Development of a relational model of disability

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Abstract

Age-specific rates of particular disability types are important for planning purposes and are a valuable input to estimates and projections of populations with different disabilities. However, survey estimates of schedules of disability rates display evidence of sampling variability and sub-national disability schedules are often unavailable for reasons of disclosure protection. This paper develops and evaluates a method to smooth sampling variability in national schedules of disability using a technique that has applicability to sub-national estimation of age-specific disability rates. Relational models are used to adjust the limiting long term illness schedule for England (Census 2001) to represent different disability schedules (Health Survey for England 2000/01) smoothing sampling fluctuations. For hearing disability a simple Brass relational model involving two parameters provides a good fit. For other disability types a modified version of the Ewbank relational model with 3 parameters is required. This paper illustrates that relational models can accurately capture the relationship between age-specific rates of limiting long term illness and various disability types.

Keywords: Relational model, disability, limiting long term illness, estimation, schedules.

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1. Introduction

This paper extends relational models beyond their original application, the estimation of mortality to the estimation of schedules of various disability types. The England curve of age-specific limiting long term illness (LLTI) rates from the census (2001) is used as the 'standard' schedule which is adjusted to represent schedules of various disability types, smoothing the variability in national age-specific rates of disability from the Health Survey for England (2000/01). Whilst disability schedules share a similar age pattern to that of LLTI (see figure 1) with low rates across the younger ages that rise with age there are differences in levels of curves as well as the increase in rates with age. The question we address in this paper is whether relational models are sufficiently flexible to accurately represent disability schedules (England) despite these differences.

Estimates of the population with disabilities that distinguish disability type and severity are important for planning purposes to inform the provision of specialist services, equipment, and support (Field 1987; Siegel 2002). Disability schedules are useful partly because the nature of disability service provision varies and is structured by age (Marshall 2009), but also because many disability types follow the same general age pattern: low rates across the younger ages that rise with age reaching the highest levels at the oldest ages (see Figure 1). Knowledge of the population size and age structure and how these are changing provides an indication of the size of the disabled population and how it too might change.

In the UK, national survey estimates of disability distinguishing disability type and severity are subject to sampling variability once disaggregated by age resulting in a ragged curve particularly at the oldest ages (see figure 1). Alternative data sources on disability that enumerate the total population are not subject to the issue of sampling error to the same extent but they do suffer from other weaknesses. For example, the 1991 and 2001 censuses record the numbers of people who are limited in work or everyday activities due to an illness, disability, or health problem but do not provide any information on the nature or severity of the limiting condition. Administrative sources such as disability registers and statistics of benefit claimants are compromised because they do not count those disabled people who do not register or use disability benefits (Macfarlane and Head 1999; Bajekal et al. 2003).

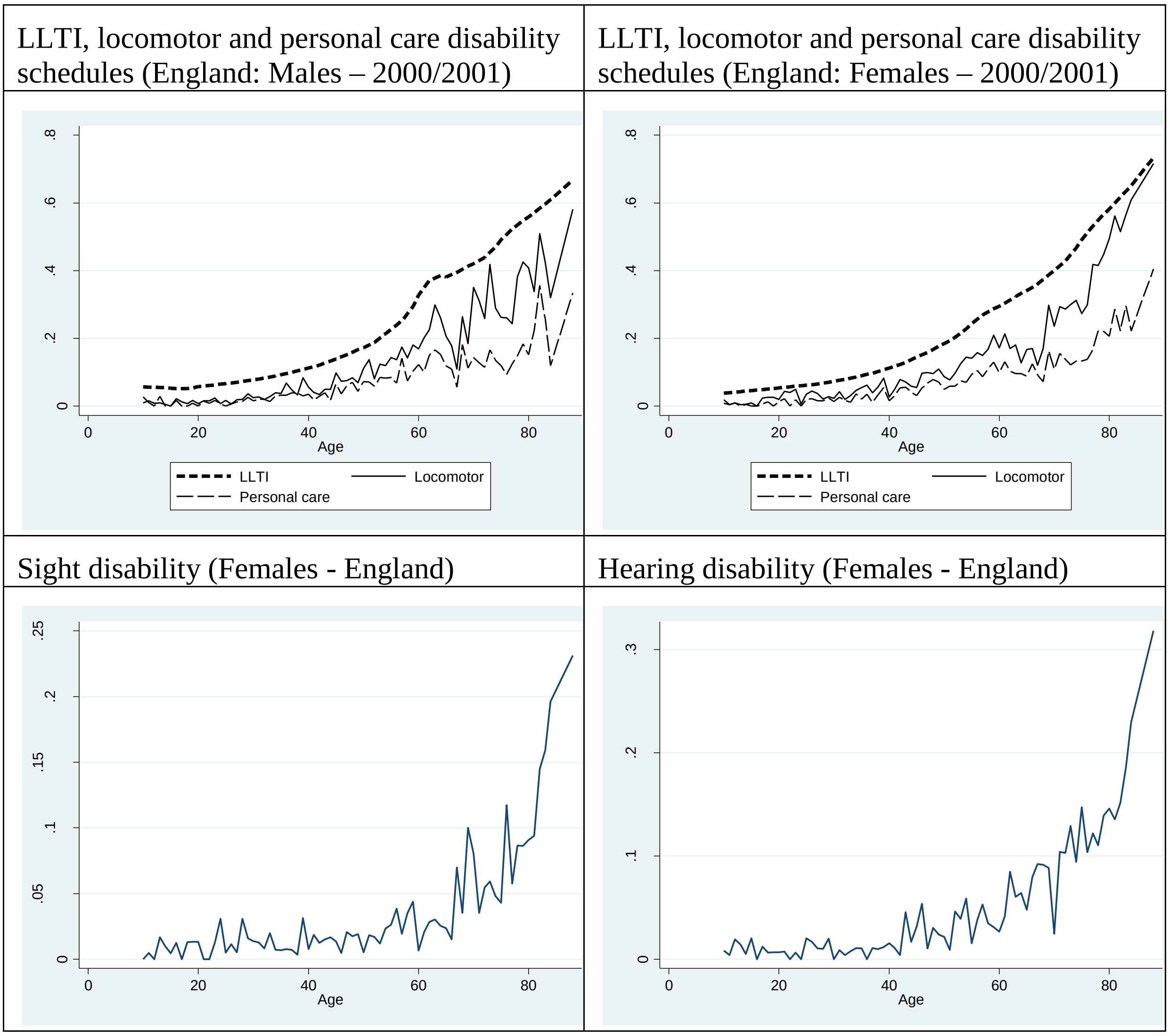


Fig 1 Limiting Long Term Illness (LLTI) and selected disability schedules for England

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Relational models comprise a (reliable) standard schedule of rates and a mathematical rule that maps the standard schedule to another schedule in a population where information may be incomplete or unreliable (Preston et al. 2001). Relational models were originally developed for the estimation of mortality schedules (Brass 1971) and a key advantage of the approach is that the complexity of the mortality age pattern is captured in the standard schedule and a small number of parameters then quantify the deviation from this standard. Relational models require fewer parameters than mathematical mortality functions and can flexibly reproduce sets of model life tables using two suitably chosen parameters and a standard schedule (Keyfitz 1982; Preston et al. 2001).

The original Brass (1971) relational model is based on a logit transformation of l(x), the probability of surviving to age x in the population of interest.

$$Y(x)\frac{1}{2}\ln\left[\frac{l(x)}{1-l(x)}\right] \qquad \mathbf{1}$$

The logit transformation of l(x) is valuable because the relationship between two logit mortality schedules turns out to be remarkably linear (Newall 1988). On the basis of this linear relationship, Brass proposed a simple relational formula involving two parameters, α and β, to predict Y(x) from the logit of $l^s(x)$, $Y^s(x)$, in the standard population:

$$Y(x) = \alpha + \beta * Y^s(x) \qquad \mathbf{2}$$

When α=0 and β=1 then $Y(x)$ and $Y^s(x)$ are identical. Altering α affects the level of mortality in the population of interest, whilst altering β influences the relationship between mortality at adulthood and childhood. For more on the impact of changes in the values of α and β see section 3.2 and Zaba (1979: p80).

Two features determine the success of the relational approach, these being the appropriateness of the standard schedule and the relational rule (Preston et al. 2001). The relational approach can be used successfully with any standard, but it is most effective if the standard is close to that of the population being modelled (Keyfitz 1982). There have been several extensions to the Brass relational function allowing more accurate representations of mortality particularly at the oldest and youngest ages. For example, Zaba (1979) and Ewbank et al. (1983) propose relational models with two additional parameters that significantly improve the fit compared with the Brass relational model (Newall 1988). Murray et al. (2003) note the difficulty associated with the empirical estimation of parameters in the Ewbank et al. (1983) and Zaba (1979) relational models and develop an alternative model which uses two additional age-specific correction factors based on mortality levels among children and adults, relative to the standard.

Relational models have been developed for particular countries (e.g. for Peru by Kamara and Lamsana (2001)) and have also been successfully extended to other demographic characteristics. For example, Brass (1981) developed a relational model for fertility schedules based on the Gompertz function, noting the utility of this approach in terms of its simplicity and the quality of fit of model rates. Zaba (1985; 1987) developed a relational model for schedules of immigration and emigration that involves three parameters. Booth (2006) documents the utilisation and development of these relational models of migration and of fertility in her excellent review of techniques of demographic forecasting (1980-2005).

Relational models are particularly appropriate for estimation of disability schedules because of the strong age pattern of prevalence rates which, like mortality schedules, are low at the youngest ages and rise with age. This pattern holds over time, place and for many disability types (see Figure 1). In this paper, the LLTI schedule for England (2001 census) is used as a reliable 'standard' schedule which is then adjusted to represent the schedules of particular disability types (Health Survey for England 2000/2001) smoothing fluctuations that are attributable to sampling error. Figure 2 compares the logit rates of LLTI (census) with logit schedules of locomotor (mobility) disability (Health Survey for England) at each single year of age illustrating the approximately linear relationship that is fundamental to the relational approach.

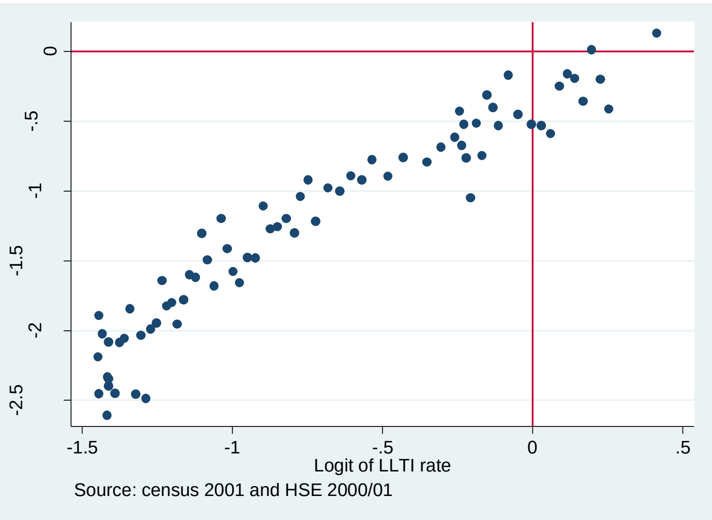


Fig 2 Scatterplot of the relationship between the logit LLTI schedule and the logit mobility disability schedule (Males – England)

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

In addition to relational models there are alternative methods by which curves could be fitted to disability schedules, such as parametric graduation or graduation using spline functions. However, the relational model of disability that is developed here is motivated by its potential to fill an important information gap; the lack of disability estimates locally either because these are extremely unreliable due to small sample sizes once disaggregated by age, or because the release of local information is suppressed for reasons of disclosure protection (Purdam et al. 2008). The use of a curve of age specific rates of LLTI from the census as the standard schedule is proposed because age-specific LLTI rates are reliably available from the census for sub-national areas. Thus census LLTI schedules could act as a proxy for the level of disability in a neighbourhood with adjustments informed by relational models fitted for England as a whole. We do not evaluate sub-national relational models of disability here as this is undertaken in a separate paper (Marshall 2012b). We return to this local application of the techniques developed here in the discussion.

It is important to note some differences between relational models of mortality and disability that may complicate the proposed modelling of disability schedules. Whilst relational models of mortality predict the logit of survivorship rates, the relational model of disability proposed here predicts the logit of disability prevalence rates. Survivorship rates start at 1 at birth and decrease monotonically to approach zero at the oldest age. In contrast disability rates start close to zero at the youngest ages and rise to between 0.7 and 0.2 depending on disability type. Increases in disability rates need not be monotonic, because, for example, people can recover from disability. Figure 1 shows that the rate of increase in LLTI for males stops around retirement age (60-65) before increasing again throughout the older ages. Features such as this ‘retirement kink’ in the LLTI schedule are likely to be preserved in the relational model disability schedules and we return to the appropriateness of this particular feature in the discussion.

After this introduction the paper is divided into six sections. First, the data sources that are used in the paper are discussed. Second, the four relational models that are fitted are defined. Third, the approach to evaluating the success of each model is outlined. Fourth, the findings of the model evaluation are stated. Fifth, the findings are discussed and finally, conclusions are drawn.

2. Data

The analysis in this paper combines data from two sources. The 2001 census provides reliable LLTI schedules and the Health Survey for England (2000/2001) provides detailed information on disability, distinguishing the nature of the disability.

2.1 The census of population (UK)

The census has been carried out since 1801, during which time sporadic questions on health and disability have been asked (Charlton 2000). The main advantage of the census as a source of data on disability is its almost complete enumeration of the population and the fine geographical detail at which data are reliably available. In 2001 the census included a question on limiting long term illness that records any illnesses, health problems, or disabilities that limit an individual in their daily activities. A very similar question had been asked in 1991. The question on LLTI features a prompt for elderly people to include problems that are due to old age. This is useful because it is known that the elderly tend to discount some health problems as being a result of ageing (Bajekal et al. 2003). There is some undercount in the census which is larger in some areas of the country and for certain population groups (Cook 2004). However, these problems are small compared to the uncertainty associated with sample data and we do not address this undercount here.

Box 1 Limiting long term illness question – census 2001.

Do you have any long-term illness, health problem or disability which limits your daily activities or the work that you can do? Include problems which are due to old age. (Yes/No)

Source: 2001 Census household questionnaire. Available at
http://www.statistics.gov.uk/census2001/pdfs/H1.pdf

In terms of the general utility of self-reported limiting long term illness, a large body of work supports the validity of self-assessed health (Mitchell 2005) with LLTI found to be most strongly associated with general health perceptions, more serious health conditions (Manor et al. 2001) and physical limitations rather than with psychological health (Cohen et al. 1995). There are strong relationships between LLTI and other health outcomes including all cause and cause-specific mortality (Charlton et al. 1994; Bentham et al. 1995; Idler & Benyamini 1997) as well as sickness benefits claims from different health conditions (Bambra and Norman 2006; Norman and Bambra 2007). The 2001 census data on LLTI is downloaded from table ST16 which records the population with (and without) LLTI with age and sex detail for the household population.

2.2 The Health Survey for England

The Health Survey for England (HSE) was set up in 1991 to monitor the health of the private household population in England and the progress towards targets laid out in the *Health of the Nation* strategy (DoH 2007). The survey follows a multistage, stratified probability sampling design. The sample size was increased from around 4,000 to 16,000 in 1994 enabling analysis for Health Authority Regions and between socio-economic groups. From 1995 a sample of 4,000 children between the ages of 2 and 15 were included in the sample (Bajekal 2000). Each year of the HSE has a particular focus, with a module measuring disability included in 1995, 2000, 2001, and 2005. In this paper the data on disability in 2000 and 2001 are combined to increase sample sizes and to overlap the data collection date of the 2001 census, a feature that is particularly useful for the models that combine HSE and census data. The 2000 survey focused on disability amongst the elderly with a boosted sample of elderly people including the elderly living in residential and care homes along with a reduced sample of the general population (Bajekal and Prescott 2003).

Presence of a disability is identified along five domains: locomotion (mobility), personal care, sight, hearing, and communication. A person is classed as having no disability or a disability at a lower or higher level for each of the five domains based on their answers to questions on ability to perform everyday tasks (see appendix). The highest score for any of the five types of disabilities is taken as the overall disability score. A score of 1 indicates a lower severity disability, a score of 2 indicates a higher severity disability and a score of 0 indicates no disability. In this paper model rates are produced for overall disability and each disability type with the exception of communication disability which is omitted because it does not display the strong age pattern necessary for the relational models developed here. Severity of disability is not distinguished and so the rates of disability include those with either a higher or lower severity disability. The HSE allows respondents to take into account the use of aids for hearing and sight disabilities, however, for the other domains the use of aids to perform tasks are not allowed. Data are collected using face-to-face computer assisted personal interviewing and the disability module applies to all people aged 10 or over. Proxy answers are not permitted for adults but parents answer for children under the age of 13 (Bajekal and Prescott 2003).

2.3 Data preparation

The analysis of the merged 2000-01 HSE datasets requires the use of two types of weights to ensure that estimates are representative of the target population. First, child weights are needed to compensate for the sample design at these ages which involves limiting the number of children interviewed in each household to two. Second, the HSE in 2000 includes weights to account for the oversampling of the elderly and institutional population. In order to compensate for the lack of older people living in institutions in 2001, the weights associated with people living in institutions in 2000 are doubled.

All models use rates by single year of age up to the age of 84 with an age of 88 to represent all those aged over 84. The use of 88 as the upper age limit is based upon the average age of the population aged over 84 as calculated using the 2001 census Sample of Anonymised Records (a 3% sample of individual records from the 2001 census). Census tabulations of LLTI are only released with quinary age detail (from the age of 20 upwards) and in order to generate single year estimates, these five year rates are smoothed using an Excel based tool developed by users of the Popgroup population projection software (http://www.ccsr.ac.uk/popgroup/index.html) specifically for this purpose. The Excel smoothing tool and more information on the smoothing approach are available at: http://www.ccsr.ac.uk/popgroup/about/manuals.html

3. Models

3.1 Notation

The subscript notation that is used in the model specifications throughout this paper is detailed in Table 1.

Table 1 Subscript notation

Notation	Range	Notes
i = individual	1 … N	
x = age	x = 10, 11, 12, … , 84, 88	88 is the age used to represent the 85+ age group
d = disability type	d = 1 … 5	Overall disability, locomotor, personal care, hearing and sight
l = limiting long term illness	0=no llti 1=llti	

3.2 Brass relational model

The Brass relational model used in this analysis is defined below:

Let:

p_{xd} = prevalence of disability d at age x in England (HSE00/01)

p_{xl} = prevalence of LLTI (*l*) at age x in England (census01)

Then:

$$\frac{1}{2}\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)=\alpha+\beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)+e_x \qquad \textbf{3}$$

The impact of varying values of α and β on the LLTI schedule are illustrated in Figure 3. A negative value of α shifts the LLTI schedule downwards whilst a positive value shifts it upwards. A value of β above 1 decreases rates of LLTI at the youngest ages and increases them at the oldest ages with the converse being true for values of β below 1.

Impact of altering values of α	Impact of altering values of β

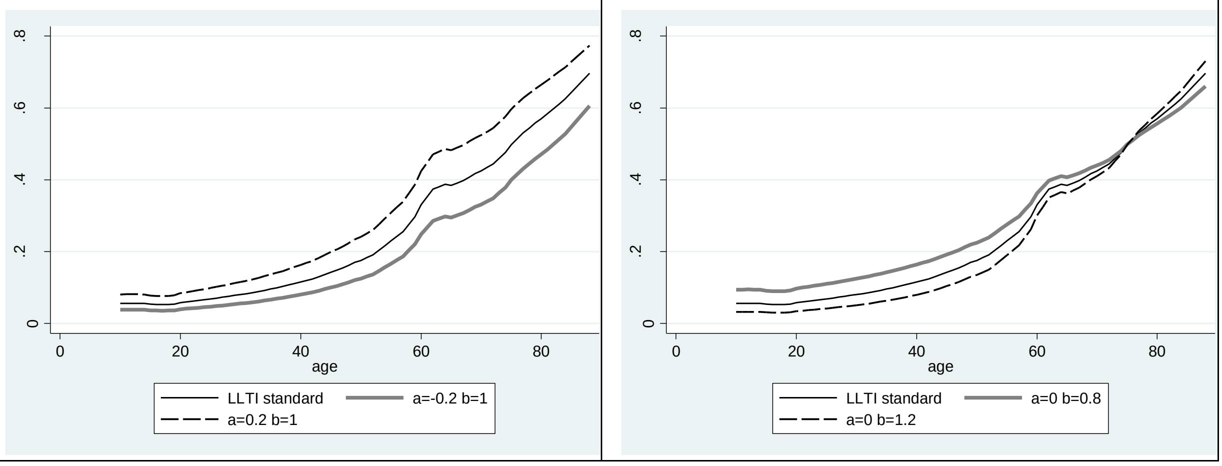


Fig 3 Brass relational model - Impact of altering values of α and β on the LLTI standard schedule (Males – England)

Source: Authors' calculations using data from the census (2001)

3.3 Ewbank relational model

Ewbank, with colleagues, Gomez de Leon and Stoto, develop a more complex relational rule with four parameters that allows more twisting of the reference schedule at the oldest and youngest ages (Ewbank et al. 1983). This four parameter system is an extension of Brass' two parameter relational model. The Ewbank model that is fitted to derive local disability schedules is defined below:

First, define function T (which comprises the two additional parameters λ and κ) as:

$$T(p_{xl};\lambda)=\frac{\left(\frac{p_{xl}}{1-p_{xl}}\right)^{\lambda}-1}{2\lambda}\ \text{ if } p_{xl}\geq 0.5 \qquad \textbf{4}$$

$$T(p_{xl};\kappa)=\frac{1-\left(\frac{1-p_{xl}}{p_{xl}}\right)^{\kappa}}{2\kappa}\ \text{ if } p_{xl}<0.5 \qquad \textbf{5}$$

Let:

ω =1 if p_{xl} ≥0.5 and 0 otherwise

χ =1 if p_{xl} <0.5 and 0 otherwise

Then:

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right)=\omega\left(\alpha+\beta\ T(p_{xl}:\lambda)\right)+\chi_{.}\left(\alpha+\beta\ T(p_{xl}:\kappa)\right)+e_x \qquad \textbf{6}$$

The part of function *T* (see equations 4 and 5) that is fitted is determined by the rates of LLTI (p_{xl}) and the age at which these rates pass 0.5. For both males and females, rates of LLTI first exceed 0.5 at age 76 so equation 4 (involving κ) is fitted between the ages of 10 to 75, whilst equation 5 (involving λ) is fitted to the ages of 76 and above. This is not altogether different to the cut-offs that might be used for a Ewbank model of mortality. For example, in the UK lifetable (2008-10) published by the Office for National Statistics, survivorship probabilities pass 0.5 at the age of 80.

The Ewbank relational model has several useful features for the purposes of this paper. First, it allows more flexibility in the adjustment of LLTI schedules than the Brass model which is likely to be necessary for disability types that deviate most from the LLTI age pattern. Second, the transformation T approaches the logit transformation (and thus a Brass relational model) as λ and κ tend to zero. This can be shown by expanding the transformation T into a series. This property of 'nested' models is useful for the model comparison and is also exploited to develop a 'Reduced Ewbank' model. Third, the transformation introduces the biggest changes at the most extreme ages where the logit transformation is most likely to be unsatisfactory. So, λ only affects estimates at the oldest ages and κ only affects estimates at the youngest ages.

3.4 Reduced Ewbank model

A criticism of the Ewbank model in the literature is the difficulty in estimating the additional parameters which complicate the application of this model (Murray et al. 20003). Congdon (1993) discusses the problem of overparameterisation when fitting relational models, where a range of parameter estimates are associated with a similar model fit, and recommends a model with fewer parameters when overparameterisation occurs. A reduced version of the Ewbank model (from here on known as the Reduced Ewbank model) is developed in this paper to provide a more flexible alternative to the Brass model whilst avoiding the issues identified by Murray et al. (2003) and Congdon (1993). There are some parallels between the reduced Ewbank model in this paper and that adopted by Kamara and Lansana (2001) for the estimation of mortality schedules in Peru where the four parameter relational model developed by Zaba (1979) is altered to derive a simpler relational model also involving three parameters.

There is a strong argument that α and β should remain in the Reduced Ewbank model as both are included in the Brass model and the full Ewbank model. This means that either κ or λ might be dropped from the model. Clearly setting one of these parameters to zero would remove the effect of β for at least part of the model (see equations 4 to 6) as the function T would equal zero. However, we know that the function *T* approaches a logit transformation when κ and λ tend to zero and so it is proposed that where one of these variables is dropped the *T* function should be replaced by a logit transformation. The Reduced Ewbank model is shown in equations 7 and 8.

Let:

ω =1 if $p_{xl} \geq 0.5$ and 0 otherwise

χ =1 if $p_{xl} < 0.5$ and 0 otherwise

Parameter kappa dropped from the model

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right) = \varpi \ \left(\alpha + \beta T(p_{xl} : \lambda)\right) + \chi \ \left(\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + e_x \qquad \mathbf{7}$$

Parameter lambda dropped from the model

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right) = \omega \ \left(\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + \chi_{.}\left(\alpha \ + \beta \ \ T(p_{xl} : \kappa \ )\right) + e_x \qquad \mathbf{8}$$

3.5 Piecewise relational model (sight disability)

Examination of the sight disability schedules (see figure 1) shows that age pattern is flat and very low up to the age of 60 with a steady increase in prevalence occurring thereafter. Epidemiological research confirms the rarity of sight disabilities at the younger and working ages and reveals that the causes are often congenital in nature (Munier et al. 1998; Rahi and Dezateux 1998). Sight disabilities occur with increasing frequency at the oldest ages with causes linked to the ageing process. Macular degeneration, glaucoma and cataracts account for three quarters of sight problems for those aged over 80 (Munier et al. 1998).

The shape of the sight schedule may be better modelled with a piecewise approach using an average prevalence rate up to the age of 60 and a relational model above the age of 60 where an age pattern emerges. This approach acknowledges the rarity of the mainly congenital sight disabilities under the age of 60 and the more typical disability age pattern at the older ages as sight problems that stem from the aging process emerge.

The piecewise Brass relational model is defined below:

Let:

γ_r =1 if x_r >59 and 0 otherwise

υ_r =1 if $x_r \leq 59$ and 0 otherwise

$$\frac{1}{2}\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right) = \upsilon_{.}\,\delta + \gamma_{.}\left((\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + e_x \qquad \textbf{9}$$

In the model specification above the parameter δ is constrained as below:

$$\delta = \frac{1}{2}\log\left(\frac{p_{10-60}}{1-p_{10-60}}\right) \qquad \textbf{10}$$

The α and β parameters have the same interpretation as in previous relational models adjusting the level and shape of the LLTI schedule to estimate the sight schedule, however in this model they only have an effect over the age of 60. The piecewise model is less desirable than the other relational models that apply across the whole age range in that it might lead to discontinuities in the model schedules on either side of the break in the piecewise function. However, such an approach may be required for sight disability which deviates most from the shape of the LLTI curve.

3.6 Model estimation

The statistical computer package STATA is used to fit the relational models using least squares regression. The regress (linear regression) command is used to fit the Brass and piecewise relational models. The nl (non-linear regression) command is used to fit the Ewbank and Reduced Ewbank models with starting values of 1 given to all parameters. Experimentation with other starting values did not alter the final parameter estimates.

An alternative method of model estimation is to use weighted least squares regression with weights based on an assumption that the rates of disability follow a binomial distribution (Congden 1993). Fitting relational models of disability in this way led to a poor fit compared to unweighted analysis; the weights gave too much attention to the low rates of disability at the youngest ages and too little attention to the higher rates at the middle and older ages. A similar issue is noted by Hoem et al. (1981) who also opt for ordinary least squares to fit relational models of fertility.

4. Comparing models

The procedure to select the most appropriate relational model for each disability type (separately for males and females) involves two stages. The first determines whether the improvement in model fit, compared to the next simplest model, reaches statistical significance. Second, the stability of parameter estimates from the chosen model is assessed and if evidence of overparameterisation is discovered the model is discarded.

4.1 Improvement in model fit

It is almost always the case that a more complicated model will fit the data better (have a lower residual sum of squares) than a simpler one. So, for example, the Ewbank model will generally fit the data better than the Reduced Ewbank model which in turn will have a lower residual sum of squares than the Brass model. The extra sum of squares F test (detailed in Motulsky and Christopoulos (2004) and Norman et al. (2012)) is based upon the difference in residual sum of squares (from here on referred to as sum of squares) from two models and controls for the number of data points and the number of parameters in each model. It uses this information as shown in Table 2 to calculate a ratio that follows an F distribution under the null hypothesis that there is no evidence to accept the more complicated model (i.e. the residual sum of squares in each model are identical after accounting for improvements attributable to additional parameters). We can use the F-ratio to calculate an associated p-value that gives the probability that the improvement in model fit associated with the more complicated model (after accounting for improvements attributable to additional parameters) is actually a result of the sampling process rather than any ‘real’ improvement. For the purposes of this research a threshold of $p=0.05$ is used to determine whether the more complex model gives a better fit than the simpler model.

Table 2 Extra sum of squares F test - calculations

Model	Sum of squares (SS)	Degrees of freedom (df)
Null hypothesis	SS_{null}	DF_{null}
Alternative hypothesis	SS_{alt}	DF_{alt}
Difference	$SS_{null}-SS_{alt}$	$DF_{null}-DF_{alt}$
Relative difference	$(SS_{null}-SS_{alt})/SS_{alt}$	$(DF_{null}-DF_{alt})/ DF_{alt}$
Ratio ($F_{DF1-DF2,DF2}$)	$\frac{(SS_{null} - SS_{alt})/SS_{alt}}{(DF_{null} - DF_{alt})/ DF_{alt}}$	

Source: Motulsky and Christopoulos (2004)

*Note the null sum of squares relates to the simpler model (e.g. Brass) and the alternative to a more complex model (e.g. reduced Ewbank).

As the two versions of the Reduced Ewbank model have the same number of parameters, then the decision as to whether *k* or *l* is dropped is made on the basis of the model with the lowest residual sum of squares.

4.2 Model stability

In addition to the extra sum of squares F test it is also important to check parameter estimates for signs of overparameterisation, where a range of parameter values are associated with a similar model fit. The symptoms of overparameterisation are standard errors that 'explode' estimates of parameters outside 'normal' ranges and high estimated correlations between parameters (virtual collinearity). A useful test of overparameterisation is to compare estimates of α and β from a Reduced or full Ewbank model to those from a Brass model that is restricted to the ages not seriously affected by the additional parameters (λ/κ). If the model is not overparameterised then we would expect estimates of α and β to be similar in each model.

5. Findings

Before displaying results from the model comparison outlined in the previous section it is worth noting the generally good fit of the relational models to the observed disability schedules. R squared statistics for all relational models are above 0.9 for overall, locomotor and personal care (females) disability and are around or above 0.8 for personal care (males) sight (females) and hearing disability. The improvement in R-squared values in the Reduced Ewbank and Ewbank models compared to the Brass model is greatest for sight disability, in particular for males.

Table 3 R^2 statistics for relational models

Males				
Disability	R^2			
	Brass	Reduced Ewbank	Ewbank	Piecewise
Overall disability	0.94	0.94	0.94	n/a
Locomotor	0.94	0.94	0.94	n/a
Personal care	0.85	0.87	0.87	n/a
Hearing	0.85	0.86	0.85	n/a
Sight	0.55	0.69	0.69	
Females				
Disability	R^2			
	Brass	Reduced Ewbank	Ewbank	Piecewise
Overall disability	0.95	0.96	0.96	n/a
Locomotor	0.91	0.92	0.92	n/a
Personal care	0.90	0.93	0.93	n/a
Hearing	0.81	0.82	0.82	n/a
Sight	0.67	0.73	0.74	

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Table 4 displays the results of the extra sum of squares F tests under the Brass, Reduced Ewbank, and Ewbank models. For both males and females, the Reduced Ewbank model offers an improvement in fit over the Brass model that is statistically significant for all disability types with the exception of hearing disability. The same version of the Reduced Ewbank model is selected for males and females for each disability type; the additional parameter κ is required for locomotor, personal care and sight disability and the additional parameter λ is required for overall disability. The improvement in model fit under the Ewbank model does not achieve statistical significance for any of the disability types.

Table 4 Residual sums of squares and F ratio p-values (from extra sum of squares F test)

MALES						
Disability	Brass	Reduced Ewbank[1]		Ewbank		Total SOS
	SOS	SOS	F ratio p value[2]	SOS	F ratio p value[2]	
Disability	1.68	1.58 (l)	0.04 (l)	1.56	0.3	27.27
Locomotor	3.28	2.35 (k)	<0.0000 (k)	2.34	0.6	39.70
Personal care	2.96	2.58 (k)	0.002 (k)	2.57	0.53	20.07
Hearing	3.68	3.67 (l)	0.61 (l)	3.68	0.96	25.35
Sight	7.41	5.25 (k)	<0.0000 (k)	5.22	0.59	16.81
FEMALES						
Disability	Brass	Reduced Ewbank[1]		Ewbank		Total SOS
	SOS	SOS	F ratio p value[2]	SOS	F ratio p value[2]	
Disability	1.39	1.28 (l)	0.01 (l)	1.22	0.06	28.55
Locomotor	3.53	3.21 (k)	0.0009 (k)	3.19	0.34	38.56
Personal care	2.25	1.76 (k)	<0.0000 (k)	1.75	0.71	23.51
Hearing	4.73	4.64 (l)	0.26 (l)	4.35	0.06	25.45
Sight	6.30	5.05 (k)	0.0001 (k)	4.86	0.10	18.90

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

A further weakness of the Ewbank models of disability is that model estimates reveal clear evidence of overparameterisation. The $\hat{\beta}$ parameter estimate is often much higher than in the Brass models and many of the parameter estimates have very high standard errors. For example, for overall disability (males) $\hat{\beta}$ is equal to 37.7 with a standard error of 54.9). Examination of estimated correlations between parameters from the Ewbank model reveals virtual collinearity between the $\hat{\beta}$ and $\hat{\kappa}$ parameters and the $\hat{\beta}$ and $\hat{\lambda}$.

[1] The parameter that is kept in the reduced Ewbank model (k or l) is determined by which of the models has the lowest residual sum of squares.

[2] The F test compares a model with the next simplest (in terms of number of parameters). The reduced Ewbank model is always compared to the Brass model. The Ewbank model is compared to the Brass model or to the reduced Ewbank model if the reduced Ewbank model offers a better fit than the Brass model. The shaded cells correspond to the model that gives the best fit according to the extra sum of squares F test.

The parameter statistics from the Brass and Reduced Ewbank models (see table 5) recommended by the extra sum of squares F test suggest that overparameterisation is less of an issue for these models. All parameter estimates achieve statistical significance and appear to be within 'normal' ranges.

Whilst our parameter estimates appear more stable in the Reduced Ewbank models, some evidence of overparameterisation remains, particularly where the κ parameter is retained. There are strong estimated correlations between the $\hat{\beta}$ and $\hat{\kappa}$ parameters in Reduced Ewbank models (estimated correlation ranges between 0.94 to 0.99). The issue of overparameterisation is noted as a potential weakness of the relational models of disability developed here. However, the stability of the Reduced Ewbank models is much improved compared to the full Ewbank models. The tendency for the same form of Reduced Ewbank model to be selected for both males and females for all disability types is an encouraging sign of the stability of Reduced Ewbank models. This suggests that a similar weakness of the Brass model is addressed in a consistent way for both female and male disability schedules.

Table 5 Parameter statistics from the relational models selected through the extra sum-of squares F test (see table 4)

Males							
	Parameter	Parameter estimate	Std. Err.	t	P>t	95% Confidence interval	
Overall disability	α	-0.16	0.03	-4.85	<0.0000	-0.23	-0.10
	β	1.01	0.04	28.76	<0.0000	0.94	1.08

	λ	1.09	0.10	11.10	<0.0000	0.90	1.29
Locomotor	α	-0.47	0.03	-14.85	<0.0000	-0.53	-0.41
	β	1.71	0.30	5.68	<0.0000	1.11	2.32
	κ	0.55	0.03	15.86	<0.0000	0.48	0.62
Personal care	α	-0.91	0.04	-25.30	<0.0000	-0.98	-0.83
	β	1.47	0.35	4.20	<0.0000	0.77	2.16
	κ	0.53	0.05	11.16	<0.0000	0.43	0.62
Hearing	α	0.98	0.05	20.57	<0.0000	0.88	1.07
	β	-0.86	0.04	-19.85	<0.0000	-0.95	-0.77
Sight	α	-1.62	0.10	-15.79	<0.0000	-1.83	-1.42
	β	3.03	0.77	3.95	<0.0000	1.50	4.57
	κ	-0.62	0.17	-3.74	<0.0000	-0.95	-0.29
Females							
	Parameter	Parameter estimate	Std. Err.	t	P>t	95% Confidence interval	
Overall disability	α	-0.15	0.03	-4.95	<0.0000	-0.21	-0.09
	β	0.99	0.03	32.27	<0.0000	0.93	1.05
	λ	1.02	0.06	17.47	<0.0000	0.90	1.13
Locomotor	α	-0.51	0.04	-13.25	<0.0000	-0.58	-0.43
	β	2.33	0.28	8.33	<0.0000	1.77	2.89
	κ	0.46	0.02	20.43	<0.0000	0.42	0.51
Personal care	α	-0.90	0.03	-31.67	<0.0000	-0.96	-0.84
	β	1.45	0.20	7.15	<0.0000	1.05	1.86
	κ	0.52	0.03	19.08	<0.0000	0.47	0.58
Hearing	α	0.93	0.05	17.38	<0.0000	0.82	1.04
	β	-1.07	0.05	-21.67	<0.0000	-1.17	-0.97
Sight	α	-1.33	0.11	-11.61	<0.0000	-1.56	-1.10
	β	1.84	0.48	3.83	<0.0000	0.88	2.79
	κ	-1.05	0.31	-3.38	<0.0000	-1.66	-0.43

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

A remaining question is whether there are benefits to modelling the sight disability schedule using the piecewise model. The R^2 statistics in table 3 confirm that sight disability is least well modelled and as the sight disability curve remains flat until the age of 60 it could be that the piecewise approach in equations 9 and 10 offer a more appropriate approach than the other relational models. However, the residual sums of squares associated with the piecewise Brass relational model (5.83 for males and 5.80 for females) are higher than for the Reduced

Ewbank model (5.25 for males and 5.05 for females). We do not select the piecewise Brass relational model because it does not appear to offer an improvement in model fit compared to a Reduced Ewbank model.

The models recommended by the findings here are shown in Table 6 below. The model schedules themselves, along with the observed survey rates, are displayed in figures 4, 5 and 6. It is encouraging that model fit appears to be reasonable for each disability type across the age range.

Table 6 Relational models recommended for each disability type (males and females)

Disability type	Males	Females
Overall disability	Reduced Ewbank (λ)	Reduced Ewbank (λ)
Locomotor disability	Reduced Ewbank (κ)	Reduced Ewbank (κ)
Personal care disability	Reduced Ewbank (κ)	Reduced Ewbank (κ)
Hearing disability	Brass	Brass
Sight disability	Reduced Ewbank (κ)	Reduced Ewbank (κ)

Males	Females

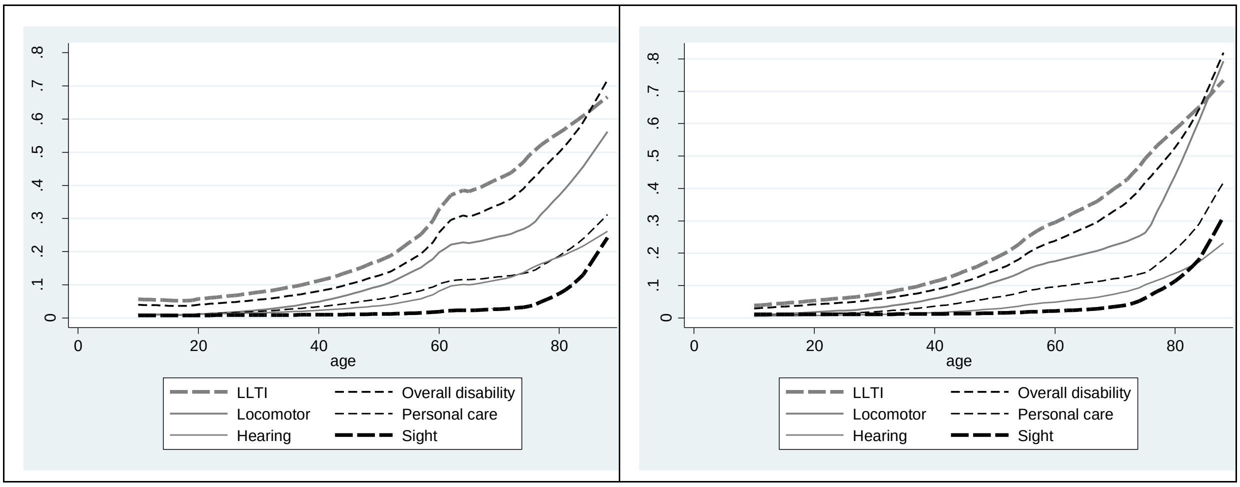
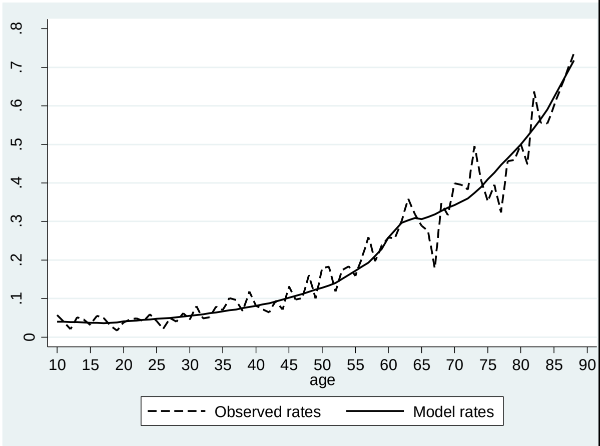
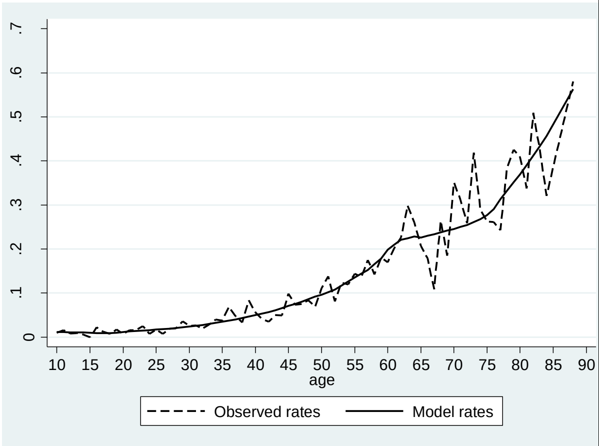


Fig 4 LLTI schedule (census) and model disability schedules (Males and females – England)

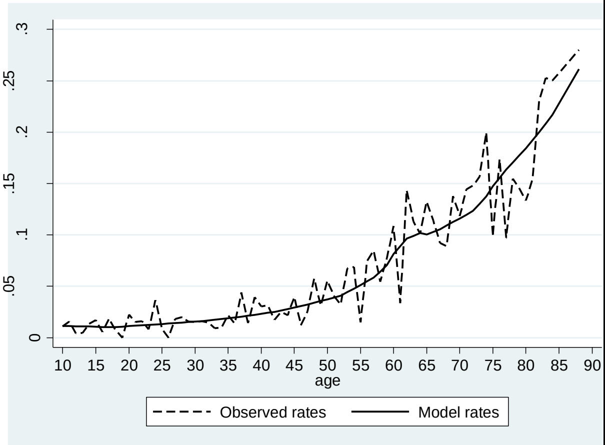
Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Overall disability	Locomotor





Hearing	Sight



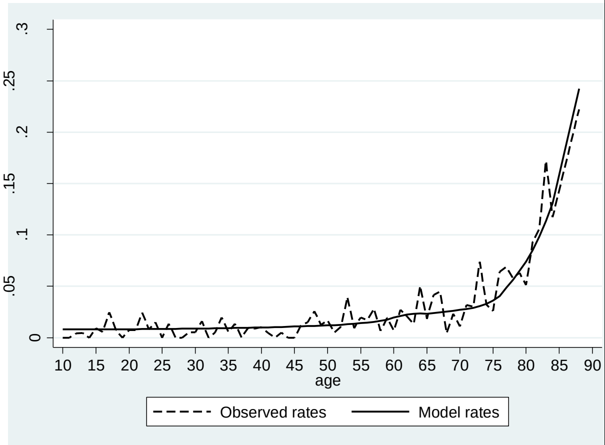


Fig 5 Observed and model disability schedules – Overall, locomotor, hearing and sight disability (Males)

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Overall disability	Locomotor

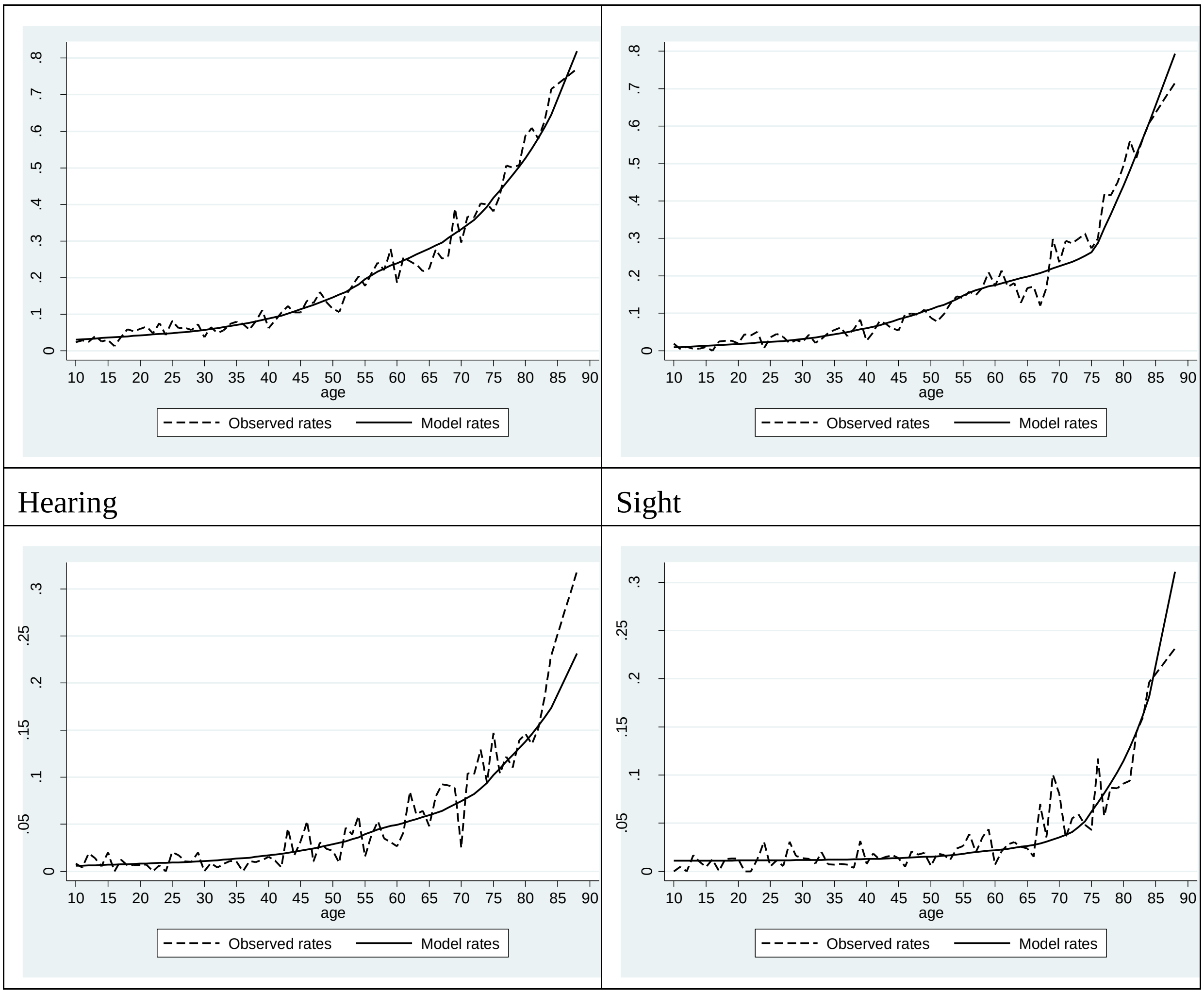


Fig 6 Observed and model disability schedules – Overall, locomotor, hearing and sight disability (Females)

Source: Authors' calculations using data from the Health Survey for England (2000/2001) and the census (2001)

6. Discussion

The findings of the previous section illustrate the success of relational models in representing disability schedules by adjusting the level and shape of LLTI curves. We now examine the model schedules (see Figures 4 to 6) commenting on their robustness in light of other research on disability. We also return to the potential application of relational models for local estimation of disability noted in the Introduction.

Interestingly, the rates of overall disability exceed census LLTI rates at the very oldest ages for both males and females. Although this seems counterintuitive, as LLTI includes a broader range of limiting conditions than the Health Survey for England measure of disability, it is supported in the literature. Bajekal and Prescott (2003) note a 'crossover' effect in their report on disability in the 2001 Health Survey for England where survey rates of disability exceeded survey rates of LLTI at the oldest ages. This crossover effect is attributed to older people under-reporting limiting longstanding illness because they consider activity limitation a normal consequence of ageing. Additionally, surveys with a health focus, such as the Health Survey for England, tend to give higher levels of illness and disability than other surveys, such as the census, where information on a range of topics is collected. Finally, whilst the census data used here excludes older people in care and residential homes this population group are included in the HSE sample. Older people in care homes are more likely to have a disability than the private household population providing another reason for the higher model rates of disability compared to census rates of LLTI at the oldest ages

A key difference between the male and female model schedules is a kink in many of the model disability schedules for males between the ages of 60 and 65 where the increase in rates with age slows before increasing again. This kink is found in the male LLTI curve and is preserved for many of the male disability schedules during the modelling process. The LLTI retirement kink is a feature noted by other researchers. For example, Bellaby (2006) finds a tailing off in the increase in LLTI after retirement ages, particularly for those in manual occupations, and a similar result from clinical assessments of health using standardised methods (e.g. forced expiratory volume (FEV1), blood pressure (allowing for control by medication), and body mass index). Westerlund et al. (2009) report a retirement related improvement in self-reported health, particularly for those in poor work environments, in a longitudinal study of employees of the French national gas and electric company. A comparison of the observed and model disability schedules (see Figure 5) suggests that the transfer of the retirement kink to specific disability types is reasonable. Additionally the relational approach appears to have the flexibility to suppress the kink for disabilities where it might be less appropriate (e.g. sight disability).

As noted in the Introduction, the relational model specification developed in this paper is partly motivated by the data availability in the UK and the potential application of relational models to address an information gap; the lack of sub-national estimates of disability. For sub-national areas survey data on specific disabilities are either very unreliable due to small sample sizes or are unavailable for reasons of disclosure protection. Traditionally relational models involve a fixed standard schedule and varying parameter estimates but here sub-national schedules could be derived from varying standards (local census LLTI schedules) with fixed parameter estimates (from national relational models). The underlying assumption is that the relationship between LLTI and disability schedules remains constant between areas. Marshall (2012b) tests this assumption using national relational parameter estimates to generate estimates of HSE disabilities for the nine regions in England. These relational model estimates successfully capture the variability in the HSE observed regional disability prevalence providing evidence to suggest that the relational models developed in this paper have not only applicability to smooth national disability schedules but also as a means to fill a local disability information gap. More generally, relational models have applicability to the estimation of schedules of other disability types, combinations of disability types, severity of disability, or of other health problems that display a strong mortality-like age pattern.

7. Conclusion

This paper illustrates that relational models can accurately capture the relationship between age-specific rates of LLTI and various disability types. The Brass or Reduced Ewbank relational models have sufficient flexibility to adjust the census LLTI schedule to give an accurate representation of schedules of overall, locomotor, personal care, hearing, and sight disability from the Health Survey for England. This is valuable as relational models offer a means to generate a more reliable set of age-specific rates in situations where there is evidence of instability in rates directly estimated from survey data. The local availability of reliable census LLTI schedules provides a means to derive local disability schedules (where direct estimates are unavailable) using relational parameters estimated at higher geographies.

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Appendix: Health survey for England - disability module

Disability Type	Survey Question	Response	Disability score

Disability Type	Survey Question	Response	Disability score
Locomotor	What is the furthest you can walk on your own without stopping and without discomfort?	Only a few steps More than a few steps but less than 200m More than 200m	2 1 0
	Can you walk up and down a flight of 12 stairs without resting?	Not at all Only if hold on and take rests Yes	2 1 0
	Can you, when standing, bend down and pick up a shoe from the floor?	No Yes	1 0
Personal care	Can you get in and out of bed on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you get in and out of a chair on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you dress and undress yourself on you own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you wash your face and hands on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you feed yourself, including cutting up food?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you get to and use the toilet on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
Seeing	Can you see well enough to recognise a friend at a distance of four metres (across the road)? If no can you see well enough to recognise a friend at a distance of one metre (at arms length)	Cannot recognise a friend at 1m Can recognise a friend at 1m but not at 4m Can recognise a friend at 4m	2 1 0
Hearing	Is your hearing good enough to follow a TV programme at a volume others find acceptable? If not, can you follow a TV programme with volume turned up?	Cannot follow a TV programme even with the volume turned up Can follow a TV programme with the volume turned up Can follow a TV programme at normal volume	2 1 0

Disability Type	Survey Question	Response	Disability score
Communication	Can you speak without difficulty?	Yes	1
		No	0
	Do you have problems communicating with other people?	Difficulty communicating with close relatives	2
		Difficulty communicating with other people	1
		No communication problem	0

Table 7: Disability Scores in the Health Survey for England (2001).

Source: Disability report: Health Survey for England 2001 (Bajekal and Prescott 2003)