Entropy Balancing: A maximum-entropy reweighting scheme to adjust for coverage error

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Abstract: *Propensity score adjustment* is gaining prominence within social science and epidemiology as a means to diagnose and adjust for selection bias in non-probabilistically selected samples. As its application has proliferated, so too has awareness of its shortcomings. Hainmueller has recently proposed an alternative method – *entropy balancing* – which builds upon the propensity score method while addressing its limitations. Here we extend this innovative reweighting procedure and demonstrate its application through an example using the *Young Lives Project* survey for rural Andhra Pradesh, South India. We summarise the potential of this procedure to contribute to robust survey-based research more widely.

Key words: causal inference, coverage error, design based inference, entropy balance, propensity score adjustment

1. Introduction

In many fields of survey analysis, generalisation from sample to population rests on design based inference, the plausibility of which depends on the adoption of randomisation procedures in sample selection and the application of survey weights to produce estimates that are unbiased, or at least "approximately unbiased" (Kalton 2002: 129). Under this mode of inference, survey procedures are ideally designed and implemented to permit generalisation of findings beyond the surveyed sample of respondents "n", to a defined population of interest "N". Various sources of error can undermine this ideal and so compromise the external validity of findings. Here our interest is in newly available techniques to correct for bias originating in coverage error.

The advent of internet-based surveys has led to an increase in methodological work to correct for coverage error (see for example Schonlau *et al*. 2009, Steinmetz and Tijdens 2009). Techniques developed in this setting have applicability to any sample design that employs purposive methods to select respondents where selection bias may be an issue (Stuart *et al.* 2010). The method has relevance for research utilising small and medium n data for countries in the Global South, where sampling frames are commonly inadequate or absent (UN 2006, Wilson *et al* 2006). Although still not widely implemented, *propensity score adjustment* (PSA) is gaining prominence within social science and epidemiology applications as an innovative means to adjust for selection bias in non-probabilistically selected samples. To date, it has been applied to substantive research problems based on survey samples in which selection bias originating in coverage error is present or suspected. Examples include work by Isakson and Forsman (2003) and Duffy *et al* (2005) to predict election results from non-probability sample surveys canvassing political opinions, by Yoshimura (2004) to generalise findings on consumption patterns beyond an internet survey sample, by Frölich (2007) for analysis of the UK gender wage gap, and by Stuart *et al.* (2010) to assess the generalizabilty of results from a randomised trial to evaluate the impact of an education intervention on student behaviour and exam results. In each of these examples, the aim is the analysis of one or more outcome(s) in the non-random sample, given the distribution of covariates in the target population.

The use of PSA in this setting extends established and widely used pre-processing methods developed for analysis of causal effects in non-experimental studies. In its traditional evaluative application, the propensity score, P(X), is calculated as the conditional probability e(x) of each observation, i, being exposed to a

"treatment", z = 1, as a function of a vector of observed covariates, x. Under Conditional Independence (CI), z is independent of x, and the propensity score is constant (Rosenbaum and Rubin, 1983):

$$e(x) = pr(z = 1|x) \quad (1)$$

In traditional evaluative applications of binary treatment effects, propensity scores are used to reweight or remove survey units to equate (or "balance") the distribution of covariates in treatment and control groups. The process orthogonalizes the treatment indicator to the covariate moments included in the reweighting - in theory reducing model dependence prior to treatment effect estimation (see Sekhon 2009, Abadie and Imbens 2011 for example applications of this principle). The propensity score approach developed as a response to the "curse of dimensionality" and diminishing cell counts encountered when matching or weighting on a large number of discrete covariates (Heckman *et al* 1998). In their seminal 1983 paper Rosenbaum and Rubin formally demonstrate that (under CI) balance across a large number of covariates can be achieved by weighting or matching on the propensity score alone, so substituting a potentially large vector of covariates with a one-dimensional probability.

The more recent adoption of PSA to adjust for coverage error in non-random samples is motivated by the theoretical appeal of applying a one-dimensional measure to balance a large number of covariates. In traditional post-stratification weighting schemes the set of auxiliary information is usually – in practice - limited to a small number of known totals, since weighting on a large number of discrete variables can lead to small or zero cell counts (Rivers 2007, Hainmueller 2012). This can be a shortcoming in cases where the plausibility of the CI assumption demands the inclusion of many confounders. Where propensity score adjustment differs from traditional post-stratification weighting schemes is in its ability to incorporate complete auxiliary information without incurring high dimensionality.

The use of PSA to adjust for coverage error in non-random samples follows the general approach developed by Rosenbaum and Rubin (1983) for the evaluation of causal effects and shares its key assumptions. The application of PSA in this setting characteristically relies on the availability of probability-sampled survey

data which is representative of the target population and gathered reasonably contemporaneously with the non-random sample data (Lee 2006). This, the *reference sample*, provides the benchmark covariate distributions for the non-random sample. Typically it includes extensive auxillary data for the population of interest and variables related to participation in / selection into the non-random sample, but lacks the key variable(s) of interest included in the non-random sample. The two samples are merged to form a single sample. In practice, propensity scores are typically calculated via a logistic or probit regression model of selection into the non-random sample based on a set of observable characteristics common to both datasets[1]. The inverse of an observation's probability of selection is then applied as a weight to adjust for differences in the sample distributions to reduce bias on observed (and associated unobserved) characteristics.

Ideally, the application of PSA weights balances the covariate distributions in the non-probability sample to match those of the reference sample. In common with traditional survey weighting techniques, the expectation is that estimates for unknown characteristics of the target population (represented by the reference sample) can be improved through the introduction of auxiliary information about the target population's known characteristics. The PSA method departs from classical post-stratification weighting techniques in its use of representative survey data in place of census-based known population totals. This departure is commonly motivated by an absence of key auxillary data, or of variables thought indicative of selection into the non-random sample, in available census data. While unavoidable in many applications, the use of weighted representative survey data necessarily increases the variance associated with the estimates. This risks undermining any gains from improved balance and associated bias-reduction.

[1]Where the propensity score is estimated non-parametrically, however, the curse-of-dimensionality is nevertheless incurred. Where estimated parametrically, the sensitivity of the estimated treatment effects to the specifications of the propensity score becomes an issue. This is rarely acknowledged in the applied literature on PSA (Zhao 2005)

The importance of variance is rarely acknowledged in the applied literature on PSA weighting. As its applications have extended, however, other practical difficulties have increasingly gained attention. Whereas traditional post-stratification weighting methods directly adjust sampling weights to exactly reproduce known population totals, propensity score adjustment involves the researcher in a back-and-forwards process of propensity score estimation, matching, and balance checking in an attempt to identify the algorithm that results in the most balanced covariate distribution. This rarely succeeds in simultaneously balancing all of the covariates, with improved balance on one covariate often at the cost of that of another (Ho *et al*. 2007, Stuart *et al.* 2010, Hainmueller 2012). Hainmueller (2012) has proposed an alternative method – that of entropy balancing – to build upon the propensity score method while addressing its limitations.

2. Entropy Balance Adjustment

Hainmueller and Xu (2013) describe entropy balancing as a generalization of the propensity score adjustment approach, though in practice the procedures are the inverse of one another. Whereas propensity scores are typically calculated via a logistic or probit regression and the resulting balance assessed to see if the estimated weights equalize the covariate distributions, entropy balancing, in common with traditional survey weighting schemes, directly calculates weights to adjust for known sample distributions, integrating covariate balance directly into the weights. Like PSE, entropy balancing was developed to evaluate treatment effects in observational survey data, but can also be applied to the objective of sample adjustment.

In practice entropy balancing employs a maximum-entropy reweighting scheme to create a set of weights such that the reference and reweighted non-random samples satisfy a large set of balance constraints. The method's principal advantage over the logistic / probit algorithms typically used to calculate propensity scores in applied settings is its ability to directly implement exact balance. The weighting procedure calculates weights to be as similar as possible (in entropy terms) to base weights, optimising the twin goals of improved balance in covariate distribution and maximum retention of information (the latter is enhanced by the entropy approach's ability to vary weights smoothly across units). Balance can be introduced on the first (mean) second (variance), and - possibly - third (skewness) moments of the covariate distributions, and

the procedure can be set to iterate repeatedly until the variance of the weights cannot be reduced further without undermining the balance constraints.

As detailed in Hainmueller (2012), entropy balancing weights are calculated through the minimisation of the loss function H(w). The entropy balancing procedure generates a set of unit adjustment weights W = $[w_i,...,w_{n0}]$' to minimize the entropy distance, measured as directed entropy divergence ($H(w_i) = w_i \, log(w_i / q_i)$) between W and the vector of base weights Q = $[q_i,...,q_{n0}]$'. The loss function measures the distance between the distribution of estimated control weights defined by the vector W = $[w_i,...,w_{n0}]$' and the distribution of the base weights specified by the vector Q = $[q_i,...,q_{n0}]$'. An appealing feature of the entropy metric (in comparison with alternative distance metrics belonging to the Cressie-Read set) is that it constrains weights to be non-negative, $w_i \geq 0$ for all i such that D = 0. The non-negative loss function decreases as the proximity of W to Q increases (to equal zero if W = Q).

The weights are adjusted as much as needed to accommodate the balance constraints, while remaining as close to the base weights as possible to retain maximum information in the reweighted data. Base weights can be set to uniform weights, $q_i = 1 / n_1$ (where n_1 is the reference sample). Alternatively, where the reference survey has existing sample weights, these can be specified as base weights, $q_i = n_w$. Entropy derived adjustment weights can be calculated to permit alignment of a non-probability sample (n_0) with a reference sample (n_1), where n_1 represents the distribution observed in the reference sample through the following reweighting scheme, where w_i is the entropy balancing weight calculated for each sample unit; $q_i = n_w$ is a base weight; and $Cr_i (X_i) = m_r$ describes a set of R balance constraints imposed on the covariate moments of the reweighted control group. D is a dummy variable indicating whether an observation belongs to the reference sample (D =1) or the non-probability sample (D = 0)[2].

$$\min_{w_i} H(w) = \sum_{\{i \mid D = 0\}} w_i \log(w_i/q_i) \qquad 2$$

[2] Refer to Hainmueller (2012) for a comprehensive presentation of the theoretical framework.

Subject to the following balance (equation 3) and normalisation (equation 4) constraints:

$$\sum_{\{i \mid D = 0\}} w_i \, Cr_i (X_i) = m_r \text{ with } r \, \varepsilon \, 1...,...R \qquad 3$$

and:

$$\sum_{\{i \mid D = 0\}} w_i = 1 \qquad 4$$

Comparing the distribution observed in the reference sample (n_1) of a characteristic with the non-representative sample distribution (n_0) enables an assessment to be made regarding the latter sample's representativeness in terms of the characteristics in question. To correct for selectivity diagnosed in this way, adjustment weights can be calculated to align n_0 with n_1. Where traditional survey weighting techniques usually necessarily limit the size of the vector of auxiliary information to circumvent the "curse of dimensionality", the entropy balance reweighting procedure (potentially) permits all available data from the reference sample to be incorporated (including co-moments as interaction effects), generating a prodigious vector of moment conditions. This permits that the density of *X* in the reweighted non-probability sample can be made to mirror very closely that of the reference sample. In addition, and in contrast to the PSA method, by directly adjusting weights to known sample moments, entropy balancing precludes balance decreases on the specified moments, a problem commonly encountered with PSA. Application of the entropy balance weights to the non-probability sample results in more weight being given to under-represented groups and less weight to over-represented groups, adjusting for unequal probability of sample selection and creating a "pseudo-population" with characteristics in line with the reference sample.

Although his emphasis is on the "evaluation problem", Heinmueller proposes a modified entropy balancing procedure to reweight a single sample to some known features of the target population (Heinmueller 2012, Heinmueller and Xu 2013). His modification entails manually specifying mean covariate values for the target population. This approach may be useful when key census characteristics are known but a suitable reference sample detailing a wider range of comparable covariates is unavailable. Where a reference sample is available the original entropy-based method can be applied. This has two advantages. First, it permits the higher moments of covariate distributions (variance and skewness) to be included in the balancing procedure.

Second, it permits the inclusion of the survey weights pre-assigned to the reference sample to be specified in the calculation of all moment conditions for reweighting. This ability to directly and simply accommodate survey weights is a further important advantage of the entropy balance procedure over alternative PSA methods (which require more involved procedures).

The remainder of this article will illustrate the application of the entropy balancing procedure to reweight a non-probabilistically sampled survey to a reference sample representative of its target population, before evaluating the effectiveness of the propensity score weighting technique in adjusting for biases originating from sample selection[3].

3. Example problem

We draw on two independent sample datasets. The Young Lives Project sample (n_0) is a non-probabilistically sampled survey separately undertaken in Ethiopia, Peru, Vietnam, and Andhra Pradesh (AP), South India in four planned rounds of data collection.[4] The dataset that we are using is the second round for rural AP - collected in 2005/6. The dataset includes information for 2,196 households and 14,110 individuals[5]. The data are primarily intended to provide a means to study the changing dynamics of childhood and household wellbeing. The population of interest is families with young children. The YLP provides a rich source of data on household demographics and individual characteristics, assets, market and non-market labour activities, and attitudes. The substantive purpose of our own study was to use this dataset

[3] All analysis is conducted in STATA 11 software; Hainmueller's "ebalance" suite of commands to perform the entropy balance procedure can be imported to STATA in the usual manner, i.e. "ssc install ebalance, all replace".

[4] The survey was sponsored by the UK *Department for International Development* (DFID), and is led by the *Oxford Department of International Development* at the *University of Oxford*, in collaboration with academic institutions in each of the four project countries.

[5] In the second round of data collection all individuals resident in a selected household were included in the survey.

to analyse the relationships among rural women's participation in poverty amelioration schemes, gender norms, and labour profiles at the individual, household, and community levels.

The YLP survey's sampling procedure is described in Wilson *et al* (2006). The use on non-probabilistic sampling was prompted by the absence of "effective, accessible and accurate sampling frames of households with qualifying children in [the] study countries" (Wilson *et al.* 2006: 356). Consequently, the study adopted a *sentinel site surveillance system* (Kumra 2008)[6]. In AP, 20 study sites (5 urban and 15 rural), each an administrative zone, or *mandal*, were selected across the State's three agro-climactic regions. Here we limit analysis to the 15 rural sites. Sites were selected on the basis of relative wealth, in line with the study's aim to oversample the poor, while enabling comparisons to be made between poor and non-poor (Wilson *et al.* 2006).

The reference sample is drawn from the *All India National Sample Survey* (n_1). The NSS is a weighted, probabilistically sampled survey representative of the national population. The survey is designed and collected by the *National Sample Survey Organisation* (NSSO), a department of the *Ministry of Statistics and Programme Implementation* (MSPI). The NSS has been conducted annually since 1950. Here we utilise the employment and unemployment schedule (schedule ten) as, importantly for our purposes, it contains information relevant to the selection mechanisms informing inclusion in the YLP sample. Schedule 10 is incorporated quinquennially. We use round 61 of the NSS. Data for this survey year was collected in 2004 - 2005, overlapping with data collection for round two of the YLP. The NSS data-set includes information for 5,550 households and 22,591 individuals in rural Andhra Pradesh[7]. The survey employs a probabilistic stratified, multi-stage sample design. Briefly, the NSS stratifies by geographic region, urban-rural area, population density, and household affluence; with each stratum designed to be non-overlapping and proportional (based on projected population figures from the 2001 national census taking into account

[6]Anderson (1996) discusses the general method of sentinel site sampling in some detail.

[7]At the all India level a total of 124,680 households and 602,833 individuals took part in the survey for schedule 10 of the 61st round of the NSS.

decadal growth rated between 1991 and 2001) (MSPI 2006: 82). Full details of the sampling methodology can be found in the NSSO's documentation for the 61st round (NSSO 2004). The NSSO, in line with the practice of most nationally representative sample survey organisations uses adjustment weights at the household level based on extrapolations of the 2001 census to account for unequal sampling rates in the strata. Samples are selected from each stratum independently. Unequal sampling rates in the strata are corrected for (in order to produce an unbiased mean estimator). In this example, the appropriate sampling weights are drawn from probabilities of selection (MSPI 2008). The weights are uniform within households since all individuals resident in a household are included in the survey.

4. The application of entropy balancing

As a first step, we define a subpopulation comparable with the YLP's target population within the NSS sample to include only households in AP with children in the target population age range. Next, covariates common to both datasets are identified and operationalised. Table one presents the covariates common to the two datasets and their values across the two datasets. The entropy balancing scheme permits the inclusion of both continuous and categorical data, taking advantage of all available information.

The densities of characteristics recorded in the YLP sample can be seen to deviate substantially from those of the target population. The YLP sample has selected a roughly even number of households from each of the State's three agro-climactic regions, with households in Rayalaseema oversampled relative to those in Coastal Andhra and Telangana. "Forward" caste households are significantly under-represented, likely as a result of the oversampling of poor households, Adivasi households are significantly over-represented. Over-sampling is practiced inconsistently, however, with religious minorities substantially under-sampled. Casual daily wage labour households are very under-represented, while marginal and mid-size farming households are over-represented[8]. Households are generally larger in the YLP sample than the target population. Heads of household are younger, disproportionately male, and more literate in the YLP than the target population.

[8] Household class is calculated on the basis of household landholding and dominant labour relations.

Table 1: Sample characteristics prior to entropy balance procedure

	NSS (reference sample, n_1)				YLP (non-random sample, n_0)				Difference	
	mean	S.E	[95% CI]		mean	S.E	[95% CI]		value	p
Household head gender (women)	9.175	0.009	7.313	11.037	7.016	0.005	5.947	8.085	-2.159	0.000
Household head age	38.079	0.424	37.247	38.911	40.123	0.245	39.643	40.603	2.044	0.000
Household head literate	41.831	0.014	39.005	44.656	50.410	0.011	48.318	52.502	8.579	0.000
Household size (adjusted)	4.932	0.007	4.919	4.945	6.425	0.058	6.313	6.538	1.493	0.000
“Forward” castes	21.471	0.012	19.163	23.779	14.299	0.007	12.834	15.763	-7.173	0.000
Dalit	20.159	0.012	17.846	22.473	21.220	0.009	19.510	22.931	1.061	0.232
Adivasi	12.440	0.011	10.281	14.599	15.073	0.008	13.576	16.570	2.633	0.001
"Other backward" castes (base)	45.927	0.002	45.585	46.268	49.408	0.011	47.316	51.500	3.481	0.001
Muslim	6.207	0.006	4.945	7.470	2.368	0.003	1.732	3.004	-3.840	0.000
Christian	2.047	0.004	1.300	2.794	0.865	0.002	0.478	1.253	-1.182	0.000
Hindu (base)	91.741	0.001	91.552	91.929	96.767	0.004	96.027	97.507	5.026	0.000
Household class: non-farm PCP, service, trade	13.425	0.009	11.693	15.156	14.390	0.007	12.921	15.858	0.965	0.204
Household class: marginal farming	3.935	0.006	2.733	5.137	18.670	0.008	17.040	20.301	14.736	0.000
Household class: small-scale farming	6.582	0.007	5.168	7.996	7.969	0.006	6.836	9.102	1.387	0.017
Household class: mid-size farming	13.836	0.011	11.738	15.934	23.087	0.009	21.325	24.850	9.252	0.000
Household class: capitalist farming	3.738	0.005	2.769	4.707	4.508	0.004	3.640	5.376	0.770	0.085
Household class: regular salaried employment	5.408	0.006	4.230	6.586	6.421	0.005	5.395	7.446	1.013	0.058
Household class: casual daily wage labour (base)	53.078	0.002	52.736	53.419	24.954	0.009	23.144	26.765	-28.123	0.000
Household landholding (acres)	2.076	0.091	1.897	2.254	2.196	0.080	2.039	2.352	0.120	0.140
Household landholding (log acres)	-0.484	0.049	-0.579	-0.388	-0.190	0.033	-0.255	-0.125	0.293	0.000
Region: Coastal	42.968	0.014	40.138	45.798	34.335	0.010	32.349	36.322	-8.633	0.000
Region: Rayalaseema	17.877	0.011	15.788	19.966	32.423	0.010	30.464	34.381	14.546	0.000
Region: Telengana (base)	39.149	0.002	38.815	39.483	33.242	0.010	31.272	35.213	-5.907	0.000

Source: Data Sources: All India National Sample Survey 2004 / 2005: round 55 / schedule 10: Employment & Unemployment & Young Lives Project; round two (2005 / 2006) n = 4172 (NSS n = 1,976) (YLP n = 2,196). Satterthwaite t-tests are applied to calculate the p value of the equality of means in the two samples (recommended when the population variances cannot be assumed to be equal)

Table 2a: Variable moment conditions prior to entropy balance procedure

	NSS (reference sample, n_1)			YLP (non-random sample, n_0)			Difference			
	mean	variance	skewness	mean	variance	skewness	mean	variance	skewness	P
Household head gender (women)	9.175	0.083	2.828	7.016	0.065	3.366	-2.159	-0.018	0.538	0.000
Household head age	38.080	178.200	0.598	40.120	131.600	1.206	2.040	-46.600	0.608	0.000
Household head literate	41.830	0.243	0.331	50.390	0.250	-0.015	8.560	0.007	-0.347	0.000
Household size (adjusted)	4.932	3.699	1.720	6.425	7.266	2.021	1.493	3.567	0.301	0.000
"Forward" castes	21.470	0.169	1.390	14.310	0.123	2.039	-7.160	-0.046	0.649	0.000
Dalit	20.160	0.161	1.488	21.230	0.167	1.407	1.070	0.006	-0.081	0.232
Adivasi	12.440	0.109	2.276	15.080	0.128	1.952	2.640	0.019	-0.324	0.001
Muslim	6.207	0.058	3.630	2.369	0.023	6.264	-3.838	-0.035	2.634	0.000
Christian	2.047	0.020	6.773	0.866	0.009	10.610	-1.181	-0.011	3.837	0.000
Hh class: non-farm pcp, services, trade	13.420	0.116	2.146	14.350	0.123	2.034	0.930	0.007	-0.112	0.204
Household class: marginal farming	3.935	0.038	4.739	18.680	0.152	1.607	14.745	0.114	-3.132	0.000
Household class: small-scale farming	6.582	0.062	3.502	7.973	0.073	3.103	1.391	0.012	-0.399	0.017
Household class: mid-size farming	13.840	0.119	2.095	23.100	0.178	1.277	9.260	0.058	-0.818	0.000
Household class: capitalist farming	3.738	0.036	4.878	4.510	0.043	4.384	0.772	0.007	-0.494	0.085
Hh class: regular salaried employment	5.408	0.051	3.943	6.424	0.060	3.555	1.016	0.009	-0.388	0.058
Household landholding (acres)	2.076	14.540	7.269	2.197	14.010	4.885	0.121	-0.530	-2.384	0.140
Household landholding (log acres)	-0.484	2.834	0.321	-0.189	2.429	-0.024	0.294	-0.405	-0.345	0.000
Region: Coastal	42.970	0.245	0.284	34.310	0.226	0.661	-8.660	-0.020	0.377	0.000
Region: Rayalaseema	17.880	0.147	1.677	32.440	0.219	0.750	14.560	0.072	-0.927	0.000
"Forward" castes *Coastal	10.940	0.097	2.504	3.235	0.031	5.287	-7.705	-0.066	2.783	0.000
Dalit*Coastal	8.502	0.078	2.976	3.508	0.034	5.054	-4.994	-0.044	2.078	0.000
Adivasi*Coastal	4.074	0.039	4.646	10.300	0.092	2.613	6.226	0.053	-2.033	0.000
"Forward" castes *Rayalaseema	5.908	0.056	3.740	7.927	0.073	3.115	2.019	0.017	-0.625	0.001
Dalit caste*Rayalaseema	3.305	0.032	5.224	8.702	0.079	2.930	5.397	0.048	-2.294	0.000
Adivasi*Rayalaseema	0.638	0.006	12.400	0.866	0.009	10.610	0.228	0.002	-1.790	0.258
"Forward" castes *household landholding	0.696	8.765	14.360	0.512	6.793	10.650	-0.184	-1.972	-3.710	0.001
Dalit*household landholding	0.186	1.066	10.780	0.241	0.938	8.417	0.054	-0.128	-2.363	0.010
Adivasi*household landholding	0.291	1.846	6.781	0.291	1.454	7.829	-0.001	-0.392	1.048	0.959

Source: Data Sources: All India National Sample Survey 2004 / 2005: round 55 / schedule 10: Employment & Unemployment & Young Lives Project; round two 2005 / 2006 n = 4172 (NSS n = 1,976) (YLP n = 2,196).

Table 2b: Variable moment conditions after entropy balance procedure

	NSS (reference sample, n_1)			YLP (non-random sample, n_0)			Difference			
	mean	variance	skewness	mean	variance	skewness	mean	variance	skewness	P
Household head gender (women)	9.175	0.083	2.828	9.174	0.083	2.829	-0.001	0.000	0.001	1.000
Household head age	38.080	178.200	0.598	38.080	178.200	0.598	0.000	0.000	0.000	0.999
Household head literate	41.830	0.243	0.331	41.830	0.243	0.331	0.000	0.000	0.000	0.999
Household size (adjusted)	4.932	3.699	1.720	4.933	3.704	1.722	0.001	0.005	0.002	0.995
"Forward" castes	21.470	0.169	1.390	21.480	0.169	1.389	0.010	0.000	-0.001	0.998
Dalit	20.160	0.161	1.488	20.160	0.161	1.488	0.000	0.000	0.000	0.999
Adivasi	12.440	0.109	2.276	12.440	0.109	2.276	0.000	0.000	0.000	0.999
Muslim	6.207	0.058	3.630	6.209	0.058	3.629	0.002	0.000	-0.001	0.999
Christian	2.047	0.020	6.773	2.046	0.020	6.774	-0.001	0.000	0.001	1.000
Hh class: non-farm pcp, services, trade	13.420	0.116	2.146	13.430	0.116	2.146	0.010	0.000	0.000	1.000
Household class: marginal farming	3.935	0.038	4.739	3.934	0.038	4.739	-0.001	0.000	0.000	0.999
Household class: small-scale farming	6.582	0.062	3.502	6.582	0.062	3.502	0.000	0.000	0.000	1.000
Household class: mid-size farming	13.840	0.119	2.095	13.840	0.119	2.095	0.000	0.000	0.000	0.999
Household class: capitalist farming	3.738	0.036	4.878	3.749	0.036	4.870	0.011	0.000	-0.008	0.991
Hh class: regular salaried employment	5.408	0.051	3.943	5.406	0.051	3.944	-0.002	0.000	0.001	0.999
Household landholding (acres)	2.076	14.540	7.269	2.077	14.550	7.263	0.001	0.010	-0.006	0.995
Household landholding (log acres)	-0.484	2.834	0.321	-0.483	2.835	0.362	0.001	0.001	0.040	0.996
Region: Coastal	42.970	0.245	0.284	42.970	0.245	0.284	0.000	0.000	0.000	0.999
Region: Rayalaseema	17.880	0.147	1.677	17.880	0.147	1.677	0.000	0.000	0.000	1.000
"Forward" castes*Coastal	10.940	0.097	2.504	10.940	0.097	2.503	0.000	0.000	-0.001	0.999
Dalit*Coastal	8.502	0.078	2.976	8.501	0.078	2.976	-0.001	0.000	0.000	0.999
Adivasi*Coastal	4.074	0.039	4.646	4.074	0.039	4.647	0.000	0.000	0.001	0.999
"Forward" castes*Rayalaseema	5.908	0.056	3.740	5.910	0.056	3.739	0.002	0.000	-0.001	0.999
Dalit caste*Rayalaseema	3.305	0.032	5.224	3.304	0.032	5.225	-0.001	0.000	0.001	0.999
Adivasi*Rayalaseema	0.638	0.006	12.400	0.638	0.006	12.400	0.000	0.000	0.000	1.000
"Forward" castes*household landholding	0.696	8.765	14.360	0.697	8.772	14.340	0.001	0.007	-0.020	0.995
Dalit*household landholding	0.186	1.066	10.780	0.186	1.066	10.780	0.000	0.000	0.000	1.000
Adivasi*household landholding	0.291	1.846	6.781	0.291	1.846	6.781	0.000	0.000	0.000	1.000

Source: Data Sources: All India National Sample Survey 2004 / 2005: round 55 / schedule 10: Employment & Unemployment & Young Lives Project; round two 2005 / 2006 n = 4172 (NSS n = 1,976) (YLP n = 2,196).

In order to apply the entropy reweighting scheme, a single indicator variable is generated in both the calibration and non-probability datasets, coded 1 for all observations in the NSS and 0 for all those in the YLP. The two datasets are then merged to form a single dataset, necessary for the calculation of entropy balance weights across higher moments. As detailed above, a set of balance constraints can now be specified for each of the covariates, equating the moments of the covariate distribution between the calibration and target samples. Recall that possible moment constraints include the mean (the first moment), variance (the second moment), and skewness (the third moment). In the case of binary variables (for example, gender of household head) adjustment of the first moment is, in practice, sufficient to match higher moments. Moment constraints may be separately defined for each covariate. The specification of interaction terms allows covariates to be balanced across key subsample groups (in this case, caste).

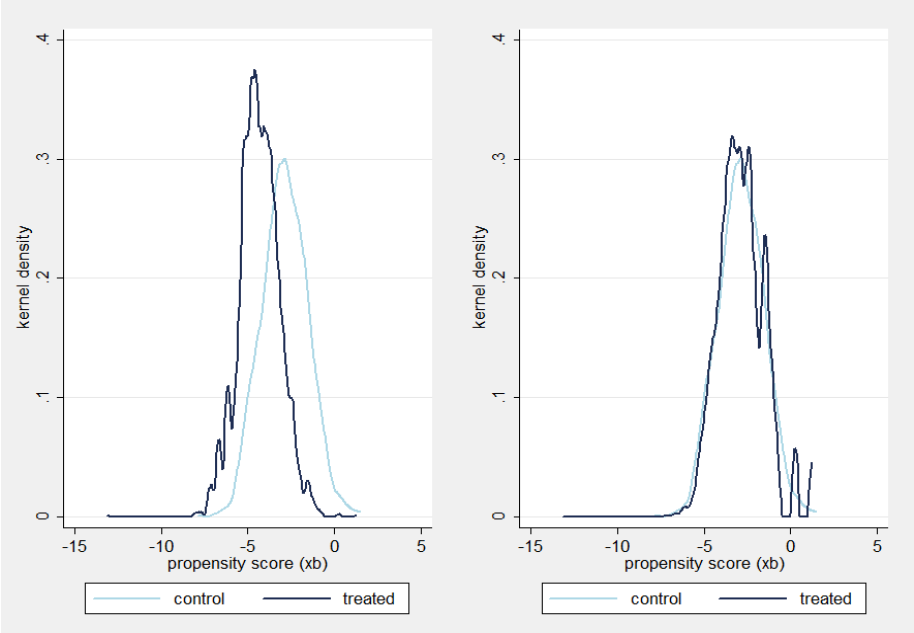
The ebalance algorithm computes the values of the specified moments in the reference sample (n_1), in this case the NSS, and seeks a set of entropy weights to adjust the YLP sample to match the reference sample. Convergence occurs once all the specified moments are matched across the data sources within the specified number of iterations and tolerance level[9]. The inclusion of too many collinear moment constraints may in theory prevent convergence, but this it seems is rare in practice. Specifying fewer moment constraints, either via the removal of implicated covariates or a reduction in their specified moment constraints (mean, variance, skewness), can remedy this. Alternatively (or additionally), the tolerance level can be relaxed. Tables 2a and 2b present the results of the entropy balance procedure.

Figure one presents the distribution of the linear predictor of a propensity score calculated via logistic regression on the two data samples prior to and following the entropy balance reweighting procedure. It demonstrates the propensity score is balanced in the reweighted data.

Figure two presents measures of the standardised differences in means for the two data samples before and after entropy balance reweighting.

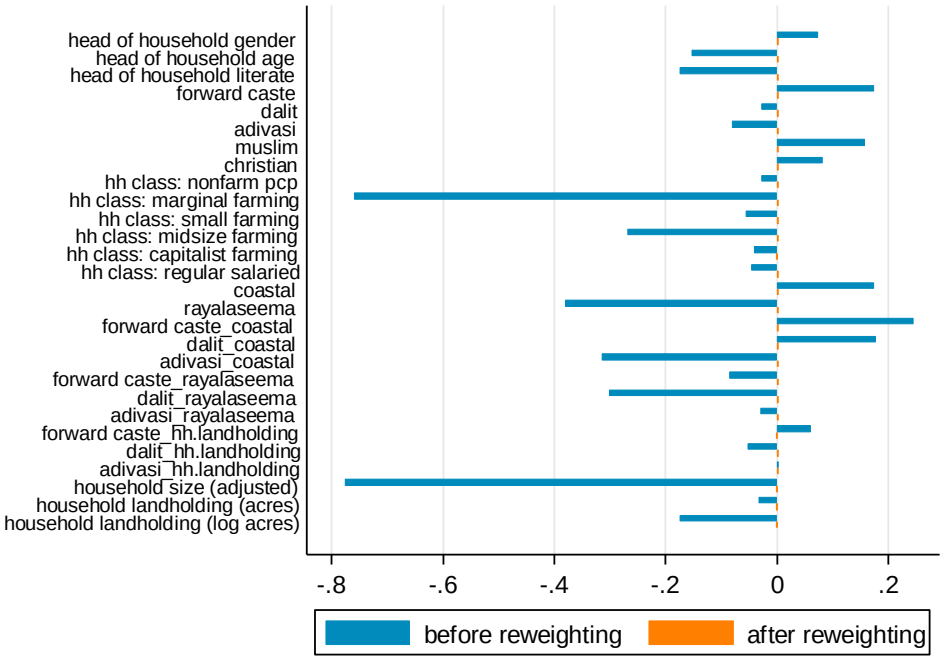
[9] The default iteration number is 20, the default tolerance level 0.015, and both can be increased if convergence fails.

Figure 1: Propensity scores before and after entropy balance reweighting



Source: Data Sources: All India National Sample Survey 2004 / 2005: round 55 / schedule 10: Employment & Unemployment & Young Lives Project; round two 2005 / 2006 n = 4172 (NSS n = 1,976) (YLP n = 2,196).

Figure 2: Covariate balance for all moment conditions before and after entropy balance reweighting



Source: Data Sources: All India National Sample Survey 2004 / 2005: round 55 / schedule 10: Employment & Unemployment & Young Lives Project; round two 2005 / 2006 n = 4172 (NSS n = 1,976) (YLP n = 2,196).

The results demonstrate that the adjustment has a dramatic effect. The entropy balance derived weights have adjusted the YLP sample's distribution such that it now reflects rural AP's population densities as reported in the weighted reference sample. Following the reweighting procedure, differences between the non-probability and reference samples, across all moment conditions for all matching variables are now effectively zero and are non-significant.

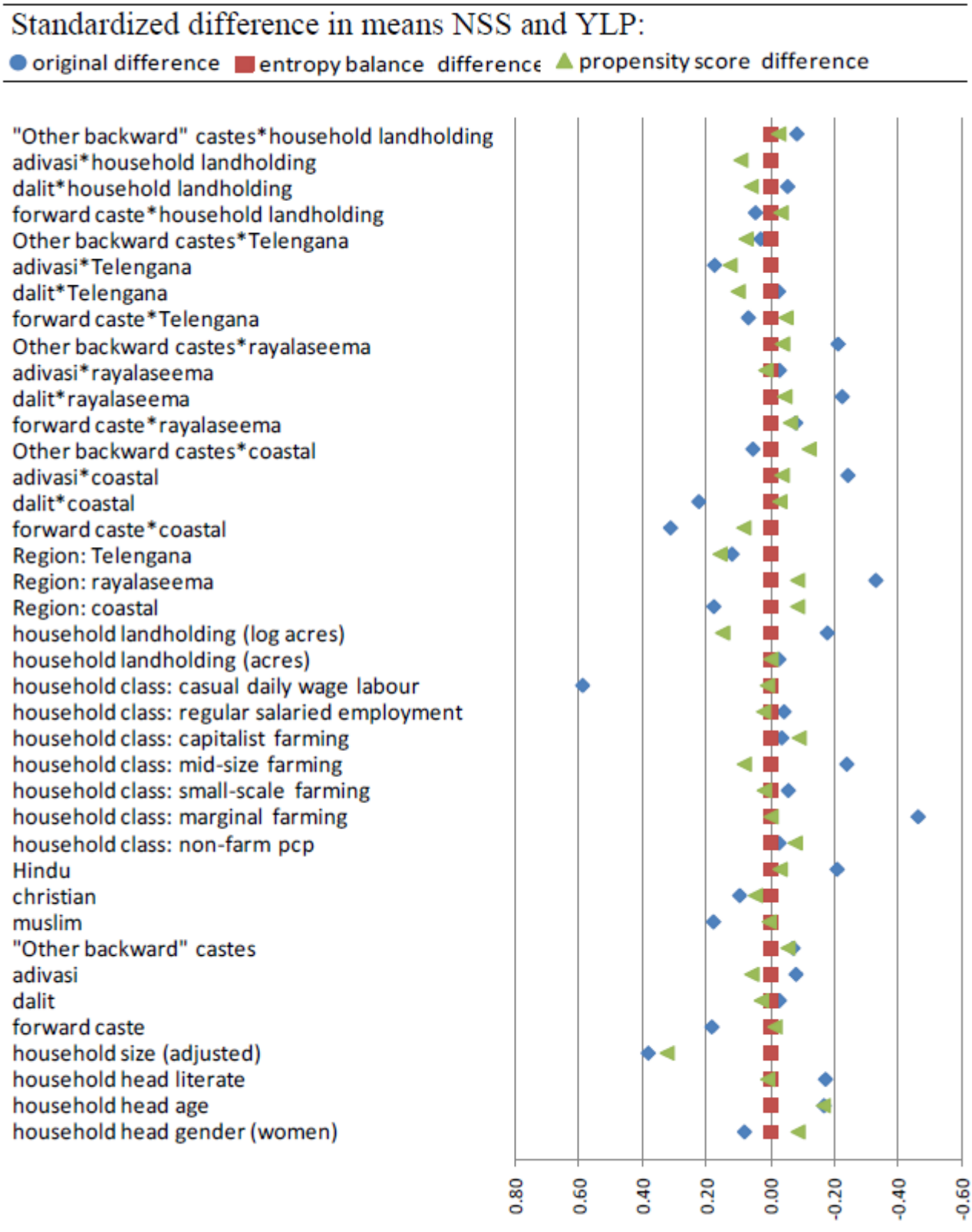
Figure three compares the results obtained through the entropy balance reweighting procedure with those obtained via the PSA method, demonstrating the superior results achieved with the former. The reported PSA results are the best obtained through an extensive back-and-forwards process of estimation, matching, and balance checking. The weights derived from the propensity score procedure improve balance on some covariates (specifically religion, head of household literacy rates, and some categories of household class), but this comes at the expense of balance on other covariates. Notably the propensity score derived weight exacerbates the extent and / or significance of the original differences in some cases. In contrast, the entropy balance derived weights result in simultaneous balance across all of the specified covariates.

Table 3: Target estimates in the non-weighted and weighted non-random sample

Target estimate	unweighted YLP data				weighted YLP data			
	n	mean	S.E	[95% CI]	mean	S.E	[95% CI]	
% of women (over 15) participating in rural self help groups	4,242	28.41	2.09	24.03 32.78	38.91	4.22	30.07	47.76
% of hhs with children (under 15's) undertaking market labour	2,196	23.63	1.54	20.42 26.85	17.04	2.21	12.41	21.67
% of children (12year cohort) with moderate or severe stunting†	745	34.50	1.88	30.55 38.44	28.99	4.69	19.15	38.84
% of children (4year old cohort) with moderate or severe stunting†	1,451	30.46	1.21	28.09 32.83	25.34	4.58	15.74	34.93

Source: Young Lives Project; round two 2005 / 2006 (YLP n = 2,196 households, 14,110 individuals) † "moderate or severe stunting" is defined by a height to weight z-score of below -2.

Figure 3: Comparison of equivalent results obtained by entropy balance and propensity score weighting



Source: Young Lives Project; round two 2005 / 2006 (YLP n = 2,196)

Table three presents the effects of the ebalance weighting procedure on key outcome variables in the pre-adjusted and adjusted non-random sample data. The results demonstrate that the application of the ebalance derived weights modifies the distribution of key outcome mean estimates. There is, however, a trade-off between bias reduction and variance increase. The weighted estimates have increased standard errors and substantially increased confidence errors in comparison with the unweighted data.

5. Discussion

In this article we have introduced, and applied in modified form, an innovative means to adjust for selection bias in non-probabilistically selected samples. We have demonstrated its benefits in relation to the more widely known propensity score adjustment method.

The entropy balance reweighting scheme permits many of the difficulties encountered with propensity score reweighting to be overcome, negating the need for the time consuming and often ultimately unsatisfactory iterative process of matching, and balance checking. Whereas the propensity score reweighting procedure rarely succeeds in simultaneously balancing all of the covariates, entropy balancing directly calculates weights to adjust for known sample distributions, integrating covariate balance directly into the weights. The entropy balance reweighting procedure (potentially) permits all available parallel data from a reference sample to be incorporated in calculating the non-probability samples population weights. This enables the density of X in the reweighted non-probability sample to be made to mirror very closely that of the reference sample. The potential to incorporate exact balance on the first, second, and (possibly) third moments of the covariate distributions is an important advantage over alternative weighting schemes. This ability to include a large set of moment conditions results in a covariate density for the reweighted sample consistent with the population of interest (as defined by the reference sample). The extent of the trade-off exacted between bias reduction and variance increase remains an important consideration, however. By incorporating design weights for the reference sample we introduce a source of variance which, though shared by the PSA approach, is absent in traditional calibration procedures utilising census counts. We should not discount the possibility that some of the increase in variance in fact corrects for bias present in the unweighted non-

random sample, however. Since we are balancing on the second moment condition it may be that the increased standard errors represent less a loss of precision than a correction for inaccurate estimates of precision in the original data.

As with all reweighting schemes, the effectiveness of the process will depend ultimately on the quality and applicability of the reference sample. Similarly, the entropy balance scheme can only correct for bias resulting from unobserved confounders to the extent that they are associated with the recorded balance constraints. The extent (and degree) of covariate equivalence across the reference and non-probability samples needs to be assessed on a case by case basis, and a sufficient number of units must be available in each to permit adequate overlap in the covariate distributions. Whilst it is possible to increase the iteration number and tolerance level in the pursuit of convergence, it is important the balance constraints are realistic and consistent. Bearing in mind these caveats, the example application demonstrates the remarkable results that can be obtained through the entropy balance reweighting scheme. Following the reweighting procedure, differences between the non-probability and reference samples, across all moment conditions for all matching variables are reduced to effectively zero. It is anticipated that similar results can be obtained for any sample dataset where coverage error is known or suspected and an appropriate reference sample is available.

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