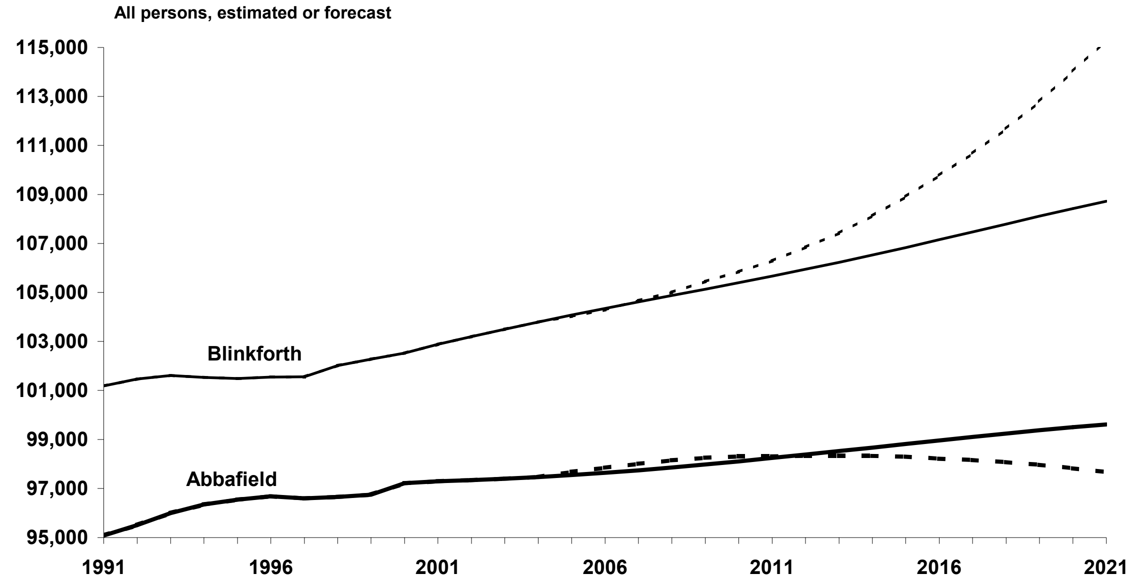
There may also be errors in the age distribution of the base population of 1991, following the ad-hoc estimation of census undercount for that year (Simpson et al 1997; Heady et al., 1994).

Integrating housing plans into a projection

The final example is the integration of the third type of constraint, a population derivative, the expected change in the number of dwellings. The impact on population of planned housing development is an integral part of land use planning in Britain. Planning constraints on provisions for housing, imposed by local government within national rules, have implications for school, housing and employment policies. The example here is taken from the training guide for POPGROUP, representing the two local Districts Abbafield and Blinkforth within the county of Fluteshire (Simpson and Heyward, 2002).

To relate housing change to population, the user must also supply household headship rates, numbers or rates of the population not in households, and a conversion rate between households and dwellings that may be expressed as rates of sharing, holiday homes and vacant dwellings. As described in Section 4, POPGROUP computes the household impact of a single migrant without the constraint, in order to adjust the number of migrants to meet the constraint. Figure 7 shows the population before and after constraint to a growing number of dwellings in Blinkforth, and a steady number of dwellings in Abbafield. The age and sex composition of the constrained population helps to examine the impact on other services of the planned housing. The term 'population derivative' is not wholly appropriate when describing a constraint, for it is the population that is now derived from the housing target.

Figure 7. Population of two districts before and after constraint to planned numbers of dwellings.



Note: broken lines represent the forecast after constraint to a housing programme.
Source: Andelin and Simpson, 2002.

7. Summary and discussion

Population projections based on the application of demographic rates to the cohorts of a base year can be considerably improved by the integration of independent information since that base year. This paper has addressed three types of such information: flows of the component events of population change – births, deaths and migrants – that may have been measured or estimated with full or partial age-sex detail for a varying number of years; estimates of population stock derived independently from the projection, again with full or partial age-sex detail; and finally population derivatives such as housing or employment which a projection scenario must be consistent with. Solutions have been presented for each of these three cases, and illustrated with the projections software POPGROUP.

Inclusion of external constraints provides a smooth integration of estimates and projections within a single information base. The early part of a projection is filled with estimated population flows and stocks where they are known. The gaps are completed by adjusting the initial detailed age-schedules of each component rate. The projection benefits from what can be considered a 'training phase'. The adjustments to demographic rates imposed by the estimated data during the training phase provide evidence of recent trends. This evidence may be used to revise the assumptions for subsequent projection years. The incorporation of estimates improves the projection not only in the early years but in the whole projection period.

The solutions presented in this paper for integrating estimated constraints within a projection are straightforward and effective. They allow integration of commonly held estimates of the three types of constraint and have shown to be useful in practice for local area and social demography. However, there will be further gains to be made from relaxing the ways in which consistency is gained between different information sources. Algorithms might apply a population constraint in such a way that the resulting age-schedule of migration is smoothed over more than one time period, or that discontinuities are minimised at the cost of only approximately meeting the constraints. The stipulation that only one of births, deaths or migration should be adjusted could be relaxed in circumstances where births and deaths are not reliably estimated. In similar vein, the consistency of regional projections to a national projection requires consistency of each component as Keilman (1985) advised. In the different context where housing and population estimates conflict, these and household formation will need to be adjusted to gain consistency. Where there is a possibility that estimates themselves carry significant error, their integration into a forecast may prejudice the results, leading the forecast astray rather than training it well. In practice one should not put much weight on approximate estimates, and be sure to include sensitivity analyses in the process and the reporting of the projection.

For a general solution, routines that include estimates within projection models should leave the user in control of which levers are to be used to achieve consistency between the various inputs. Forecasting at its best, in demography and in other fields, is an iterative review of assumptions and output from both estimates and forecasts. By reviewing both estimates and projections, the validity of each can be assessed in ways that independent assessments may miss. The divergence of population and household estimates in Britain in the 1990s could have indicated changing household formation or population figures that were not robust. Similarly, the divergence of the 2001 Census from projected population figures could have been the results of excess under-enumeration in the Census or of un-monitored emigration omitted from the projection. In both cases the difference alerts one to validate the robustness of each data source.

Where estimates are included in the early years of a forecast, the notion of a ‘base year’ is arguable, just as the distinction between estimate and projection becomes ambiguous. This is a reasonable reflection of the reality in which new information challenges and revises our existing views. Projection models which integrate potentially conflicting information help to clarify and validate the projection assumptions, leading to a more accurate picture of the past and the future.

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